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Question 1

Four married couples sit around a circular table. Calculate the following number of arrangements:

(A)   The total number of arrangements.

(B)   Each married couple sitting next to each other.

(C)   Each married couple sitting opposite each other.

(D) All the males grouped together.

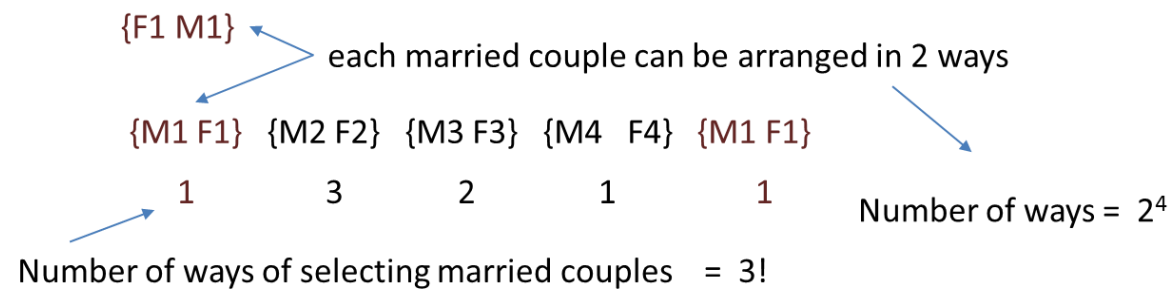(E) No two males or two females are next to each other.

When n objects are arranged around a circle, there is no first or last object. Therefore, we can always place one object (A) at a reference position and then arrange the other $(n-1)$ objects in $(n-1)!$ ways.

We can think of the arrangement in a circle as a straight line arrangement with object A at the reference position

For $n=8$ A x x x x x x x x A   total number of arrangements $N = (8-1)! = 7! = 5040$

(A)   Total number of arrangements $N = (8-1)! = 7! = 5040$

(B)

{F1 M1}

each married couple can be arranged in 2 ways

{M1 F1} {M2 F2} {M3 F3} {M4 F4} {M1 F1}

  1   3   2   1   1   Number of ways = 2^4

Number of ways of selecting married couples   = 3!

Number of arrangements with married couples seated together = $(2^4)(3!) = 96$

(C)

M1

Reference couple only 1 choice / positions are not swapped

M2   choice of 3 couples / positions of F & M can be swapped

M3   choice of 2 couples / positions of F & M can be swapped
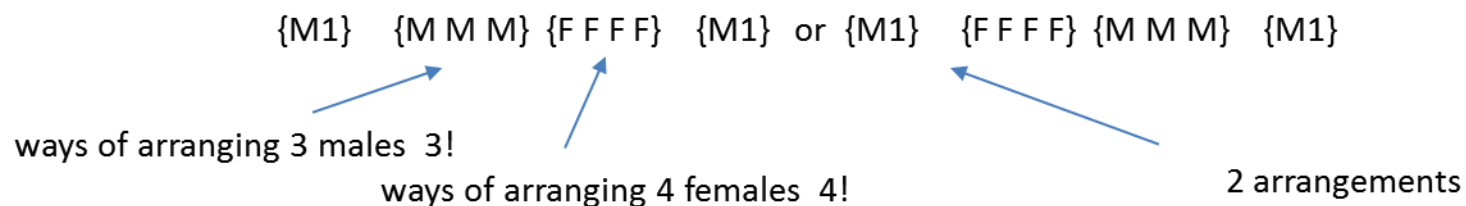
M4   choice of 1 couples / positions of F & M can be swapped

F4

F3

F2

F1
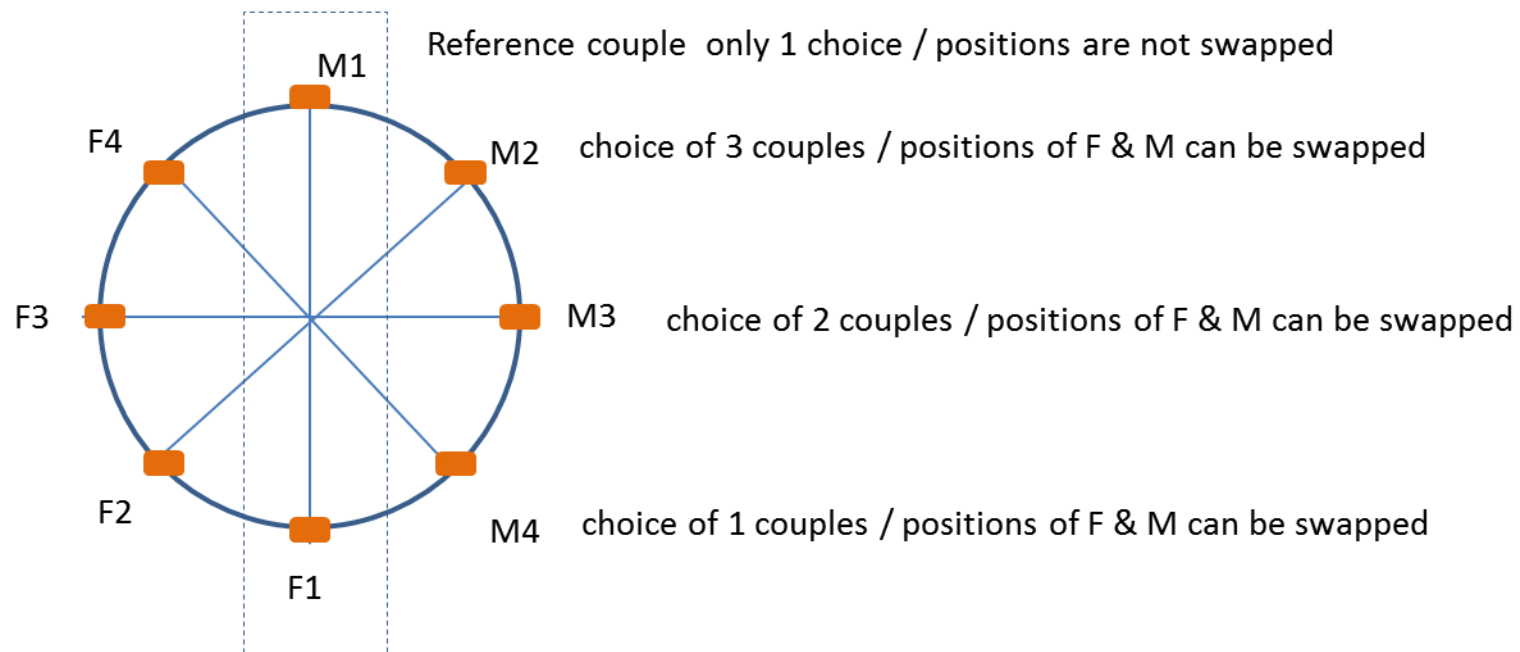
Number of arrangements $N = (3!)\,(2^3) = 48$

(D)

Males grouped together / females grouped together: arrangements

{M1}   {M M M} {F F F F}   {M1}   or   {M1}   {F F F F} {M M M}   {M1}

ways of arranging 3 males 3!

ways of arranging 4 females 4!

2 arrangements

Total number of seating arrangements N = (2)(3!)(4!) = 288

(E)

Males and females alternate {M1} {F} {M} {F} {M} {F} {M} {F} {M1}

ways of arranging 4 females 4!

ways of arranging 3 males 3!

Total number of seating arrangements N = (3!)(4!) = 144