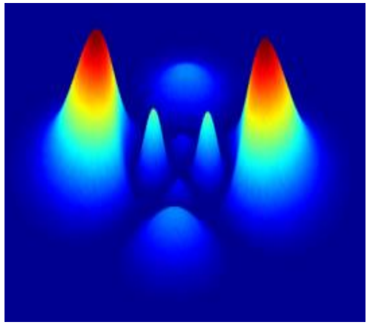


MATHEMATICS EXTENSION 2

4 UNIT MATHEMATICS

DIFFERENTIATION

Essential Background Topic

Differentiation is concerned with the rates of change of physical quantities. It is a fundamental topic in mathematics, physics, chemistry, engineering etc.

Consider a continuous and single value function $y = f(x)$. The the rate of change of y with respect to x at the point x_1 is called the **derivative** and equals the slope of the tangent to the curve $y = f(x)$ at the point x_1. The process of finding the derivative of a function is called **differentiation**.

Take two points P(x_1, y_1) and Q(x_2, y_2) on the curve $y = f(x)$. We require the slope of the tangent at the point P(x_1, y_1). The slope of the straight line (chord) joining the points P and Q is

(1) $$\text{slope PQ} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

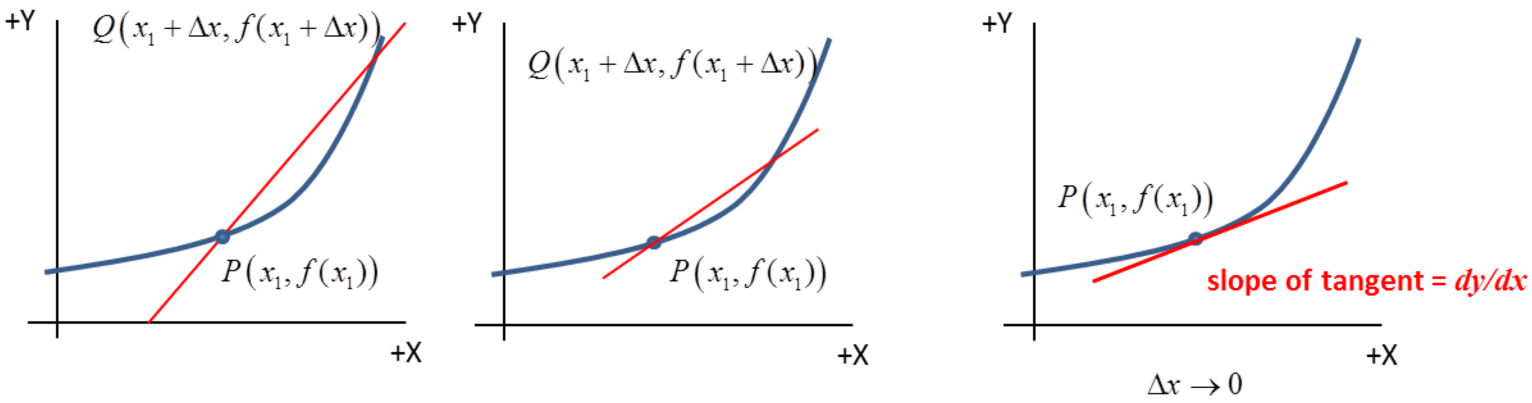
Let $x_2 = x_1 + \Delta x$, $\Delta x = x_2 - x_1$ and $f(x_2) = f(x_1 + \Delta x)$. The point Q approaches the point P as $\Delta x \to 0$ and the slope of the chord approaches the slope of the tangent at the point x_1. Intuitively, we can say that the slope of the tangent at P will be given by the limit of the slope of the chord as $\Delta x \to 0$

(2) $$\text{slope at P} = \underset{x \to 0}{\text{limit}}\left(\frac{f(x_1 + \Delta x) - f(x_1)}{\Delta x}\right)$$

Given a curve $y = f(x)$ and a point P on the curve, the slope of the curve at P is the limit of the slope of lines between P and Q on the curve as Q approaches P. The slope of a curve $y = f(x)$ is the rate at which y is changing as x changes or it is the rate of change of y with respect to x. This slope is known as the derivative of the function y with respect to x. It is given by the special symbols

$$\frac{dy}{dx} \qquad \frac{d\,f(x)}{dx} \qquad f'(x) \qquad \dot{y}$$

(3) $$\frac{dy}{dx} = \underset{x \to 0}{\text{limit}}\left(\frac{f(x + \Delta x) - f(x)}{\Delta x}\right)$$



$$\text{slope chord PQ} = \frac{f(x_1 + \Delta x) - f(x_1)}{x_2 - x_1}$$