# ISE/DSA 5113
# Advanced Analytics and Metaheuristics

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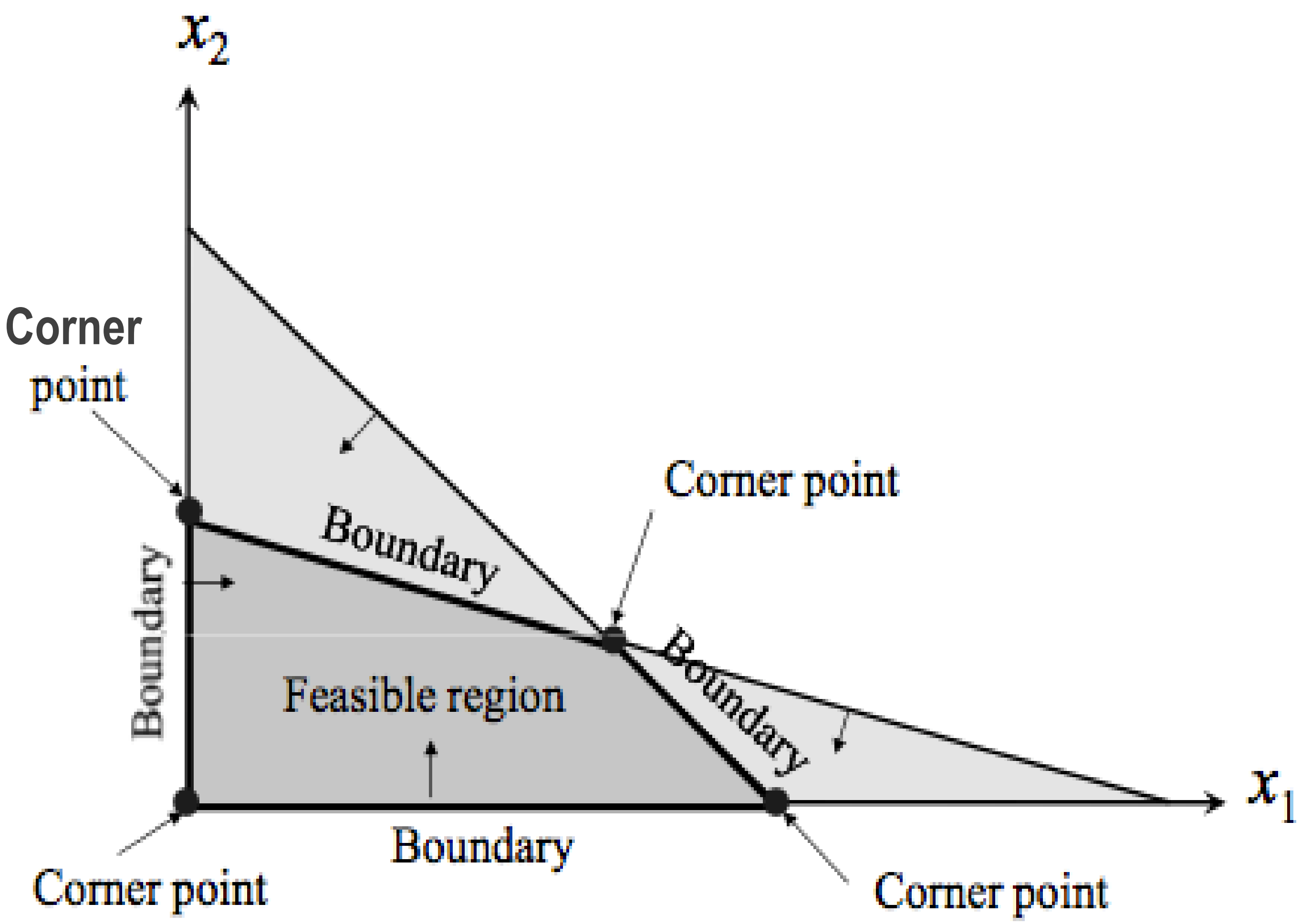
Lecture Week 2

# LINEAR PROGRAMMING

# linear programming (LP) problem

$$\text{maximize } \mathbf{c}^T\mathbf{x}$$
$$\text{subject to } \mathbf{A}\mathbf{x} \leq \mathbf{b}$$
$$\mathbf{x} \geq 0$$

- many problems can be formulated as LP
- no analytical formula for solution
- efficient solution techniques exist
- provide useful “what if” analysis



# LP assumptions

- objective function appropriateness
- decision variable appropriateness
- constraint appropriateness
- additivity
- proportionality
- divisibility
- certainty

## The Galaxy Industries Production Problem

Galaxy manufactures two toy guns:

1. Space Rays

   $8 dollar profit per unit

2. Zappers

   $5 dollar profit per unit

Management is seeking a production schedule that will increase the company’s profit.

The current heuristic production plan:

- Produce as much of the more profitable product, Space Ray’s
- Use left over resources to produce Zappers

**constraints**

- Resources are limited:
  - 1000 pounds of special plastic
  - 40 hours of production time per week
- Space Rays requires 2 pounds of plastic and 3 minutes of labor per unit.
- Zappers requires 1 pound of plastic and 4 minutes of labor per unit.
- Total production cannot exceed 700 units.
- Number of units of Space Rays cannot exceed Zappers by more than 350.

Develop a mathematical model for this problem.

# The Galaxy Linear Programming Model

Decision variables:

$X_1$ = Weekly production level of Space Rays

$X_2$ = Weekly production level of Zappers

Objective Function: **maximize weekly profit**

# The Galaxy Linear Programming Model

Max: $8X_1 + 5X_2$ (Weekly profit)

subject to

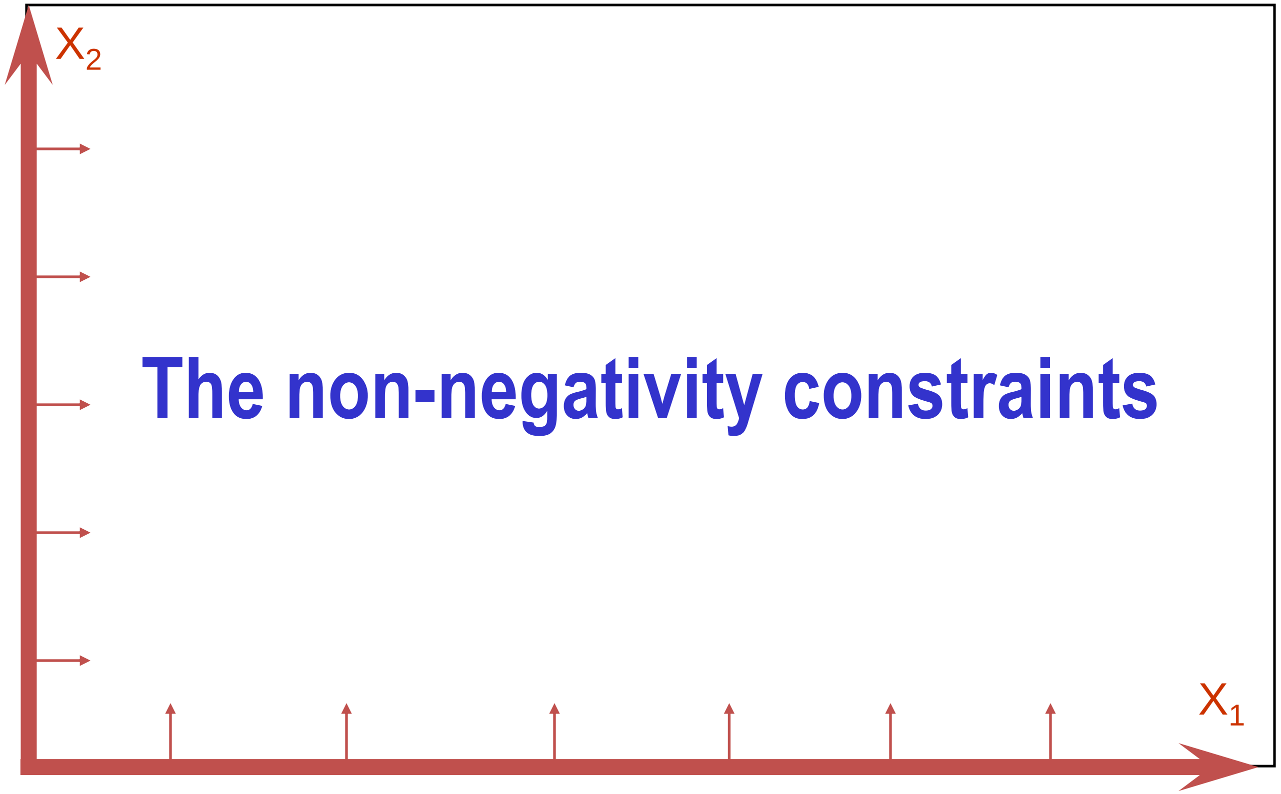
$2X_1 + 1X_2 \leq 1000$ (Plastic)

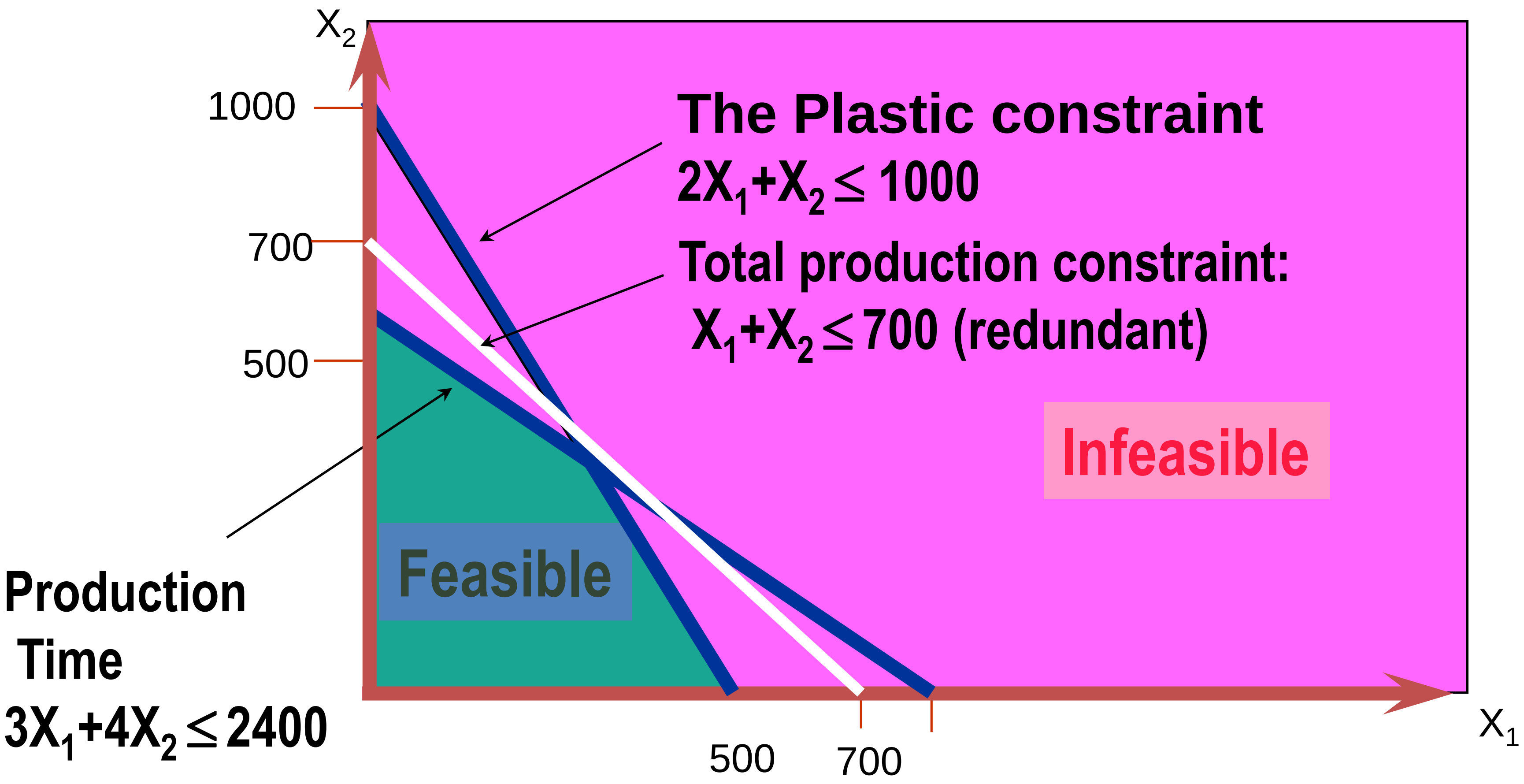
$3X_1 + 4X_2 \leq 2400$ (Production Time)

$X_1 + X_2 \leq 700$ (Total production)

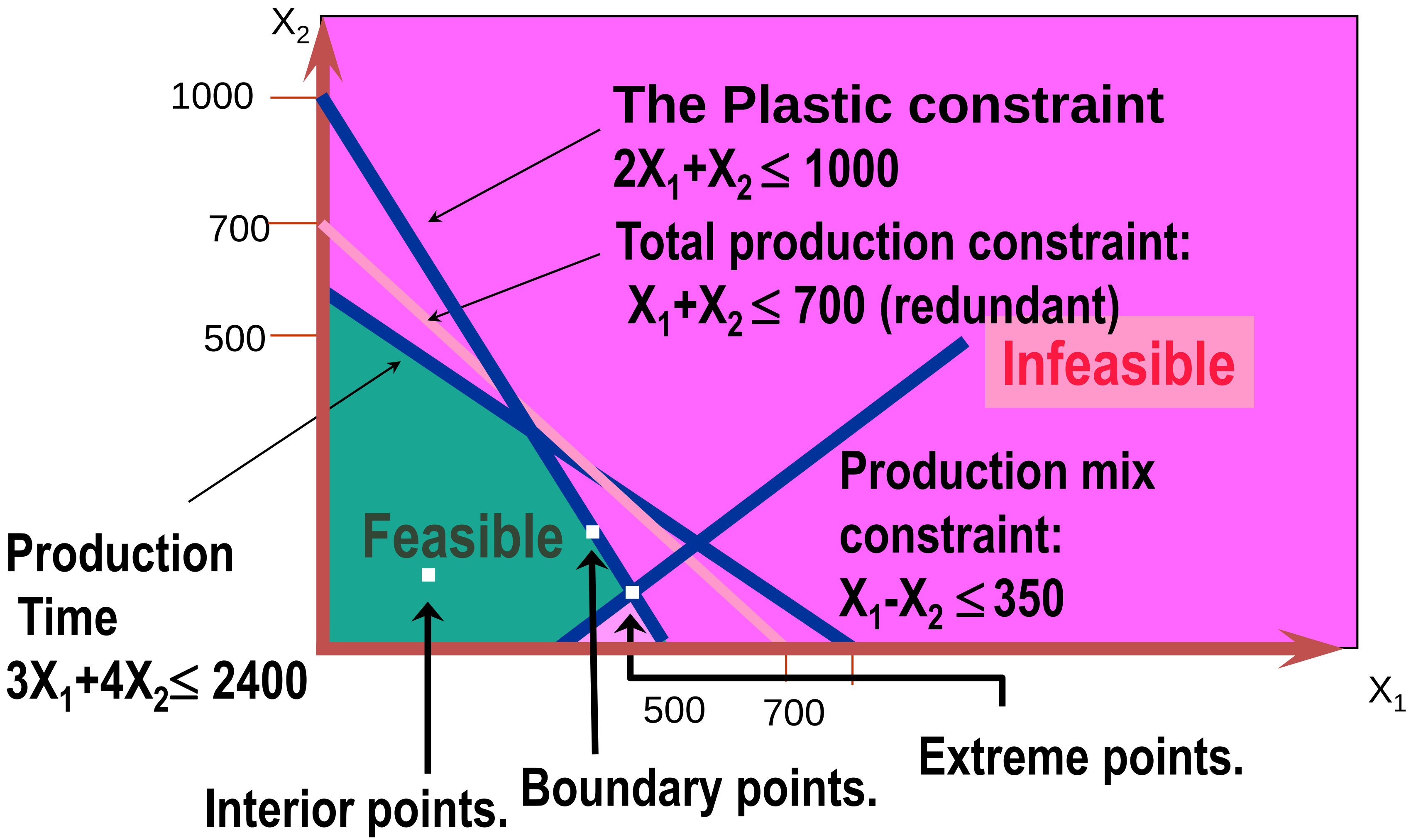
$X_1 - X_2 \leq 350$ (Mix)

$X_j >= 0, \; j = 1,2$ (Nonnegativity)


$X_2$
The non-negativity constraints
$X_1$



X2
1000
700
500
The Plastic constraint
2X1+X2 ≤ 1000
Total production constraint:
X1+X2 ≤ 700 (redundant)
Infeasible
Feasible
Production
Time
3X1+4X2 ≤ 2400
500
700
X1



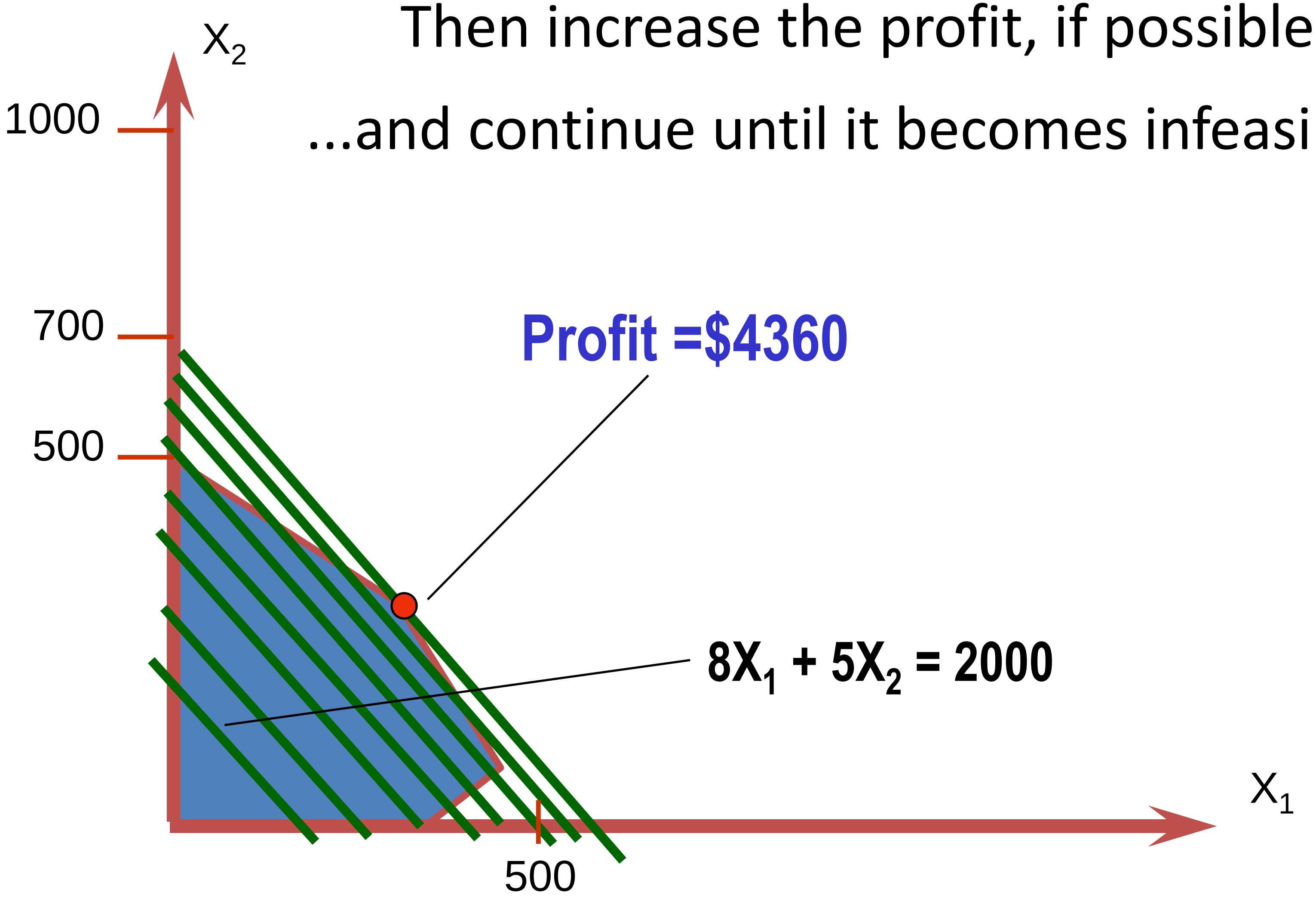

There are three types of feasible points

# search for an optimal solution

Start at some arbitrary profit, say profit = $2,000...

Then increase the profit, if possible...

...and continue until it becomes infeasible



## summary of the optimal solution

Space Rays = 320 units

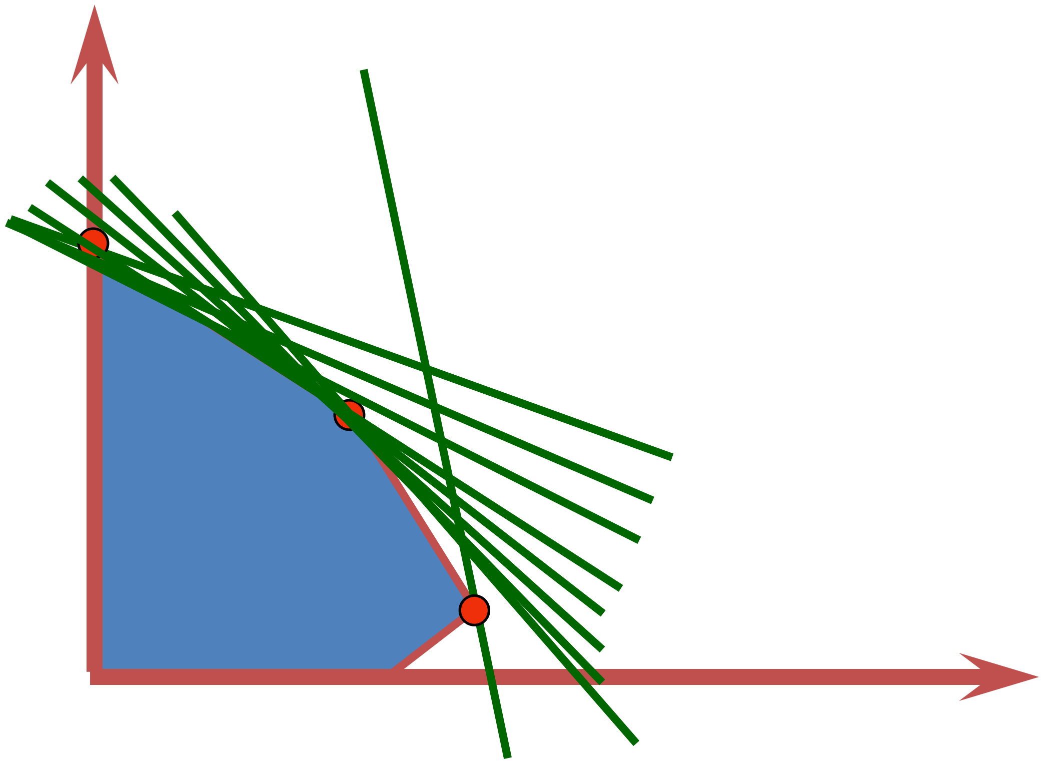
Zappers = 360 units

Profit = $4360

- This solution utilizes all the plastic and all the production hours.
- Total production is only 680 (not 700).
- Zappers exceed Space Rays.

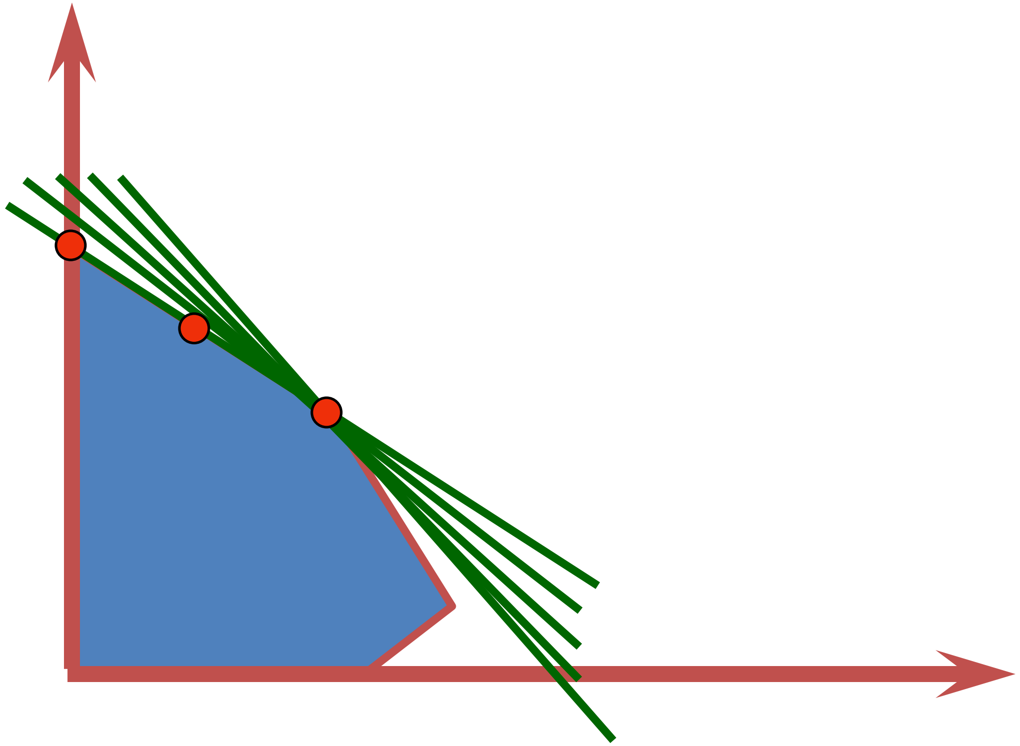
## extreme points and optimal solutions

If a linear programming problem has an optimal solution, an extreme point is optimal.



# multiple optimal solutions

For multiple optimal solutions to exist, the objective function must be parallel to one of the constraints



# Let’s look as doing this with AMPL

# basic AMPL commands

- **reset** : clears all information from memory
- **model** : loads a model from a file to memory
- **data** : loads data from file to memory
- **solve** : solves the current model in memory
- **option** : sets a variety of options
  - including the solver and info to pass to the solver
  - available solvers include CPLEX, Gurobi, Minos
- **display, show, printf, expand**
  - various commands for printing or display model information

# basic AMPL commands

- **var** : to declare decision variables
- **minimize** : to define objective function
- **maximize** : to define objective function
- **subject to** : to define constraint

**Free AMPL book:**
http://ampl.com/resources/the-ampl-book/

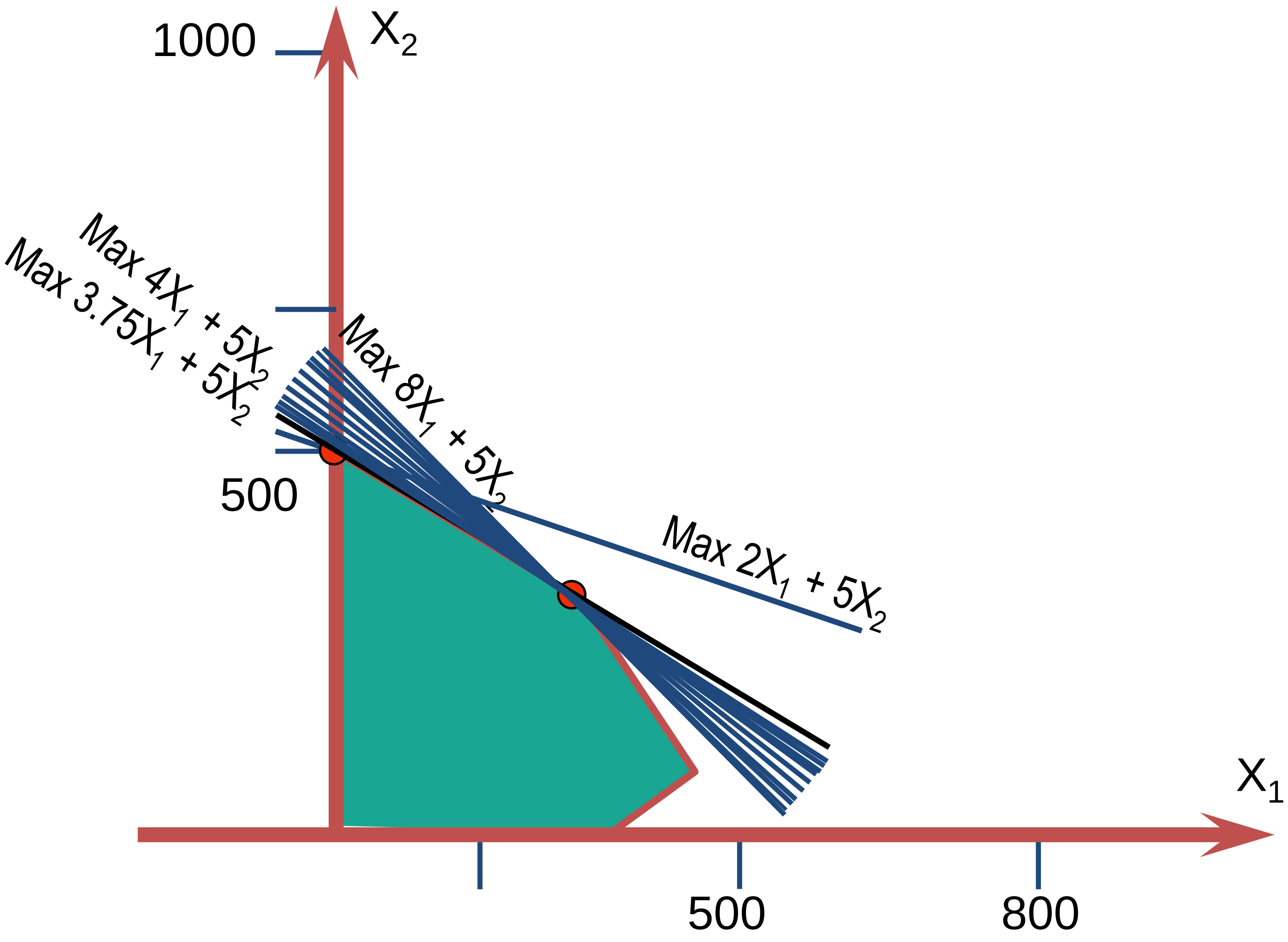
**See video lecture and files for code.**

# Is the optimal solution sensitive to changes in input parameters?

## sensitivity analysis of ***objective function coefficients***

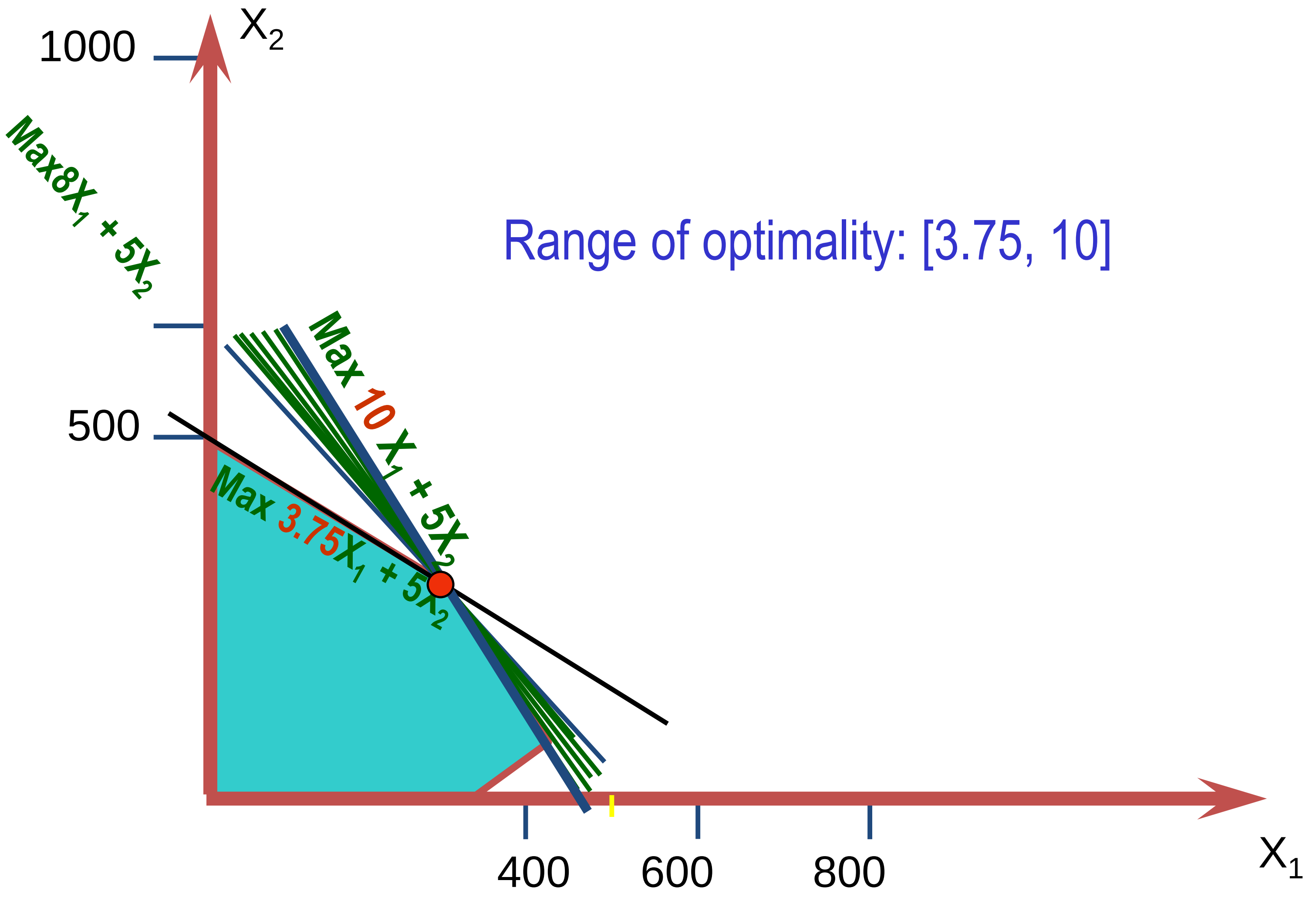
The *optimal solution* will remain unchanged as long as:

- An objective function coefficient lies within its *range of optimality*
- There are no changes in any other input parameters.



1000
$X_2$
Max $4X_1 + 5X_2$
Max $3.75X_1 + 5X_2$
Max $8X_1 + 5X_2$
500
Max $2X_1 + 5X_2$
$X_1$
500
800

Range of optimality: [3.75, 10]

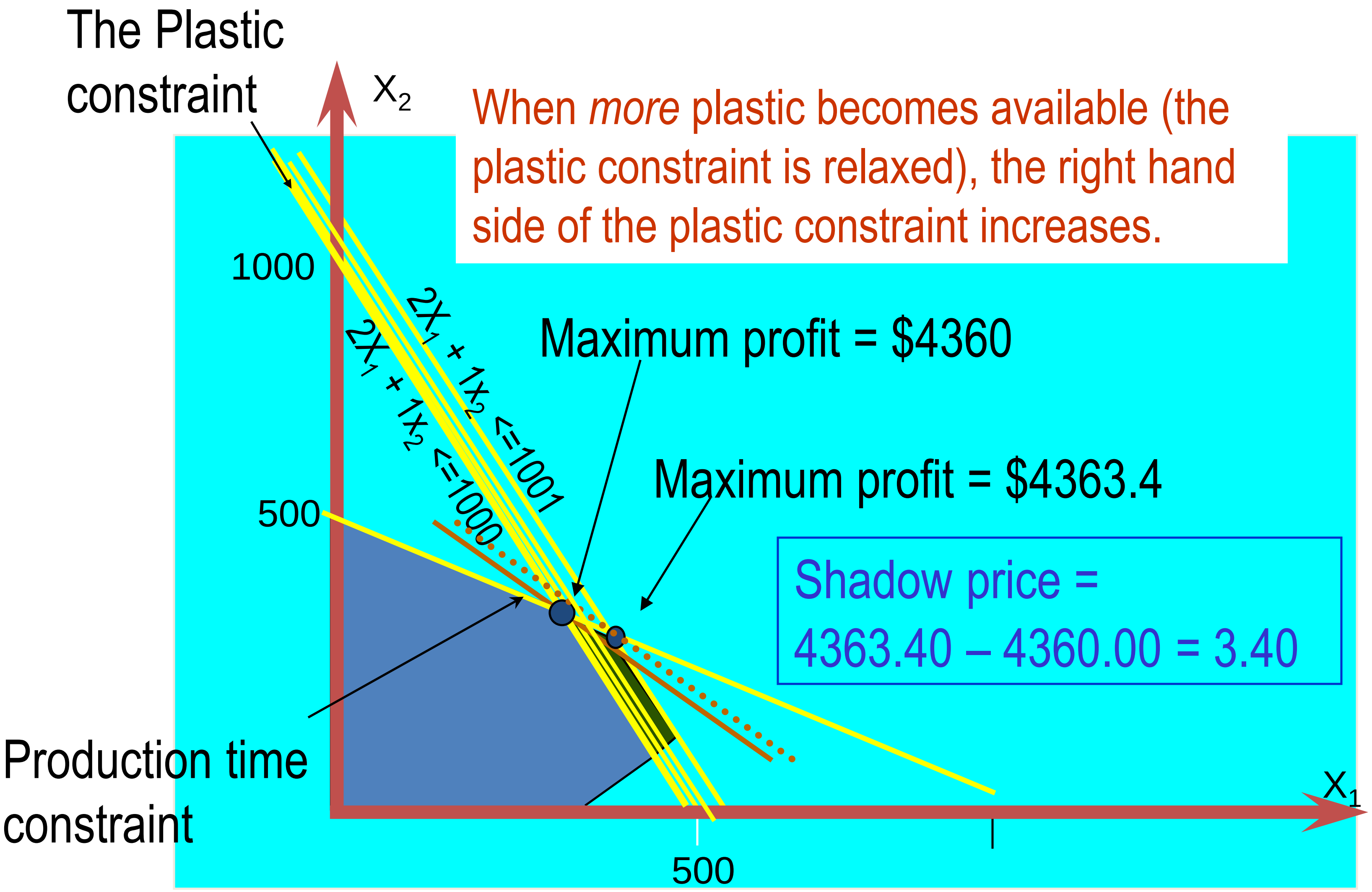


**sensitivity analysis of *right-hand side* values**

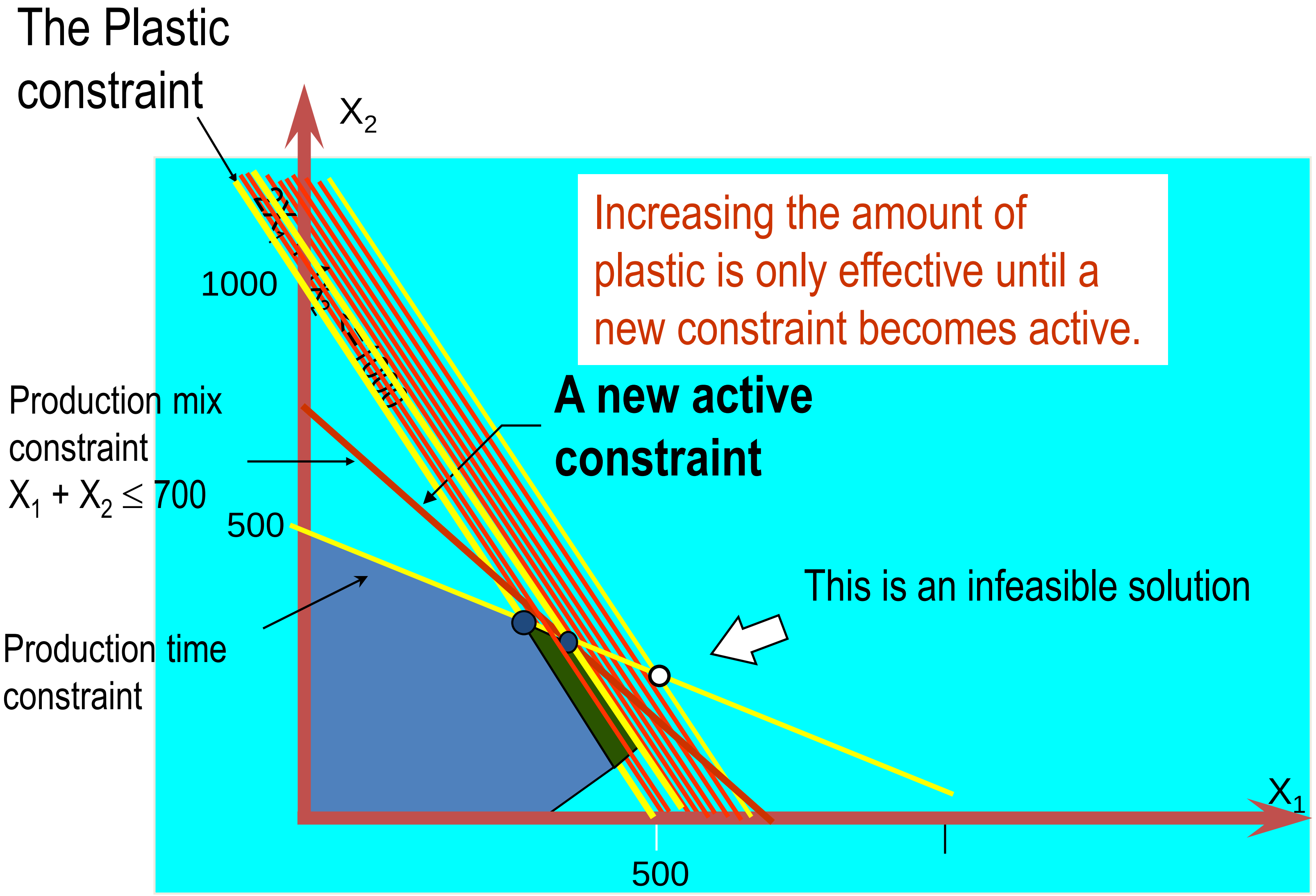
- Keeping all other factors the same, how much would the optimal value of the objective function change if the right-hand side of a constraint changed by one unit?

- For how many additional or fewer units will this per unit change be valid?

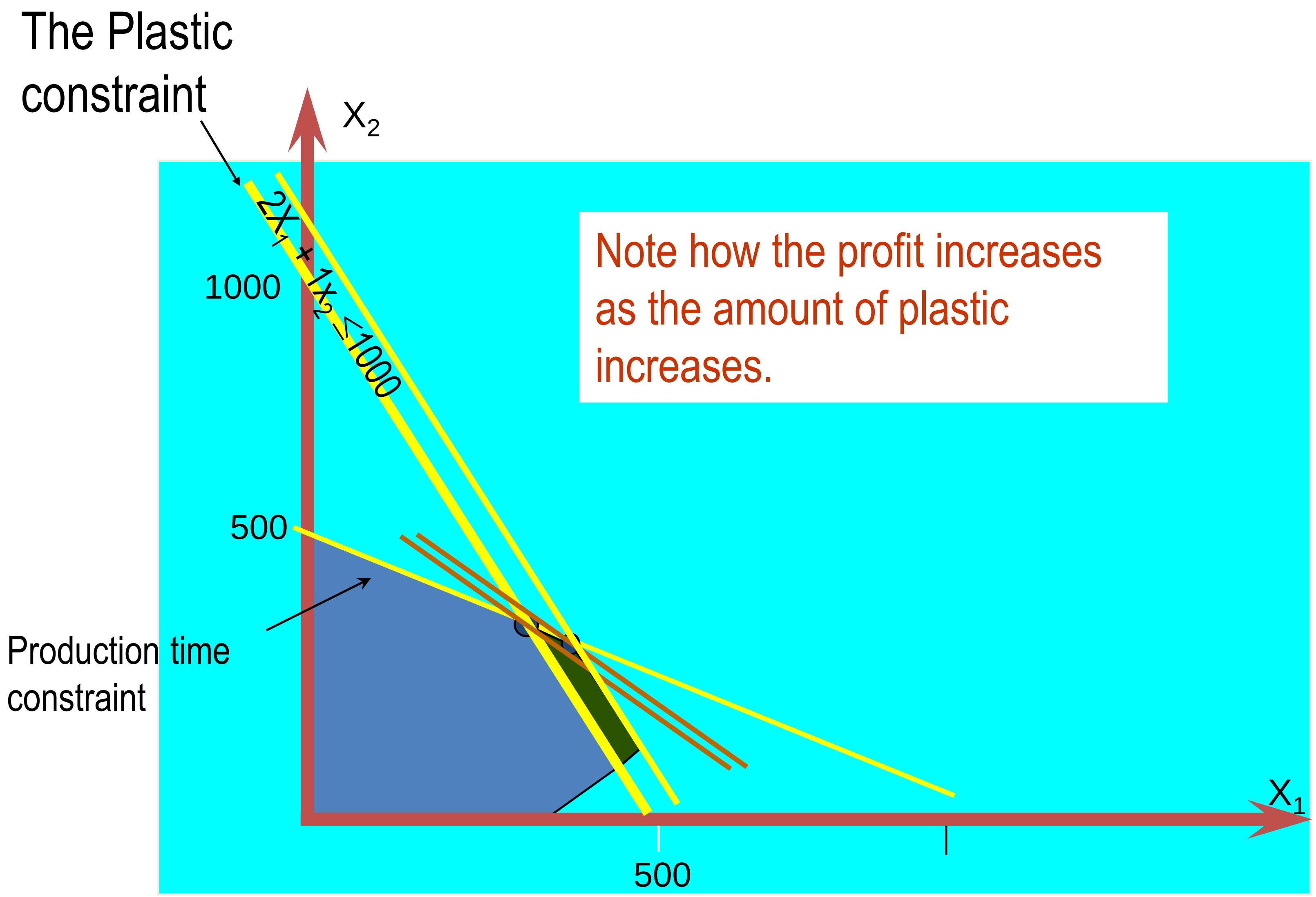
# Shadow Price – graphical demonstration



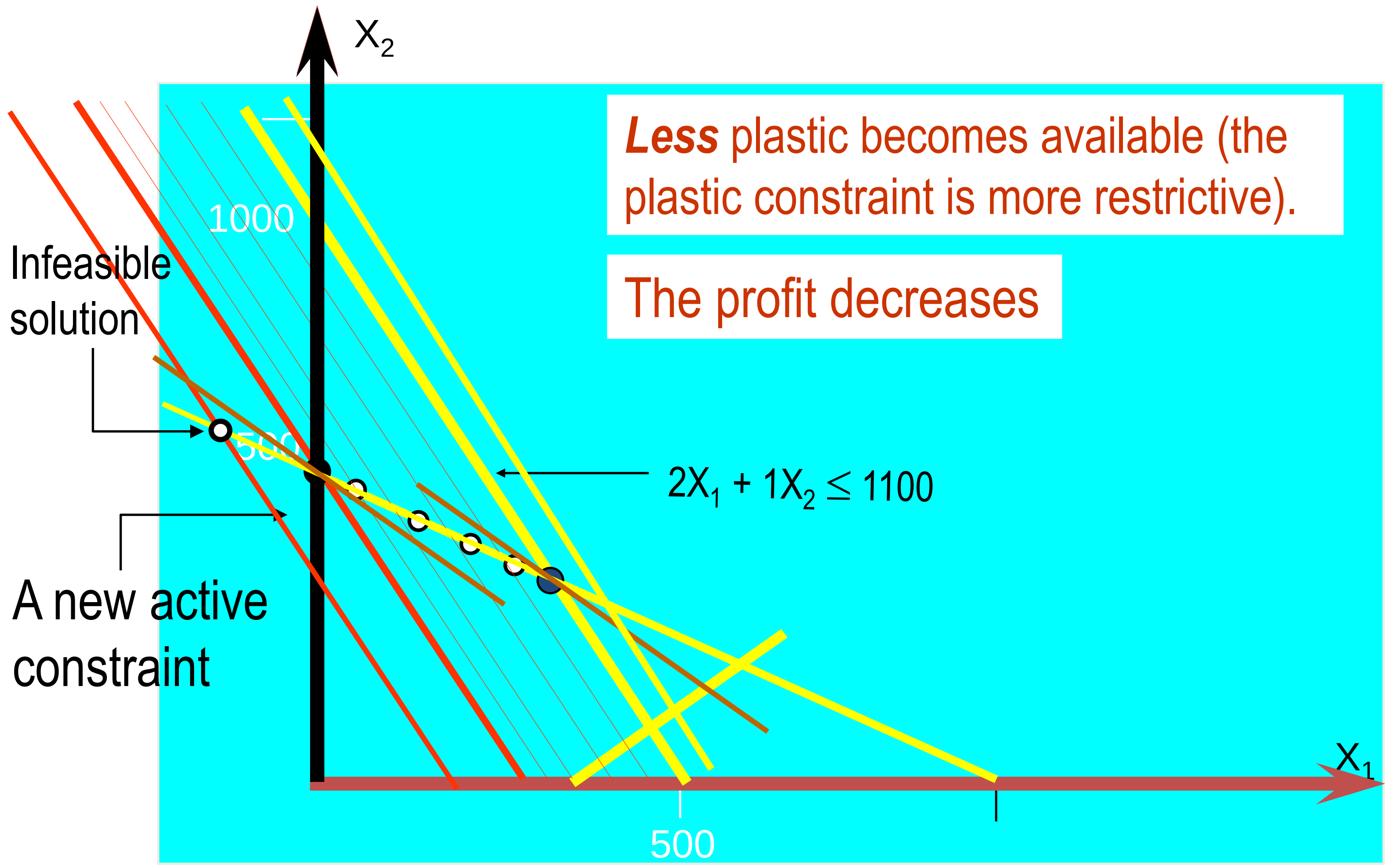
# Range of Feasibility



# Range of Feasibility



# Range of Feasibility



# basic AMPL commands for sensitivity analysis

**display <var name>.down; display <var name>.up : lower/upper endpoints of the range of feasibility of objective-function coefficient of variable**

**display <constr name> : shadow price of constraint**

**display <constr name>.down; display <constr name>.up : lower/upper endpoints of the range of feasibility of objective-function coefficient of variable**

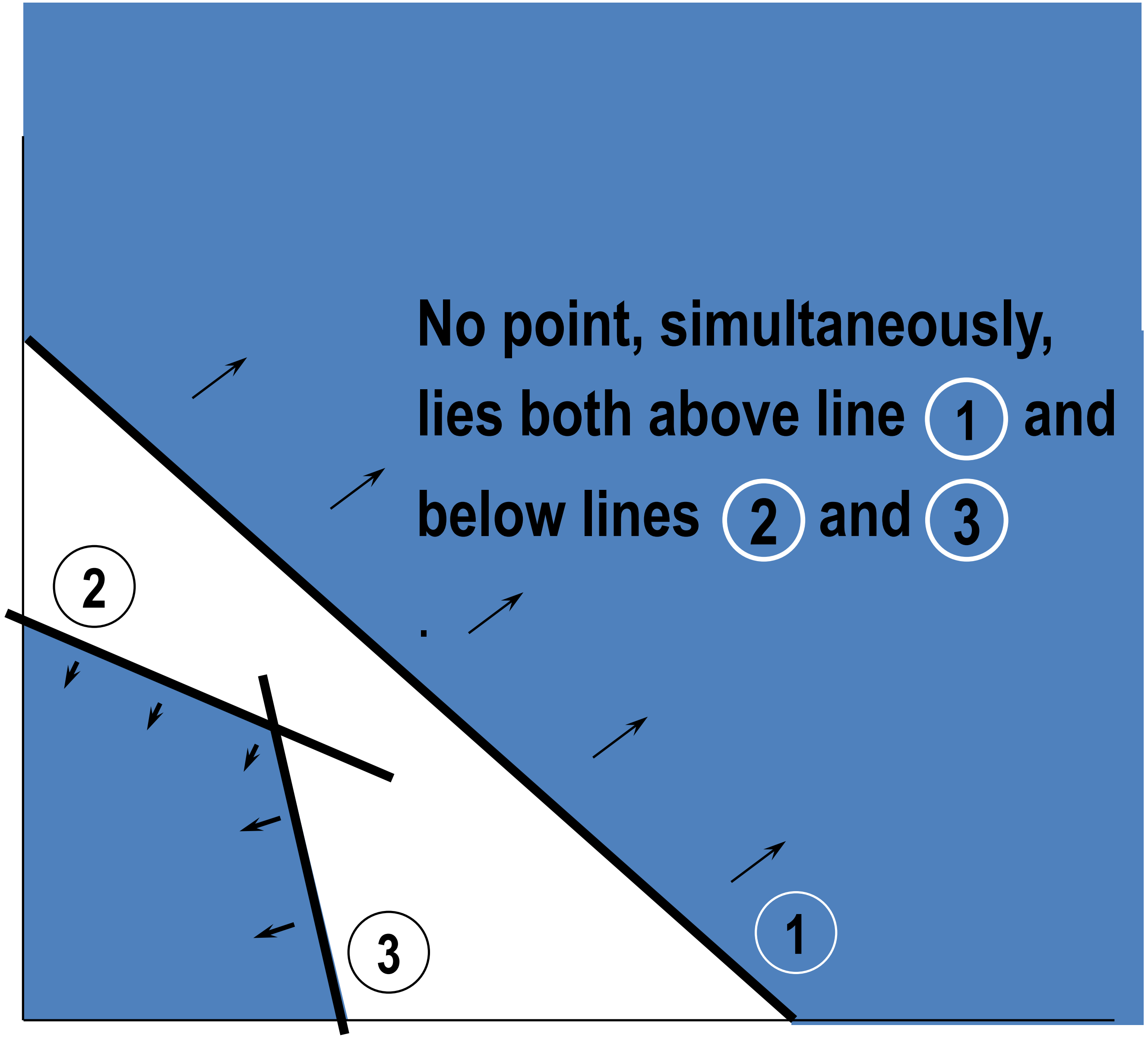
**display <constr name>.slack : slack in the constraint**

**display <var name>.rc : reduced cost of variable**

***<u>Infeasibility</u>*****:** Occurs when a model has no feasible point

***<u>Unboundness:</u>*** Occurs when the objective can become infinitely large (max), or infinitely small (min)

# Infeasible Model



# Unbounded solution