EXERCISE 40 SYLLABUS EXAMLES

Part (A)

Evaluate the integral $\int \left(\frac{1+x}{x}\right)^2 dx$

Part (B)

Evaluate the integral $I = \int \left(\frac{x^2}{x^2+1}\right) dx$

Part (C)

Evaluate the integral $I = \int x\left(1+x^2\right)^4 dx$

Part (D)

Evaluate the integral

$$I = \int \left(\frac{x}{\sqrt{1-x}} \right) dx$$

Part (E)

Evaluate the integral

$$I = \int_0^{\pi/4} \sin^2(2x)\, dx$$

Part (F)

Evaluate the integral

$$I = \int \sin^2(x)\cos(x)\, dx$$

Part (G)

Evaluate the integral

$$I = \int \sin^2(x)\cos^3(x)\, dx$$

Part (H)

Evaluate the integral

$$I = \int \frac{dx}{a^2 + x^2}$$

Part (I)

Evaluate the integral

$$\int \sin^{-1} x \, dx$$

Answer Part (A)

$$I = \int \left(\frac{1+x}{x}\right)^2 dx$$

$$I = \int \left(\frac{1+x}{x}\right)^2 dx = I = \int \left(1 + \frac{2}{x} + \frac{1}{x^2}\right) dx$$

$$I = x + 2\log_e(x) - \frac{1}{x} + K$$

Answer Part (B)

$$I = \int \left(\frac{x^2}{x^2+1} \right) dx$$

$$N = \frac{x^2}{x^2+1} = A + \frac{B}{x^2+1} = \frac{Ax^2 + A + B}{x^2+1}$$

$$A = 1 \quad B = -1$$

$$I = \int \left(1 - \frac{1}{x^2+1} \right) dx$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a} \right)$$

$$I = x - \tan^{-1}(x) + K$$

Answer Part (C)

$$I = \int x\left(1 + x^2\right)^4 dx$$

$$u = 1 + x^2 \qquad du = 2x\,dx \qquad x\,dx = \frac{u}{2}$$

$$I = \left(\frac{1}{2}\right)\int u^4\,du$$

$$I = \left(\frac{1}{2}\right)\left(\frac{1}{5}\right)u^5 + K$$

$$I = \left(\frac{1}{10}\right)\left(1 + x^2\right)^5 + K$$

Answer Part (D)

$$I = \int \left(\frac{x}{\sqrt{1-x}} \right) dx$$

$$u = \sqrt{1-x} \quad u^2 = 1 - x \quad x = 1 - u^2 \quad dx = -2u\,du$$

$$I = -2\int \left(\frac{1-u^2}{u} \right) u\,du = -2\int \left(1-u^2\right) du$$

$$I = -2\left(u - u^3/3\right) + K =$$

$$I = -\left(2/3\right)u\left(3-u^2\right) + K$$

$$I = -\left(2/3\right)\sqrt{1-x}\left(3-1+x\right) + K$$

$$I = -\left(2/3\right)\sqrt{1-x}\left(2+x\right) + K$$

Answer Part (E)

$$I = \int_0^{\pi/4} \sin^2(2x)\,dx$$

$$\cos(2x) = \cos^2(x) - \sin^2(x) = 1 - 2\sin^2(x)$$

$$\sin^2(2x) = \frac{1}{2}\left(1 - \cos(4x)\right)$$

$$I = \frac{1}{2}\int_0^{\pi/4} \left(1 - \cos(4x)\right)dx$$

$$I = \frac{1}{2}\left[x - \tfrac{1}{4}\sin(4x)\right]_0^{\pi/4}$$

$$I = \pi / 8$$

Answer Part (F)

$$I = \int \sin^2(x)\cos(x)\,dx$$

$$I = \frac{1}{3}\sin^3(x) + K$$

Answer Part (G)

$$I = \int \sin^2(x)\cos^3(x)\,dx$$

$$I = \int \cos(x)\sin^2(x)\cos^2(x)\,dx \qquad \sin^2(x)+\cos^2(x)=1$$

$$I = \int \cos(x)\sin^2(x)\left(1-\sin^2(x)\right)dx$$

$$I = \int \left(\cos(x)\sin^2(x)-\cos(x)\sin^4(x)\right)dx$$

$$I = \frac{1}{3}\sin^3(x)-\frac{1}{5}\sin^5(x)+K$$

Answer Part (H)

$$I = \int \frac{dx}{a^2 + x^2}$$

$$x = a \tan\theta \quad dx = a \sec^2\theta \, d\theta \quad \theta = \tan^{-1}\left(\frac{x}{a}\right)$$

$$I = \int \frac{a \sec^2\theta \, d\theta}{a^2 + a^2 \tan^2\theta} = \frac{1}{a}\int \, d\theta$$

$$I = \frac{\theta}{a}$$

$$I = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + K$$

Answer Part(I)

$$I = \int \sin^{-1} x \, dx$$

$$\theta = \sin^{-1} x \quad \sin\theta = x \quad \cos\theta \, d\theta = dx$$

$$I = \int \theta \cos\theta \, d\theta$$

integrate by parts

$$u = \theta \quad du = d\theta \quad dv = \cos\theta \, d\theta \quad v = \sin\theta$$

$$\int u \, dv = uv - \int v \, du$$

$$I = \theta \sin\theta - \int \sin\theta \, d\theta$$

$$I = \theta \sin\theta + \cos\theta + K$$

$$\sin\theta = x \quad \sin^2\theta + \cos^2\theta = 1 \quad \cos^2\theta = 1 - \sin^2\theta \quad \cos\theta = \sqrt{1 - x^2}$$

$$I = x \sin^{-1} x + \sqrt{1 - x^2} + K$$