ONLINE: MATHEMATICS EXTENSION 2

Topic 2 COMPLEX NUMBERS

EXERCISE p2401

p001

Show the region on an Argand diagram that satisfy the conditions

$$\mathrm{Re}(z) \leq 3 \qquad \mathrm{Re}(z) > -3$$
$$\mathrm{Im}(z) < 2 \qquad \mathrm{Re}(z) > -3$$

p002

Show on an Argand diagram the complex numbers z that satisfy the condition

$$|z - z_1| = |z - z_2| \qquad z_1 = 1 + i \qquad z_2 = -3 + i$$

p003

If $z_1 = 1 + i$ then show on an Argand diagram

$$z - z_1 = 2\left(\cos\left(\pi/4\right) + i\sin\left(\pi/4\right)\right)$$

p004

If $z_1 = -1 + i$ then show on an Argand diagram

$$\left|z - z_1\right| = 2$$

p005

If $z_1 = -1 + i$ then show on an Argand diagram

$$\theta = Arg\left(z - z_1\right) = \pi/4$$

p006

Show that

$$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|} \qquad \arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$$

$$Arg(z - z_1) - Arg(z - z_2) = Arg\left(\frac{z - z_1}{z - z_2}\right)$$

Sketch and comment on the locus of

$$Arg(z - z_1) - Arg(z - z_2) = Arg\left(\frac{z - z_1}{z - z_2}\right)$$

where

$$z_1 = \cos(\pi/4) + i\sin(\pi/4)$$

$$z_2 = 2\left(\cos(5\pi/6) + i\sin(5\pi/6)\right)$$

p007

Sketch the region on an Argand diagram for the expression

$$2(z + \overline{z}) - z\,\overline{z} > 8$$

p008

Sketch the allowed region defined by the relationships

$$-\frac{\pi}{4} < Arg(z) < \frac{\pi}{4} \qquad |z-i| < 3$$

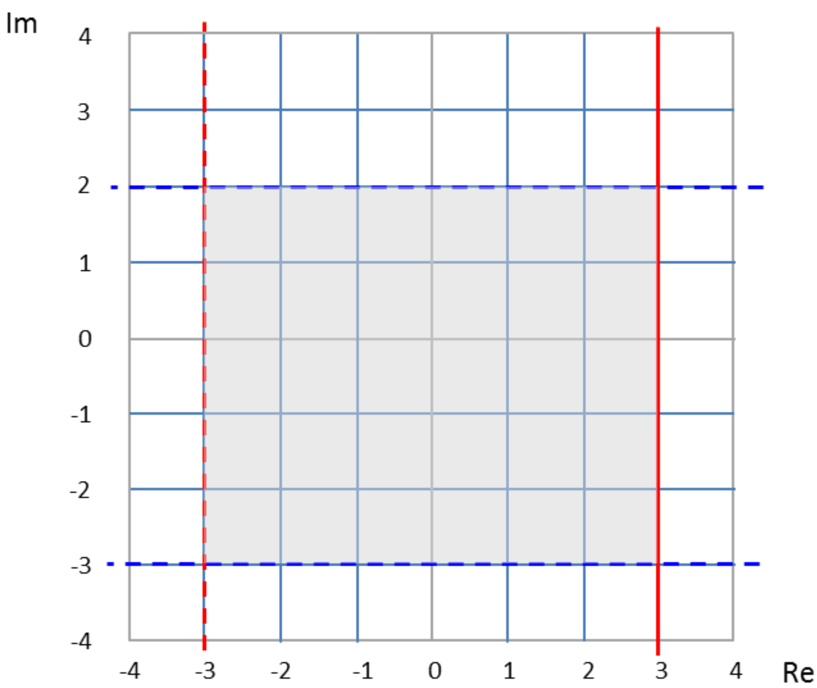
ANSWERS

a001

Show the region on an Argand diagram that satisfy the conditions

$\mathrm{Re}(z) \leq 3 \qquad \mathrm{Re}(z) > -3$

$\mathrm{Im}(z) < 2 \qquad \mathrm{Re}(z) > -3$



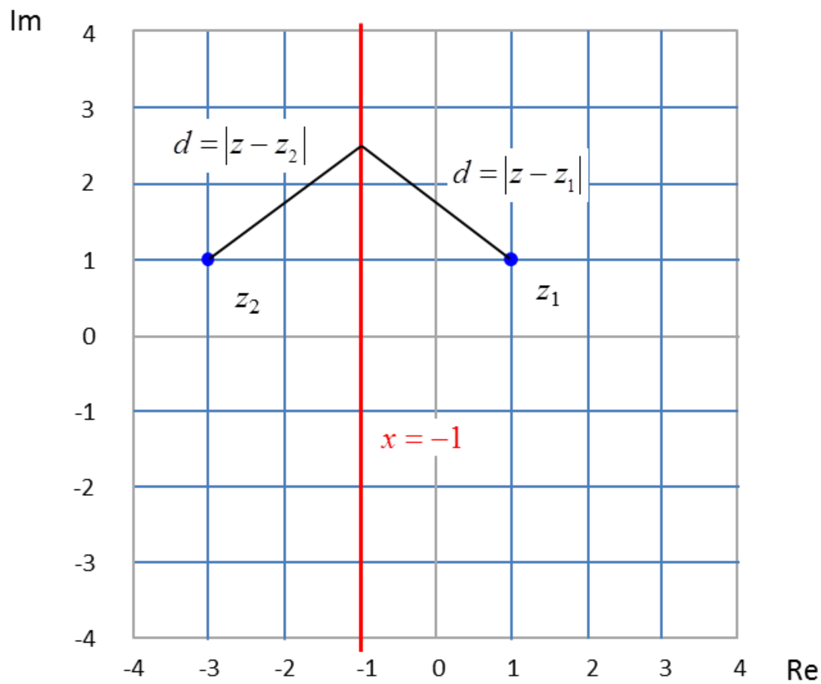
a002

Show on an Argand diagram the complex numbers z that satisfy the condition

$$\left|z-z_1\right|=\left|z-z_2\right| \qquad z_1=1+i \qquad z_2=-3+i$$

$$\left|(x-1)+i\,(y-1)\right|=\left|(x-1)+i\,(y-1)\right|$$
$$(x-1)^2+(y-1)^2=(x+3)^2+(y-1)^2$$
$$x^2-2x+1=x_2+6x+9$$
$$x=-1$$

The line x = -1 corresponds to the perpendicular bisector of the two points z_1 and z_2.


Im
Re
$d = |z - z_2|$
$d = |z - z_1|$
z_2
z_1
$x = -1$
4
3
2
1
0
-1
-2
-3
-4

a003

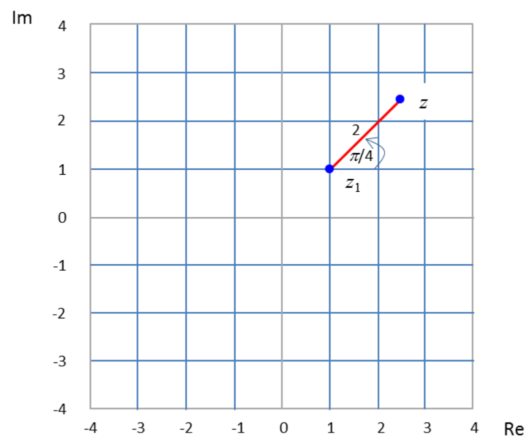
If $z_1 = 1 + i$ then show on an Argand diagram

$$z - z_1 = 2\left(\cos\left(\pi/4\right) + i\sin\left(\pi/4\right)\right)$$

$$z - z_1 = 2\left(\cos\left(\pi/4\right) + i\sin\left(\pi/4\right)\right)$$

$$\left|z - z_1\right| = 2 \qquad \theta = Arg\left(z - z_1\right) = \pi/4$$

Therefore all the points z lie on the straight line drawn from (1, 1) of length 2 and at an angle of $\pi/4$ with respect to the horizontal.



a004

If z_1 = -1 + i then show on an Argand diagram

$$|z - z_1| = 2$$

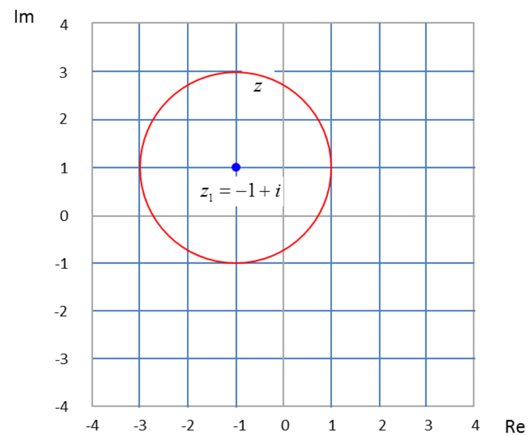
$$z = x + i\ y$$

$$z_1 = -1 + i$$

$$z - z_1 = (x+1) + i\,(y-1)$$

$$|z - z_1| = (x+1)^2 + (y-1)^2 = 2$$

This corresponds to a circle with centre (-1,1) and radius 2.

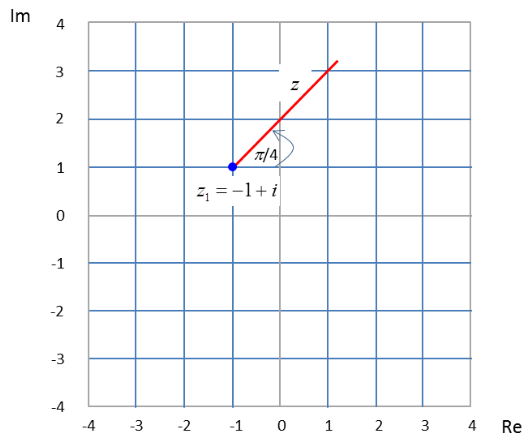


a005

If z_1 = -1 + i then show on an Argand diagram

$$\theta = Arg\left(z - z_1\right) = \pi / 4$$

$$z = x + i\ y$$
$$z_1 = -1 + i$$
$$z - z_1 = \left(x+1\right) + i\left(y-1\right)$$
$$\left|z - z_1\right| = \left(x+1\right)^2 + \left(y-1\right)^2 = 2$$
$$z = x + i\ y \quad z_1 = -1 + i$$
$$z - z_1 = \left(x+1\right) + i\left(y-1\right)$$
$$\theta = Arg\left(z - z_1\right) = \text{atan}\left(\frac{y-1}{x+1}\right) = \pi / 4$$
$$\tan\theta = \left(\frac{y-1}{x+1}\right)$$



The locus is the straight line from the point z_1(-1, 1) but not including the point z_1 to the points z which makes an angle of π/4 with respect to the horizontal.

a006

Show that

$$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|} \qquad \arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$$

$$Arg(z - z_1) - Arg(z - z_2) = Arg\left(\frac{z - z_1}{z - z_2}\right)$$

Sketch and comment on the locus of

$$Arg(z - z_1) - Arg(z - z_2) = Arg\left(\frac{z - z_1}{z - z_2}\right) = \frac{\pi}{6}$$

where

$$z_1 = 1 + i \qquad z_2 = -2 + i$$

$$z_1 = R_1\, e^{i\,\theta_1} \qquad z_2 = R_2\, e^{i\,\theta_2}$$

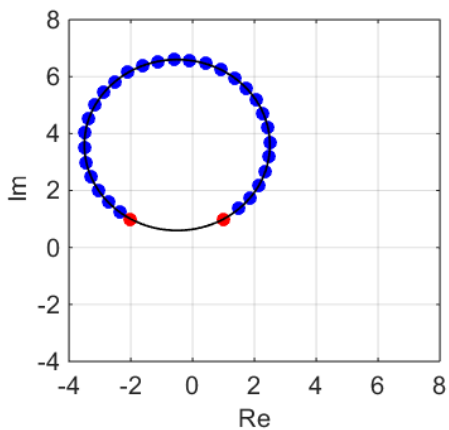
$$|z_1| = R_1 \qquad |z_2| = R_2$$

$$Arg\left(z_1\right) = \theta_1 \qquad Arg\left(z_2\right) = \theta_2$$

$$\frac{z_1}{z_2} = \frac{R_1}{R_2}\, e^{i\,(\theta_1 - \theta_2)}$$

$$\left|\frac{z_1}{z_2}\right| = \frac{R_1}{R_2} = \frac{|z_1|}{|z_2|}$$

$$Arg\left(\frac{z_1}{z_2}\right) = \left(\theta_1 - \theta_2\right) = Arg\left(z_1\right) - Arg\left(z_2\right)$$



a007

Sketch the region on an Argand diagram for the expression

$$2\left(z+\overline{z}\right)-z\,\overline{z}>8$$

$$2\left(z+\overline{z}\right)-z\,\overline{z}>-5$$

$$z=x+i\ y \quad \overline{z}=x-i\ y$$

$$z+\overline{z}=2\,x \quad z\,\overline{z}=x^2+y^2$$

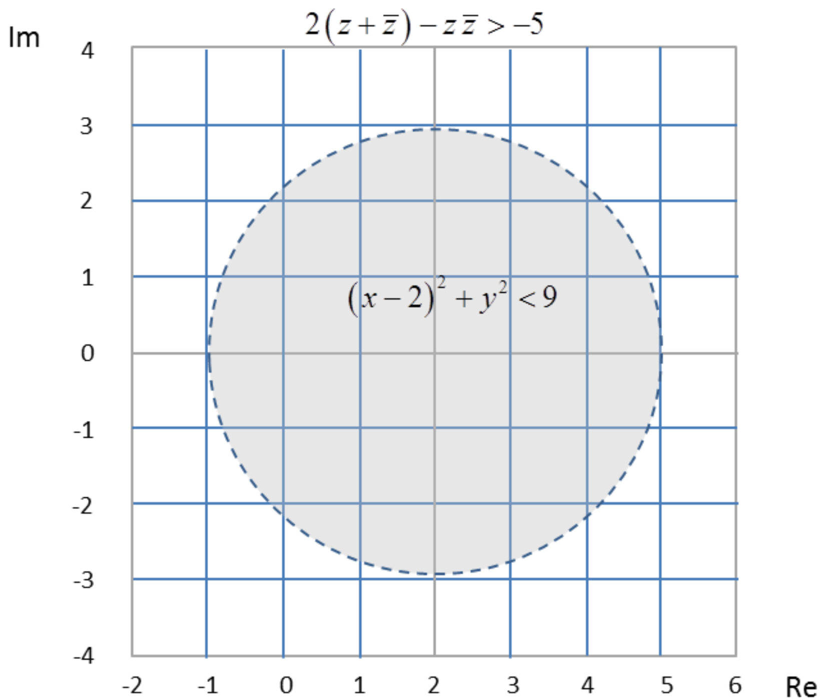
$$-\left(x^2+y^2\right)+2\,x>-5$$

$$x^2+y^2-4\,x<5$$

$$x^2-4x+4+y^2<5+4$$

$$\left(x-2\right)^2+y^2<9$$

Therefore, the allowed region is inside the circle with centre (2, 0) and radius 3. The circle is shown with a dotted line to show that the region does not include the circumference of the circle.


$2(z+\bar{z}) - z\bar{z} > -5$
$(x-2)^2 + y^2 < 9$
Im
4
3
2
1
0
-1
-2
-3
-4
-2
-1
0
1
2
3
4
5
6
Re

a008

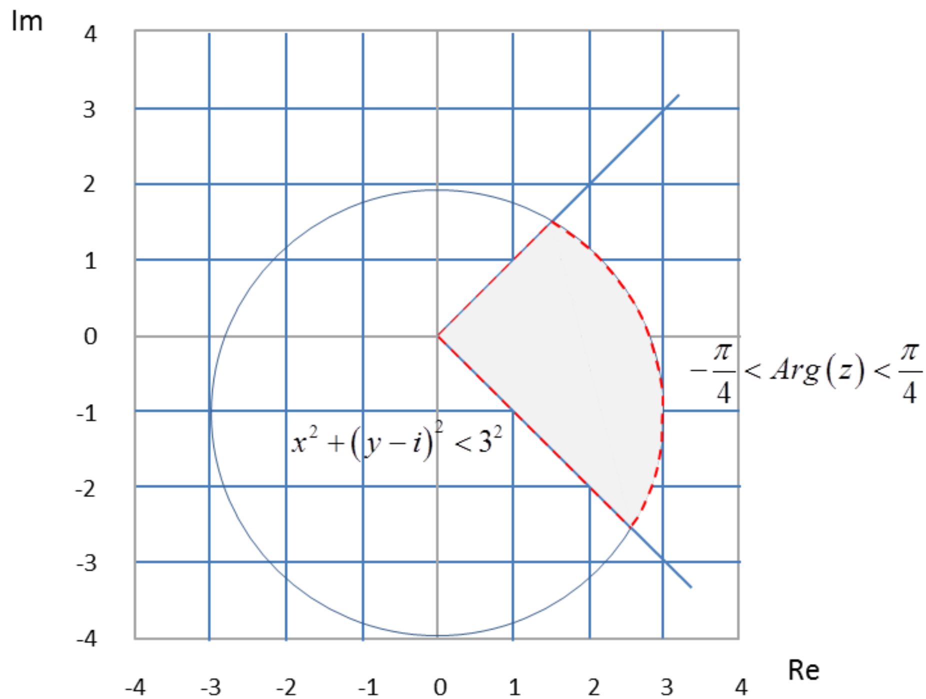
Sketch the allowed region defined by the relationships

$-\frac{\pi}{4} < Arg(z) < \frac{\pi}{4} \qquad |z-i| < 3$

$|z-i| < 3 \quad x^2 + (y-i)^2 < 3^2$ region inside a circle of centre (0, 1) and radius 3

$-\frac{\pi}{4} < Arg(z) < \frac{\pi}{4}$

z must lie between the lines drawn from (0, 0) and making angles of - π /4 and + π /4 with respect to the real axis.


Im
4
3
2
1
0
-1
-2
-3
-4
-4
-3
-2
-1
0
1
2
3
4
Re
$-\frac{\pi}{4} < Arg(z) < \frac{\pi}{4}$
$x^2 + (y-i)^2 < 3^2$