EXERCISE 44 SYLLABUS EXAMLES

Evaluate the following integrals

Part (A) $$I = \int \frac{dx}{x^2 - 4x - 1}$$

Part (B) $$I = \int \frac{dx}{3x^2 + 6x + 10}$$

Part (C) $$I = \int \frac{3x + 2}{x^2 - 4x + 1}\, dx$$

Answer Part (A)

$$I = \int \frac{dx}{x^2 - 4x - 1}$$

let $x^2 - 4x - 1 = (x - A)^2 + B = x^2 - 2Ax + A^2 + B$

$A = 2 \quad 4 + B = -1 \quad B = -5 \quad a = \sqrt{5} \quad B = -a^2$

$z = x - A = x - 2 \quad dx = dz$

$$\frac{1}{x^2 - 4x - 1} = \frac{1}{z^2 - a^2} = \frac{1}{2a}\left(\frac{1}{z - a} - \frac{1}{z + a}\right)$$

$$I = \frac{1}{2a}\int\left(\frac{1}{z - a} - \frac{1}{z + a}\right)dz$$

$$I = \frac{1}{2a}\left(\log_e(z - a) - \log_e(z + a)\right) + K$$

$$I = \frac{1}{2\sqrt{5}}\left(\log_e\left(\frac{x - 2 - \sqrt{5}}{x - 2 + \sqrt{5}}\right)\right) + K$$

Answer Part (B)

$$I = \int \frac{dx}{3x^2 + 6x + 10}$$

let $\quad 3x^2 + 6x + 10 = 3\left(x^2 + 2x + 10/3\right) = 3\left(x^2 + 2x + 1 + 10/3 - 1\right)$

$$3x^2 + 6x + 10 = 3\left((x+1)^2 + 7/3\right)$$

$$z = x + 1 \qquad dx = dz \qquad a = \sqrt{7/3}$$

$$I = \frac{1}{3}\int \frac{dx}{z^2 + a^2}$$

$$I = \frac{1}{3a}\tan^{-1}\left(\frac{z}{a}\right) + K$$

$$I = \frac{1}{\sqrt{21}}\tan^{-1}\left(\sqrt{\frac{3}{7}}(x+1)\right) + K$$

$$I = \frac{\sqrt{21}}{21}\tan^{-1}\left(\frac{\sqrt{21}}{7}(x+1)\right) + K$$

Answer Part (C)

$$I = \int \frac{3x+2}{x^2-4x+1}\,dx$$

let $y = x^2-4x+1$ $\quad dy/dx = 2x-4$ $\quad 3x+2 = \left(\frac{3}{2}\right)(2x-4)+8$

$$I = \frac{3}{2}\int\left(\frac{2x-4}{x^2-4x+1}\right)dx + \frac{3}{2}\int\frac{8}{x^2-4x+1}dx$$

$$I = \frac{3}{2}\log_e\left(x^2-4x+1\right)+12\int\frac{dx}{x^2-4x+1}+K$$

$x^2-4x+1 = (x-2)^2-3$ $\quad z = x-2$ $\quad a = \sqrt{3}$

$$I_1 = \int\frac{dx}{x^2-4x+1} = \int\frac{dx}{z^2-a^2}$$

$$\frac{1}{z^2-a^2} = \left(\frac{1}{2a}\right)\left(\frac{1}{z-a}-\frac{1}{z+a}\right)$$

$$I_1 = \left(\frac{1}{2a}\right)\int\left(\left(\frac{1}{z-a}-\frac{1}{z+a}\right)\right)dx = \left(\frac{1}{2a}\right)\log_e\left(\frac{z-a}{z+a}\right)$$

$$I_1 = \left(\frac{1}{2\sqrt{3}}\right)\log_e\left(\frac{x-2-\sqrt{3}}{x-2+\sqrt{3}}\right)$$

$$I = \frac{3}{2}\log_e\left(x^2-4x+1\right)+2\sqrt{3}\log_e\left(\frac{x-2-\sqrt{3}}{x-2+\sqrt{3}}\right)+K$$