Parallelizing for Performance with OpenMP

Stanford University, ME344 — Summer 2018

Andrey Vladimirov, Colfax International

Today

- Review example: numerical integration
  - Parallel loops
  - Mutexes
  - Reduction
- Advanced example: LU decomposition
  - Parallel dependencies
  - Tasks
  - Semaphores

Example 1: Numerical Integration

$$I(a,b) = \int_0^a f(x)\,\mathrm{d}x \approx \sum_{i=0}^{n-1} f\left(x_{\mathrm{i}+\frac{1}{2}}\right)\Delta x,$$

where

$$\Delta x = \frac{a}{n}, \quad x_{\mathrm{i}+\frac{1}{2}} = \left(i+\frac{1}{2}\right)\Delta x.$$

Serial Implementation

```
float IntegrateMyFunction(int const n, float const a, float  const b) {

  float const dx = (b-a)/float(n);              // Integration interval
  float I = 0.0f;                               // Running sum of the integral

  for (int i = 0; i < n; i++) {                 // Loop through the integration range
    float const x = a + dx*(float(i) + 0.5f); // Midpoint of the integration interval
    float const f = 1.0f/sqrtf(x);              // Function value at the midpoint
    I += f;                                     // Incrementing the running sum
  }
  I *= dx; // Scale according to the integration interval

  return I;
}
```

Incorrect Parallelization

```
float IntegrateMyFunction(int const n, float const a, float  const b) {

  float const dx = (b-a)/float(n);            // Integration interval
  float I = 0.0f;                             // Running sum of the integral

  #pragma omp parallel for
  for (int i = 0; i < n; i++) {               // Loop through the integration range
    float const x = a + dx*(float(i) + 0.5f); // Midpoint of the integration interval
    float const f = 1.0f/sqrtf(x);            // Function value at the midpoint
    I += f;                                   // Incrementing the running sum
  }
  I *= dx; // Scale according to the integration interval

  return I;
}
```

Critical Section: Correct but Very Slow

```
float IntegrateMyFunction(int const n, float const a, float  const b) {

  float const dx = (b-a)/float(n);                  // Integration interval
  float I = 0.0f;                                   // Running sum of the integral

  #pragma omp parallel for
  for (int i = 0; i < n; i++) {                     // Loop through the integration range
    float const x = a + dx*(float(i) + 0.5f); // Midpoint of the integration interval
    float const f = 1.0f/sqrtf(x);                  // Function value at the midpoint
    #pragma omp critical
    {
      I += f;                                       // Incrementing the running sum
    }
  }
  I *= dx; // Scale according to the integration interval

  return I;
}
```

Atomic Mutex: Correct but Slow

```
float IntegrateMyFunction(int const n, float const a, float  const b) {

  float const dx = (b-a)/float(n);                  // Integration interval
  float I = 0.0f;                                   // Running sum of the integral

  #pragma omp parallel for
  for (int i = 0; i < n; i++) {                     // Loop through the integration range
    float const x = a + dx*(float(i) + 0.5f); // Midpoint of the integration interval
    float const f = 1.0f/sqrtf(x);                  // Function value at the midpoint
    #pragma omp atomic
    I += f;                                       // Incrementing the running sum
  }
  I *= dx; // Scale according to the integration interval

  return I;
}
```

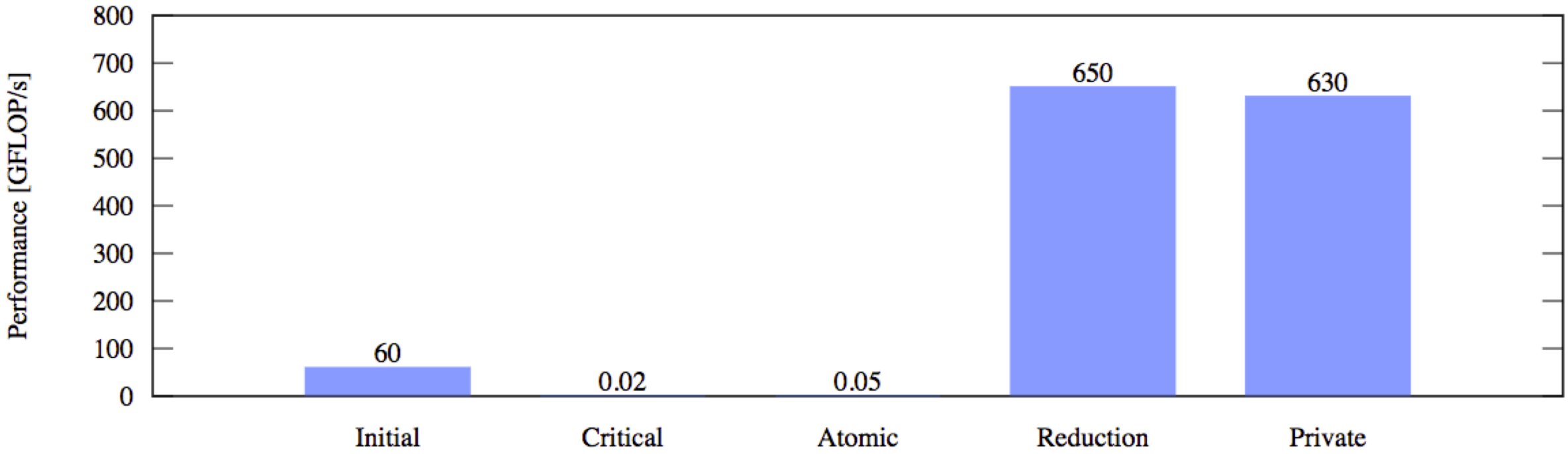
Reduction Clause: Correct and Fast

```
float IntegrateMyFunction(int const n, float const a, float  const b) {

  float const dx = (b-a)/float(n);              // Integration interval
  float I = 0.0f;                               // Running sum of the integral

  #pragma omp parallel for reduction(+: I)
  for (int i = 0; i < n; i++) {                 // Loop through the integration range
    float const x = a + dx*(float(i) + 0.5f); // Midpoint of the integration interval
    float const f = 1.0f/sqrtf(x);              // Function value at the midpoint
    I += f;                                    // Incrementing the running sum
  }
  I *= dx; // Scale according to the integration interval

  return I;
}
```

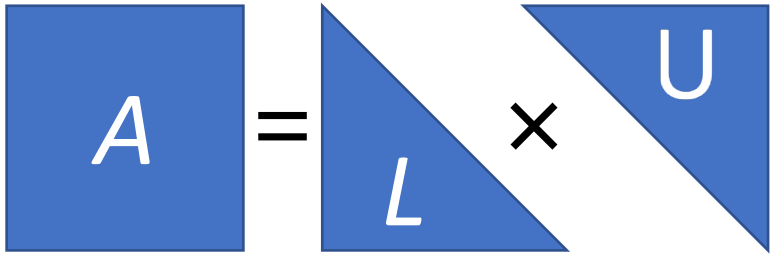
Private Container: Correct and Fast

```
float IntegrateMyFunction(int const n, float const a, float  const b) {

  float const dx = (b-a)/float(n);              // Integration interval
  float I = 0.0f;                               // Running sum of the integral

  #pragma omp parallel
  {
    float I_partial = 0.0f;                       // Private to the current thread
    #pragma omp for
    for (int i = 0; i < n; i++) {                 // Loop through the integration range
      float const x = a + dx*(float(i) + 0.5f); // Midpoint of the integration interval
      float const f = 1.0f/sqrtf(x);              // Function value at the midpoint
      I_partial += f;                             // Incrementing the running sum
    }
    #pragma omp atomic
    I += I_partial;                               // Reduction into a shared counter

  }
  I *= dx; // Scale according to the integration interval
```

Performance

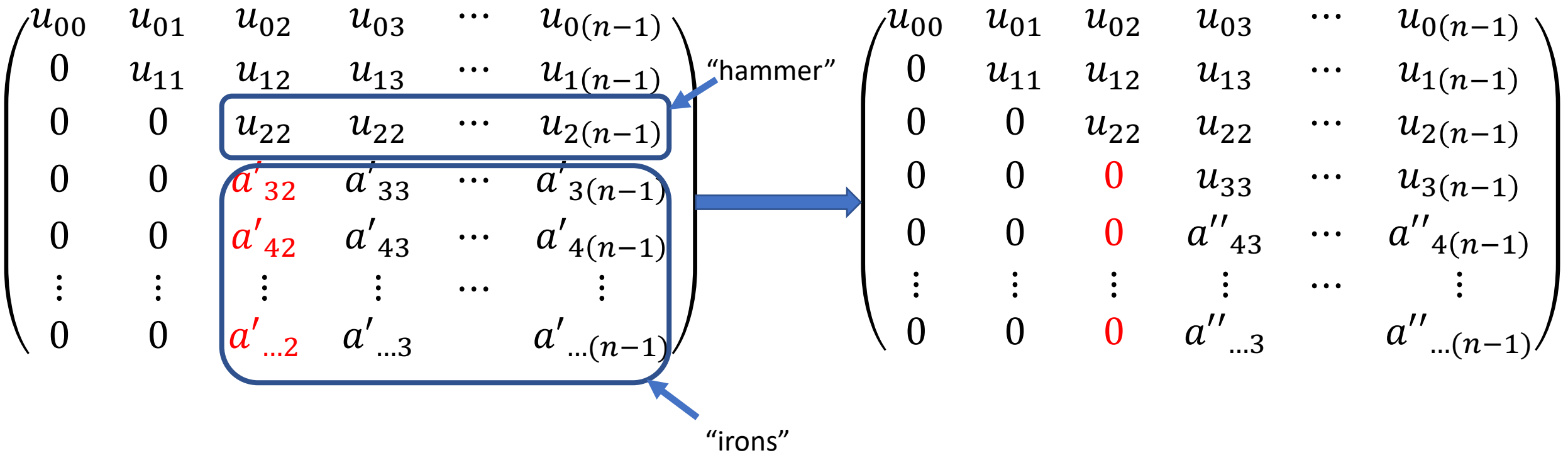


Example 2: LU Decomposition



$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix} = LU = \begin{pmatrix} l_{11} & 0 & \cdots & 0 \\ l_{21} & l_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ l_{n1} & l_{n2} & \cdots & l_{nn} \end{pmatrix} \begin{pmatrix} 1 & u_{12} & \cdots & u_{1n} \\ 0 & 1 & \cdots & u_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}$$

Mnemonic: Hammers and Irons



Serial Implementation (ikj)

```
void LU_decomp(int const n, int const lda, double* const A) {

  int i, j, k;

  for (i = 1; i < n; i++) {
    double * const Ai = A + i*lda;   // Pointer to row i
    for (k = 0; k < i; k++) {        // For all "hammer" rows
      double * const Ak = A + k*lda; // Pointer to row k
      Ai[k] /= Ak[k];                // Compute the scaling factor (and the element of L)
      for (j = k + 1; j < n; j++)    // Hit row "iron" row i with "hammer" row k
        Ai[j] -= Ai[k]*Ak[j];

    }
  }
}
```

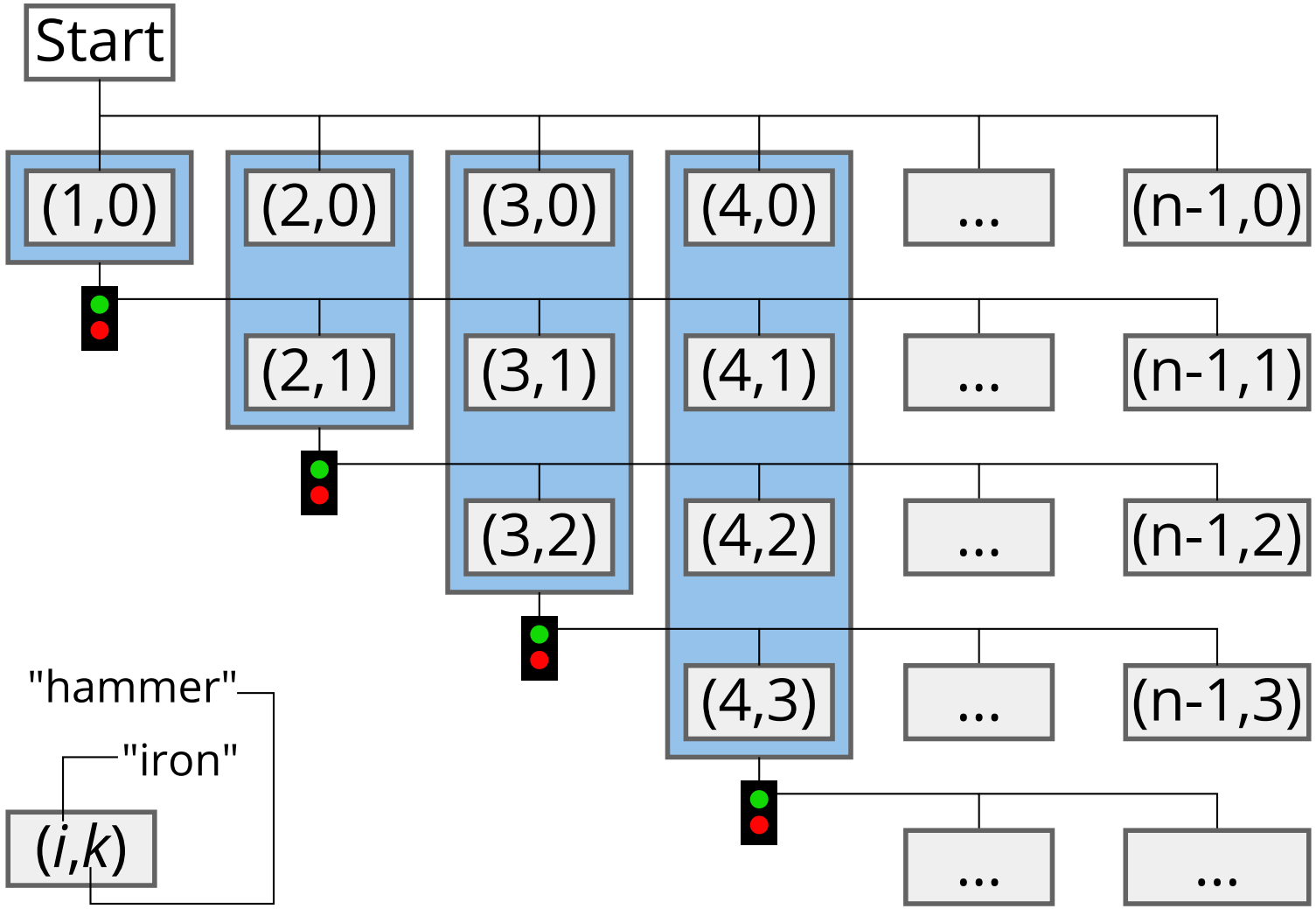
Permuted Implementation (kij)

```
void LU_decomp(int const n, int const lda, double* const A) {

  int i, j, k;

  for (k = 0; k < n; k++) {          // For all "hammer" rows
    double * const Ak = A + k*lda; // Pointer to row k
    for (i = k + 1; i < n; i++) {
      double * const Ai = A + i*lda;   // Pointer to row i
      Ai[k] /= Ak[k];                  // Compute the scaling factor (and the element of L)
      for (j = k + 1; j < n; j++)    // Hit row "iron" row i with "hammer" row k
        Ai[j] -= Ai[k]*Ak[j];

    }
  }
}
```

Parallel Implementation in kij

```
void LU_decomp(int const n, int const lda, double* const A) {

  int i, j, k;

  for (k = 0; k < n; k++) {          // For all "hammer" rows
    double * const Ak = A + k*lda; // Pointer to row k
    #pragma omp parallel for private(i)
    for (i = k + 1; i < n; i++) {
      double * const Ai = A + i*lda;   // Pointer to row i
      Ai[k] /= Ak[k];                  // Compute the scaling factor (and the element of L)
      for (j = k + 1; j < n; j++)    // Hit row "iron" row i with "hammer" row k
        Ai[j] -= Ai[k]*Ak[j];

    }
  }
}
```

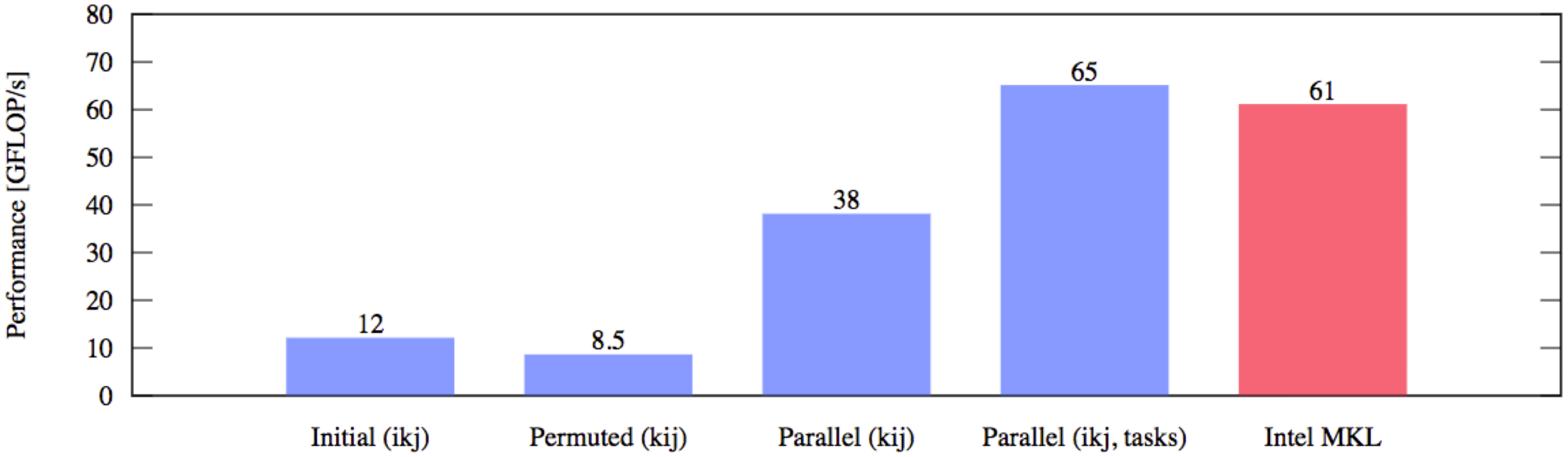
Task Dependencies



Parallel Implementation in ikj

```
#pragma omp parallel private(i, j, k)     // Start up all threads
{
  #pragma omp master                      // Restrict execution to one thread
  { for (i = 1; i < n; i++) {             // For all "iron" rows
      #pragma omp task firstprivate(i)    // Spawn async task for each "iron"
      { double * const Ai = A + i*lda;    // Pointer to row i
        for (k = 0; k < i; k++) {         // For all "hammer" rows
          double * const Ak = A + k*lda;  // Pointer to row k
          while (!row_is_ready[k]) {      // Wait for "hammer" number k to become ready
            #pragma omp taskyield         // Yield to other tasks until we can proceed
          }
          Ai[k] /= Ak[k];                 // Compute the scaling factor (and the element of L)
          #pragma omp simd
          for (j = k + 1; j < n; j++)     // Hit row "iron" row i with "hammer" row k
            Ai[j] -= Ai[k]*Ak[j];
        }
        row_is_ready[i] = 1;              // Done processing the "iron" i. Now it can become a "hammer"
        #pragma omp flush
      }
    }
  }
}
```

Performance



Efficient Parallelism With OpenMP

- Expose parallelism of code as far out in the loop hierarchy as possible
- Avoid excessive synchronization through reduction
- Set thread affinity
- Use highly-optimized libraries for common math

Today's code:
https://github.com/ColfaxResearch/ME344-2018/

Next Week

- Using Intel Parallel Studio XE tools for performance tuning.
- Access materials at https://mc2learning.com/account/
- Voucher code: (shown in class)

Learn more: HowSeries.com

About Colfax: ColfaxResearch.com