

Cheat Sheet for PFPL

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I. Judgements and Rules

1 Abstract Syntax

N	P	Expression	Name	Say	Meaning
1	5	\mathcal{P}	Script P	Proposition	Something to be proved
2	5	$\mathcal{P}(a)$	Script P of a	Proposition about tree a	Something to be proved about AST a
3	5	\mathcal{O}	Script O	Operator	An operator that can be used in an AST
4	5	$\mathcal{O}(a)$	Script O of a	Operator of arity a	An operator of a given arity
5	5	\mathcal{X}_s	Script X sub s	Variables x of sort s	Variables x of sort s
6	5	S	S	A set of sorts	A set of sorts
7	5	$\{X_s\}_{s \in \mathcal{S}}$	Family	Family X of s	A sort-indexed family of disjoint finite sets X_s of variables x of sort s
8	6	$[b/x] a$	Substitution	Substitute b for x in a	Substitute b for x in a
9	7	$x_1, \dots, x_n.a$	Abstractor	Bind variables x_n to expression a	Bind variables x_n to expression a
10	8	\vec{x}	X arrow	List of x s	x_1, \dots, x_n
11	8	$\rho : \vec{x} \leftrightarrow \vec{x}'$	Fresh renaming	Freshen x using renaming ρ	A bijection between \vec{x} and \vec{x}' where \vec{x}' is fresh.
12	8	$\hat{\rho}_i(a_i)$	Rho hat sub i	Rename result	The result of applying the renaming ρ_i to a_i
13	8	$x =_\alpha y$	Equal alpha	α -equivalence	Trees x and y equal up to renaming
14	9	$x \stackrel{\Delta}{=} y$	Delta equals	Replacement	Replace expression x with expression y

2 Inductive Definitions

N	P	Expression	Name	Say	Meaning
15	13	τ type	Type	Type τ	Judgement that τ is a type
16	13	$e : \tau$	Colon	e is of type τ	Judgement that expression e is of type τ
17	13	$e \Downarrow v$	Down arrow	e has value v	Judgement that expression e has value v
18	14	$\frac{J_1 \dots J_k}{J}$	Surfboard	Infers	Judgements $J_1 \dots J_k$ infer judgement J

3 Hypothetical and General Judgements

N	P	Expression	Name	Say	Meaning
19	23	$J_1 \dots J_k \vdash_{\mathcal{R}} \mathcal{K}$	Turnstile	Entails	Given \mathcal{R} and J infer \mathcal{K}
20	23	Γ	Gamma	Judgements Gamma	A finite set of judgements
21	23	Δ	Delta	Judgements Delta	A finite set of type judgements
22	25	$\Gamma \vdash_R J$	Double turnstile	Admissible	$\vdash_R \Gamma$ implies $\vdash_R J$
23	28	∇	Down triangle	Generic derivation	Generic derivation

II. Statics and Dynamics

4 Statics

N	P	Expression	Name	Say	Meaning
24	36	$n ::= s$	Colon equals	The syntax of n is s	Specifies the syntax of n
25	36	$;$	Semicolon	And	Separates arguments to expression- sin abstact notation

5 Dynamics

N	P	Expression	Name	Say	Meaning
26	41	$s \longmapsto s'$	Bar arrow	Transistion	State s transitions to state s'
27	42	$s \longmapsto^* s'$	Bar arrow star	Iterated transistion	State s transitions to state s' over more than zero transitions
28	42	$s \longmapsto^n s'$	Bar arrow n	N times iterated transis- tion	State s transitions to state s' over n transitions
29	44	\mathcal{E}	Script E	Expression context	Expression context
30	45	\circ	Circle	Hole	Placeholder to put an instruction
31	46	$e \equiv e'$	Equivalent	Definitional equivalence	e is definitionally equivalent to e'

6 Type Safety

N	P	Expression	Name	Say	Meaning
32	58	$e??$	Wrong	E goes wrong	Expression e goes wrong

7 Evaluation Dynamics

N	P	Expression	Name	Say	Meaning
33	58	$e \Downarrow^k v$	Downarrow k	E evaluates in k steps	Expression e evaluates to v in k steps

III. Total Functions

8 Function Definitions and Values

N	P	Expression	Name	Say	Meaning
34	63	$\{f\}$	Brace brackets	Function	Surround function f in abstract notation
35	63	$f.e$	Dot	Dot	Introduces the scope e of a function f in abstract notation
36	64	$f(\tau_1) : \tau_2$	Function	Function definition	A function taking an argument of type τ_1 and returning a value of type τ_2
37	64	$\llbracket x.e/f \rrbracket e'$	Script bracket	Function substitution	Function substitution
38	65	$\tau_1 \rightarrow \tau_2$	Right arrow	Maps to	A total function that maps elements of type τ_1 to elements of type τ_2
39	65	λ	Lambda	Lambda	Abstraction

9 System T of Higher-Order Recursion

N	P	Expression	Name	Say	Meaning
40	71	\hookrightarrow	Hook arrow	Select	Selector (used in System T recursion, sum types, and product types)
41	71	$ $	Bar	Either	A choice
42	71	\overline{n}	Overline	Church numbering	The Church numbering
43	76	$\lceil n \rceil$	Divided hat	Gödel numbering	The Gödel numbering

IV. Finite Data Types

10 Product Types

N	P	Expression	Name	Say	Meaning
44	81	$<>$	Angle brackets	Null tuple	Null tuple
45	81	$< e_1, e_2 >$	Angle brackets	Ordered pair	Ordered pair
46	81	$e.l$	Left	Left projection	Select left member of the ordered pair
47	81	$e.r$	Right	Right projection	Select right member of the ordered pair

11 Sum Types

N	P	Expression	Name	Say	Meaning
48	87	$l.e$	Left	Left injection	Create sum type element using left type
49	87	$r.e$	Right	Right injection	Create sum type element using right type
50	93	\triangleq	Delta equals	Delta equals	Replacement

VI. Infinite Data Types

14 Generic Programming

N	P	Expression	Name	Say	Meaning
51	121	$t.\tau$	Dot	Type operator	Bind t to type τ

15 Inductive and Coinductive Types

N	P	Expression	Name	Say	Meaning
52	133	\cong	Tilde equal	Isomorphism	Isomorphism

VII. Variable Types

17 Abstract Types

N	P	Expression	Name	Say	Meaning
53	149	$\exists(t.\tau)$	Existential quantifier	Exists	Defines an interface
54	153	$\forall(t.\tau \longrightarrow \tau_2)$	Universal quantifier	For all	Defines universal type

18 Higher Kinds

N	P	Expression	Name	Say	Meaning
55	157	$::$	Colon colon	Kind type constructor	Maps types to types

VIII. Partiality and Recursive Types

19 System PCF of Recursive Functions

N	P	Expression	Name	Say	Meaning
56	166	\mapsto	Short bar arrow	Maps to	Function definition
57	166	\perp	Bottom	Bottom	Totally undefined partial function
58	167	$\tau_1 \multimap \tau_2$	Harpoon	Partial function	Partial function

20 System FPC of Recursive Types

N	P	Expression	Name	Say	Meaning
59	177	$_$	Underscore	Underscore	Unfree variable

IX. Dynamic Types

21 The Untyped λ -Calculus

N	P	Expression	Name	Say	Meaning
60	185	Λ	Lambda	Lambda calculus	The lambda calculus
61	188	Y	Y	Y Combinator	The Y combinator
62	190	x^\dagger	Superscript cross	Superscript cross	Language isomorphism

X. Subtyping

24 Structural Subtyping

N	P	Expression	Name	Say	Meaning
63	213	$\tau' <: \tau$	Subtype	τ' is a subtype of τ	τ' is a subtype of τ

XI. Dynamic Dispatch

27 Inheritance

N	P	Expression	Name	Say	Meaning
64	252	$()^\ddagger$	Isomorphism	Isomorphism	$()^\ddagger$ is a method isomorphism

XII. Control Flow

28 Control Stacks

N	P	Expression	Name	Say	Meaning
65	257	$k \triangleright e$	Right triangle	Evaluation state	Evaluate e on k
66	257	$k \triangleleft e$	Left triangle	Return state	Evaluate k on e
67	258	ϵ	Epsilon	Empty frame	Empty frame
68	258	(—)	Frame hole	Frame hole	A place to put an evaluated expression into a frame
69	258	$k; f$	Stack with frame	Stack k with frame f	Stack k has frame f at the bottom
70	259	$k \triangleleft: \tau$	Triangle colon	Stack k expects value of type τ	Stack k expects value of type τ
71	259	$f : \tau \rightsquigarrow \tau'$	Squiggle arrow	Transform	Frame f transforms expression of type τ into expression of type τ'
72	261	$s \looparrowright e$	Loop arrow	Unravel	State s goes to expression e
73	261	$k \bowtie e = e'$	Bowtie	Goes to	Stack k and expression e goes to expression e'

29 Exceptions

N	P	Expression	Name	Say	Meaning
74	266	$k \blacklozenge$	Dark left triangle	Failed	Stack k is in a failed state

XIII. Symbolic Data

31 Symbols

N	P	Expression	Name	Say	Meaning
75	282	$a \sim \tau$	Tilde	Define a	Define symbol a as type τ
76	282	Σ	Sigma	Symbol context	Set of symbol definitions
77	284	$'a$	Quote	Symbol reference	Symbol reference

32 Fluid Binding

N	P	Expression	Name	Say	Meaning
78	290	$\mu' \otimes a \hookrightarrow e$	Tensor product	Extended by	Map symbol a to expression e
79	290	$\mu' \otimes a \hookrightarrow \bullet$	Tensor product	Extended by	Symbol a is undefined
80	290	$\mu' \otimes a \hookrightarrow _$	Tensor product	Extended by	Symbol a is indeterminate

XIV. Mutable State

34 Modernized Algol

N	P	Expression	Name	Say	Meaning
81	308	$x \leftarrow e; m$	Left arrow	Sequence	Execute command m with e substituted for x
82	308	$a := e$	Colon equals	Assign	Assign e to a
83	308	$*a$	Star	Reference	Get the value of a
84	309	$m \mu$	State	State	Command m with memory map μ
85	314	$m \sim \tau$	Dotted tilde	Dotted tilde	Command m returns a value of type τ