Cheat Sheet for PFPL

October 28, 2016

I. Judgements and Rules

1 Abstract Syntax

| N | Р | Expression | Name | Say | Meaning |
|----|---|--|-----------------|--|--|
| 1 | 5 | ${\cal P}$ | Script P | Proposition | Something to be proved |
| 2 | 5 | $\mathcal{P}(a)$ | Script P of a | Proposition about tree a | Something to be proved about AST a |
| 3 | 5 | \mathcal{O} | Script O | Operator | An operator that can be used in an AST |
| 4 | 5 | $\mathcal{O}(a)$ | Script O of a | Operator of arity a | An operator of a given arity |
| 5 | 5 | \mathcal{X}_s | Script X sub s | Variables x of sort s | Variables x of sort s |
| 6 | 5 | S | S | A set of sorts | A set of sorts |
| 7 | 5 | $\{X_s\}_{s\in\mathcal{S}}$ | Family | Family X of s | A sort-indexed family of disjoint finite sets X_s of variables x of sort s |
| 8 | 6 | [b/x] a | Substitution | Substitute b for x in a | Substitute b for x in a |
| 9 | 7 | $x_1, \ldots, x_n.a$ | Abstractor | Bind variables x_n to expression a | Bind variables x_n to expression a |
| 10 | 8 | \overrightarrow{x} | X arrow | List of xs | $x_1,, x_n$ |
| 11 | 8 | $\rho: \overrightarrow{x} \leftrightarrow \overrightarrow{x}'$ | Fresh renaming | Freshen x using renam- | A bijection between \overrightarrow{x} and \overrightarrow{x}' where \overrightarrow{x}' is fresh. |
| 10 | 0 | â(a) | Dha hat auh i | $\log ho$ | |
| 12 | 8 | $\widehat{ ho}_i(a_i)$ | Rho hat sub i | Rename result | The result of applying the renaming ρ_i to a_i |
| 13 | 8 | $x =_{\alpha} y$ | Equal alpha | α -equivalence | Trees x and y equal up to renaming |
| 14 | 9 | $x \stackrel{\Delta}{=} y$ | Delta equals | Replacement | Replace expression x with expression \mathbf{y} |

2 Inductive Definitions

| N | Р | Expression | Name | Say | Meaning |
|----------|----------|--------------------|---------------|-----------------------------------|---|
| 15 16 | 13 13 | au type $e:	au$ | Type Colon | Type τ e is of type τ | Judgement that τ is a type Judgement that expression e is of type τ |
| 17 | 13 | $e \Downarrow v$ | Down arrow | e has value v | Judgement that expression e has value v |
| 18 | 14 | $\frac{J_1J_k}{J}$ | Surfboard | Infers | Judgements J_1J_k infer judgement J |

3 Hypothetical and General Judgements

| N | Р | Expression | Name | Say | Meaning |
|----|----|---|------------------|--|---|
| | | | | , and the second | |
| 19 | 23 | $J_1J_k \vdash_{\mathcal{R}} \mathcal{K}$ | Turnstile | Entails | Given \mathcal{R} and J infer \mathcal{K} |
| 20 | 23 | Γ | Gamma | Judgements Gamma | A finite set of judgements |
| 21 | 23 | Δ | Delta | Judgements Delta | A finite set of type judgements |
| 22 | 25 | $\Gamma \models_R J$ | Double turnstile | Admissible | $\vdash_R \Gamma \text{ implies } \vdash_R J$ |
| 23 | 28 | ∇ | Down triangle | Generic derivation | Generic derivation |

II. Statics and Dynamics

4 Statics

| N | Р | Expression | Name | | Say | Meaning |
|----|----|------------|-----------|------|--------------------------|---|
| 24 | 36 | n ::= s | Colon co | olon | The syntax of n is s | Specifies the syntax of n |
| 25 | 36 | ; | Semicolon | | And | Separates arguments to expressionsin abstact notation |

5 Dynamics

| N | Р | Expression | Name | Say | Meaning |
|----------|----|---|-----------------------------|----------------------------------|---|
| 26 27 | | $s \longmapsto s'$ $s \longmapsto^* s'$ | Bar arrow Bar arrow star | Transistion Iterated transistion | State s transitions to state s' State s transitions to state s' over |
| 28 | | $s \longmapsto^n s'$ | Bar arrow n | N times iterated transis- | more than zero transitions State s transitions to state s' over n |
| | | | | tion | transitions |
| 29 | 44 | ${\cal E}$ | Script E | Expression context | Expression context |
| 30 | 45 | 0 | Circle | Hole | Placeholder to put an instruction |
| 31 | 46 | $e \equiv e'$ | Equivalent | Definitional equivalence | e is definitionally equivalent to e' |
| | | | | | |

6 Type Safety

| IN | Р | Expression | Name | Say | Meaning |
|----|----|------------|-------|--------------|---------------------------|
| 32 | 58 | e?? | Wrong | E goes wrong | Expression e goes wrong |

7 Evaluation Dynamics

| N | Р | Expression | Name | Say | Meaning |
|----|----|--------------------|-------------|------------------------|--|
| 33 | 58 | $e \Downarrow^k v$ | Downarrow k | E evaluates in k steps | Expression e evaluates to v in k |
| | | | | | steps |

III. Total Functions

8 Function Definitions and Values

| N | Р | Expression | Name | Say | Meaning |
|----|----|--------------------------|----------------|-----------------------|---|
| 34 | 63 | $\{f\}$ | Brace brackets | Function | Surround function f in abstract notation |
| 35 | 63 | f.e | Dot | Dot | Introduces the scope e of a function |
| 36 | 64 | $f(au_1):	au_2$ | Function | Function definition | f in abstract notation A function taking an argument of type τ_1 and returning a value of type τ_2 |
| 37 | 64 | [x.e/f]e' | Script bracket | Function substitution | Function substitution |
| 38 | 65 | $	au_1 ightarrow 	au_2$ | Right arrow | Maps to | A total function that maps elements |
| 39 | 65 | λ | Lambda | Lambda | of type τ_1 to elements of type τ_2 Abstraction |

9 System T of Higher-Order Recursion

| N | Р | Expression | Name | Say | Meaning |
|----|----|-------------------|-------------|---------------------------|---|
| 40 | 71 | \hookrightarrow | Hook arrow | Select | Selector (used in System T recursion, sum types, and product types) |
| 41 | 71 | | Bar | Either | A choice |
| 42 | 71 | \overline{n} | Overline | Church numbering | The Church numbering |
| 43 | 76 | $\lceil n \rceil$ | Divided hat | $G\ddot{o}$ del numbering | The $G\ddot{o}$ del numbering |

IV. Finite Data Types

10 Product Types

| N | Р | Expression | Name | Say | Meaning |
|----------|----------|-----------------------|-------------------------------|----------------------------|--|
| 44 45 | 81 81 | $<> < < < e_1, e_2 >$ | Angle brackets Angle brackets | Null tuple Ordered pair | Null tuple Ordered pair |
| 46 | 81 | e.l | Left | Left projection | Select left member of the ordered |
| 47 | 81 | e.r | Right | Right projection | pair Select right member of the ordered pair |

11 Sum Types

| N | Р | Expression | Name | Say | Meaning |
|----|----|------------|--------------|-----------------|--|
| 48 | 87 | 1.e | Left | Left injection | Create sum type element using left type |
| 49 | 87 | r.e | Right | Right injection | Create sum type element using right type |
| 50 | 93 | <u>Δ</u> | Delta equals | Delta equals | Replacement |

VI. Infinite Data Types

14 Generic Programming

| N | Р | Expression | Name | Say | Meaning |
|----|-----|------------|------|---------------|-------------------------|
| 51 | 121 | t.	au | Dot | Type operator | Bind t to type τ |

15 Inductive and Coinductive Types

| N | Р | Expression | Name | Say | Meaning |
|----|-----|------------|-------------|-------------|-------------|
| 52 | 133 | \cong | Tilde equal | Isomorphism | Isomorphism |

VII. Variable Types

17 Abstract Types

| N | Р | Expression | Name | Say | Meaning |
|----|-----|---|------------------------|---------|------------------------|
| 53 | 149 | $\exists (t.	au)$ | Existential quantifier | Exists | Defines an interface |
| 54 | 153 | $\forall (t.\tau \longrightarrow \tau_2)$ | Universal quantifier | For all | Defines universal type |

18 Higher Kinds

| N | P Ex | xpression | Name | Say | Meaning |
|----|--------|-----------|-------------|-----------------------|---------------------|
| 55 | 157 :: | | Colon colon | Kind type constructor | Maps types to types |

VIII. Partiality and Recursive Types

19 System PCF of Recursive Functions

| N | P Ex | rpression | Name | Say | Meaning |
|----------------|--|-----------|--------------------------------------|---------------------------------------|---|
| 56 57 58 | $\begin{array}{ccc} 166 & \mapsto \\ 166 & \bot \\ 167 & \tau_1 \end{array}$ | | Short bar arrow Bottom Harpoon | Maps to Bottom Partial function | Function definition Totally undefined partial function Partial function |

20 System FPC of Recursive Types

| N | Р | Expression | Name | Say | Meaning |
|----|-----|------------|------------|------------|-----------------|
| 59 | 177 | _ | Underscore | Underscore | Unfree variable |

IX. Dynamic Types

21 The Untyped λ -Calculus

| N | Р | Expression | Name | Say | Meaning |
|----------------|-------------------|------------|----------------------------------|--|---|
| 60 61 62 | 185 188 190 | Y | Lambda Y Superscript cross | Lambda calculus Y Combinator Superscript cross | The lambda calculus The Y combinator Language isomorphism |

X. Subtyping

24 Structural Subtyping

| N | Р | Expression | Name | Say | Meaning |
|----|-----|------------|---------|--------------------------------|--------------------------------|
| 63 | 213 | au' <: 	au | Subtype | τ' is a subtype of τ | τ' is a subtype of τ |

XI. Dynamic Dispatch

27 Inheritance

| N | Р | Expression | Name | Say | Meaning |
|----|-----|------------|-------------|-------------|---|
| 64 | 252 | ()‡ | Isomorphism | Isomorphism | $()^{\ddagger}$ is a method isomorphism |

XII. Control Flow

28 Control Stacks

| N | Р | Expression | Name | Say | Meaning |
|----|-----|--------------------------------------|------------------|--|--|
| 65 | 257 | $k \rhd e$ | Right triangle | Evaluation state | Evaluate e on k |
| 66 | 257 | $k \triangleleft e$ | Left triangle | Return state | Evaluate k on e |
| 67 | 258 | ϵ | Epsilon | Empty frame | Empty frame |
| 68 | 258 | () | Frame hole | Frame hole | A place to put an evaluated expression into a frame |
| 69 | 258 | k; f | Stack with frame | Stack k with frame f | Stack k has frame f at the bottom |
| 70 | 259 | $k \mathrel{\vartriangleleft}: \tau$ | Triangle colon | Stack k expects value of type τ | Stack k expects value of type τ |
| 71 | 259 | $f:\tau\leadsto\tau'$ | Squiggle arrow | Transform | Frame f transforms expression of type τ into expression of type τ' |
| 72 | 261 | $s \hookrightarrow e$ | Loop arrow | Unravel | State s goes to expression e |
| 73 | 261 | $k \bowtie e = e'$ | Bowtie | Goes to | Stack k and expression e goes to expression e' |

29 Exceptions

| N | Р | Expression | Name | Say | Meaning |
|----|-----|----------------|--------------------|--------|--------------------------------|
| 74 | 266 | $k \spadesuit$ | Dark left triangle | Failed | Stack k is in a failed state |

XIII. Symbolic Data

31 Symbols

| N | Р | Expression | Name | Say | Meaning |
|----------------|-------------------|------------|-------------------------|--|---|
| 75 76 77 | 282 282 284 | | Tilde Sigma Quote | Define a Symbol context Symbol reference | Define symbol a as type τ Set of symbol definitions Symbol reference |

32 Fluid Binding

| N | Р | Expression | Name | Say | Meaning |
|-----|-----|---|---------------|-------------|----------------------------------|
| - ' | - | | 1 (0.1110 | | 11100111110 |
| | | | | | |
| 78 | 290 | $\mu' \otimes a \hookrightarrow e$ | Circle times | Extended by | Map symbol a to expression e |
| 79 | 200 | $\mu' \otimes a \hookrightarrow \bullet$ | Circle times | Extended by | Symbol a is undefined |
| 19 | 290 | $\mu \otimes a \rightarrow ullet$ | Officie times | Extended by | Symbol a is undefined |
| 80 | 290 | $\mu' \otimes a \hookrightarrow \underline{\ }$ | Circle times | Extended by | Symbol a is indeterminate |

XIV. Mutable State

34 Modernized Algol

| N | P Expression | Name | Say | Meaning |
|----|-------------------------|--------------|--------------|--|
| 81 | 308 $x \leftarrow e; m$ | Left arrow | Sequence | Execute command m with e substituted for x |
| 82 | $308 \ a := e$ | Colon equals | Assign | Assign e to a |
| 83 | 308 *a | Star | Contents | Get the value of a |
| 84 | $309 \ m \parallel \mu$ | State | State | Command m with memory map μ |
| 85 | $314 \ m \sim \tau$ | Dotted tilde | Dotted tilde | Command m returns a value of type |
| | | | | au |

35 Assignable References

| N | Р | Expression | Name | Say | Meaning | |
|--------------------|-----|-------------------|------------------|-----------|------------------------------|--|
| 86 | 320 | &a | Ampersand | Reference | Reference to assignable | |
| 36 Lazy Evaluation | | | | | | |
| N | Р | Expression | Name | Say | Meaning | |
| 87 | 333 | \Longrightarrow | Long right arrow | Implies | Implication (low precedence) | |

XV. Parallelism

37 Nested Parallelism

| N | Р | Expression | Name | Say | Meaning |
|----------------|-----|--|---------------------------------------|---|---|
| 88 88 90 | 344 | $c_1 \otimes c_2$ $c_1 \oplus c_2$ $a_1 \hookrightarrow s_1 \otimes$ $a_2 \hookrightarrow s_2$ | Circle times Circle plus Circle times | Parallel cost Sequential cost Parallel join | Cost of executing in parallel Cost of executing in sequence Join local states |

XVI. Concurrency and Distribution

39 Process Calculus

| N | P Expression | Name | Say | Meaning |
|----|-----------------------|-----------------------|-------------|--|
| 01 | 20T 0 D | D . II . | C - 1 | C 1 2 2 2 2 1 1 1 2 2 2 2 2 2 2 2 2 2 2 |
| 91 | $365 \ \$E$ | Dollar | Synchronize | Synchronize parallel processes |
| 92 | $365 E_1 + E_2$ | Plus | Choice | Pick one of 2 expressions |
| 93 | 365 ?a; P | Query | Query | Query a channel |
| 94 | 365 !a; P | Bang | Signal | Signal a channel |
| 95 | $367 P_1 \otimes P_2$ | Circle times | Composition | Compose 2 concurrent processes |
| 96 | 369 *P | Star | Replication | Replicate process P |
| 97 | 373 $!a(e; P)$ | Bang | Send | Send expression e on channel a to process P |
| 98 | $373 \ ?a(x.P)$ | Query | Receive | Receive expression e on channel a in process P |
| 99 | 377 @ | At | Map | Maps lambda expressions to PIC |