

# Reference Manual

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## Chapter 1

# Modular arbitrary-order ocean-atmosphere model: MAOOAM -- Fortran implementation

### Lyapunov exponents computation version

#### About

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This software is provided as supplementary material with:

- De Cruz, L., Demaeyer, J. and Vannitsem, S.: The Modular Arbitrary-Order Ocean-Atmosphere Model: MAOOAM v1.0, Geosci. Model Dev., 9, 2793-2808, [doi:10.5194/gmd-9-2793-2016](#), 2016.

#### Please cite this article if you use (a part of) this software for a publication.

The authors would appreciate it if you could also send a reprint of your paper to [lesley.decruz@meteo.be](mailto:lesley.decruz@meteo.be), [jonathan.demaeyer@meteo.be](mailto:jonathan.demaeyer@meteo.be) and [svn@meteo.be](mailto:svn@meteo.be).

Consult the MAOOAM [code repository](#) for updates, and [our website](#) for additional resources.

A pdf version of this manual is available [here](#).

#### Note on the Lyapunov exponents computation

This version of the code allows for the computation of the backward Lyapunov exponents. The method used is described in

- Benettin, G., Galgani, L., Giorgilli, A., and Strelcyn, J. M. : Lyapunov characteristic exponents for smooth dynamical systems; a method for computing all of them. Part 2: Numerical application. Meccanica, 15, 21-30, [doi:10.1007/BF02128237](#), 1980.

This version has been used to compute the Lyapunov exponents of MAOOAM in the following manuscript:

- De Cruz, L., Schubert, S., Demaeyer, J., Lucarini, V., and Vannitsem, S.: Exploring the Lyapunov instability properties of high-dimensional atmospheric and climate models, Nonlin. Processes Geophys. Discuss., [doi:10.5194/npg-2017-76](#), in review, 2018.

## Installation

The program can be installed with Makefile. We provide configuration files for two compilers : gfortran and ifort.

By default, gfortran is selected. To select one or the other, simply modify the Makefile accordingly or pass the CO<sub>MPILER</sub> flag to `make`. If gfortran is selected, the code should be compiled with gfortran 4.7+ (allows for allocatable arrays in namelists). If ifort is selected, the code has been tested with the version 14.0.2 and we do not guarantee compatibility with older compiler version.

To install, unpack the archive in a folder or clone with git:

```
1 git clone https://github.com/Climdyn/MAOOAM.git
2 cd MAOOAM
```

and run:

```
1 make
```

By default, the inner products of the basis functions, used to compute the coefficients of the ODEs, are not stored in memory. If you want to enable the storage in memory of these inner products, run make with the following flag:

```
1 make RES=store
```

Depending on the chosen resolution, storing the inner products may result in a huge memory usage and is not recommended unless you need them for a specific purpose.

Remark: The command "make clean" removes the compiled files.

For Windows users, a minimalistic GNU development environment (including gfortran and make) is available at [www.mingw.org](http://www.mingw.org).

## Description of the files

The model tendencies are represented through a tensor called aotensor which includes all the coefficients. This tensor is computed once at the program initialization.

- [maooam.f90](#) : Main program.
- [maooam\\_lyap.f90](#) : Version of the main program with the computation of the Lyapunov exponents.
- [maooam\\_lyap\\_div.f90](#) : Version of the main program with the computation of the first Lyapunov exponents with the divergence method.
- [aotensor\\_def.f90](#) : Tensor aotensor computation module.
- [lyap\\_vectors.f90](#) : Module for the computation of Lyapunov exponents and vectors
- [IC\\_def.f90](#) : A module which loads the user specified initial condition.
- [inprod\\_analytic.f90](#) : Inner products computation module.
- [rk2\\_integrator.f90](#) : A module which contains the Heun integrator for the model equations.
- [rk4\\_integrator.f90](#) : A module which contains the RK4 integrator for the model equations.
- Makefile : The Makefile.

- [params.f90](#) : The model parameters module.
- [tl\\_ad\\_tensor.f90](#) : Tangent Linear (TL) and Adjoint (AD) model tensors definition module
- [rk2\\_tl\\_ad\\_integrator.f90](#) : Heun Tangent Linear (TL) and Adjoint (AD) model integrators module
- [rk4\\_tl\\_ad\\_integrator.f90](#) : RK4 Tangent Linear (TL) and Adjoint (AD) model integrators module
- [test\\_tl\\_ad.f90](#) : Tests for the Tangent Linear (TL) and Adjoint (AD) model versions
- README.md : A read me file.
- [LICENSE.txt](#) : The license text of the program.
- [util.f90](#) : A module with various useful functions.
- [tensor.f90](#) : Tensor utility module.
- [stat.f90](#) : A module for statistic accumulation.
- [params.nml](#) : A namelist to specify the model parameters.
- [int\\_params.nml](#) : A namelist to specify the integration parameters.
- [modeselection.nml](#) : A namelist to specify which spectral decomposition will be used.

## Usage

The user first has to fill the [params.nml](#) and [int\\_params.nml](#) namelist files according to their needs. Indeed, model and integration parameters can be specified respectively in the [params.nml](#) and [int\\_params.nml](#) namelist files. Some examples related to already published article are available in the [params](#) folder.

The parameters related to the Lyapunov exponents computation are located in the [int\\_params.nml](#) namelist file.

The [modeselection.nml](#) namelist can then be filled :

- NBOC and NBATM specify the number of blocks that will be used in respectively the ocean and the atmosphere. Each block corresponds to a given x and y wavenumber.
- The OMS and AMS arrays are integer arrays which specify which wavenumbers of the spectral decomposition will be used in respectively the ocean and the atmosphere. Their shapes are  $OMS(NBOC,2)$  and  $AMS(NBATM,2)$ .
- The first dimension specifies the number attributed by the user to the block and the second dimension specifies the x and the y wavenumbers.
- The VDDG model, described in Vannitsem et al. (2015) is given as an example in the archive.
- Note that the variables of the model are numbered according to the chosen order of the blocks.

The Makefile allows to change the integrator being used for the time evolution. The user should modify it according to its need. By default a RK2 scheme is selected.

Finally, the [IC.nml](#) file specifying the initial condition should be defined. To obtain an example of this configuration file corresponding to the model you have previously defined, simply delete the current [IC.nml](#) file (if it exists) and run the main program :

```
./maoam
```

It will generate a new one and start with the 0 initial condition. If you want another initial condition, stop the program, fill the newly generated file and restart :

```
./maooam
```

It will generate two files :

- evol\_field.dat : the recorded time evolution of the variables.
- mean\_field.dat : the mean field (the climatology)

The tangent linear and adjoint models of MAOOAM are provided in the [tl\\_ad\\_tensor](#), rk2\_tl\_ad\_integrator and rk4\_tl\_ad\_integrator modules. It is documented [here](#).

The computation of the Lyapunov exponents can be done by running the program :

```
./maooam_lyap
```

It will generate four files:

- evol\_field.dat : the recorded time evolution of the variables.
- mean\_field.dat : the mean field (the climatology)
- lyapunov\_exponents.dat : the recorded time evolution of the Lyapunov exponents
- mean\_lyapunov.dat : the mean Lyapunov exponents as well as their variance.

Alternatively, the first Lyapunov exponent can be computed by the divergence method, by running :

```
./maooam_lyap_div
```

It will generate three files :

- evol\_field.dat : the recorded time evolution of the variables.
- mean\_field.dat : the mean field (the climatology)
- lyapunov\_exponents\_div.dat : the recorded time evolution of the first Lyapunov exponent

## Implementation notes

As the system of differential equations is at most bilinear in  $y_j$  ( $j = 1..n$ ),  $\mathbf{y}$  being the array of variables, it can be expressed as a tensor contraction :

$$\frac{dy_i}{dt} = \sum_{j,k=0}^{ndim} \mathcal{T}_{i,j,k} y_k y_j$$

with  $y_0 = 1$ .

The tensor [aotensor\\_def::aotensor](#) is the tensor  $\mathcal{T}$  that encodes the differential equations is composed so that:

- $\mathcal{T}_{i,j,k}$  contains the contribution of  $dy_i/dt$  proportional to  $y_j y_k$ .
- Furthermore,  $y_0$  is always equal to 1, so that  $\mathcal{T}_{i,0,0}$  is the constant contribution to  $dy_i/dt$
- $\mathcal{T}_{i,j,0} + \mathcal{T}_{i,0,j}$  is the contribution to  $dy_i/dt$  which is linear in  $y_j$ .

Ideally, the tensor [aotensor\\_def::aotensor](#) is composed as an upper triangular matrix (in the last two coordinates).

The tensor for this model is composed in the [aotensor\\_def](#) module and uses the inner products defined in the [inprod\\_analytic](#) module.

## Final Remarks

The authors would like to thank Kris for help with the lua2fortran project. It has greatly reduced the amount of (error-prone) work.

No animals were harmed during the coding process.

## Chapter 2

# Modular arbitrary-order ocean-atmosphere model: The Tangent Linear and Adjoint model

### Description :

The Tangent Linear and Adjoint model are implemented in the same way as the nonlinear model, with a tensor storing the different terms. The Tangent Linear (TL) tensor  $\mathcal{T}_{i,j,k}^{TL}$  is defined as:

$$\mathcal{T}_{i,j,k}^{TL} = \mathcal{T}_{i,k,j} + \mathcal{T}_{i,j,k}$$

while the Adjoint (AD) tensor  $\mathcal{T}_{i,j,k}^{AD}$  is defined as:

$$\mathcal{T}_{i,j,k}^{AD} = \mathcal{T}_{j,k,i} + \mathcal{T}_{j,i,k}.$$

where  $\mathcal{T}_{i,j,k}$  is the tensor of the nonlinear model.

These two tensors are used to compute the trajectories of the models, with the equations

$$\frac{d\delta y_i}{dt} = \sum_{j=1}^{ndim} \sum_{k=0}^{ndim} \mathcal{T}_{i,j,k}^{TL} y_k^* \delta y_j.$$

$$-\frac{d\delta y_i}{dt} = \sum_{j=1}^{ndim} \sum_{k=0}^{ndim} \mathcal{T}_{i,j,k}^{AD} y_k^* \delta y_j.$$

where  $y^*$  is the point where the Tangent model is defined (with  $y_0^* = 1$ ).

### Implementation :

The two tensors are implemented in the module [tl\\_ad\\_tensor](#) and must be initialized (after calling [params::init\\_↵](#) [params](#) and [aotensor\\_def::aotensor](#)) by calling [tl\\_ad\\_tensor::init\\_tlensor\(\)](#) and [tl\\_ad\\_tensor::init\\_adtensor\(\)](#). The tendencies are then given by the routine [tl\(t,ystar,deltay,buf\)](#) and [ad\(t,ystar,deltay,buf\)](#). An integrator with the Heun method is available in the module [rk2\\_tl\\_ad\\_integrator](#) and a fourth-order Runge-Kutta integrator in [rk4\\_tl\\_ad\\_↵](#) [integrator](#). An example on how to use it can be found in the test file [test\\_tl\\_ad.f90](#)



## Chapter 3

# Modules Index

### 3.1 Modules List

Here is a list of all modules with brief descriptions:

|                                  |  |    |
|----------------------------------|--|----|
| <a href="#">aotensor_def</a>     | The equation tensor for the coupled ocean-atmosphere model with temperature which allows for an extensible set of modes in the ocean and in the atmosphere . . . . .   | 13 |
| <a href="#">ic_def</a>           | Module to load the initial condition . . . . .   | 17 |
| <a href="#">icdelta_def</a>      | Module to load the perturbation initial condition . . . . .  | 20 |
| <a href="#">inprod_analytic</a>  | Inner products between the truncated set of basis functions for the ocean and atmosphere streamfunction fields. These are partly calculated using the analytical expressions from Cehelsky, P., & Tung, K. K. : Theories of multiple equilibria and weather regimes-A critical reexamination. Part II: Baroclinic two-layer models. Journal of the atmospheric sciences, 44(21), 3282-3303, 1987 . . . . . | 22 |
| <a href="#">integrator</a>       | Module with the integration routines . . . . .   | 32 |
| <a href="#">lyap_stat</a>        | Statistics accumulators for the Lyapunov exponents . . . . .   | 35 |
| <a href="#">lyap_vectors</a>     | Module for computation of Lyapunov exponents and vectors . . . . .   | 38 |
| <a href="#">params</a>           | The model parameters module . . . . .  | 42 |
| <a href="#">stat</a>             | Statistics accumulators . . . . .  | 58 |
| <a href="#">tensor</a>           | Tensor utility module . . . . .  | 61 |
| <a href="#">tl_ad_integrator</a> | Tangent Linear (TL) and Adjoint (AD) model versions of MAOOAM. Integrators module . . . .  | 71 |
| <a href="#">tl_ad_tensor</a>     | Tangent Linear (TL) and Adjoint (AD) model versions of MAOOAM. Tensors definition module .   | 82 |
| <a href="#">util</a>             | Utility module . . . . .   | 90 |





## Chapter 4

# Data Type Index

### 4.1 Data Types List

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| <a href="#">inprod_analytic::atm_wavenum</a>                                      |    |
| Atmospheric bloc specification type . . . . .                                     | 94 |
| <a href="#">tensor::coolist</a>   |    |
| Coordinate list. Type used to represent the sparse tensor . . . . .               | 96 |
| <a href="#">tensor::coolist_elem</a>  |    |
| Coordinate list element type. Elementary elements of the sparse tensors . . . . . | 96 |
| <a href="#">inprod_analytic::ocean_tensors</a>                                    |    |
| Type holding the oceanic inner products tensors . . . . .                         | 98 |
| <a href="#">inprod_analytic::ocean_wavenum</a>                                    |    |
| Oceanic bloc specification type . . . . .   | 99 |



## Chapter 5

# File Index

### 5.1 File List

Here is a list of all files with brief descriptions:

|  |     |
|--|-----|
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| <a href="#">ic_def.f90</a>               | 102 |
| <a href="#">icdelta_def.f90</a>          | 102 |
| <a href="#">inprod_analytic.f90</a>      | 103 |
| <a href="#">lyap_stat.f90</a>            | 109 |
| <a href="#">lyap_vectors.f90</a>         | 110 |
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## Chapter 6

# Module Documentation

### 6.1 aotensor\_def Module Reference

The equation tensor for the coupled ocean-atmosphere model with temperature which allows for an extensible set of modes in the ocean and in the atmosphere.

#### Functions/Subroutines

- integer function **psi** (i)  
*Translate the  $\psi_{a,i}$  coefficients into effective coordinates.*
- integer function **theta** (i)  
*Translate the  $\theta_{a,i}$  coefficients into effective coordinates.*
- integer function **a** (i)  
*Translate the  $\psi_{o,i}$  coefficients into effective coordinates.*
- integer function **t** (i)  
*Translate the  $\delta T_{o,i}$  coefficients into effective coordinates.*
- integer function **kdelta** (i, j)  
*Kronecker delta function.*
- subroutine **coeff** (i, j, k, v)  
*Subroutine to add element in the **aotensor**  $\mathcal{T}_{i,j,k}$  structure.*
- subroutine **add\_count** (i, j, k, v)  
*Subroutine to count the elements of the **aotensor**  $\mathcal{T}_{i,j,k}$ . Add +1 to `count_elems(i)` for each value that is added to the tensor *i*-th component.*
- subroutine **compute\_aotensor** (func)  
*Subroutine to compute the tensor **aotensor**.*
- subroutine, public **init\_aotensor**  
*Subroutine to initialise the **aotensor** tensor.*

#### Variables

- integer, dimension(:), allocatable **count\_elems**  
*Vector used to count the tensor elements.*
- real(kind=8), parameter **real\_eps** = 2.2204460492503131e-16  
*Epsilon to test equality with 0.*
- type(**coolist**), dimension(:), allocatable, public **aotensor**  
 *$\mathcal{T}_{i,j,k}$  - Tensor representation of the tendencies.*

### 6.1.1 Detailed Description

The equation tensor for the coupled ocean-atmosphere model with temperature which allows for an extensible set of modes in the ocean and in the atmosphere.

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#### Remarks

Generated Fortran90/95 code from aotensor.lua

### 6.1.2 Function/Subroutine Documentation

#### 6.1.2.1 integer function aotensor\_def::a ( integer $i$ ) [private]

Translate the  $\psi_{o,i}$  coefficients into effective coordinates.

Definition at line 76 of file aotensor\_def.f90.

```
76      INTEGER :: i,a
77      a = i + 2 * natm
```

#### 6.1.2.2 subroutine aotensor\_def::add\_count ( integer, intent(in) $i$ , integer, intent(in) $j$ , integer, intent(in) $k$ , real(kind=8), intent(in) $v$ ) [private]

Subroutine to count the elements of the [aotensor](#)  $\mathcal{T}_{i,j,k}$ . Add +1 to count\_elems( $i$ ) for each value that is added to the tensor  $i$ -th component.

#### Parameters

|     |                          |
|-----|--------------------------|
| $i$ | tensor $i$ index         |
| $j$ | tensor $j$ index         |
| $k$ | tensor $k$ index         |
| $v$ | value that will be added |

Definition at line 124 of file aotensor\_def.f90.

```
124      INTEGER, INTENT(IN) :: i,j,k
125      REAL(KIND=8), INTENT(IN) :: v
126      IF (abs(v) .ge. real_eps) count_elems(i)=count_elems(i)+1
```

#### 6.1.2.3 subroutine aotensor\_def::coeff ( integer, intent(in) $i$ , integer, intent(in) $j$ , integer, intent(in) $k$ , real(kind=8), intent(in) $v$ ) [private]

Subroutine to add element in the [aotensor](#)  $\mathcal{T}_{i,j,k}$  structure.

## Parameters

|     |                  |
|-----|------------------|
| $i$ | tensor $i$ index |
| $j$ | tensor $j$ index |
| $k$ | tensor $k$ index |
| $v$ | value to add     |

Definition at line 99 of file aotensor\_def.f90.

```

99      INTEGER, INTENT(IN) :: i,j,k
100      REAL(KIND=8), INTENT(IN) :: v
101      INTEGER :: n
102      IF (.NOT. ALLOCATED(aotensor)) stop "*** coeff routine : tensor not yet allocated ***"
103      IF (.NOT. ALLOCATED(aotensor(i)%elems)) stop "*** coeff routine : tensor not yet allocated ***"
104      IF (abs(v) .ge. real_eps) THEN
105          n=(aotensor(i)%elems)+1
106          IF (j .LE. k) THEN
107              aotensor(i)%elems(n)%j=j
108              aotensor(i)%elems(n)%k=k
109          ELSE
110              aotensor(i)%elems(n)%j=k
111              aotensor(i)%elems(n)%k=j
112          END IF
113          aotensor(i)%elems(n)%v=v
114          aotensor(i)%elems=n
115      END IF

```

#### 6.1.2.4 subroutine aotensor\_def::compute\_aotensor ( external func ) [private]

Subroutine to compute the tensor [aotensor](#).

## Parameters

|             |                              |
|-------------|------------------------------|
| <i>func</i> | External function to be used |
|-------------|------------------------------|

Definition at line 132 of file aotensor\_def.f90.

#### 6.1.2.5 subroutine, public aotensor\_def::init\_aotensor ( )

Subroutine to initialise the [aotensor](#) tensor.

## Remarks

This procedure will also call [params::init\\_params\(\)](#) and [inprod\\_analytic::init\\_inprod\(\)](#) . It will finally call [inprod\\_analytic::deallocate\\_inprod\(\)](#) to remove the inner products, which are not needed anymore at this point.

Definition at line 203 of file aotensor\_def.f90.

```

203      INTEGER :: i
204      INTEGER :: allocstat
205
206      CALL init_params ! Iniatialise the parameter
207
208      CALL init_inprod ! Initialise the inner product tensors
209
210      ALLOCATE(aotensor(ndim),count_elems(ndim), stat=allocstat)
211      IF (allocstat /= 0) stop "*** Not enough memory ! ***"

```

```

212     count_elems=0
213
214     CALL compute_aotensor(add_count)
215
216     DO i=1,ndim
217         ALLOCATE(aotensor(i)%elems(count_elems(i)), stat=allocstat)
218         IF (allocstat /= 0) stop "*** Not enough memory ! ***"
219     END DO
220
221     DEALLOCATE(count_elems, stat=allocstat)
222     IF (allocstat /= 0) stop "*** Deallocation problem ! ***"
223
224     CALL compute_aotensor(coeff)
225
226     CALL simplify(aotensor)
227

```

#### 6.1.2.6 integer function aotensor\_def::kdelta ( integer *i*, integer *j* ) [private]

Kronecker delta function.

Definition at line 88 of file aotensor\_def.f90.

```

88     INTEGER :: i,j,kdelta
89     kdelta=0
90     IF (i == j) kdelta = 1

```

#### 6.1.2.7 integer function aotensor\_def::psi ( integer *i* ) [private]

Translate the  $\psi_{a,i}$  coefficients into effective coordinates.

Definition at line 64 of file aotensor\_def.f90.

```

64     INTEGER :: i,psi
65     psi = i

```

#### 6.1.2.8 integer function aotensor\_def::t ( integer *i* ) [private]

Translate the  $\delta T_{o,i}$  coefficients into effective coordinates.

Definition at line 82 of file aotensor\_def.f90.

```

82     INTEGER :: i,t
83     t = i + 2 * natm + noc

```

#### 6.1.2.9 integer function aotensor\_def::theta ( integer *i* ) [private]

Translate the  $\theta_{a,i}$  coefficients into effective coordinates.

Definition at line 70 of file aotensor\_def.f90.

```

70     INTEGER :: i,theta
71     theta = i + natm

```



### 6.1.3 Variable Documentation

#### 6.1.3.1 `type(coolist), dimension(:), allocatable, public aotensor_def::aotensor`

$\mathcal{T}_{i,j,k}$  - Tensor representation of the tendencies.

Definition at line 45 of file aotensor\_def.f90.

```
45  TYPE(coolist), DIMENSION(:), ALLOCATABLE, PUBLIC :: aotensor
```

#### 6.1.3.2 `integer, dimension(:), allocatable aotensor_def::count_elems [private]`

Vector used to count the tensor elements.

Definition at line 37 of file aotensor\_def.f90.

```
37  INTEGER, DIMENSION(:), ALLOCATABLE :: count_elems
```

#### 6.1.3.3 `real(kind=8), parameter aotensor_def::real_eps = 2.2204460492503131e-16 [private]`

Epsilon to test equality with 0.

Definition at line 40 of file aotensor\_def.f90.

```
40  REAL(KIND=8), PARAMETER :: real_eps = 2.2204460492503131e-16
```

## 6.2 ic\_def Module Reference

Module to load the initial condition.

### Functions/Subroutines

- subroutine, public [load\\_ic](#)

*Subroutine to load the initial condition if IC.nml exists. If it does not, then write IC.nml with 0 as initial condition.*

### Variables

- logical [exists](#)  
*Boolean to test for file existence.*
- `real(kind=8), dimension(:), allocatable, public ic`  
*Initial condition vector.*

## 6.2.1 Detailed Description

Module to load the initial condition.

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## 6.2.2 Function/Subroutine Documentation

### 6.2.2.1 subroutine, public ic\_def::load\_ic ( )

Subroutine to load the initial condition if IC.nml exists. If it does not, then write IC.nml with 0 as initial condition.

Definition at line 32 of file ic\_def.f90.

```

32     INTEGER :: i,allocstat,j
33     CHARACTER(len=20) :: fm
34     REAL(KIND=8) :: size_of_random_noise
35     INTEGER, DIMENSION(:), ALLOCATABLE :: seed
36     CHARACTER(LEN=4) :: init_type
37     namelist /iclist/ ic
38     namelist /rand/ init_type,size_of_random_noise,seed
39
40
41     fm(1:6)=' (F3.1)'
42
43     CALL random_seed(size=j)
44
45     IF (ndim == 0) stop "*** Number of dimensions is 0! ***"
46     ALLOCATE(ic(0:ndim),seed(j), stat=allocstat)
47     IF (allocstat /= 0) stop "*** Not enough memory ! ***"
48
49     INQUIRE(file='./IC.nml',exist=exists)
50
51     IF (exists) THEN
52         OPEN(8, file="IC.nml", status='OLD', recl=80, delim='APOSTROPHE')
53         READ(8,nml=iclist)
54         READ(8,nml=rand)
55         CLOSE(8)
56         SELECT CASE (init_type)
57             CASE ('seed')
58                 CALL random_seed(put=seed)
59                 CALL random_number(ic)
60                 ic=2*(ic-0.5)
61                 ic=ic*size_of_random_noise*10.d0
62                 ic(0)=1.0d0
63                 WRITE(6,*) "*** IC.nml namelist written. Starting with 'seeded' random initial condition !***"
64             CASE ('rand')
65                 CALL init_random_seed()
66                 CALL random_seed(get=seed)
67                 CALL random_number(ic)
68                 ic=2*(ic-0.5)
69                 ic=ic*size_of_random_noise*10.d0
70                 ic(0)=1.0d0
71                 WRITE(6,*) "*** IC.nml namelist written. Starting with random initial condition !***"
72             CASE ('zero')
73                 CALL init_random_seed()
74                 CALL random_seed(get=seed)
75                 ic=0
76                 ic(0)=1.0d0
77                 WRITE(6,*) "*** IC.nml namelist written. Starting with initial condition in IC.nml !***"
78             CASE ('read')
79                 CALL init_random_seed()
80                 CALL random_seed(get=seed)
81                 ic(0)=1.0d0
82                 ! except IC(0), nothing has to be done IC has already the right values
83                 WRITE(6,*) "*** IC.nml namelist written. Starting with initial condition in IC.nml !***"
84         END SELECT
85     ELSE
86         CALL init_random_seed()
87         CALL random_seed(get=seed)
88         ic=0
89         ic(0)=1.0d0

```

```

90      init_type="zero"
91      size_of_random_noise=0.d0
92      WRITE(6,*) "*** IC.nml namelist written. Starting with 0 as initial condition !***"
93  END IF
94  OPEN(8, file="IC.nml", status='REPLACE')
95  WRITE(8,'(a)') "!-------!"
96  WRITE(8,'(a)') " ! Namelist file : !"
97  WRITE(8,'(a)') " ! Initial condition. !"
98  WRITE(8,'(a)') "!-------!"
99  WRITE(8,*) ""
100  WRITE(8,'(a)') "&ICLIST"
101  WRITE(8,*) " ! psi variables"
102  DO i=1,natm
103      WRITE(8,*) " IC("//trim(str(i))//") = ",ic(i)," ! typ= "&
104      & //awavenum(i)%typ//", Nx= "//trim(rstr(awavenum(i)&
105      & %Nx, fm))//", Ny= "//trim(rstr(awavenum(i)%Ny, fm))
106  END DO
107  WRITE(8,*) " ! theta variables"
108  DO i=1,natm
109      WRITE(8,*) " IC("//trim(str(i+natm))//") = ",ic(i+natm)," ! typ= "&
110      & //awavenum(i)%typ//", Nx= "//trim(rstr(awavenum(i)&
111      & %Nx, fm))//", Ny= "//trim(rstr(awavenum(i)%Ny, fm))
112  END DO
113
114  WRITE(8,*) " ! A variables"
115  DO i=1,noc
116      WRITE(8,*) " IC("//trim(str(i+2*natm))//") = ",ic(i+2*natm)," ! Nx&
117      & = "//trim(rstr(owavenum(i)%Nx, fm))//", Ny= "&
118      & //trim(rstr(owavenum(i)%Ny, fm))
119  END DO
120  WRITE(8,*) " ! T variables"
121  DO i=1,noc
122      WRITE(8,*) " IC("//trim(str(i+noc+2*natm))//") = ",ic(i+2*natm+noc)," &
123      & ! Nx= "//trim(rstr(owavenum(i)%Nx, fm))//", Ny= "&
124      & //trim(rstr(owavenum(i)%Ny, fm))
125  END DO
126
127  WRITE(8,'(a)') "&END"
128  WRITE(8,*) ""
129  WRITE(8,'(a)') "!-------!"
130  WRITE(8,'(a)') " ! Initialisation type. !"
131  WRITE(8,'(a)') "!-------!"
132  WRITE(8,'(a)') " ! type = 'read': use IC above (will generate a new seed);"
133  WRITE(8,'(a)') " ! 'rand': random state (will generate a new seed);"
134  WRITE(8,'(a)') " ! 'zero': zero IC (will generate a new seed);"
135  WRITE(8,'(a)') " ! 'seed': use the seed below (generate the same IC)"
136  WRITE(8,*) ""
137  WRITE(8,'(a)') "&RAND"
138  WRITE(8,'(a)') " init_type= "//init_type//""
139  WRITE(8,'(a,d15.7)') " size_of_random_noise = ",size_of_random_noise
140  DO i=1,j
141      WRITE(8,*) " seed("//trim(str(i))//") = ",seed(i)
142  END DO
143  WRITE(8,'(a)') "&END"
144  WRITE(8,*) ""
145  CLOSE(8)
146

```

## 6.2.3 Variable Documentation

### 6.2.3.1 logical ic\_def::exists [private]

Boolean to test for file existence.

Definition at line 21 of file ic\_def.f90.

```
21  LOGICAL :: exists !< Boolean to test for file existence.
```

### 6.2.3.2 real(kind=8), dimension(:), allocatable, public ic\_def::ic

Initial condition vector.

Definition at line 23 of file ic\_def.f90.

```
23  REAL(KIND=8), DIMENSION(:), ALLOCATABLE, PUBLIC :: ic !< Initial condition vector
```

## 6.3 icdelta\_def Module Reference

Module to load the perturbation initial condition.

### Functions/Subroutines

- subroutine, public [load\\_icdelta](#)

*Subroutine to load the initial condition if ICdelta.nml exists. If it does not, then write ICdelta.nml with random initial condition.*

### Variables

- logical [exists](#)  
*Boolean to test for file existence.*
- real(kind=8), dimension(:), allocatable, public [icdelta](#)  
*Initial condition vector.*

### 6.3.1 Detailed Description

Module to load the perturbation initial condition.

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### 6.3.2 Function/Subroutine Documentation

#### 6.3.2.1 subroutine, public icdelta\_def::load\_icdelta ( )

Subroutine to load the initial condition if ICdelta.nml exists. If it does not, then write ICdelta.nml with random initial condition.

Definition at line 32 of file icdelta\_def.f90.

```

32     INTEGER :: i, allocstat
33     CHARACTER(len=20) :: fm
34     REAL(KIND=8) :: size_of_random_noise
35     CHARACTER(LEN=4) :: init_type
36     namelist /iclist/ icdelta
37     namelist /rand/ init_type, size_of_random_noise
38
39
40
41     fm(1:6)='(F3.1)'
42
43     IF (ndim == 0) stop "*** Number of dimensions is 0! ***"
44     ALLOCATE(icdelta(0:ndim), stat=allocstat)
45     IF (allocstat /= 0) stop "*** Not enough memory ! ***"
46
47     INQUIRE(file='./ICdelta.nml', exist=exists)
48
49     IF (exists) THEN
50         OPEN(8, file="ICdelta.nml", status='OLD', recl=80, delim='APOSTROPHE')
51         READ(8, nml=iclist)
52         READ(8, nml=rand)
53         SELECT CASE (init_type)

```

```

54      CASE ('rand')
55        CALL random_number(icdelta)
56        icdelta=2*(icdelta-0.5)
57        icdelta=icdelta*size_of_random_noise*10.d0
58        icdelta(0)=1.0d0
59        WRITE(6,*) "*** ICdelta.nml namelist written. Starting with random initial condition !***"
60      CASE ('zero')
61        icdelta=0
62        icdelta(0)=1.0d0
63        WRITE(6,*) "*** ICdelta.nml namelist written. Starting with initial condition in ICdelta.nml
!***"
64      CASE ('read')
65        !nothing has to be done ICdelta has already the right values
66        WRITE(6,*) "*** ICdelta.nml namelist written. Starting with initial condition in ICdelta.nml
!***"
67      END SELECT
68    CLOSE(8)
69  ELSE
70    CALL random_number(icdelta)
71    icdelta=2*(icdelta-0.5)
72    size_of_random_noise=1.d-3
73    icdelta=icdelta*size_of_random_noise*10.d0
74    icdelta(0)=1.0d0
75    init_type="rand"
76    WRITE(6,*) "*** ICdelta.nml namelist written. Starting with 0 as initial condition !***"
77
78  END IF
79  OPEN(8, file="ICdelta.nml", status='REPLACE')
80  WRITE(8,'(a)') "!-------!"
81  WRITE(8,'(a)') " ! Namelist file :                                !"
82  WRITE(8,'(a)') " ! Initial condition.                                !"
83  WRITE(8,'(a)') "!-------!"
84  WRITE(8,*) ""
85  WRITE(8,'(a)') "&ICLIST"
86  WRITE(8,*) " ! psi variables"
87  DO i=1,natm
88    WRITE(8,*) " ICdelta(//trim(str(i))//) = ",icdelta(i+natm)," ! typ= "&
89    &//awavenum(i)%typ//", Nx= "//trim(rstr(awavenum(i)&
90    &%Nx,fm))//", Ny= "//trim(rstr(awavenum(i)%Ny,fm))
91  END DO
92  WRITE(8,*) " ! theta variables"
93  DO i=1,natm
94    WRITE(8,*) " ICdelta(//trim(str(i+natm))//) = ",icdelta(i+natm)," ! typ= "&
95    &//awavenum(i)%typ//", Nx= "//trim(rstr(awavenum(i)&
96    &%Nx,fm))//", Ny= "//trim(rstr(awavenum(i)%Ny,fm))
97  END DO
98
99  WRITE(8,*) " ! A variables"
100  DO i=1,noc
101    WRITE(8,*) " ICdelta(//trim(str(i+2*natm))//) = ",icdelta(i+2*natm)," ! Nx&
102    &= "//trim(rstr(owavenum(i)%Nx,fm))//", Ny= "&
103    &//trim(rstr(owavenum(i)%Ny,fm))
104  END DO
105  WRITE(8,*) " ! T variables"
106  DO i=1,noc
107    WRITE(8,*) " ICdelta(//trim(str(i+noc+2*natm))//) = ",icdelta(i+2*natm+noc)," &
108    &! Nx= "//trim(rstr(owavenum(i)%Nx,fm))//", Ny= "&
109    &//trim(rstr(owavenum(i)%Ny,fm))
110  END DO
111
112  WRITE(8,'(a)') "&END"
113  WRITE(8,*) ""
114  WRITE(8,'(a)') "!-------!"
115  WRITE(8,'(a)') " ! Initialisation type.                                !"
116  WRITE(8,'(a)') "!-------!"
117  WRITE(8,'(a)') " ! type = 'read': use ICdelta; 'rand': random state; 'zero': zero condition "
118  WRITE(8,'(a)') " ! The seed is specified in IC.nml"
119  WRITE(8,*) ""
120  WRITE(8,'(a)') "&RAND"
121  WRITE(8,'(a)') " init_type= "//init_type//""
122  WRITE(8,'(a,d15.7)') " size_of_random_noise = ",size_of_random_noise
123  WRITE(8,'(a)') "&END"
124  WRITE(8,*) ""
125  CLOSE(8)

```

### 6.3.3 Variable Documentation

#### 6.3.3.1 logical icdelta\_def::exists [private]

Boolean to test for file existence.

Definition at line 21 of file icdelta\_def.f90.

```
21 LOGICAL :: exists !< Boolean to test for file existence.
```

### 6.3.3.2 `real(kind=8), dimension(:), allocatable, public icdelta_def::icdelta`

Initial condition vector.

Definition at line 23 of file `icdelta_def.f90`.

```
23 REAL(KIND=8), DIMENSION(:), ALLOCATABLE, PUBLIC :: icdelta !< Initial condition vector
```

## 6.4 `inprod_analytic` Module Reference

Inner products between the truncated set of basis functions for the ocean and atmosphere streamfunction fields. These are partly calculated using the analytical expressions from Cehelsky, P., & Tung, K. K. : Theories of multiple equilibria and weather regimes-A critical reexamination. Part II: Baroclinic two-layer models. Journal of the atmospheric sciences, 44(21), 3282-3303, 1987.

### Data Types

- type `atm_tensors`  
*Type holding the atmospheric inner products tensors.*
- type `atm_wavenum`  
*Atmospheric bloc specification type.*
- type `ocean_tensors`  
*Type holding the oceanic inner products tensors.*
- type `ocean_wavenum`  
*Oceanic bloc specification type.*

### Functions/Subroutines

- `real(kind=8)` function `b1` (Pi, Pj, Pk)  
*Cehelsky & Tung Helper functions.*
- `real(kind=8)` function `b2` (Pi, Pj, Pk)  
*Cehelsky & Tung Helper functions.*
- `real(kind=8)` function `delta` (r)  
*Integer Dirac delta function.*
- `real(kind=8)` function `flambda` (r)  
*"Odd or even" function*
- `real(kind=8)` function `s1` (Pj, Pk, Mj, Hk)  
*Cehelsky & Tung Helper functions.*
- `real(kind=8)` function `s2` (Pj, Pk, Mj, Hk)  
*Cehelsky & Tung Helper functions.*
- `real(kind=8)` function `s3` (Pj, Pk, Hj, Hk)  
*Cehelsky & Tung Helper functions.*
- `real(kind=8)` function `s4` (Pj, Pk, Hj, Hk)  
*Cehelsky & Tung Helper functions.*

- real(kind=8) function [calculate\\_a](#) (i, j)  
*Eigenvalues of the Laplacian (atmospheric)*
- real(kind=8) function [calculate\\_b](#) (i, j, k)  
*Streamfunction advection terms (atmospheric)*
- real(kind=8) function [calculate\\_c\\_atm](#) (i, j)  
*Beta term for the atmosphere.*
- real(kind=8) function [calculate\\_d](#) (i, j)  
*Forcing of the ocean on the atmosphere.*
- real(kind=8) function [calculate\\_g](#) (i, j, k)  
*Temperature advection terms (atmospheric)*
- real(kind=8) function [calculate\\_s](#) (i, j)  
*Forcing (thermal) of the ocean on the atmosphere.*
- real(kind=8) function [calculate\\_k](#) (i, j)  
*Forcing of the atmosphere on the ocean.*
- real(kind=8) function [calculate\\_m](#) (i, j)  
*Forcing of the ocean fields on the ocean.*
- real(kind=8) function [calculate\\_n](#) (i, j)  
*Beta term for the ocean.*
- real(kind=8) function [calculate\\_o](#) (i, j, k)  
*Temperature advection term (passive scalar)*
- real(kind=8) function [calculate\\_c\\_oc](#) (i, j, k)  
*Streamfunction advection terms (oceanic)*
- real(kind=8) function [calculate\\_w](#) (i, j)  
*Short-wave radiative forcing of the ocean.*
- subroutine, public [init\\_inprod](#)  
*Initialisation of the inner product.*

## Variables

- type([atm\\_wavenum](#)), dimension(:), allocatable, public [awavenum](#)  
*Atmospheric blocs specification.*
- type([ocean\\_wavenum](#)), dimension(:), allocatable, public [owavenum](#)  
*Oceanic blocs specification.*
- type([atm\\_tensors](#)), public [atmos](#)  
*Atmospheric tensors.*
- type([ocean\\_tensors](#)), public [ocean](#)  
*Oceanic tensors.*

### 6.4.1 Detailed Description

Inner products between the truncated set of basis functions for the ocean and atmosphere streamfunction fields. These are partly calculated using the analytical expressions from Cehelsky, P., & Tung, K. K. : Theories of multiple equilibria and weather regimes-A critical reexamination. Part II: Baroclinic two-layer models. Journal of the atmospheric sciences, 44(21), 3282-3303, 1987.

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## Remarks

Generated Fortran90/95 code from inprod\_analytic.lua

## 6.4.2 Function/Subroutine Documentation

### 6.4.2.1 `real(kind=8) function inprod_analytic::b1 ( integer $P_i$ , integer $P_j$ , integer $P_k$ ) [private]`

Cehelsky & Tung Helper functions.

Definition at line 100 of file `inprod_analytic.f90`.

```
100      INTEGER :: pi,pj,pk
101      b1 = (pk + pj) / REAL(pi)
```

### 6.4.2.2 `real(kind=8) function inprod_analytic::b2 ( integer $P_i$ , integer $P_j$ , integer $P_k$ ) [private]`

Cehelsky & Tung Helper functions.

Definition at line 106 of file `inprod_analytic.f90`.

```
106      INTEGER :: pi,pj,pk
107      b2 = (pk - pj) / REAL(pi)
```

### 6.4.2.3 `real(kind=8) function inprod_analytic::calculate_a ( integer, intent(in) $i$ , integer, intent(in) $j$ ) [private]`

Eigenvalues of the Laplacian (atmospheric)

$$a_{i,j} = (F_i, \nabla^2 F_j).$$

Definition at line 164 of file `inprod_analytic.f90`.

```
164      INTEGER, INTENT(IN) :: i, j
165      TYPE(atm_wavenum) :: ti
166
167      calculate_a = 0.d0
168      IF (i==j) THEN
169          ti = awavenum(i)
170          calculate_a = -(n**2) * ti%Nx**2 - ti%Ny**2
171      END IF
```

### 6.4.2.4 `real(kind=8) function inprod_analytic::calculate_b ( integer, intent(in) $i$ , integer, intent(in) $j$ , integer, intent(in) $k$ ) [private]`

Streamfunction advection terms (atmospheric)

$$b_{i,j,k} = (F_i, J(F_j, \nabla^2 F_k)).$$

Definition at line 178 of file `inprod_analytic.f90`.

```
178      INTEGER, INTENT(IN) :: i, j, k
179
180      calculate_b = calculate_a(k,k) * calculate_g(i,j,k)
181
```



#### 6.4.2.5 `real(kind=8) function inprod_analytic::calculate_c_atm ( integer, intent(in) i, integer, intent(in) j ) [private]`

Beta term for the atmosphere.

$$c_{i,j} = (F_i, \partial_x F_j) .$$

Definition at line 188 of file inprod\_analytic.f90.

```

188     INTEGER, INTENT(IN) :: i, j
189     TYPE(atm_wavenum) :: ti, tj
190
191     ti = awavenum(i)
192     tj = awavenum(j)
193     calculate_c_atm = 0.d0
194     IF ((ti%typ == "K") .AND. (tj%typ == "L")) THEN
195         calculate_c_atm = n * ti%M * delta(ti%M - tj%H) * delta(ti%P - tj%P)
196     ELSE IF ((ti%typ == "L") .AND. (tj%typ == "K")) THEN
197         ti = awavenum(j)
198         tj = awavenum(i)
199         calculate_c_atm = - n * ti%M * delta(ti%M - tj%H) * delta(ti%P - tj%P)
200     END IF
201
```

#### 6.4.2.6 `real(kind=8) function inprod_analytic::calculate_c_oc ( integer, intent(in) i, integer, intent(in) j, integer, intent(in) k ) [private]`

Streamfunction advection terms (oceanic)

$$C_{i,j,k} = (\eta_i, J(\eta_j, \nabla^2 \eta_k)) .$$

Definition at line 412 of file inprod\_analytic.f90.

```

412     INTEGER, INTENT(IN) :: i, j, k
413
414     calculate_c_oc = calculate_m(k,k) * calculate_o(i,j,k)
415
```

#### 6.4.2.7 `real(kind=8) function inprod_analytic::calculate_d ( integer, intent(in) i, integer, intent(in) j ) [private]`

Forcing of the ocean on the atmosphere.

$$d_{i,j} = (F_i, \nabla^2 \eta_j) .$$

Definition at line 208 of file inprod\_analytic.f90.

```

208     INTEGER, INTENT(IN) :: i, j
209
210     calculate_d=calculate_s(i,j) * calculate_m(j,j)
211
```

#### 6.4.2.8 real(kind=8) function inprod\_analytic::calculate\_g ( integer, intent(in) i, integer, intent(in) j, integer, intent(in) k ) [private]

Temperature advection terms (atmospheric)

$$g_{i,j,k} = (F_i, J(F_j, F_k)).$$

Definition at line 218 of file inprod\_analytic.f90.

```

218  INTEGER, INTENT(IN) :: i,j,k
219  TYPE(atm_wavenum) :: ti,tj,tk
220  REAL(KIND=8) :: val,vb1, vb2, vs1, vs2, vs3, vs4
221  INTEGER, DIMENSION(3) :: a,b
222  INTEGER, DIMENSION(3,3) :: w
223  CHARACTER, DIMENSION(3) :: s
224  INTEGER :: par
225
226  ti = awavenum(i)
227  tj = awavenum(j)
228  tk = awavenum(k)
229
230  a(1)=i
231  a(2)=j
232  a(3)=k
233
234  val=0.d0
235
236  IF ((ti%typ == "L") .AND. (tj%typ == "L") .AND. (tk%typ == "L")) THEN
237
238      CALL piksrt(3,a,par)
239
240      ti = awavenum(a(1))
241      tj = awavenum(a(2))
242      tk = awavenum(a(3))
243
244      vs3 = s3(tj%P,tk%P,tj%H,tk%H)
245      vs4 = s4(tj%P,tk%P,tj%H,tk%H)
246      val = vs3 * ((delta(tk%H - tj%H - ti%H) - delta(tk%H &
247      &- tj%H + ti%H)) * delta(tk%P + tj%P - ti%P) +&
248      & delta(tk%H + tj%H - ti%H) * (delta(tk%P - tj%P&
249      & + ti%P) - delta(tk%P - tj%P - ti%P))) + vs4 *&
250      & ((delta(tk%H + tj%H - ti%H) * delta(tk%P - tj&
251      &%P - ti%P)) + (delta(tk%H - tj%H + ti%H) -&
252      & delta(tk%H - tj%H - ti%H)) * (delta(tk%P - tj&
253      &%P - ti%P) - delta(tk%P - tj%P + ti%P)))
254  ELSE
255
256      s(1)=ti%typ
257      s(2)=tj%typ
258      s(3)=tk%typ
259
260      w(1,:)=isin("A",s)
261      w(2,:)=isin("K",s)
262      w(3,:)=isin("L",s)
263
264      IF (any(w(1,:)/=0) .AND. any(w(2,:)/=0) .AND. any(w(3,:)/=0)) THEN
265          b=w(:,1)
266          ti = awavenum(a(b(1)))
267          tj = awavenum(a(b(2)))
268          tk = awavenum(a(b(3)))
269          call piksrt(3,b,par)
270          vb1 = b1(ti%P,tj%P,tk%P)
271          vb2 = b2(ti%P,tj%P,tk%P)
272          val = -2 * sqrt(2.) / pi * tj%M * delta(tj%M - tk%H) * flambda(ti%P + tj%P + tk%P)
273          IF (val /= 0.d0) val = val * (vb1**2 / (vb1**2 - 1) - vb2**2 / (vb2**2 - 1))
274      ELSEIF ((w(2,2)/=0) .AND. (w(2,3)==0) .AND. any(w(3,:)/=0)) THEN
275          ti = awavenum(a(w(2,1)))
276          tj = awavenum(a(w(2,2)))
277          tk = awavenum(a(w(3,1)))
278          b(1)=w(2,1)
279          b(2)=w(2,2)
280          b(3)=w(3,1)
281          call piksrt(3,b,par)
282          vs1 = s1(tj%P,tk%P,tj%M,tk%H)
283          vs2 = s2(tj%P,tk%P,tj%M,tk%H)
284          val = vs1 * (delta(ti%M - tk%H - tj%M) * delta(ti%P -&
285          & tk%P + tj%P) - delta(ti%M- tk%H - tj%M) *&
286          & delta(ti%P + tk%P - tj%P) + (delta(tk%H - tj%M&
287          & + ti%M) + delta(tk%H - tj%M - ti%M)) *&
288          & delta(tk%P + tj%P - ti%P)) + vs2 * (delta(ti%M&
289          & - tk%H - tj%M) * delta(ti%P - tk%P - tj%P) +&

```

```

290          & (delta(tk%H - tj%M - ti%M) + delta(ti%M + tk%H&
291          & - tj%M)) * (delta(ti%P - tk%P + tj%P) -&
292          & delta(tk%P - tj%P + ti%P)))
293      ENDIF
294  ENDIF
295  calculate_g=par*val*n
296

```

#### 6.4.2.9 real(kind=8) function inprod\_analytic::calculate\_k ( integer, intent(in) i, integer, intent(in) j ) [private]

Forcing of the atmosphere on the ocean.

$$K_{i,j} = (\eta_i, \nabla^2 F_j).$$

Definition at line 336 of file inprod\_analytic.f90.

```

336  INTEGER, INTENT(IN) :: i, j
337
338  calculate_k = calculate_s(j,i) * calculate_a(j,j)

```

#### 6.4.2.10 real(kind=8) function inprod\_analytic::calculate\_m ( integer, intent(in) i, integer, intent(in) j ) [private]

Forcing of the ocean fields on the ocean.

$$M_{i,j} = (eta_i, \nabla^2 \eta_j).$$

Definition at line 345 of file inprod\_analytic.f90.

```

345  INTEGER, INTENT(IN) :: i, j
346  TYPE(ocean_wavenum) :: di
347
348  calculate_m=0.d0
349  IF (i==j) THEN
350      di = owavenum(i)
351      calculate_m = -(n**2) * di%Nx**2 - di%Ny**2
352  END IF

```

#### 6.4.2.11 real(kind=8) function inprod\_analytic::calculate\_n ( integer, intent(in) i, integer, intent(in) j ) [private]

Beta term for the ocean.

$$N_{i,j} = (\eta_i, \partial_x \eta_j).$$

Definition at line 359 of file inprod\_analytic.f90.

```

359  INTEGER, INTENT(IN) :: i, j
360  TYPE(ocean_wavenum) :: di, dj
361  REAL(KIND=8) :: val
362
363  di = owavenum(i)
364  dj = owavenum(j)
365  calculate_n = 0.d0
366  IF (dj%H/=di%H) THEN
367      val = delta(di%P - dj%P) * flambda(di%H + dj%H)
368      calculate_n = val * (-2) * dj%H * di%H * n / ((dj%H**2 - di%H**2) * pi)
369  ENDIF
370

```

#### 6.4.2.12 `real(kind=8) function inprod_analytic::calculate_o ( integer, intent(in) i, integer, intent(in) j, integer, intent(in) k )` [private]

Temperature advection term (passive scalar)

$$O_{i,j,k} = (\eta_i, J(\eta_j, \eta_k)).$$

Definition at line 377 of file inprod\_analytic.f90.

```

377  INTEGER, INTENT(IN) :: i,j,k
378  TYPE(ocean_wavenum) :: di,dj,dk
379  REAL(KIND=8) :: vs3,vs4,val
380  INTEGER, DIMENSION(3) :: a
381  INTEGER :: par
382
383  val=0.d0
384
385  a(1)=i
386  a(2)=j
387  a(3)=k
388
389  CALL piksrt(3,a,par)
390
391  di = owavenum(a(1))
392  dj = owavenum(a(2))
393  dk = owavenum(a(3))
394
395  vs3 = s3(dj%P,dk%P,dj%H,dk%H)
396  vs4 = s4(dj%P,dk%P,dj%H,dk%H)
397  val = vs3*((delta(dk%H - dj%H - di%H) - delta(dk%H - dj%
398    &%H + di%H)) * delta(dk%P + dj%P - di%P) + delta(dk%
399    &%H + dj%H - di%H) * (delta(dk%P - dj%P + di%P) - &
400    & delta(dk%P - dj%P - di%P))) + vs4 * ((delta(dk%H &
401    &+ dj%H - di%H) * delta(dk%P - dj%P - di%P)) + &
402    & (delta(dk%H - dj%H + di%H) - delta(dk%H - dj%H - &
403    & di%H)) * (delta(dk%P - dj%P - di%P) - delta(dk%P &
404    &- dj%P + di%P)))
405  calculate_o = par * val * n / 2

```

#### 6.4.2.13 `real(kind=8) function inprod_analytic::calculate_s ( integer, intent(in) i, integer, intent(in) j )` [private]

Forcing (thermal) of the ocean on the atmosphere.

$$s_{i,j} = (F_i, \eta_j).$$

Definition at line 303 of file inprod\_analytic.f90.

```

303  INTEGER, INTENT(IN) :: i,j
304  TYPE(atm_wavenum) :: ti
305  TYPE(ocean_wavenum) :: dj
306  REAL(KIND=8) :: val
307
308  ti = awavenum(i)
309  dj = owavenum(j)
310  val=0.d0
311  IF (ti%typ == "A") THEN
312    val = flambda(dj%H) * flambda(dj%P + ti%P)
313    IF (val /= 0.d0) THEN
314      val = val*8*sqrt(2.)*dj%P/(pi**2 * (dj%P**2 - ti%P**2) * dj%H)
315    END IF
316  ELSEIF (ti%typ == "K") THEN
317    val = flambda(2 * ti%M + dj%H) * delta(dj%P - ti%P)
318    IF (val /= 0.d0) THEN
319      val = val*4*dj%H/(pi * (-4 * ti%M**2 + dj%H**2))
320    END IF
321  ELSEIF (ti%typ == "L") THEN
322    val = delta(dj%P - ti%P) * delta(2 * ti%H - dj%H)
323  END IF
324  calculate_s=val
325

```

**6.4.2.14** `real(kind=8) function inprod_analytic::calculate_w ( integer, intent(in) i, integer, intent(in) j ) [private]`

Short-wave radiative forcing of the ocean.

$$W_{i,j} = (\eta_i, F_j) .$$

Definition at line 422 of file inprod\_analytic.f90.

```
422  INTEGER, INTENT(IN) :: i,j
423
424  calculate_w = calculate_s(j,i)
425
```

**6.4.2.15** `real(kind=8) function inprod_analytic::delta ( integer r ) [private]`

Integer Dirac delta function.

Definition at line 112 of file inprod\_analytic.f90.

```
112  INTEGER :: r
113  IF (r==0) THEN
114      delta = 1.d0
115  ELSE
116      delta = 0.d0
117  ENDIF
```

**6.4.2.16** `real(kind=8) function inprod_analytic::flambda ( integer r ) [private]`

"Odd or even" function

Definition at line 122 of file inprod\_analytic.f90.

```
122  INTEGER :: r
123  IF (mod(r,2)==0) THEN
124      flambda = 0.d0
125  ELSE
126      flambda = 1.d0
127  ENDIF
```

**6.4.2.17** `subroutine, public inprod_analytic::init_inprod ( )`

Initialisation of the inner product.

Definition at line 436 of file inprod\_analytic.f90.

```

436     INTEGER :: i,j
437     INTEGER :: allocstat
438
439     IF (natm == 0 ) THEN
440         stop "*** Problem : natm==0 ! ***"
441     ELSEIF (noc == 0) then
442         stop "*** Problem : noc==0 ! ***"
443     END IF
444
445
446     ! Definition of the types and wave numbers tables
447
448     ALLOCATE(owavenum(noc),awavenum(natm), stat=allocstat)
449     IF (allocstat /= 0) stop "*** Not enough memory ! ***"
450
451     j=0
452     DO i=1,nbatm
453         IF (ams(i,1)==1) THEN
454             awavenum(j+1)%typ=' A'
455             awavenum(j+2)%typ=' K'
456             awavenum(j+3)%typ=' L'
457
458             awavenum(j+1)%P=ams(i,2)
459             awavenum(j+2)%M=ams(i,1)
460             awavenum(j+2)%P=ams(i,2)
461             awavenum(j+3)%H=ams(i,1)
462             awavenum(j+3)%P=ams(i,2)
463
464             awavenum(j+1)%Ny=REAL(ams(i,2))
465             awavenum(j+2)%Nx=REAL(ams(i,1))
466             awavenum(j+2)%Ny=REAL(ams(i,2))
467             awavenum(j+3)%Nx=REAL(ams(i,1))
468             awavenum(j+3)%Ny=REAL(ams(i,2))
469
470             j=j+3
471         ELSE
472             awavenum(j+1)%typ=' K'
473             awavenum(j+2)%typ=' L'
474
475             awavenum(j+1)%M=ams(i,1)
476             awavenum(j+1)%P=ams(i,2)
477             awavenum(j+2)%H=ams(i,1)
478             awavenum(j+2)%P=ams(i,2)
479
480             awavenum(j+1)%Nx=REAL(ams(i,1))
481             awavenum(j+1)%Ny=REAL(ams(i,2))
482             awavenum(j+2)%Nx=REAL(ams(i,1))
483             awavenum(j+2)%Ny=REAL(ams(i,2))
484
485             j=j+2
486         ENDIF
487     ENDDO
488
489     DO i=1,noc
490         owavenum(i)%H=oms(i,1)
491         owavenum(i)%P=oms(i,2)
492
493         owavenum(i)%Nx=oms(i,1)/2.d0
494         owavenum(i)%Ny=oms(i,2)
495     ENDDO
496
497
498     ! Pointing to the atmospheric inner products functions
499
500     atmos%a => calculate_a
501     atmos%g => calculate_g
502     atmos%s => calculate_s
503     atmos%b => calculate_b
504     atmos%d => calculate_d
505     atmos%c => calculate_c_atm
506
507
508     ! Pointing to the oceanic inner products functions
509
510     ocean%M => calculate_m
511     ocean%N => calculate_n
512     ocean%O => calculate_o
513     ocean%C => calculate_c_oc
514     ocean%W => calculate_w
515     ocean%K => calculate_k
516

```

**6.4.2.18** `real(kind=8) function inprod_analytic::s1 ( integer Pj, integer Pk, integer Mj, integer Hk ) [private]`

Cehelsky & Tung Helper functions.

Definition at line 132 of file inprod\_analytic.f90.

```
132    INTEGER :: pk,pj,mj,hk
133    s1 = -((pk * mj + pj * hk)) / 2.d0
```

**6.4.2.19** `real(kind=8) function inprod_analytic::s2 ( integer Pj, integer Pk, integer Mj, integer Hk ) [private]`

Cehelsky & Tung Helper functions.

Definition at line 138 of file inprod\_analytic.f90.

```
138    INTEGER :: pk,pj,mj,hk
139    s2 = (pk * mj - pj * hk) / 2.d0
```

**6.4.2.20** `real(kind=8) function inprod_analytic::s3 ( integer Pj, integer Pk, integer Hj, integer Hk ) [private]`

Cehelsky & Tung Helper functions.

Definition at line 144 of file inprod\_analytic.f90.

```
144    INTEGER :: pj,pk,hj,hk
145    s3 = (pk * hj + pj * hk) / 2.d0
```

**6.4.2.21** `real(kind=8) function inprod_analytic::s4 ( integer Pj, integer Pk, integer Hj, integer Hk ) [private]`

Cehelsky & Tung Helper functions.

Definition at line 150 of file inprod\_analytic.f90.

```
150    INTEGER :: pj,pk,hj,hk
151    s4 = (pk * hj - pj * hk) / 2.d0
```

## 6.4.3 Variable Documentation

**6.4.3.1** `type(atm_tensors), public inprod_analytic::atmos`

Atmospheric tensors.

Definition at line 78 of file inprod\_analytic.f90.

```
78    TYPE(atm_tensors), PUBLIC :: atmos
```

#### 6.4.3.2 `type(atm_wavenum)`, `dimension(:)`, `allocatable`, `public inprod_analytic::awavenum`

Atmospheric blocs specification.

Definition at line 73 of file `inprod_analytic.f90`.

```
73  TYPE(atm_wavenum), DIMENSION(:), ALLOCATABLE, PUBLIC :: awavenum
```

#### 6.4.3.3 `type(ocean_tensors)`, `public inprod_analytic::ocean`

Oceanic tensors.

Definition at line 80 of file `inprod_analytic.f90`.

```
80  TYPE(ocean_tensors), PUBLIC :: ocean
```

#### 6.4.3.4 `type(ocean_wavenum)`, `dimension(:)`, `allocatable`, `public inprod_analytic::owavenum`

Oceanic blocs specification.

Definition at line 75 of file `inprod_analytic.f90`.

```
75  TYPE(ocean_wavenum), DIMENSION(:), ALLOCATABLE, PUBLIC :: owavenum
```

## 6.5 integrator Module Reference

Module with the integration routines.

### Functions/Subroutines

- subroutine, `public` [init\\_integrator](#)  
*Routine to initialise the integration buffers.*
- subroutine [tendencies](#) (`t`, `y`, `res`)  
*Routine computing the tendencies of the model.*
- subroutine, `public` [step](#) (`y`, `t`, `dt`, `res`)  
*Routine to perform an integration step (Heun algorithm). The incremented time is returned.*

### Variables

- `real(kind=8)`, `dimension(:)`, `allocatable` [buf\\_y1](#)  
*Buffer to hold the intermediate position (Heun algorithm)*
- `real(kind=8)`, `dimension(:)`, `allocatable` [buf\\_f0](#)  
*Buffer to hold tendencies at the initial position.*
- `real(kind=8)`, `dimension(:)`, `allocatable` [buf\\_f1](#)  
*Buffer to hold tendencies at the intermediate position.*
- `real(kind=8)`, `dimension(:)`, `allocatable` [buf\\_ka](#)  
*Buffer A to hold tendencies.*
- `real(kind=8)`, `dimension(:)`, `allocatable` [buf\\_kb](#)  
*Buffer B to hold tendencies.*



### 6.5.1 Detailed Description

Module with the integration routines.

Module with the RK4 integration routines.

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#### Remarks

This module actually contains the Heun algorithm routines. The user can modify it according to its preferred integration scheme. For higher-order schemes, additional buffers will probably have to be defined.

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#### Remarks

This module actually contains the RK4 algorithm routines. The user can modify it according to its preferred integration scheme. For higher-order schemes, additional buffers will probably have to be defined.

### 6.5.2 Function/Subroutine Documentation

#### 6.5.2.1 subroutine public integrator::init\_integrator ( )

Routine to initialise the integration buffers.

Definition at line 37 of file rk2\_integrator.f90.

```
37  INTEGER :: allocstat
38  ALLOCATE(buf_y1(0:ndim),buf_f0(0:ndim),buf_f1(0:ndim) ,stat=allocstat)
39  IF (allocstat /= 0) stop "*** Not enough memory ! ***"
```

#### 6.5.2.2 subroutine public integrator::step ( real(kind=8), dimension(0:ndim), intent(in) y, real(kind=8), intent(inout) t, real(kind=8), intent(in) dt, real(kind=8), dimension(0:ndim), intent(out) res )

Routine to perform an integration step (Heun algorithm). The incremented time is returned.

Routine to perform an integration step (RK4 algorithm). The incremented time is returned.

#### Parameters

|            |                             |
|------------|-----------------------------|
| <i>y</i>   | Initial point.              |
| <i>t</i>   | Actual integration time     |
| <i>dt</i>  | Integration timestep.       |
| <i>res</i> | Final point after the step. |

Definition at line 61 of file rk2\_integrator.f90.

```

61     REAL(KIND=8), DIMENSION(0:ndim), INTENT(IN) :: y
62     REAL(KIND=8), INTENT(INOUT) :: t
63     REAL(KIND=8), INTENT(IN) :: dt
64     REAL(KIND=8), DIMENSION(0:ndim), INTENT(OUT) :: res
65
66     CALL tendencies(t,y,buf_f0)
67     buf_y1 = y+dt*buf_f0
68     CALL tendencies(t+dt,buf_y1,buf_f1)
69     res=y+0.5*(buf_f0+buf_f1)*dt
70     t=t+dt

```

**6.5.2.3** subroutine integrator::tendencies ( real(kind=8), intent(in) *t*, real(kind=8), dimension(0:ndim), intent(in) *y*, real(kind=8), dimension(0:ndim), intent(out) *res* ) [private]

Routine computing the tendencies of the model.

#### Parameters

|            |   |
|------------|---|
| <i>t</i>   | Time at which the tendencies have to be computed. Actually not needed for autonomous systems. |
| <i>y</i>   | Point at which the tendencies have to be computed.  |
| <i>res</i> | vector to store the result.   |

#### Remarks

Note that it is NOT safe to pass *y* as a result buffer, as this operation does multiple passes.

Definition at line 49 of file rk2\_integrator.f90.

```

49     REAL(KIND=8), INTENT(IN) :: t
50     REAL(KIND=8), DIMENSION(0:ndim), INTENT(IN) :: y
51     REAL(KIND=8), DIMENSION(0:ndim), INTENT(OUT) :: res
52     CALL sparse_mul3(aotensor, y, y, res)

```

### 6.5.3 Variable Documentation

**6.5.3.1** real(kind=8), dimension(:), allocatable integrator::buf\_f0 [private]

Buffer to hold tendencies at the initial position.

Definition at line 28 of file rk2\_integrator.f90.

```

28     REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: buf_f0 !< Buffer to hold tendencies at the initial position

```

**6.5.3.2** real(kind=8), dimension(:), allocatable integrator::buf\_f1 [private]

Buffer to hold tendencies at the intermediate position.

Definition at line 29 of file rk2\_integrator.f90.

```

29     REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: buf_f1 !< Buffer to hold tendencies at the intermediate
    position

```

**6.5.3.3** `real(kind=8), dimension(:), allocatable integrator::buf_ka` [private]

Buffer A to hold tendencies.

Definition at line 28 of file rk4\_integrator.f90.

```
28  REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: buf_ka !< Buffer A to hold tendencies
```

**6.5.3.4** `real(kind=8), dimension(:), allocatable integrator::buf_kb` [private]

Buffer B to hold tendencies.

Definition at line 29 of file rk4\_integrator.f90.

```
29  REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: buf_kb !< Buffer B to hold tendencies
```

**6.5.3.5** `real(kind=8), dimension(:), allocatable integrator::buf_y1` [private]

Buffer to hold the intermediate position (Heun algorithm)

Definition at line 27 of file rk2\_integrator.f90.

```
27  REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: buf_y1 !< Buffer to hold the intermediate position (Heun
    algorithm)
```

## 6.6 lyap\_stat Module Reference

Statistics accumulators for the Lyapunov exponents.

### Functions/Subroutines

- subroutine, public [lyap\\_init\\_stat](#)  
*Initialise the accumulators.*
- subroutine, public [lyap\\_acc](#) (x)  
*Accumulate one state.*
- `real(kind=8)` function, `dimension(0:ndim)`, public [lyap\\_mean](#) ()  
*Function returning the mean.*
- `real(kind=8)` function, `dimension(0:ndim)`, public [lyap\\_var](#) ()  
*Function returning the variance.*
- integer function, public [lyap\\_iter](#) ()  
*Function returning the number of data accumulated.*
- subroutine, public [lyap\\_reset](#)  
*Routine resetting the accumulators.*

## Variables

- integer `i` =0  
*Number of stats accumulated.*
- real(kind=8), dimension(:), allocatable `m`  
*Vector storing the inline mean.*
- real(kind=8), dimension(:), allocatable `mprev`  
*Previous mean vector.*
- real(kind=8), dimension(:), allocatable `v`  
*Vector storing the inline variance.*
- real(kind=8), dimension(:), allocatable `mtmp`

### 6.6.1 Detailed Description

Statistics accumulators for the Lyapunov exponents.

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### 6.6.2 Function/Subroutine Documentation

#### 6.6.2.1 subroutine, public lyap\_stat::lyap\_acc ( real(kind=8), dimension(0:ndim), intent(in) x )

Accumulate one state.

Definition at line 48 of file lyap\_stat.f90.

```

48      IMPLICIT NONE
49      REAL(KIND=8), DIMENSION(0:ndim), INTENT(IN) :: x
50      i=i+1
51      mprev=m+(x-m)/i
52      mtmp=mprev
53      mprev=m
54      m=mtmp
55      v=v+(x-mprev)*(x-m)

```

#### 6.6.2.2 subroutine, public lyap\_stat::lyap\_init\_stat ( )

Initialise the accumulators.

Definition at line 35 of file lyap\_stat.f90.

```

35      INTEGER :: allocstat
36
37      ALLOCATE(m(0:ndim),mprev(0:ndim),v(0:ndim),mtmp(0:ndim), stat=allocstat)
38      IF (allocstat /= 0) stop '*** Not enough memory ***'
39      m=0.d0
40      mprev=0.d0
41      v=0.d0
42      mtmp=0.d0
43

```

**6.6.2.3 integer function, public lyap\_stat::lyap\_iter ( )**

Function returning the number of data accumulated.

Definition at line 72 of file lyap\_stat.f90.

```
72      INTEGER :: lyap_iter
73      lyap_iter=i
```

**6.6.2.4 real(kind=8) function, dimension(0:ndim), public lyap\_stat::lyap\_mean ( )**

Function returning the mean.

Definition at line 60 of file lyap\_stat.f90.

```
60      REAL(KIND=8), DIMENSION(0:ndim) :: lyap_mean
61      lyap_mean=m
```

**6.6.2.5 subroutine, public lyap\_stat::lyap\_reset ( )**

Routine resetting the accumulators.

Definition at line 78 of file lyap\_stat.f90.

```
78      m=0.d0
79      mprev=0.d0
80      v=0.d0
81      i=0
```

**6.6.2.6 real(kind=8) function, dimension(0:ndim), public lyap\_stat::lyap\_var ( )**

Function returning the variance.

Definition at line 66 of file lyap\_stat.f90.

```
66      REAL(KIND=8), DIMENSION(0:ndim) :: lyap_var
67      lyap_var=v/(i-1)
```

**6.6.3 Variable Documentation****6.6.3.1 integer lyap\_stat::i =0 [private]**

Number of stats accumulated.

Definition at line 20 of file lyap\_stat.f90.

```
20      INTEGER :: i=0 !< Number of stats accumulated
```

### 6.6.3.2 `real(kind=8), dimension(:), allocatable lyap_stat::m` [private]

Vector storing the inline mean.

Definition at line 23 of file `lyap_stat.f90`.

```
23  REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: m      !< Vector storing the inline mean
```

### 6.6.3.3 `real(kind=8), dimension(:), allocatable lyap_stat::mprev` [private]

Previous mean vector.

Definition at line 24 of file `lyap_stat.f90`.

```
24  REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: mprev !< Previous mean vector
```

### 6.6.3.4 `real(kind=8), dimension(:), allocatable lyap_stat::mtmp` [private]

Definition at line 26 of file `lyap_stat.f90`.

```
26  REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: mtmp
```

### 6.6.3.5 `real(kind=8), dimension(:), allocatable lyap_stat::v` [private]

Vector storing the inline variance.

Definition at line 25 of file `lyap_stat.f90`.

```
25  REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: v      !< Vector storing the inline variance
```

## 6.7 `lyap_vectors` Module Reference

Module for computation of Lyapunov exponents and vectors.

### Functions/Subroutines

- subroutine, public [init\\_lyap](#)  
*Initialize Lyapunov computation (possibly also vectors in later version) and initializes also a random orthogonal matrix for the matrix ensemble.*
- subroutine, public [multiply\\_prop](#) (`prop_mul`)  
*Multiplies `prop_mul` from the left with the `prop` matrix defined in this module and saves the result to `prop_mul`.*
- subroutine, public [benettin\\_step](#)  
*Performs the benettin step in integration. Multiplies the aggregated propagators in `prop` with ensemble and performs QR decomposition (Gram-Schmidt orthogonalization gives `Q` and upper triangular matrix `R`). Computes also the Lyapunov exponents via the diagonal of `R`. WATCH OUT: `prop` is changed during the subroutine and restored to a unit matrix.*
- subroutine, public [get\\_lyap\\_state](#) (`prop_ret`, `ensemble_ret`)  
*Routine that returns the current global propagator and ensemble of lyapunov vectors.*

## Variables

- real(kind=8), dimension(:), allocatable, public [loclap](#)  
*Buffer containing the local Lyapunov exponent.*
- real(kind=8), dimension(:), allocatable, public [lyapunov](#)  
*Buffer containing the averaged Lyapunov exponent.*
- real(kind=8), dimension(:, :), allocatable, public [ensemble](#)  
*Buffer containing the QR decomposition of the ensemble.*
- real(kind=8), dimension(:, :), allocatable [prop](#)  
*Buffer holding the propagator matrix.*
- integer [lwork](#)
- real(kind=8), dimension(:), allocatable [work](#)  
*Temporary buffer for QR decomposition.*
- real(kind=8), dimension(:), allocatable [work2](#)  
*Temporary buffer for QR decomposition.*
- real(kind=8), dimension(:), allocatable [tau](#)  
*Temporary buffer for QR decomposition.*
- real(kind=8), dimension(:, :), allocatable [prop\\_buf](#)  
*Buffer holding the local propagator matrix.*

### 6.7.1 Detailed Description

Module for computation of Lyapunov exponents and vectors.

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#### Remarks

This module contains the necessary tools to perform the Benettin steps to compute the lyapunov exponents. (Ginelli for CLV will be added later)

References : Benettin, G., Galgani, L., Giorgilli, A., & Strelcyn, J. M. (1980). Lyapunov characteristic exponents for smooth dynamical systems; a method for computing all of them. Part 2: Numerical application. *Meccanica* , 15, 21-30.

### 6.7.2 Function/Subroutine Documentation

#### 6.7.2.1 subroutine, public lyap\_vectors::benettin\_step ( )

Performs the benettin step in integration. Multiplies the aggregated propagators in prop with ensemble and performs QR decomposition (Gram-Schmidt orthogonalization gives Q and upper triangular matrix R). Computes also the Lyapunov exponents via the diagonal of R. WATCH OUT: prop is changed during the subroutine and restored to a unit matrix.

Definition at line 99 of file lyap\_vectors.f90.

```

99      INTEGER :: info,k
100
101      ! Multiply the Propagator prop from the right side with the non transposed q matrix
102      ! from the qr decomposition which is stored in ensemble.
103      CALL dorm2r("r","n",ndim,ndim,ndim,ensemble,ndim,tau,prop,ndim,work2,info)
104      ! prop contains prop*ensemble but QR decomposed(tau is needed for that as
105      ! well !) => copy to ensemble
106      ensemble=prop
107
108      ! From here on ensemble contains the new information prop*ensemble
109      CALL dgeqrf(ndim,ndim,ensemble,ndim,tau,work,lwork, info) ! qr decomposition
110
111      DO k=1,ndim
112          loclyap(k)=log(abs(ensemble(k,k)))/rescaling_time
113      END DO
114
115      !
116      ! Add here save for
117      !
118
119      ! Initialise prop again with unit matrix
120      CALL init_one(prop)
121

```

**6.7.2.2** subroutine, public lyap\_vectors::get\_lyap\_state ( real(kind=8), dimension(ndim,ndim), intent(out) *prop\_ret*, real(kind=8), dimension(ndim,ndim), intent(out) *ensemble\_ret* )

Routine that returns the current global propagator and ensemble of lyapunov vectors.

Definition at line 127 of file lyap\_vectors.f90.

```

127      REAL(KIND=8), DIMENSION(ndim,ndim),INTENT(OUT) :: prop_ret,ensemble_ret
128      prop_ret=prop
129      ensemble_ret=ensemble

```

**6.7.2.3** subroutine, public lyap\_vectors::init\_lyap ( )

Initialize Lyapunov computation (possibly also vectors in later version) and initializes also a random orthogonal matrix for the matrix ensemble.

Definition at line 69 of file lyap\_vectors.f90.

```

69      INTEGER :: allocstat,ilaenv,info
70      lwork=ilaenv(1,"dgeqrf"," ",ndim,ndim,ndim,-1)
71      lwork=ndim*lwork
72      ALLOCATE(prop_buf(ndim,ndim),lyapunov(ndim),loclyap(ndim),ensemble(ndim,ndim),tau(ndim),prop(ndim,ndim)
73      , &
74      & work2(ndim),work(lwork),stat=allocstat)
75      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
76
77      lyapunov=0.0d0
78      loclyap=0.0d0
79      CALL init_one(prop)
80      CALL random_number(ensemble)
81      ensemble=2*(ensemble-0.5)
82      CALL dgeqrf(ndim,ndim,ensemble,ndim,tau,work,lwork, info) ! qr decomposition

```

**6.7.2.4** subroutine, public lyap\_vectors::multiply\_prop ( real(kind=8), dimension(ndim,ndim), intent(in) *prop\_mul* )

Multiplies prop\_mul from the left with the prop matrix defined in this module and saves the result to prop\_mul.



## Parameters

|                 |  |
|-----------------|--|
| <i>prop_mul</i> | local propagator to multiply with the global one |
|-----------------|--|

Definition at line 88 of file lyap\_vectors.f90.

```
88     REAL(KIND=8), DIMENSION(ndim,ndim), INTENT(IN) :: prop_mul
89     prop_buf=prop
90     CALL dgemm('n','n', ndim, ndim, ndim, 1.0d0, prop_mul, ndim, prop_buf, ndim, 0.0d0, prop, ndim)
```

### 6.7.3 Variable Documentation

#### 6.7.3.1 real(kind=8), dimension(:, :), allocatable, public lyap\_vectors::ensemble

Buffer containing the QR decompoition of the ensemble.

Definition at line 42 of file lyap\_vectors.f90.

```
42     REAL(KIND=8), DIMENSION(:, :), ALLOCATABLE :: ensemble !< Buffer containing the QR decompoition of the
      ensemble
```

#### 6.7.3.2 real(kind=8), dimension(:), allocatable, public lyap\_vectors::loclyap

Buffer containing the local Lyapunov exponent.

Definition at line 40 of file lyap\_vectors.f90.

```
40     REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: loclyap !< Buffer containing the local Lyapunov exponent
```

#### 6.7.3.3 integer lyap\_vectors::lwork [private]

Definition at line 45 of file lyap\_vectors.f90.

```
45     INTEGER :: lwork
```

#### 6.7.3.4 real(kind=8), dimension(:), allocatable, public lyap\_vectors::lyapunov

Buffer containing the averaged Lyapunov exponent.

Definition at line 41 of file lyap\_vectors.f90.

```
41     REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: lyapunov !< Buffer containing the averaged Lyapunov exponent
```

#### 6.7.3.5 `real(kind=8), dimension(:,:), allocatable lyap_vectors::prop` [private]

Buffer holding the propagator matrix.

Definition at line 43 of file lyap\_vectors.f90.

```
43  REAL(KIND=8), DIMENSION(:,:), ALLOCATABLE :: prop      !< Buffer holding the propagator matrix
```

#### 6.7.3.6 `real(kind=8), dimension(:,:), allocatable lyap_vectors::prop_buf` [private]

Buffer holding the local propagator matrix.

Definition at line 49 of file lyap\_vectors.f90.

```
49  REAL(KIND=8), DIMENSION(:,:), ALLOCATABLE :: prop_buf !< Buffer holding the local propagator matrix
```

#### 6.7.3.7 `real(kind=8), dimension(:), allocatable lyap_vectors::tau` [private]

Temporary buffer for QR decomposition.

Definition at line 48 of file lyap\_vectors.f90.

```
48  REAL(kind=8), DIMENSION(:), ALLOCATABLE :: tau        !< Temporary buffer for QR decomposition
```

#### 6.7.3.8 `real(kind=8), dimension(:), allocatable lyap_vectors::work` [private]

Temporary buffer for QR decomposition.

Definition at line 46 of file lyap\_vectors.f90.

```
46  REAL(kind=8), DIMENSION(:), ALLOCATABLE :: work      !< Temporary buffer for QR decomposition
```

#### 6.7.3.9 `real(kind=8), dimension(:), allocatable lyap_vectors::work2` [private]

Temporary buffer for QR decomposition.

Definition at line 47 of file lyap\_vectors.f90.

```
47  REAL(kind=8), DIMENSION(:), ALLOCATABLE :: work2     !< Temporary buffer for QR decomposition
```

## 6.8 params Module Reference

The model parameters module.

## Functions/Subroutines

- subroutine, private [init\\_nml](#)  
*Read the basic parameters and mode selection from the namelist.*
- subroutine [init\\_params](#)  
*Parameters initialisation routine.*

## Variables

- real(kind=8) [n](#)  
 $n = 2L_y / L_x$  - Aspect ratio
- real(kind=8) [phi0](#)  
*Latitude in radian.*
- real(kind=8) [rra](#)  
*Earth radius.*
- real(kind=8) [sig0](#)  
 $\sigma_0$  - Non-dimensional static stability of the atmosphere.
- real(kind=8) [k](#)  
*Bottom atmospheric friction coefficient.*
- real(kind=8) [kp](#)  
 $k'$  - Internal atmospheric friction coefficient.
- real(kind=8) [r](#)  
*Frictional coefficient at the bottom of the ocean.*
- real(kind=8) [d](#)  
*Mechanical coupling parameter between the ocean and the atmosphere.*
- real(kind=8) [f0](#)  
 $f_0$  - Coriolis parameter
- real(kind=8) [gp](#)  
 $g'$  Reduced gravity
- real(kind=8) [h](#)  
*Depth of the active water layer of the ocean.*
- real(kind=8) [phi0\\_npi](#)  
*Latitude exprimed in fraction of pi.*
- real(kind=8) [lambda](#)  
 $\lambda$  - Sensible + turbulent heat exchange between the ocean and the atmosphere.
- real(kind=8) [co](#)  
 $C_a$  - Constant short-wave radiation of the ocean.
- real(kind=8) [go](#)  
 $\gamma_o$  - Specific heat capacity of the ocean.
- real(kind=8) [ca](#)  
 $C_a$  - Constant short-wave radiation of the atmosphere.
- real(kind=8) [to0](#)  
 $T_o^0$  - Stationary solution for the 0-th order ocean temperature.
- real(kind=8) [ta0](#)  
 $T_a^0$  - Stationary solution for the 0-th order atmospheric temperature.
- real(kind=8) [epsa](#)  
 $\epsilon_a$  - Emissivity coefficient for the grey-body atmosphere.
- real(kind=8) [ga](#)  
 $\gamma_a$  - Specific heat capacity of the atmosphere.
- real(kind=8) [rr](#)

- $R$  - Gas constant of dry air
  - real(kind=8) [scale](#)
- $L_y = L \pi$  - The characteristic space scale.
  - real(kind=8) [pi](#)
- $\pi$ 
  - real(kind=8) [lr](#)
- $L_R$  - Rossby deformation radius
  - real(kind=8) [g](#)
- $\gamma$ 
  - real(kind=8) [rp](#)
- $r'$  - Frictional coefficient at the bottom of the ocean.
  - real(kind=8) [dp](#)
- $d'$  - Non-dimensional mechanical coupling parameter between the ocean and the atmosphere.
  - real(kind=8) [kd](#)
- $k_d$  - Non-dimensional bottom atmospheric friction coefficient.
  - real(kind=8) [kdp](#)
- $k'_d$  - Non-dimensional internal atmospheric friction coefficient.
  - real(kind=8) [cpo](#)
- $C'_a$  - Non-dimensional constant short-wave radiation of the ocean.
  - real(kind=8) [lpo](#)
- $\lambda'_o$  - Non-dimensional sensible + turbulent heat exchange from ocean to atmosphere.
  - real(kind=8) [cpa](#)
- $C'_a$  - Non-dimensional constant short-wave radiation of the atmosphere.
  - real(kind=8) [lpa](#)
- $\lambda'_a$  - Non-dimensional sensible + turbulent heat exchange from atmosphere to ocean.
  - real(kind=8) [sbpo](#)
- $\sigma'_{B,o}$  - Long wave radiation lost by ocean to atmosphere & space.
  - real(kind=8) [sbpa](#)
- $\sigma'_{B,a}$  - Long wave radiation from atmosphere absorbed by ocean.
  - real(kind=8) [lsbpo](#)
- $S'_{B,o}$  - Long wave radiation from ocean absorbed by atmosphere.
  - real(kind=8) [lsbpa](#)
- $S'_{B,a}$  - Long wave radiation lost by atmosphere to space & ocean.
  - real(kind=8) [l](#)
- $L$  - Domain length scale
  - real(kind=8) [sc](#)
- Ratio of surface to atmosphere temperature.
  - real(kind=8) [sb](#)
- Stefan–Boltzmann constant.
  - real(kind=8) [betp](#)
- $\beta'$  - Non-dimensional beta parameter
  - real(kind=8) [nua](#) =0.D0
- Dissipation in the atmosphere.
  - real(kind=8) [nuo](#) =0.D0
- Dissipation in the ocean.
  - real(kind=8) [nuap](#)
- Non-dimensional dissipation in the atmosphere.
  - real(kind=8) [nuop](#)
- Non-dimensional dissipation in the ocean.
  - real(kind=8) [t\\_trans](#)
- Transient time period.

- real(kind=8) `t_run`  
*Effective intergration time (length of the generated trajectory)*
- real(kind=8) `dt`  
*Integration time step.*
- real(kind=8) `tw`  
*Write all variables every tw time units.*
- logical `writeout`  
*Write to file boolean.*
- real(kind=8) `rescaling_time`  
*Rescaling time for the Lyapunov computation.*
- integer `nboc`  
*Number of atmospheric blocks.*
- integer `nbatm`  
*Number of oceanic blocks.*
- integer `natm` =0  
*Number of atmospheric basis functions.*
- integer `noc` =0  
*Number of oceanic basis functions.*
- integer `ndim`  
*Number of variables (dimension of the model)*
- integer, dimension(:, :), allocatable `oms`  
*Ocean mode selection array.*
- integer, dimension(:, :), allocatable `ams`  
*Atmospheric mode selection array.*

### 6.8.1 Detailed Description

The model parameters module.

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#### Remarks

Once the `init_params()` subroutine is called, the parameters are loaded globally in the main program and its subroutines and function

### 6.8.2 Function/Subroutine Documentation

#### 6.8.2.1 subroutine, private `params::init_nml ( )` [`private`]

Read the basic parameters and mode selection from the namelist.

Definition at line 99 of file `params.f90`.

```

99      INTEGER :: allocstat
100
101      namelist /aoscale/  scale,f0,n,rra,phi0_npi
102      namelist /oparams/  gp,r,h,d,nuo
103      namelist /aparams/  k,kp,sig0,nua
104      namelist /toparams/ go,co,to0
105      namelist /taparams/ ga,ca,epsa,ta0
106      namelist /otparams/ sc,lambdarr,rr,sb
107
108      namelist /modeselection/ oms,ams
109      namelist /numblocs/  nboc,nbatm
110
111      namelist /int_params/ t_trans,t_run,dt,tw,writeout
112      namelist /lyap_params/ rescaling_time
113
114      OPEN(8, file="params.nml", status='OLD', recl=80, delim='APOSTROPHE')
115
116      READ(8,nml=aoscale)
117      READ(8,nml=oparams)
118      READ(8,nml=aparams)
119      READ(8,nml=toparams)
120      READ(8,nml=taparams)
121      READ(8,nml=otparams)
122
123      CLOSE(8)
124
125      OPEN(8, file="modeselection.nml", status='OLD', recl=80, delim='APOSTROPHE')
126      READ(8,nml=numblocs)
127
128      ALLOCATE(oms(nboc,2),ams(nbatm,2), stat=allocstat)
129      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
130
131      READ(8,nml=modeselection)
132      CLOSE(8)
133
134      OPEN(8, file="int_params.nml", status='OLD', recl=80, delim='APOSTROPHE')
135      READ(8,nml=int_params)
136      READ(8,nml=lyap_params)
137

```

### 6.8.2.2 subroutine params::init\_params ( )

Parameters initialisation routine.

Definition at line 142 of file params.f90.

```

142      INTEGER, DIMENSION(2) :: s
143      INTEGER :: i
144      CALL init_nml
145
146      !-----!
147      !
148      ! Computation of the dimension of the atmospheric
149      ! and oceanic components
150      !
151      !-----!
152
153      natm=0
154      DO i=1,nbatm
155          IF (ams(i,1)==1) THEN
156              natm=natm+3
157          ELSE
158              natm=natm+2
159          ENDIF
160      ENDDO
161      s=shape(oms)
162      noc=s(1)
163
164      ndim=2*natm+2*noc
165
166      !-----!
167      !
168      ! Some general parameters (Domain, beta, gamma, coupling)
169      !
170      !-----!
171
172      pi=dacos(-1.d0)
173      l=scale/pi
174      phi0=phi0_npi*pi

```

```

175     lr=sqrt(gp*h)/f0
176     g=-1**2/lr**2
177     betp=1/rra*cos(phi0)/sin(phi0)
178     rp=r/f0
179     dp=d/f0
180     kd=k*2
181     kdp=kp
182
183     !-----!
184     !                                     !
185     ! DERIVED QUANTITIES                 !
186     !                                     !
187     !-----!
188
189     cpo=co/(go*f0) * rr/(f0**2*1**2)
190     lpo=lambda/(go*f0)
191     cpa=ca/(ga*f0) * rr/(f0**2*1**2)/2 ! Cpa acts on psil-psi3, not on theta
192     lpa=lambda/(ga*f0)
193     sbpo=4*sb*to0**3/(go*f0) ! long wave radiation lost by ocean to atmosphere space
194     sbpa=8*epsa*sb*ta0**3/(go*f0) ! long wave radiation from atmosphere absorbed by ocean
195     lsbo=2*epsa*sb*to0**3/(ga*f0) ! long wave radiation from ocean absorbed by atmosphere
196     lsba=8*epsa*sb*ta0**3/(ga*f0) ! long wave radiation lost by atmosphere to space & ocea
197     nuap=nua/(f0*1**2)
198     nuop=nuo/(f0*1**2)
199

```

### 6.8.3 Variable Documentation

#### 6.8.3.1 integer, dimension(:, :), allocatable params::ams

Atmospheric mode selection array.

Definition at line 89 of file params.f90.

```

89     INTEGER, DIMENSION(:, :), ALLOCATABLE :: ams    !< Atmospheric mode selection array

```

#### 6.8.3.2 real(kind=8) params::betp

$\beta'$  - Non-dimensional beta parameter

Definition at line 67 of file params.f90.

```

67     REAL(KIND=8) :: betp    !< \f$\beta'$\f$ - Non-dimensional beta parameter

```

#### 6.8.3.3 real(kind=8) params::ca

$C_a$  - Constant short-wave radiation of the atmosphere.

Definition at line 40 of file params.f90.

```

40     REAL(KIND=8) :: ca    !< \f$C_a\f$ - Constant short-wave radiation of the atmosphere.

```

#### 6.8.3.4 `real(kind=8) params::co`

$C_a$  - Constant short-wave radiation of the ocean.

Definition at line 38 of file params.f90.

```
38  REAL(KIND=8) :: co      !< \f$C_a\f$ - Constant short-wave radiation of the ocean.
```

#### 6.8.3.5 `real(kind=8) params::cpa`

$C'_a$  - Non-dimensional constant short-wave radiation of the atmosphere.

##### Remarks

Cpa acts on psi1-psi3, not on theta.

Definition at line 58 of file params.f90.

```
58  REAL(KIND=8) :: cpa      !< \f$C'_a\f$ - Non-dimensional constant short-wave radiation of the
    atmosphere. @remark Cpa acts on psi1-psi3, not on theta.
```

#### 6.8.3.6 `real(kind=8) params::cpo`

$C'_a$  - Non-dimensional constant short-wave radiation of the ocean.

Definition at line 56 of file params.f90.

```
56  REAL(KIND=8) :: cpo      !< \f$C'_a\f$ - Non-dimensional constant short-wave radiation of the ocean.
```

#### 6.8.3.7 `real(kind=8) params::d`

Mechanical coupling parameter between the ocean and the atmosphere.

Definition at line 31 of file params.f90.

```
31  REAL(KIND=8) :: d      !< Mechanical coupling parameter between the ocean and the atmosphere.
```

#### 6.8.3.8 `real(kind=8) params::dp`

$d'$  - Non-dimensional mechanical coupling parameter between the ocean and the atmosphere.

Definition at line 52 of file params.f90.

```
52  REAL(KIND=8) :: dp      !< \f$d'\f$ - Non-dimensional mechanical coupling parameter between the ocean
    and the atmosphere.
```



**6.8.3.9 real(kind=8) params::dt**

Integration time step.

Definition at line 77 of file params.f90.

```
77  REAL(KIND=8) :: dt           !< Integration time step
```

**6.8.3.10 real(kind=8) params::epsa**

$\epsilon_a$  - Emissivity coefficient for the grey-body atmosphere.

Definition at line 43 of file params.f90.

```
43  REAL(KIND=8) :: epsa       !< \f$\epsilon_a\f$ - Emissivity coefficient for the grey-body atmosphere.
```

**6.8.3.11 real(kind=8) params::f0**

$f_0$  - Coriolis parameter

Definition at line 32 of file params.f90.

```
32  REAL(KIND=8) :: f0         !< \f$f_0\f$ - Coriolis parameter
```

**6.8.3.12 real(kind=8) params::g**

$\gamma$

Definition at line 50 of file params.f90.

```
50  REAL(KIND=8) :: g         !< \f$\gamma\f$
```

**6.8.3.13 real(kind=8) params::ga**

$\gamma_a$  - Specific heat capacity of the atmosphere.

Definition at line 44 of file params.f90.

```
44  REAL(KIND=8) :: ga       !< \f$\gamma_a\f$ - Specific heat capacity of the atmosphere.
```

**6.8.3.14 real(kind=8) params::go**

$\gamma_o$  - Specific heat capacity of the ocean.

Definition at line 39 of file params.f90.

```
39  REAL(KIND=8) :: go          !< \f$\gamma_o\f$ - Specific heat capacity of the ocean.
```

**6.8.3.15 real(kind=8) params::gp**

$g'$  Reduced gravity

Definition at line 33 of file params.f90.

```
33  REAL(KIND=8) :: gp          !< \f$g'\f$Reduced gravity
```

**6.8.3.16 real(kind=8) params::h**

Depth of the active water layer of the ocean.

Definition at line 34 of file params.f90.

```
34  REAL(KIND=8) :: h          !< Depth of the active water layer of the ocean.
```

**6.8.3.17 real(kind=8) params::k**

Bottom atmospheric friction coefficient.

Definition at line 28 of file params.f90.

```
28  REAL(KIND=8) :: k          !< Bottom atmospheric friction coefficient.
```

**6.8.3.18 real(kind=8) params::kd**

$k_d$  - Non-dimensional bottom atmospheric friction coefficient.

Definition at line 53 of file params.f90.

```
53  REAL(KIND=8) :: kd          !< \f$k_d\f$ - Non-dimensional bottom atmospheric friction coefficient.
```

**6.8.3.19 real(kind=8) params::kdp**

$k'_d$  - Non-dimensional internal atmospheric friction coefficient.

Definition at line 54 of file params.f90.

```
54  REAL(KIND=8) :: kdp          !< \f$k'_d\f$ - Non-dimensional internal atmospheric friction coefficient.
```

**6.8.3.20 real(kind=8) params::kp**

$k'$  - Internal atmospheric friction coefficient.

Definition at line 29 of file params.f90.

```
29  REAL(KIND=8) :: kp          !< \f$k'\f$ - Internal atmospheric friction coefficient.
```

**6.8.3.21 real(kind=8) params::l**

$L$  - Domain length scale

Definition at line 64 of file params.f90.

```
64  REAL(KIND=8) :: l          !< \f$L\f$ - Domain length scale
```

**6.8.3.22 real(kind=8) params::lambda**

$\lambda$  - Sensible + turbulent heat exchange between the ocean and the atmosphere.

Definition at line 37 of file params.f90.

```
37  REAL(KIND=8) :: lambda      !< \f$\lambda\f$ - Sensible + turbulent heat exchange between the ocean and the
    atmosphere.
```

**6.8.3.23 real(kind=8) params::lpa**

$\lambda'_a$  - Non-dimensional sensible + turbulent heat exchange from atmosphere to ocean.

Definition at line 59 of file params.f90.

```
59  REAL(KIND=8) :: lpa        !< \f$\lambda'_a\f$ - Non-dimensional sensible + turbulent heat exchange from
    atmosphere to ocean.
```

### 6.8.3.24 real(kind=8) params::lpo

$\lambda'_o$  - Non-dimensional sensible + turbulent heat exchange from ocean to atmosphere.

Definition at line 57 of file params.f90.

```
57  REAL(KIND=8) :: lpo      !< \f$\lambda'_o\f$ - Non-dimensional sensible + turbulent heat exchange from
    ocean to atmosphere.
```

### 6.8.3.25 real(kind=8) params::lr

$L_R$  - Rossby deformation radius

Definition at line 49 of file params.f90.

```
49  REAL(KIND=8) :: lr      !< \f$L_R\f$ - Rossby deformation radius
```

### 6.8.3.26 real(kind=8) params::lsbpa

$S'_{B,a}$  - Long wave radiation lost by atmosphere to space & ocean.

Definition at line 63 of file params.f90.

```
63  REAL(KIND=8) :: lsbpa   !< \f$S'_{B,a}\f$ - Long wave radiation lost by atmosphere to space & ocean.
```

### 6.8.3.27 real(kind=8) params::lsbpo

$S'_{B,o}$  - Long wave radiation from ocean absorbed by atmosphere.

Definition at line 62 of file params.f90.

```
62  REAL(KIND=8) :: lsbpo   !< \f$S'_{B,o}\f$ - Long wave radiation from ocean absorbed by atmosphere.
```

### 6.8.3.28 real(kind=8) params::n

$n = 2L_y/L_x$  - Aspect ratio

Definition at line 24 of file params.f90.

```
24  REAL(KIND=8) :: n      !< \f$n = 2 L_y / L_x\f$ - Aspect ratio
```

**6.8.3.29 integer params::natm =0**

Number of atmospheric basis functions.

Definition at line 85 of file params.f90.

```
85  INTEGER :: natm=0 !< Number of atmospheric basis functions
```

**6.8.3.30 integer params::nbatm**

Number of oceanic blocks.

Definition at line 84 of file params.f90.

```
84  INTEGER :: nbatm !< Number of oceanic blocks
```

**6.8.3.31 integer params::nboc**

Number of atmospheric blocks.

Definition at line 83 of file params.f90.

```
83  INTEGER :: nboc !< Number of atmospheric blocks
```

**6.8.3.32 integer params::ndim**

Number of variables (dimension of the model)

Definition at line 87 of file params.f90.

```
87  INTEGER :: ndim !< Number of variables (dimension of the model)
```

**6.8.3.33 integer params::noc =0**

Number of oceanic basis functions.

Definition at line 86 of file params.f90.

```
86  INTEGER :: noc=0 !< Number of oceanic basis functions
```

#### 6.8.3.34 `real(kind=8) params::nua =0.D0`

Dissipation in the atmosphere.

Definition at line 69 of file `params.f90`.

```
69  REAL(KIND=8) :: nua=0.d0  !< Dissipation in the atmosphere
```

#### 6.8.3.35 `real(kind=8) params::nuap`

Non-dimensional dissipation in the atmosphere.

Definition at line 72 of file `params.f90`.

```
72  REAL(KIND=8) :: nuap      !< Non-dimensional dissipation in the atmosphere
```

#### 6.8.3.36 `real(kind=8) params::nuo =0.D0`

Dissipation in the ocean.

Definition at line 70 of file `params.f90`.

```
70  REAL(KIND=8) :: nuo=0.d0  !< Dissipation in the ocean
```

#### 6.8.3.37 `real(kind=8) params::nuop`

Non-dimensional dissipation in the ocean.

Definition at line 73 of file `params.f90`.

```
73  REAL(KIND=8) :: nuop      !< Non-dimensional dissipation in the ocean
```

#### 6.8.3.38 `integer, dimension(:,,:), allocatable params::oms`

Ocean mode selection array.

Definition at line 88 of file `params.f90`.

```
88  INTEGER, DIMENSION(:,,:), ALLOCATABLE :: oms  !< Ocean mode selection array
```

**6.8.3.39 real(kind=8) params::phi0**

Latitude in radian.

Definition at line 25 of file params.f90.

```
25  REAL(KIND=8) :: phi0      !< Latitude in radian
```

**6.8.3.40 real(kind=8) params::phi0\_npi**

Latitude exprimed in fraction of pi.

Definition at line 35 of file params.f90.

```
35  REAL(KIND=8) :: phi0_npi !< Latitude exprimed in fraction of pi.
```

**6.8.3.41 real(kind=8) params::pi**

$\pi$

Definition at line 48 of file params.f90.

```
48  REAL(KIND=8) :: pi      !< \f$\pi\f$
```

**6.8.3.42 real(kind=8) params::r**

Frictional coefficient at the bottom of the ocean.

Definition at line 30 of file params.f90.

```
30  REAL(KIND=8) :: r      !< Frictional coefficient at the bottom of the ocean.
```

**6.8.3.43 real(kind=8) params::rescaling\_time**

Rescaling time for the Lyapunov computation.

Definition at line 81 of file params.f90.

```
81  REAL(KIND=8) :: rescaling_time !< Rescaling time for the Lyapunov computation
```

**6.8.3.44 real(kind=8) params::rp**

$r'$  - Frictional coefficient at the bottom of the ocean.

Definition at line 51 of file params.f90.

```
51  REAL(KIND=8) :: rp          !< \f$r'\f$ - Frictional coefficient at the bottom of the ocean.
```

**6.8.3.45 real(kind=8) params::rr**

$R$  - Gas constant of dry air

Definition at line 45 of file params.f90.

```
45  REAL(KIND=8) :: rr          !< \f$R\f$ - Gas constant of dry air
```

**6.8.3.46 real(kind=8) params::rra**

Earth radius.

Definition at line 26 of file params.f90.

```
26  REAL(KIND=8) :: rra          !< Earth radius
```

**6.8.3.47 real(kind=8) params::sb**

Stefan–Boltzmann constant.

Definition at line 66 of file params.f90.

```
66  REAL(KIND=8) :: sb          !< Stefan-Boltzmann constant
```

**6.8.3.48 real(kind=8) params::sbpa**

$\sigma'_{B,a}$  - Long wave radiation from atmosphere absorbed by ocean.

Definition at line 61 of file params.f90.

```
61  REAL(KIND=8) :: sbpa          !< \f$\sigma'_{B,a}\f$ - Long wave radiation from atmosphere absorbed by ocean.
```



**6.8.3.49 real(kind=8) params::sbpo**

$\sigma'_{B,o}$  - Long wave radiation lost by ocean to atmosphere & space.

Definition at line 60 of file params.f90.

```
60  REAL(KIND=8) :: sbpo      !< \f$\sigma'_{B,o}\f$ - Long wave radiation lost by ocean to atmosphere &
    space.
```

**6.8.3.50 real(kind=8) params::sc**

Ratio of surface to atmosphere temperature.

Definition at line 65 of file params.f90.

```
65  REAL(KIND=8) :: sc      !< Ratio of surface to atmosphere temperature.
```

**6.8.3.51 real(kind=8) params::scale**

$L_y = L \pi$  - The characteristic space scale.

Definition at line 47 of file params.f90.

```
47  REAL(KIND=8) :: scale   !< \f$L_y = L \, \pi\f$ - The characteristic space scale.
```

**6.8.3.52 real(kind=8) params::sig0**

$\sigma_0$  - Non-dimensional static stability of the atmosphere.

Definition at line 27 of file params.f90.

```
27  REAL(KIND=8) :: sig0    !< \f$\sigma_0\f$ - Non-dimensional static stability of the atmosphere.
```

**6.8.3.53 real(kind=8) params::t\_run**

Effective intergration time (length of the generated trajectory)

Definition at line 76 of file params.f90.

```
76  REAL(KIND=8) :: t_run   !< Effective intergration time (length of the generated trajectory)
```

**6.8.3.54 real(kind=8) params::t\_trans**

Transient time period.

Definition at line 75 of file params.f90.

```
75  REAL(KIND=8) :: t_trans    !< Transient time period
```

**6.8.3.55 real(kind=8) params::ta0**

$T_a^0$  - Stationary solution for the 0-th order atmospheric temperature.

Definition at line 42 of file params.f90.

```
42  REAL(KIND=8) :: ta0      !< \f$T_a^0\f$ - Stationary solution for the 0-th order atmospheric
    temperature.
```

**6.8.3.56 real(kind=8) params::to0**

$T_o^0$  - Stationary solution for the 0-th order ocean temperature.

Definition at line 41 of file params.f90.

```
41  REAL(KIND=8) :: to0      !< \f$T_o^0\f$ - Stationary solution for the 0-th order ocean temperature.
```

**6.8.3.57 real(kind=8) params::tw**

Write all variables every tw time units.

Definition at line 78 of file params.f90.

```
78  REAL(KIND=8) :: tw      !< Write all variables every tw time units
```

**6.8.3.58 logical params::writeout**

Write to file boolean.

Definition at line 79 of file params.f90.

```
79  LOGICAL :: writeout     !< Write to file boolean
```

**6.9 stat Module Reference**

Statistics accumulators.

## Functions/Subroutines

- subroutine, public [init\\_stat](#)  
*Initialise the accumulators.*
- subroutine, public [acc](#) (x)  
*Accumulate one state.*
- real(kind=8) function, dimension(0:ndim), public [mean](#) ()  
*Function returning the mean.*
- real(kind=8) function, dimension(0:ndim), public [var](#) ()  
*Function returning the variance.*
- integer function, public [iter](#) ()  
*Function returning the number of data accumulated.*
- subroutine, public [reset](#)  
*Routine resetting the accumulators.*

## Variables

- integer [i](#) =0  
*Number of stats accumulated.*
- real(kind=8), dimension(:), allocatable [m](#)  
*Vector storing the inline mean.*
- real(kind=8), dimension(:), allocatable [mprev](#)  
*Previous mean vector.*
- real(kind=8), dimension(:), allocatable [v](#)  
*Vector storing the inline variance.*
- real(kind=8), dimension(:), allocatable [mtmp](#)

### 6.9.1 Detailed Description

Statistics accumulators.

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### 6.9.2 Function/Subroutine Documentation

#### 6.9.2.1 subroutine, public stat::acc ( real(kind=8), dimension(0:ndim), intent(in) x )

Accumulate one state.

Definition at line 48 of file stat.f90.

```

48      IMPLICIT NONE
49      REAL(KIND=8), DIMENSION(0:ndim), INTENT(IN) :: x
50      i=i+1
51      mprev=m+(x-m)/i
52      mtmp=mprev
53      mprev=m
54      m=mtmp
55      v=v+(x-mprev)*(x-m)
```

### 6.9.2.2 subroutine, public stat::init\_stat ( )

Initialise the accumulators.

Definition at line 35 of file stat.f90.

```

35      INTEGER :: allocstat
36
37      ALLOCATE(m(0:ndim),mprev(0:ndim),v(0:ndim),mtmp(0:ndim), stat=allocstat)
38      IF (allocstat /= 0) stop '*** Not enough memory ***'
39      m=0.d0
40      mprev=0.d0
41      v=0.d0
42      mtmp=0.d0
43
```

### 6.9.2.3 integer function, public stat::iter ( )

Function returning the number of data accumulated.

Definition at line 72 of file stat.f90.

```

72      INTEGER :: iter
73      iter=i
```

### 6.9.2.4 real(kind=8) function, dimension(0:ndim), public stat::mean ( )

Function returning the mean.

Definition at line 60 of file stat.f90.

```

60      REAL(KIND=8), DIMENSION(0:ndim) :: mean
61      mean=m
```

### 6.9.2.5 subroutine, public stat::reset ( )

Routine resetting the accumulators.

Definition at line 78 of file stat.f90.

```

78      m=0.d0
79      mprev=0.d0
80      v=0.d0
81      i=0
```

### 6.9.2.6 real(kind=8) function, dimension(0:ndim), public stat::var ( )

Function returning the variance.

Definition at line 66 of file stat.f90.

```

66      REAL(KIND=8), DIMENSION(0:ndim) :: var
67      var=v/(i-1)
```

### 6.9.3 Variable Documentation

#### 6.9.3.1 integer stat::i =0 [private]

Number of stats accumulated.

Definition at line 20 of file stat.f90.

```
20  INTEGER :: i=0 !< Number of stats accumulated
```

#### 6.9.3.2 real(kind=8), dimension(:), allocatable stat::m [private]

Vector storing the inline mean.

Definition at line 23 of file stat.f90.

```
23  REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: m !< Vector storing the inline mean
```

#### 6.9.3.3 real(kind=8), dimension(:), allocatable stat::mprev [private]

Previous mean vector.

Definition at line 24 of file stat.f90.

```
24  REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: mprev !< Previous mean vector
```

#### 6.9.3.4 real(kind=8), dimension(:), allocatable stat::mtmp [private]

Definition at line 26 of file stat.f90.

```
26  REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: mtmp
```

#### 6.9.3.5 real(kind=8), dimension(:), allocatable stat::v [private]

Vector storing the inline variance.

Definition at line 25 of file stat.f90.

```
25  REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: v !< Vector storing the inline variance
```

## 6.10 tensor Module Reference

Tensor utility module.

## Data Types

- type `coolist`  
*Coordinate list. Type used to represent the sparse tensor.*
- type `coolist_elem`  
*Coordinate list element type. Elementary elements of the sparse tensors.*

## Functions/Subroutines

- subroutine, public `copy_coo` (src, dst)  
*Routine to copy a coolist.*
- subroutine, public `mat_to_coo` (src, dst)  
*Routine to convert a matrix to a tensor.*
- subroutine, public `sparse_mul3` (coolist\_ijk, arr\_j, arr\_k, res)

*Sparse multiplication of a tensor with two vectors:* 
$$\sum_{j,k=0}^{ndim} \mathcal{T}_{i,j,k} a_j b_k.$$

- subroutine, public `jsparse_mul` (coolist\_ijk, arr\_j, jcoo\_ij)  
*Sparse multiplication of two tensors to determine the Jacobian:*

$$J_{i,j} = \sum_{k=0}^{ndim} (\mathcal{T}_{i,j,k} + \mathcal{T}_{i,k,j}) a_k.$$

*It's implemented slightly differently: for every  $\mathcal{T}_{i,j,k}$ , we add to  $J_{i,j}$  as follows:*

$$J_{i,j} = J_{i,j} + \mathcal{T}_{i,j,k} a_k \quad J_{i,k} = J_{i,k} + \mathcal{T}_{i,j,k} a_j$$

*This version return a coolist (sparse tensor).*

- subroutine, public `jsparse_mul_mat` (coolist\_ijk, arr\_j, jcoo\_ij)  
*Sparse multiplication of two tensors to determine the Jacobian:*

$$J_{i,j} = \sum_{k=0}^{ndim} (\mathcal{T}_{i,j,k} + \mathcal{T}_{i,k,j}) a_k.$$

*It's implemented slightly differently: for every  $\mathcal{T}_{i,j,k}$ , we add to  $J_{i,j}$  as follows:*

$$J_{i,j} = J_{i,j} + \mathcal{T}_{i,j,k} a_k \quad J_{i,k} = J_{i,k} + \mathcal{T}_{i,j,k} a_j$$

*This version return a matrix.*

- subroutine, public `sparse_mul2` (coolist\_ij, arr\_j, res)

*Sparse multiplication of a 2d sparse tensor with a vector:* 
$$\sum_{j=0}^{ndim} \mathcal{T}_{i,j,k} a_j.$$

- subroutine, public `simplify` (tensor)  
*Routine to simplify a coolist (sparse tensor). For each index  $i$ , it upper triangularize the matrix*

$$\mathcal{T}_{i,j,k} \quad 0 \leq j, k \leq ndim.$$

- subroutine, public `add_elem` (t, i, j, k, v)  
*Subroutine to add element to a coolist.*
- subroutine, public `add_check` (t, i, j, k, v, dst)  
*Subroutine to add element to a coolist and check for overflow. Once the t buffer tensor is full, add it to the destination buffer.*
- subroutine, public `add_to_tensor` (src, dst)  
*Routine to add a rank-3 tensor to another one.*
- subroutine, public `print_tensor` (t, s)  
*Routine to print a rank 3 tensor coolist.*
- subroutine, public `write_tensor_to_file` (s, t)  
*Load a rank-4 tensor coolist from a file definition.*
- subroutine, public `load_tensor_from_file` (s, t)  
*Load a rank-4 tensor coolist from a file definition.*

## Variables

- `real(kind=8)`, parameter `real_eps = 2.2204460492503131e-16`  
*Parameter to test the equality with zero.*

### 6.10.1 Detailed Description

Tensor utility module.

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### 6.10.2 Function/Subroutine Documentation

**6.10.2.1** `subroutine, public tensor::add_check ( type(coolist), dimension(ndim), intent(inout) t, integer, intent(in) i, integer, intent(in) j, integer, intent(in) k, real(kind=8), intent(in) v, type(coolist), dimension(ndim), intent(inout) dst )`

Subroutine to add element to a coolist and check for overflow. Once the t buffer tensor is full, add it to the destination buffer.

#### Parameters

|            |  |
|------------|--|
| <i>t</i>   | temporary buffer tensor for the destination tensor |
| <i>i</i>   | tensor <i>i</i> index                              |
| <i>j</i>   | tensor <i>j</i> index                              |
| <i>k</i>   | tensor <i>k</i> index                              |
| <i>v</i>   | value to add                                       |
| <i>dst</i> | destination tensor                                 |

Definition at line 303 of file tensor.f90.

```

303     TYPE(coolist), DIMENSION(ndim), INTENT(INOUT) :: t
304     TYPE(coolist), DIMENSION(ndim), INTENT(INOUT) :: dst
305     INTEGER, INTENT(IN) :: i,j,k
306     REAL(KIND=8), INTENT(IN) :: v
307     INTEGER :: n
308     CALL add_elem(t,i,j,k,v)
309     IF (t(i)%nelems==size(t(i)%elems)) THEN
310         CALL add_to_tensor(t,dst)
311         DO n=1,ndim
312             t(n)%nelems=0
313         ENDDO
314     ENDIF

```

**6.10.2.2** `subroutine, public tensor::add_elem ( type(coolist), dimension(ndim), intent(inout) t, integer, intent(in) i, integer, intent(in) j, integer, intent(in) k, real(kind=8), intent(in) v )`

Subroutine to add element to a coolist.

## Parameters

|          |                       |
|----------|-----------------------|
| <i>t</i> | destination tensor    |
| <i>i</i> | tensor <i>i</i> index |
| <i>j</i> | tensor <i>j</i> index |
| <i>k</i> | tensor <i>k</i> index |
| <i>v</i> | value to add          |

Definition at line 281 of file tensor.f90.

```

281  TYPE(coolist), DIMENSION(ndim), INTENT(INOUT) :: t
282  INTEGER, INTENT(IN) :: i,j,k
283  REAL(KIND=8), INTENT(IN) :: v
284  INTEGER :: n
285  IF (abs(v) .ge. real_eps) THEN
286    n=(t(i)%nelems)+1
287    t(i)%elems(n)%j=j
288    t(i)%elems(n)%k=k
289    t(i)%elems(n)%v=v
290    t(i)%nelems=n
291  END IF

```

### 6.10.2.3 subroutine, public tensor::add\_to\_tensor ( type(coolist), dimension(ndim), intent(in) src, type(coolist), dimension(ndim), intent(inout) dst )

Routine to add a rank-3 tensor to another one.

## Parameters

|            |                    |
|------------|--------------------|
| <i>src</i> | Tensor to add      |
| <i>dst</i> | Destination tensor |

Definition at line 321 of file tensor.f90.

```

321  TYPE(coolist), DIMENSION(ndim), INTENT(IN) :: src
322  TYPE(coolist), DIMENSION(ndim), INTENT(INOUT) :: dst
323  TYPE(coolist_elem), DIMENSION(:), ALLOCATABLE :: celems
324  INTEGER :: i,j,n,allocstat
325
326  DO i=1,ndim
327    IF (src(i)%nelems/=0) THEN
328      IF (dst(i)%nelems==0) THEN
329        IF (ALLOCATED(dst(i)%elems)) THEN
330          DEALLOCATE(dst(i)%elems, stat=allocstat)
331          IF (allocstat /= 0) stop "*** Deallocation problem ! ***"
332        ENDIF
333        ALLOCATE(dst(i)%elems(src(i)%nelems), stat=allocstat)
334        IF (allocstat /= 0) stop "*** Not enough memory ! ***"
335        n=0
336      ELSE
337        n=dst(i)%nelems
338        ALLOCATE(celems(n), stat=allocstat)
339        DO j=1,n
340          celems(j)%j=dst(i)%elems(j)%j
341          celems(j)%k=dst(i)%elems(j)%k
342          celems(j)%v=dst(i)%elems(j)%v
343        ENDDO
344        DEALLOCATE(dst(i)%elems, stat=allocstat)
345        IF (allocstat /= 0) stop "*** Deallocation problem ! ***"
346        ALLOCATE(dst(i)%elems(src(i)%nelems+n), stat=allocstat)
347        IF (allocstat /= 0) stop "*** Not enough memory ! ***"
348        DO j=1,n
349          dst(i)%elems(j)%j=celems(j)%j
350          dst(i)%elems(j)%k=celems(j)%k
351          dst(i)%elems(j)%v=celems(j)%v

```



```

352         ENDDO
353         DEALLOCATE(celems, stat=allocstat)
354         IF (allocstat /= 0) stop "*** Deallocation problem ! ***"
355     ENDF
356     DO j=1,src(i)%nelems
357         dst(i)%elems(n+j)%j=src(i)%elems(j)%j
358         dst(i)%elems(n+j)%k=src(i)%elems(j)%k
359         dst(i)%elems(n+j)%v=src(i)%elems(j)%v
360     ENDDO
361     dst(i)%nelems=src(i)%nelems+n
362 ENDF
363 ENDDO
364

```

#### 6.10.2.4 subroutine, public tensor::copy\_coo ( type(coolist), dimension(ndim), intent(in) src, type(coolist), dimension(ndim), intent(out) dst )

Routine to copy a coolist.

##### Parameters

|            |                     |
|------------|---------------------|
| <i>src</i> | Source coolist      |
| <i>dst</i> | Destination coolist |

##### Remarks

The destination tensor have to be an empty tensor, i.e. with unallocated list of elements and nelems set to 0.

Definition at line 45 of file tensor.f90.

```

45     TYPE(coolist), DIMENSION(ndim), INTENT(IN) :: src
46     TYPE(coolist), DIMENSION(ndim), INTENT(OUT) :: dst
47     INTEGER :: i,j,allocstat
48
49     DO i=1,ndim
50         IF (dst(i)%nelems/=0) stop "*** copy_coo : Destination coolist not empty ! ***"
51         ALLOCATE(dst(i)%elems(src(i)%nelems), stat=allocstat)
52         IF (allocstat /= 0) stop "*** Not enough memory ! ***"
53         DO j=1,src(i)%nelems
54             dst(i)%elems(j)%j=src(i)%elems(j)%j
55             dst(i)%elems(j)%k=src(i)%elems(j)%k
56             dst(i)%elems(j)%v=src(i)%elems(j)%v
57         ENDDO
58         dst(i)%nelems=src(i)%nelems
59     ENDDO

```

#### 6.10.2.5 subroutine, public tensor::jsparse\_mul ( type(coolist), dimension(ndim), intent(in) coolist\_ijk, real(kind=8), dimension(0:ndim), intent(in) arr\_j, type(coolist), dimension(ndim), intent(out) jcoo\_ij )

Sparse multiplication of two tensors to determine the Jacobian:

$$J_{i,j} = \sum_{k=0}^{ndim} (\mathcal{T}_{i,j,k} + \mathcal{T}_{i,k,j}) a_k.$$

It's implemented slightly differently: for every  $\mathcal{T}_{i,j,k}$ , we add to  $J_{i,j}$  as follows:

$$J_{i,j} = J_{i,j} + \mathcal{T}_{i,j,k} a_k J_{i,k} = J_{i,k} + \mathcal{T}_{i,j,k} a_j$$

This version return a coolist (sparse tensor).

## Parameters

|                    |   |
|--------------------|---|
| <i>coolist_ijk</i> | a coordinate list (sparse tensor) of which index 2 or 3 will be contracted. |
| <i>arr_j</i>       | the vector to be contracted with index 2 and then index 3 of ffi_coo_ijk    |
| <i>jcoo_ij</i>     | a coolist (sparse tensor) to store the result of the contraction            |

Definition at line 124 of file tensor.f90.

```

124  TYPE(coolist), DIMENSION(ndim), INTENT(IN):: coolist_ijk
125  TYPE(coolist), DIMENSION(ndim), INTENT(OUT):: jcoo_ij
126  REAL(KIND=8), DIMENSION(0:ndim), INTENT(IN) :: arr_j
127  REAL(KIND=8) :: v
128  INTEGER :: i,j,k,n,nj,allocstat
129  DO i=1,ndim
130      IF (jcoo_ij(i)%nelems/=0) stop "*** jsparse_mul : Destination coolist not empty ! ***"
131      nj=2*coolist_ijk(i)%nelems
132      ALLOCATE(jcoo_ij(i)%elems(nj), stat=allocstat)
133      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
134      nj=0
135      DO n=1,coolist_ijk(i)%nelems
136          j=coolist_ijk(i)%elems(n)%j
137          k=coolist_ijk(i)%elems(n)%k
138          v=coolist_ijk(i)%elems(n)%v
139          IF (j /=0) THEN
140              nj=nj+1
141              jcoo_ij(i)%elems(nj)%j=j
142              jcoo_ij(i)%elems(nj)%k=0
143              jcoo_ij(i)%elems(nj)%v=v*arr_j(k)
144          END IF
145
146          IF (k /=0) THEN
147              nj=nj+1
148              jcoo_ij(i)%elems(nj)%j=k
149              jcoo_ij(i)%elems(nj)%k=0
150              jcoo_ij(i)%elems(nj)%v=v*arr_j(j)
151          END IF
152      END DO
153      jcoo_ij(i)%nelems=nj
154  END DO

```

**6.10.2.6** subroutine, public `tensor::jsparse_mul_mat ( type(coolist), dimension(ndim), intent(in) coolist_ijk, real(kind=8), dimension(0:ndim), intent(in) arr_j, real(kind=8), dimension(ndim,ndim), intent(out) jcoo_ij )`

Sparse multiplication of two tensors to determine the Jacobian:

$$J_{i,j} = \sum_{k=0}^{ndim} (\mathcal{T}_{i,j,k} + \mathcal{T}_{i,k,j}) a_k.$$

It's implemented slightly differently: for every  $\mathcal{T}_{i,j,k}$ , we add to  $J_{i,j}$  as follows:

$$J_{i,j} = J_{i,j} + \mathcal{T}_{i,j,k} a_k J_{i,k} = J_{i,k} + \mathcal{T}_{i,j,k} a_j$$

This version return a matrix.

## Parameters

|                    |   |
|--------------------|---|
| <i>coolist_ijk</i> | a coordinate list (sparse tensor) of which index 2 or 3 will be contracted. |
| <i>arr_j</i>       | the vector to be contracted with index 2 and then index 3 of ffi_coo_ijk    |
| <i>jcoo_ij</i>     | a matrix to store the result of the contraction                             |

Definition at line 167 of file tensor.f90.

```

167  TYPE(coolist), DIMENSION(ndim), INTENT(IN) :: coolist_ijk
168  REAL(KIND=8), DIMENSION(ndim,ndim), INTENT(OUT) :: jcoo_ij
169  REAL(KIND=8), DIMENSION(0:ndim), INTENT(IN) :: arr_j
170  REAL(KIND=8) :: v
171  INTEGER :: i,j,k,n
172  jcoo_ij=0.d0
173  DO i=1,ndim
174      DO n=1,coolist_ijk(i)%nelems
175          j=coolist_ijk(i)%elems(n)%j
176          k=coolist_ijk(i)%elems(n)%k
177          v=coolist_ijk(i)%elems(n)%v
178          IF (j /=0) jcoo_ij(i,j)=jcoo_ij(i,j)+v*arr_j(k)
179          IF (k /=0) jcoo_ij(i,k)=jcoo_ij(i,k)+v*arr_j(j)
180      END DO
181  END DO

```

**6.10.2.7** subroutine, public tensor::load\_tensor\_from\_file ( character (len=\*) , intent(in) s, type(coolist), dimension(ndim), intent(out) t )

Load a rank-4 tensor coolist from a file definition.

Parameters

|          |  |
|----------|--|
| <i>s</i> | Filename of the tensor definition file |
| <i>t</i> | The loaded coolist                     |

Remarks

The destination tensor have to be an empty tensor, i.e. with unallocated list of elements and nelems set to 0.

Definition at line 416 of file tensor.f90.

```

416  CHARACTER (LEN=*) , INTENT(IN) :: s
417  TYPE(coolist), DIMENSION(ndim), INTENT(OUT) :: t
418  INTEGER :: i,ir,j,k,n,allocstat
419  REAL(KIND=8) :: v
420  OPEN(30,file=s,status='old')
421  DO i=1,ndim
422      READ(30,*) ir,n
423      IF (n /= 0) THEN
424          ALLOCATE(t(i)%elems(n), stat=allocstat)
425          IF (allocstat /= 0) stop "*** Not enough memory ! ***"
426          t(i)%nelems=n
427      ENDIF
428      DO n=1,t(i)%nelems
429          READ(30,*) ir,j,k,v
430          t(i)%elems(n)%j=j
431          t(i)%elems(n)%k=k
432          t(i)%elems(n)%v=v
433      ENDDO
434  END DO
435  CLOSE(30)

```

**6.10.2.8** subroutine, public tensor::mat\_to\_coo ( real(kind=8), dimension(0:ndim,0:ndim), intent(in) src, type(coolist), dimension(ndim), intent(out) dst )

Routine to convert a matrix to a tensor.

## Parameters

|            |                    |
|------------|--------------------|
| <i>src</i> | Source matrix      |
| <i>dst</i> | Destination tensor |

## Remarks

The destination tensor have to be an empty tensor, i.e. with unallocated list of elements and nelems set to 0.

Definition at line 67 of file tensor.f90.

```

67      REAL(KIND=8), DIMENSION(0:ndim,0:ndim), INTENT(IN) :: src
68      TYPE(coolist), DIMENSION(ndim), INTENT(OUT) :: dst
69      INTEGER :: i,j,n,allocstat
70      DO i=1,ndim
71          n=0
72          DO j=1,ndim
73              IF (abs(src(i,j))>real_eps) n=n+1
74          ENDDO
75          IF (dst(i)%nelems/=0) stop "*** mat_to_coo : Destination coolist not empty ! ***"
76          ALLOCATE(dst(i)%elems(n), stat=allocstat)
77          IF (allocstat /= 0) stop "*** Not enough memory ! ***"
78          n=0
79          DO j=1,ndim
80              IF (abs(src(i,j))>real_eps) THEN
81                  n=n+1
82                  dst(i)%elems(n)%j=j
83                  dst(i)%elems(n)%k=0
84                  dst(i)%elems(n)%v=src(i,j)
85              ENDIF
86          ENDDO
87          dst(i)%nelems=n
88      ENDDO

```

#### 6.10.2.9 subroutine, public tensor::print\_tensor ( type(coolist), dimension(ndim), intent(in) t, character, intent(in), optional s )

Routine to print a rank 3 tensor coolist.

## Parameters

|          |                  |
|----------|------------------|
| <i>t</i> | coolist to print |
|----------|------------------|

Definition at line 370 of file tensor.f90.

```

370      USE util, only: str
371      TYPE(coolist), DIMENSION(ndim), INTENT(IN) :: t
372      CHARACTER, INTENT(IN), OPTIONAL :: s
373      CHARACTER :: r
374      INTEGER :: i,n,j,k
375      IF (PRESENT(s)) THEN
376          r=s
377      ELSE
378          r="t"
379      END IF
380      DO i=1,ndim
381          DO n=1,t(i)%nelems
382              j=t(i)%elems(n)%j
383              k=t(i)%elems(n)%k
384              IF ( abs(t(i)%elems(n)%v) .GE. real_eps) THEN
385                  write(*,"(A,ES12.5)") s//"["//trim(str(i))//"]["//trim(str(j)) &
386                      & //"["//trim(str(k))//"] = ",t(i)%elems(n)%v
387              END IF
388          END DO
389      END DO

```

6.10.2.10 subroutine, public `tensor::simplify ( type(coolist), dimension(ndim), intent(inout) tensor )`

Routine to simplify a coolist (sparse tensor). For each index  $i$ , it upper triangularize the matrix

$$\mathcal{T}_{i,j,k} \quad 0 \leq j, k \leq ndim.$$

## Parameters

|               |   |
|---------------|---|
| <i>tensor</i> | a coordinate list (sparse tensor) which will be simplified. |
|---------------|---|

Definition at line 209 of file tensor.f90.

```

209  TYPE(coolist), DIMENSION(ndim), INTENT(INOUT):: tensor
210  INTEGER :: i,j,k
211  INTEGER :: li,lil,lili,n
212  DO i= 1,ndim
213    n=tensor(i)%elems
214    DO li=n,2,-1
215      j=tensor(i)%elems(li)%j
216      k=tensor(i)%elems(li)%k
217      DO lil=li-1,1,-1
218        IF ((j==tensor(i)%elems(lil)%j).AND.(k==tensor(i)%
219          &%elems(lil)%k)).OR.((j==tensor(i)%elems(lil)%k).AND.(k==
tensor(i)%elems(lil)%j))) THEN
220          ! Found another entry with the same i,j,k: merge both into
221          ! the one listed first (of those two).
222          tensor(i)%elems(lil)%v=tensor(i)%elems(lil)%v+tensor(i)%elems(li)%v
223          IF (j>k) THEN
224            tensor(i)%elems(lil)%j=tensor(i)%elems(li)%k
225            tensor(i)%elems(lil)%k=tensor(i)%elems(li)%j
226          ENDIF
227
228          ! Shift the rest of the items one place down.
229          DO lili=li+1,n
230            tensor(i)%elems(lili-1)%j=tensor(i)%elems(lil)%j
231            tensor(i)%elems(lili-1)%k=tensor(i)%elems(lil)%k
232            tensor(i)%elems(lili-1)%v=tensor(i)%elems(lil)%v
233          END DO
234          tensor(i)%elems=tensor(i)%elems-1
235          ! Here we should stop because the li no longer points to the
236          ! original i,j,k element
237          EXIT
238        ENDIF
239      ENDDO
240    ENDDO
241    n=tensor(i)%elems
242    DO li=1,n
243      ! Clear new "almost" zero entries and shift rest of the items one place down.
244      ! Make sure not to skip any entries while shifting!
245      DO WHILE (abs(tensor(i)%elems(li)%v) < real_eps)
246        DO lili=li+1,n
247          tensor(i)%elems(lili-1)%j=tensor(i)%elems(li)%j
248          tensor(i)%elems(lili-1)%k=tensor(i)%elems(li)%k
249          tensor(i)%elems(lili-1)%v=tensor(i)%elems(li)%v
250        ENDDO
251        tensor(i)%elems=tensor(i)%elems-1
252        if (li > tensor(i)%elems) THEN
253          EXIT
254        ENDIF
255      ENDDO
256    ENDDO
257
258    n=tensor(i)%elems
259    DO li=1,n
260      ! Upper triangularize
261      j=tensor(i)%elems(li)%j
262      k=tensor(i)%elems(li)%k
263      IF (j>k) THEN
264        tensor(i)%elems(li)%j=k
265        tensor(i)%elems(li)%k=j
266      ENDIF
267    ENDDO
268
269  ENDDO
270
```

**6.10.2.11** subroutine, public `tensor::sparse_mul2 ( type(coolist), dimension(ndim), intent(in) coolist_ij, real(kind=8), dimension(0:ndim), intent(in) arr_j, real(kind=8), dimension(0:ndim), intent(out) res )`

Sparse multiplication of a 2d sparse tensor with a vector:  $\sum_{j=0}^{ndim} \mathcal{T}_{i,j,k} a_j$ .

#### Parameters

|                         |  |
|-------------------------|--|
| <code>coolist_ij</code> | a coordinate list (sparse tensor) of which index 2 will be contracted. |
| <code>arr_j</code>      | the vector to be contracted with index 2 of <code>coolist_ijk</code>   |
| <code>res</code>        | vector (buffer) to store the result of the contraction                 |

#### Remarks

Note that it is NOT safe to pass `arr_j` as a result buffer, as this operation does multiple passes.

Definition at line 192 of file `tensor.f90`.

```

192  TYPE(coolist), DIMENSION(ndim), INTENT(IN) :: coolist_ij
193  REAL(KIND=8), DIMENSION(0:ndim), INTENT(IN) :: arr_j
194  REAL(KIND=8), DIMENSION(0:ndim), INTENT(OUT) :: res
195  INTEGER :: i, j, n
196  res=0.d0
197  DO i=1,ndim
198      DO n=1,coolist_ij(i)%elems
199          j=coolist_ij(i)%elems(n)%j
200          res(i) = res(i) + coolist_ij(i)%elems(n)%v * arr_j(j)
201      END DO
202  END DO

```

**6.10.2.12** subroutine, public `tensor::sparse_mul3 ( type(coolist), dimension(ndim), intent(in) coolist_ijk, real(kind=8), dimension(0:ndim), intent(in) arr_j, real(kind=8), dimension(0:ndim), intent(in) arr_k, real(kind=8), dimension(0:ndim), intent(out) res )`

Sparse multiplication of a tensor with two vectors:  $\sum_{j,k=0}^{ndim} \mathcal{T}_{i,j,k} a_j b_k$ .

#### Parameters

|                          |  |
|--------------------------|--|
| <code>coolist_ijk</code> | a coordinate list (sparse tensor) of which index 2 and 3 will be contracted. |
| <code>arr_j</code>       | the vector to be contracted with index 2 of <code>coolist_ijk</code>         |
| <code>arr_k</code>       | the vector to be contracted with index 3 of <code>coolist_ijk</code>         |
| <code>res</code>         | vector (buffer) to store the result of the contraction                       |

#### Remarks

Note that it is NOT safe to pass `arr_j/arr_k` as a result buffer, as this operation does multiple passes.

Definition at line 100 of file `tensor.f90`.

```

100     TYPE(coolist), DIMENSION(ndim), INTENT(IN) :: coolist_ijk
101     REAL(KIND=8), DIMENSION(0:ndim), INTENT(IN)  :: arr_j, arr_k
102     REAL(KIND=8), DIMENSION(0:ndim), INTENT(OUT) :: res
103     INTEGER :: i,j,k,n
104     res=0.d0
105     DO i=1,ndim
106         DO n=1,coolist_ijk(i)%elems
107             j=coolist_ijk(i)%elems(n)%j
108             k=coolist_ijk(i)%elems(n)%k
109             res(i) = res(i) + coolist_ijk(i)%elems(n)%v * arr_j(j)*arr_k(k)
110         END DO
111     END DO

```

#### 6.10.2.13 subroutine, public tensor::write\_tensor\_to\_file ( character (len=\*) intent(in) s, type(coolist), dimension(ndim), intent(in) t )

Load a rank-4 tensor coolist from a file definition.

##### Parameters

|          |                      |
|----------|----------------------|
| <i>s</i> | Destination filename |
| <i>t</i> | The coolist to write |

Definition at line 396 of file tensor.f90.

```

396     CHARACTER (LEN=*), INTENT(IN) :: s
397     TYPE(coolist), DIMENSION(ndim), INTENT(IN) :: t
398     INTEGER :: i,j,k,n
399     OPEN(30,file=s)
400     DO i=1,ndim
401         WRITE(30,*) i,t(i)%elems
402         DO n=1,t(i)%elems
403             j=t(i)%elems(n)%j
404             k=t(i)%elems(n)%k
405             WRITE(30,*) i,j,k,t(i)%elems(n)%v
406         END DO
407     END DO
408     CLOSE(30)

```

## 6.10.3 Variable Documentation

### 6.10.3.1 real(kind=8), parameter tensor::real\_eps = 2.2204460492503131e-16

Parameter to test the equality with zero.

Definition at line 33 of file tensor.f90.

```

33     REAL(KIND=8), PARAMETER :: real_eps = 2.2204460492503131e-16

```

## 6.11 tl\_ad\_integrator Module Reference

Tangent Linear (TL) and Adjoint (AD) model versions of MAOOAM. Integrators module.

## Functions/Subroutines

- subroutine `tendencies` (t, y, res)  
*Routine computing the tendencies of the nonlinear model.*
- subroutine, public `init_tl_ad_integrator`  
*Routine to initialise the integration buffers.*
- subroutine, public `ad_step` (y, ystar, t, dt, res)  
*Routine to perform an integration step (Heun algorithm) of the adjoint model. The incremented time is returned.*
- subroutine, public `evolve_ad_step` (y, deltat, t, dt, ynew, deltatnew)  
*Routine to perform a simultaneous integration step (RK4 algorithm) of the nonlinear and adjoint together. The incremented time is returned.*
- subroutine, public `tl_step` (y, ystar, t, dt, res)  
*Routine to perform an integration step (Heun algorithm) of the tangent linear model. The incremented time is returned.*
- subroutine, public `evolve_tl_step` (y, deltat, t, dt, ynew, deltatnew)  
*Routine to perform a simultaneous integration step (RK4 algorithm) of the nonlinear and the tangent linear model together. The incremented time is returned.*
- subroutine, public `prop_step` (y, propagator, t, dt, ynew, adjoint)  
*Routine to perform a simultaneously an integration step (Heun algorithm) of the nonlinear and computes the Heun tangent linear propagator. The boolean variable adjoint allows for an adjoint forward integration. The incremented time is returned.*

## Variables

- real(kind=8), dimension(:), allocatable `buf_y1`  
*Buffer to hold the intermediate position (Heun algorithm) of the tangent linear model.*
- real(kind=8), dimension(:), allocatable `buf_f0`  
*Buffer to hold tendencies at the initial position of the tangent linear model.*
- real(kind=8), dimension(:), allocatable `buf_f1`  
*Buffer to hold tendencies at the intermediate position of the tangent linear model.*
- real(kind=8), dimension(:), allocatable `buf_y11`  
*Buffer to hold the intermediate position (Heun algorithm) of the tangent linear model.*
- real(kind=8), dimension(:), allocatable `buf_f00`  
*Buffer to hold tendencies at the initial position of the tangent linear model.*
- real(kind=8), dimension(:), allocatable `buf_f11`  
*Buffer to hold tendencies at the intermediate position of the tangent linear model.*
- real(kind=8), dimension(:, :), allocatable `buf_j1`  
*Buffer to hold jacobians in the RK4 scheme for the tangent linear model.*
- real(kind=8), dimension(:, :), allocatable `buf_j2`  
*Buffer to hold jacobians in the RK4 scheme for the tangent linear model.*
- real(kind=8), dimension(:, :), allocatable `buf_j1h`  
*Buffer to hold jacobians in the RK4 scheme for the tangent linear model.*
- real(kind=8), dimension(:, :), allocatable `buf_j2h`  
*Buffer to hold jacobians in the RK4 scheme for the tangent linear model.*
- real(kind=8), dimension(:, :), allocatable `one`  
*unit matrix*
- real(kind=8), dimension(:), allocatable `buf_ka`  
*Buffer to hold tendencies in the RK4 scheme for the tangent linear model.*
- real(kind=8), dimension(:), allocatable `buf_kb`  
*Buffer to hold tendencies in the RK4 scheme for the tangent linear model.*
- real(kind=8), dimension(:), allocatable `buf_kc`  
*Buffer to hold tendencies in the RK4 scheme for the tangent linear model.*



- `real(kind=8), dimension(:), allocatable buf_kd`  
*Buffer to hold tendencies in the RK4 scheme for the tangent linear model.*
- `real(kind=8), dimension(:, :), allocatable buf_j3`  
*Buffer to hold jacobians in the RK4 scheme for the tangent linear model.*
- `real(kind=8), dimension(:, :), allocatable buf_j4`  
*Buffer to hold jacobians in the RK4 scheme for the tangent linear model.*
- `real(kind=8), dimension(:, :), allocatable buf_j3h`  
*Buffer to hold jacobians in the RK4 scheme for the tangent linear model.*
- `real(kind=8), dimension(:, :), allocatable buf_j4h`  
*Buffer to hold jacobians in the RK4 scheme for the tangent linear model.*
- `real(kind=8), dimension(:), allocatable buf_kaa`  
*Buffer to hold tendencies in the RK4 scheme for the tangent linear model.*
- `real(kind=8), dimension(:), allocatable buf_kbb`  
*Buffer to hold tendencies in the RK4 scheme for the tangent linear model.*

### 6.11.1 Detailed Description

Tangent Linear (TL) and Adjoint (AD) model versions of MAOOAM. Integrators module.

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#### Remarks

This module actually contains the Heun algorithm routines. The user can modify it according to its preferred integration scheme. For higher-order schemes, additional buffers will probably have to be defined.

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#### Remarks

This module actually contains the RK4 algorithm routines. The user can modify it according to its preferred integration scheme. For higher-order schemes, additional buffers will probably have to be defined.

### 6.11.2 Function/Subroutine Documentation

**6.11.2.1** `subroutine public tl_ad_integrator::ad_step ( real(kind=8), dimension(0:ndim), intent(in) y, real(kind=8), dimension(0:ndim), intent(in) ystar, real(kind=8), intent(inout) t, real(kind=8), intent(in) dt, real(kind=8), dimension(0:ndim), intent(out) res )`

Routine to perform an integration step (Heun algorithm) of the adjoint model. The incremented time is returned.

Routine to perform an integration step (RK4 algorithm) of the adjoint model. The incremented time is returned.

## Parameters

|              |   |
|--------------|---|
| <i>y</i>     | Initial point.                            |
| <i>ystar</i> | Adjoint model at the point <i>ystar</i> . |
| <i>t</i>     | Actual integration time                   |
| <i>dt</i>    | Integration timestep.                     |
| <i>res</i>   | Final point after the step.               |

Definition at line 92 of file rk2\_tl\_ad\_integrator.f90.

```

92      REAL(KIND=8), DIMENSION(0:ndim), INTENT(IN) :: y, ystar
93      REAL(KIND=8), INTENT(INOUT) :: t
94      REAL(KIND=8), INTENT(IN) :: dt
95      REAL(KIND=8), DIMENSION(0:ndim), INTENT(OUT) :: res
96
97      CALL ad(t, ystar, y, buf_f0)
98      buf_y1 = y + dt * buf_f0
99      CALL ad(t + dt, ystar, buf_y1, buf_f1)
100     res = y + 0.5 * (buf_f0 + buf_f1) * dt
101     t = t + dt

```

**6.11.2.2** `subroutine public tl_ad_integrator::evolve_ad_step ( real(kind=8), dimension(0:ndim), intent(in) y, real(kind=8), dimension(0:ndim), intent(in) deltay, real(kind=8), intent(inout) t, real(kind=8), intent(in) dt, real(kind=8), dimension(0:ndim), intent(out) ynew, real(kind=8), dimension(0:ndim), intent(out) deltaynew )`

Routine to perform a simultaneous integration step (RK4 algorithm) of the nonlinear and adjoint together. The incremented time is returned.

Routine to perform a simultaneous integration step (RK4 algorithm) of the nonlinear and adjoint model together. The incremented time is returned.

## Parameters

|                  |   |
|------------------|---|
| <i>y</i>         | Model variable at time <i>t</i>             |
| <i>deltay</i>    | Perturbation at time <i>t</i>               |
| <i>t</i>         | Actual integration time                     |
| <i>dt</i>        | Integration timestep.                       |
| <i>ynew</i>      | Model variable at time <i>t</i> + <i>dt</i> |
| <i>deltaynew</i> | Perturbation at time <i>t</i> + <i>dt</i>   |

Definition at line 112 of file rk2\_tl\_ad\_integrator.f90.

```

112     REAL(KIND=8), DIMENSION(0:ndim), INTENT(IN) :: y, deltay
113     REAL(KIND=8), INTENT(INOUT) :: t
114     REAL(KIND=8), INTENT(IN) :: dt
115     REAL(KIND=8), DIMENSION(0:ndim), INTENT(OUT) :: ynew, deltaynew
116
117     CALL tendencies(t, y, buf_f0)
118     CALL ad(t, y, deltay, buf_f00)
119
120     buf_y1 = y + dt * buf_f0
121     buf_y11 = deltay + dt * buf_f00
122
123     CALL tendencies(t + dt, buf_y1, buf_f1)
124     CALL ad(t + dt, buf_y1, buf_y11, buf_f11)
125
126     t = t + dt
127     ynew = y + 0.5 * (buf_f0 + buf_f1) * dt
128     deltaynew = deltay + 0.5 * (buf_f00 + buf_f11) * dt

```

**6.11.2.3** subroutine public tl\_ad\_integrator::evolve\_tl\_step ( real(kind=8), dimension(0:ndim), intent(in) *y*, real(kind=8), dimension(0:ndim), intent(in) *deltay*, real(kind=8), intent(inout) *t*, real(kind=8), intent(in) *dt*, real(kind=8), dimension(0:ndim), intent(out) *ynew*, real(kind=8), dimension(0:ndim), intent(out) *deltaynew* )

Routine to perform a simultaneous integration step (RK4 algorithm) of the nonlinear and the tangent linear model together. The incremented time is returned.

Routine to perform a simultaneous integration step (RK4 algorithm) of the nonlinear and tangent linear model together. The incremented time is returned.

#### Parameters

|                  |   |
|------------------|---|
| <i>y</i>         | Model variable at time <i>t</i>             |
| <i>deltay</i>    | Perturbation at time <i>t</i>               |
| <i>t</i>         | Actual integration time                     |
| <i>dt</i>        | Integration timestep.                       |
| <i>ynew</i>      | Model variable at time <i>t</i> + <i>dt</i> |
| <i>deltaynew</i> | Perturbation at time <i>t</i> + <i>dt</i>   |

Definition at line 165 of file rk2\_tl\_ad\_integrator.f90.

```

165     REAL(KIND=8), DIMENSION(0:ndim), INTENT(IN) :: y,deltay
166     REAL(KIND=8), INTENT(INOUT) :: t
167     REAL(KIND=8), INTENT(IN) :: dt
168     REAL(KIND=8), DIMENSION(0:ndim), INTENT(OUT) :: ynew,deltaynew
169
170     CALL tendencies(t,y,buf_f0)
171     CALL ad(t,y,deltay,buf_f00)
172
173     buf_y1 = y + dt*buf_f0
174     buf_y11 = deltay + dt*buf_f00
175
176     CALL tendencies(t+dt,buf_y1,buf_f1)
177     CALL ad(t+dt,buf_y1,buf_y11,buf_f11)
178
179     t=t+dt
180     ynew=y+0.5*(buf_f0+buf_f1)*dt
181     deltaynew=deltay+0.5*(buf_f00+buf_f11)*dt

```

**6.11.2.4** subroutine public tl\_ad\_integrator::init\_tl\_ad\_integrator ( )

Routine to initialise the integration buffers.

Routine to initialise the TL-AD integration buffers.

Definition at line 68 of file rk2\_tl\_ad\_integrator.f90.

```

68     INTEGER :: allocstat
69     ALLOCATE(buf_y1(0:ndim),buf_f0(0:ndim),buf_f1(0:ndim), &
70             &buf_y11(0:ndim),buf_f00(0:ndim),buf_f11(0:ndim), &
71             &buf_j1(ndim,ndim),buf_j2(ndim,ndim),one(ndim,ndim), &
72             &buf_j1h(ndim,ndim),buf_j2h(ndim,ndim),stat=allocstat)
73     IF (allocstat /= 0) stop "*** Not enough memory ! ***"
74     CALL init_one(one)

```

6.11.2.5 subroutine public tl\_ad\_integrator::prop\_step ( real(kind=8), dimension(0:ndim), intent(in) y, real(kind=8), dimension(ndim,ndim), intent(out) *propagator*, real(kind=8), intent(inout) t, real(kind=8), intent(in) dt, real(kind=8), dimension(0:ndim), intent(out) ynew, logical, intent(in) *adjoint* )

Routine to perform a simultaneously an integration step (Heun algorithm) of the nonlinear and computes the Heun tangent linear propagator. The boolean variable *adjoint* allows for an adjoint forward integration. The incremented time is returned.

Routine to perform a simultaneously an integration step (RK4 algorithm) of the nonlinear and computes the RK4 tangent linear propagator. The boolean variable *adjoint* allows for an adjoint forward integration. The incremented time is returned.

#### Parameters

|                   |   |
|-------------------|---|
| <i>y</i>          | Model variable at time t  |
| <i>propagator</i> | Propagator at time t  |
| <i>t</i>          | Actual integration time   |
| <i>dt</i>         | Integration timestep.   |
| <i>ynew</i>       | Model variable at time t+dt   |
| <i>adjoint</i>    | If true, compute the propagator of the adjoint model (AD) instead of the tangent one (TL) |
| <i>y</i>          | Model variable at time t  |
| <i>propagator</i> | Propagator at time t  |
| <i>t</i>          | Actual integration time   |
| <i>dt</i>         | Integration timestep  |
| <i>ynew</i>       | Model variable at time t+dt   |
| <i>adjoint</i>    | If true, compute the propagator of the adjoint model (AD) instead of the tangent one (TL) |

Definition at line 193 of file rk2\_tl\_ad\_integrator.f90.

```

193  REAL(KIND=8), INTENT(INOUT) :: t
194  REAL(KIND=8), INTENT(IN) :: dt
195  REAL(KIND=8), DIMENSION(0:ndim), INTENT(IN) :: y
196  LOGICAL, INTENT(IN) :: adjoint
197  REAL(KIND=8), DIMENSION(ndim,ndim), INTENT(OUT) :: propagator
198  REAL(KIND=8), DIMENSION(0:ndim), INTENT(OUT) :: ynew
199
200  CALL tendencies(t,y,buf_f0)
201  buf_j1=jacobian_mat(y)
202
203  buf_y1 = y + dt*buf_f0
204
205  CALL tendencies(t+dt,buf_y1,buf_f1)
206  buf_j2=jacobian_mat(buf_y1)
207
208  buf_j1h=buf_j1
209  buf_j2h=buf_j2
210  CALL dgemm ('n', 'n', ndim, ndim, ndim, dt, buf_j2, ndim,buf_j1h, ndim,1.0d0, buf_j2h, ndim)
211
212  ynew=y + dt/2.0d0*(buf_f0 + buf_f1)
213  IF (adjoint) THEN
214      propagator=one - dt/2.0d0*(buf_j1h + buf_j2h)
215  ELSE
216      propagator=one + dt/2.0d0*(buf_j1h + buf_j2h)
217  END IF
218  t=t+dt
219
```

6.11.2.6 subroutine tl\_ad\_integrator::tendencies ( real(kind=8), intent(in) t, real(kind=8), dimension(0:ndim), intent(in) y, real(kind=8), dimension(0:ndim), intent(out) res ) [private]

Routine computing the tendencies of the nonlinear model.

## Parameters

|            |   |
|------------|---|
| <i>t</i>   | Time at which the tendencies have to be computed. Actually not needed for autonomous systems. |
| <i>y</i>   | Point at which the tendencies have to be computed.  |
| <i>res</i> | vector to store the result.   |

## Remarks

Note that it is NOT safe to pass *y* as a result bufer, as this operation does multiple passes.

Definition at line 60 of file rk2\_tl\_ad\_integrator.f90.

```

60     REAL(KIND=8), INTENT(IN) :: t
61     REAL(KIND=8), DIMENSION(0:ndim), INTENT(IN) :: y
62     REAL(KIND=8), DIMENSION(0:ndim), INTENT(OUT) :: res
63     CALL sparse_mul3(aotensor, y, y, res)

```

**6.11.2.7** subroutine public tl\_ad\_integrator::tl\_step ( real(kind=8), dimension(0:ndim), intent(in) *y*, real(kind=8), dimension(0:ndim), intent(in) *ystar*, real(kind=8), intent(inout) *t*, real(kind=8), intent(in) *dt*, real(kind=8), dimension(0:ndim), intent(out) *res* )

Routine to perform an integration step (Heun algorithm) of the tangent linear model. The incremented time is returned.

Routine to perform an integration step (RK4 algorithm) of the tangent linear model. The incremented time is returned.

## Parameters

|              |                                   |
|--------------|-----------------------------------|
| <i>y</i>     | Initial point.                    |
| <i>ystar</i> | Adjoint model at the point ystar. |
| <i>t</i>     | Actual integration time           |
| <i>dt</i>    | Integration timestep.             |
| <i>res</i>   | Final point after the step.       |

Definition at line 145 of file rk2\_tl\_ad\_integrator.f90.

```

145     REAL(KIND=8), DIMENSION(0:ndim), INTENT(IN) :: y, ystar
146     REAL(KIND=8), INTENT(INOUT) :: t
147     REAL(KIND=8), INTENT(IN) :: dt
148     REAL(KIND=8), DIMENSION(0:ndim), INTENT(OUT) :: res
149
150     CALL tl(t, ystar, y, buf_f0)
151     buf_y1 = y + dt * buf_f0
152     CALL tl(t + dt, ystar, buf_y1, buf_f1)
153     res = y + 0.5 * (buf_f0 + buf_f1) * dt
154     t = t + dt

```

## 6.11.3 Variable Documentation

**6.11.3.1** real(kind=8), dimension(:), allocatable tl\_ad\_integrator::buf\_f0 [private]

Buffer to hold tendencies at the initial position of the tangent linear model.

Definition at line 35 of file rk2\_tl\_ad\_integrator.f90.

```
35  REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: buf_f0 !< Buffer to hold tendencies at the initial position of
    the tangent linear model
```

#### 6.11.3.2 `real(kind=8), dimension(:), allocatable tl_ad_integrator::buf_f00` [private]

Buffer to hold tendencies at the initial position of the tangent linear model.

Definition at line 39 of file `rk2_tl_ad_integrator.f90`.

```
39  REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: buf_f00 !< Buffer to hold tendencies at the initial position
    of the tangent linear model
```

#### 6.11.3.3 `real(kind=8), dimension(:), allocatable tl_ad_integrator::buf_f1` [private]

Buffer to hold tendencies at the intermediate position of the tangent linear model.

Definition at line 36 of file `rk2_tl_ad_integrator.f90`.

```
36  REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: buf_f1 !< Buffer to hold tendencies at the intermediate
    position of the tangent linear model
```

#### 6.11.3.4 `real(kind=8), dimension(:), allocatable tl_ad_integrator::buf_f11` [private]

Buffer to hold tendencies at the intermediate position of the tangent linear model.

Definition at line 40 of file `rk2_tl_ad_integrator.f90`.

```
40  REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: buf_f11 !< Buffer to hold tendencies at the intermediate
    position of the tangent linear model
```

#### 6.11.3.5 `real(kind=8), dimension(:, :), allocatable tl_ad_integrator::buf_j1` [private]

Buffer to hold jacobians in the RK4 scheme for the tangent linear model.

Definition at line 42 of file `rk2_tl_ad_integrator.f90`.

```
42  REAL(KIND=8), DIMENSION(:, :), ALLOCATABLE :: buf_j1 !< Buffer to hold jacobians in the RK4 scheme for the
    tangent linear model
```

#### 6.11.3.6 `real(kind=8), dimension(:, :), allocatable tl_ad_integrator::buf_j1h` [private]

Buffer to hold jacobians in the RK4 scheme for the tangent linear model.

Definition at line 44 of file `rk2_tl_ad_integrator.f90`.

```
44  REAL(KIND=8), DIMENSION(:, :), ALLOCATABLE :: buf_j1h !< Buffer to hold jacobians in the RK4 scheme for
    the tangent linear model
```

**6.11.3.7** `real(kind=8), dimension(:,,:), allocatable tl_ad_integrator::buf_j2` [private]

Buffer to hold jacobians in the RK4 scheme for the tangent linear model.

Definition at line 43 of file rk2\_tl\_ad\_integrator.f90.

```
43  REAL(KIND=8), DIMENSION(:,,:), ALLOCATABLE :: buf_j2 !< Buffer to hold jacobians in the RK4 scheme for the
    tangent linear model
```

**6.11.3.8** `real(kind=8), dimension(:,,:), allocatable tl_ad_integrator::buf_j2h` [private]

Buffer to hold jacobians in the RK4 scheme for the tangent linear model.

Definition at line 45 of file rk2\_tl\_ad\_integrator.f90.

```
45  REAL(KIND=8), DIMENSION(:,,:), ALLOCATABLE :: buf_j2h !< Buffer to hold jacobians in the RK4 scheme for
    the tangent linear model
```

**6.11.3.9** `real(kind=8), dimension(:,,:), allocatable tl_ad_integrator::buf_j3` [private]

Buffer to hold jacobians in the RK4 scheme for the tangent linear model.

Definition at line 42 of file rk4\_tl\_ad\_integrator.f90.

```
42  REAL(KIND=8), DIMENSION(:,,:), ALLOCATABLE :: buf_j3 !< Buffer to hold jacobians in the RK4 scheme for the
    tangent linear model
```

**6.11.3.10** `real(kind=8), dimension(:,,:), allocatable tl_ad_integrator::buf_j3h` [private]

Buffer to hold jacobians in the RK4 scheme for the tangent linear model.

Definition at line 46 of file rk4\_tl\_ad\_integrator.f90.

```
46  REAL(KIND=8), DIMENSION(:,,:), ALLOCATABLE :: buf_j3h !< Buffer to hold jacobians in the RK4 scheme for
    the tangent linear model
```

**6.11.3.11** `real(kind=8), dimension(:,,:), allocatable tl_ad_integrator::buf_j4` [private]

Buffer to hold jacobians in the RK4 scheme for the tangent linear model.

Definition at line 43 of file rk4\_tl\_ad\_integrator.f90.

```
43  REAL(KIND=8), DIMENSION(:,,:), ALLOCATABLE :: buf_j4 !< Buffer to hold jacobians in the RK4 scheme for the
    tangent linear model
```

**6.11.3.12** `real(kind=8), dimension(:, :), allocatable tl_ad_integrator::buf_j4h` [private]

Buffer to hold jacobians in the RK4 scheme for the tangent linear model.

Definition at line 47 of file `rk4_tl_ad_integrator.f90`.

```
47  REAL(KIND=8), DIMENSION(:, :), ALLOCATABLE :: buf_j4h !< Buffer to hold jacobians in the RK4 scheme for
    the tangent linear model
```

**6.11.3.13** `real(kind=8), dimension(:), allocatable tl_ad_integrator::buf_ka` [private]

Buffer to hold tendencies in the RK4 scheme for the tangent linear model.

Definition at line 35 of file `rk4_tl_ad_integrator.f90`.

```
35  REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: buf_ka !< Buffer to hold tendencies in the RK4 scheme for the
    tangent linear model
```

**6.11.3.14** `real(kind=8), dimension(:), allocatable tl_ad_integrator::buf_kaa` [private]

Buffer to hold tendencies in the RK4 scheme for the tangent linear model.

Definition at line 48 of file `rk4_tl_ad_integrator.f90`.

```
48  REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: buf_kaa !< Buffer to hold tendencies in the RK4 scheme for the
    tangent linear model
```

**6.11.3.15** `real(kind=8), dimension(:), allocatable tl_ad_integrator::buf_kb` [private]

Buffer to hold tendencies in the RK4 scheme for the tangent linear model.

Definition at line 36 of file `rk4_tl_ad_integrator.f90`.

```
36  REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: buf_kb !< Buffer to hold tendencies in the RK4 scheme for the
    tangent linear model
```

**6.11.3.16** `real(kind=8), dimension(:), allocatable tl_ad_integrator::buf_kbb` [private]

Buffer to hold tendencies in the RK4 scheme for the tangent linear model.

Definition at line 49 of file `rk4_tl_ad_integrator.f90`.

```
49  REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: buf_kbb !< Buffer to hold tendencies in the RK4 scheme for the
    tangent linear model
```



**6.11.3.17** `real(kind=8), dimension(:), allocatable tl_ad_integrator::buf_kc` [private]

Buffer to hold tendencies in the RK4 scheme for the tangent linear model.

Definition at line 38 of file rk4\_tl\_ad\_integrator.f90.

```
38  REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: buf_kc !< Buffer to hold tendencies in the RK4 scheme for the
    tangent linear model
```

**6.11.3.18** `real(kind=8), dimension(:), allocatable tl_ad_integrator::buf_kd` [private]

Buffer to hold tendencies in the RK4 scheme for the tangent linear model.

Definition at line 39 of file rk4\_tl\_ad\_integrator.f90.

```
39  REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: buf_kd !< Buffer to hold tendencies in the RK4 scheme for the
    tangent linear model
```

**6.11.3.19** `real(kind=8), dimension(:), allocatable tl_ad_integrator::buf_y1` [private]

Buffer to hold the intermediate position (Heun algorithm) of the tangent linear model.

Buffer to hold the intermediate position of the tangent linear model.

Definition at line 34 of file rk2\_tl\_ad\_integrator.f90.

```
34  REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: buf_y1 !< Buffer to hold the intermediate position (Heun
    algorithm) of the tangent linear model
```

**6.11.3.20** `real(kind=8), dimension(:), allocatable tl_ad_integrator::buf_y11` [private]

Buffer to hold the intermediate position (Heun algorithm) of the tangent linear model.

Buffer to hold the intermediate position of the tangent linear model.

Definition at line 38 of file rk2\_tl\_ad\_integrator.f90.

```
38  REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: buf_y11 !< Buffer to hold the intermediate position (Heun
    algorithm) of the tangent linear model
```

**6.11.3.21** `real(kind=8), dimension(:, :), allocatable tl_ad_integrator::one` [private]

unit matrix

Definition at line 46 of file rk2\_tl\_ad\_integrator.f90.

```
46  REAL(KIND=8), DIMENSION(:, :), ALLOCATABLE :: one          !< unit matrix
```

## 6.12 tl\_ad\_tensor Module Reference

Tangent Linear (TL) and Adjoint (AD) model versions of MAOOAM. Tensors definition module.

### Functions/Subroutines

- type([coolist](#)) function, dimension(ndim) [jacobian](#) (ystar)  
*Compute the Jacobian of MAOOAM in point ystar.*
- real(kind=8) function, dimension(ndim, ndim), public [jacobian\\_mat](#) (ystar)  
*Compute the Jacobian of MAOOAM in point ystar.*
- subroutine, public [init\\_tlensor](#)  
*Routine to initialize the TL tensor.*
- subroutine [compute\\_tlensor](#) (func)  
*Routine to compute the TL tensor from the original MAOOAM one.*
- subroutine [tl\\_add\\_count](#) (i, j, k, v)  
*Subroutine used to count the number of TL tensor entries.*
- subroutine [tl\\_coeff](#) (i, j, k, v)  
*Subroutine used to compute the TL tensor entries.*
- subroutine, public [init\\_adtensor](#)  
*Routine to initialize the AD tensor.*
- subroutine [compute\\_adtensor](#) (func)  
*Subroutine to compute the AD tensor from the original MAOOAM one.*
- subroutine [ad\\_add\\_count](#) (i, j, k, v)  
*Subroutine used to count the number of AD tensor entries.*
- subroutine [ad\\_coeff](#) (i, j, k, v)
- subroutine, public [init\\_adtensor\\_ref](#)  
*Alternate method to initialize the AD tensor from the TL tensor.*
- subroutine [compute\\_adtensor\\_ref](#) (func)  
*Alternate subroutine to compute the AD tensor from the TL one.*
- subroutine [ad\\_add\\_count\\_ref](#) (i, j, k, v)  
*Alternate subroutine used to count the number of AD tensor entries from the TL tensor.*
- subroutine [ad\\_coeff\\_ref](#) (i, j, k, v)  
*Alternate subroutine used to compute the AD tensor entries from the TL tensor.*
- subroutine, public [ad](#) (t, ystar, deltay, buf)  
*Tendencies for the AD of MAOOAM in point ystar for perturbation deltay.*
- subroutine, public [tl](#) (t, ystar, deltay, buf)  
*Tendencies for the TL of MAOOAM in point ystar for perturbation deltay.*

### Variables

- real(kind=8), parameter [real\\_eps](#) = 2.2204460492503131e-16  
*Epsilon to test equality with 0.*
- integer, dimension(:), allocatable [count\\_elems](#)  
*Vector used to count the tensor elements.*
- type([coolist](#)), dimension(:), allocatable, public [tlensor](#)  
*Tensor representation of the Tangent Linear tendencies.*
- type([coolist](#)), dimension(:), allocatable, public [adtensor](#)  
*Tensor representation of the Adjoint tendencies.*

### 6.12.1 Detailed Description

Tangent Linear (TL) and Adjoint (AD) model versions of MAOOAM. Tensors definition module.

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#### Remarks

The routines of this module should be called only after [params::init\\_params\(\)](#) and [aotensor\\_def::init\\_↵aotensor\(\)](#) have been called !

### 6.12.2 Function/Subroutine Documentation

**6.12.2.1** subroutine, public `tl_ad_tensor::ad ( real(kind=8), intent(in) t, real(kind=8), dimension(0:ndim), intent(in) ystar, real(kind=8), dimension(0:ndim), intent(in) deltax, real(kind=8), dimension(0:ndim), intent(out) buf )`

Tendencies for the AD of MAOOAM in point ystar for perturbation deltax.

#### Parameters

|               |   |
|---------------|---|
| <i>t</i>      | time  |
| <i>ystar</i>  | vector with the variables (current point in trajectory) |
| <i>deltax</i> | vector with the perturbation of the variables at time t |
| <i>buf</i>    | vector (buffer) to store derivatives.                   |

Definition at line 384 of file `tl_ad_tensor.f90`.

```

384     REAL(KIND=8), INTENT(IN) :: t
385     REAL(KIND=8), DIMENSION(0:ndim), INTENT(IN) :: ystar,deltax
386     REAL(KIND=8), DIMENSION(0:ndim), INTENT(OUT) :: buf
387     CALL sparse_mul3(adtensor,deltax,ystar,buf)

```

**6.12.2.2** subroutine `tl_ad_tensor::ad_add_count ( integer, intent(in) i, integer, intent(in) j, integer, intent(in) k, real(kind=8), intent(in) v ) [private]`

Subroutine used to count the number of AD tensor entries.

#### Parameters

|          |                          |
|----------|--------------------------|
| <i>i</i> | tensor <i>i</i> index    |
| <i>j</i> | tensor <i>j</i> index    |
| <i>k</i> | tensor <i>k</i> index    |
| <i>v</i> | value that will be added |

Definition at line 243 of file `tl_ad_tensor.f90`.

```

243     INTEGER, INTENT(IN) :: i,j,k
244     REAL(KIND=8), INTENT(IN) :: v
245     IF ((abs(v) .ge. real_eps).AND.(i /= 0)) THEN
246         IF (k /= 0) count_elems(k)=count_elems(k)+1
247         IF (j /= 0) count_elems(j)=count_elems(j)+1
248     ENDIF

```

**6.12.2.3** subroutine `tl_ad_tensor::ad_add_count_ref` ( integer, intent(in) *i*, integer, intent(in) *j*, integer, intent(in) *k*, real(kind=8), intent(in) *v* ) [private]

Alternate subroutine used to count the number of AD tensor entries from the TL tensor.

#### Parameters

|          |                          |
|----------|--------------------------|
| <i>i</i> | tensor <i>i</i> index    |
| <i>j</i> | tensor <i>j</i> index    |
| <i>k</i> | tensor <i>k</i> index    |
| <i>v</i> | value that will be added |

Definition at line 346 of file `tl_ad_tensor.f90`.

```

346     INTEGER, INTENT(IN) :: i,j,k
347     REAL(KIND=8), INTENT(IN) :: v
348     IF ((abs(v) .ge. real_eps).AND.(j /= 0)) count_elems(j)=count_elems(j)+1

```

**6.12.2.4** subroutine `tl_ad_tensor::ad_coeff` ( integer, intent(in) *i*, integer, intent(in) *j*, integer, intent(in) *k*, real(kind=8), intent(in) *v* ) [private]

#### Parameters

|          |                       |
|----------|-----------------------|
| <i>i</i> | tensor <i>i</i> index |
| <i>j</i> | tensor <i>j</i> index |
| <i>k</i> | tensor <i>k</i> index |
| <i>v</i> | value to add          |

Definition at line 257 of file `tl_ad_tensor.f90`.

```

257     INTEGER, INTENT(IN) :: i,j,k
258     REAL(KIND=8), INTENT(IN) :: v
259     INTEGER :: n
260     IF (.NOT. ALLOCATED(adtensor)) stop "*** ad_coeff routine : tensor not yet allocated ***"
261     IF ((abs(v) .ge. real_eps).AND.(i /=0)) THEN
262         IF (k /=0) THEN
263             IF (.NOT. ALLOCATED(adtensor(k)%elems)) stop "*** ad_coeff routine : tensor not yet allocated ***"
264             n=(adtensor(k)%elems)+1
265             adtensor(k)%elems(n)%j=i
266             adtensor(k)%elems(n)%k=j
267             adtensor(k)%elems(n)%v=v
268             adtensor(k)%elems=n
269         END IF
270         IF (j /=0) THEN
271             IF (.NOT. ALLOCATED(adtensor(j)%elems)) stop "*** ad_coeff routine : tensor not yet allocated ***"
272             n=(adtensor(j)%elems)+1
273             adtensor(j)%elems(n)%j=i
274             adtensor(j)%elems(n)%k=k

```

```

275         adtensor(j)%elems(n)%v=v
276         adtensor(j)%elems=n
277     END IF
278 END IF

```

**6.12.2.5** subroutine `tl_ad_tensor::ad_coeff_ref` ( integer, intent(in) *i*, integer, intent(in) *j*, integer, intent(in) *k*, real(kind=8), intent(in) *v* ) [private]

Alternate subroutine used to compute the AD tensor entries from the TL tensor.

#### Parameters

|          |                       |
|----------|-----------------------|
| <i>i</i> | tensor <i>i</i> index |
| <i>j</i> | tensor <i>j</i> index |
| <i>k</i> | tensor <i>k</i> index |
| <i>v</i> | value to add          |

Definition at line 358 of file `tl_ad_tensor.f90`.

```

358     INTEGER, INTENT(IN) :: i,j,k
359     REAL(KIND=8), INTENT(IN) :: v
360     INTEGER :: n
361     IF (.NOT. ALLOCATED(adtensor)) stop "*** ad_coeff_ref routine : tensor not yet allocated ***"
362     IF ((abs(v) .ge. real_eps).AND.(j /=0)) THEN
363         IF (.NOT. ALLOCATED(adtensor(j)%elems)) stop "*** ad_coeff_ref routine : tensor not yet allocated ***"
364         n=(adtensor(j)%elems)+1
365         adtensor(j)%elems(n)%j=i
366         adtensor(j)%elems(n)%k=k
367         adtensor(j)%elems(n)%v=v
368         adtensor(j)%elems=n
369     END IF

```

**6.12.2.6** subroutine `tl_ad_tensor::compute_adtensor` ( external *func* ) [private]

Subroutine to compute the AD tensor from the original MAOOAM one.

#### Parameters

|             |                                       |
|-------------|---------------------------------------|
| <i>func</i> | subroutine used to do the computation |
|-------------|---------------------------------------|

Definition at line 217 of file `tl_ad_tensor.f90`.

**6.12.2.7** subroutine `tl_ad_tensor::compute_adtensor_ref` ( external *func* ) [private]

Alternate subroutine to compute the AD tensor from the TL one.

#### Parameters

|             |                                       |
|-------------|---------------------------------------|
| <i>func</i> | subroutine used to do the computation |
|-------------|---------------------------------------|

Definition at line 318 of file `tl_ad_tensor.f90`.

#### 6.12.2.8 subroutine `tl_ad_tensor::compute_tltensor ( external func )` [private]

Routine to compute the TL tensor from the original MAOOAM one.

##### Parameters

|                   |                                       |
|-------------------|---------------------------------------|
| <code>func</code> | subroutine used to do the computation |
|-------------------|---------------------------------------|

Definition at line 121 of file `tl_ad_tensor.f90`.

#### 6.12.2.9 subroutine, public `tl_ad_tensor::init_adtensor ( )`

Routine to initialize the AD tensor.

Definition at line 193 of file `tl_ad_tensor.f90`.

```

193     INTEGER :: i
194     INTEGER :: allocstat
195     ALLOCATE (adtensor(ndim),count_elems(ndim), stat=allocstat)
196     IF (allocstat /= 0) stop "*** Not enough memory ! ***"
197     count_elems=0
198     CALL compute_adtensor(ad_add_count)
199
200     DO i=1,ndim
201         ALLOCATE (adtensor(i)%elems(count_elems(i)), stat=allocstat)
202         IF (allocstat /= 0) stop "*** Not enough memory ! ***"
203     END DO
204
205     DEALLOCATE(count_elems, stat=allocstat)
206     IF (allocstat /= 0) stop "*** Deallocation problem ! ***"
207
208     CALL compute_adtensor(ad_coeff)
209
210     CALL simplify (adtensor)
211
```

#### 6.12.2.10 subroutine, public `tl_ad_tensor::init_adtensor_ref ( )`

Alternate method to initialize the AD tensor from the TL tensor.

##### Remarks

The `tltensor` must be initialised before using this method.

Definition at line 294 of file `tl_ad_tensor.f90`.

```

294     INTEGER :: i
295     INTEGER :: allocstat
296     ALLOCATE (adtensor(ndim),count_elems(ndim), stat=allocstat)
297     IF (allocstat /= 0) stop "*** Not enough memory ! ***"
298     count_elems=0
299     CALL compute_adtensor_ref(ad_add_count_ref)
300
301     DO i=1,ndim
302         ALLOCATE (adtensor(i)%elems(count_elems(i)), stat=allocstat)
303         IF (allocstat /= 0) stop "*** Not enough memory ! ***"
304     END DO
305
306     DEALLOCATE(count_elems, stat=allocstat)
307     IF (allocstat /= 0) stop "*** Deallocation problem ! ***"
308
309     CALL compute_adtensor_ref(ad_coeff_ref)
310
311     CALL simplify (adtensor)
312
```

## 6.12.2.11 subroutine, public tl\_ad\_tensor::init\_tltensor ( )

Routine to initialize the TL tensor.

Definition at line 97 of file tl\_ad\_tensor.f90.

```

97     INTEGER :: i
98     INTEGER :: allocstat
99     ALLOCATE(tltensor(ndim),count_elems(ndim), stat=allocstat)
100     IF (allocstat /= 0) stop "*** Not enough memory ! ***"
101     count_elems=0
102     CALL compute_tltensor(tl_add_count)
103
104     DO i=1,ndim
105         ALLOCATE(tltensor(i)%elems(count_elems(i)), stat=allocstat)
106         IF (allocstat /= 0) stop "*** Not enough memory ! ***"
107     END DO
108
109     DEALLOCATE(count_elems, stat=allocstat)
110     IF (allocstat /= 0) stop "*** Deallocation problem ! ***"
111
112     CALL compute_tltensor(tl_coeff)
113
114     CALL simplify(tltensor)
115

```

6.12.2.12 type(coolist) function, dimension(ndim) tl\_ad\_tensor::jacobian ( real(kind=8), dimension(0:ndim), intent(in) ystar )  
[private]

Compute the Jacobian of MAOOAM in point ystar.

## Parameters

|              |   |
|--------------|---|
| <i>ystar</i> | array with variables in which the jacobian should be evaluated. |
|--------------|---|

## Returns

Jacobian in coolist-form (table of tuples {i,j,0,value})

Definition at line 75 of file tl\_ad\_tensor.f90.

```

75     REAL(KIND=8), DIMENSION(0:ndim), INTENT(IN) :: ystar
76     TYPE(coolist), DIMENSION(ndim) :: jacobian
77     CALL jsparse_mul(aotensor,ystar,jacobian)

```

6.12.2.13 real(kind=8) function, dimension(ndim,ndim), public tl\_ad\_tensor::jacobian\_mat ( real(kind=8), dimension(0:ndim),  
intent(in) ystar )

Compute the Jacobian of MAOOAM in point ystar.

## Parameters

|              |   |
|--------------|---|
| <i>ystar</i> | array with variables in which the jacobian should be evaluated. |
|--------------|---|

## Returns

Jacobian in matrix form

Definition at line 84 of file `tl_ad_tensor.f90`.

```
84     REAL(KIND=8), DIMENSION(0:ndim), INTENT(IN) :: ystar
85     REAL(KIND=8), DIMENSION(ndim,ndim) :: jacobian_mat
86     CALL jsparse_mul_mat(aotensor,ystar,jacobian_mat)
```

**6.12.2.14** subroutine, public `tl_ad_tensor::tl` ( `real(kind=8)`, intent(in) *t*, `real(kind=8)`, dimension(0:ndim), intent(in) *ystar*, `real(kind=8)`, dimension(0:ndim), intent(in) *deltay*, `real(kind=8)`, dimension(0:ndim), intent(out) *buf* )

Tendencies for the TL of MAOOAM in point ystar for perturbation deltay.

## Parameters

|               |   |
|---------------|---|
| <i>t</i>      | time  |
| <i>ystar</i>  | vector with the variables (current point in trajectory) |
| <i>deltay</i> | vector with the perturbation of the variables at time t |
| <i>buf</i>    | vector (buffer) to store derivatives.                   |

Definition at line 396 of file `tl_ad_tensor.f90`.

```
396     REAL(KIND=8), INTENT(IN) :: t
397     REAL(KIND=8), DIMENSION(0:ndim), INTENT(IN) :: ystar,deltay
398     REAL(KIND=8), DIMENSION(0:ndim), INTENT(OUT) :: buf
399     CALL sparse_mul3(tltensor,deltay,ystar,buf)
```

**6.12.2.15** subroutine `tl_ad_tensor::tl_add_count` ( `integer`, intent(in) *i*, `integer`, intent(in) *j*, `integer`, intent(in) *k*, `real(kind=8)`, intent(in) *v* ) [`private`]

Subroutine used to count the number of TL tensor entries.

## Parameters

|          |                          |
|----------|--------------------------|
| <i>i</i> | tensor <i>i</i> index    |
| <i>j</i> | tensor <i>j</i> index    |
| <i>k</i> | tensor <i>k</i> index    |
| <i>v</i> | value that will be added |

Definition at line 147 of file `tl_ad_tensor.f90`.

```
147     INTEGER, INTENT(IN) :: i,j,k
148     REAL(KIND=8), INTENT(IN) :: v
149     IF (abs(v) .ge. real_eps) THEN
150         IF (j /= 0) count_elems(i)=count_elems(i)+1
151         IF (k /= 0) count_elems(i)=count_elems(i)+1
152     ENDIF
```



**6.12.2.16** subroutine `tl_ad_tensor::tl_coeff` ( integer, intent(in) *i*, integer, intent(in) *j*, integer, intent(in) *k*, real(kind=8), intent(in) *v* ) [private]

Subroutine used to compute the TL tensor entries.

#### Parameters

|          |                       |
|----------|-----------------------|
| <i>i</i> | tensor <i>i</i> index |
| <i>j</i> | tensor <i>j</i> index |
| <i>k</i> | tensor <i>k</i> index |
| <i>v</i> | value to add          |

Definition at line 161 of file `tl_ad_tensor.f90`.

```

161  INTEGER, INTENT(IN) :: i,j,k
162  REAL(KIND=8), INTENT(IN) :: v
163  INTEGER :: n
164  IF (.NOT. ALLOCATED(tltensor)) stop "*** tl_coeff routine : tensor not yet allocated ***"
165  IF (.NOT. ALLOCATED(tltensor(i)%elems)) stop "*** tl_coeff routine : tensor not yet allocated ***"
166  IF (abs(v) .ge. real_eps) THEN
167      IF (j /=0) THEN
168          n=(tltensor(i)%elems)+1
169          tltensor(i)%elems(n)%j=j
170          tltensor(i)%elems(n)%k=k
171          tltensor(i)%elems(n)%v=v
172          tltensor(i)%elems=n
173      END IF
174      IF (k /=0) THEN
175          n=(tltensor(i)%elems)+1
176          tltensor(i)%elems(n)%j=k
177          tltensor(i)%elems(n)%k=j
178          tltensor(i)%elems(n)%v=v
179          tltensor(i)%elems=n
180      END IF
181  END IF

```

### 6.12.3 Variable Documentation

**6.12.3.1** type(`coolist`), dimension(:), allocatable, public `tl_ad_tensor::adtensor`

Tensor representation of the Adjoint tendencies.

Definition at line 44 of file `tl_ad_tensor.f90`.

```

44  TYPE(coolist), DIMENSION(:), ALLOCATABLE, PUBLIC :: adtensor

```

**6.12.3.2** integer, dimension(:), allocatable `tl_ad_tensor::count_elems` [private]

Vector used to count the tensor elements.

Definition at line 38 of file `tl_ad_tensor.f90`.

```

38  INTEGER, DIMENSION(:), ALLOCATABLE :: count_elems

```

### 6.12.3.3 `real(kind=8), parameter tl_ad_tensor::real_eps = 2.2204460492503131e-16` [private]

Epsilon to test equality with 0.

Definition at line 35 of file `tl_ad_tensor.f90`.

```
35  REAL(KIND=8), PARAMETER :: real_eps = 2.2204460492503131e-16
```

### 6.12.3.4 `type(coolist), dimension(:), allocatable, public tl_ad_tensor::tltensor`

Tensor representation of the Tangent Linear tendencies.

Definition at line 41 of file `tl_ad_tensor.f90`.

```
41  TYPE(coolist), DIMENSION(:), ALLOCATABLE, PUBLIC :: tltensor
```

## 6.13 util Module Reference

Utility module.

### Functions/Subroutines

- `character(len=20)` function, public `str` (k)  
*Convert an integer to string.*
- `character(len=40)` function, public `rstr` (x, fm)  
*Convert a real to string with a given format.*
- `integer` function, `dimension(size(s))`, public `isin` (c, s)  
*Determine if a character is in a string and where.*
- subroutine, public `init_random_seed` ()  
*Random generator initialization routine.*
- subroutine, public `piksort` (k, arr, par)  
*Simple card player sorting function.*
- subroutine, public `init_one` (A)  
*Initialize a square matrix A as a unit matrix.*

### 6.13.1 Detailed Description

Utility module.

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## 6.13.2 Function/Subroutine Documentation

### 6.13.2.1 subroutine, public util::init\_one ( real(kind=8), dimension(:, :), intent(inout) A )

Initialize a square matrix A as a unit matrix.

Definition at line 137 of file util.f90.

```

137      REAL(KIND=8), DIMENSION(:, :), INTENT(INOUT) :: a
138      INTEGER :: i, n
139      n=size(a, 1)
140      a=0.0d0
141      DO i=1, n
142          a(i, i)=1.0d0
143      END DO
144  
```

### 6.13.2.2 subroutine, public util::init\_random\_seed ( )

Random generator initialization routine.

Definition at line 62 of file util.f90.

### 6.13.2.3 integer function, dimension(size(s)), public util::isin ( character, intent(in) c, character, dimension(:), intent(in) s )

Determine if a character is in a string and where.

#### Remarks

: return positions in a vector if found and 0 vector if not found

Definition at line 45 of file util.f90.

```

45      CHARACTER, INTENT(IN) :: c
46      CHARACTER, DIMENSION(:), INTENT(IN) :: s
47      INTEGER, DIMENSION(size(s)) :: isin
48      INTEGER :: i, j
49
50      isin=0
51      j=0
52      DO i=size(s), 1, -1
53          IF (c==s(i)) THEN
54              j=j+1
55              isin(j)=i
56          END IF
57      END DO
  
```

### 6.13.2.4 subroutine, public util::piksort ( integer, intent(in) k, integer, dimension(k), intent(inout) arr, integer, intent(out) par )

Simple card player sorting function.

Definition at line 116 of file util.f90.

```

116     INTEGER, INTENT(IN) :: k
117     INTEGER, DIMENSION(k), INTENT(INOUT) :: arr
118     INTEGER, INTENT(OUT) :: par
119     INTEGER :: i, j, a, b
120
121     par=1
122
123     DO j=2, k
124         a=arr(j)
125         DO i=j-1, 1, -1
126             IF (arr(i).le.a) EXIT
127             arr(i+1)=arr(i)
128             par=-par
129         END DO
130         arr(i+1)=a
131     ENDDO
132     RETURN
  
```

#### 6.13.2.5 `character(len=40)` function, public `util::rstr ( real(kind=8), intent(in) x, character(len=20), intent(in) fm )`

Convert a real to string with a given format.

Definition at line 36 of file `util.f90`.

```
36      REAL(KIND=8), INTENT(IN) :: x
37      CHARACTER(len=20), INTENT(IN) :: fm
38      WRITE (rstr, trim(adjustl(fm))) x
39      rstr = adjustl(rstr)
```

#### 6.13.2.6 `character(len=20)` function, public `util::str ( integer, intent(in) k )`

Convert an integer to string.

Definition at line 29 of file `util.f90`.

```
29      INTEGER, INTENT(IN) :: k
30      WRITE (str, *) k
31      str = adjustl(str)
```

## Chapter 7

# Data Type Documentation

### 7.1 inprod\_analytic::atm\_tensors Type Reference

Type holding the atmospheric inner products tensors.

#### Private Attributes

- procedure([calculate\\_a](#)), pointer, nopass [a](#)
- procedure([calculate\\_b](#)), pointer, nopass [b](#)
- procedure([calculate\\_c\\_atm](#)), pointer, nopass [c](#)
- procedure([calculate\\_d](#)), pointer, nopass [d](#)
- procedure([calculate\\_g](#)), pointer, nopass [g](#)
- procedure([calculate\\_s](#)), pointer, nopass [s](#)

#### 7.1.1 Detailed Description

Type holding the atmospheric inner products tensors.

Definition at line 53 of file inprod\_analytic.f90.

#### 7.1.2 Member Data Documentation

##### 7.1.2.1 procedure([calculate\\_a](#)), pointer, nopass inprod\_analytic::atm\_tensors::a [private]

Definition at line 54 of file inprod\_analytic.f90.

```
54      PROCEDURE(calculate_a), POINTER, NOPASS :: a
```

##### 7.1.2.2 procedure([calculate\\_b](#)), pointer, nopass inprod\_analytic::atm\_tensors::b [private]

Definition at line 55 of file inprod\_analytic.f90.

```
55      PROCEDURE(calculate_b), POINTER, NOPASS :: b
```

### 7.1.2.3 `procedure(calculate_c_atm), pointer, nopass inprod_analytic::atm_tensors::c` [private]

Definition at line 56 of file `inprod_analytic.f90`.

```
56      PROCEDURE(calculate_c_atm), POINTER, NOPASS :: c
```

### 7.1.2.4 `procedure(calculate_d), pointer, nopass inprod_analytic::atm_tensors::d` [private]

Definition at line 57 of file `inprod_analytic.f90`.

```
57      PROCEDURE(calculate_d), POINTER, NOPASS :: d
```

### 7.1.2.5 `procedure(calculate_g), pointer, nopass inprod_analytic::atm_tensors::g` [private]

Definition at line 58 of file `inprod_analytic.f90`.

```
58      PROCEDURE(calculate_g), POINTER, NOPASS :: g
```

### 7.1.2.6 `procedure(calculate_s), pointer, nopass inprod_analytic::atm_tensors::s` [private]

Definition at line 59 of file `inprod_analytic.f90`.

```
59      PROCEDURE(calculate_s), POINTER, NOPASS :: s
```

The documentation for this type was generated from the following file:

- [inprod\\_analytic.f90](#)

## 7.2 `inprod_analytic::atm_wavenum` Type Reference

Atmospheric bloc specification type.

### Private Attributes

- character `typ`
- integer `m` =0
- integer `p` =0
- integer `h` =0
- real(kind=8) `nx` =0.
- real(kind=8) `ny` =0.

### 7.2.1 Detailed Description

Atmospheric bloc specification type.

Definition at line 40 of file inprod\_analytic.f90.

### 7.2.2 Member Data Documentation

#### 7.2.2.1 integer inprod\_analytic::atm\_wavenum::h =0 [private]

Definition at line 42 of file inprod\_analytic.f90.

#### 7.2.2.2 integer inprod\_analytic::atm\_wavenum::m =0 [private]

Definition at line 42 of file inprod\_analytic.f90.

```
42      INTEGER :: m=0, p=0, h=0
```

#### 7.2.2.3 real(kind=8) inprod\_analytic::atm\_wavenum::nx =0. [private]

Definition at line 43 of file inprod\_analytic.f90.

```
43      REAL(KIND=8) :: nx=0., ny=0.
```

#### 7.2.2.4 real(kind=8) inprod\_analytic::atm\_wavenum::ny =0. [private]

Definition at line 43 of file inprod\_analytic.f90.

#### 7.2.2.5 integer inprod\_analytic::atm\_wavenum::p =0 [private]

Definition at line 42 of file inprod\_analytic.f90.

#### 7.2.2.6 character inprod\_analytic::atm\_wavenum::typ [private]

Definition at line 41 of file inprod\_analytic.f90.

```
41      CHARACTER :: typ
```

The documentation for this type was generated from the following file:

- [inprod\\_analytic.f90](#)

## 7.3 `tensor::coolist` Type Reference

Coordinate list. Type used to represent the sparse tensor.

### Public Attributes

- `type(coolist_elem)`, `dimension(:)`, allocatable `elems`  
*Lists of elements [tensor::coolist\\_elem](#).*
- integer `nelems` = 0  
*Number of elements in the list.*

### 7.3.1 Detailed Description

Coordinate list. Type used to represent the sparse tensor.

Definition at line 27 of file `tensor.f90`.

### 7.3.2 Member Data Documentation

#### 7.3.2.1 `type(coolist_elem)`, `dimension(:)`, allocatable `tensor::coolist::elems`

Lists of elements [tensor::coolist\\_elem](#).

Definition at line 28 of file `tensor.f90`.

```
28      TYPE(coolist_elem), DIMENSION(:), ALLOCATABLE :: elems !< Lists of elements tensor::coolist_elem
```

#### 7.3.2.2 integer `tensor::coolist::nelems` = 0

Number of elements in the list.

Definition at line 29 of file `tensor.f90`.

```
29      INTEGER :: nelems = 0 !< Number of elements in the list.
```

The documentation for this type was generated from the following file:

- [tensor.f90](#)

## 7.4 `tensor::coolist_elem` Type Reference

Coordinate list element type. Elementary elements of the sparse tensors.



## Private Attributes

- integer [j](#)  
*Index  $j$  of the element.*
- integer [k](#)  
*Index  $k$  of the element.*
- real(kind=8) [v](#)  
*Value of the element.*

### 7.4.1 Detailed Description

Coordinate list element type. Elementary elements of the sparse tensors.

Definition at line 20 of file tensor.f90.

### 7.4.2 Member Data Documentation

#### 7.4.2.1 integer tensor::coolist\_elem::j [private]

Index  $j$  of the element.

Definition at line 21 of file tensor.f90.

```
21      INTEGER :: j !< Index \f$j\f$ of the element
```

#### 7.4.2.2 integer tensor::coolist\_elem::k [private]

Index  $k$  of the element.

Definition at line 22 of file tensor.f90.

```
22      INTEGER :: k !< Index \f$k\f$ of the element
```

#### 7.4.2.3 real(kind=8) tensor::coolist\_elem::v [private]

Value of the element.

Definition at line 23 of file tensor.f90.

```
23      REAL(KIND=8) :: v !< Value of the element
```

The documentation for this type was generated from the following file:

- [tensor.f90](#)

## 7.5 inprod\_analytic::ocean\_tensors Type Reference

Type holding the oceanic inner products tensors.

### Private Attributes

- procedure([calculate\\_k](#)), pointer, nopass [k](#)
- procedure([calculate\\_m](#)), pointer, nopass [m](#)
- procedure([calculate\\_c\\_oc](#)), pointer, nopass [c](#)
- procedure([calculate\\_n](#)), pointer, nopass [n](#)
- procedure([calculate\\_o](#)), pointer, nopass [o](#)
- procedure([calculate\\_w](#)), pointer, nopass [w](#)

### 7.5.1 Detailed Description

Type holding the oceanic inner products tensors.

Definition at line 63 of file inprod\_analytic.f90.

### 7.5.2 Member Data Documentation

#### 7.5.2.1 procedure([calculate\\_c\\_oc](#)), pointer, nopass inprod\_analytic::ocean\_tensors::c [private]

Definition at line 66 of file inprod\_analytic.f90.

```
66      PROCEDURE(calculate_c_oc), POINTER, NOPASS :: c
```

#### 7.5.2.2 procedure([calculate\\_k](#)), pointer, nopass inprod\_analytic::ocean\_tensors::k [private]

Definition at line 64 of file inprod\_analytic.f90.

```
64      PROCEDURE(calculate_k), POINTER, NOPASS :: k
```

#### 7.5.2.3 procedure([calculate\\_m](#)), pointer, nopass inprod\_analytic::ocean\_tensors::m [private]

Definition at line 65 of file inprod\_analytic.f90.

```
65      PROCEDURE(calculate_m), POINTER, NOPASS :: m
```

#### 7.5.2.4 procedure([calculate\\_n](#)), pointer, nopass inprod\_analytic::ocean\_tensors::n [private]

Definition at line 67 of file inprod\_analytic.f90.

```
67      PROCEDURE(calculate_n), POINTER, NOPASS :: n
```

7.5.2.5 `procedure(calculate_o), pointer, nopass inprod_analytic::ocean_tensors::o` [private]

Definition at line 68 of file inprod\_analytic.f90.

```
68      PROCEDURE(calculate_o), POINTER, NOPASS :: o
```

7.5.2.6 `procedure(calculate_w), pointer, nopass inprod_analytic::ocean_tensors::w` [private]

Definition at line 69 of file inprod\_analytic.f90.

```
69      PROCEDURE(calculate_w), POINTER, NOPASS :: w
```

The documentation for this type was generated from the following file:

- [inprod\\_analytic.f90](#)

## 7.6 inprod\_analytic::ocean\_wavenum Type Reference

Oceanic bloc specification type.

### Private Attributes

- integer `p`
- integer `h`
- real(kind=8) `nx`
- real(kind=8) `ny`

### 7.6.1 Detailed Description

Oceanic bloc specification type.

Definition at line 47 of file inprod\_analytic.f90.

### 7.6.2 Member Data Documentation

7.6.2.1 `integer inprod_analytic::ocean_wavenum::h` [private]

Definition at line 48 of file inprod\_analytic.f90.

7.6.2.2 `real(kind=8) inprod_analytic::ocean_wavenum::nx` [private]

Definition at line 49 of file inprod\_analytic.f90.

```
49      REAL(KIND=8) :: nx, ny
```

**7.6.2.3** `real(kind=8) inprod_analytic::ocean_wavenum::ny` `[private]`

Definition at line 49 of file `inprod_analytic.f90`.

**7.6.2.4** `integer inprod_analytic::ocean_wavenum::p` `[private]`

Definition at line 48 of file `inprod_analytic.f90`.

```
48      INTEGER :: p,h
```

The documentation for this type was generated from the following file:

- [inprod\\_analytic.f90](#)

## Chapter 8

# File Documentation

### 8.1 aotensor\_def.f90 File Reference

#### Modules

- module [aotensor\\_def](#)

*The equation tensor for the coupled ocean-atmosphere model with temperature which allows for an extensible set of modes in the ocean and in the atmosphere.*

#### Functions/Subroutines

- integer function [aotensor\\_def::psi](#) (i)  
*Translate the  $\psi_{a,i}$  coefficients into effective coordinates.*
- integer function [aotensor\\_def::theta](#) (i)  
*Translate the  $\theta_{a,i}$  coefficients into effective coordinates.*
- integer function [aotensor\\_def::a](#) (i)  
*Translate the  $\psi_{o,i}$  coefficients into effective coordinates.*
- integer function [aotensor\\_def::t](#) (i)  
*Translate the  $\delta T_{o,i}$  coefficients into effective coordinates.*
- integer function [aotensor\\_def::kdelta](#) (i, j)  
*Kronecker delta function.*
- subroutine [aotensor\\_def::coeff](#) (i, j, k, v)  
*Subroutine to add element in the [aotensor](#)  $\mathcal{T}_{i,j,k}$  structure.*
- subroutine [aotensor\\_def::add\\_count](#) (i, j, k, v)  
*Subroutine to count the elements of the [aotensor](#)  $\mathcal{T}_{i,j,k}$ . Add +1 to `count_elems(i)` for each value that is added to the tensor  $i$ -th component.*
- subroutine [aotensor\\_def::compute\\_aotensor](#) (func)  
*Subroutine to compute the tensor [aotensor](#).*
- subroutine, public [aotensor\\_def::init\\_aotensor](#)  
*Subroutine to initialise the [aotensor](#) tensor.*

## Variables

- integer, dimension(:), allocatable [aotensor\\_def::count\\_elems](#)  
*Vector used to count the tensor elements.*
- real(kind=8), parameter [aotensor\\_def::real\\_eps](#) = 2.2204460492503131e-16  
*Epsilon to test equality with 0.*
- type(coolist), dimension(:), allocatable, public [aotensor\\_def::aotensor](#)  
 $\mathcal{T}_{i,j,k}$  - Tensor representation of the tendencies.

## 8.2 doc/gen\_doc.md File Reference

## 8.3 doc/tl\_ad\_doc.md File Reference

## 8.4 ic\_def.f90 File Reference

### Modules

- module [ic\\_def](#)  
*Module to load the initial condition.*

### Functions/Subroutines

- subroutine, public [ic\\_def::load\\_ic](#)  
*Subroutine to load the initial condition if IC.nml exists. If it does not, then write IC.nml with 0 as initial condition.*

### Variables

- logical [ic\\_def::exists](#)  
*Boolean to test for file existence.*
- real(kind=8), dimension(:), allocatable, public [ic\\_def::ic](#)  
*Initial condition vector.*

## 8.5 icdelta\_def.f90 File Reference

### Modules

- module [icdelta\\_def](#)  
*Module to load the perturbation initial condition.*

### Functions/Subroutines

- subroutine, public [icdelta\\_def::load\\_icdelta](#)  
*Subroutine to load the initial condition if ICdelta.nml exists. If it does not, then write ICdelta.nml with random initial condition.*

## Variables

- logical `icdelta_def::exists`  
*Boolean to test for file existence.*
- real(kind=8), dimension(:), allocatable, public `icdelta_def::icdelta`  
*Initial condition vector.*

## 8.6 inprod\_analytic.f90 File Reference

### Data Types

- type `inprod_analytic::atm_wavenum`  
*Atmospheric bloc specification type.*
- type `inprod_analytic::ocean_wavenum`  
*Oceanic bloc specification type.*
- type `inprod_analytic::atm_tensors`  
*Type holding the atmospheric inner products tensors.*
- type `inprod_analytic::ocean_tensors`  
*Type holding the oceanic inner products tensors.*

### Modules

- module `inprod_analytic`  
*Inner products between the truncated set of basis functions for the ocean and atmosphere streamfunction fields. These are partly calculated using the analytical expressions from Cehelsky, P., & Tung, K. K. : Theories of multiple equilibria and weather regimes-A critical reexamination. Part II: Baroclinic two-layer models. Journal of the atmospheric sciences, 44(21), 3282-3303, 1987.*

### Functions/Subroutines

- real(kind=8) function `inprod_analytic::b1` (Pi, Pj, Pk)  
*Cehelsky & Tung Helper functions.*
- real(kind=8) function `inprod_analytic::b2` (Pi, Pj, Pk)  
*Cehelsky & Tung Helper functions.*
- real(kind=8) function `inprod_analytic::delta` (r)  
*Integer Dirac delta function.*
- real(kind=8) function `inprod_analytic::flambda` (r)  
*"Odd or even" function*
- real(kind=8) function `inprod_analytic::s1` (Pj, Pk, Mj, Hk)  
*Cehelsky & Tung Helper functions.*
- real(kind=8) function `inprod_analytic::s2` (Pj, Pk, Mj, Hk)  
*Cehelsky & Tung Helper functions.*
- real(kind=8) function `inprod_analytic::s3` (Pj, Pk, Hj, Hk)  
*Cehelsky & Tung Helper functions.*
- real(kind=8) function `inprod_analytic::s4` (Pj, Pk, Hj, Hk)  
*Cehelsky & Tung Helper functions.*
- real(kind=8) function `inprod_analytic::calculate_a` (i, j)  
*Eigenvalues of the Laplacian (atmospheric)*

- real(kind=8) function [inprod\\_analytic::calculate\\_b](#) (i, j, k)  
*Streamfunction advection terms (atmospheric)*
- real(kind=8) function [inprod\\_analytic::calculate\\_c\\_atm](#) (i, j)  
*Beta term for the atmosphere.*
- real(kind=8) function [inprod\\_analytic::calculate\\_d](#) (i, j)  
*Forcing of the ocean on the atmosphere.*
- real(kind=8) function [inprod\\_analytic::calculate\\_g](#) (i, j, k)  
*Temperature advection terms (atmospheric)*
- real(kind=8) function [inprod\\_analytic::calculate\\_s](#) (i, j)  
*Forcing (thermal) of the ocean on the atmosphere.*
- real(kind=8) function [inprod\\_analytic::calculate\\_k](#) (i, j)  
*Forcing of the atmosphere on the ocean.*
- real(kind=8) function [inprod\\_analytic::calculate\\_m](#) (i, j)  
*Forcing of the ocean fields on the ocean.*
- real(kind=8) function [inprod\\_analytic::calculate\\_n](#) (i, j)  
*Beta term for the ocean.*
- real(kind=8) function [inprod\\_analytic::calculate\\_o](#) (i, j, k)  
*Temperature advection term (passive scalar)*
- real(kind=8) function [inprod\\_analytic::calculate\\_c\\_oc](#) (i, j, k)  
*Streamfunction advection terms (oceanic)*
- real(kind=8) function [inprod\\_analytic::calculate\\_w](#) (i, j)  
*Short-wave radiative forcing of the ocean.*
- subroutine, public [inprod\\_analytic::init\\_inprod](#)  
*Initialisation of the inner product.*

## Variables

- type(atm\_wavenum), dimension(:), allocatable, public [inprod\\_analytic::awavenum](#)  
*Atmospheric blocs specification.*
- type(ocean\_wavenum), dimension(:), allocatable, public [inprod\\_analytic::owavenum](#)  
*Oceanic blocs specification.*
- type(atm\_tensors), public [inprod\\_analytic::atmos](#)  
*Atmospheric tensors.*
- type(ocean\_tensors), public [inprod\\_analytic::ocean](#)  
*Oceanic tensors.*

## 8.7 LICENSE.txt File Reference

### Functions

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## 8.8 lyap\_stat.f90 File Reference

### Modules

- module [lyap\\_stat](#)  
*Statistics accumulators for the Lyapunov exponents.*

## Functions/Subroutines

- subroutine, public [lyap\\_stat::lyap\\_init\\_stat](#)  
*Initialise the accumulators.*
- subroutine, public [lyap\\_stat::lyap\\_acc](#) (x)  
*Accumulate one state.*
- real(kind=8) function, dimension(0:ndim), public [lyap\\_stat::lyap\\_mean](#) ()  
*Function returning the mean.*
- real(kind=8) function, dimension(0:ndim), public [lyap\\_stat::lyap\\_var](#) ()  
*Function returning the variance.*
- integer function, public [lyap\\_stat::lyap\\_iter](#) ()  
*Function returning the number of data accumulated.*
- subroutine, public [lyap\\_stat::lyap\\_reset](#)  
*Routine resetting the accumulators.*

## Variables

- integer [lyap\\_stat::i](#) =0  
*Number of stats accumulated.*
- real(kind=8), dimension(:), allocatable [lyap\\_stat::m](#)  
*Vector storing the inline mean.*
- real(kind=8), dimension(:), allocatable [lyap\\_stat::mprev](#)  
*Previous mean vector.*
- real(kind=8), dimension(:), allocatable [lyap\\_stat::v](#)  
*Vector storing the inline variance.*
- real(kind=8), dimension(:), allocatable [lyap\\_stat::mtmp](#)

## 8.9 lyap\_vectors.f90 File Reference

### Modules

- module [lyap\\_vectors](#)  
*Module for computation of Lyapunov exponents and vectors.*

### Functions/Subroutines

- subroutine, public [lyap\\_vectors::init\\_lyap](#)  
*Initialize Lyapunov computation (possibly also vectors in later version) and initializes also a random orthogonal matrix for the matrix ensemble.*
- subroutine, public [lyap\\_vectors::multiply\\_prop](#) (prop\_mul)  
*Multiplies prop\_mul from the left with the prop matrix defined in this module and saves the result to prop\_mul.*
- subroutine, public [lyap\\_vectors::benettin\\_step](#)  
*Performs the benettin step in integration. Multiplies the aggregated propagators in prop with ensemble and performs QR decomposition (Gram-Schmidt orthogonalization gives Q and upper triangular matrix R). Computes also the Lyapunov exponents via the diagonal of R. WATCH OUT: prop is changed during the subroutine and restored to a unit matrix.*
- subroutine, public [lyap\\_vectors::get\\_lyap\\_state](#) (prop\_ret, ensemble\_ret)  
*Routine that returns the current global propagator and ensemble of lyapunov vectors.*

## Variables

- real(kind=8), dimension(:), allocatable, public [lyap\\_vectors::loclap](#)  
*Buffer containing the local Lyapunov exponent.*
- real(kind=8), dimension(:), allocatable, public [lyap\\_vectors::lyapunov](#)  
*Buffer containing the averaged Lyapunov exponent.*
- real(kind=8), dimension(:, :), allocatable, public [lyap\\_vectors::ensemble](#)  
*Buffer containing the QR decompsition of the ensemble.*
- real(kind=8), dimension(:, :), allocatable [lyap\\_vectors::prop](#)  
*Buffer holding the propagator matrix.*
- integer [lyap\\_vectors::lwork](#)
- real(kind=8), dimension(:), allocatable [lyap\\_vectors::work](#)  
*Temporary buffer for QR decomposition.*
- real(kind=8), dimension(:), allocatable [lyap\\_vectors::work2](#)  
*Temporary buffer for QR decomposition.*
- real(kind=8), dimension(:), allocatable [lyap\\_vectors::tau](#)  
*Temporary buffer for QR decomposition.*
- real(kind=8), dimension(:, :), allocatable [lyap\\_vectors::prop\\_buf](#)  
*Buffer holding the local propagator matrix.*

## 8.10 maoam.f90 File Reference

### Functions/Subroutines

- program [maoam](#)  
*Fortran 90 implementation of the modular arbitrary-order ocean-atmosphere model MAOOAM.*

### 8.10.1 Function/Subroutine Documentation

#### 8.10.1.1 program maoam ( )

Fortran 90 implementation of the modular arbitrary-order ocean-atmosphere model MAOOAM.

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Definition at line 13 of file maoam.f90.

## 8.11 maoam\_lyap.f90 File Reference

### Functions/Subroutines

- program [maoam\\_lyap](#)  
*Fortran 90 implementation of the modular arbitrary-order ocean-atmosphere model MAOOAM computing the Lyapunov spectrum.*

### 8.11.1 Function/Subroutine Documentation

#### 8.11.1.1 program maoam\_lyap ( )

Fortran 90 implementation of the modular arbitrary-order ocean-atmosphere model MAOOAM computing the Lyapunov spectrum.

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Definition at line 13 of file maoam\_lyap.f90.

## 8.12 maoam\_lyap\_div.f90 File Reference

### Functions/Subroutines

- program [maoam\\_lyap\\_div](#)

*Fortran 90 implementation of the modular arbitrary-order ocean-atmosphere model MAOOAM computing the first Lyapunov exponent with the divergence method.*

### 8.12.1 Function/Subroutine Documentation

#### 8.12.1.1 program maoam\_lyap\_div ( )

Fortran 90 implementation of the modular arbitrary-order ocean-atmosphere model MAOOAM computing the first Lyapunov exponent with the divergence method.

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Definition at line 14 of file maoam\_lyap\_div.f90.

## 8.13 params.f90 File Reference

### Modules

- module [params](#)

*The model parameters module.*

### Functions/Subroutines

- subroutine, private [params::init\\_nml](#)

*Read the basic parameters and mode selection from the namelist.*

- subroutine [params::init\\_params](#)

*Parameters initialisation routine.*



## Variables

- real(kind=8) [params::n](#)  
 $n = 2L_y / L_x$  - Aspect ratio
- real(kind=8) [params::phi0](#)  
Latitude in radian.
- real(kind=8) [params::rra](#)  
Earth radius.
- real(kind=8) [params::sig0](#)  
 $\sigma_0$  - Non-dimensional static stability of the atmosphere.
- real(kind=8) [params::k](#)  
Bottom atmospheric friction coefficient.
- real(kind=8) [params::kp](#)  
 $k'$  - Internal atmospheric friction coefficient.
- real(kind=8) [params::r](#)  
Frictional coefficient at the bottom of the ocean.
- real(kind=8) [params::d](#)  
Mechanical coupling parameter between the ocean and the atmosphere.
- real(kind=8) [params::f0](#)  
 $f_0$  - Coriolis parameter
- real(kind=8) [params::gp](#)  
 $g'$  Reduced gravity
- real(kind=8) [params::h](#)  
Depth of the active water layer of the ocean.
- real(kind=8) [params::phi0\\_npi](#)  
Latitude exprimed in fraction of pi.
- real(kind=8) [params::lambda](#)  
 $\lambda$  - Sensible + turbulent heat exchange between the ocean and the atmosphere.
- real(kind=8) [params::co](#)  
 $C_a$  - Constant short-wave radiation of the ocean.
- real(kind=8) [params::go](#)  
 $\gamma_o$  - Specific heat capacity of the ocean.
- real(kind=8) [params::ca](#)  
 $C_a$  - Constant short-wave radiation of the atmosphere.
- real(kind=8) [params::to0](#)  
 $T_o^0$  - Stationary solution for the 0-th order ocean temperature.
- real(kind=8) [params::ta0](#)  
 $T_a^0$  - Stationary solution for the 0-th order atmospheric temperature.
- real(kind=8) [params::epsa](#)  
 $\epsilon_a$  - Emissivity coefficient for the grey-body atmosphere.
- real(kind=8) [params::ga](#)  
 $\gamma_a$  - Specific heat capacity of the atmosphere.
- real(kind=8) [params::rr](#)  
 $R$  - Gas constant of dry air
- real(kind=8) [params::scale](#)  
 $L_y = L \pi$  - The characteristic space scale.
- real(kind=8) [params::pi](#)  
 $\pi$
- real(kind=8) [params::lr](#)  
 $L_R$  - Rossby deformation radius
- real(kind=8) [params::g](#)

- $\gamma$ 
  - real(kind=8) [params::rp](#)  
*r'* - Frictional coefficient at the bottom of the ocean.
- real(kind=8) [params::dp](#)  
*d'* - Non-dimensional mechanical coupling parameter between the ocean and the atmosphere.
- real(kind=8) [params::kd](#)  
*k<sub>d</sub>* - Non-dimensional bottom atmospheric friction coefficient.
- real(kind=8) [params::kdp](#)  
*k'<sub>d</sub>* - Non-dimensional internal atmospheric friction coefficient.
- real(kind=8) [params::cpo](#)  
*C'<sub>a</sub>* - Non-dimensional constant short-wave radiation of the ocean.
- real(kind=8) [params::lpo](#)  
*λ'<sub>o</sub>* - Non-dimensional sensible + turbulent heat exchange from ocean to atmosphere.
- real(kind=8) [params::cpa](#)  
*C'<sub>a</sub>* - Non-dimensional constant short-wave radiation of the atmosphere.
- real(kind=8) [params::lpa](#)  
*λ'<sub>a</sub>* - Non-dimensional sensible + turbulent heat exchange from atmosphere to ocean.
- real(kind=8) [params::sbpo](#)  
*σ'<sub>B,o</sub>* - Long wave radiation lost by ocean to atmosphere & space.
- real(kind=8) [params::sbpa](#)  
*σ'<sub>B,a</sub>* - Long wave radiation from atmosphere absorbed by ocean.
- real(kind=8) [params::lsbpo](#)  
*S'<sub>B,o</sub>* - Long wave radiation from ocean absorbed by atmosphere.
- real(kind=8) [params::lsbpa](#)  
*S'<sub>B,a</sub>* - Long wave radiation lost by atmosphere to space & ocean.
- real(kind=8) [params::l](#)  
*L* - Domain length scale
- real(kind=8) [params::sc](#)  
Ratio of surface to atmosphere temperature.
- real(kind=8) [params::sb](#)  
Stefan–Boltzmann constant.
- real(kind=8) [params::betp](#)  
*β'* - Non-dimensional beta parameter
- real(kind=8) [params::nua](#) =0.D0  
Dissipation in the atmosphere.
- real(kind=8) [params::nuo](#) =0.D0  
Dissipation in the ocean.
- real(kind=8) [params::nuap](#)  
Non-dimensional dissipation in the atmosphere.
- real(kind=8) [params::nuop](#)  
Non-dimensional dissipation in the ocean.
- real(kind=8) [params::t\\_trans](#)  
Transient time period.
- real(kind=8) [params::t\\_run](#)  
Effective intergration time (length of the generated trajectory)
- real(kind=8) [params::dt](#)  
Integration time step.
- real(kind=8) [params::tw](#)  
Write all variables every tw time units.
- logical [params::writeout](#)  
Write to file boolean.

- real(kind=8) [params::rescaling\\_time](#)  
*Rescaling time for the Lyapunov computation.*
- integer [params::nboc](#)  
*Number of atmospheric blocks.*
- integer [params::nbatm](#)  
*Number of oceanic blocks.*
- integer [params::natm](#) =0  
*Number of atmospheric basis functions.*
- integer [params::noc](#) =0  
*Number of oceanic basis functions.*
- integer [params::ndim](#)  
*Number of variables (dimension of the model)*
- integer, dimension(:,:), allocatable [params::oms](#)  
*Ocean mode selection array.*
- integer, dimension(:,:), allocatable [params::ams](#)  
*Atmospheric mode selection array.*

## 8.14 rk2\_integrator.f90 File Reference

### Modules

- module [integrator](#)  
*Module with the integration routines.*

### Functions/Subroutines

- subroutine, public [integrator::init\\_integrator](#)  
*Routine to initialise the integration buffers.*
- subroutine [integrator::tendencies](#) (t, y, res)  
*Routine computing the tendencies of the model.*
- subroutine, public [integrator::step](#) (y, t, dt, res)  
*Routine to perform an integration step (Heun algorithm). The incremented time is returned.*

### Variables

- real(kind=8), dimension(:), allocatable [integrator::buf\\_y1](#)  
*Buffer to hold the intermediate position (Heun algorithm)*
- real(kind=8), dimension(:), allocatable [integrator::buf\\_f0](#)  
*Buffer to hold tendencies at the initial position.*
- real(kind=8), dimension(:), allocatable [integrator::buf\\_f1](#)  
*Buffer to hold tendencies at the intermediate position.*

## 8.15 rk2\_tl\_ad\_integrator.f90 File Reference

### Modules

- module [tl\\_ad\\_integrator](#)  
*Tangent Linear (TL) and Adjoint (AD) model versions of MAOOAM. Integrators module.*

## Functions/Subroutines

- subroutine [tl\\_ad\\_integrator::tendencies](#) (t, y, res)  
*Routine computing the tendencies of the nonlinear model.*
- subroutine, public [tl\\_ad\\_integrator::init\\_tl\\_ad\\_integrator](#)  
*Routine to initialise the integration buffers.*
- subroutine, public [tl\\_ad\\_integrator::ad\\_step](#) (y, ystar, t, dt, res)  
*Routine to perform an integration step (Heun algorithm) of the adjoint model. The incremented time is returned.*
- subroutine, public [tl\\_ad\\_integrator::evolve\\_ad\\_step](#) (y, deltax, t, dt, ynew, deltaxnew)  
*Routine to perform a simultaneous integration step (RK4 algorithm) of the nonlinear and adjoint together. The incremented time is returned.*
- subroutine, public [tl\\_ad\\_integrator::tl\\_step](#) (y, ystar, t, dt, res)  
*Routine to perform an integration step (Heun algorithm) of the tangent linear model. The incremented time is returned.*
- subroutine, public [tl\\_ad\\_integrator::evolve\\_tl\\_step](#) (y, deltax, t, dt, ynew, deltaxnew)  
*Routine to perform a simultaneous integration step (RK4 algorithm) of the nonlinear and the tangent linear model together. The incremented time is returned.*
- subroutine, public [tl\\_ad\\_integrator::prop\\_step](#) (y, propagator, t, dt, ynew, adjoint)  
*Routine to perform a simultaneously an integration step (Heun algorithm) of the nonlinear and computes the Heun tangent linear propagator. The boolean variable adjoint allows for an adjoint forward integration. The incremented time is returned.*

## Variables

- real(kind=8), dimension(:), allocatable [tl\\_ad\\_integrator::buf\\_y1](#)  
*Buffer to hold the intermediate position (Heun algorithm) of the tangent linear model.*
- real(kind=8), dimension(:), allocatable [tl\\_ad\\_integrator::buf\\_f0](#)  
*Buffer to hold tendencies at the initial position of the tangent linear model.*
- real(kind=8), dimension(:), allocatable [tl\\_ad\\_integrator::buf\\_f1](#)  
*Buffer to hold tendencies at the intermediate position of the tangent linear model.*
- real(kind=8), dimension(:), allocatable [tl\\_ad\\_integrator::buf\\_y11](#)  
*Buffer to hold the intermediate position (Heun algorithm) of the tangent linear model.*
- real(kind=8), dimension(:), allocatable [tl\\_ad\\_integrator::buf\\_f00](#)  
*Buffer to hold tendencies at the initial position of the tangent linear model.*
- real(kind=8), dimension(:), allocatable [tl\\_ad\\_integrator::buf\\_f11](#)  
*Buffer to hold tendencies at the intermediate position of the tangent linear model.*
- real(kind=8), dimension(:, :), allocatable [tl\\_ad\\_integrator::buf\\_j1](#)  
*Buffer to hold jacobians in the RK4 scheme for the tangent linear model.*
- real(kind=8), dimension(:, :), allocatable [tl\\_ad\\_integrator::buf\\_j2](#)  
*Buffer to hold jacobians in the RK4 scheme for the tangent linear model.*
- real(kind=8), dimension(:, :), allocatable [tl\\_ad\\_integrator::buf\\_j1h](#)  
*Buffer to hold jacobians in the RK4 scheme for the tangent linear model.*
- real(kind=8), dimension(:, :), allocatable [tl\\_ad\\_integrator::buf\\_j2h](#)  
*Buffer to hold jacobians in the RK4 scheme for the tangent linear model.*
- real(kind=8), dimension(:, :), allocatable [tl\\_ad\\_integrator::one](#)  
*unit matrix*

## 8.16 rk4\_integrator.f90 File Reference

### Modules

- module [integrator](#)  
*Module with the integration routines.*

## Functions/Subroutines

- subroutine, public [integrator::init\\_integrator](#)  
*Routine to initialise the integration buffers.*
- subroutine [integrator::tendencies](#) (t, y, res)  
*Routine computing the tendencies of the model.*
- subroutine, public [integrator::step](#) (y, t, dt, res)  
*Routine to perform an integration step (Heun algorithm). The incremented time is returned.*

## Variables

- real(kind=8), dimension(:), allocatable [integrator::buf\\_ka](#)  
*Buffer A to hold tendencies.*
- real(kind=8), dimension(:), allocatable [integrator::buf\\_kb](#)  
*Buffer B to hold tendencies.*

## 8.17 rk4\_tl\_ad\_integrator.f90 File Reference

### Modules

- module [tl\\_ad\\_integrator](#)  
*Tangent Linear (TL) and Adjoint (AD) model versions of MAOOAM. Integrators module.*

### Functions/Subroutines

- subroutine [tl\\_ad\\_integrator::tendencies](#) (t, y, res)  
*Routine computing the tendencies of the nonlinear model.*
- subroutine, public [tl\\_ad\\_integrator::init\\_tl\\_ad\\_integrator](#)  
*Routine to initialise the integration buffers.*
- subroutine, public [tl\\_ad\\_integrator::ad\\_step](#) (y, ystar, t, dt, res)  
*Routine to perform an integration step (Heun algorithm) of the adjoint model. The incremented time is returned.*
- subroutine, public [tl\\_ad\\_integrator::evolve\\_ad\\_step](#) (y, deltat, t, dt, ynew, deltatnew)  
*Routine to perform a simultaneous integration step (RK4 algorithm) of the nonlinear and adjoint together. The incremented time is returned.*
- subroutine, public [tl\\_ad\\_integrator::tl\\_step](#) (y, ystar, t, dt, res)  
*Routine to perform an integration step (Heun algorithm) of the tangent linear model. The incremented time is returned.*
- subroutine, public [tl\\_ad\\_integrator::evolve\\_tl\\_step](#) (y, deltat, t, dt, ynew, deltatnew)  
*Routine to perform a simultaneous integration step (RK4 algorithm) of the nonlinear and the tangent linear model together. The incremented time is returned.*
- subroutine, public [tl\\_ad\\_integrator::prop\\_step](#) (y, propagator, t, dt, ynew, adjoint)  
*Routine to perform a simultaneously an integration step (Heun algorithm) of the nonlinear and computes the Heun tangent linear propagator. The boolean variable adjoint allows for an adjoint forward integration. The incremented time is returned.*

## Variables

- real(kind=8), dimension(:), allocatable [tl\\_ad\\_integrator::buf\\_ka](#)  
*Buffer to hold tendencies in the RK4 scheme for the tangent linear model.*
- real(kind=8), dimension(:), allocatable [tl\\_ad\\_integrator::buf\\_kb](#)  
*Buffer to hold tendencies in the RK4 scheme for the tangent linear model.*
- real(kind=8), dimension(:), allocatable [tl\\_ad\\_integrator::buf\\_kc](#)  
*Buffer to hold tendencies in the RK4 scheme for the tangent linear model.*
- real(kind=8), dimension(:), allocatable [tl\\_ad\\_integrator::buf\\_kd](#)  
*Buffer to hold tendencies in the RK4 scheme for the tangent linear model.*
- real(kind=8), dimension(:, :), allocatable [tl\\_ad\\_integrator::buf\\_j3](#)  
*Buffer to hold jacobians in the RK4 scheme for the tangent linear model.*
- real(kind=8), dimension(:, :), allocatable [tl\\_ad\\_integrator::buf\\_j4](#)  
*Buffer to hold jacobians in the RK4 scheme for the tangent linear model.*
- real(kind=8), dimension(:, :), allocatable [tl\\_ad\\_integrator::buf\\_j3h](#)  
*Buffer to hold jacobians in the RK4 scheme for the tangent linear model.*
- real(kind=8), dimension(:, :), allocatable [tl\\_ad\\_integrator::buf\\_j4h](#)  
*Buffer to hold jacobians in the RK4 scheme for the tangent linear model.*
- real(kind=8), dimension(:), allocatable [tl\\_ad\\_integrator::buf\\_kaa](#)  
*Buffer to hold tendencies in the RK4 scheme for the tangent linear model.*
- real(kind=8), dimension(:), allocatable [tl\\_ad\\_integrator::buf\\_kbb](#)  
*Buffer to hold tendencies in the RK4 scheme for the tangent linear model.*

## 8.18 stat.f90 File Reference

### Modules

- module [stat](#)  
*Statistics accumulators.*

### Functions/Subroutines

- subroutine, public [stat::init\\_stat](#)  
*Initialise the accumulators.*
- subroutine, public [stat::acc](#) (x)  
*Accumulate one state.*
- real(kind=8) function, dimension(0:ndim), public [stat::mean](#) ()  
*Function returning the mean.*
- real(kind=8) function, dimension(0:ndim), public [stat::var](#) ()  
*Function returning the variance.*
- integer function, public [stat::iter](#) ()  
*Function returning the number of data accumulated.*
- subroutine, public [stat::reset](#)  
*Routine resetting the accumulators.*

## Variables

- integer `stat::i` = 0  
*Number of stats accumulated.*
- real(kind=8), dimension(:), allocatable `stat::m`  
*Vector storing the inline mean.*
- real(kind=8), dimension(:), allocatable `stat::mprev`  
*Previous mean vector.*
- real(kind=8), dimension(:), allocatable `stat::v`  
*Vector storing the inline variance.*
- real(kind=8), dimension(:), allocatable `stat::mtmp`

## 8.19 tensor.f90 File Reference

### Data Types

- type `tensor::coolist_elem`  
*Coordinate list element type. Elementary elements of the sparse tensors.*
- type `tensor::coolist`  
*Coordinate list. Type used to represent the sparse tensor.*

### Modules

- module `tensor`  
*Tensor utility module.*

### Functions/Subroutines

- subroutine, public `tensor::copy_coo` (src, dst)  
*Routine to copy a coolist.*
- subroutine, public `tensor::mat_to_coo` (src, dst)  
*Routine to convert a matrix to a tensor.*
- subroutine, public `tensor::sparse_mul3` (coolist\_ijk, arr\_j, arr\_k, res)  
*Sparse multiplication of a tensor with two vectors:  $\sum_{j,k=0}^{ndim} \mathcal{T}_{i,j,k} a_j b_k$ .*
- subroutine, public `tensor::jsparse_mul` (coolist\_ijk, arr\_j, jcoo\_ij)  
*Sparse multiplication of two tensors to determine the Jacobian:*

$$J_{i,j} = \sum_{k=0}^{ndim} (\mathcal{T}_{i,j,k} + \mathcal{T}_{i,k,j}) a_k.$$

*It's implemented slightly differently: for every  $\mathcal{T}_{i,j,k}$ , we add to  $J_{i,j}$  as follows:*

$$J_{i,j} = J_{i,j} + \mathcal{T}_{i,j,k} a_k \quad J_{i,k} = J_{i,k} + \mathcal{T}_{i,j,k} a_j$$

*This version return a coolist (sparse tensor).*

- subroutine, public `tensor::jsparse_mul_mat` (coolist\_ijk, arr\_j, jcoo\_ij)

*Sparse multiplication of two tensors to determine the Jacobian:*

$$J_{i,j} = \sum_{k=0}^{ndim} (\mathcal{T}_{i,j,k} + \mathcal{T}_{i,k,j}) a_k.$$

*It's implemented slightly differently: for every  $\mathcal{T}_{i,j,k}$ , we add to  $J_{i,j}$  as follows:*

$$J_{i,j} = J_{i,j} + \mathcal{T}_{i,j,k} a_k J_{i,k} = J_{i,k} + \mathcal{T}_{i,j,k} a_j$$

*This version return a matrix.*

- subroutine, public [tensor::sparse\\_mul2](#) (coolist\_ij, arr\_j, res)

*Sparse multiplication of a 2d sparse tensor with a vector:  $\sum_{j=0}^{ndim} \mathcal{T}_{i,j,k} a_j$ .*

- subroutine, public [tensor::simplify](#) (tensor)

*Routine to simplify a coolist (sparse tensor). For each index  $i$ , it upper triangularize the matrix*

$$\mathcal{T}_{i,j,k} \quad 0 \leq j, k \leq ndim.$$

- subroutine, public [tensor::add\\_elem](#) (t, i, j, k, v)

*Subroutine to add element to a coolist.*

- subroutine, public [tensor::add\\_check](#) (t, i, j, k, v, dst)

*Subroutine to add element to a coolist and check for overflow. Once the t buffer tensor is full, add it to the destination buffer.*

- subroutine, public [tensor::add\\_to\\_tensor](#) (src, dst)

*Routine to add a rank-3 tensor to another one.*

- subroutine, public [tensor::print\\_tensor](#) (t, s)

*Routine to print a rank 3 tensor coolist.*

- subroutine, public [tensor::write\\_tensor\\_to\\_file](#) (s, t)

*Load a rank-4 tensor coolist from a file definition.*

- subroutine, public [tensor::load\\_tensor\\_from\\_file](#) (s, t)

*Load a rank-4 tensor coolist from a file definition.*

## Variables

- real(kind=8), parameter [tensor::real\\_eps](#) = 2.2204460492503131e-16

*Parameter to test the equality with zero.*

## 8.20 test\_aotensor.f90 File Reference

### Functions/Subroutines

- program [test\\_aotensor](#)

*Small program to print the inner products.*

### 8.20.1 Function/Subroutine Documentation

#### 8.20.1.1 program test\_aotensor ( )

Small program to print the inner products.

### Copyright

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Definition at line 13 of file test\_aotensor.f90.



## 8.21 test\_inprod\_analytic.f90 File Reference

### Functions/Subroutines

- program [inprod\\_analytic\\_test](#)  
*Small program to print the inner products.*

### 8.21.1 Function/Subroutine Documentation

#### 8.21.1.1 program inprod\_analytic\_test ( )

Small program to print the inner products.

#### Copyright

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#### Remarks

Print in the same order as test\_inprod.lua

Definition at line 18 of file test\_inprod\_analytic.f90.

## 8.22 test\_tl\_ad.f90 File Reference

### Functions/Subroutines

- program [test\\_tl\\_ad](#)  
*Tests for the Tangent Linear (TL) and Adjoint (AD) model versions of MAOOAM.*
- real(kind=8) function [gasdev](#) (idum)
- real(kind=8) function [ran2](#) (idum)

### 8.22.1 Function/Subroutine Documentation

#### 8.22.1.1 real(kind=8) function gasdev ( integer idum )

Definition at line 149 of file test\_tl\_ad.f90.

```

149  INTEGER :: idum
150  REAL(KIND=8) :: gasdev, ran2
151  !    USES ran2
152  INTEGER :: iset
153  REAL(KIND=8) :: fac, gset, rsq, v1, v2
154  SAVE iset, gset
155  DATA iset/0/
156  if (idum.lt.0) iset=0
157  if (iset.eq.0) then
158  1    v1=2.d0*ran2(idum)-1.
159      v2=2.d0*ran2(idum)-1.
160      rsq=v1**2+v2**2
161      if (rsq.ge.1.d0.or.rsq.eq.0.d0) goto 1
162      fac=sqrt(-2.*log(rsq)/rsq)
163      gset=v1*fac
164      gasdev=v2*fac
165      iset=1
166  else
167      gasdev=gset
168      iset=0
169  endif
170  return

```

### 8.22.1.2 `real(kind=8) function ran2 ( integer idum )`

Definition at line 174 of file `test_tl_ad.f90`.

```

174  INTEGER :: idum,im1,im2,imm1,ia1,ia2,iq1,iq2,ir1,ir2,ntab,ndiv
175  REAL(KIND=8) :: ran2,am,eps,rnm
176  parameter(im1=2147483563,im2=2147483399,am=1.d0/im1,imm1=im1-1&
177    &,ia1=40014,ia2=40692,iq1=53668,iq2=52774,ir1=12211,ir2&
178    &=3791,ntab=32,ndiv=1+imm1/ntab,eps=1.2d-7,rnm=1.d0-eps)
179  INTEGER :: idum2,j,k,iv(ntab),iy
180  SAVE iv,iy,idum2
181  DATA idum2/123456789/, iv/ntab*0/, iy/0/
182  if (idum.le.0) then
183    idum=max(-idum,1)
184    idum2=idum
185    do j=ntab+8,1,-1
186      k=idum/iq1
187      idum=ia1*(idum-k*iq1)-k*ir1
188      if (idum.lt.0) idum=idum+im1
189      if (j.le.ntab) iv(j)=idum
190    enddo
191    iy=iv(1)
192  endif
193  k=idum/iq1
194  idum=ia1*(idum-k*iq1)-k*ir1
195  if (idum.lt.0) idum=idum+im1
196  k=idum2/iq2
197  idum2=ia2*(idum2-k*iq2)-k*ir2
198  if (idum2.lt.0) idum2=idum2+im2
199  j=1+iy/ndiv
200  iy=iv(j)-idum2
201  iv(j)=idum
202  if (iy.lt.1) iy=iy+imm1
203  ran2=min(am*iy,rnm)
204  return

```

### 8.22.1.3 `program test_tl_ad ( )`

Tests for the Tangent Linear (TL) and Adjoint (AD) model versions of MAOOAM.

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Definition at line 14 of file `test_tl_ad.f90`.

## 8.23 `tl_ad_tensor.f90` File Reference

#### Modules

- module `tl_ad_tensor`

*Tangent Linear (TL) and Adjoint (AD) model versions of MAOOAM. Tensors definition module.*

## Functions/Subroutines

- type(coolist) function, dimension(ndim) [tl\\_ad\\_tensor::jacobian](#) (ystar)  
*Compute the Jacobian of MAOOAM in point ystar.*
- real(kind=8) function, dimension(ndim, ndim), public [tl\\_ad\\_tensor::jacobian\\_mat](#) (ystar)  
*Compute the Jacobian of MAOOAM in point ystar.*
- subroutine, public [tl\\_ad\\_tensor::init\\_tltensor](#)  
*Routine to initialize the TL tensor.*
- subroutine [tl\\_ad\\_tensor::compute\\_tltensor](#) (func)  
*Routine to compute the TL tensor from the original MAOOAM one.*
- subroutine [tl\\_ad\\_tensor::tl\\_add\\_count](#) (i, j, k, v)  
*Subroutine used to count the number of TL tensor entries.*
- subroutine [tl\\_ad\\_tensor::tl\\_coeff](#) (i, j, k, v)  
*Subroutine used to compute the TL tensor entries.*
- subroutine, public [tl\\_ad\\_tensor::init\\_adtensor](#)  
*Routine to initialize the AD tensor.*
- subroutine [tl\\_ad\\_tensor::compute\\_adtensor](#) (func)  
*Subroutine to compute the AD tensor from the original MAOOAM one.*
- subroutine [tl\\_ad\\_tensor::ad\\_add\\_count](#) (i, j, k, v)  
*Subroutine used to count the number of AD tensor entries.*
- subroutine [tl\\_ad\\_tensor::ad\\_coeff](#) (i, j, k, v)
- subroutine, public [tl\\_ad\\_tensor::init\\_adtensor\\_ref](#)  
*Alternate method to initialize the AD tensor from the TL tensor.*
- subroutine [tl\\_ad\\_tensor::compute\\_adtensor\\_ref](#) (func)  
*Alternate subroutine to compute the AD tensor from the TL one.*
- subroutine [tl\\_ad\\_tensor::ad\\_add\\_count\\_ref](#) (i, j, k, v)  
*Alternate subroutine used to count the number of AD tensor entries from the TL tensor.*
- subroutine [tl\\_ad\\_tensor::ad\\_coeff\\_ref](#) (i, j, k, v)  
*Alternate subroutine used to compute the AD tensor entries from the TL tensor.*
- subroutine, public [tl\\_ad\\_tensor::ad](#) (t, ystar, deltat, buf)  
*Tendencies for the AD of MAOOAM in point ystar for perturbation deltat.*
- subroutine, public [tl\\_ad\\_tensor::tl](#) (t, ystar, deltat, buf)  
*Tendencies for the TL of MAOOAM in point ystar for perturbation deltat.*

## Variables

- real(kind=8), parameter [tl\\_ad\\_tensor::real\\_eps](#) = 2.2204460492503131e-16  
*Epsilon to test equality with 0.*
- integer, dimension(:), allocatable [tl\\_ad\\_tensor::count\\_elems](#)  
*Vector used to count the tensor elements.*
- type(coolist), dimension(:), allocatable, public [tl\\_ad\\_tensor::tltensor](#)  
*Tensor representation of the Tangent Linear tendencies.*
- type(coolist), dimension(:), allocatable, public [tl\\_ad\\_tensor::adtensor](#)  
*Tensor representation of the Adjoint tendencies.*

## 8.24 tr\_jacob\_mat.f90 File Reference

### Functions/Subroutines

- program [tr\\_jacob\\_mat](#)  
*Tests to obtain the trace of the Jacobian matrix.*

## 8.24.1 Function/Subroutine Documentation

### 8.24.1.1 program tr\_jacob\_mat ( )

Tests to obtain the trace of the Jacobian matrix.

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Definition at line 13 of file tr\_jacob\_mat.f90.

## 8.25 util.f90 File Reference

### Modules

- module [util](#)  
*Utility module.*

### Functions/Subroutines

- character(len=20) function, public [util::str](#) (k)  
*Convert an integer to string.*
- character(len=40) function, public [util::rstr](#) (x, fm)  
*Convert a real to string with a given format.*
- integer function, dimension(size(s)), public [util::isin](#) (c, s)  
*Determine if a character is in a string and where.*
- subroutine, public [util::init\\_random\\_seed](#) ()  
*Random generator initialization routine.*
- integer function [lcg](#) (s)
- subroutine, public [util::piksort](#) (k, arr, par)  
*Simple card player sorting function.*
- subroutine, public [util::init\\_one](#) (A)  
*Initialize a square matrix A as a unit matrix.*

## 8.25.1 Function/Subroutine Documentation

### 8.25.1.1 integer function init\_random\_seed::lcg ( integer(int64) s )

Definition at line 102 of file util.f90.

```

102     integer :: lcg
103     integer(int64) :: s
104     IF (s == 0) THEN
105         s = 104729
106     ELSE
107         s = mod(s, 4294967296_int64)
108     END IF
109     s = mod(s * 279470273_int64, 4294967291_int64)
110     lcg = int(mod(s, int(huge(0), int64)), kind(0))
111 END FUNCTION lcg

```

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