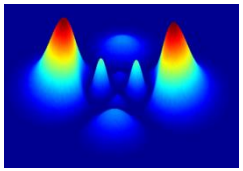


MATHEMATICS EXTENSION 2

4 UNIT MATHEMATICS

TOPIC 5: VOLUMES

Exercise vol5_p003

Consider the region $\mathcal{R}$ formed by the part of a circle of radius a in the first quadrant and the X-axis. Revolution about the Y-axis of the region $\mathcal{R}$ produces a solid hemisphere. Find the volume V of the hemisphere by the cylindrical shell method and the disk method.

How to approach the problem:
Sketch the function and the solid.
Give the equations for the shape of the solid. Find the upper and lower limits for the bounded region.
Evaluation the definite integral to find the volume.

Solution

Volume of solid of revolution about the Y-axis using the **cylindrical shell method** is

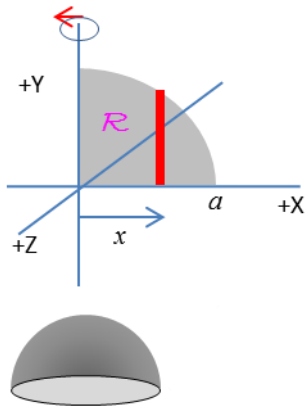
$$V = 2\pi \int_{x_a}^{x_b} x\, y\, dx$$

The equation of the circle is $x^2 + y^2 = a^2$

For the region of the circle in the first quadrant

$$y = \left(a^2 - x^2\right)^{1/2}$$

and the limits of integration are $x_a = 0$ and $x_b = 0$



The volume is

$$V = 2\pi \int_0^a x\,y\,dx = \int_0^a x\left(a^2 - x^2\right)^{1/2} dx$$

$$u = a^2 - x^2 \quad du = -2\,x\,dx \quad x\,dx = \left(-1/2\right)du$$

$$x = 0 \quad u = a^2 \qquad x = a \quad u = 0$$

$$V = -\pi \int_{a^2}^0 u^{1/2} du = \pi \int_0^{a^2} u^{1/2} du$$

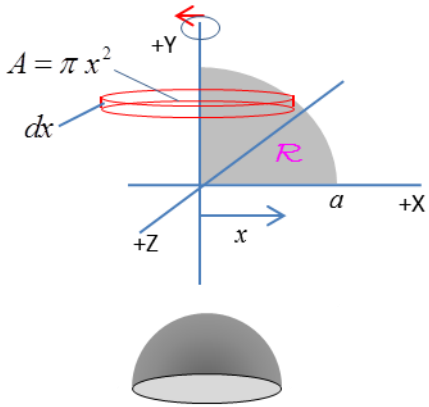
$$V = \left(\frac{2\pi}{3}\right)a^3$$

QED

Volume of solid of revolution about the Y-axis using the **disk method** is

$$V = \pi \int_{x_a}^{x_b} x^2\,dy$$

The equation of the circle is $x^2 + y^2 = a^2$



For the region of the circle in the first quadrant

$$x^2 = \left(a^2 - y^2\right)$$

and the limits of integration are $'a' = 0$ and $'b' = 0$

The volume is

$$V = \pi \int_0^a \left(a^2 - y^2\right)^2 dy$$

$$V = \pi \left[a^2 y - \tfrac{1}{3} y^3\right]_0^a = \pi\left(a^3 - \tfrac{1}{3}a^3\right)$$

$$V = \left(\frac{2\pi}{3}\right)a^3$$

QED