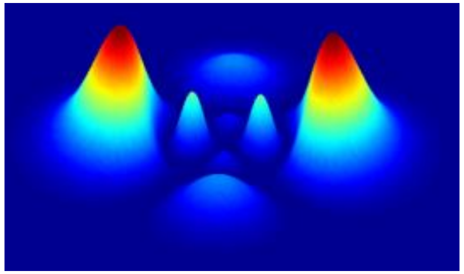


MATHEMATICS EXTENSION 2

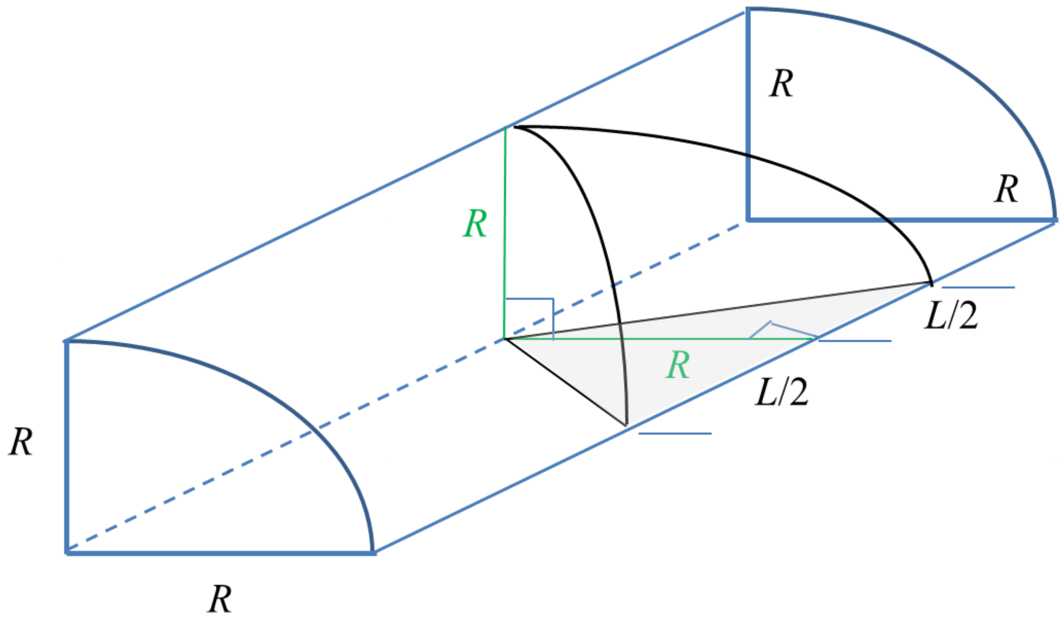
TOPIC 5: VOLUMES

Exercise vol5_p007

A solid is cut from a quarter cylinder of radius R. The solid's base is an isosceles triangle. The width of the isosceles triangle is L and its height is R.

Show that the volume of the solid is

$$V = \frac{1}{3} L\, R^3$$



Solution

The volume of the solid can be calculated from the formula

$$V = \int_{x_a}^{x_b} A(x)\,dx$$

where the cross-sections of the solid are rectangles in the YZ plane. The area of the rectangle at x is

$$A(x) = (2\,y)\,z$$

The base of the solid is an isosceles triangle of height R and width L. From similar triangles at x

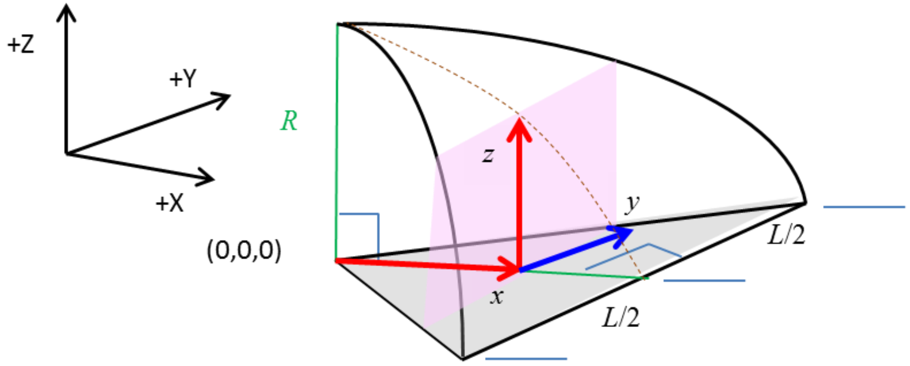
$$y = \left(\frac{L}{2}\right)x$$

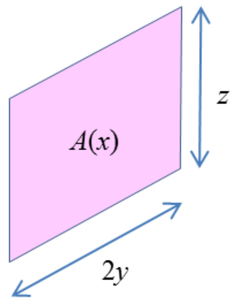
In the XZ plane, the height of the rectangle at x is

$$z = \left(R^2 - x^2\right)^{1/2}$$

Area of the rectangle at x is

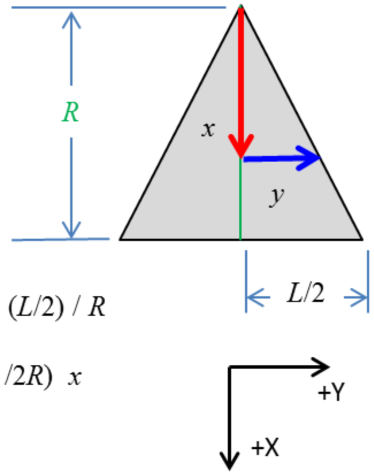
$$A(x) = \left(\frac{L}{R}\right)x\left(R^2 - x^2\right)^{1/2}$$





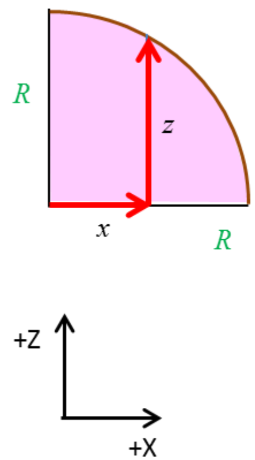
$A(x) = (2\,y)\,z$

$A(x) = (L / R)\,x\,(R^2 - x^2)^{1/2}$



$y / x = (L/2) / R$

$y = (L/2R)\;x$



$x^2 + z^2 = R^2$

$z = (R^2 - x^2)^{1/2}$

The limits of the integration are $x_a = 0$ and $x_b = R$.

The volume of the solid is

$$V = \int_{x_a}^{x_b} A(x)\, dx$$

$$V = \int_0^R \left(\frac{L}{R}\right) x\left(R^2 - x^2\right)^{1/2} dx$$

$$V = \left(\frac{L}{R}\right)\left[\left(\frac{2}{3}\right)\left(\frac{-1}{2}\right)\left(R^2 - x^2\right)^{3/2}\right]_0^R$$

$$V = \frac{1}{3} L\, R^2$$

QED