TOPIC 18 PERMUTATIONS COMBINATIONS PROBABILITY

EXERCISE ex34u18556

A guesthouse has four bedrooms.

(A) Calculate how many ways that six people can be accommodated in the four rooms.

(B) Calculate how many ways that 6, 5, 4, 3 and 2 people can be accommodated in the rooms.

(C) How many different ways can the six people be accommodated in the four rooms if there is a maximum of four people to a room?

HSC EXT2 2013 Q10

Answers: there are different approaches to answering this type of question, we will consider only one way.

(A)

Each of the 6 people can be assigned to anyone of the 4 rooms, therefore, the number of possible arrangments is

$$N_{total} = (4)(4)(4)(4)(4)(4) = 4^6 = 4096$$

(B)

If you want a given number of people in at least one room than the number arrangements N is determined from

$$N = \begin{pmatrix} \text{number of combinations for} \\ \text{placing people in groups} \end{pmatrix} \begin{pmatrix} \text{number of permutations for} \\ \text{placing groups of people in rooms} \end{pmatrix}$$

$$\text{people in group} \quad N_{pg} = \begin{pmatrix} \text{number of combinations for} \\ \text{placing people in groups} \end{pmatrix}$$

$$\text{groups in rooms} \quad N_{gr} = \begin{pmatrix} \text{number of permutations for} \\ \text{placing groups of people in rooms} \end{pmatrix}$$

$$N = N_{pg} \; N_{gr}$$

A spreadsheet is a useful tool to use in answering such problems. A section of a spreadsheet is shown below. In using MS EXCEL, you can use functions for calculating the factorial $n, \;\; n!$ and the binomial coefficients $^{n}C_k$, for example, $5! \;\rightarrow\; = fact(5)$ $\quad {}^{6}C_2 \rightarrow \; = combin(6,2)$

No. of people N =	6			No. of rooms n =		4						
Total number of arrangments of six people in 4 rooms = (4)(4)(4)(4)(4)(4)= 409								<--- each person can be assigned to one of four rooms				
No. people in each group / room				Max No.	No. groups	Same group no.		Arrangements	Arrangements	# total arrangment		Arrangements: people
Combinations of people in rooms				in room				groups in rooms	people			ways of selecting people into groups
1	**2**	**3**	**4**		**k**	**n1**	**n2**			**4096**	**4096**	
6	0	0	0	**6**	**1**			4	1	4	**4**	COMBIN(6,6)
5	1	0	0	**5**	**2**			12	6	72	**72**	COMBIN(6,5)
4	2	0	0	**4**	**2**			12	15	180	**540**	COMBIN(6,4)
4	1	1	0	**4**	**3**	2		12	30	360		COMBIN(6,4)*COMBIN(2,1)
3	3	0	0	**3**	**2**	**2**		6	20	120	**2040**	COMBIN(6,3)
3	2	1	0	**3**	3			24	60	1440		COMBIN(6,3)*COMBIN(3,2)
3	1	1	1	**3**	**4**	3		4	120	480		COMBIN(6,3)*COMBIN(3,1)*COMBIN(2,1)
2	2	2	0	**2**	**3**	3		4	90	360	**1440**	COMBIN(6,2)*COMBIN(4,2)
2	2	1	1	**2**	**4**	**2**	2	6	180	1080		COMBIN(6,2)*COMBIN(4,2)*COMBIN(2,1)
								(1)	(2)	(1) * (2)		

^nP_k or $^nP_k / (n1! * n2!)$

Carefully examine the spreadsheet to make sure you understand the principles behind the calculations and then do the calculations. An explanation for the calculations for the shaded rows is given below.

Consider the arrangement in which 3 people must go to one room. This combinations are

3	3	0	0
3	2	1	0
3	1	1	1

where the number represents the people in a room and 0 corresponds to an empty room.

Row 1 [3 3 0 0] : the number of ways 3 people can be selected from 6 is ${}^{6}C_{3}$ for the first group of three and 1 for the second group of three

$$N_{gp} = \left({}^{6}C_{3} \right)(1) = 20$$

The number of permutation of allocating the two groups to the four rooms is

$$N_{gr} = \frac{{}^{4}P_{2}}{2!} = \frac{4!}{2!\,2!} = 6$$

since the two groups both have 3 people in it.

Hence the number of arrangments is $N = N_{pg}\ N_{gr} = (20)(6) = 120$

Row 2 [3 2 1 0]: 3 people in one room, two people in one room, one person in one room and one empty room. The number of ways of selecting the first group of 3 people is ${}^6C_3$; then we select two people from 3 in ${}^3C_2$ ways, therefore,

$$N_{gp} = \left({}^6C_3 \right)\left({}^3C_2 \right) = (20)(3) = 60$$

The number of permutation of allocating the three groups to the four rooms is

$$N_{gr} = {}^4P_3 = 24$$

Hence, the number of arrangments is $N = N_{pg}\ N_{gr} = (60)(24) = 1440$

Row 3 [3 1 1 1]: 3 people in one room, one person in one of three rooms. The number of ways of selecting the first group of 3 people is ${}^6C_3$; then we select one person from three in ${}^3C_1$ ways and then one person from 2 in ${}^2C_1$ ways, therefore,

$$N_{gp} = \left({}^6C_3 \right)\left({}^3C_1 \right)\left({}^2C_1 \right) = (20)(3)(2) = 120$$

The number of permutation of allocating the three groups to the four rooms is

$$N_{gr} = \frac{{}^4P_3}{3!} = 4$$

since there is one person in each of three rooms. Hence, the number of arrangments is

$$N = N_{pg}\ N_{gr} = (120)(4) = 480$$

The total number of allocations with three people in one room = 120 + 1440 + 480 = 2040.

(C)

We can add the numbers from the table in the spreadsheet for the combinations of 2, 3 and 4 people so that there is a maximum of four people allocated to a room

$$N = 540 + 2040 + 1440 = 4020$$

A quicker way is to calclate the ways in which 5 or 6 people can be in a room and substrct these numbers from the total number of arrangments

$$N = 4096 - 72 - 4 = 4020$$