# Cheat Sheet for PFPL

# November 11, 2016

# I. Judgements and Rules

#### 1 Abstract Syntax

N	Р	Expression	Name	Say	Meaning
1	5	${\cal P}$	Script P	Proposition	Something to be proved
2	5	$\mathcal{P}(a)$	Script P of a	Proposition about tree $a$	Something to be proved about AST $a$
3	5	$\mathcal{O}$	Script O	Operator	An operator that can be used in an AST
4	5	$\mathcal{O}(a)$	Script O of $a$	Operator of arity $a$	An operator of a given arity
5	5	$\mathcal{X}_s$	Script X sub s	Variables $x$ of sort $s$	Variables $x$ of sort $s$
6	5	S	S	A set of sorts	A set of sorts
7	5	$\{X_s\}_{s\in\mathcal{S}}$	Family	Family $X$ of $s$	A sort-indexed family of disjoint finite sets $X_s$ of variables $x$ of sort $s$
8	6	[b/x] a	Substitution	Substitute $b$ for $x$ in $a$	Substitute $b$ for $x$ in $a$
9	7	$x_1, \ldots, x_n.a$	Abstractor	Bind variables $x_n$ to expression $a$	Bind variables $x_n$ to expression $a$
10	8	$\overrightarrow{x}$	X arrow	List of xs	$x_1,, x_n$
11	8	$\rho: \overrightarrow{x} \leftrightarrow \overrightarrow{x}'$	Fresh renaming	Freshen $x$ using renam-	A bijection between $\overrightarrow{x}$ and $\overrightarrow{x}'$ where $\overrightarrow{x}'$ is fresh.
10	0	â(a)	Dha hat auh i	$\log  ho$	
12	8	$\widehat{ ho}_i(a_i)$	Rho hat sub i	Rename result	The result of applying the renaming $\rho_i$ to $a_i$
13	8	$x =_{\alpha} y$	Equal alpha	$\alpha$ -equivalence	Trees x and y equal up to renaming
14	9	$x \stackrel{\Delta}{=} y$	Delta equals	Replacement	Replace expression <b>x</b> with expression $\mathbf{y}$

#### 2 Inductive Definitions

N	Р	Expression	Name	Say	Meaning
15 16	13 13	au type $e: au$	Type Colon	Type $\tau$ $e$ is of type $\tau$	Judgement that $\tau$ is a type Judgement that expression $e$ is of type $\tau$
17	13	$e \Downarrow v$	Down arrow	e has value $v$	Judgement that expression $e$ has value $v$
18	14	$\frac{J_1J_k}{J}$	Surfboard	Infers	Judgements $J_1J_k$ infer judgement $J$

## 3 Hypothetical and General Judgements

N	Р	Expression	Name	Say	Meaning
				, and the second	
19	23	$J_1J_k \vdash_{\mathcal{R}} \mathcal{K}$	Turnstile	Entails	Given $\mathcal{R}$ and $J$ infer $\mathcal{K}$
20	23	Γ	Gamma	Judgements Gamma	A finite set of judgements
21	23	$\Delta$	Delta	Judgements Delta	A finite set of type judgements
22	25	$\Gamma \models_R J$	Double turnstile	Admissible	$\vdash_R \Gamma \text{ implies } \vdash_R J$
23	28	$\nabla$	Down triangle	Generic derivation	Generic derivation

# II. Statics and Dynamics

#### 4 Statics

N	Р	Expression	Name		Say	Meaning
24	36	n ::= s	Colon co	olon	The syntax of $n$ is $s$	Specifies the syntax of $n$
25	36	;	Semicolon		And	Separates arguments to expressionsin abstact notation

#### 5 Dynamics

N	Р	Expression	Name	Say	Meaning
26 27		$s \longmapsto s'$ $s \longmapsto^* s'$	Bar arrow Bar arrow star	Transistion Iterated transistion	State $s$ transitions to state $s'$ State $s$ transitions to state $s'$ over
28		$s \longmapsto^n s'$	Bar arrow n	N times iterated transis-	more than zero transitions State $s$ transitions to state $s'$ over n
				tion	transitions
29	44	${\cal E}$	Script E	Expression context	Expression context
30	45	0	Circle	Hole	Placeholder to put an instruction
31	46	$e \equiv e'$	Equivalent	Definitional equivalence	e is definitionally equivalent to $e'$

#### 6 Type Safety

IN	Р	Expression	Name	Say	Meaning
32	58	e??	Wrong	E goes wrong	Expression $e$ goes wrong

#### 7 Evaluation Dynamics

N	Р	Expression	Name	Say	Meaning
33	58	$e \Downarrow^k v$	Downarrow k	E evaluates in k steps	Expression $e$ evaluates to $v$ in $k$
					steps

# III. Total Functions

#### 8 Function Definitions and Values

N	Р	Expression	Name	Say	Meaning
34	63	$\{f\}$	Brace brackets	Function	Surround function $f$ in abstract notation
35	63	f.e	Dot	Dot	Introduces the scope $e$ of a function
36	64	$f( au_1): au_2$	Function	Function definition	f in abstract notation A function taking an argument of type $\tau_1$ and returning a value of type $\tau_2$
37	64	[x.e/f]e'	Script bracket	Function substitution	Function substitution
38	65	$ au_1  ightarrow  au_2$	Right arrow	Maps to	A total function that maps elements
39	65	$\lambda$	Lambda	Lambda	of type $\tau_1$ to elements of type $\tau_2$ Abstraction

## 9 System T of Higher-Order Recursion

N	Р	Expression	Name	Say	Meaning
40	71	$\hookrightarrow$	Hook arrow	Select	Selector (used in System T recursion, sum types, and product types)
41	71		Bar	Either	A choice
42	71	$\overline{n}$	Overline	Church numbering	The Church numbering
43	76	$\lceil n \rceil$	Divided hat	$G\ddot{o}$ del numbering	The $G\ddot{o}$ del numbering

# IV. Finite Data Types

## 10 Product Types

N	Р	Expression	Name	Say	Meaning
44 45	81 81	$<> < < < e_1, e_2 >$	Angle brackets Angle brackets	Null tuple Ordered pair	Null tuple Ordered pair
46	81	e.l	Left	Left projection	Select left member of the ordered
47	81	e.r	Right	Right projection	pair Select right member of the ordered pair

## 11 Sum Types

N	Р	Expression	Name	Say	Meaning
48	87	1.e	Left	Left injection	Create sum type element using left type
49	87	r.e	Right	Right injection	Create sum type element using right type
50	93	<u>Δ</u>	Delta equals	Delta equals	Replacement

## VI. Infinite Data Types

#### 14 Generic Programming

N	Р	Expression	Name	Say	Meaning
51	121	t. au	Dot	Type operator	Bind $t$ to type $\tau$

#### 15 Inductive and Coinductive Types

N	Р	Expression	Name	Say	Meaning
52	133	$\cong$	Tilde equal	Isomorphism	Isomorphism

## VII. Variable Types

#### 17 Abstract Types

N	Р	Expression	Name	Say	Meaning
53	149	$\exists (t. au)$	Existential quantifier	Exists	Defines an interface
54	153	$\forall (t.\tau \longrightarrow \tau_2)$	Universal quantifier	For all	Defines universal type

#### 18 Higher Kinds

N	P Ex	xpression	Name	Say	Meaning
55	157 ::		Colon colon	Kind type constructor	Maps types to types

## VIII. Partiality and Recursive Types

#### 19 System PCF of Recursive Functions

N	P Ex	rpression	Name	Say	Meaning
56 57 58	$\begin{array}{ccc} 166 & \mapsto \\ 166 & \bot \\ 167 & \tau_1 \end{array}$		Short bar arrow Bottom Harpoon	Maps to Bottom Partial function	Function definition Totally undefined partial function Partial function

#### 20 System FPC of Recursive Types

N	Р	Expression	Name	Say	Meaning
59	177	_	Underscore	Underscore	Unfree variable

## IX. Dynamic Types

#### 21 The Untyped $\lambda$ -Calculus

N	Р	Expression	Name	Say	Meaning
60 61 62	185 188 190	Y	Lambda Y Superscript cross	Lambda calculus Y Combinator Superscript cross	The lambda calculus The Y combinator Language isomorphism

# X. Subtyping

## 24 Structural Subtyping

N	Р	Expression	Name	Say	Meaning
63	213	au' <:  au	Subtype	$\tau'$ is a subtype of $\tau$	$\tau'$ is a subtype of $\tau$

# XI. Dynamic Dispatch

#### 27 Inheritance

N	Р	Expression	Name	Say	Meaning
64	252	()‡	Isomorphism	Isomorphism	$()^{\ddagger}$ is a method isomorphism

## XII. Control Flow

#### 28 Control Stacks

N	Р	Expression	Name	Say	Meaning
65	257	$k \rhd e$	Right triangle	Evaluation state	Evaluate $e$ on $k$
66	257	$k \triangleleft e$	Left triangle	Return state	Evaluate $k$ on $e$
67	258	$\epsilon$	Epsilon	Empty frame	Empty frame
68	258	(—)	Frame hole	Frame hole	A place to put an evaluated expression into a frame
69	258	k; f	Stack with frame	Stack $k$ with frame $f$	Stack $k$ has frame $f$ at the bottom
70	259	$k \mathrel{\vartriangleleft}: \tau$	Triangle colon	Stack $k$ expects value of type $\tau$	Stack $k$ expects value of type $\tau$
71	259	$f:\tau\leadsto\tau'$	Squiggle arrow	Transform	Frame $f$ transforms expression of type $\tau$ into expression of type $\tau'$
72	261	$s \hookrightarrow e$	Loop arrow	Unravel	State $s$ goes to expression $e$
73	261	$k \bowtie e = e'$	Bowtie	Goes to	Stack $k$ and expression $e$ goes to expression $e'$

#### 29 Exceptions

N	Р	Expression	Name	Say	Meaning
74	266	$k \spadesuit$	Dark left triangle	Failed	Stack $k$ is in a failed state

# XIII. Symbolic Data

## 31 Symbols

N	Р	Expression	Name	Say	Meaning
75 76 77	282 282 284		Tilde Sigma Quote	Define $a$ Symbol context Symbol reference	Define symbol $a$ as type $\tau$ Set of symbol definitions Symbol reference

## 32 Fluid Binding

N	Р	Expression	Name	Say	Meaning
- '	-		1 (0.1110		11100111110
78	290	$\mu' \otimes a \hookrightarrow e$	Circle times	Extended by	Map symbol $a$ to expression $e$
79	200	$\mu' \otimes a \hookrightarrow \bullet$	Circle times	Extended by	Symbol a is undefined
19	290	$\mu \otimes a \rightarrow ullet$	Officie times	Extended by	Symbol $a$ is undefined
80	290	$\mu' \otimes a \hookrightarrow \underline{\ }$	Circle times	Extended by	Symbol $a$ is indeterminate

#### XIV. Mutable State

#### 34 Modernized Algol

N	P Expression	Name	Say	Meaning
81	308 $x \leftarrow e; m$	Left arrow	Sequence	Execute command $m$ with $e$ substituted for $x$
82	$308 \ a := e$	Colon equals	Assign	Assign $e$ to $a$
83	308 *a	Star	Contents	Get the value of $a$
84	$309 \ m \parallel \mu$	State	State	Command $m$ with memory map $\mu$
85	$314 \ m \sim \tau$	Dotted tilde	Dotted tilde	Command $m$ returns a value of type
				au

#### 35 Assignable References

N	Р	Expression	Name	Say	Meaning	
86	320	&a	Ampersand	Reference	Reference to assignable	
36 Lazy Evaluation						
N	Р	Expression	Name	Say	Meaning	
87	333	$\Longrightarrow$	Long right arrow	Implies	Implication (low precedence)	

#### XV. Parallelism

#### 37 Nested Parallelism

N	Р	Expression	Name	Say	Meaning
88 88 90	344	$c_1 \otimes c_2$ $c_1 \oplus c_2$ $a_1 \hookrightarrow s_1 \otimes$ $a_2 \hookrightarrow s_2$	Circle times Circle plus Circle times	Parallel cost Sequential cost Parallel join	Cost of executing in parallel Cost of executing in sequence Join local states

# XVI. Concurrency and Distribution

## 39 Process Calculus

N	Р	Expression	Name	Say	Meaning
91	365	\$E	Dollar	Synchronize	Synchronize parallel processes
92	365	$E_1 + E_2$	Plus	Choice	Pick one of 2 expressions
93	365	?a;P	Query	Query	Query a channel
94	365	!a;P	Bang	Signal	Signal a channel
95	367	$P_1 \otimes P_2$	Circle times	Composition	Compose 2 concurrent processes
96	367	$P \xrightarrow{\alpha} P'$	Maps to	Steps	$P$ steps to $P'$ if action $\alpha$ is permissible
97	369	*P	Star	Replication	Replicate process $P$
98	370	$\nu a.P$	Nu	New channel	Declare new channel $a$ in process $P$
99	371	$P \xrightarrow{\Sigma} P'$	Maps to	Steps	$P$ steps to $P'$ if action $\alpha$ is permissible in context $\Sigma$
100	373	!a(e;P)	Bang	Send	Send expression $e$ on channel $a$ to process $P$
101	373	?a(x.P)	Query	Receive	Receive expression $e$ on channel $a$ in process $P$
102	376	$\tau$ chan	Tau	Channel type	Channel of type $\tau$
103	376	&a	Ampersand	Channel reference	Reference to channel $a$
104	376	$!!(e_1; e_2; P)$	Bang bang	Send	Send message on channel $e_1$
105	376	??(e; x.P)	Query query	Receive	Receive message on channel $e$
106	377	@	At	Map	Maps lambda expressions to PiC

## 40 Concurrent Algol

N	Р	Expression	Name	Say	Meaning
107	382	$P_1 \otimes P_2$	Circle times	Concurrent	Run processes $P_1$ and $P_2$ concurrently
108	382	$\nu a \ \tau.p$	Nu	New channel	Create a new chanell
109	383	$m \qquad \Longrightarrow_{\Sigma}$ $\nu \Sigma' \{ m' \otimes p' \}$	Fat right arrow	Transitions	Command $m$ transitions to command $m'$ while creating new channels $\Sigma$ and new processes $p'$ . The action $\alpha$ specifies the interactions of which $m$ is capable when executed.

## 41 Distributed Algol

N	Р	Expression	Name	Say	Meaning
110	392	$a \tau @ w$	Tilde	Declare	$a$ is a channel at site $w$ carrying payloads of type $\tau$