Special topics in Logic and Security I

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The Doley-Yao attacker

Recall that we defined the attacker knowledge using a deductive systems, e.g.

```
\begin{array}{lll} \text{if} & M \vdash t \text{ and } M \vdash k & \text{then} & M \vdash \{\mid t\mid\}_k \\ \text{if} & M \vdash t_1 \text{ and } M \vdash t_2 & \text{then} & M \vdash (t_1, t_2) \\ \text{if} & M \vdash \{\mid t\mid\}_k \text{ and } M \vdash k & \text{then} & M \vdash t \\ \text{if} & M \vdash (t_1, t_2) & \text{then} & M \vdash t1 \text{ and } M \vdash t_2 \end{array}
```

where $M \vdash t$ means that t can be deduced knowing M

The Doley-Yao attacker

Recall that a Horn clause has the form

$$H_1 \wedge \cdots \wedge H_n \rightarrow C$$

where H_1, \ldots, H_n and C are atomic formulas in FOL.

We consider a unary predicate att such that

att(t) means that the attacker knows t.

The attacker deduction system can be defined using implications as follows:

$$\begin{array}{l} \textit{att}(t) \land \textit{att}(k) \rightarrow \textit{att}(\{\mid t\mid\}_k) \\ \textit{att}(t_1) \land \textit{att}(t_2) \rightarrow \textit{att}((t_1, t_2)) \\ \textit{att}(\{\mid t\mid\}_k) \land \textit{att}(k) \rightarrow \textit{att}(t) \\ \textit{att}(t_1, t_2) \rightarrow \textit{att}(t_1) \\ \textit{att}(t_1, t_2) \rightarrow \textit{att}(t_2) \end{array}$$

which are obviously Horn clauses.

The Dolev-Yao attacker

```
att(t) \land att(k) \rightarrow att(\{\mid t \mid \}_k)
att(t_1) \land att(t_2) \rightarrow att((t_1, t_2))
att(\{\mid t \mid \}_k) \land att(k) \rightarrow att(t)
att(t_1, t_2) \rightarrow att(t_1)
att(t_1, t_2) \rightarrow att(t_2)
```

These can be expressed in Prolog as follows:

```
att(senc(X, Y)) :- att(X), att(Y).
att(pair(X,Y)) :- att(X), att(Y).
att(X) :- att(senc(X, Y)), att(Y).
att(X) :- att(pair(X, _))
att(Y) :- att(pair(_, Y)).
```

Note that Prolog (compound) terms - pair(X,Y) and senc(X,Y) - are used in order to define pairing and symmetric ecryption.

Prolog: the Dolev-Yao attacker

```
Is Prolog suitable for such inferences?
att(ka).
att(kb).
att(senc(secret,pair(ka,kb))).
att(senc(X, Y)) := att(X), att(Y).
att(pair(X,Y)) := att(X), att(Y).
att(X) := att(senc(X, Y)), att(Y).
att(X) := att(pair(X, _)).
att(Y) :- att(pair(_, Y)).
?- att(secret).
true
```

Prolog: the Dolev-Yao attacker

```
?- trace.
true.
[trace] ?- att(secret).
   Call: (10) att(secret) ? creep
   Call: (11) att(senc(secret, _21612)) ? creep
   Exit: (11) att(senc(secret, pair(ka, kb))) ? creep
   Call: (11) att(pair(ka, kb)) ? creep
   Call: (12) att(ka) ? creep
   Exit: (12) att(ka) ? creep
   Call: (12) att(kb) ? creep
   Exit: (12) att(kb) ? creep
   Exit: (11) att(pair(ka, kb)) ? creep
  Exit: (10) att(secret) ? creep
true .
```

Prolog: the Dolev-Yao attacker

We can further add rules for public encryption and signing:

```
att(pk(X)) := att(X).
att(X) := att(aenc(X, pk(Y))), att(Y).
att(aenc(X, Y)) := att(X), att(Y).
att(X) := att(sign(X, Y)).
att(sign(X,Y)) := att(X), att(Y).
att(X) := att(senc(X, Y)), att(Y).
att(senc(X, Y)) := att(X), att(Y).
att(X) := att(pair(X, _)).
att(Y) := att(pair(_, Y)).
att(pair(X,Y)) := att(X), att(Y).
```

Can we follow the same ideas for protocols?

We consider the Denning-Sacco protocol (without timestamps):

- $(1) A \longrightarrow B : \{ \{ k \}_{sk(A)} \}_{pk(B)}$
- (2) $B \longrightarrow A : \{ | secret | \}_k$

The protocol is vulnerable to the following attack

```
(att1) A \longrightarrow E : \{ \{ \{ k \}_{sk(A)} \}_{pk(E)} \}
(att2) E \longrightarrow B : \{ \{ \{ k \}_{sk(A)} \}_{pk(B)} \}
(att3) B \longrightarrow A : \{ \{ secret \}_k \}
```

Since E is a corrupted agent, by (att1) the attacker knows the key k so, by (att3), he knows the *secret*.

We consider the Denning-Sacco protocol (without timestamps):

- $(1) A \longrightarrow B : \{ |\{ k \}_{sk(A)} \}_{pk(B)}$
- (2) $B \longrightarrow A : \{ | secret | \}_k$

The communication between A and B cand be represented (from the attacker's point of view) by:

```
att(senc(secret, K)) :- att(aenc(sign(K, skA), pk(skB))).
```

If B receives what (s)he expects, then (s)he behaves following the protocols and the messages are "added" to the adversary knowledge. Note that the agents are identified with their secret keys.

- $(1) \ A \longrightarrow B : \{ \mid \{ \mid k \mid \}_{sk(A)} \mid \}_{pk(B)}$
- (2) $B \longrightarrow A : \{ | secret | \}_k$

We've defined the fact that B acts when s(he) receives a message, but we still need to define that fact that A starts the protocol. To do this we can think that "A wants to speak to any principal (who's public key is known)":

```
att(aenc(sign(k, skA), pk(X))) :- att(pk(X)).
```

Moreover, we can express the fact that k is a key that depends on the communication partner:

```
att(aenc(sign(k(pk(X)), skA), pk(X))) :- att(pk(X)).
```

We've shown that the protocol

```
(1) A \longrightarrow B : \{ |\{ k \}_{sk(A)} \}_{pk(B)}
```

(2) $B \longrightarrow A : \{ | secret | \}_k$

can be defined in Prolog by:

```
att(aenc(sign(K, skA), pk(X))) :- att(pk(X)).
att(senc(secret, K)) :- att(aenc(sign(K, skA), pk(skB))).
```

Can we find the attack? This means that:

```
?- att(secret).
true
```

In order to analyze the protocol we need to represent the attacker inference system, as well as the adversary initial knowledge:

% initial knowledge

% the attacker inference system

att(skI).
att(pk(skA)).
att(pk(skB)).

```
att(pk(X)) :- att(X).
att(X) :- att(aenc(X, pk(Y))), att(Y).
att(aenc(X, Y)) :- att(X), att(Y).
att(X) :- att(sign(X, Y)).
att(sign(X,Y)) :- att(X), att(Y).
att(X) :- att(senc(X, Y)), att(Y).
att(senc(X, Y)) :- att(X), att(Y).

%the protocol
att(aenc(sign(K, skA), pk(X))) :- att(pk(X)).
att(senc(secret, K)) :- att(aenc(sign(K, skA), pk(skB))).
```

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```
att(skI).
att(pk(skA)).
att(pk(skB)).
att(aenc(sign(K, skA), pk(X))) :- att(pk(X)).
att(senc(secret, K)) :- att(aenc(sign(K, skA), pk(skB))).
att(pk(X)) := att(X).
att(X) := att(senc(X, Y)), att(Y).
att(aenc(X, Y)) := att(X), att(Y).
att(sign(X,Y)) := att(X), att(Y).
att(X) := att(aenc(X, pk(Y))), att(Y).
att(X) := att(sign(X, Y)).
att(senc(X, Y)) := att(X), att(Y).
?- att(secret).
true
```

```
?- trace.
[trace] ?- att(secret).
  Call: (10) att(secret) ? creep
   Call: (11) att(senc(secret, _16792)) ? creep
   Call: (12) att(aenc(sign(_16792, skA), pk(skB))) ? creep
   Call: (13) att(pk(skB)) ? creep
   Exit: (13) att(pk(skB)) ? creep
   Exit: (12) att(aenc(sign(_16792, skA), pk(skB))) ? creep
   Exit: (11) att(senc(secret, _16792)) ? creep
   Call: (11) att(_16792) ? creep
   Exit: (11) att(skI) ? creep
  Exit: (10) att(secret) ? creep
true
```

Why "naive"?

- The clause ordering affects the results or it can provide false attacks.
- The clause ordering affects termination.

However, the idea of using Horn clauses for modeling and analyzing protocols is valuable, and few tools and theories are known.

"The ProVerif tool takes protocols written in a variant of the applied pi calculus as input together with a security property to verify. The protocol is then automatically translated into a set of first-order Horn clauses and the properties are translated into derivability queries. The verification algorithm is based on a dedicated Horn clause resolution procedure."

[1] Véronique Cortier, Steve Kremer. Formal Models and Techniques for Analyzing Security Protocols: A Tutorial. Foundations and Trends in Programming Languages, Now Publishers, 2014, 1 (3), pp.117.

https://hal.archives-ouvertes.fr/hal-01090874

References

- 1 Véronique Cortier, Steve Kremer. Formal Models and Techniques for Analyzing Security Protocols: A Tutorial. Foundations and Trends in Programming Languages, Now Publishers, 2014, 1 (3), pp.117. https://hal.archives-ouvertes.fr/hal-01090874
- 2 Bruno Blanchet. Modeling and Verifying Security Protocols with the Applied Pi Calculus and ProVerif. Foundations and Trends® in Privacy and Security , Now publishers inc, 2016, 1 (1-2), pp.1 135.https://hal.inria.fr/hal-01423760/
- 3 Bruno Blanchet. Using Horn Clauses for Analyzing Security Protocols. In Véronique Cortier and Steve Kremer, editors, Formal Models and Techniques for Analyzing Security Protocols, volume 5 of Cryptology and Information Security Series, pages 86-111. IOS Press, March 2011.
- 4 Abadi, M., B. Blanchet, and C. Fournet. The Applied Pi Calculus: Mobile Values, New Names, and Secure Communication. Journal of the ACM Vol. 65Issue 1February 2018 Article No.: 1pp 1–41. Available at http://arxiv.org/abs/1609. 03003v1.

ProVerif: protocols as Horn clauses

Let $\mathcal F$ be a *signature* (a set of function symbols together with their arity), $\mathcal N$ a set of *names* (representing nonces, keys, identities) and $\mathcal X$ a set of variables.

• The terms are defined by:

- The *predicate* is *att*(*t*), which is *true* if the attacker knows the message modelled by *t*.
- The Horn clauses are:
 - facts: att(t)
 - rules: $att(t_1) \wedge \cdots \wedge att(t_n) \rightarrow att(t)$

ProVerif: the attacker

```
att(x) \land att(y) \rightarrow att(senc(x, y))
att(senc(x, y)) \land att(y) \rightarrow att(x)
att(x) \rightarrow att(pk(x))
att(x) \land att(y) \rightarrow att(aenc(x, y))
att(aenc(x, pk(y))) \land att(y) \rightarrow att(x)
att(x) \land att(y) \rightarrow att(sign(x, y))
att(sign(x, y)) \rightarrow att(x)
att(x) \land att(y) \rightarrow att((x, y))
att(x, y) \rightarrow att(x)
att(x, y) \rightarrow att(y)
att(t) for any term t in the initial knowledge of the attacker
att(a[]) for at least one name a
```

ProVerif: Horn clauses for the Denning-Sacco protocol

```
proverif2.04\examples\horn\secr
http://proverif.inria.fr
(* Initialization *)
c:c[]:
c:pk(sA[]);
c:pk(sB[]);
(* The attacker *)
c:x & c:encrypt(m,pk(x)) -> c:m;
c:x & c:sencrypt(m,x) -> c:m;
c:x \rightarrow c:pk(x);
c:x \& c:y \rightarrow c:encrypt(x,y);
c:x & c:y -> c:sencrypt(x,y);
c:sign(x,y) \rightarrow c:x;
c:x \& c:y \rightarrow c:sign(x,y);
(* The protocol *)
(* A *)
c:pk(x) \rightarrow c:encrypt(sign(k[pk(x)], sA[]), pk(x));
(*B*)
c:encrypt(sign(k, sA[]), pk(sB[])) -> c:sencrypt(secret[], k).
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```

ProVerif: Horn clauses for the Denning-Sacco protocol

```
proverif2.04\examples\horn\secr
http://proverif.inria.fr
Clause 15: c:x & c:encrypt(m,pk(x)) \rightarrow c:m
Clause 14: c:x \rightarrow c:pk(x)
Clause 13: c:x & c:y -> c:encrypt(x,y)
Clause 12: c:sign(x,y) \rightarrow c:x
Clause 11: c:x \& c:y \rightarrow c:sign(x,y)
Clause 10: c:pk(x) \rightarrow c:encrypt(sign(k[pk(x)],sA[]),pk(x))
Clause 9: c:encrypt(sign(k_1,sA[]),pk(sB[])) ->
                             c:encrypt(secret[],pk(k_1))
Clause 8: c:c[]
Clause 7: c:pk(sA[])
Clause 6: c:pk(sB[])
Clause 5: c:x & c:encrypt(m,pk(x)) \rightarrow c:m
Clause 4: c:x \rightarrow c:pk(x)
Clause 3: c:x & c:y -> c:encrypt(x,y)
Clause 2: c:sign(x,y) \rightarrow c:x
```

ProVerif: Horn clauses for the Denning-Sacco protocol

```
proverif2.04\examples\horn\secr
http://proverif.inria.fr
Derivation:
Abbreviations:
k_1 = k[pk(x)]
clause 15 c:secret[]
    clause 12 c:k_1
        duplicate c:sign(k_1,sA[])
    clause 9 c:encrypt(secret[],pk(k_1))
        clause 13 c:encrypt(sign(k_1,sA[]),pk(sB[]))
            clause 15 c:sign(k_1,sA[])
                duplicate c:x
                clause 10 c:encrypt(sign(k_1,sA[]),pk(x))
                    clause 14 c:pk(x)
                         any c:x
            clause 6 c:pk(sB[])
RESULT goal reachable: c:secret[]
```

ProVerif

- The protocols are specified in a variant of π calculus.
- The description of the protocol is automatically translated in a set of Horn clauses.
- The security property is checked by performing a particular version of resolution, developed for this particular setting.

A π -calculus for protocols

Let $\mathcal F$ be a signature, $\mathcal N$ a set of names and $\mathcal X$ a set of variables such that $\mathcal N$ and $\mathcal X$ contain special subsets of channel names and channel variables, respectively.

• The *terms* are defined by:

$$\begin{array}{lll} \textit{u}, \textit{v} & ::= & \textit{x}, \textit{y}, \textit{z} & \text{variables from } \mathcal{X} \\ & \textit{a}, \textit{b}, \textit{c} & \text{names from } \mathcal{N} \\ & & \textit{f}\left(t_1, \dots, t_n\right) & \text{function application for } \textit{f} \text{ from } \mathcal{F} \end{array}$$

• The *processes* are defined by:

$$\begin{array}{lll} P,Q & ::= & 0 & & \text{the process that does nothing} \\ & & in(u,x).P & & \text{the input on channel u is bound to x} \\ & & out(u,t).P & & \text{the output on channel u is t} \\ & & let \ x = t \ in \ P & & \text{local definition} \\ & & if \ t_1 = t_2 \ then \ P \ else \ Q & \text{conditional} \\ & & \nu \ n.P & & \text{name restriction} \\ & & P \ | Q & & \text{parallel composition} \\ & & !P & & \text{replication, infinite parallel composition} \end{array}$$

where n is a name, x is a variable, t, t_1 , t_2 are terms and u stands for a channel name or a channel variable.

A π -calculus for protocols: example

For the Horn clause representation we've used the signature $\mathcal{F} = \{pk, aenc, senc, sign, pair\}$, which is a signature of constructors.

We can express the first exchange in NSPK

$$A \longrightarrow B: \{ A, na \}_{pk(B)}$$
 by

$$\nu$$
 n.out(c, aenc(pair(skA, n), pk(skB)))

where n, skA and skB, $c \in \mathcal{N}$ and c is a name for channels.

The second exchange in NSPK is:

$$B \longrightarrow A : \{ | na, nb | \}_{pk(A)}$$

and we note that B has to obtain n_a from the initial message, which means to perform decryption and to extract the second component of the decrypted message.

In order to define B's actions, we add destructors, as well as equations for constructors and destructors.

Equational theory

- for asymmetric encryption the destructor is adec and the ecuation is adec(aenc(x, pk(y)), y) = x
- for the constructor senc the destructor is sdec and the equational theory for symmetric encryption is sdec(senc(x, y), y) = x
- for the constructors sign, the destructor can be check and the equational theory is check(sign(x, y), pk(y)) = x
- for the constructor *pair* the destructors are *fst* and *snd*, which extract the first and the second component.

Consequently, $\mathcal{F} = \{pk, aenc, senc, sign, pair\} \cup \{adec, sdec, check, fst, snd\}$. More operations (constructors, destructors) and equations can be considered, depending on the protocol we specify.

A π -calculus for protocols: remarks

$$\begin{array}{lll} P,Q & ::= & 0 & & \text{the process that does nothing} \\ & & in(u,x).P & & \text{the input on channel u is bound to x} \\ & & out(u,t).P & & \text{the output on channel u is t} \\ & & let \ x = t \ in \ P & & \text{local definition} \\ & & if \ t_1 = t_2 \ then \ P \ else \ Q & \text{conditional} \\ & & \nu \ n.P & & \text{name restriction} \\ & & P \mid Q & & \text{parallel composition} \\ & & !P & & \text{replication, infinite parallel composition} \end{array}$$

- Names and variables have scopes, defined by restrictions, inputs and local definitions and one can define the free (bound) names and variables.
- The expressions defining processes are equal modulo renaming of bound names and variables.
- The formal semantics is defined by *reduction*:

$$out(u, t).P|in(u, x).Q \rightarrow P|Q\{t/x\}$$

if $t = t$ then P else $Q \rightarrow P$
if $t_1 = t_2$ then P else $Q \rightarrow Q$

where $\{t/x\}$ is the substitution that replaces the free occurrences of x with t and $t_1=t_2$ is not provable in the corresponding equational theory.

We refer to [4] for details.

The Denning-Sacco protocol revisited

```
A \longrightarrow B: {| {| k |}<sub>sk(A)</sub> |}<sub>pk(B)</sub>

B \longrightarrow A: {| secret |}<sub>k</sub>
```

In order to model our protocol we define

- a process for each role and
- a process that brings them toghether.

```
P_A(skA, pkB) = v \ k.out(c, aenc(sign(k, skA), pkB)).

in(c, x).

let \ z = sdec(x, k) \ in \ 0
```

```
A \longrightarrow B: \{ \{ k \}_{sk(A)} \}_{pk(B)} 

B \longrightarrow A: \{ \{ secret \}_k \}
```

a process for each role:

$$P_A(skA, pkB) = v \ k.out(c, aenc(sign(k, skA), pkB)).$$
 $in(c, x).$
 $let \ z = sdec(x, k) \ in \ 0$
 $P_B(skB, pkA) = in(c, y).$
 $let \ z = adec(y, skB) \ in$
 $if \ check(z, pkA) = xk \ then$
 $v \ s.out(c, senc(s, xk))$

a process that brings them together.

$$P_{DS} = \nu \, skA.\nu \, skB.P_A(skA, pk(skB)) \mid P_B(skB, pk(skA))$$

A π -calculus for protocols: NSPK

For the Needham-Scroeder Public Key Protocol (NSPK):

```
A \longrightarrow B: \{ A, na \}_{pk(B)}

B \longrightarrow A: \{ na, nb \}_{pk(A)}

A \longrightarrow B: \{ nb \}_{pk(B)}
```

the process corresponding to the initiator can be defined by:

$$P_A(skA, pkB) = \nu n.out(c, aenc(pair(skA, n), pkB)).$$

$$in(c, x).$$

$$let \ z = adec(x, skA) \ in$$

$$if \ fst(z) = n \ then \ out(c, aenc(snd(z), pkB)) \ else \ 0$$

The Denning-Sacco protocol continued

We can improve the representation of the protocol by:

• allowing multiple parallel executions for each role: $P_A(skA, pkB) = ! \nu \ k.out(c, aenc(sign(k, skA), pkB)).$ in(c, x). $let \ z = sdec(x, k) \ in \ 0$ $P_B(skB, pkA) = ! in(c, y).$ $let \ z = adec(y, skB) \ in$ $if \ check(z, pkA) = xk \ then$ $\nu \ s.out(c, senc(s, xk))$

making the public keys available to a potential attacker:

```
P_{DS} = v skA.v skB.
let pkA = pk(skA) in
let pkB = pk(skB) in
out(c, pk(skA)).out(c, pk(skB)).
P_A(skA, pk(skB)) | P_B(skB, pk(skA))
```

The Denning-Sacco protocol continued

In the above representation, the (honest) participants talk only to each other. In order to allow the attacker to interfere we can modify the initiator process such that the public key of the communication partner is received through the channel c:

```
P_A(skA) = !in(c, x_{pk}).
             \nu k.out(c, aenc(sign(k, skA), x_{nk}).
              in(c,x).
              let z = sdec(x, k) in 0
P_B(skB, pkA) = !in(c, y).
                   let z = adec(y, skB) in
                     if check(z, pkA) = xk then
                       \nu s.out(c, senc(s, xk))
P_{DS} = \nu \, skA.\nu \, skB.
          let pkA = pk(skA) in
           let pkB = pk(skB) in
             out(c, pk(skA)).out(c, pk(skB)).
              P_A(skA, pk(skB)) \mid P_B(skB, pk(skA))
```

ProVerif

- The ProVerif language extends the π calculus for protocols.
- The description of the protocol is automatically translated in a set of Horn clauses.
- The security property is checked by performing a particular version of resolution, developed for this particular setting.

https://bblanche.gitlabpages.inria.fr/proverif/

Thank you!

References

- 1 Véronique Cortier, Steve Kremer. Formal Models and Techniques for Analyzing Security Protocols: A Tutorial. Foundations and Trends in Programming Languages, Now Publishers, 2014, 1 (3), pp.117. https://hal.archives-ouvertes.fr/hal-01090874
- 2 Bruno Blanchet. Modeling and Verifying Security Protocols with the Applied Pi Calculus and ProVerif. Foundations and Trends® in Privacy and Security , Now publishers inc, 2016, 1 (1-2), pp.1 135.https://hal.inria.fr/hal-01423760/
- 3 Bruno Blanchet. Using Horn Clauses for Analyzing Security Protocols. In Véronique Cortier and Steve Kremer, editors, Formal Models and Techniques for Analyzing Security Protocols, volume 5 of Cryptology and Information Security Series, pages 86-111. IOS Press, March 2011.
- 4 Abadi, M., B. Blanchet, and C. Fournet. The Applied Pi Calculus: Mobile Values, New Names, and Secure Communication. Journal of the ACM Vol. 65Issue 1February 2018 Article No.: 1pp 1–41. Available at http://arxiv.org/abs/1609. 03003v1.