EXERCISE 41 SYLLABUS EXAMLES

Evaluate the following integrals

Part (A) $I = \int \left(\frac{1+x}{x} \right)^2 dx$

Part (B) $I = \int \left(\frac{x^2}{x^2+1} \right) dx$

Part (C) $I = \int x\left(1+x^2\right)^4 dx$

Part (D) $I = \int \left(\frac{x}{\sqrt{1-x}} \right) dx$

Part (E) $I = \int_0^{\pi/4} \sin^2(2x)\, dx$

Answer Part (A)

$$I = \int \left(\frac{1+x}{x} \right)^2 dx$$

$$I = \int \left(\frac{1+x}{x} \right)^2 dx = I = \int \left(1 + \frac{2}{x} + \frac{1}{x^2} \right) dx$$

$$I = x + 2\log_e(x) - \frac{1}{x} + K$$

Answer Part (B)

$$I = \int \left(\frac{x^2}{x^2+1} \right) dx$$

$$N = \frac{x^2}{x^2+1} = A + \frac{B}{x^2+1} = \frac{Ax^2 + A + B}{x^2+1}$$

$$A = 1 \quad B = -1$$

$$I = \int \left(1 - \frac{1}{x^2+1} \right) dx$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a} \right)$$

$$I = x - \tan^{-1}(x) + K$$

Answer Part (C)

$$I = \int x\left(1+x^2\right)^4 dx$$

$$u = 1+x^2 \quad du = 2x\,dx \quad x\,dx = \frac{u}{2}$$

$$I = \left(\frac{1}{2}\right)\int u^4\,du$$

$$I = \left(\frac{1}{2}\right)\left(\frac{1}{5}\right)u^5 + K$$

$$I = \left(\frac{1}{10}\right)\left(1+x^2\right)^5 + K$$

Answer Part (D)

$$I = \int \left(\frac{x}{\sqrt{1-x}} \right) dx$$

$$u = \sqrt{1-x} \quad u^2 = 1-x \quad x = 1-u^2 \quad dx = -2u\,du$$

$$I = -2\int \left(\frac{1-u^2}{u} \right) u\,du = -2\int \left(1-u^2\right) du$$

$$I = -2\left(u - u^3/3\right) + K =$$

$$I = -(2/3)u\left(3-u^2\right) + K$$

$$I = -(2/3)\sqrt{1-x}\left(3-1+x\right) + K$$

$$I = -(2/3)\sqrt{1-x}\left(2+x\right) + K$$

Answer Part (E)

$$I = \int_0^{\pi/4} \sin^2(2x)\,dx$$

$$\cos(2x) = \cos^2(x) - \sin^2(x) = 1 - 2\sin^2(x)$$

$$\sin^2(2x) = \frac{1}{2}\left(1 - \cos(4x)\right)$$

$$I = \frac{1}{2}\int_0^{\pi/4} \left(1 - \cos(4x)\right)dx$$

$$I = \frac{1}{2}\left[x - \tfrac{1}{4}\sin(4x)\right]_0^{\pi/4}$$

$$I = \pi / 8$$