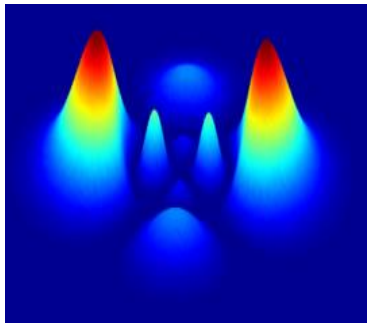

HSC MATHEMATICS: MATHEMATICS EXTENSION 1

3 UNIT MATHEMATICS

TOPIC 18 EXERCISE 01

PERMUTATIONS COMBINATIONS PROBABILITY

Question 88

There are 6 red, 6 blue, 6 green and 6 yellow balls in a box. A sample of 6 balls are removed from the box. Calculate (A) the probability that all the balls are the same colour; (B) the sample contains four balls of one colour and two balls of another colour and (C) three sets of 2 balls each of the same colours.

Answer 88

Number of balls $n = 24$ number of balls selected $k = 6$ $n_{red} = n_{blue} = n_{green} = n_{yellow} = 6$

The total number of ways (combinations) of selecting 6 balls from 24 is

$$N = {}^{24}C_6 = \frac{24!}{(6!)\,(18!)} = 134596$$

(A)

Draw the first ball (it could be any colour), then the other 5 balls have to be the same as the first ball. After the first selection there are only 23 balls in the box and only 5 balls which are the same colour as the first ball selected. Using the same argument for the selection of the other balls, the probability is

$$\text{Prob(balls same colour)} = (1)\left(\frac{5}{23}\right)\left(\frac{4}{22}\right)\left(\frac{3}{21}\right)\left(\frac{2}{20}\right)\left(\frac{1}{19}\right) = 2.9719 \times 10^{-5}$$

Alternatively: there are only 4 possible ways of selecting all the balls to be the same colour since we have only four different colours

$$\text{Prob(balls same colour)} = \frac{4}{{}^{24}C_6} = \frac{4}{134596} = 2.9719 \times 10^{-5}$$

(B)

There are 4 ways of selecting the first colour and 3 ways of selecting the second colour, hence the number of ways of selecting only two colours is $N_{colour} = (4)(3) = 12$.

The numbers of ways of choosing 4 balls of the one colour from 6 balls is $N_4 = {}^6C_4 = \frac{6!}{(4!)\,(2!)} = 15$

The numbers of ways of choosing 2 balls of the one colour from 6 balls is $N_2 = {}^6C_2 = \frac{6!}{(2!)\,(4!)} = 15$

$$\text{Prob(4 same colour / 2 same colour)} = \frac{(12)(15)(15)}{{}^{24}C_6} = \frac{2700}{134596} = 0.0201$$

(C)

There are 4 ways of selecting the first colour, 3 ways of selecting the second colour and 2 ways of selecting the third colour, hence, the number of ways of selecting only three colours is $N_{colour} = (4)(3)(2) = 24$.

The numbers of ways of choosing 2 balls of the one colour from 6 balls is $N_2 = {}^6C_2 = \dfrac{6!}{(2!)\ (4!)} = 15$

$$\text{Prob}(2 \text{ same colour} / 2 \text{ same colour} / 2 \text{ same colour}) = \frac{(24)(15)(15)(15)}{{}^{24}C_6} = \frac{81000}{134596} = 0.6018$$