Faculty of Aerospace Engineering **Technion- Israel Institute of Technology**

Numerical Methods in Aerospace Engineering Spring Semester 2019

Examples Sheet 2

Question 1 **Please submit your solution to this question according to the instructions on the course's site.**

The following equation describes the motion of an infinite oscillating cylinder in an incompressible Newtonian fluid at rest:

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}, \quad 0\le r<1$$

subject to the following initial and boundary conditions:

u is bounded as $r\to 0$; $u(1,t)=sin(\omega t)$; $u(r,0)=0$.

Solve this problem numerically using a method of your choice, for $0\le t\le 10$ when $\omega=1$ and $\omega=2$.

Question 2

(a) **Develop** the completely implicit method for the numerical solution of the partial differential equation $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$. (Hint: try making use of the formal solution developed in my lecture: $u(x,t+\delta t)=exp\left[\delta t D^2{}_x\right]u(x,t)$).

(b) Consider the nonlinear partial differential equation $\frac{\partial u}{\partial t}=\frac{\partial}{\partial x}\left[F(u)\frac{\partial u}{\partial x}\right]$. What is the formal solution of this equation? **Develop** the completely implicit method for this solving this equation numerically.

(c) For the p.d.e of (b) explain how you would actually solve if you were using the fully implicit method.

Question 3

Find the local truncation error for the fully implicit method when solving the p.d.e. $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$.

Good luck!!