EXERCISE 43 SYLLABUS EXAMLES

Evaluate the following integrals

Part (A) $I = \int x^n \log_e(x)\, dx$

Part (B) $I = \int x^n e^x\, dx$ and evaluate the integral when n = 3.

Part (C) $I = \int \cos^n x\, dx$ and evaluate the integral when n = 4.

Answer Part (A)

$$I = \int x^n \log_e(x)\, dx \qquad \log_e(x) \equiv \ln(x)$$

Integrate by parts $\int u\, dv = u\, v - \int v\, du$

$$u = \log_e(x) \quad du = \frac{dx}{x} \qquad dv = x^n \quad v = \frac{1}{n+1}x^{n+1}$$

$$I = \frac{1}{n+1}x^{n+1}\log_e(x) - \frac{1}{n+1}\int x^n\, dx$$

$$I = \frac{x^{n+1}}{n+1}\log_e(x) - \frac{x^{n+1}}{(n+1)^2} + K$$

$$I = \frac{x^{n+1}}{(n+1)^2}\left((n+1)\log_e(x) - 1\right) + K$$

Answer Part (B)

$$I_n = \int x^n \, e^x \, dx$$

Integrate by parts $\int u \, dv = u \, v - \int v \, du$

$u = x^n \quad du = n \, x^{n-1} \, dx \qquad dv = e^x \quad v = e^x$

$$I_n = x^n \, e^x - n \int x^{n-1} \, e^x dx$$

$$I_n = x^n \, e^x - n \, I_{n-1}$$

$$I_0 = \int e^x \, dx = e^x$$

$$I_1 = x \, e^x - e^x = e^x \left(x - 1 \right)$$

$$I_2 = x^2 \, e^x - 2e^x \left(x - 1 \right) = e^x \left(x^2 - 2x + 2 \right)$$

$$I_3 = x^3 \, e^x - 3e^x \left(x^2 - 2x + 2 \right) = e^x \left(x^3 - 3x^2 + 6x - 6 \right)$$

Answer Part (C)

$$I_n = \int \cos^n x \, dx$$

integrate by parts $\int u \, dv = u\,v - \int v \, du$

$$u = \cos^{n-1} x \quad du = -(n-1) \sin x \cos^{n-2} x \, dx$$

$$dv = \cos x \, dx \quad v = \sin x$$

$$I_n = \sin x \cos^{n-1} x + (n-1) \int \sin^2 x \cos^{n-2} x \, dx$$

$$\sin^2 x + \cos^2 x = 1 \quad \sin^2 x = 1 - \cos^2 x$$

$$I_n = \sin x \cos^{n-1} x + (n-1) \int \left(\cos^{n-2} x - \cos^n x\right) dx$$

$$I_n = \sin x \cos^{n-1} x + (n-1) \left(I_{n-2} - I_n\right) + K$$

$$I_n \left(1 + n - 1\right) = \sin x \cos^{n-1} x + (n-1) \, I_{n-2}$$

$$I_n = \left(\frac{1}{n}\right) \sin x \cos^{n-1} x + \left(\frac{n-1}{n}\right) I_{n-2}$$

$$n = 4$$

$$I_4 = \int \cos^4 x\, dx$$

$$I_4 = \left(\frac{1}{4}\right)\sin x \cos^3 x + \left(\frac{3}{4}\right) I_2 + K$$

$$I_2 = \left(\frac{1}{2}\right)\sin x \cos x + \left(\frac{1}{2}\right) I_0 \qquad I_0 = x$$

$$I_4 = \left(\frac{1}{4}\right)\sin x \cos^3 x + \left(\frac{3}{4}\right)\left(\left(\frac{1}{2}\right)\sin x \cos x + \left(\frac{1}{2}\right)x\right) + K$$

$$I_4 = \left(\frac{1}{4}\right)\sin x \cos^3 x + \left(\frac{3}{8}\right)\left(\sin x \cos x + x\right) + K$$

$$I_4 = \left(\frac{1}{8}\right)2\sin x \cos x \cos^2 x + \left(\frac{3}{16}\right)\left(2\sin x \cos x + 2x\right) + K$$

$$I_4 = \left(\frac{1}{8}\right)\sin(2x)\cos^2 x + \left(\frac{3}{16}\right)\sin(2x) + \left(\frac{3x}{8}\right) + K$$