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**Protocol specification**

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$$\begin{aligned} \text{RoleTerm} ::= & \text{Var} \mid \text{Fresh} \mid \text{Role} \mid \text{Func}(\text{RoleTerm}^*) \\ & \mid (\text{RoleTerm}, \text{RoleTerm}) \mid \{\!\{ \text{RoleTerm} \}\!\}_{\text{RoleTerm}} \\ & \mid \text{sk}(\text{RoleTerm}) \mid \text{pk}(\text{RoleTerm}) \mid k(\text{RoleTerm}, \text{RoleTerm}) \end{aligned}$$

$$\begin{aligned} \text{RoleEvent}_R ::= & \text{send}_{\text{Label}}(R, \text{Role}, \text{RoleTerm}) \\ & \mid \text{recv}_{\text{Label}}(\text{Role}, R, \text{RoleTerm}) \\ & \mid \text{claim}_{\text{Label}}(R, \text{Claim}, [\text{RoleTerm}]) \end{aligned}$$

$$\text{RoleEvent} = \bigcup_{R \in \text{Role}} \text{RoleEvent}_R$$

$$P(R) = (KN_0(R), s) \in \mathcal{P}(\text{RoleTerm}) \times \text{RoleEvent}_R^*$$

$$\begin{aligned} \text{RoleSpec} = & \{ (kn, s) \mid kn \in \mathcal{P}(\text{RoleTerm}) \wedge \forall rt (rt \in kn \rightarrow \text{vars}(rt) = \emptyset) \\ & \wedge s \in \text{RoleEvent}^* \wedge \text{wellformed}(s) \} \end{aligned}$$

$$\text{Protocol} = \text{Role} \rightarrow \text{RoleSpec}$$

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**Deduction on terms**

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$M \vdash t$  means that  $t$  can be deduced knowing  $M$

$\vdash$  is the least relation with the following properties:

$$\begin{aligned} \text{if } & t \in M \quad \text{then } M \vdash t \\ \text{if } & M \vdash t_1 \text{ and } M \vdash t_2 \quad \text{then } M \vdash (t_1, t_2) \\ \text{if } & M \vdash (t_1, t_2) \quad \text{then } M \vdash t_1 \text{ and } M \vdash t_2 \\ \text{if } & M \vdash t \text{ and } M \vdash k \quad \text{then } M \vdash \{\!\{ t \}\!\}_k \\ \text{if } & M \vdash \{\!\{ t \}\!\}_k \text{ and } M \vdash k^{-1} \quad \text{then } M \vdash t \\ \text{if } & M \vdash t_1 \text{ and } \dots \text{ and } M \vdash t_n \quad \text{then } M \vdash f(t_1, \dots, t_n) \end{aligned}$$

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**Protocol execution**

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$$\begin{aligned} \text{RunTerm} ::= & \text{Var}^{\#RID} \mid \text{Fresh}^{\#RID} \mid \text{Role}^{\#RID} \mid \text{Agent} \mid \text{Func}(\text{RunTerm}^*) \\ & \mid (\text{RunTerm}, \text{RunTerm}) \mid \{\!\{ \text{RunTerm} \}\!\}_{\text{RunTerm}} \\ & \mid \text{AdversaryFresh} \\ & \mid \text{sk}(\text{RunTerm}) \mid \text{pk}(\text{RunTerm}) \mid k(\text{RunTerm}, \text{RunTerm}) \end{aligned}$$

$$\text{Inst} = RID \times (\text{Role} \rightarrow \text{Agent}) \times (\text{Var} \rightarrow \text{RunTerm}) \quad \text{inst} = (\theta, \rho, \sigma) \in \text{Inst}$$

$$\text{Match} \subseteq \text{Inst} \times \text{RoleTerm} \times \text{RunTerm} \times \text{Inst}$$

$\text{Match}(\text{inst}, pt, m, \text{inst}')$  holds if

$$\text{inst} = (\theta, \rho, \sigma), \text{inst}' = (\theta, \rho, \sigma'), \text{dom}(\sigma') = \text{dom}(\sigma) \cup \text{vars}(pt),$$

$$\text{inst}'(pt) = m \text{ for } pt \in \text{RoleTerm} \text{ and } m \in \text{RunTerm},$$

$$\sigma \subseteq \sigma' \text{ and } \sigma'(v) \in \text{type}(v) \text{ for any } v \in \text{dom}(\sigma'),$$

where  $\text{vars}(pt)$  is the set of variables from  $\text{Var}$  which appear in  $pt$ , and  $\text{type}(v)$  is a function that depends on the agent model.

$$Run = Inst \times RoleEvent^* \quad runsof : Protocol \times Roles \rightarrow \mathcal{P}(Run)$$

$$runsof(P, R) = \{(inst, s) \mid \text{there exists } kn \text{ such that } P(R) = (kn, s) \\ inst = (\theta, \rho, \sigma) \text{ with } dom(\rho) = roles(s)\} \text{ where } R \in dom(P).$$

For  $F \subseteq Run$  we define  $runIds(F) = \{\theta \mid ((\theta, \rho, \sigma), s) \in F \text{ for some } \rho, \sigma, s\}$

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### Operational semantics

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$$State = \mathcal{P}(RunTerm) \times \mathcal{P}(Run)$$

$st = \langle\langle AKN, F \rangle\rangle \in State$  where  $AKN$  is the adversary knowledge and  $F \subseteq Run$  are the runs that has to be executed.

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$$RunEvent = Inst \times (RoleEvent \cup \{create(R) \mid R \in Role\})$$

Labeled Transition System for Operational Semantics:  $(State, RunEvent, \rightarrow, st_0(P))$   
 where  $st_0(P) = \langle\langle AKN_0(P), \emptyset \rangle\rangle$  where  $AKN_0(P)$  is the initial adversary knowledge.

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### The Needham-Schroeder protocol

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$$\begin{aligned} NS(i) = & (\{i, r, ni, sk(i), pk(i), pk(r)\}, \\ & [send_1(i, r, \{\!\! \{ ni, i \}\!\! \}_{pk(r)}), \\ & recv_2(r, i, \{\!\! \{ ni, V \}\!\! \}_{pk(i)}), \\ & send_3(i, r, \{\!\! \{ V \}\!\! \}_{pk(r)}), \\ & claim_4(i, synch)]) \\ NS(r) = & (\{i, r, nr, sk(r), pk(r), pk(i)\}, \\ & [recv_1(i, r, \{\!\! \{ W, i \}\!\! \}_{pk(r)}), \\ & send_2(r, i, \{\!\! \{ W, nr \}\!\! \}_{pk(i)}), \\ & recv_3(i, r, \{\!\! \{ nr \}\!\! \}_{pk(r)}), \\ & claim_5(r, synch)]) \end{aligned}$$

$$AKN_0(NS) = AdversaryFresh \cup Agent \cup \{pk(A) \mid A \in Agent\} \cup \{sk(A) \mid A \in Agent_C\}$$