EXERCISE 45   SYLLABUS EXAMLES

Evaluate the following integrals

Part (A)
$$I = \int \frac{9x - 2}{2x^2 - 7x + 3} \, dx$$

Part (B)
$$I = \int \frac{3x^2 - 2x + 1}{\left(x^2 + 1\right)\left(x^2 + 2\right)} \, dx$$

Part (C)
$$I = \int \frac{2x^2 + 3x - 1}{x^3 - x^2 + x - 1} \, dx$$

$$I = \int \frac{9x-2}{2x^2 - 7x + 3}\,dx$$

$$\frac{9x-2}{2x^2-7x+3} = \frac{9x-2}{(x-3)(2x-1)} = \frac{A}{x-3} + \frac{B}{2x-1}$$

$$2Ax - A + Bx - 3B = 9x - 2$$

$$A = 5 \quad B = -1$$

$$I = \int \left(\frac{5}{x-3} - \frac{1}{2\left(x-\frac{1}{2}\right)} \right) dx$$

$$I = 5\log_e(x-3) - \tfrac{1}{2}\log_e\left(x-\tfrac{1}{2}\right) + K$$

$$I = \int \frac{3x^2 - 2x + 1}{\left(x^2 + 1\right)\left(x^2 + 2\right)} dx$$

$$\frac{3x^2 - 2x + 1}{\left(x^2 + 1\right)\left(x^2 + 2\right)} = \frac{A + Bx}{x^2 + 1} + \frac{C + Dx}{x^2 + 2}$$

$$3x^2 - 2x + 1 = Ax^2 + 2A + Bx^3 + 2Bx + Cx^2 + Dx^3 + C + Dx$$

$$3x^2 - 2x + 1 = (B + D)x^3 + (A + C)x^2 + (2B + D)x + 2A + C$$

$$D = -B \quad C = 3 - A \quad B = -2 \quad D = 2 \quad A = -2 \quad C = 5$$

$$I = \int \left(-2\left(\frac{1 + x}{x^2 + 1} \right) + \frac{5 + 2x}{x^2 + 2} \right) dx$$

$$I = \int \left(-2\left(\frac{1}{x^2 + 1} + \frac{x}{x^2 + 1} \right) + \frac{5}{x^2 + 2} + \frac{2x}{x^2 + 2} \right) dx$$

$$I = \log_e\left(x^2 + 2\right) - \log_e\left(x^2 + 1\right) + \frac{5}{\sqrt{2}} \tan^{-1}\left(\frac{x}{\sqrt{2}} \right) - 2\tan^{-1}(x) + K$$

$$I = \int \frac{2x^2 + 3x - 1}{x^3 - x^2 + x - 1} \, dx$$

$$x^3 - x^2 + x - 1 = (x-1)(x^2 + 1)$$

$$\frac{2x^2 + 3x - 1}{x^3 - x^2 + x - 1} = \frac{A}{x-1} + \frac{B}{x^2 + 1}$$

$$2x^2 + 3x - 1 = Ax^2 + A + Bx - B$$

$$A = 2 \quad B = 3$$

$$I = \int \left(2\left(\frac{1}{x-1}\right) + 3\left(\frac{1}{x^2+1}\right) \right) dx$$

$$I = 2\log_e (x-1) + 3\tan^{-1}(x) + K$$