ECE 09495/09595

Advanced Emerging Topics in Computational Intelligence, Machine Learning and Data Mining
Emerging Topics in Computational Intelligence, Machine Learning and Data Mining

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Lecture 3 and 4

Housekeeping

- Google ML Crash Course
- Projects
- Google Colab

Data Preparation

May take up to 70-80% of the teams' effort initially

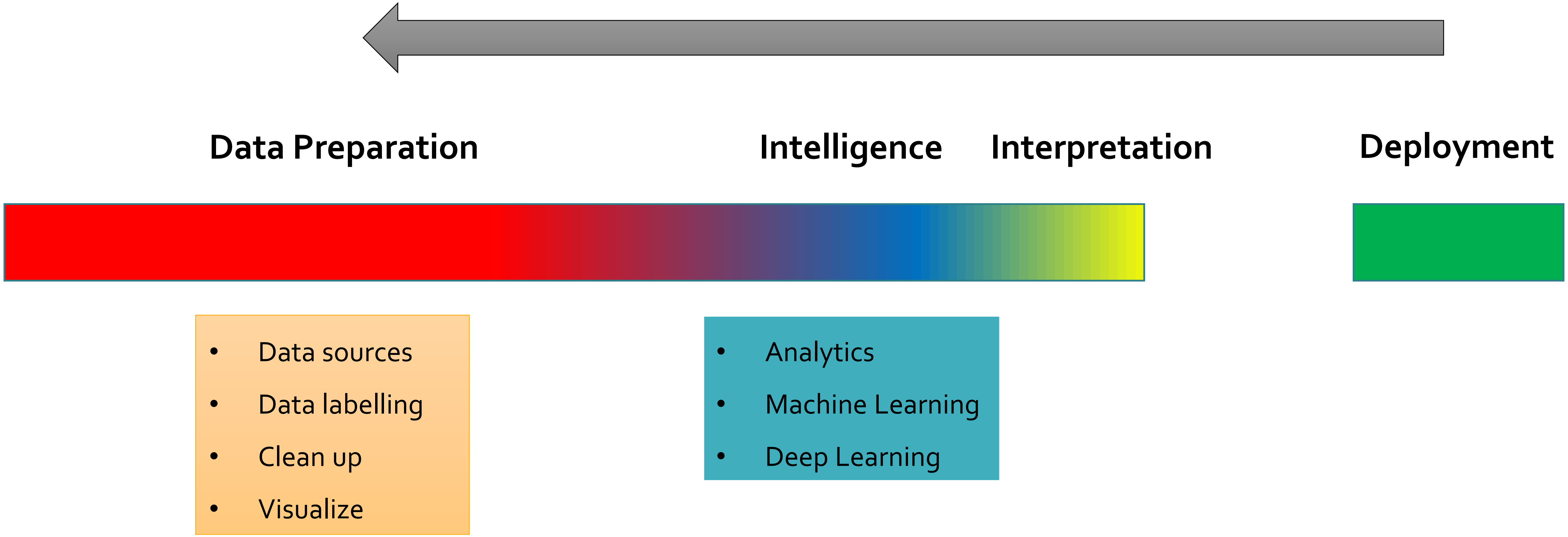
Data Preparation **Intelligence** **Interpretation** **Deployment**

- Data sources
- Data labelling
- Clean up
- Visualize

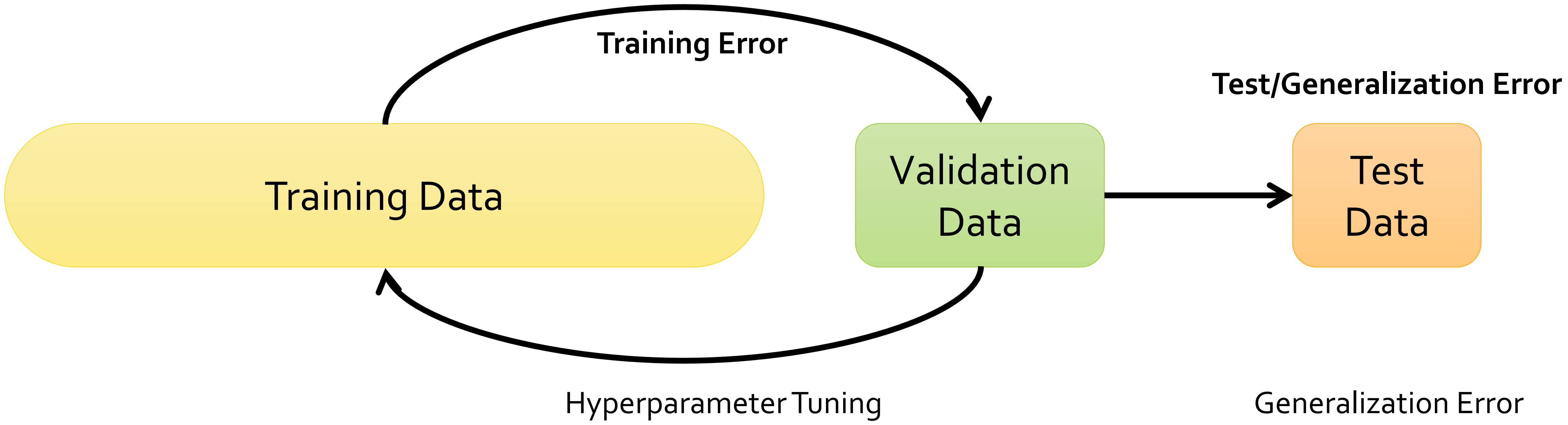
- Analytics
- Machine Learning
- Deep Learning

Data Preparation

Evaluation and design iteration



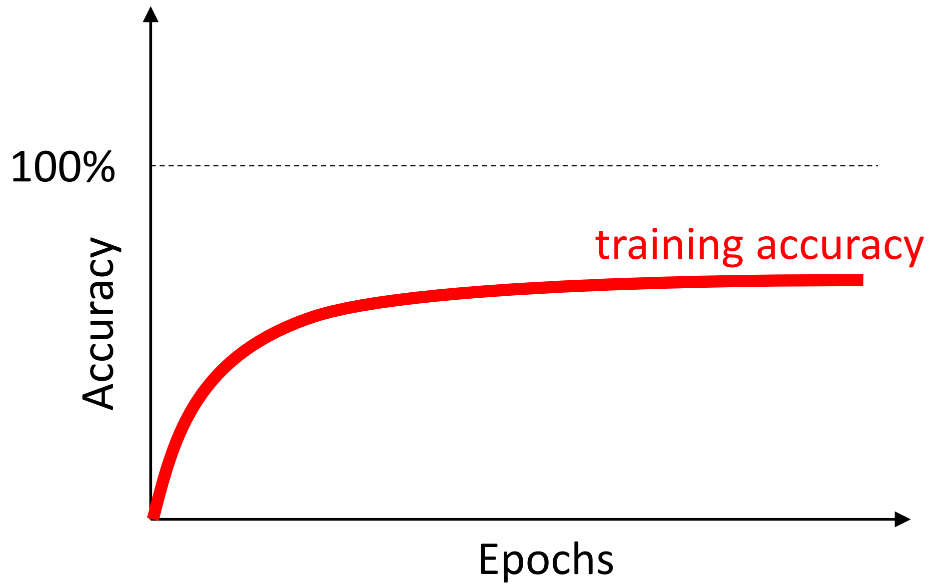
Data Distribution



- **Make the training error small ->** **Model performs well**
- **Make the gap between training and test error small ->** **Model generalizes well**

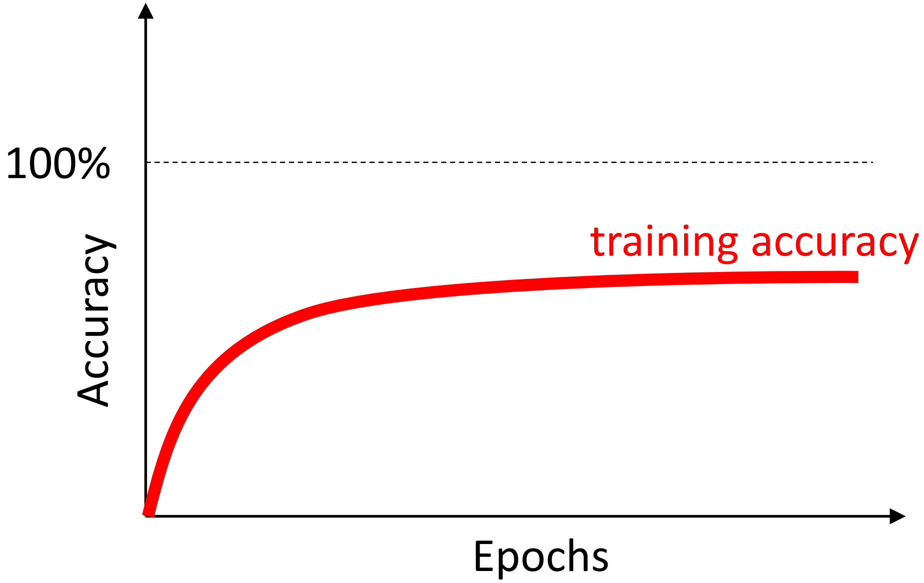
Underfitting

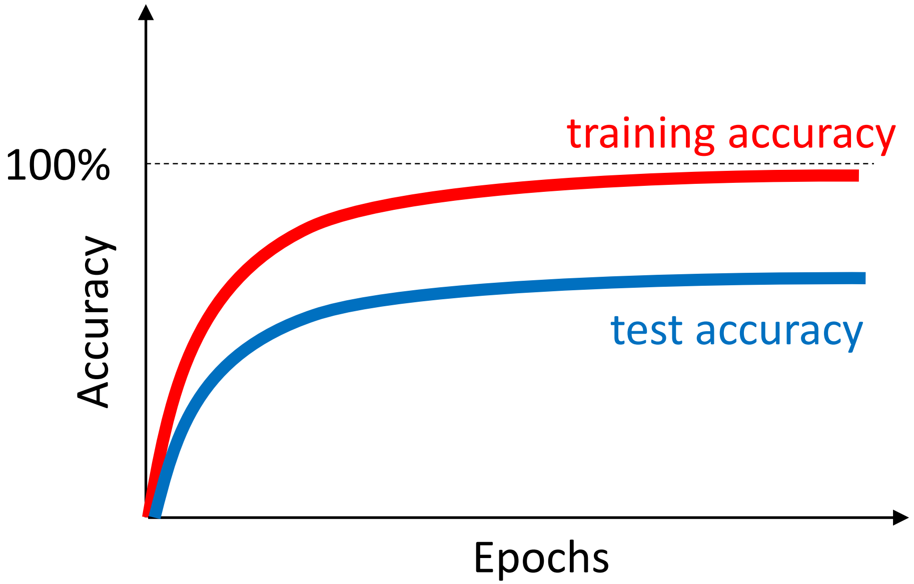
Underfitting occurs when the model is not able to obtain a sufficiently low error value on the training set.



Underfitting and Overfitting

Underfitting occurs when the model is not able to obtain a sufficiently low error value on the training set.





Overfitting occurs when the gap between the training accuracy and test accuracy is too large.

Regularization – Dropout, L1, L2, Weight Decay

Linear Models - Why?

Supervised Learning

Linear Regression → Logistic Regression → Neuron

- Labels are real numbers
- Interpretability
- Closed-form solution
- Gradient descent

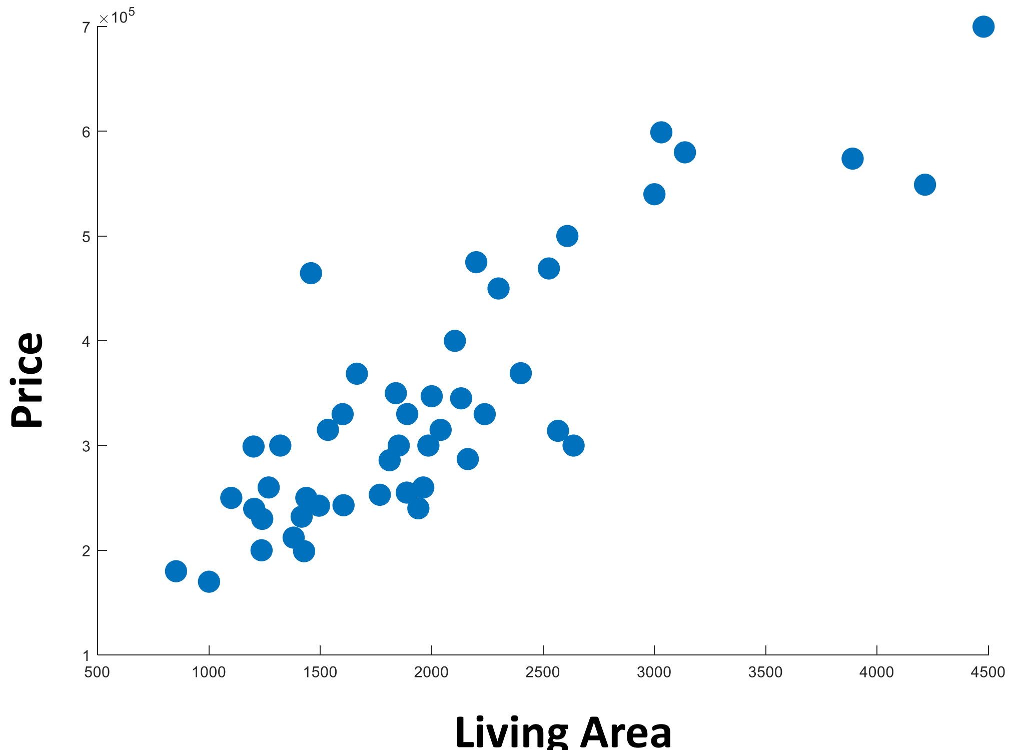
- Labels are categories
- Classification problem
- Nonlinearity
- Gradient descent

Neural networks are stacked layers of neurons

Linear Regression

- We have a dataset giving the living areas and prices of houses

Living Area	Price $K
2104	400
1600	330
2400	369
⋮	⋮

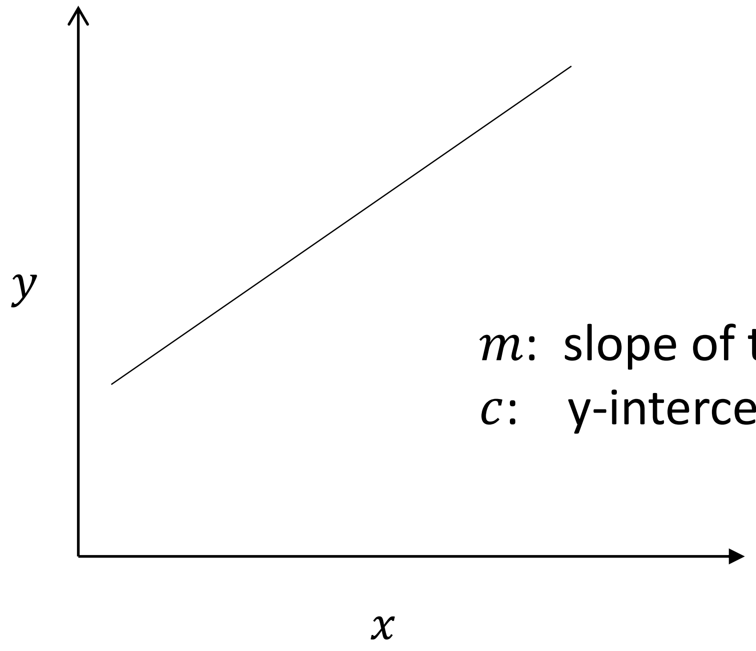


Linear Model

- Lets try to make a linear model

$$h_w(x) = w_0 + w_1 x$$

$$y = mx + c$$



m: slope of the line
c: y-intercept

Notations

- $x^{(i)}$ is the i-th input example (feature) with $y^{(i)}$ as the target or the label.
- Together $(x^{(i)}, y^{(i)})$ are referred to as the i-th training example.
- $\{(x^{(i)}, y^{(i)}); i = 1, \cdots, m\}$ is the training data set.
- $\mathcal{X}$ is the space of input values, and $\mathcal{Y}$ the space of labels/targets, $\mathcal{X} = \mathcal{Y} = \mathbb{R}$

Our goal

$$h : \mathcal{X} \mapsto \mathcal{Y}$$

so that $h(x)$ is a good predictor of y

More features

- We may have more features, living area and number of bedrooms, so we have $x_1^{(i)}$, and $x_2^{(i)}$. Feature space is now two-dimensional, $x \in \mathbb{R}^2$.
- Who decides how many features we have?

Lets try to make a linear model

$$h_w(x) = w_0 + w_1 x_1 + w_2 x_2$$

$$h(x) = \sum_{i=0}^{n} w_i x_i = w^T x$$

x_0 is set to 1.

w_i are the parameters or weights and w is the vector of parameters/weights. x is the vector on inputs.

What are known and what are unknowns?

The unknown w and the Least-Squares

Given training set $\{(x^{(i)}, y^{(i)}), i = 1, \cdots, m\}$, how do we learn the parameters w such that $h(x)$ is close to the ground truth y.

Choose some w randomly and test all examples and compare $h(x)$ and y.

Lets define cost function

$$J(w) = \frac{1}{2}\sum_{i=1}^{m}\left(h\left(x^{(i)}\right) - y^{i}\right)^2$$

This is a least-squares cost function and referred to as the ordinary least-squares (OLS) regression.

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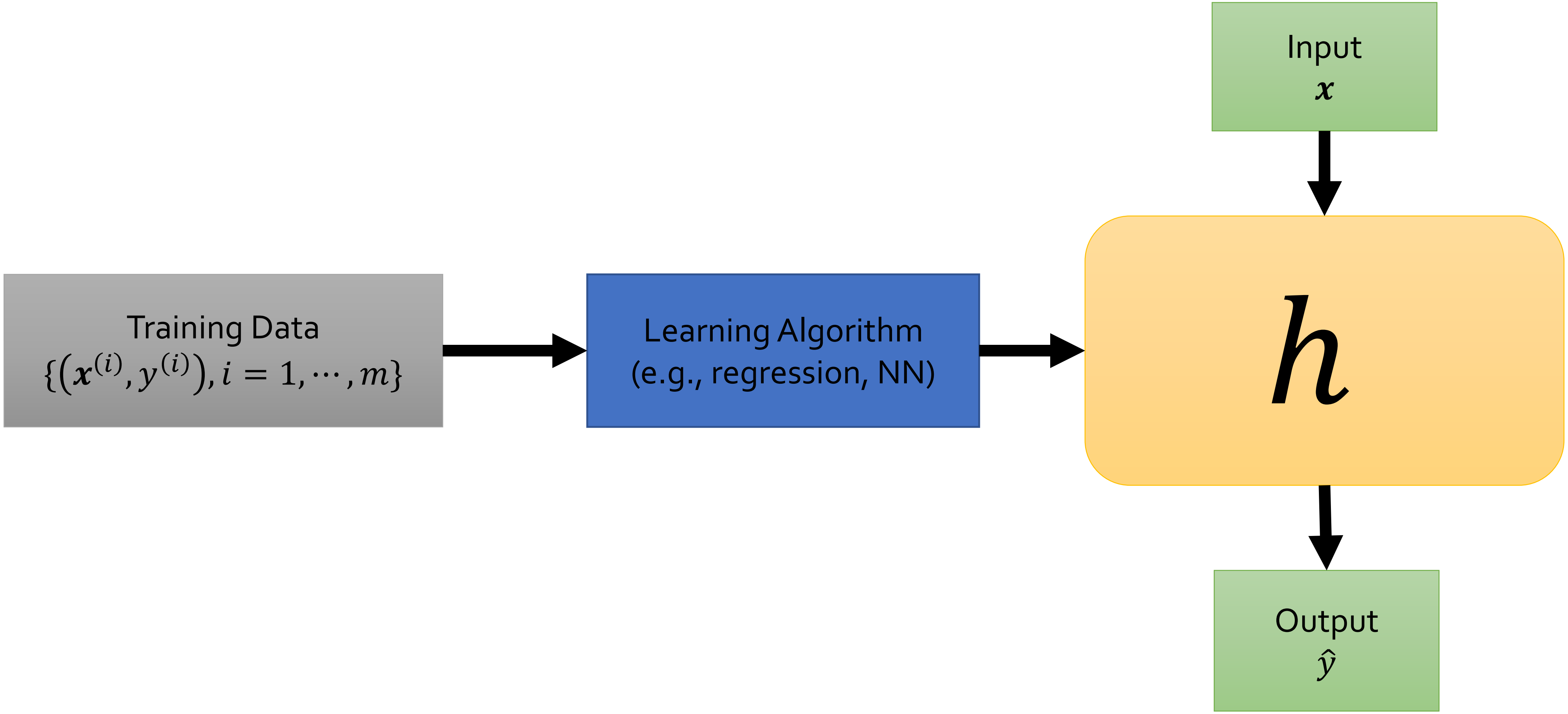
Machine Learning - Supervised Learning



Linear Regression - cost function

$$J(w) = \frac{1}{2}\boxed{\sum_{i=1}^{m}\boxed{\left(\boxed{h\left(x^{(i)}\right)} - \boxed{y^{i}}\right)^{2}}}$$

$$h(x) = \sum_{i=0}^{n} w_i x_i = w^T x$$

Square Error

Ground truth

Sum over all given examples

Goal : Minimize $J(w)$

Gradient Descent

We want to choose w so as to minimize $J(w)$

Starts with some “guess” for w, and then repeatedly change w to make $J(w)$ smaller.

The gradient descent algorithm starts with some initial w, and repeatedly performs the update:

$$w_j := w_j - \alpha \frac{\partial}{\partial w_j} J(w) \qquad j = 1, \cdots, n$$

α is the learning rate (more on this later)

For a single training example, we may solve and get:

$$w_j := w_j - \alpha\left(h_w\left(x^{(i)}\right) - y^{(i)}\right)x_j^{(i)}$$

LMS (Least Mean Squares) Update Rule

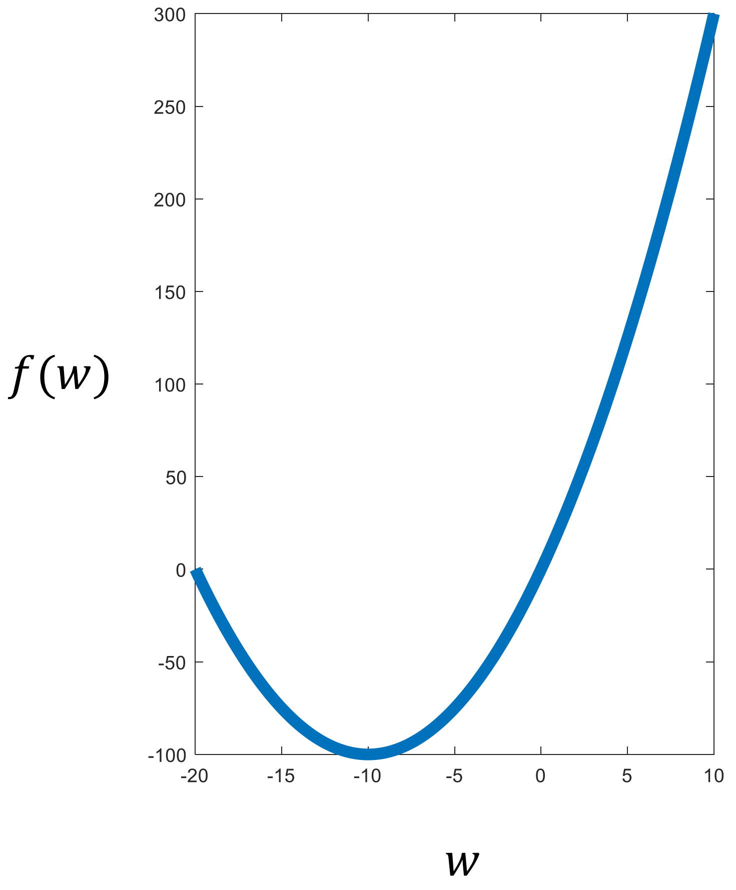
Gradient descent: But how?

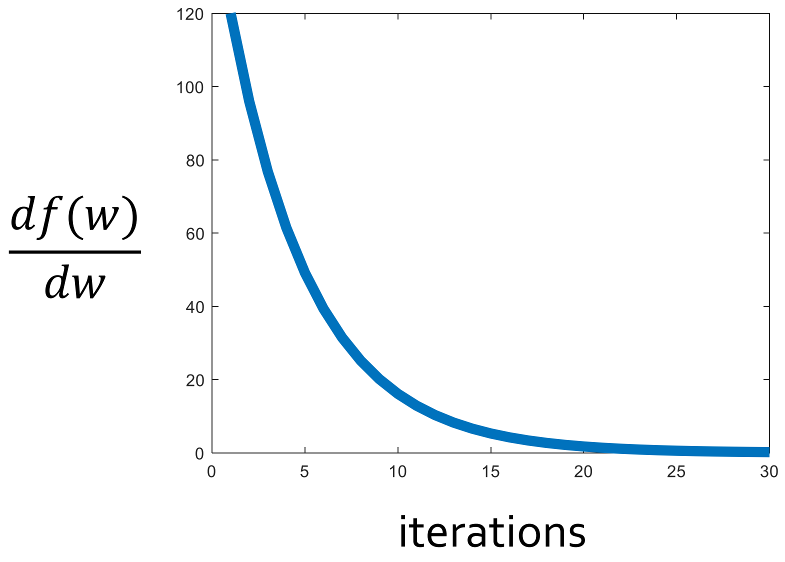
$$f(w) = w^2 + 20w$$

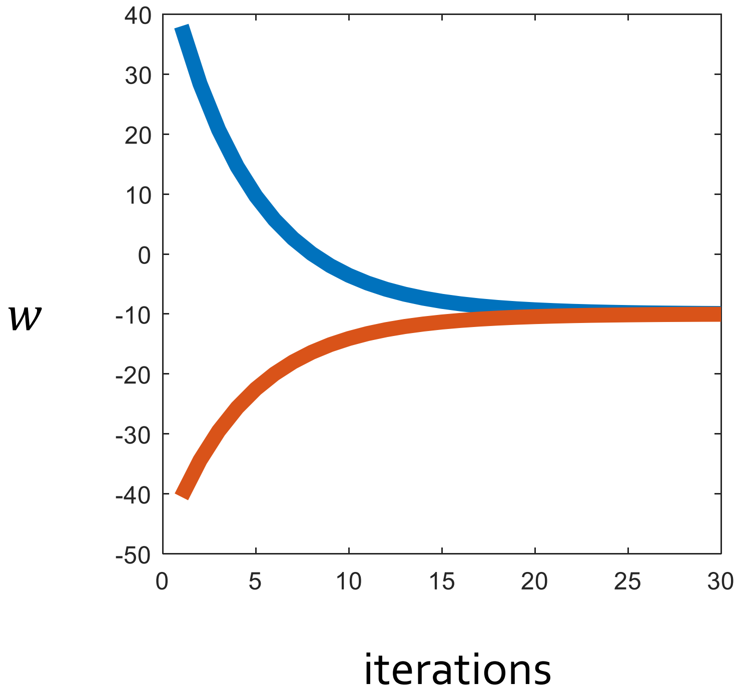
$$\frac{df(w)}{dw} = 2w + 20$$

$$\alpha = 0.1$$

$$w := w + \alpha\left(-\frac{df(w)}{dw}\right)$$







Batch gradient descent

Loop till convergence:

$$w_j := w_j - \alpha \sum_{i=1}^{m} \boxed{\left(h_w\left(x^{(i)}\right) - y^{(i)}\right) x_j^{(i)}}$$

$$\frac{\partial}{\partial w_j} J(w)$$

This method looks at every example in the entire training set on every step

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Stochastic gradient descent

Loop:{

for $i = 1{:}m$ {

$$w_j := w_j - \alpha\left(h_w\left(x^{(i)}\right) - y^{(i)}\right)x_j^{(i)} \quad \text{for all } j$$

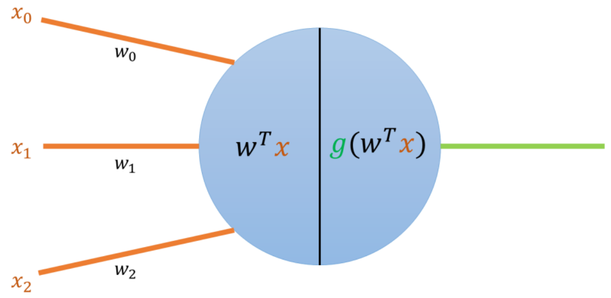
}

}

Batch gradient descent has to scan through the entire training set before taking a single step of updating w – a costly operation if m is large

Stochastic gradient descent can start making progress right away, and continues to make progress with each example it looks at.
Stochastic gradient descent may get w close to the minimum much faster than batch gradient descent.
It may never converge to the minimum, that is, w may keep oscillating around the minimum of $J(w)$

Logistic Regression



Labels are now discrete categories, e.g.

$y \in \{0, 1\}$ – a binary classification problem

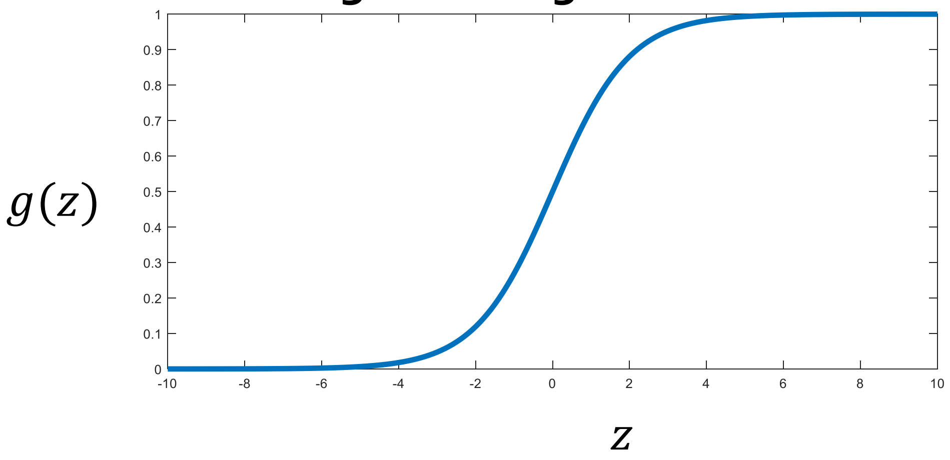
Previously : $h_w(x) = w^T x$

Now: $h_w(x) = g(w^T x)$ such that

$h_w(x) \in \{0,1\}$

$$g(z) = \frac{1}{1 + \exp(-z)}$$

Logistic or Sigmoid Function



Logistic regression: How to estimate w?

Lets start with probability assignment:

$$P(y = 1|x; w) = h_w(x) \text{ and thus } P(y = 0|x; w) = 1 - h_w(x)$$

$$p(y|x; w) = [h_w(x)]^y [1 - h_w(x)]^{1-y}$$

For m independent training examples:

$$L(w) = \prod_{i=1}^{m} \left[h_w\left(x^{(i)}\right)\right]^{y^{(i)}} \left[1 - h_w\left(x^{(i)}\right)\right]^{1-y^{(i)}}$$

$$l(w) = \log L(w) = \sum_{i=1}^{m} y^{(i)} \log h(x^{(i)}) + \left(1 - y^{(i)}\right) \log(1 - h(x^{(i)}))$$

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How do we maximize it? Gradient Ascent

Logistic Regression Loss Function

Inputs: x, y
Parameters: w and b
Output : $\hat{y}$

$$J(w,b) = -\sum_{i=1}^{m} y^{(i)} \log(\hat{y}^{(i)}) + \left(1 - y^{(i)}\right) \log(1 - \hat{y}^{(i)})$$

$$\hat{y}^{(i)} = \sigma\left(w^T x^{(i)} + b\right) = \frac{1}{1+\exp(-w^T x^{(i)}+b)}$$

w and b can be combined

$$w^T x + b = [w_1, w_2]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + b$$

$$= [b, w_1, w_2]\begin{bmatrix} 1 \\ x_1 \\ x_2 \end{bmatrix}$$

$y^{(i)} = 1$ and $h\left(x^{(i)}\right) = 0.999$ Loss ≈ 0

$y^{(i)} = 0$ and $h\left(x^{(i)}\right) = 0.001$ Loss ≈ 0

$y^{(i)} = 1$ and $h\left(x^{(i)}\right) = 0.001$ Loss ≈ 3

$y^{(i)} = 0$ and $h\left(x^{(i)}\right) = 0.999$ Loss ≈ 3

Loss function / Cost function

- Function of the unknown weights, w, and b
- Differentiable
- Must output a scalar

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Linear Regression and Logistic Regression

When the target variable (y) we are trying to predict is continuous, such as in our housing example, we call the learning problem a regression problem. We can solve this problem using linear models and this is referred to as Linear Regression.

Examples???

When y can take on only a small number of discrete values or categories, (e.g., low-range, mid-range, expensive, crazy-expensive), we call it a classification problem. We can solve this problem using linear models, this is referred to as Logistic Regression.

Examples???

Logistic Regression – 1 example case

b

$$\frac{dJ}{db} = \frac{dJ}{dz}$$

w_1

x_1

$$\frac{dJ}{dw_1} = x_1 \frac{dJ}{dz}$$

w_2

x_2

$$\frac{dJ}{dw_2} = x_2 \frac{dJ}{dz}$$

$$z = w_1 x_1 + w_2 x_2 + b$$

$$\frac{dJ}{dz} = \frac{dJ}{da}\frac{da}{dz}$$

$$\frac{dJ}{dz} = \left(-\frac{y}{a} + \frac{1-y}{1-a}\right) * a(1-a)$$

$$\frac{dJ}{dz} = a - y$$

$$a = \sigma(z)$$

$$\frac{dJ}{da} = -\frac{y}{a} + \frac{1-y}{1-a}$$

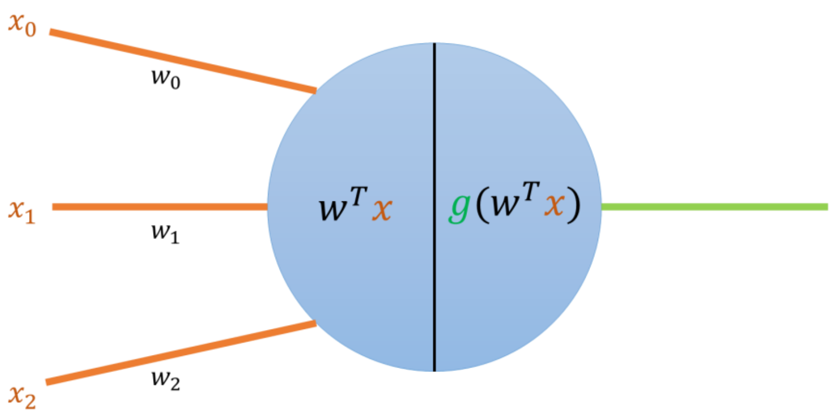
$$J(w,b) = -y\log(a) - (1-y)\log(1-a)$$

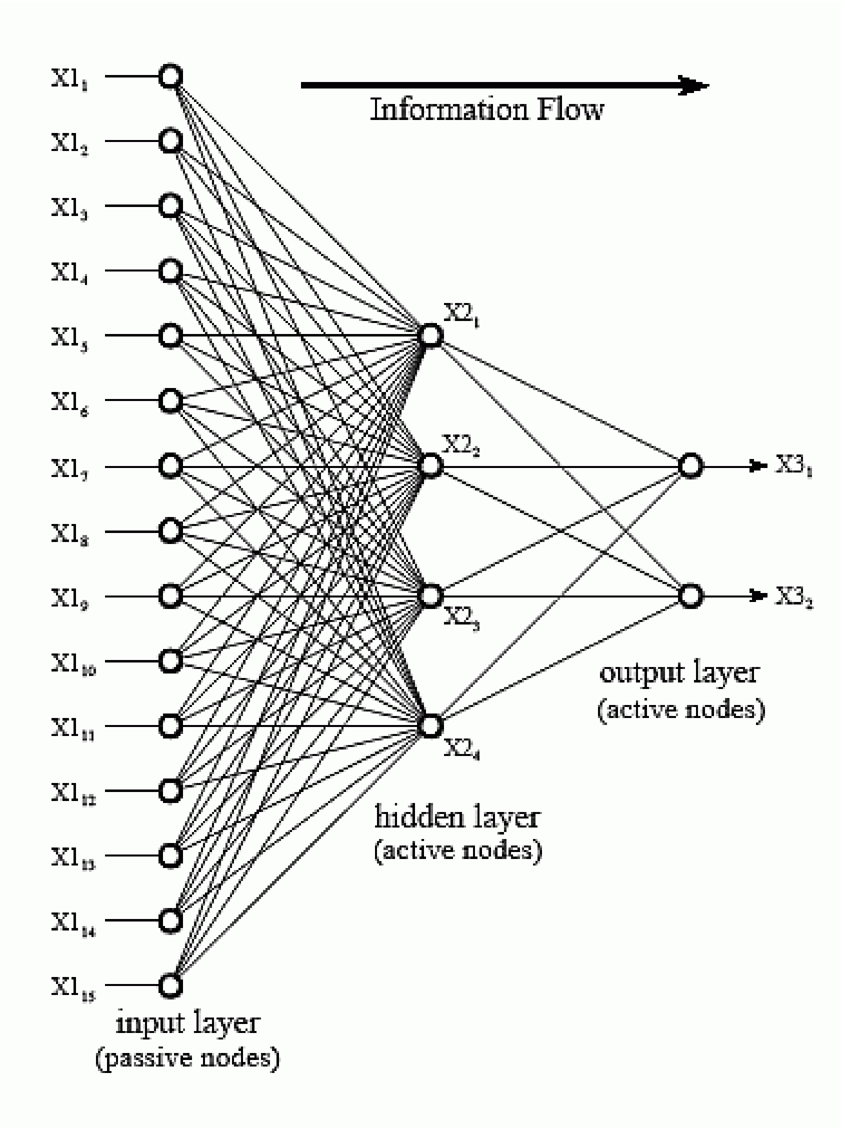
$$w_1 := w_1 - \alpha \frac{dJ}{dw_1}$$

$$w_2 := w_2 - \alpha \frac{dJ}{dw_2}$$

$$b := b - \alpha \frac{dJ}{db}$$

Logistic Regression to Neuron

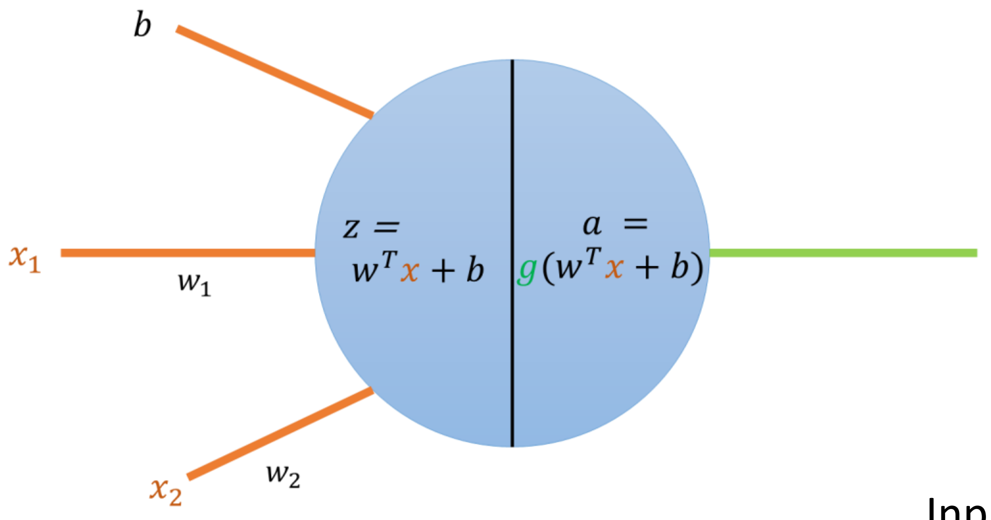


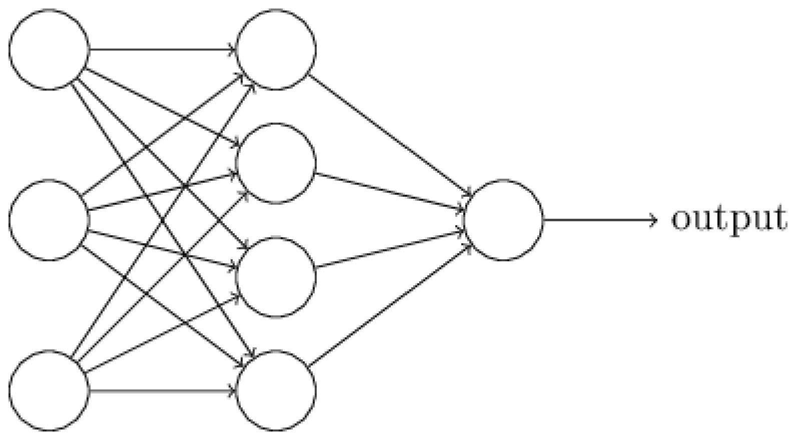


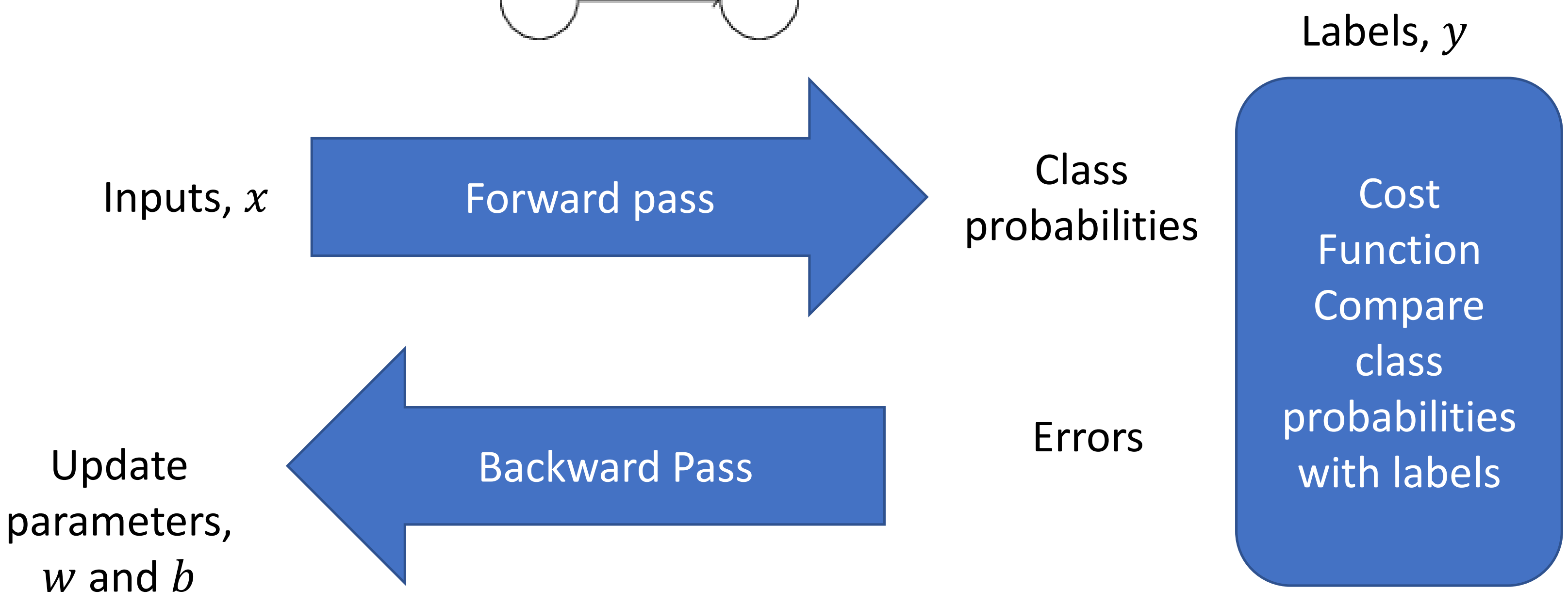
Neural Network Examples

Applications	Input	Output	Neural Network
Housing Example			Simple NN
Iris Classification			Simple NN
Image classification			CNN
Tumor Segmentation			CNN
Speech translation			RNN / LSTM
Sentiment analysis			RNN / LSTM
Autonomous cars			Customized Architectures

Neuron to Artificial neural network







http://www.dspguide.com/ch26/2.htm

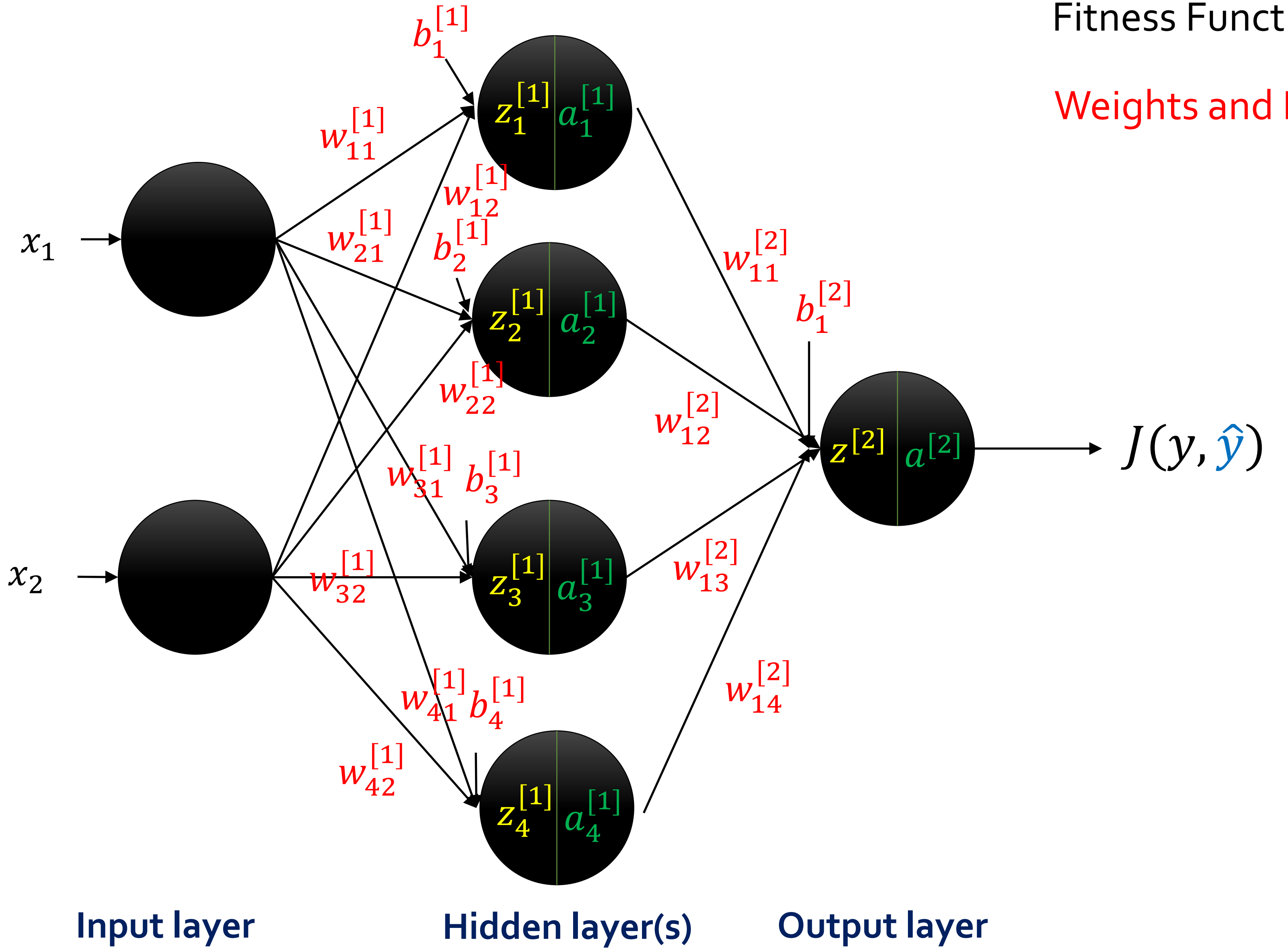
Artificial Neural Networks

Input Data = $\boldsymbol{x}, y$

Out Data = $\hat{y}$

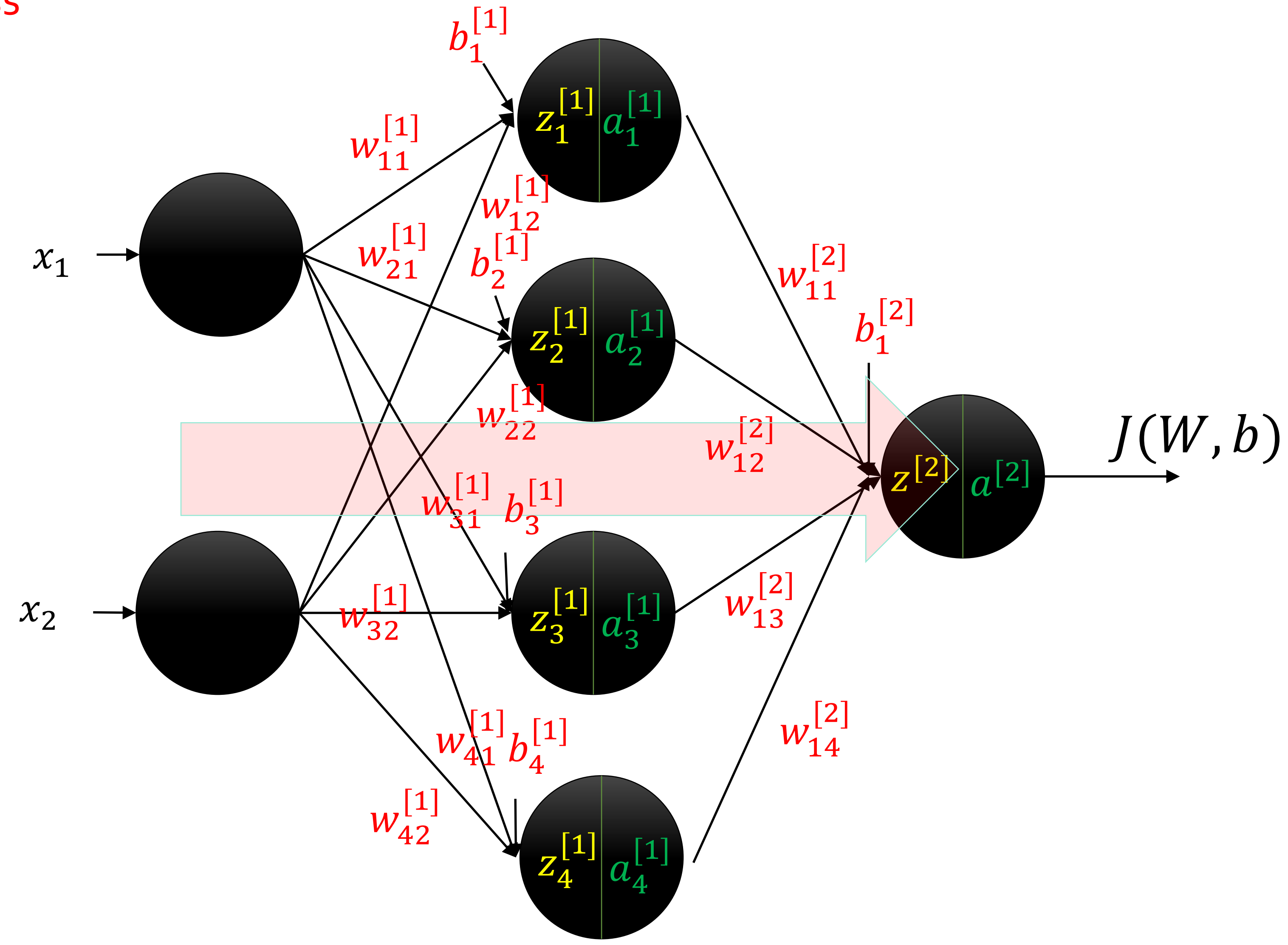
Fitness Function = $J(y, \hat{y})$

Weights and Biases = ? ? ?



Artificial Neural Networks

Forward Pass



$$Z^{[1]} = W^{[1]}X + b^{[1]}$$
$$A^{[1]} = \sigma(Z^{[1]})$$

$$Z^{[2]} = W^{[2]}A^{[1]} + b^{[2]}$$
$$A^{[2]} = \sigma(Z^{[2]})$$

$$X = \begin{bmatrix} | & | & | \\ x^{(1)} & x^{(2)} & x^{(m)} \\ | & | & | \end{bmatrix}$$

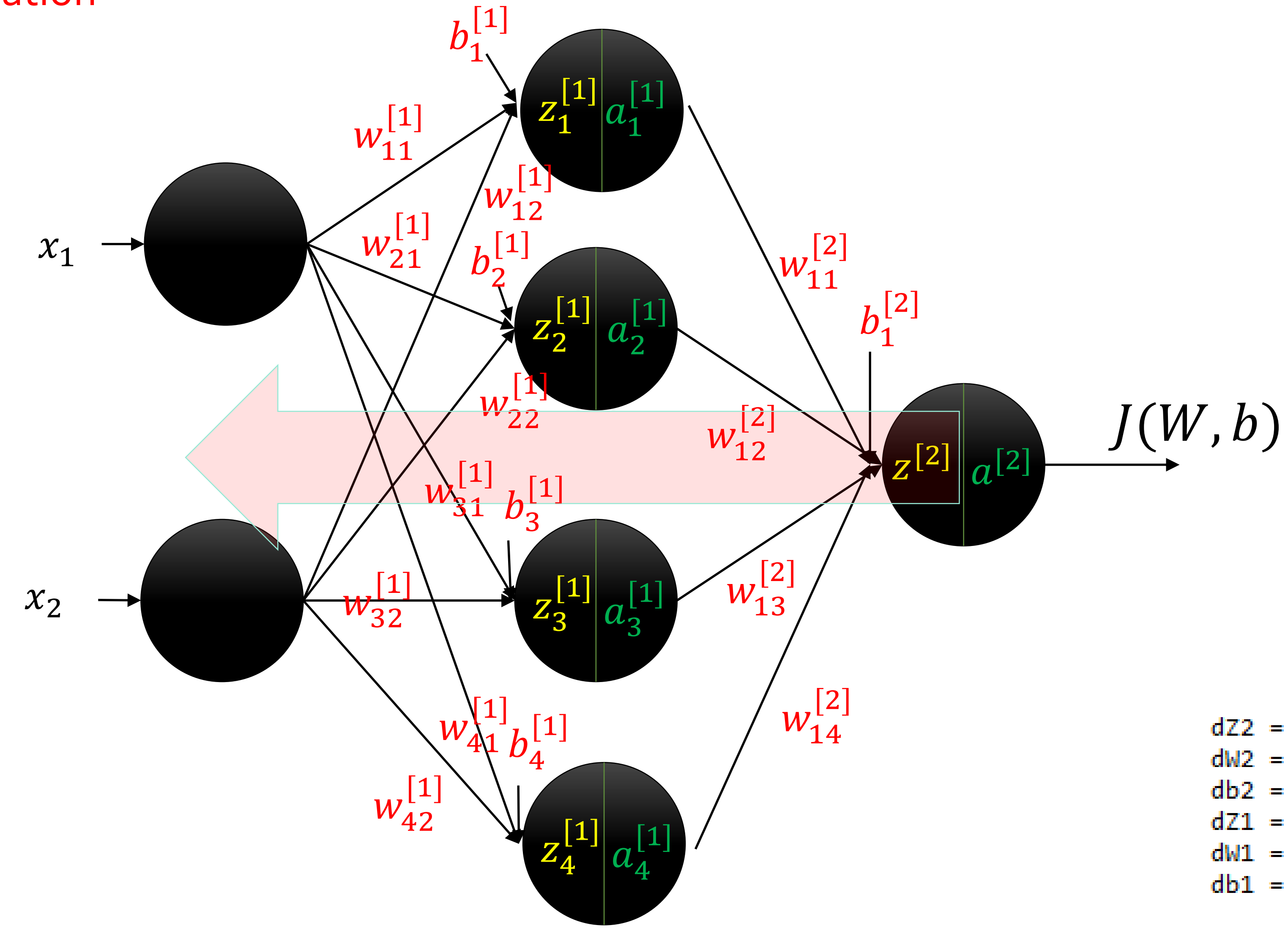
$$Z^{[1]} = \begin{bmatrix} | & | & | \\ z^{[1](1)} & z^{[1](2)} & z^{[1](m)} \\ | & | & | \end{bmatrix}$$

$$A^{[1]} = \begin{bmatrix} | & | & | \\ a^{[1](1)} & a^{[1](2)} & a^{[1](m)} \\ | & | & | \end{bmatrix}$$

```
Z1 = np.dot(W1, X) + b1
A1 = sigmoid(Z1)
Z2 = np.dot(W2, A1) + b2
A2 = sigmoid(Z2)
```

Artificial Neural Networks

Backpropagation

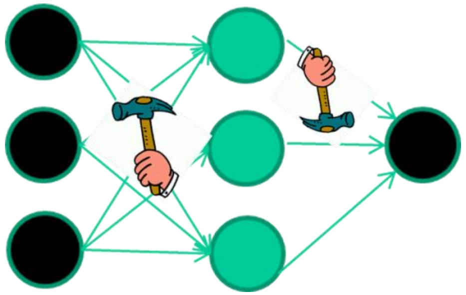


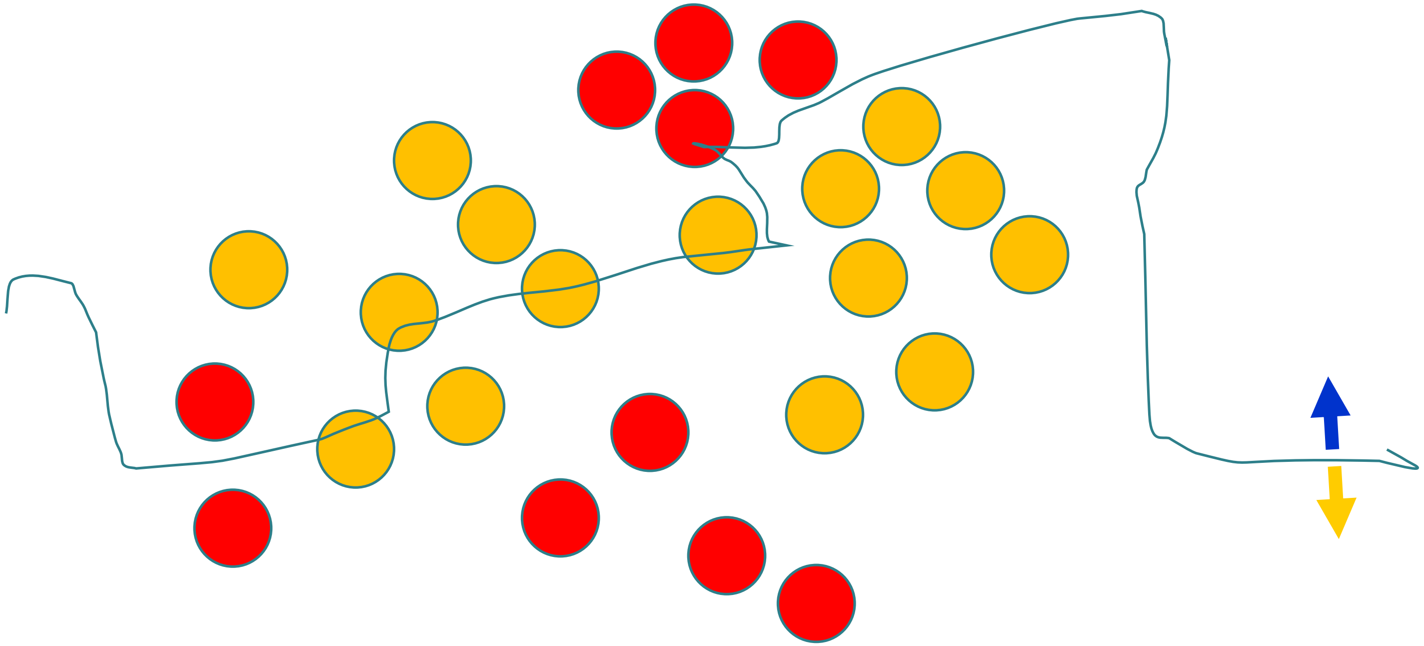
$$dZ^{[2]} = A^{[2]} - Y$$
$$dW^{[2]} = \frac{1}{m} dZ^{[2]} A^{[1]T}$$
$$db^{[2]} = \frac{1}{m} \sum dZ^{[2]}$$
$$dZ^{[1]} = W^{[2]T} dZ^{[2]} * \acute{\sigma}(Z^{[1]})$$
$$dW^{[1]} = dZ^{[1]} X^T$$
$$db^{[1]} = \frac{1}{m} \sum dZ^{[1]}$$

```
dZ2 = A2 - Y
dW2 = 1/m*(np.dot(dZ2,A1.T))
db2 = 1/m*(np.sum(dZ2, axis=1, keepdims=True))
dZ1 = np.dot(W2.T, dZ2)* (A1*(1-A1))
dW1 = 1/m*np.dot(dZ1, X.T)
db1 = (1/m) * np.sum(dZ1, axis=1, keepdims=True)
```

Artificial Neural Networks – Training

Classification Task

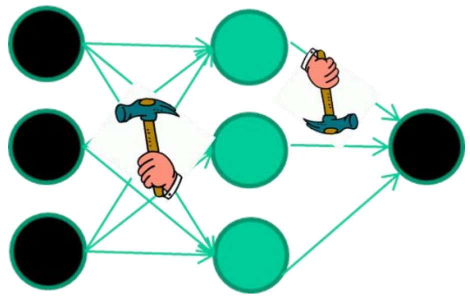


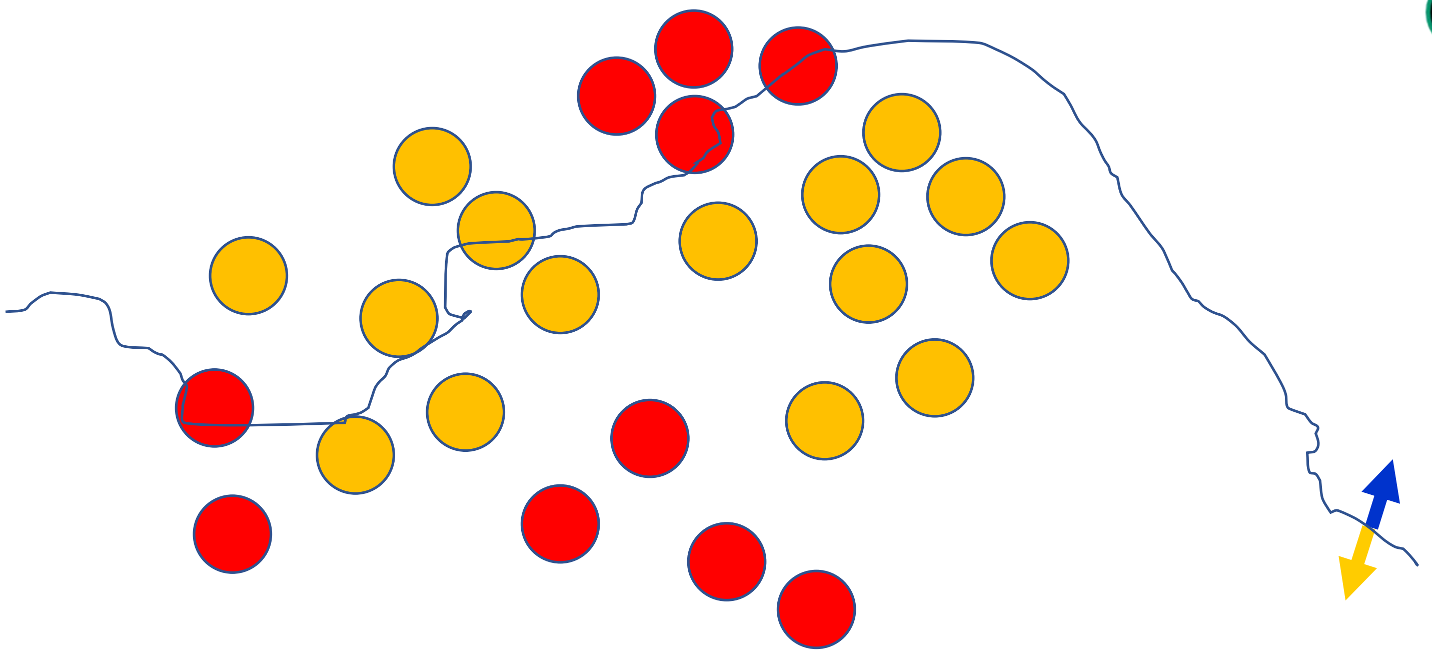


Initial random weights

Artificial Neural Networks – Training

Classification Task

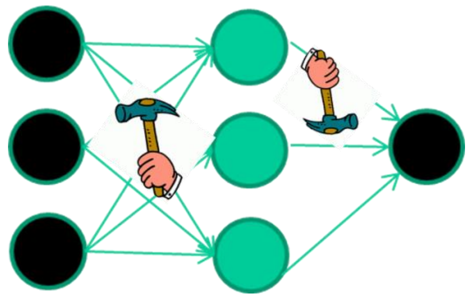


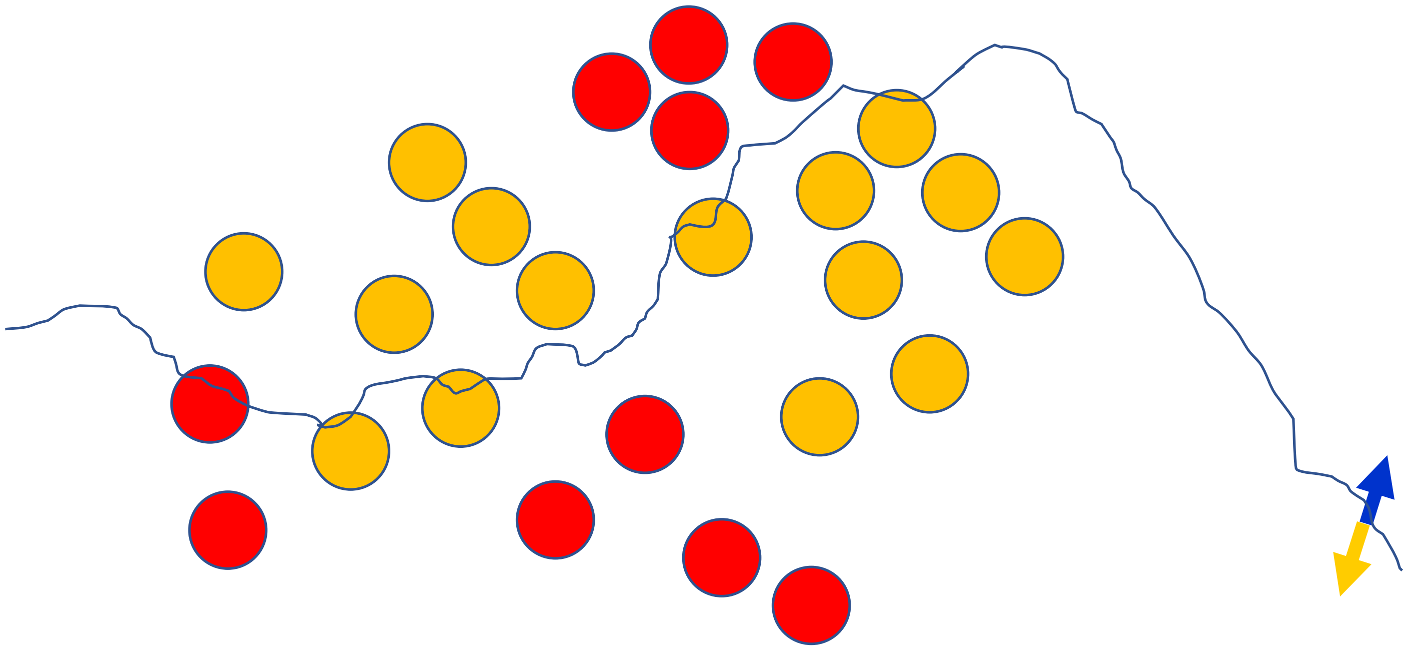


Present a training instance / adjust the weights

Artificial Neural Networks – Training

Classification Task

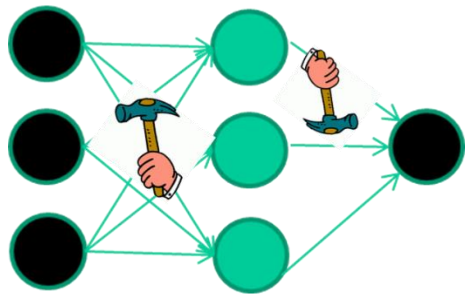


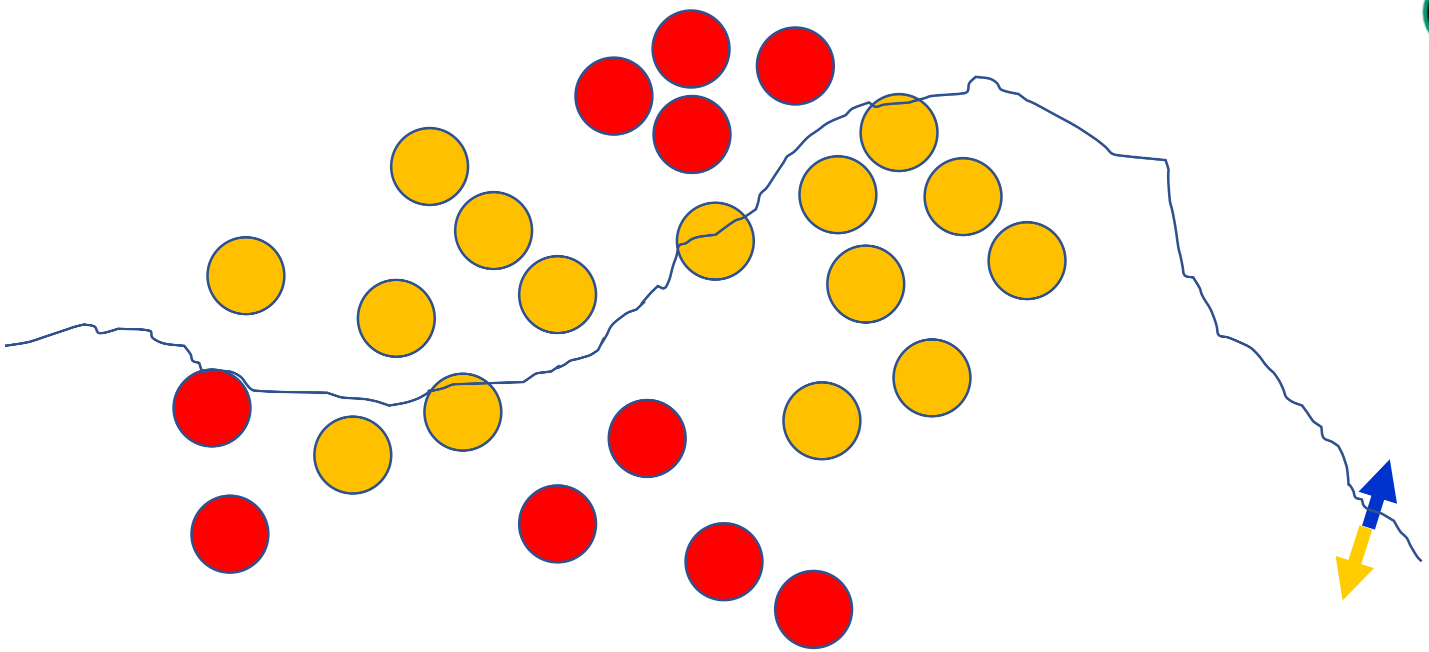


Present a training instance / adjust the weights

Artificial Neural Networks – Training

Classification Task

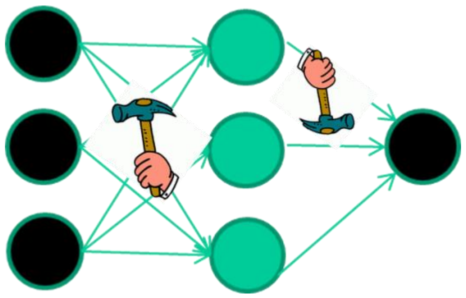


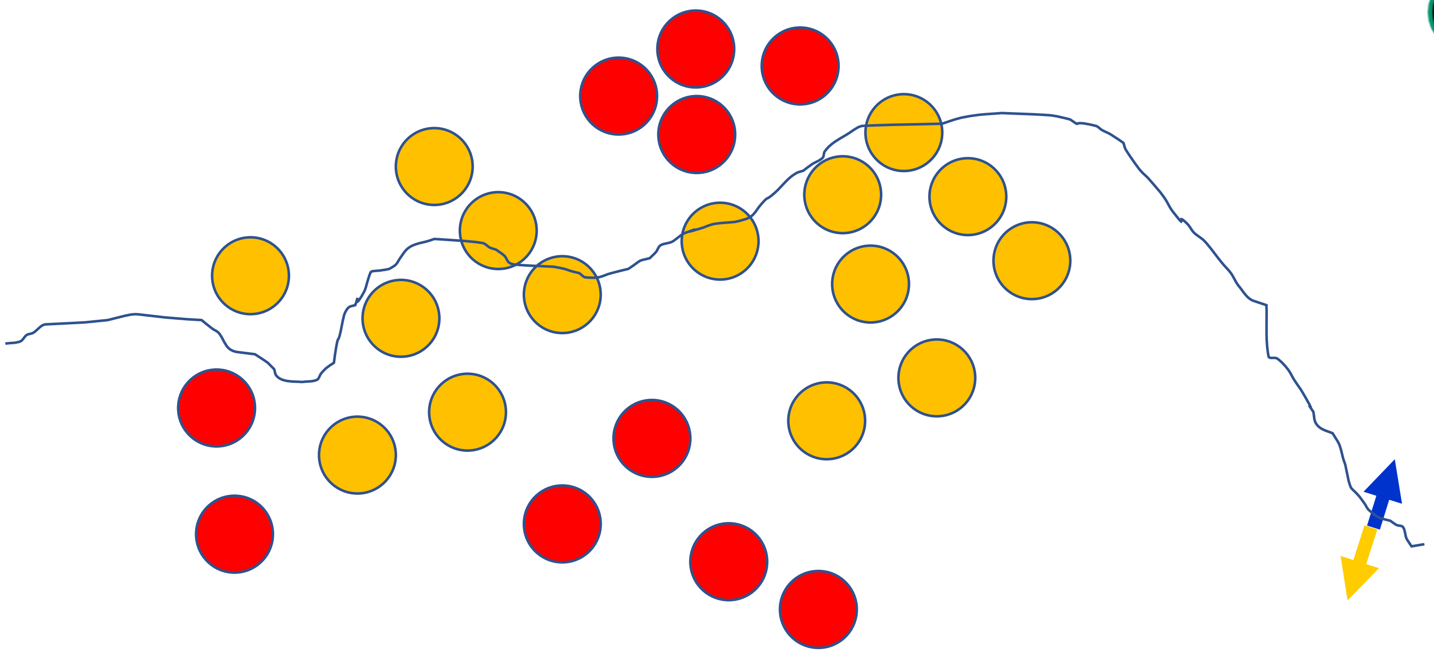


Present a training instance / adjust the weights

Artificial Neural Networks – Training

Classification Task

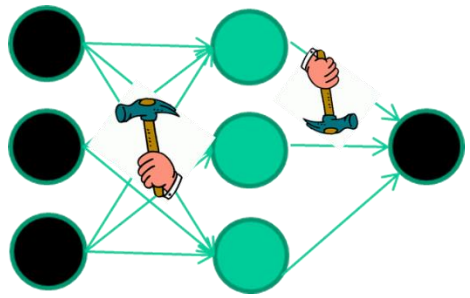


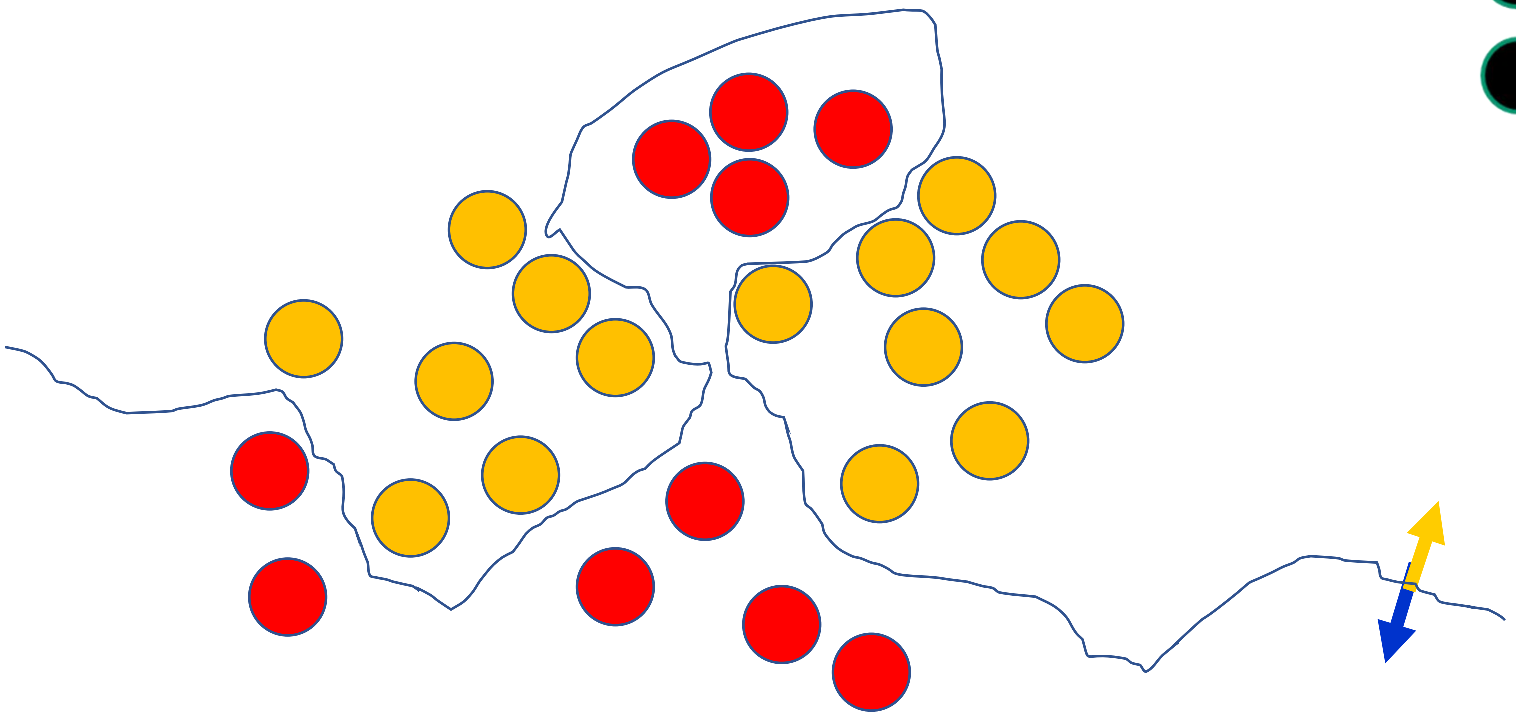


Present a training instance / adjust the weights

Artificial Neural Networks – Training

Classification Task

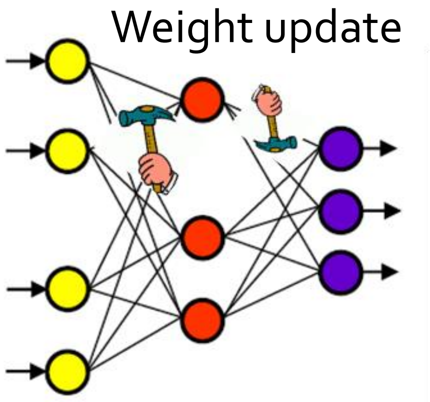




Eventually

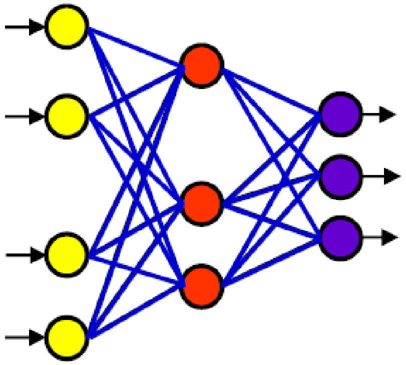
Artificial Neural Networks – Training and Testing

Train



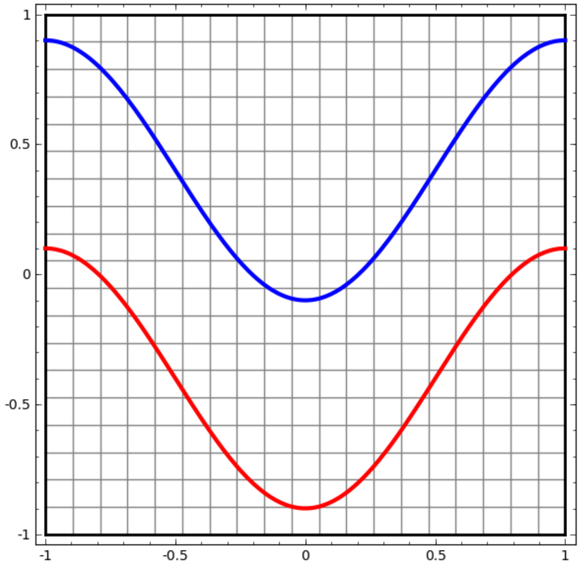
73	73	101
101	134	101
134	73	134

Test

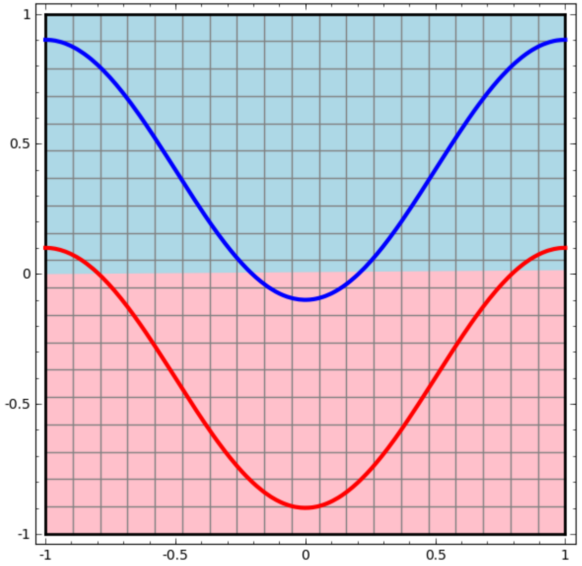


?

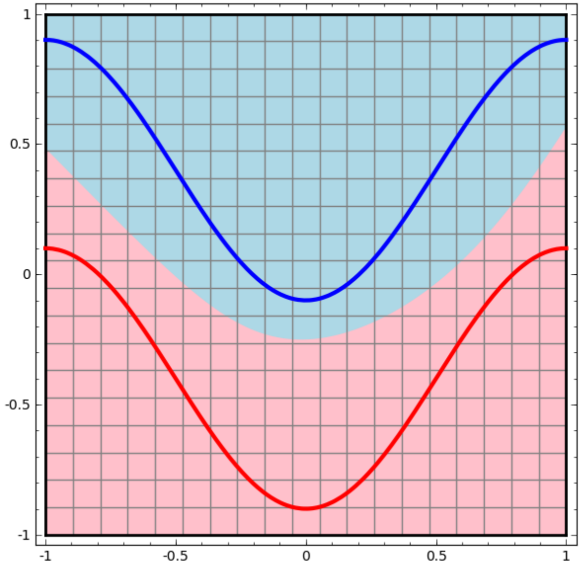
Artificial Neural Networks - Nonlinearity



The network will learn to classify points as belonging to one or the other.

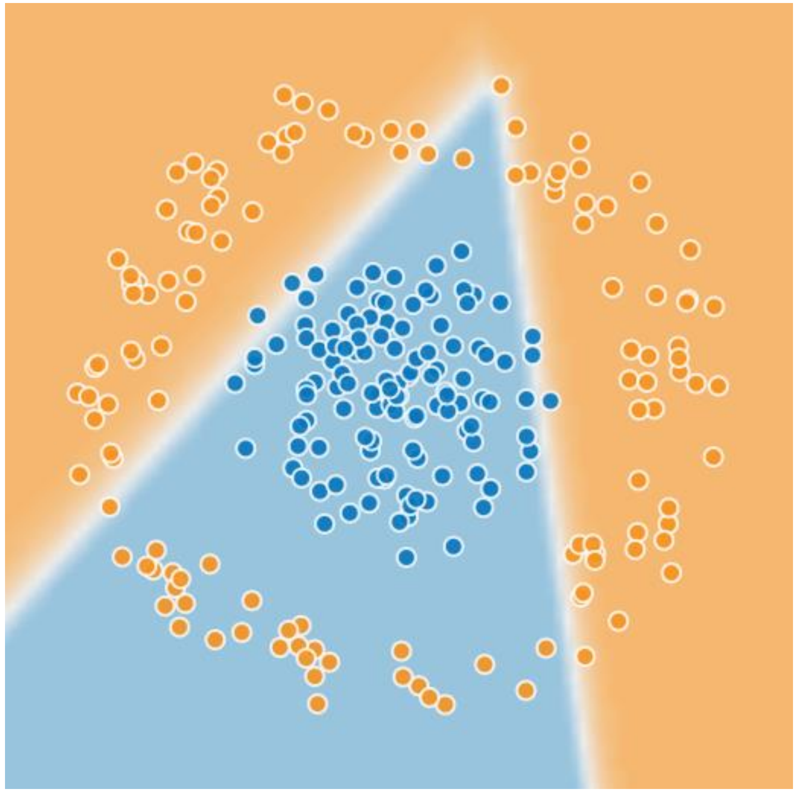


ANN with 1 input layer and 1 output layer. The network simply tries to separate the two classes of data by dividing them with a line.

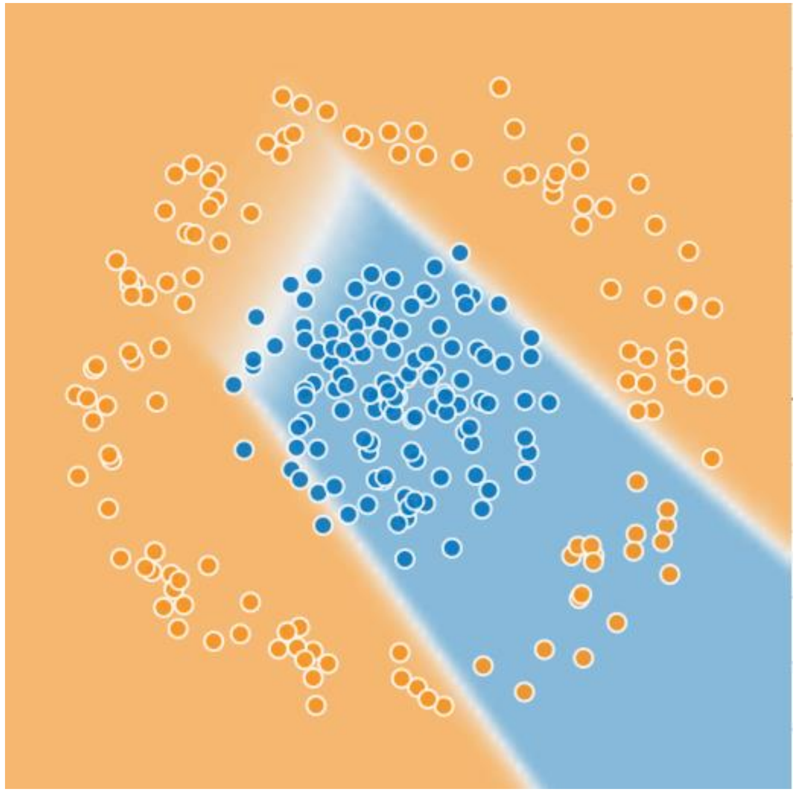


ANN with 1 hidden layer. It separates the data with a more complicated curve than a line.

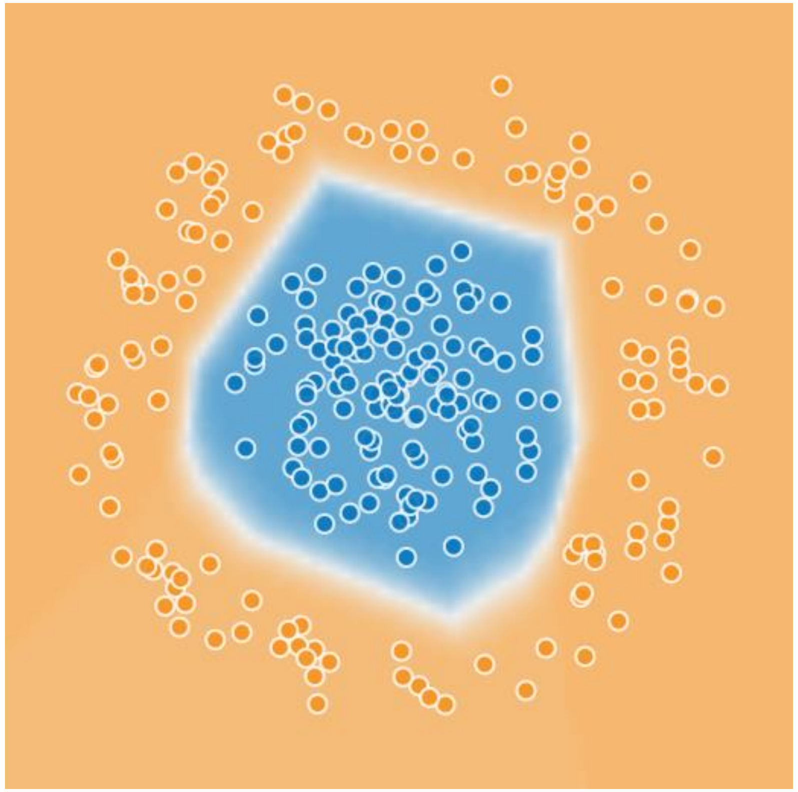
Artificial Neural Networks – Why Go Deep?



1 hidden layer – 2 neurons



2 hidden layers – 4 neurons



3 hidden layers – 9 neurons

Background Math

- The Matrix Calculus You Need For Deep Learning

https://arxiv.org/pdf/1802.01528.pdf

Coding Assignment

- Logistic Regression
- Neural Network
  - Without using any libraries
  - Keras
  - TensorFlow
- Google co-lab

https://colab.research.google.com/drive/1H_F3Fgab3aTH1gBmXHTjlvarBheAJGvy