EXERCISE 2_123

The HSC references are questions that are the same or similar to the actual HSC questions

Part (A) HSC 2013 Q11e

Sketch the region on the Argand diagram defined by the relationship

$$z^2 + \overline{z}^2 \leq 8$$

Part (B) HSC 2012 Q11b

Sketch the region on the Argand diagram defined by the inequalities

$$|z - i| \leq 1 \qquad |z + 2| \geq 2$$

Part (C) HSC 2012 Q6c

Sketch the region on the Argand diagram defined by the inequality

$$\left|\frac{1}{z} + i\right| \leq 1$$

Answer (A)

$z = x + iy \quad z^2 = x^2 - y^2 + i(2xy) \qquad \bar{z} = x - iy \quad z^2 = x^2 - y^2 - i(2xy)$

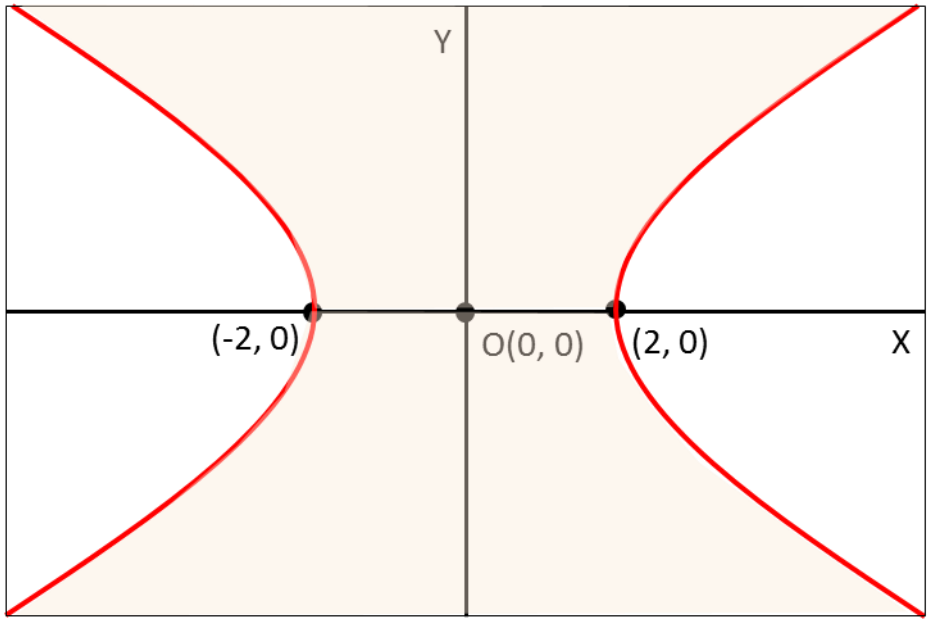
$z^2 + \bar{z}^2 = 2\left(x^2 - y^2\right) \le 8 \qquad \frac{x^2}{2^2} - \frac{y^2}{2^2} \le 1$

The equation $\frac{x^2}{2^2} - \frac{y^2}{2^2} = 1$ corresponds to a rectangular hyperbola with vertices at (-2, 0) and (2, 0).

$x = 0 \quad y = 0 \quad \Rightarrow z^2 + \bar{z}^2 \le 8$

$x > 2 \quad y = 0 \quad \Rightarrow z^2 + \bar{z}^2 > 8$

The shaded area is the defined region.



$\frac{x^2}{2^2} - \frac{y^2}{2^2} = 1$

Answer (B)

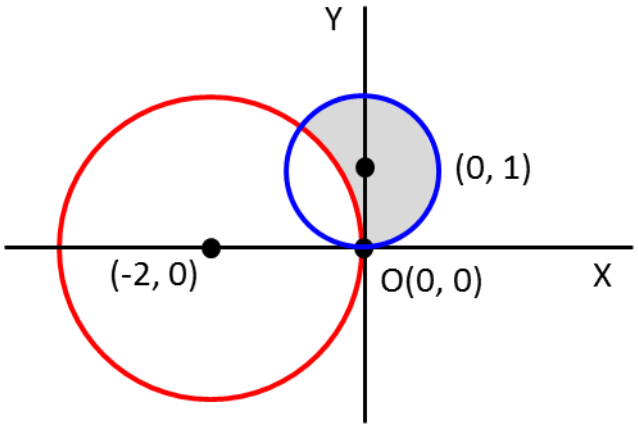
$|z-i| \le 1 \qquad |x+i(y-1)| \le 1 \qquad x^2+(y-1)^2 \le 1$

Region corresponds to the area inside a circles of radius 1 and centre (0, 1)

$|z+2| \ge 2 \qquad |(x+2)+iy| \ge 2 \qquad (x+2)^2+y^2 \ge 2$

Region corresponds to the area outside a circles of radius 2 and centre (-2, 0)

The shaded area is the region defined by $|z-i| \le 1 \qquad |z+2| \ge 2$



Answer (C)

$$\left|\frac{1}{z}+i\right|\le 1$$

$$\left|\frac{1}{z}+i\right|=\left|\frac{1+iz}{z}\right|=\frac{|1+iz|}{|z|}\le 1$$

$$|1+iz|\le|z|$$

$$z=x+iy$$

$$iz=-y+ix$$

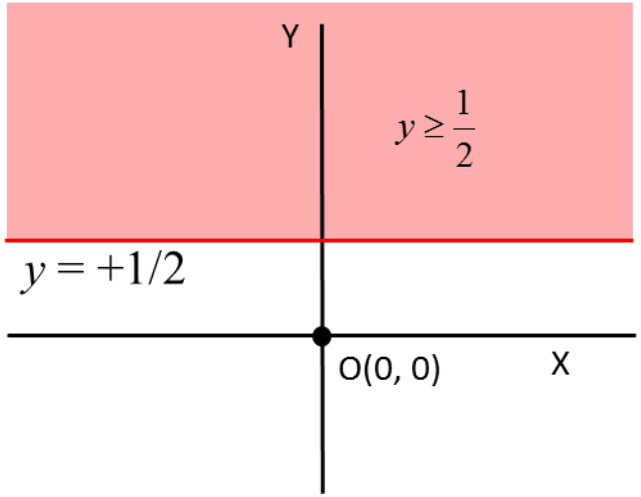
$$1+iz=1-y+ix$$

$$|z|^2=x^2+y^2$$

$$|1+iz|^2=(1-y)^2+x^2=1-2y+y^2+x^2$$

$$1-2y+y^2+x^2\le x^2+y^2$$

$$y\ge\frac{1}{2}$$



Region shown as shaded area