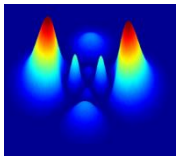

HSC MATHEMATICS: MATHEMATICS EXTENSION 1 (3 UNIT)

TOPIC 18 PERMUTATIONS COMBINATIONS PROBABILITY

EXERCISE ex3u18225 THREE DICE PROBABILITIES

Three dice are thrown together. Calculate the probabilities for the following outcomes

(A) three sixes

(B) highest number is three on any one of the three dice

(C) numbers on each dice are different

(D) numbers are consecutive

(E) different ways of getting even and odd numbers

(F) sum of the numbers is equal to 8

Answer

Each dice has the set of numbers {1 2 3 4 5 6}

In rolling 3 dice the number of distinct outcomes is $N_{total} = (6)(6)(6) = 216$

(A) three sixes

The number of ways of getting three sixes is $N = 1$ Probability 3 sixes is $\text{Prob} = N / N_{total} = 1/216$

Alternatively: $\text{Prob} = (1/6)(1/6)(1/6) = 1/216$

(B) ≤ 3

Probability of getting 1 or 2 or 3 from one dice is $3/6 = 1/2$

$\text{Prob} = (1/2)(1/2)(1/2) = 1/8$

(C)

Dice 1: can be any of the six numbers; Dice 2: can only be 1 of 5 numbers and Dice 3 can be only 1 of 4 numbers

Number of permutations $N = (6)(5)(4) = 120$

$\text{Prob} = 120/216 = 5/9$

(D)

There are only 4 possible combinations for consecutive numbers are

{1 2 3} {2 3 4} {3 4 5} {4 5 6}

Number of permutations of three numbers = $3! = 6$

Total number of permutations = $(4)(6) = 24$

$\text{Prob} = 24 / 216 = 1 / 9$

(E)

When a dice is thrown the number can only be even or odd, therefore, the probability of any permutation of even and odd numbers for the three dice is

$\text{Prob} = (1/2)(1/2)(1/2) = 1/8$

The number of distinct permutations of even (E) and odd (O) numbers is 8

{E E E} {E E O} {E O E} {0 E E} {O O E} {O E O} {O 0 E} {O O O} probability of each outcome is 1/8

$\text{Prob(all even numbers)} = 1/8$

$\text{Prob(two even \& one odd number)} = 3/8$

$\text{Prob(one even \& two odd numbers)} = 3/8$

$\text{Prob(all odd numbers)} = 1/8$

Alternatively: calculating the probability of getting two even numbers and one odd number

3 ways of getting an even number and 3 ways of getting an odd number

Number of permutations of two even numbers and one odd number = $\frac{3!}{2!} = 3$

Total number of permutations = $(3)(3)(3)\ (3) = 81$

$\text{Prob}(\text{two even \& one odd number}) = 81/216 = 3/8$

(F) sum of the numbers is 8

The outcomes and number of permutations* are

{1 1 6} 3

{1 2 5} 6

{1 3 4} 6

{2 3 3} 3

{2 4 2} 3

*Arrangement of three distinct numbers = 3! = 6

Arrangement of three numbers which two are the same = 3! / 2! = 3

Total number of permutations = 3+6+6+3+3 = 21

$\text{Prob} = 21/216 = 7/72$