EXERCISE 2_113 HSC 2013 Q11a

Consider the two complex variables

$$z_1 = 2 - \sqrt{3}\,i \quad \text{and} \quad z_2 = 1 + \sqrt{3}\,i$$

Find the following

The complex conjugates of z_1 and z_2
The magnitudes (moduli) of z_1 and z_2
The arguments of z_1 and z_2
The polar and exponential forms of z_1 and z_2

$z_1 + z_2 \qquad z_1 - \overline{z}_2 \qquad z_1 + \sqrt{3}\,i\ z_2$

$z_1\,z_2$ magnitude and argument of $z_1\,z_2$

$\dfrac{z_1}{z_2}$ magnitude and argument of $\dfrac{z_1}{z_2}$

z_1^{-12} and z_2^{24} in its simplest form

ANSWERS

$$\bar{z}_1 = 2 + \sqrt{3}\,i \qquad \bar{z}_2 = 1 - \sqrt{3}\,i$$

$$|z_1| = R_1 = \sqrt{4+3} = \sqrt{7} = 2.6458 \qquad |z_2| = R_2 = \sqrt{1+3} = 2.0000$$

$$\arg(z_1) = \theta_1 = \tan^{-1}\left(\frac{-\sqrt{3}}{2}\right) = -0.7137 \text{ rad}$$

$$\arg(z_2) = \theta_2 = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = 1.0472 \text{ rad} = \frac{\pi}{3} \text{ rad}$$

Polar form $\quad z_1 = R_1\left(\cos\theta_1 + i\sin\theta_1\right) \qquad z_2 = R_2\left(\cos\theta_2 + i\sin\theta_2\right)$

Exponential form $\quad z_1 = R_1 e^{i\theta_1} \qquad z_2 = R_2 e^{i\theta_2}$

$$R_1 = \sqrt{7} = 2.6458 \quad \theta_1 = -0.7137 \text{ rad}$$

$$R_2 = 2.0000 \quad \theta_2 = 1.0472 \text{ rad} = \frac{\pi}{3} \text{ rad}$$

$$z_1 + z_2 = 3 \qquad z_1 - \overline{z}_2 = 1$$

$$z_1 = 2 - \sqrt{3}\,i \qquad z_2 = 1 + \sqrt{3}\,i \quad \sqrt{3}\,i\,z_2 = -3 + \sqrt{3}\,i \quad z_1 + \sqrt{3}\,i\,z_2 = -1$$

$$z_1 z_2 = \left(2 - \sqrt{3}\,i\right)\left(1 + \sqrt{3}\,i\right) = 5 + \sqrt{3}\,i$$

$$\left|z_1 z_2\right| = \sqrt{25 + 3} = 5.2915 \qquad \left|z_1 z_2\right| = R_1 R_2 = \left(2.6458\right)\left(2.0000\right) = 5.2915$$

$$\arg(z_1 z_2) = \tan^{-1}\left(\frac{\sqrt{3}}{5}\right) = 0.3335 \text{ rad} \qquad \arg(z_1 z_2) = \arg(z_1) + \arg(z_2) = -0.7137 + 1.0472 = 0.3335$$

$$z_1 / z_2 = \frac{2 - \sqrt{3}\,i}{1 + \sqrt{3}\,i} = \frac{2 - \sqrt{3}\,i}{1 + \sqrt{3}\,i}\;\frac{1 - \sqrt{3}\,i}{1 - \sqrt{3}\,i} = -0.2500 - 1.2990\,i$$

$$\left|z_1 / z_2\right| = \sqrt{0.2500^2 + 1.2990^2} = 1.329 \qquad \left|z_1 / z_2\right| = R_1 / R_2 = \left(2.6458\right) / \left(2.0000\right) = 1.3229$$

$$\arg(z_1 / z_2) = \tan^{-1}\left(\frac{-1.2990}{0.2500}\right) = -1.7609 \text{ rad} \quad \arg(z_1 / z_2) = \arg(z_1) - \arg(z_2) = -0.7137 - 1.0472 = -1.7609$$

NB in the Argand diagram the complex number $\dfrac{z_1}{z_2}$ is in the third quadrant

$$z_1 = R_1 e^{i\theta_1} = 7^{1/2} e^{i(-0.7137)}$$

$$z_1^{-12} = 7^{(1/2)(-12)} e^{i(-0.7137)(-12)} = 7^{-6} e^{i(8.5644)} = 7^{-6} e^{i(8.5644-2\pi)} = 7^{-6} e^{i(2.2812)}$$

$$z_2 = R_2 e^{i\theta_2} = 2e^{i(\pi/3)}$$

$$z_2^{24} = 2^{24} e^{i(24\pi/3)} = 2^{24} e^{i(8\pi)} = 2^{24}$$