1 Setup

$$\frac{\partial T}{\partial t} + \nabla \cdot T\vec{u} - \alpha\nabla^2 T = q$$

$$\int_D (\frac{\partial T}{\partial t} + \nabla \cdot T\vec{u} - \alpha\nabla^2 T - q)\omega_i \, \mathrm{d}x \, \mathrm{d}t = 0$$

$$\psi_i(x) = \begin{cases} \dfrac{x - x_{i-1}}{x_i - x_{i-1}} & x_{i-1} \le x \le x_i \\ \dfrac{-x - x_{i+1}}{x_{i+1} - x_i} & x_i \le x \le x_{i+1} \end{cases}$$

$$\int_D (\frac{\partial T}{\partial t} + \frac{\partial T\vec{u}}{\partial t} + \frac{\partial T\vec{u}}{\partial x} + \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial t^2})\omega_i \, \mathrm{d}x \, \mathrm{d}t$$

$$\int_{t_k}^{t_{k+1}} \int_L (\frac{\partial T}{\partial t} + \frac{\partial T\vec{u}}{\partial t} + \frac{\partial T\vec{u}}{\partial x} + \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial t^2})\omega_i \, \mathrm{d}x \, \mathrm{d}t$$

2 Transient Term

$$\int_{t_k}^{t_{k+1}} \int_L \frac{\partial T}{\partial t}\omega_i \, \mathrm{d}x \, \mathrm{d}t$$

$$\int_D \frac{\partial T}{\partial t}\omega_i \, \mathrm{d}x \, \mathrm{d}t$$

$$\sum_j \int_L \phi_j T_{j,k+1}\Delta t\omega_i \, \mathrm{d}x$$

$$\sum_j T_{j,k+1}\Delta t \int_L \phi_j\omega_i \, \mathrm{d}x$$

$$j = i - 1$$

$$\sum_j T_{j,k+1}\Delta t\frac{-x_{j-1} - 5x_j}{6}$$

$$j = i$$

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$$\sum_j T_{j,k+1} \Delta t (x_{j+1} - x_{j-1})$$

$$j = i + 1$$

$$\sum_j T_{j,k+1} \Delta t \frac{-x_j - 5x_{j+1}}{6}$$

3 Convective Term

$$\int_D \frac{\partial T \vec{u}}{\partial t} \omega_i + \frac{\partial T \vec{u}}{\partial x} \omega_i \, \mathrm{d}x \, \mathrm{d}t$$

$$\int_D \frac{\partial \sum_j T_j \vec{u} \phi_j}{\partial t} \omega_i + \frac{\partial \sum_j T_j \vec{u} \phi_j}{\partial x} \omega_i \, \mathrm{d}x \, \mathrm{d}t$$

$$\sum_j T_{j,k+1} \vec{u}_{j,k+1} \Delta t \int_L \phi_j + \sum_j (x_{i+1} - x_{i-1}) \int_t T_j \vec{u}_j \phi_j$$