

# Cheat Sheet for PFPL

March 16, 2016

## I. Judgements and Rules

### 1 Abstract Syntax

N	P	Expression	Name	Say	Meaning
1	5	$\mathcal{P}$	Script P	Proposition	Something to be proved
2	5	$\mathcal{P}(a)$	Script P of $a$	Proposition about tree $a$	Something to be proved about AST $a$
3	5	$\mathcal{O}$	Script O	Operator	An operator that can be used in an AST
4	5	$\mathcal{O}(a)$	Script O of $a$	Operator of arity $a$	An operator of a given arity
5	5	$\mathcal{X}_s$	Script X sub s	Variables $x$ of sort $s$	Variables $x$ of sort $s$
6	5	$S$	S	A set of sorts	A set of sorts
7	5	$\{X_s\}_{s \in \mathcal{S}}$	Family	Family $X$ of $s$	A sort-indexed family of disjoint finite sets $X_s$ of variables $x$ of sort $s$
8	6	$[b/x] a$	Substitution	Substitute $b$ for $x$ in $a$	Substitute $b$ for $x$ in $a$
9	7	$x_1, \dots, x_n.a$	Abstractor	Bind variables $x_n$ to expression $a$	Bind variables $x_n$ to expression $a$
10	8	$\vec{x}$	X arrow	List of $x$ s	$x_1, \dots, x_n$
11	8	$\rho : \vec{x} \leftrightarrow \vec{x}'$	Fresh renaming	Freshen $x$ using renaming $\rho$	A bijection between $\vec{x}$ and $\vec{x}'$ where $\vec{x}'$ is fresh.
12	8	$\hat{\rho}_i(a_i)$	Rho hat sub i	Rename result	The result of applying the renaming $\rho_i$ to $a_i$
13	8	$x =_\alpha y$	Equal alpha	$\alpha$ -equivalence	Trees $x$ and $y$ equal up to renaming
14	9	$x \stackrel{\Delta}{=} y$	Delta equals	Replacement	Replace expression $x$ with expression $y$

### 2 Inductive Definitions

N	P	Expression	Name	Say	Meaning
15	13	$\tau$ type	Type	Type $\tau$	Judgement that $\tau$ is a type
16	13	$e : \tau$	Colon	$e$ is of type $\tau$	Judgement that expression $e$ is of type $\tau$
17	13	$e \Downarrow v$	Down arrow	$e$ has value $v$	Judgement that expression $e$ has value $v$
18	14	$\frac{J_1 \dots J_k}{J}$	Surfboard	Infers	Judgements $J_1 \dots J_k$ infer judgement $J$

### 3 Hypothetical and General Judgements

N	P	Expression	Name	Say	Meaning
19	23	$J_1 \dots J_k \vdash_{\mathcal{R}} \mathcal{K}$	Turnstile	Entails	Given $\mathcal{R}$ and $J$ infer $\mathcal{K}$
20	23	$\Gamma$	Gamma	Judgements Gamma	A finite set of judgements
21	23	$\Delta$	Delta	Judgements Delta	A finite set of type judgements
22	25	$\Gamma \vdash_R J$	Double turnstile	Admissible	$\vdash_R \Gamma$ implies $\vdash_R J$
23	28	$\nabla$	Down triangle	Generic derivation	Generic derivation

## II. Statics and Dynamics

### 4 Statics

N	P	Expression	Name	Say	Meaning
24	36	$n ::= s$	Colon equals	The syntax of $n$ is $s$	Specifies the syntax of $n$
25	36	$;$	Semicolon	And	Separates arguments to expression- sin abstact notation

### 5 Dynamics

N	P	Expression	Name	Say	Meaning
26	41	$s \longmapsto s'$	Bar arrow	Transistion	State $s$ transitions to state $s'$
27	42	$s \longmapsto^* s'$	Bar arrow star	Iterated transistion	State $s$ transitions to state $s'$ over more than zero transitions
28	42	$s \longmapsto^n s'$	Bar arrow n	N times iterated transis- tion	State $s$ transitions to state $s'$ over n transitions
29	44	$\mathcal{E}$	Script E	Expression context	Expression context
30	45	$\circ$	Circle	Hole	Placeholder to put an instruction
31	46	$e \equiv e'$	Equivalent	Definitional equivalence	$e$ is definitionally equivalent to $e'$

### 6 Type Safety

N	P	Expression	Name	Say	Meaning
32	58	$e??$	Wrong	E goes wrong	Expression $e$ goes wrong

### 7 Evaluation Dynamics

N	P	Expression	Name	Say	Meaning
33	58	$e \Downarrow^k v$	Downarrow k	E evaluates in k steps	Expression $e$ evaluates to $v$ in $k$ steps

### III. Total Functions

#### 8 Function Definitions and Values

N	P	Expression	Name	Say	Meaning
34	63	$\{f\}$	Brace brackets	Function	Surround function $f$ in abstract notation
35	63	$f.e$	Dot	Dot	Introduces the scope $e$ of a function $f$ in abstract notation
36	64	$f(\tau_1) : \tau_2$	Function	Function definition	A function taking an argument of type $\tau_1$ and returning a value of type $\tau_2$
37	64	$\llbracket x.e/f \rrbracket e'$	Script bracket	Function substitution	Function substitution
38	65	$\tau_1 \rightarrow \tau_2$	Right arrow	Maps to	A total function that maps elements of type $\tau_1$ to elements of type $\tau_2$
39	65	$\lambda$	Lambda	Lambda	Abstraction

#### 9 System T of Higher-Order Recursion

N	P	Expression	Name	Say	Meaning
40	71	$\hookrightarrow$	Hook arrow	Select	Deconstructor selector
41	71	$ $	Bar	Either	A choice
42	71	$\overline{n}$	Overline	Church numbering	The Church numbering
43	76	$\lceil n \rceil$	Divided hat	Gödel numbering	The Gödel numbering

### IV. Finite Data Types

#### 10 Product Types

N	P	Expression	Name	Say	Meaning
44	81	$<>$	Angle brackets	Null tuple	Null tuple
45	81	$< e_1, e_2 >$	Angle brackets	Ordered pair	Ordered pair
46	81	$e.l$	Left	Left projection	Select left member of the ordered pair
47	81	$e.r$	Right	Right projection	Select right member of the ordered pair

#### 11 Sum Types

N	P	Expression	Name	Say	Meaning
48	87	$l.e$	Left	Left injection	Create sum type element using left type
49	87	$r.e$	Right	Right injection	Create sum type element using right type

### VI. Infinite Data Types

#### 14 Generic Programming

N	P	Expression	Name	Say	Meaning
50	121	$t.\tau$	Dot	Type operator	Bind $t$ to type $\tau$

15 Inductive and Coinductive Types

N	P	Expression	Name	Say	Meaning
51	133	$\cong$	Tilde equal	Isomorphism	Isomorphism

VII. Variable Types

17 Abstract Types

N	P	Expression	Name	Say	Meaning
52	149	$\exists(t.\tau)$	Existential quantifier	Exists	Defines an interface
53	153	$\forall(t.\tau \longrightarrow \tau_2)$	Universal quantifier	For all	Defines universal type

18 Higher Kinds

N	P	Expression	Name	Say	Meaning
54	157	$::$	Colon colon	Kind type constructor	Maps types to types

VIII. Partiality and Recursive Types

19 System PCF of Recursive Functions

N	P	Expression	Name	Say	Meaning
55	166	$\mapsto$	Short bar arrow	Maps to	Function definition
56	166	$\perp$	Bottom	Bottom	Totally undefined partial function
57	167	$\tau_1 \rightarrow \tau_2$	Harpoon	Partial function	Partial function

20 System FPC of Recursive Types

N	P	Expression	Name	Say	Meaning
58	177	$_$	Underscore	Underscore	Unfree variable

IX. Dynamic Types

21 The Untyped  $\lambda$ -Calculus

N	P	Expression	Name	Say	Meaning
59	185	$\Lambda$	Lambda	Lambda calculus	The lambda calculus
60	188	$Y$	$Y$	$Y$ Combinator	The $Y$ combinator
61	190	$x^\dagger$	Superscript cross	Superscript cross	Language isomorphism