Generalized Network flows

elements of the Generalized Network Flow Problem

Defined on a directed network: $G = (N, A)$

where N is a set of n *nodes*: $\{1, 2, \ldots, n\}$

and A is a set of m arcs as a subset of $N \times N$

Each node i has an associated value $b(i)$

Arc (i, j) has certain characteristics:

- cost c_{ij} per unit of flow on arc (i, j)
- upper bound on flow of u_{ij} (capacity)
- lower bound on flow of ℓ_{ij} (usually 0)
- **multiplier $\mu_{ij} \geq 0$ such that if 1 unit of flow leaves node i, then μ_{ij} units arrive at node j**

Generalized network flow examples*

- **Financial networks** where nodes represent equities (e.g., stocks, bonds, current deposits, Treasury bills, etc.); and the arcs represent various investment alternatives that convert one type of equity into another. The arc multiplier represents the gain associated with the corresponding investment.
- **Mineral networks** where nodes represent mines, purification plants, refineries, ports, and final markets; arcs represent processing opportunities or flow of material through intermediate junctions. The multipliers represent loss associated with the corresponding process.
- **Energy networks** where nodes represent raw materials (e.g., crude oil, coal, uranium), and various outputs (e.g., electricity, domestic oil, gas); and the arcs represent the transformation of one raw material into an energy output; the multiplier represents the efficiency of this transformation.

*Ahuja, R., T. Magnanti, J. Orlin. 1993. Network Flows. Prentice Hall.

Generalized network formulation

$$
\begin{aligned}
\text{minimize} \quad & \sum_{(i,j)\in A} c_{ij} x_{ij} \\
\text{subject to} \quad & \sum_{j:(i,j)\in A} x_{ij} - \sum_{j:(j,i)\in A} \mu_{ji} x_{ji} = b_i \quad \forall i \in N \\
& l_{ij} \le x_{ij} \le u_{ij} \qquad \forall (i,j) \in A
\end{aligned}
$$

Generalized network AMPL model

```
set NODES;                          # nodes in the network
set ARCS within {NODES, NODES};     # arcs in the network

param b {NODES} default 0;          # supply/demand for node i
param c {ARCS}  default 0;          # cost of one of flow on arc(i,j)
param l {ARCS}  default 0;          # lower bound on flow on arc(i,j)
param u {ARCS}  default Infinity; # upper bound on flow on arc(i,j)
param mu {ARCS} default 1;          # multiplier on arc(i,j)
                                    # i.e., if one unit leaves i, mu[i,j] units arrive at j

var x {ARCS};                       # flow on arc (i,j)

minimize cost: sum{(i,j) in ARCS} c[i,j] * x[i,j];  #objective: minimize arc flow cost

# Flow Out(i) - Flow In(i) = b(i)

subject to flow_balance {i in NODES}:
sum{j in NODES: (i,j) in ARCS} x[i,j] - sum{j in NODES: (j,i) in ARCS} mu[j,i] * x[j,i] = b[i];

subject to capacity {(i,j) in ARCS}: l[i,j] <= x[i,j] <= u[i,j];
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Generalized network flow example

- An entrepreneur is starting a new business in buying/selling political action figures.   The prices for the figures are quite volatile, and she is trying to determine how to manage her purchases, sales, and storage patterns.
- She has $8500 of cash on-hand; and 750 units of her preferred political figurine. Inventory capacity each period: 1,500 units. Inventory holding costs per unit per period: $0.50
- She has good estimates of the price of the items over the next several periods: $10, $40, $80, and $50.
- Each month she can choose to put her money in the bank and earn interest (0.25%) per period.
- Due to a rodent problem, she assumes a spoilage rate of inventory of 1% each period.

Generalized network flow example

Using a generalized network flow formulation, please help the entrepreneur determine a plan for cash management, purchases, sales, and storage patterns over the next four periods, in such a way to maximize the cash-on-hand in period 5.

Assume any product on-hand during period k can be sold; the cash is available in period $k + 1$.

Similarly, assume any amount of product can be purchased in period k and is available in period $k + 1$.

Generalized network flow example

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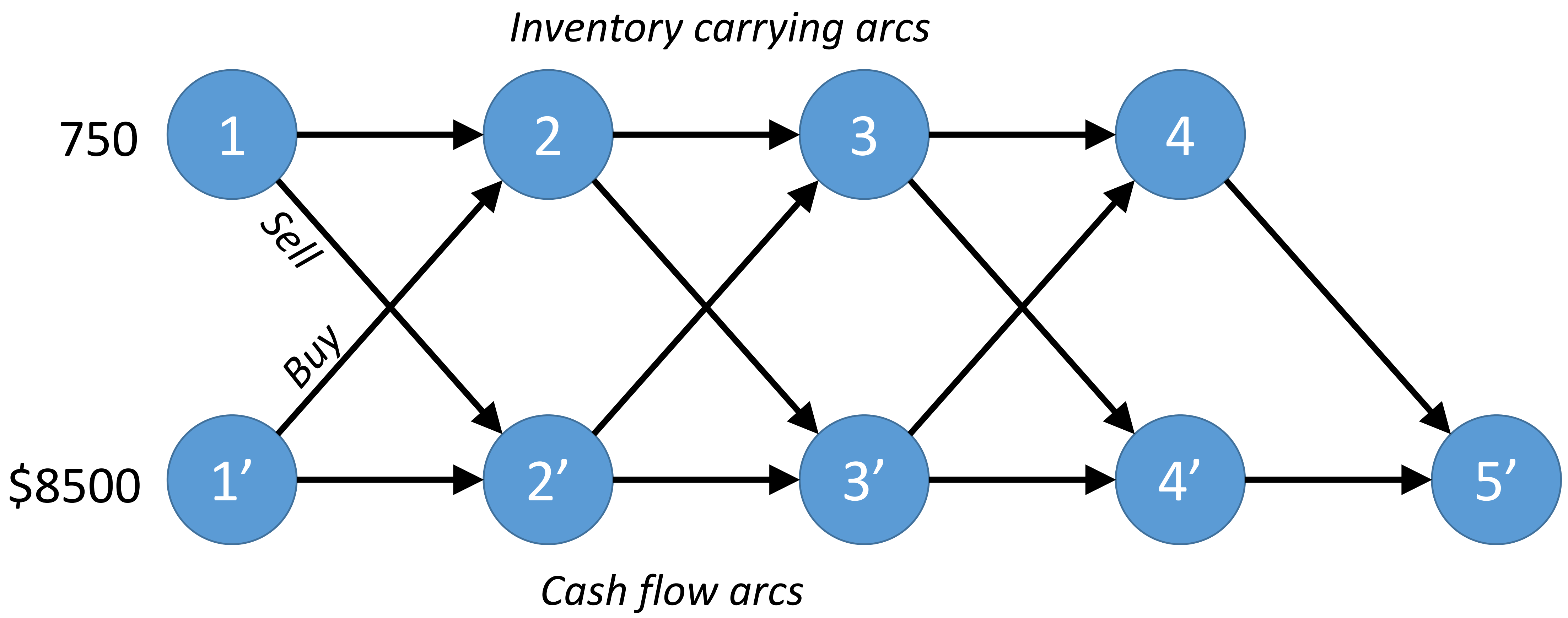
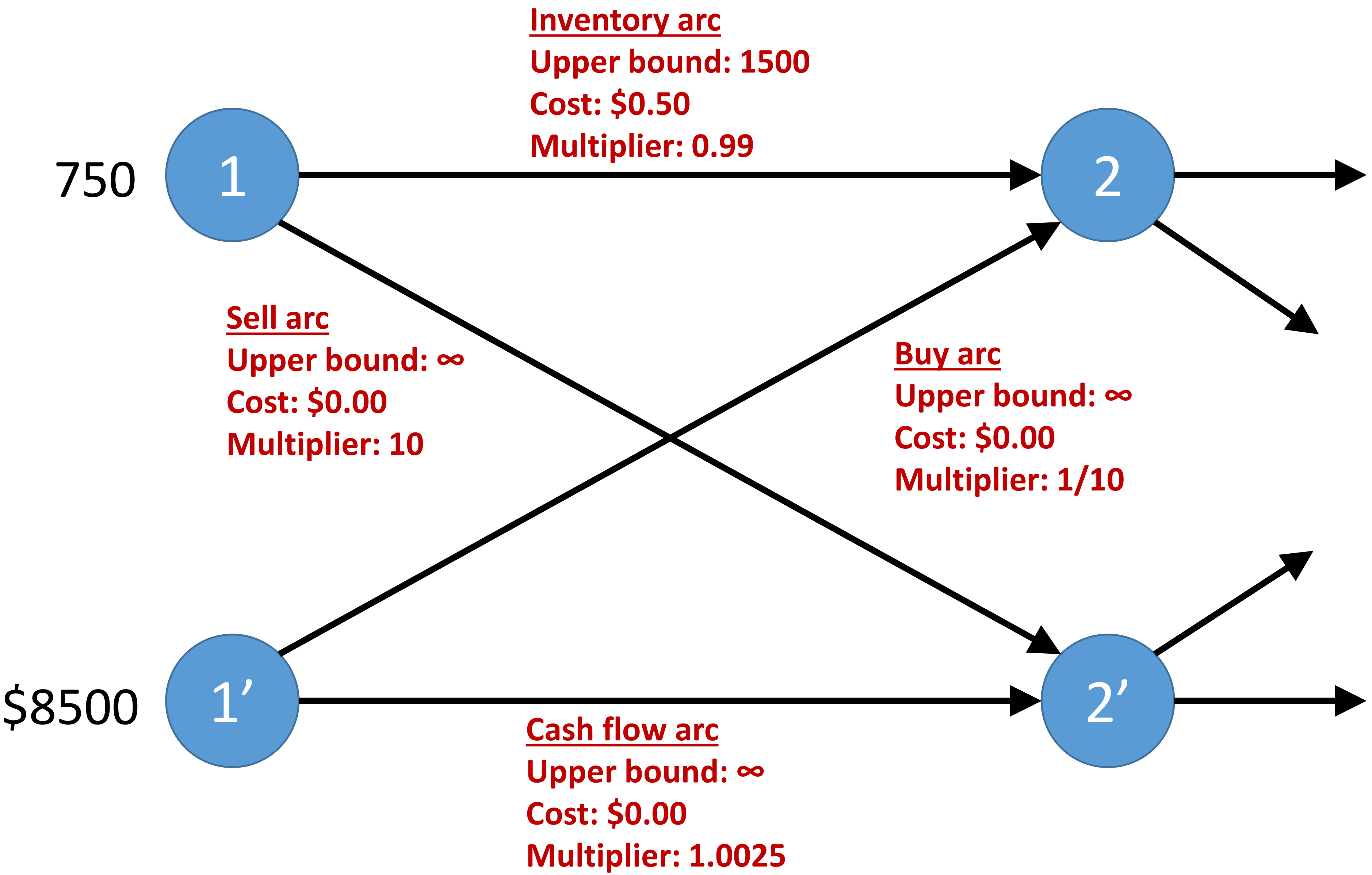


Generalized network flow example



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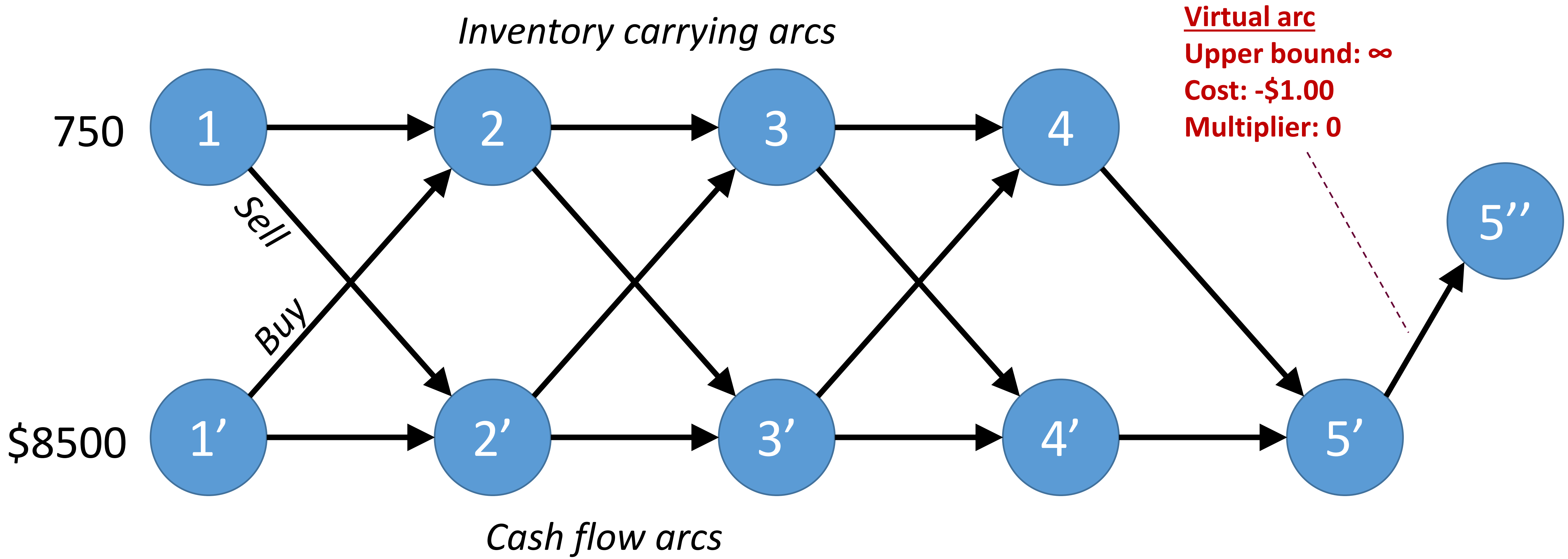

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