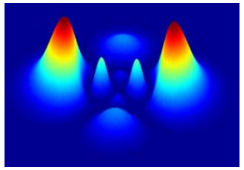


MATHEMATICS EXTENSION 2

4 UNIT MATHEMATICS

TOPIC 5: VOLUMES

Exercise vol5_p005

Find the volume V of the solid of revolution generated by the rotation about the X-axis of the region bounded by the curves

$$f_2(x) = 42 - 5x \quad \text{and} \quad f_1(x) = 2x^2 - 5x + 10$$

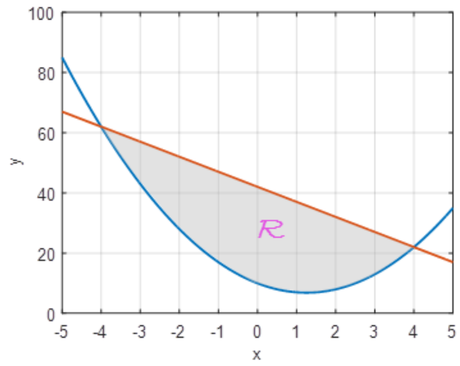
Solution

How to approach the problem:
Sketch the function and the solid.
Give the equations for the shape of the solid.
Find the upper and lower limits for the bounded region.
Evaluation the definite integral to find the volume.

We need to find the points of intersection of the two functions.

$$f_1(x) = f_2(x)$$
$$2x^2 - 5x + 10 = 42 - 5x$$
$$x^2 = 16$$
$$x = \pm 4$$

The limits of integration are x_a = - 4 and x_b = 4.



$f_2(x) = 42 - 5x$

$f_1(x) = 2x^2 - 5x + 10$

Volume of solid of revolution about the X-axis is

$$V = V_2 - V_1$$
$$V_2 = \pi \int_{x_a}^{x_b} f_2(x)^2 \, dx$$
$$f_2(x) = 42 - 5x \qquad \left(f_2(x)\right)^2 = 25x^2 - 420x + 1764$$
$$V_2 = \pi \int_{-4}^{4} \left(25x^2 - 420x + 1764\right) dx$$
$$V_2 = \pi \left[\tfrac{25}{3} x^3 - 210\, x^2 + 1764\, x\right]_{-4}^{4}$$
$$V_2 = \pi \left[\tfrac{25}{3}\left(4^3 + 4^3\right) - 210\left(4^2 - 4^2\right) x^2 + 1764\left(4 + 4\right)\right]$$
$$V_2 = 15179\,\pi$$

$$f_1(x) = 2x^2 - 5x + 10$$

$$\left(f_1(x)\right)^2 = 4x^4 - 20x^3 + 65x^2 - 100x + 100$$

$$V_1 = \pi \int_{-4}^{4} \left(4x^4 - 20x^3 + 65x^2 - 100x + 100\right) dx$$

$$V_1 = \pi \left[\tfrac{4}{5} x^5 - 5x^4 + \tfrac{65}{3} x^3 - 50x^2 + 100x \right]_{-4}^{4}$$

$$V_1 = \pi \left[\tfrac{4}{5}\left(4^5 + 4^5\right) + \tfrac{65}{3}\left(4^3 + 4^3\right) + 100\left(4 + 4\right) \right]$$

$$V_1 = 5212\pi$$

$$V = V_2 - V_1 = (15179 - 5212)\pi$$

$$V = 9967\pi$$

QED