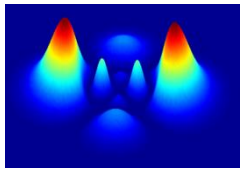


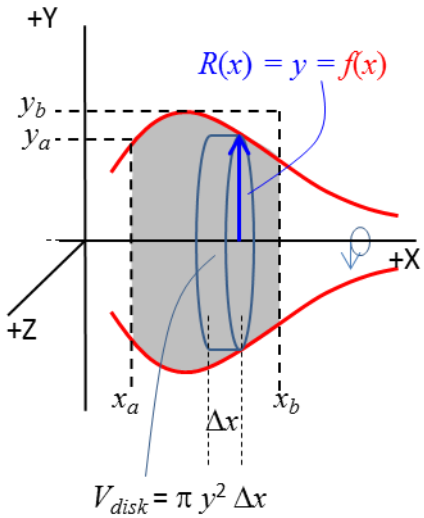
# MATHEMATICS EXTENSION 2

## 4 UNIT MATHEMATICS

## TOPIC 5: VOLUMES

## SUMMARY

A **solid of revolution** is obtained by revolving a **region** in a plane about a straight line called the **axis of revolution** that does not intersect the region.




## Rotation about the X-axis

$y = f(x)$ is a single valued function and the region ***R*** is bounded by the function $y = f(x)$, the X-axis and the vertical lines $x_a$ and $x_b$. The Y-axis limits are $y_a = f(x_a)$ $y_b = f(x_b)$

The volume $V$ of the solid of revolution obtained by revolving a region ***R*** about the X-axis is

### DISK METHOD

$$V = \int_{x_a}^{x_b} A(x)\,dx$$

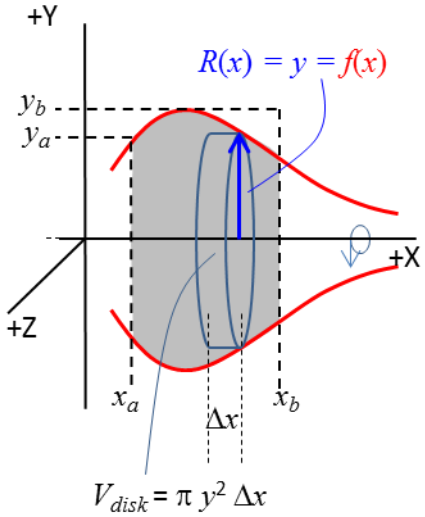
$$A(x) = \pi R(x)^2 \quad R(x) = y \quad A(x) = \pi y^2$$

$$V = \pi \int_{x_a}^{x_b} R(x)^2\,dx = \pi \int_{x_a}^{x_b} y^2\,dx$$

## CYLINDRICAL SHELL METHOD

The cylindrical shell method can be used to find the volume of revolution of **R** about the X-axis. But in this example, the disk method is much easier.
You need to use volume of the shell between the line $x_b$ and the Y-axis and subtract from this the volume between the line $x_a$ and the Y-axis

$$V = 2\pi\left(\int_0^{y_b} y\,x\,dy - x_a\int_0^{y_a} y\,dy\right)$$



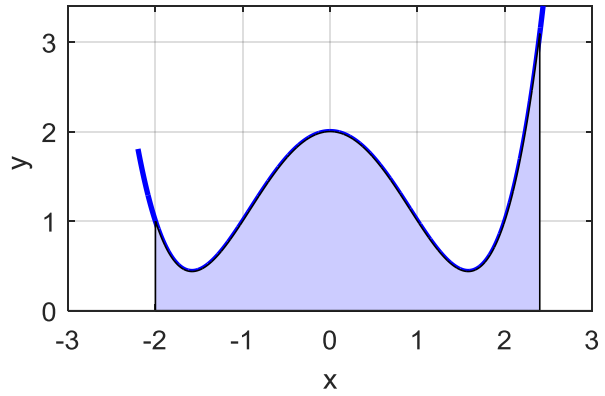
**Example**

The region $\boldsymbol{R}$ is the blue shaded area under the curve which is rotated about the X-axis. The function

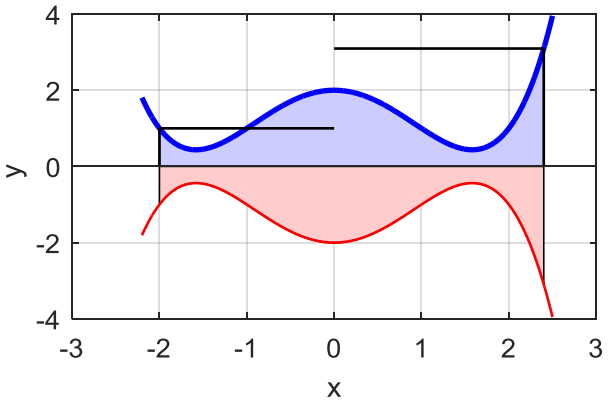
$$y = f(x) = 2 + \left(x^4 - x^2\right)/4$$

is shown by the blue curved line. The region is bounded by the X-axis and the lines $x_a$ = -2 and $x_b$ = +2.4. The Y limits are

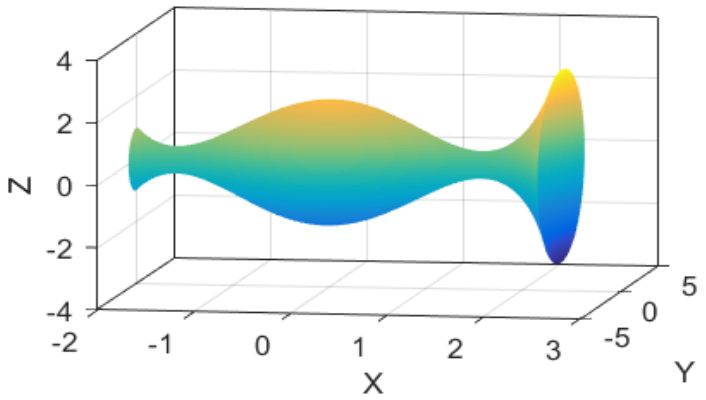
$y_a$ = +1 and $y_b$ = 3.



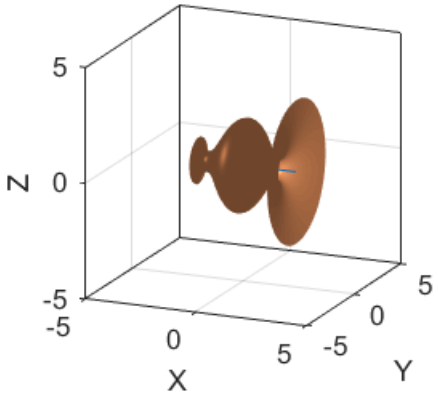

The blue shaded region is rotated around the X-axis. The red shaded area shows the reflection of the blue shaded area about the X-axis. The blue and red lines give the profile of the solid of revolution in the XY plane.



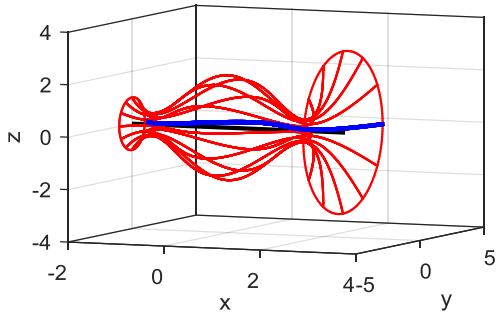
[3D] plot of the solid of revolution




[3D] plot of the solid of revolution (x, y and z values too scale)




View animation of the rotation of the function



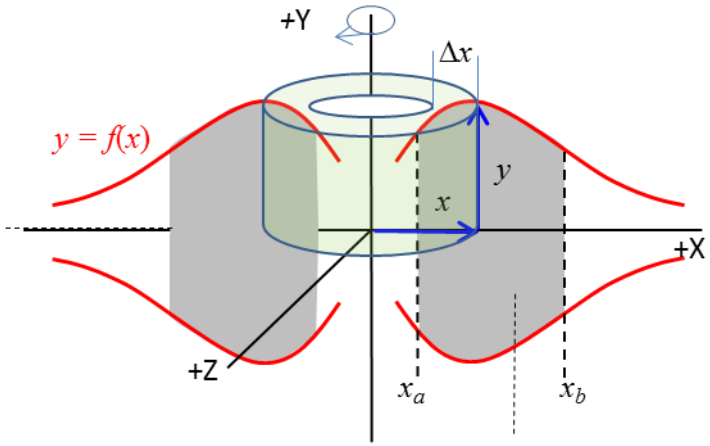
# Rotation about the Y-axis

$y = f(x)$ is a single valued function and the region $\boldsymbol{R}$ is bounded by the function $y = f(x)$, the X-axis and the vertical lines $x_a$ and $x_b$. The Y-axis limits are $y_a = f(x_a)$ $y_b = f(x_b)$

The volume $V$ of the solid of revolution obtained by revolving a region $\boldsymbol{R}$ about the Y-axis is

**CYLINDRICAL SHELL METHOD**

$$V = 2\pi \int_{x_a}^{x_b} y\, x\, dx$$




$$V_{shell} = (2\pi x)\,\Delta x\; y$$

$$V = 2\pi \int_{x_a}^{x_b} y\, x\, dx$$

This is often the best method to use for the rotation around the Y-axis of the region bounded by the X-axis and a function $y = f(x)$.