Find the volume $V$ of an ellipsoid formed by the rotation of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad \text{about the X-axis.}$$

**Solution**

Volume of solid of revolution about the X axis is

$$V = \pi \int_a^b y^2 \, dx$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$y = b\left(1 - \frac{x^2}{a_2}\right)^{1/2}$$

The limits of integration are $x_a = -a$ and $x_b = a$.

The function $y = f(x) \geq 0$ in the interval [$-a$ $a$] is

$$y = b\left(1 - \frac{x^2}{a_2}\right)^{1/2} \qquad y^2 = b^2\left(1 - \frac{x^2}{a^2}\right)$$

The volume of the ellipsoid is

$$V = \pi \int_{-a}^{a} b^2\left(1 - \frac{x^2}{a^2}\right) dx = \frac{2\pi b^2}{a^2} \int_0^a \left(a^2 - x^2\right) dx$$

$$V = \frac{2\pi b^2}{a^2}\left[a^2 x - \tfrac{1}{3}x^3\right]_0^a$$

$$V = \frac{4\pi a b^2}{3}$$

For a sphere of radius $a$ $\qquad a = b \qquad V_{sphere} = \frac{4\pi a^3}{3}$ $\qquad$ **QED**