Solutions for coursework CITY 1081

The Python program used in the report, the LTspice simulations, and some PPT slide presentations on complex signals and network analysis are available in the GitHub repository.

1. With the aid of appropriate diagrams explain the following features of waveforms:
(i) fundamental frequency, (ii) harmonics, (iii) amplitude, (iv) period and (v) phase angle.

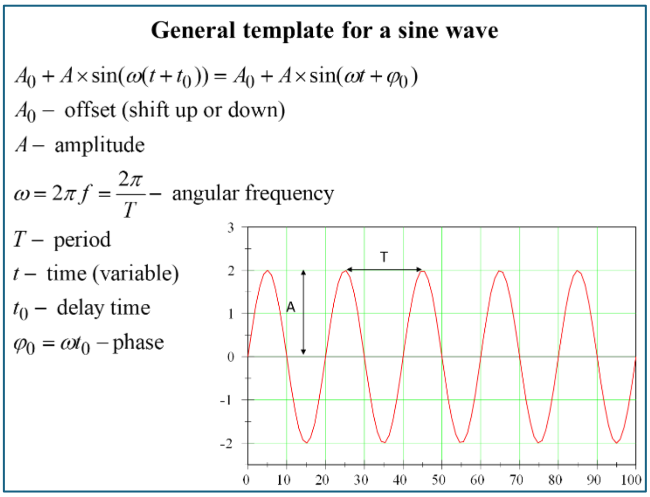
A complex current wave is represented is represented by:

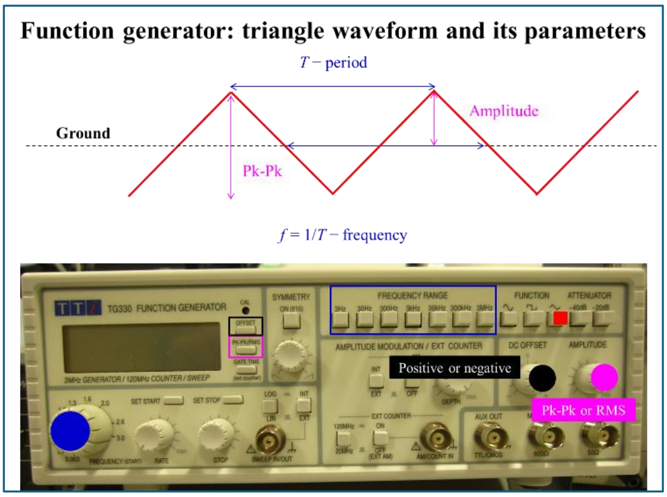
$$i = 40sin160\pi t + 16sin\left(480\pi t + \frac{\pi}{2}\right) + 4sin\left(800\pi t + \frac{\pi}{5}\right)\ A$$

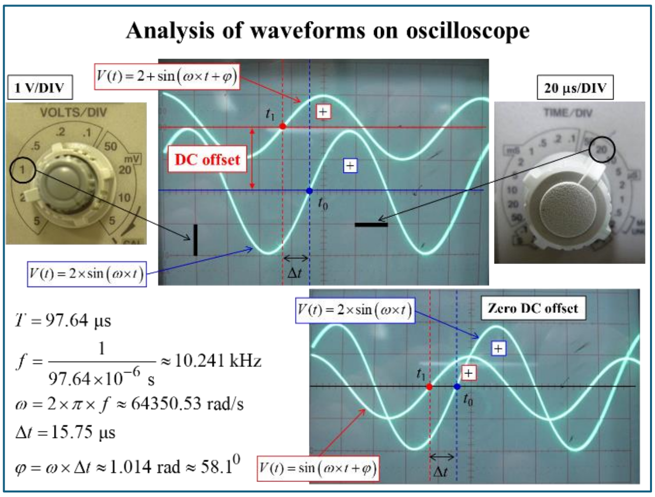
Determine (i) the frequency of the fundamental, (ii) the percentage third harmonic, (iii) the r.m.s. value of the third harmonic, (iv) the phase angles of the harmonic components and (v) the mean value of the fifth harmonic.

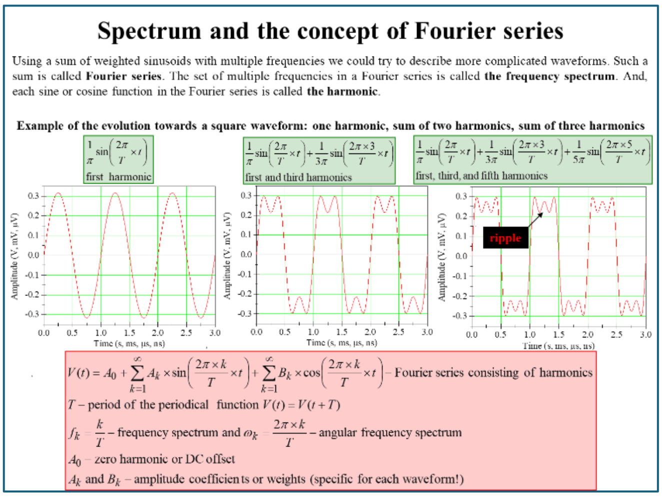
Solution:

For the first part of the question, look our lecture slides on the complex signals.









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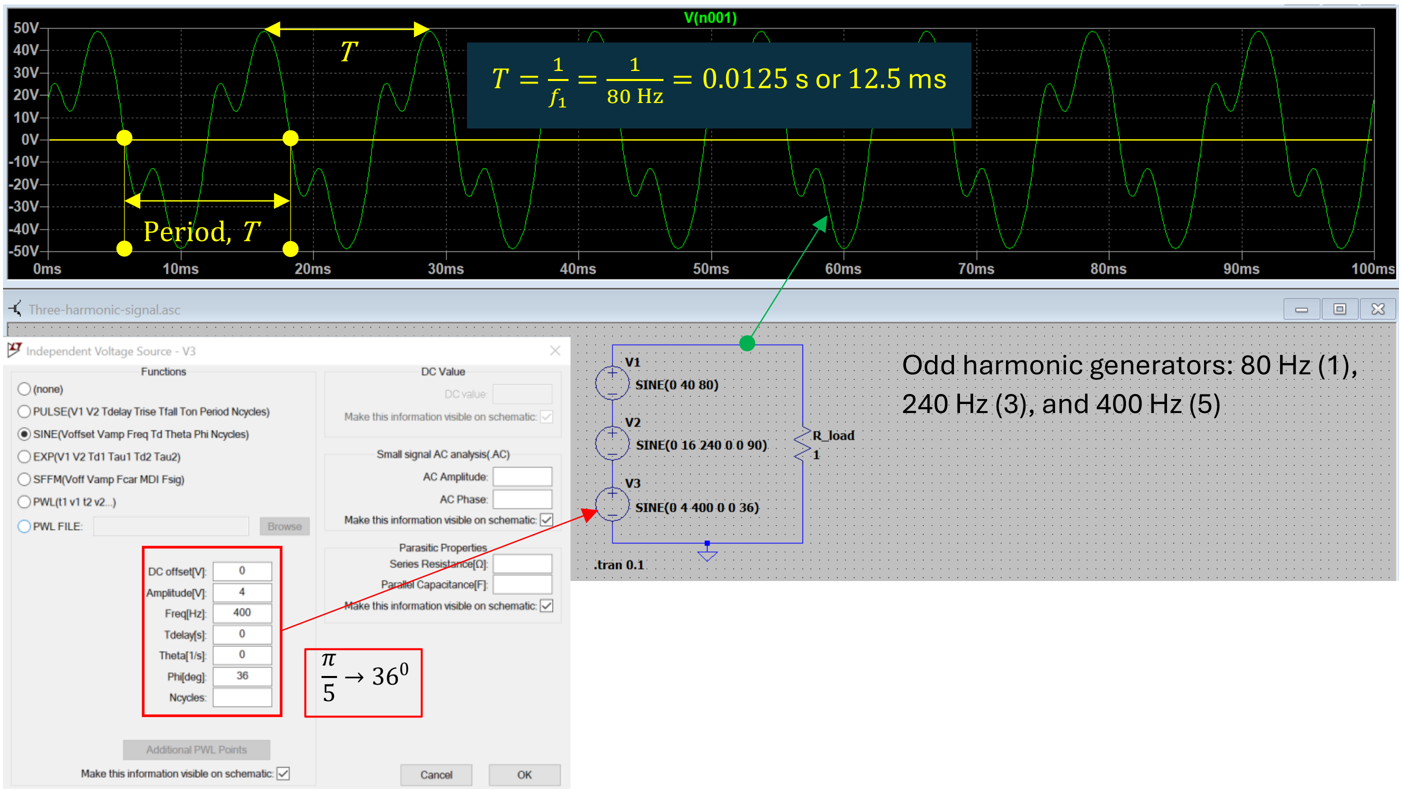
This complex waveform can be easily simulated in LTspice using three sinusoidal generators, as shown in the figures below. For each generator, we have to specify its frequency (same for all generators), amplitude, and phase in degrees. The total voltage will be measured with respect to the ground indicated in the circuit. For the load resistance, any value can be chosen, for example, 1 Ω as shown in the circuit.



(i) The fundamental frequency or first harmonic is $f_1 = 80$ Hz: $40\sin(2\pi f_1 t) = 40\sin(160\pi t)$

In precise terms, to conduct an analysis of harmonic amplitudes, the signal necessitates representation in the framework of a canonical Fourier series. This entails a format devoid of phase considerations, incorporating both sine and cosine terms.

$$V(t) = A_0 + \sum_{k=1}^{\infty} A_k \times \sin\left(\frac{2\pi \times k}{T} \times t\right) + \sum_{k=1}^{\infty} B_k \times \cos\left(\frac{2\pi \times k}{T} \times t\right) - \text{Fourier series consisting of harmonics}$$

T – period of the periodical function $V(t) = V(t+T)$

$f_k = \frac{k}{T}$ – frequency spectrum and $\omega_k = \frac{2\pi \times k}{T}$ – angular frequency spectrum

A_0 – zero harmonic or DC offset

A_k and B_k – amplitude coefficients or weights (specific for each waveform!)

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$
$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

For further details refer to the list of trigonometric identities.

$\mathrm{RMS}_k = \sqrt{\frac{A_k^2}{2} + \frac{B_k^2}{2}} = \frac{1}{\sqrt{2}} \times \sqrt{A_k^2 + B_k^2}$ is the RMS amplitude of a k-th harmonic.

(ii) The third harmonic $16\sin(6\pi f_1 t + \pi/2) = 16\cos(6\pi f_1 t) = 16\cos(480\pi t)$ has an amplitude of 16. Its percentage with respect to the first harmonic is $(16/40) \times 100\% = 40\%$. But you could use other reasonable definitions for "a harmonic percentage" that characterises its contribution to the total signal.

(iii) The RMS amplitude of the third harmonic is $16/\sqrt{2} \approx 11.314$. For the fifth harmonic we have $4\sin(10\pi f_1 t + \pi/5) = 4\cos(\pi/5)\sin(800\pi t) + 4\sin(\pi/5)\cos(800\pi t) \approx 3.236\sin(800\pi t) + 2.3514\cos(800\pi t)$. Its RMS amplitude will be $(4/\sqrt{2}) \times \sqrt{\sin^2(\pi/5) + \cos^2(\pi/5)} = 4/\sqrt{2} \approx 2.828$, i.e. the phase does not affect the RMS amplitude.

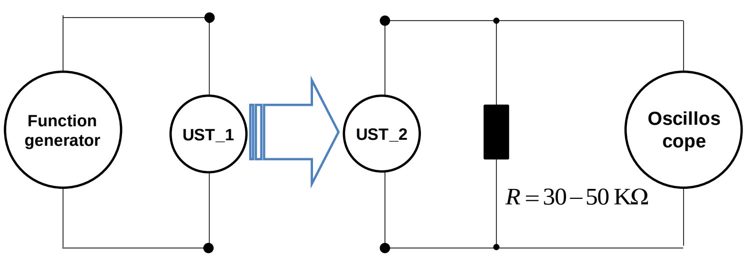
(iv) Harmonics' phase angles are $\varphi_1 = 0$, $\varphi_3 = \pi/2\ (90^0)$, and $\varphi_5 = \pi/5\ (36^0)$.

(v) The mean value of k-th harmonic is understood as the averaging integral of the module of harmonic waveform along its period T/k : $\frac{kA_k}{T}\int_0^{T/k} |\sin(\omega_k t)|\, dt = \frac{2kA_k}{T}\int_0^{T/(2k)} \sin\left(\frac{2\pi k}{T} t\right) = 2A_k/\pi$, where A_k is the amplitude of the k-th harmonic. Therefore, for the fifth harmonic we obtain $2 \times 4/\pi \approx 2.55$ A.

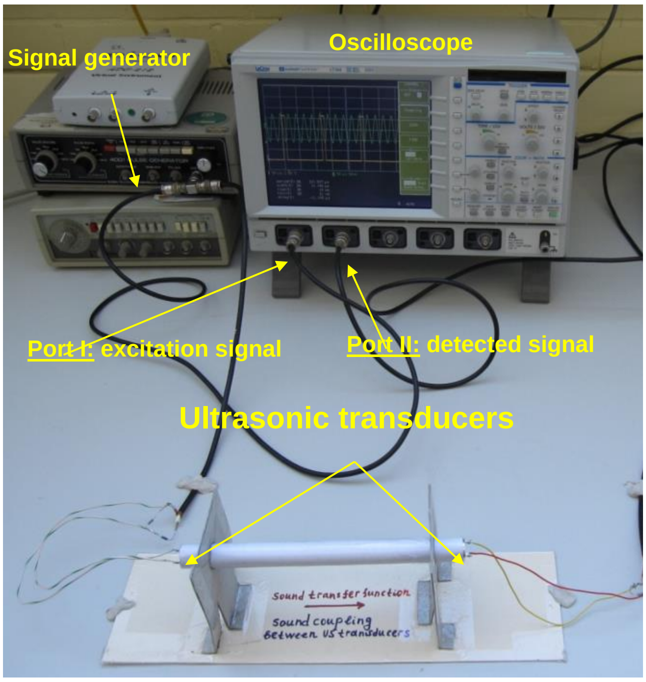
2. Describe, using relevant examples as appropriate, the significance of waveform analysis and how the Fourier series is used to yield an analytical solution.

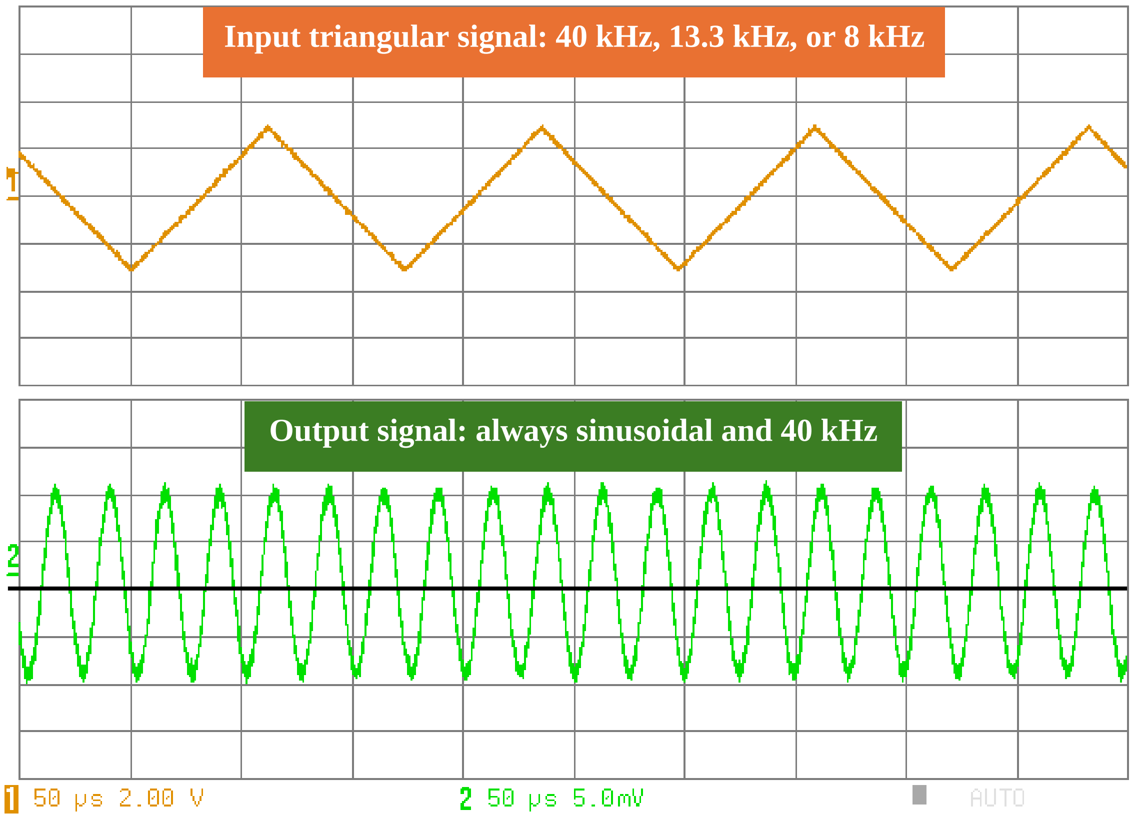
Solution:

Let us examine an acoustical system, shown in the photo and the circuit diagram below, comprising two identical ultrasonic transducers (UST), each resonating at 40 kHz. One transducer is connected to a function generator while the other is connected to an oscilloscope. The emission and absorption intensities of the transducers exhibit a narrow frequency range near 40 kHz, characterized by a −3 dB bandwidth of approximately 1 kHz. When exciting the emitting transducer (connected to the generator) with a triangular waveform, we observe that the signal detected at the receiving transducer (connected to the oscilloscope) consistently exhibits a sinusoidal shape (not triangular!) with a frequency of 40 kHz. Furthermore, signal transmission only manifests at specific frequencies within the triangular waveform, namely 40 kHz, 13.3 kHz, or 8 kHz, with decreasing amplitudes in that order. For any other frequencies, the output signal is almost zero. **How can we explain the transmission characteristics exhibited by such UST network?**



UST – ultrasonic transducer
The resistor R is used here just to suppress the noise.

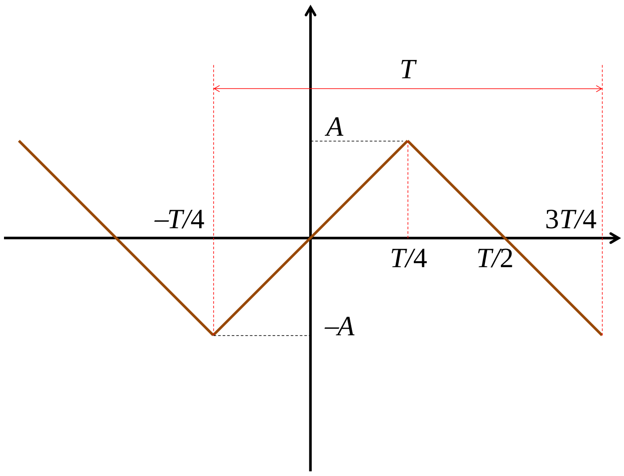




To explain the transmission properties of the UST network, we need to represent the excitation triangular waveform by its Fourier series. For the analytical representation of the waveform within the period $2l = T = 1/f$, where f is the fundamental frequency, we can use the following piecewise function (see the Figure below):

$$V_{in}(t) = \begin{cases} \frac{4A}{T}t, & -\frac{T}{4} \leq t \leq \frac{T}{4} \\ 2A - \frac{4A}{T}t, & \frac{T}{4} \leq t \leq \frac{3T}{4} \end{cases}$$

where A is the amplitude.



Here, $t_0 = -T/4$ is the reference time. Since the function is odd, $a_0 = 0$ and all $a_k = 0$. For b_k, we obtain:

$$b_k = \frac{1}{l}\int_{t_0}^{t_0+2l} V_{in}(t)\sin\left(\frac{k\pi}{l}t\right)dt = \frac{2}{T}\int_{-T/4}^{T/4}\left(\frac{4A}{T}t\right)\sin\left(\frac{2\pi k}{T}t\right)dt + \frac{2}{T}\int_{T/4}^{3T/4}\left(2A - \frac{4A}{T}t\right)\sin\left(\frac{2\pi k}{T}t\right)dt =$$

$$= \frac{8A}{T^2}\int_{-T/4}^{T/4} t\sin\left(\frac{2\pi k}{T}t\right)dt + \frac{4A}{T}\int_{T/4}^{3T/4}\sin\left(\frac{2\pi k}{T}t\right)dt - \frac{8A}{T^2}\int_{T/4}^{3T/4} t\sin\left(\frac{2\pi k}{T}t\right)dt =$$

$$= \frac{8A}{(\pi k)^2}\sin\left(\frac{\pi k}{2}\right)$$

Fourier series:

$$V_{in}(t) = \frac{8A}{\pi^2}\sum_{k=1}^{\infty}\left(\frac{1}{k^2}\sin\left(\frac{\pi k}{2}\right)\right)\sin\left(\frac{2\pi k}{T}t\right) = \frac{8A}{\pi^2}\sum_{m=1}^{\infty}\left(\frac{(-1)^{m+1}}{(2m-1)^2}\right)\sin\left(\frac{2\pi(2m-1)}{T}t\right)$$

Or, in a more compact form:

$$V_{in}(t) = \frac{8A}{\pi^2}\sum_{m=1}^{\infty}\left(\frac{(-1)^{m+1}}{(2m-1)^2}\right)\sin((2m-1)\omega t)$$

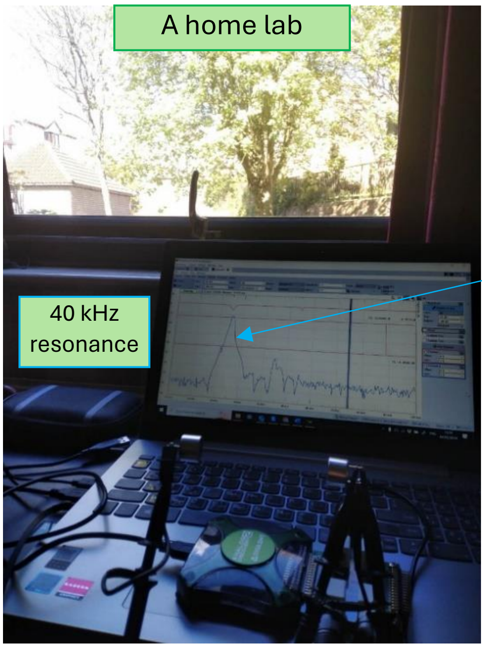
where $\omega = 2\pi f$ and $f = 1/T$ is the fundamental frequency. This Fourier series has a fast convergence due to the amplitude factor $1/k^2 = 1/(2m-1)^2$, where $m = 1, 2, 3, 4, \ldots$ are natural numbers.

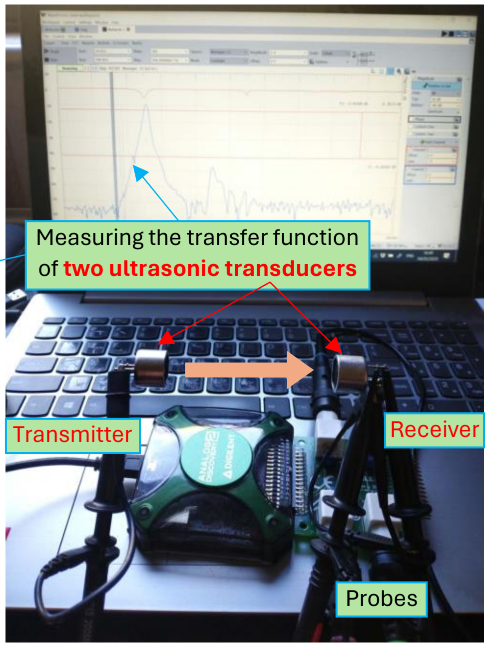
The Fourier series contains only the odd harmonics $\sin(2\pi k f t)$, where $k = 1, 3, 5, 7, .., (2m-1)$ .... are odd integers. When the excitation frequency of the triangular waveform is $f = f_{res}/k$, with an odd k and $f_{res} =$ 40 kHz, at the receiver we will observe only the k-th odd harmonic with the frequency $f_{out} = f \times k =$

$(f_{res}/k) \times k = 40$ kHz and the amplitude factor $1/k^2$. That is why the output signal observed at the receiver consistently manifests as a 40 kHz sinusoid, with any additional harmonics in the spectrum being suppressed by the linear network due to their location outside the pass band 40 ± 1 kHz. Our UST network works as a filter that selects only a k-th odd harmonic with the frequency $f_{out} = f_{res}$. We obtain the following excitation frequencies, for which an output signal can be observed at the receiver:

- **Excitation frequency $\boldsymbol{f = 40}$ KHz**, for which only the first harmonic ($k = 1$) will be passed, where $f_{out} = (f_{res}/1) \times 1 = 40$ KHz and $1/k^2 = 1$ is the amplitude factor (see the Fourier series).
- **Excitation frequency $\boldsymbol{f = 40/3 \approx 13.3}$ KHz**, for which only the third harmonic ($k = 3$) will be passed, where $f_{out} = (f_{res}/3) \times 3 = 40$ KHz and $1/3^2 = 1/9$ is the amplitude factor (much smaller in comparison with the first harmonic).
- **Excitation frequency $\boldsymbol{f = 40/5 = 8}$ KHz**, for which only the fifth harmonic ($k = 5$) will be passed, where $f_{out} = (f_{res}/5) \times 5 = 40$ KHz and $1/5^2 = 1/25$ is the amplitude factor.
- For other lower excitation frequencies $f = 40/k$ with odd $k > 5$, we will have sinusoidal output signals with $f_{out} = 40$ KHz and the amplitude factors $1/k^2$. However, these signals may be very small to be practically observed.

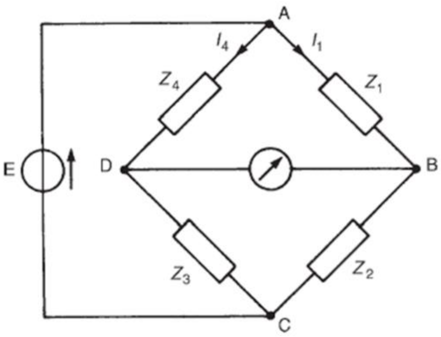
Are you able to set up such an ultrasonic measurement rig at home or in a college setting?



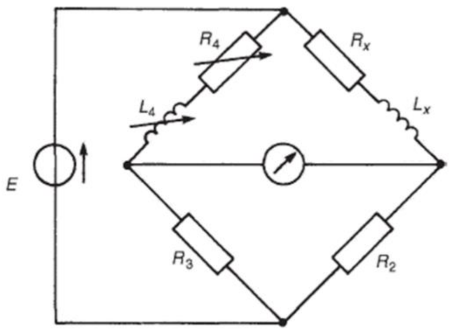


Analog Discovery 2: digital 2-channel oscilloscope and 2-channel waveform generator

Another compelling illustration showcasing the utility of Fourier series as a set of harmonics can be observed in the context of an AC bridge, shown in the figure below. The bridge arms can encompass both conventional resistors and impedance networks, with the voltage source E being an AC generator. The bridge is considered balanced for an unknown impedance Z_1 when adjustments to other impedances result in the voltmeter or oscilloscope detecting the minimum signal amplitude between points B and D. Theoretically, achieving a zero reading is always possible, but practically, some residual signal inevitably persists.

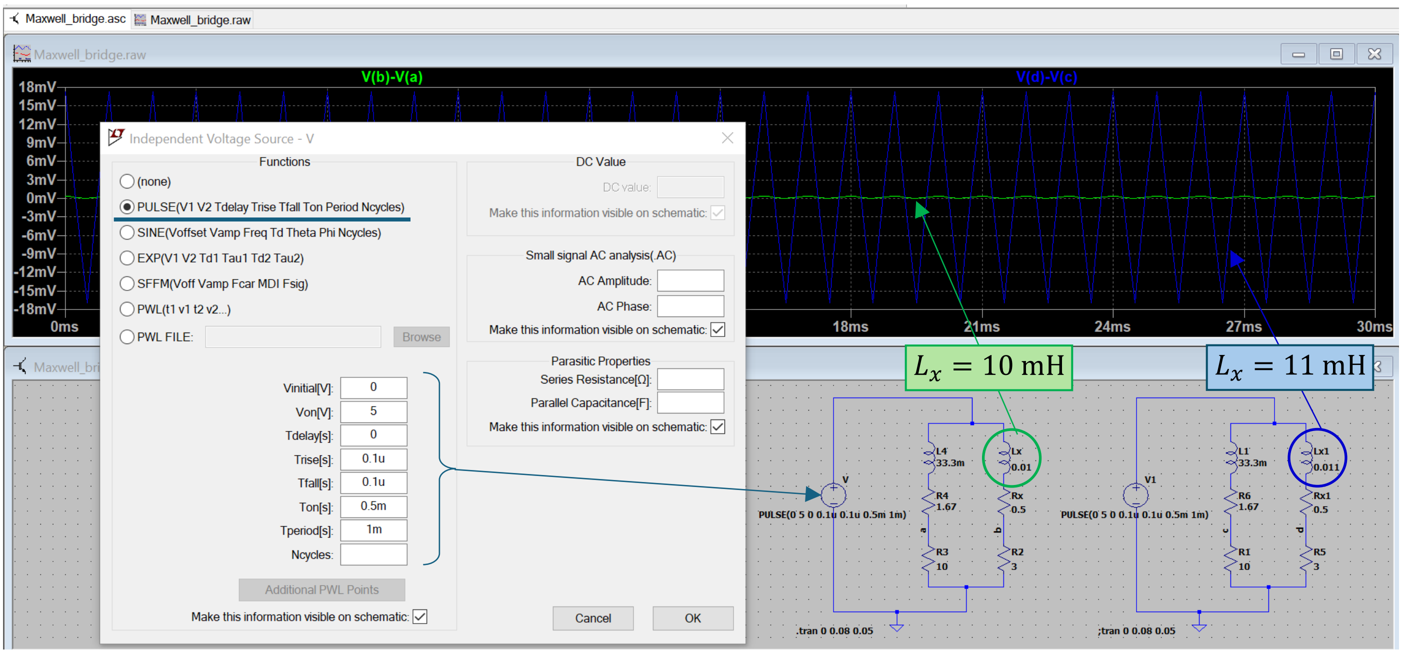


General scheme of an AC bridge.



Maxwell's bridge.

The balancing conditions, expressed in terms of complex numbers, necessitate that $Z_1 \times Z_3 = Z_2 \times Z_4$. For the Maxwell's bridge shown above, the complex balancing condition results in two conditions for the real values: $R_x = \frac{R_2 R_4}{R_3}$ and $L_x = \frac{R_2 L_4}{R_3}$. Since these conditions are independent of frequency, they will hold for any harmonic in a complex signal of any shape. Thus, any periodical waveshape can be used from the generator. This conclusion is confirmed by simulations in LTspice for the pulse excitation circuit shown below which is already balanced for $L_x = 10$ mH and $R_x = 0.5\ \Omega$.



A 10% variation in the measured inductance L_x precipitates a substantial imbalance, a characteristic hallmark of bridge circuits. The advantage of bridge circuits, where balancing conditions remain independent of frequency, lies in their insensitivity to harmonic disturbances in the sinusoidal excitation signal.

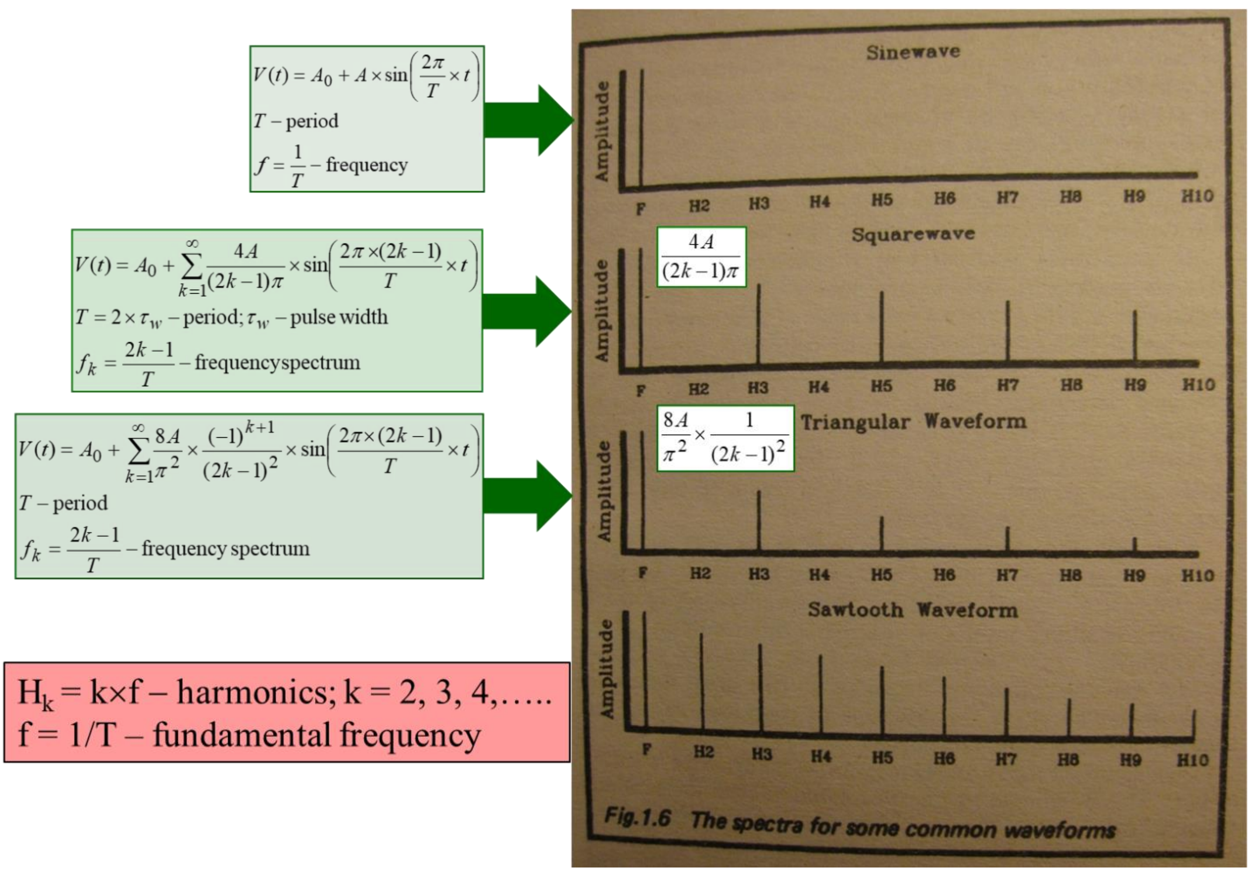
> 3. Explain how the component harmonics that comprise a complex waveform can be illustrated graphically in the time domain as well as the frequency domain. Include square, triangular and saw-tooth waveforms in your description.

Solution:

In the contemporary era of diverse simulators and graphical user interfaces (GUI), visualizing spectra and harmonics poses little challenge. A harmonic, characterized by a specific amplitude and frequency, can be readily perceived both audibly and graphically, manifesting in the time domain as a continuous periodic waveform. Alternative methodologies for visualizing the signal formation process in the time domain can be proposed, such as employing a system of interconnected circles. Here, each circle features a designated point serving as the rotational centre for the subsequent circle, with diminishing diameters and escalating frequencies.

In the frequency domain, a linearly increasing frequency is plotted along the positive horizontal axis, with amplitude depicted along the vertical axis. Each harmonic manifests as a vertical line, discretely positioned at multiple frequencies, with its length proportional to the harmonic's amplitude. It's noteworthy that contemporary computational signal processing methods often transcend graphical representation, rendering such visualizations purely illustrative.

The spectra for some common waveforms



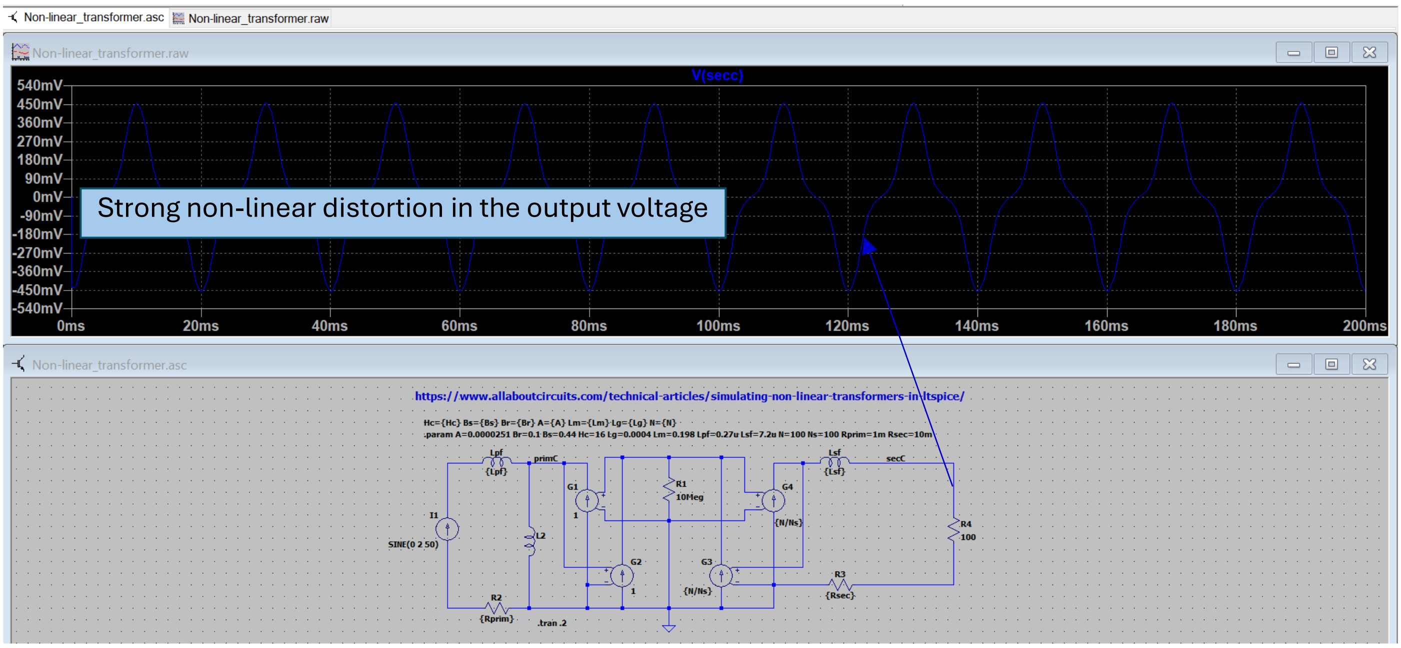
4. Discuss the main causes and effects of harmonics in electrical systems.

Solution:

The main reason for the occurrence of harmonics in electrical circuits and electronics is the presence of nonlinear elements, as well as various types of switching or modulation techniques, such as pulse width modulation (PWM). The source of non-linear distortion in electronics, and therefore harmonics, will be some kind of amplifying cascade. In electrical engineering, these can be transformers or switching power supplies. Here, we provide only one example directly related to the topics covered in our electrical modules, namely, non-linear distortions in transformers. A more detailed discussion can be found in our reports on hysteresis loops and magnetic cores.

To effectively contain magnetic flux within transformers and reduce leakage, specialized cores made of magnetic Fe-based or Co-based alloys, or ferrites, are employed. Prior to reaching saturation, the relationship between magnetic flux density B (measured in Tesla, T) and magnetizing field H (measured in amperes per meter, A/m) is roughly linear, with a proportionality coefficient known as magnetic permeability (μ). However, even within this linear range, a narrow hysteresis BH-loop occurs, resulting in minor nonlinear distortions and additional harmonics.

As H increases together with the current in the primary circuit, B approaches saturation (where its rate of change slows), causing significant nonlinear distortions in the output voltage. This state, known as the saturation regime, is best avoided in practical applications. Simulation of a non-linear transformer using LTspice with a magnetic core operating in partial saturation, shown below, demonstrates highly distorted output signals deviating from a simple sinusoid. In such scenarios, the contribution of harmonics becomes substantial.



LTspice simulation of a non-linear transformer using the Chan's hysteresis model.

The presence of harmonics in transformer circuits can have several consequences:

- **Increased Losses:** Harmonics cause additional eddy current and hysteresis losses in the transformer core material. These losses result in increased heating of the transformer, reducing its efficiency and potentially shortening its lifespan.
- **Voltage Distortion:** Harmonics can distort the output voltage waveform of the transformer. This distortion can affect the performance of connected equipment, particularly sensitive electronics, leading to malfunctions or premature failure.
- **Overloading and Voltage Drop:** Harmonics can lead to increased current flow in the transformer windings, causing overloading. Additionally, harmonic currents can create voltage drop along the distribution system, affecting the voltage stability at various points in the network.
- **Resonance:** Harmonics can cause resonant conditions in the transformer and the connected system. Resonance amplifies harmonic voltages and currents, potentially causing insulation breakdown, equipment damage, and safety hazards.
- **Interference:** Harmonics generated by transformers can interfere with communication systems and other electronic equipment connected to the same power supply. This interference manifests as noise or signal distortion, affecting the performance of sensitive devices.
- **Regulatory Compliance:** Excessive harmonics can lead to violations of regulatory limits on harmonic distortion in power systems. Non-compliance may result in penalties or restrictions on operations.

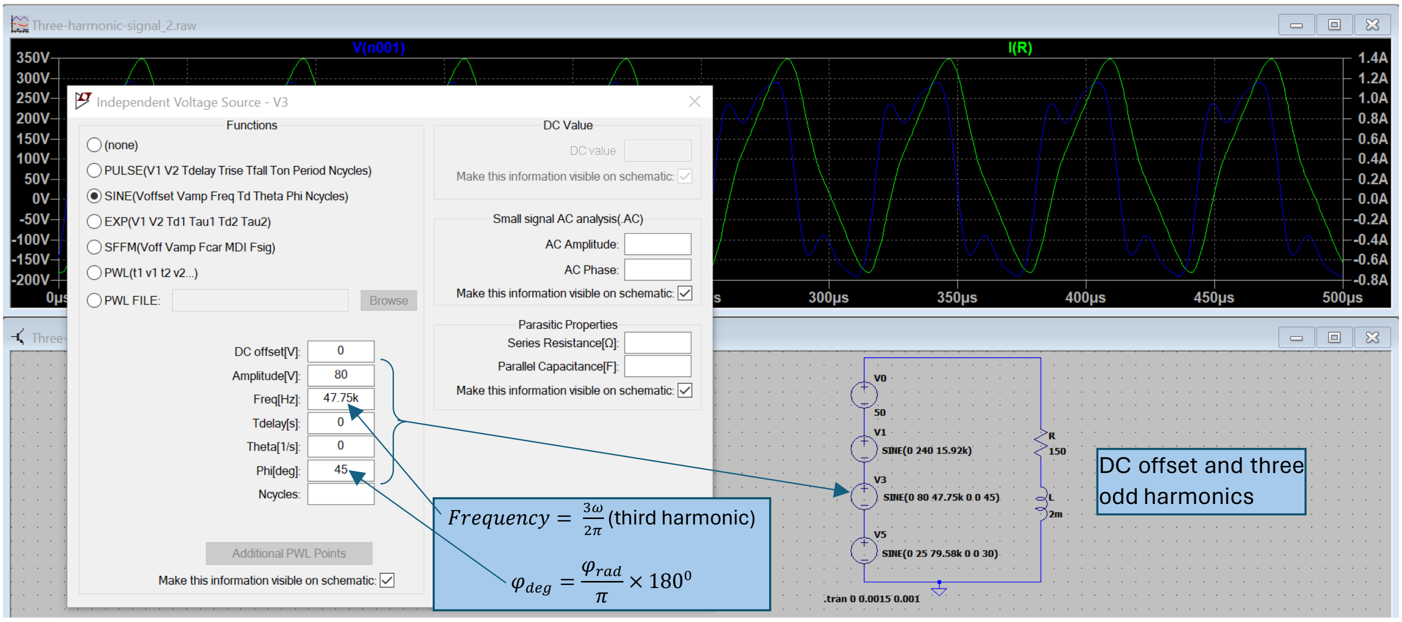
5. A circuit comprises a 150Ω resistance in series with a 2mH inductance. The supply voltage is given by

$$v = 50 + 240sin\omega t + 80sin\left(3\omega t + \frac{\pi}{4}\right) + 25sin\left(5\omega t + \frac{\pi}{6}\right)\ volts$$

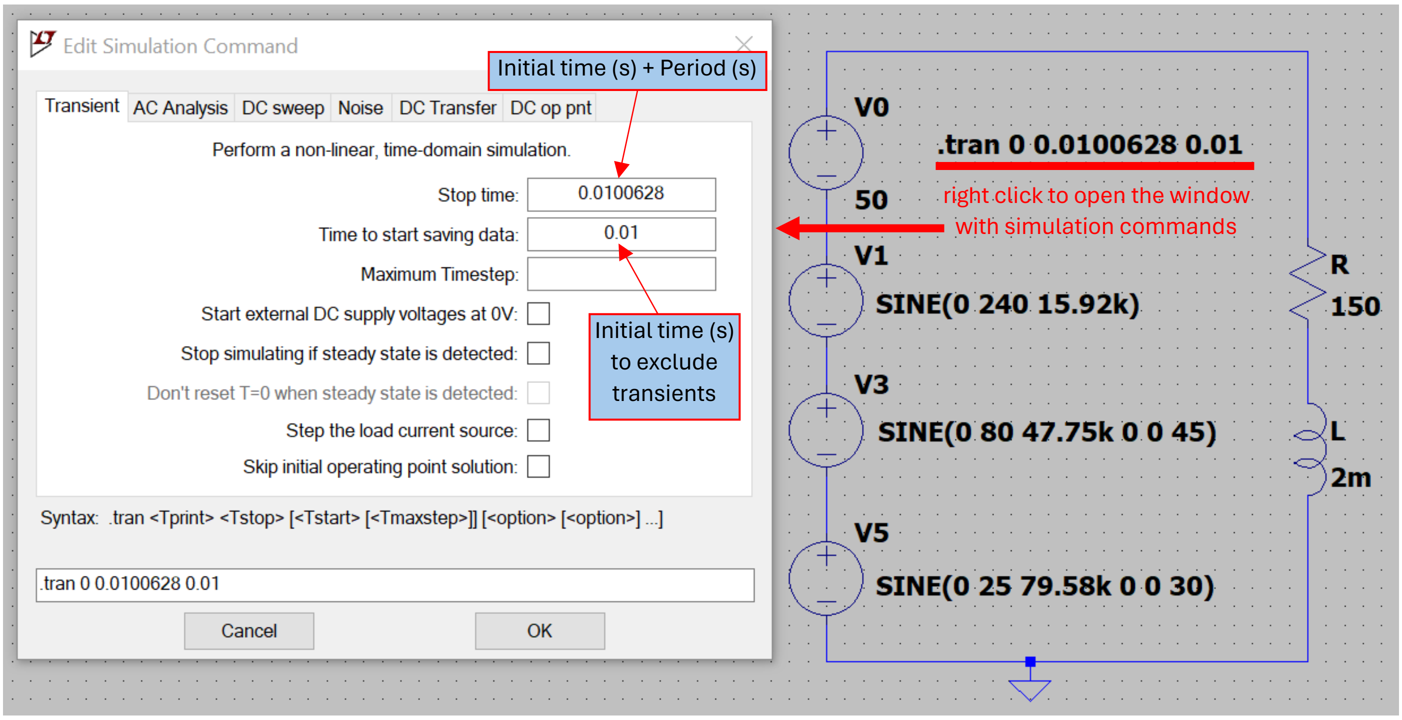
where $\omega = 10^5$ rad/s. Determine for the circuit (a) an expression to represent the current flowing, (b) the r.m.s. value of current and (c) the power dissipated.

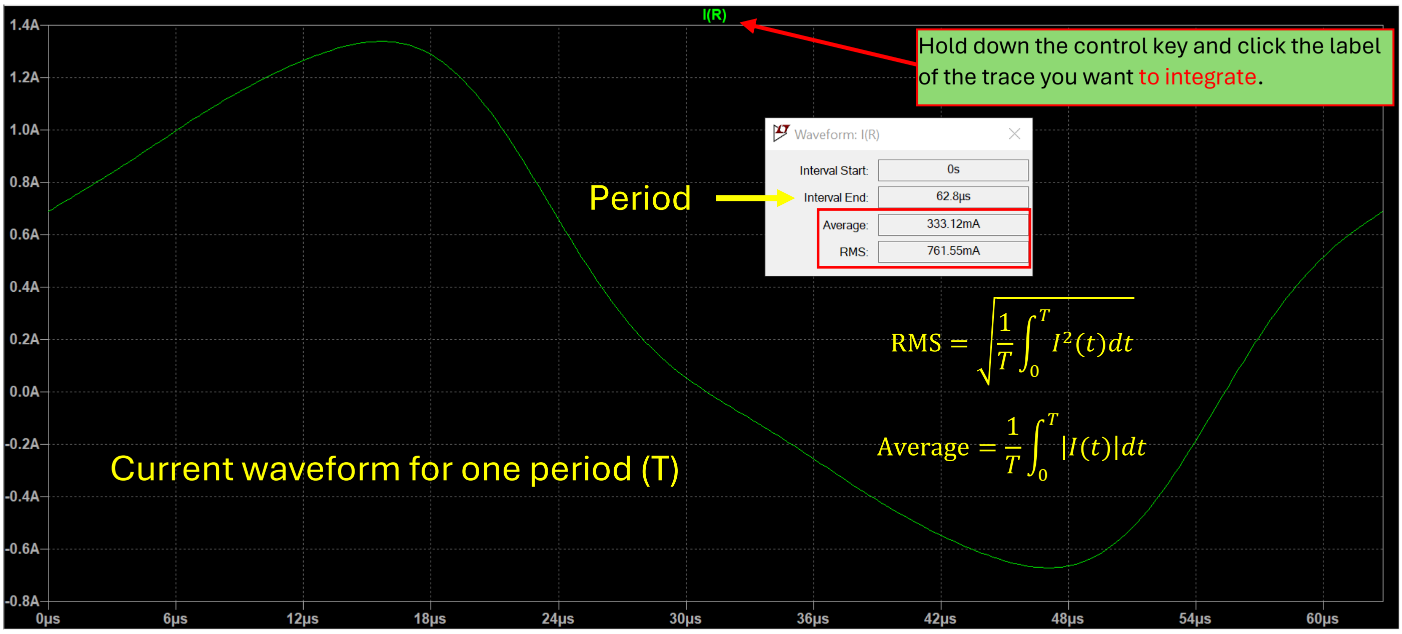
Solution:

Every harmonic $V_k = A_k \sin\left(\omega_k t + \varphi_k\right)$, where $\omega_k = k\omega$, present in the voltage signal $V = \sum_{k \geq 0} V_k$, including the DC offset ($V_0 = A_0$), can be viewed as an individual voltage source. The current (green) and voltage (blue) outputs simulated in LTspice are shown below.



In LTspice we can calculate the RMS and average values of a waveform as explained in next two slides. Initially, it would be more convenient to simulate the current waveform across a single period $T = \frac{2\pi}{\omega} \approx 62.832$ μs. In doing so, it is recommended to commence visualizing the waveform slightly later, thereby excluding transient processes at the onset, as our focus lies solely on the steady state. Following this, the starting time should be extended by one period to ascertain the cessation time, as shown below.





The impedance $Z_k = R + i\omega_k L$ of the inductive load, as it is viewed by the k-th harmonic, determines both the amplitude $A_k/|Z_k|$ and the lagging phase $\theta_k = -\tan^{-1}(\omega_k L/R)$ of the corresponding current harmonic $I_k = (A_k/|Z_k|) \times \sin(\omega_k t + \varphi_k + \theta_k)$, where $|Z_k| = \sqrt{R^2 + (\omega_k L)^2}$, $Z_0 = R$, and $I_0 = A_0/R$. The voltage and current RMS amplitudes are $V_{rms} = \sqrt{V_0^2 + \frac{1}{2}\sum_{k\geq 1} V_k^2}$ and $I_{rms} = \sqrt{I_0^2 + \frac{1}{2}\sum_{k\geq 1} I_k^2}$, respectively, where the summation is conducted over all harmonics $k \geq 1$ in the spectrum. The power dissipated in the resistor is $P_W = I_{rms}^2 \times R$.

Numerical calculation performed in the Python program:

```
Resistance (Ohm) = 150
Inductance (Henry) = 2.0e-3
Angular frequency (rad/s) = 1.0e+5
Are the harmonics odd/even/both? Enter the key word = odd
Number of harmonics in addition to the DC offset = 3
DC offset (V) = 50
Amplitude (V) of the 1-th voltage harmonic = 240
Phase (degrees) of the 1-th voltage harmonic = 0
Amplitude (V) of the 3-th voltage harmonic = 80
Phase (degrees) of the 3-th voltage harmonic = 45
Amplitude (V) of the 5-th voltage harmonic = 25
Phase (degrees) of the 5-th voltage harmonic = 30

Vrms =  186.581
Irms =  0.762
Power dissipated (W) =  87.054

Amplitudes of the current harmonics:
I[0] =  0.333
I[1] = 0.96
I[3] = 0.129
I[5] = 0.025

Impedance modules for the current harmonics:
Z[0] =  150.0
Z[1] = 250.0
Z[3] = 618.466
Z[5] = 1011.187

Phases for the current harmonics with respect to the voltage harmonics, rad (degrees):
Current phase[1] = -0.927, (-53.13)
Current phase[3] = -1.326, (-75.964)
Current phase[5] = -1.422, (-81.469)

Final phases for the current harmonics, rad (degrees):
Current phase[1] = -0.927, (-53.13)
Current phase[3] = -0.54, (-30.964)
Current phase[5] = -0.898, (-51.469)
```

Finally, we obtain the following current waveform:

$$I(t) \approx 0.333 + 0.96\sin(\omega t - 0.927) + 0.129\sin(\omega t - 0.54) + 0.025\sin(\omega t - 0.898)$$

The interactive console application in Python used for the calculation:

```
import math  # math library

pi = math.pi  # pi = 3.141592653589793238462643383279 5

print('')
R = float(input('Resistance (Ohm) = '))
L = float(input('Inductance (Henry) = '))
w = float(input('Angular frequency (rad/s) = '))
mode = (input('Are the harmonics odd/even/both? Enter the key word = ')).lower()  # lower case input string
N = int(input('Number of harmonics in addition to the DC offset = '))

# Calculating the index of a harmonic from its ordinal number, for example, "first odd harmonic = 3w",
# "second odd harmonic = 5w", etc. Or, "first even harmonic = 2w", "second even harmonic = 4w", etc.
# Here, w is the angular frequency.
def index(m, mode):
    return {'odd': 2 * m - 1, 'even': 2 * m, 'both': m}.get(mode)

V, Z, vphase, I, iphase = [0] * (2 * N + 1), [0] * (2 * N + 1), [0] * (2 * N + 1), [0] * (2 * N + 1), [0] * (2 * N + 1)
V[0] = float(input('DC offset (V) = '))
Z[0] = R  # impedance for the DC offset
I[0] = V[0] / R  # current zero harmonic
I[0] = round(I[0], 3)  # rounding to three decimal places
```

```python
for m in range(1, N + 1):
    k = index(m, mode)
    message = f'Amplitude (V) of the {k}-th voltage harmonic = '
    V[k] = float(input(message))
    message = f'Phase (degrees) of the {k}-th voltage harmonic = '
    vphase[k] = float(input(message))  # phase (degrees) of the k-th voltage harmonic
    vphase[k] = (vphase[k] * pi) / 180.0  # converting degrees to radians
    Z[k] = (R**2 + (k * w * L)**2)**0.5  # impedance module for the k-th current harmonic

# Current harmonic amplitudes and phases
for m in range(1, N + 1):
    k = index(m, mode)
    I[k] = V[k] / Z[k]
    iphase[k] = -math.atan((k * w * L) / R)  # lagging phase for an inductive load

# RMS voltage and current amplitudes, and power dissipated
Vrms = math.sqrt(sum(V[k]**2 / 2.0 for m in range(1, N + 1) for k in [index(m, mode)]) + V[0]**2)
Irms = math.sqrt(sum(I[k]**2 / 2.0 for m in range(1, N + 1) for k in [index(m, mode)]) + I[0]**2)
Pw = Irms**2 * R  # power dissipated
print('')
print('Vrms = ', round(Vrms, 3))  # rounding to three decimal places
print('Irms = ', round(Irms, 3))  # rounding to three decimal places
print('Power dissipated (W) = ', round(Pw, 3))

print('')
print('Amplitudes of the current harmonics:')
print('I[0] = ', I[0])
for m in range(1, N + 1):
    k = index(m, mode)
    value = round(I[k], 3)
    print(f'I[{k}] = {value}')

print('')
print('Impedance modules for the current harmonics:')
print('Z[0] = ', Z[0])
for m in range(1, N + 1):
    k = index(m, mode)
    value = round(Z[k], 3)
    print(f'Z[{k}] = {value}')

print('')
print('Phases for the current harmonics with respect to the voltage harmonics, rad (degrees):')
for m in range(1, N + 1):
    k = index(m, mode)
    rad = round(iphase[k], 3)
    deg = iphase[k] * 180 / pi
    deg = round(deg, 3)
    print(f'Current phase[{k}] = {rad}, ({deg})')

print('')
print('Final phases for the current harmonics, rad (degrees):')
for m in range(1, N + 1):
    k = index(m, mode)
    rad = vphase[k] + iphase[k]
    deg = rad * 180 / pi
    rad = round(rad, 3)
    deg = round(deg, 3)
    print(f'Current phase[{k}] = {rad}, ({deg})')
```

The solution of this task can also be explored through the lens of the general theory of linear networks, as elucidated in our lecture. Through this methodology, the input signal is expressed in its canonical form Fourier series, potentially comprising a limited number of harmonics. Subsequently, this representation undergoes conversion into a Fourier series of the output signal, utilizing the network transfer function within the frequency domain. In our scenario, the linear network manifests as the transfer function $\hat{F}(\omega) = \frac{1}{Z(\omega)} = \frac{1}{R+i\omega L} = \frac{R-i\omega L}{R^2+(\omega L)^2}$ governing the transformation from voltage to current across the inductive load $R + i\omega L$.

According to the general theory of linear networks, we can calculate the Fourier series of current from the Fourier series of voltage using the following equations:

$$V(t) = A_0 + \sum_{k=1}^{\infty}[a_k \cos(\omega_k t) + b_k \sin(\omega_k t)]$$

$$I(t) = \mathrm{Re}[\hat{F}(0)]\, A_0 + \sum_{k=1}^{\infty}\Big(\big(\, a_k \,\mathrm{Re}[\hat{F}(\omega_k)] + b_k \,\mathrm{Im}[\hat{F}(\omega_k)]\,\big)\cos(\omega_k t) + \big(\, b_k \,\mathrm{Re}[\hat{F}(\omega_k)] - a_k \,\mathrm{Im}[\hat{F}(\omega_k)]\,\big)\sin(\omega_k t)\Big)$$

where

$$\omega_k = \omega k$$

$$\mathrm{Re}[\hat{F}(\omega)] = \frac{R}{R^2+(\omega L)^2}$$

$$\mathrm{Re}[\hat{F}(0)] = \frac{1}{R}$$

$$\mathrm{Im}[\hat{F}(\omega)] = \frac{-\omega L}{R^2+(\omega L)^2}$$

Voltage-to-Current Linear Network

Our voltage signal has only three odd harmonics in addition to a DC offset. Let us represent this signal in the canonical form with "sin" and "cos" instead of using phases:

$$V(t) = 50 + 240\sin(\omega t) + 80\sin(3\omega t + \pi/4) + 25\sin(5\omega t + \pi/6) =$$

$$= 50 + 240\sin(\omega t) + 80\cos(\pi/4)\sin(3\omega t) + 80\sin(\pi/4)\cos(3\omega t) +$$

$$+ 25\cos(\pi/6)\sin(5\omega t) + 25\sin(\pi/6)\cos(5\omega t)$$

We obtain:

$$A_0 = 50$$

$$a_1 = 0$$

$$b_1 = 240$$

$$a_3 = 80\sin(\pi/4)$$

$$b_3 = 80\cos(\pi/4)$$

$$a_5 = 25\sin(\pi/6)$$

$$b_5 = 25\cos(\pi/6)$$

Other coefficients will be zero. Finally, we obtain for the current waveform:

$$I(t) = \frac{A_0}{R} + b_1 \,\mathrm{Im}[\hat{F}(\omega)]\cos(\omega t) + b_1 \,\mathrm{Re}[\hat{F}(\omega)]\sin(\omega t) + \big(\, a_3 \,\mathrm{Re}[\hat{F}(3\omega)] + b_3 \,\mathrm{Im}[\hat{F}(3\omega)]\,\big)\cos(3\omega t) + \big(\, b_3 \,\mathrm{Re}[\hat{F}(3\omega)] - a_3 \,\mathrm{Im}[\hat{F}(3\omega)]\,\big)\sin(3\omega t) + \big(\, a_5 \,\mathrm{Re}[\hat{F}(5\omega)] + b_5 \,\mathrm{Im}[\hat{F}(5\omega)]\,\big)\cos(5\omega t) + \big(\, b_5 \,\mathrm{Re}[\hat{F}(5\omega)] - a_5 \,\mathrm{Im}[\hat{F}(5\omega)]\,\big)\sin(5\omega t)$$

While the resultant equation may seem more complex than the previous solution, the computational framework relying on transfer functions can be readily implemented through programming, offering a versatile template applicable to diverse signals and transfer functions. Given the prevalent utilization of computer technology and programming, such methodologies hold greater appeal in contemporary engineering practices. Facilitating manual calculations is rendered redundant as they are seldom employed in practical settings. Furthermore, the evolution of artificial intelligence engenders substantial shifts in both problem formulation and solution methodologies.