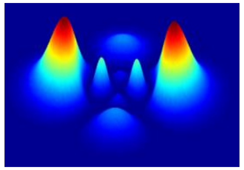


MATHEMATICS EXTENSION 2

4 UNIT MATHEMATICS

TOPIC 5: VOLUMES

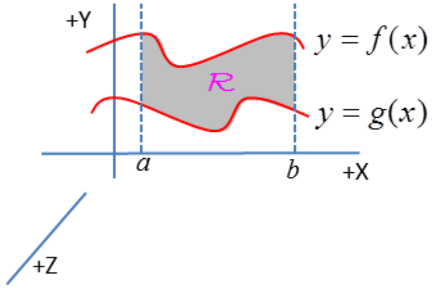
5.4 SOLIDS OF REVOLUTION (Part 2)

Ring or Annulus Method

Let $f(x)$ and $g(x)$ be continuous functions such that

$$0 \le g(x) \le f(x) \quad a \le x \le b.$$

The region $\mathcal{R}$ is the area bounded by the function $y = f(x)$ and $y = g(x)$ in the interval $[a, b]$. Revolving of the region $\mathcal{R}$ about the X-axis generates a solid called a **solid of revolution**. The solid of revolution has a volume V which is the difference between the volume of revolution generated by the region under $y = f(x)$ and the volume of the solid of revolution generated by the region under $y = g(x)$. Hence, the volume V of the solid of revolution about the X-axis is given by



$$V = \pi \int_a^b \left[\left(f(x) \right)^2 - \left(g(x)^2 \right) \right] dx$$

ring / annulus / washer formula*

* washer

cross section obtained by revolving a vertical segment has the shape of a plumbers washer.

Example

Find the volume V of the solid of revolution generated by the rotation about the X-axis of the region bounded by the curves

$$f(x) = 42 - 5x \quad \text{and} \quad g(x) = 2x^2 - 5x + 10$$

Solution

How to approach the problem:
Sketch the function and the solid. Give the equations for the shape of the solid. Find the upper and lower limits for the bounded region. Evaluation the definite integral to find the volume.

Volume of solid of revolution about the X-axis is

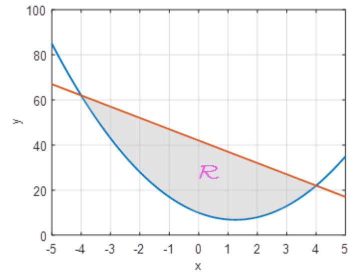
$$V = \pi \int_a^b \left[\left(f(x) \right)^2 - \left(g(x)^2 \right) \right] dx$$

$$f(x) = 42 - 5x \qquad \left(f(x) \right)^2 = 25x^2 - 420x + 1764$$

$$g(x) = 2x^2 - 5x + 10$$

$$\left(g(x) \right)^2 = 4x^4 - 20x^3 + 65x^2 - 100x + 100$$

$$\left(f(x) \right)^2 - \left(g(x) \right)^2 = -4x^4 + 20x^3 - 45x^2 - 320x + 1664$$



$f(x) = 42 - 5x$

$g(x) = 2x^2 - 5x + 10$

We need to find the points of intersection of the two functions.

$$f(x) = g(x)$$

$$2x^2 - 5x + 10 = 42 - 5x$$

$$x^2 = 16$$

$$x = \pm 4$$

The limits of integration are a = - 4 and b = 4.

The volume is

$$V = \pi \int_{-4}^{4} \left[-4x^4 + 20x^3 - 45x^2 - 320x + 1664 \right] dx = \pi \left[-\tfrac{4}{5} x^5 + 5x^4 - 15x^3 - 160x^2 + 1664x \right]_{-4}^{4}$$

$$V = 9754\pi$$

QED