EXERCISE 42 SYLLABUS EXAMLES

Evaluate the following integrals

Part (A) $I = \int \sin^2(x)\cos(x)\,dx$

Part (B) $I = \int \sin^2(x)\cos^3(x)\,dx$

Part (C) $I = \int \dfrac{dx}{a^2 + x^2}$

Part (D) $I = \int \sin^{-1} x\,dx$

Part (E) $I = \int e^{3\theta}\cos(4\theta)\,d\theta$

Answer Part (A)

$$I = \int \sin^2(x)\cos(x)\,dx$$

$$I = \frac{1}{3}\sin^3(x) + K$$

Answer Part (B)

$$I = \int \sin^2(x)\cos^3(x)\,dx$$

$$I = \int \cos(x)\sin^2(x)\cos^2(x)\,dx \qquad \sin^2(x)+\cos^2(x)=1$$

$$I = \int \cos(x)\sin^2(x)\left(1-\sin^2(x)\right)dx$$

$$I = \int \left(\cos(x)\sin^2(x)-\cos(x)\sin^4(x)\right)dx$$

$$I = \frac{1}{3}\sin^3(x)-\frac{1}{5}\sin^5(x)+K$$

Answer Part (C)

$$I = \int \frac{dx}{a^2 + x^2}$$

$$x = a\tan\theta \quad dx = a\sec^2\theta\, d\theta \quad \theta = \tan^{-1}\left(\frac{x}{a}\right)$$

$$I = \int \frac{a\sec^2\theta\, d\theta}{a^2 + a^2\tan^2\theta} = \frac{1}{a}\int d\theta$$

$$I = \frac{\theta}{a}$$

$$I = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + K$$

Answer Part (D)

$$I = \int \sin^{-1} x \, dx$$

$$\theta = \sin^{-1} x \quad \sin\theta = x \quad \cos\theta \, d\theta = dx$$

$$I = \int \theta \cos\theta \, d\theta$$

integrate by parts

$$u = \theta \quad du = d\theta \quad dv = \cos\theta \, d\theta \quad v = \sin\theta$$

$$\int u \, dv = uv - \int v \, du$$

$$I = \theta \sin\theta - \int \sin\theta \, d\theta$$

$$I = \theta \sin\theta + \cos\theta + K$$

$$\sin\theta = x \quad \sin^2\theta + \cos^2\theta = 1 \quad \cos^2\theta = 1 - \sin^2\theta \quad \cos\theta = \sqrt{1 - x^2}$$

$$I = x \sin^{-1} x + \sqrt{1 - x^2} + K$$

Answer Part (E)

$$I = \int e^{3\theta} \cos(4\theta)\, d\theta$$

Integrate by parts $\int u\, dv = u\, v - \int v\, du \quad u = e^{3\theta} \quad du = 3e^{3\theta}\, d\theta \qquad dv = \cos(4\theta)\, d\theta \quad v = \left(\frac{1}{4}\right)\sin(4\theta)$

$$I = \left(\frac{1}{4}\right)e^{3\theta}\sin(4\theta) - \left(\frac{3}{4}\right)\int e^{3\theta}\sin(4\theta)\, d\theta \qquad I_1 = \int e^{3\theta}\sin(4\theta)\, d\theta$$

Integrate by parts $\int u\, dv = u\, v - \int v\, du \quad u = e^{3\theta} \quad du = 3e^{3\theta}\, d\theta \qquad dv = \sin(4\theta)\, d\theta \quad v = \left(\frac{-1}{4}\right)\cos(4\theta)$

$$I_1 = \left(\frac{-1}{4}\right)e^{3\theta}\cos(4\theta) + \left(\frac{3}{4}\right)\int 3e^{3\theta}\cos(4\theta)\, d\theta = \left(\frac{-1}{4}\right)e^{3\theta}\cos(4\theta) + \left(\frac{3}{4}\right)I$$

$$I = \left(\frac{1}{4}\right)e^{3\theta}\sin(4\theta) - \left(\frac{3}{4}\right)\left(\left(\frac{-1}{4}\right)e^{3\theta}\cos(4\theta) + \left(\frac{3}{4}\right)I\right)$$

$$I = e^{3\theta}\left(\left(\frac{1}{4}\right)\sin(4\theta) + \left(\frac{3}{4^2}\right)\cos(4\theta)\right) - \left(\frac{3}{4}\right)^2 I$$

$$\left(1 + \left(\frac{3}{4}\right)^2\right)I = \left(\frac{1}{4}\right)e^{3\theta}\left(\sin(4\theta) + \left(\frac{3}{4}\right)\cos(4\theta)\right)$$

$$\left(\frac{4^2 + 3^2}{4^2}\right)I = \left(\frac{e^{3\theta}}{4}\right)\left(\sin(4\theta) + \left(\frac{3}{4}\right)\cos(4\theta)\right)$$

$$I = \left(\frac{1}{4^2 + 3^2}\right)e^{3\theta}\left(4\sin(4\theta) + 3\cos(4\theta)\right)$$

$$= \frac{e^{3\theta}}{25}\left(4\sin(4\theta) + 3\cos(4\theta)\right)$$