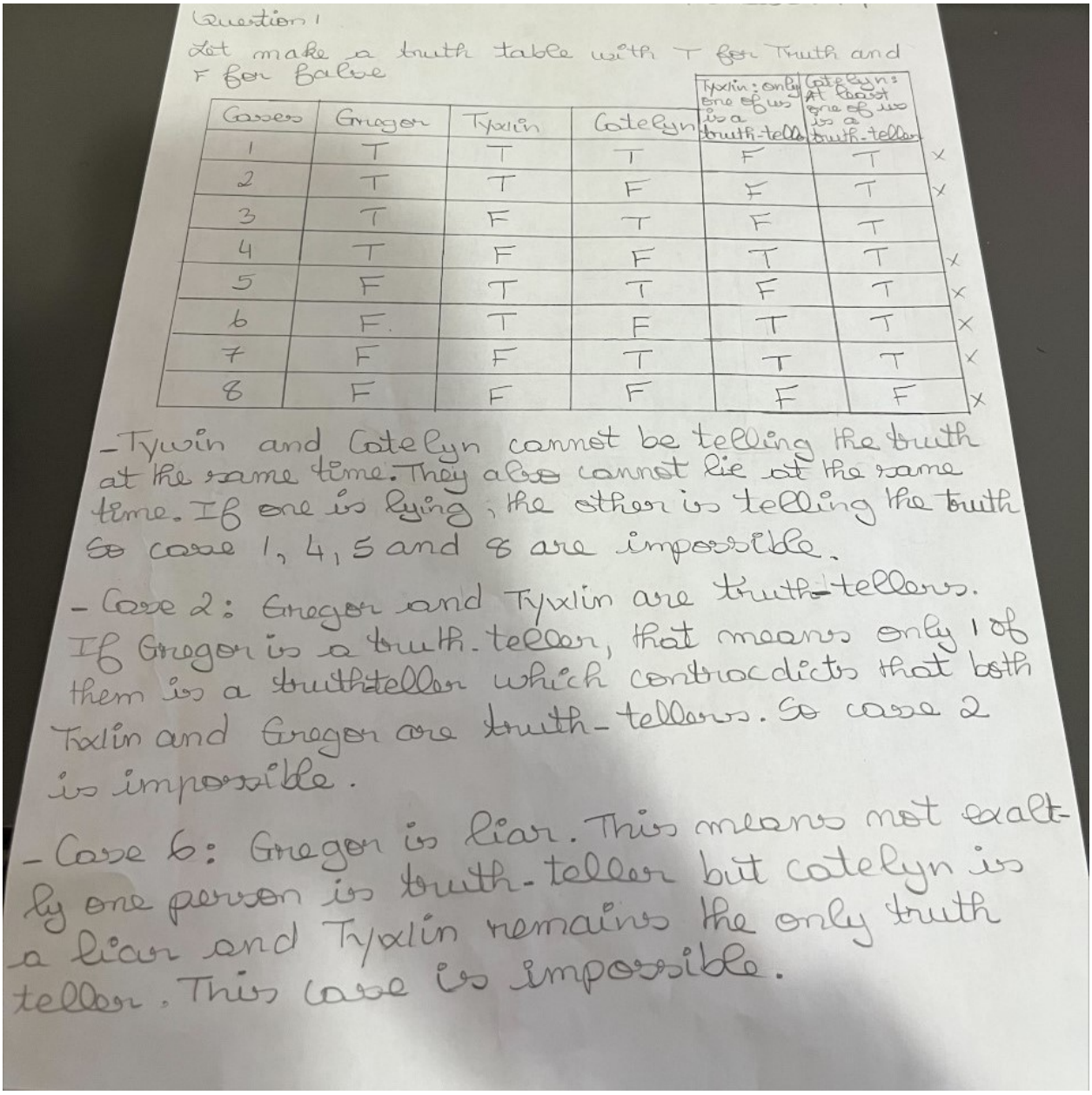
Homework 1 - Truth Tables & Linear Programming with AMPL

Daniel Carpenter and Effouehi Moody

January 2022

Contents

1 Problem 1



Question 1

Let make a truth table with T for Truth and F for False

Cases	Gregor	Tyxlin	Catelyn	Tyxlin: only one of us is a truth-teller	Catelyn: At least one of us is a truth-teller	
1	T	T	T	F	T	x
2	T	T	F	F	T	x
3	T	F	T	F	T	
4	T	F	F	T	T	x
5	F	T	T	F	T	x
6	F	T	F	T	T	x
7	F	F	T	T	T	x
8	F	F	F	F	F	x

- Tywin and Catelyn cannot be telling the truth at the same time. They also cannot lie at the same time. If one is lying, the other is telling the truth So case 1, 4, 5 and 8 are impossible.

- Case 2: Gregor and Tyxlin are truth-tellers. If Gregor is a truth-teller, that means only 1 of them is a truthteller which contradicts that both Txlin and Gregor are truth-tellers. So case 2 is impossible.

- Case 6: Gregor is liar. This means not exactly one person is truth-teller but catelyn is a liar and Tyxlin remains the only truth teller. This case is impossible.

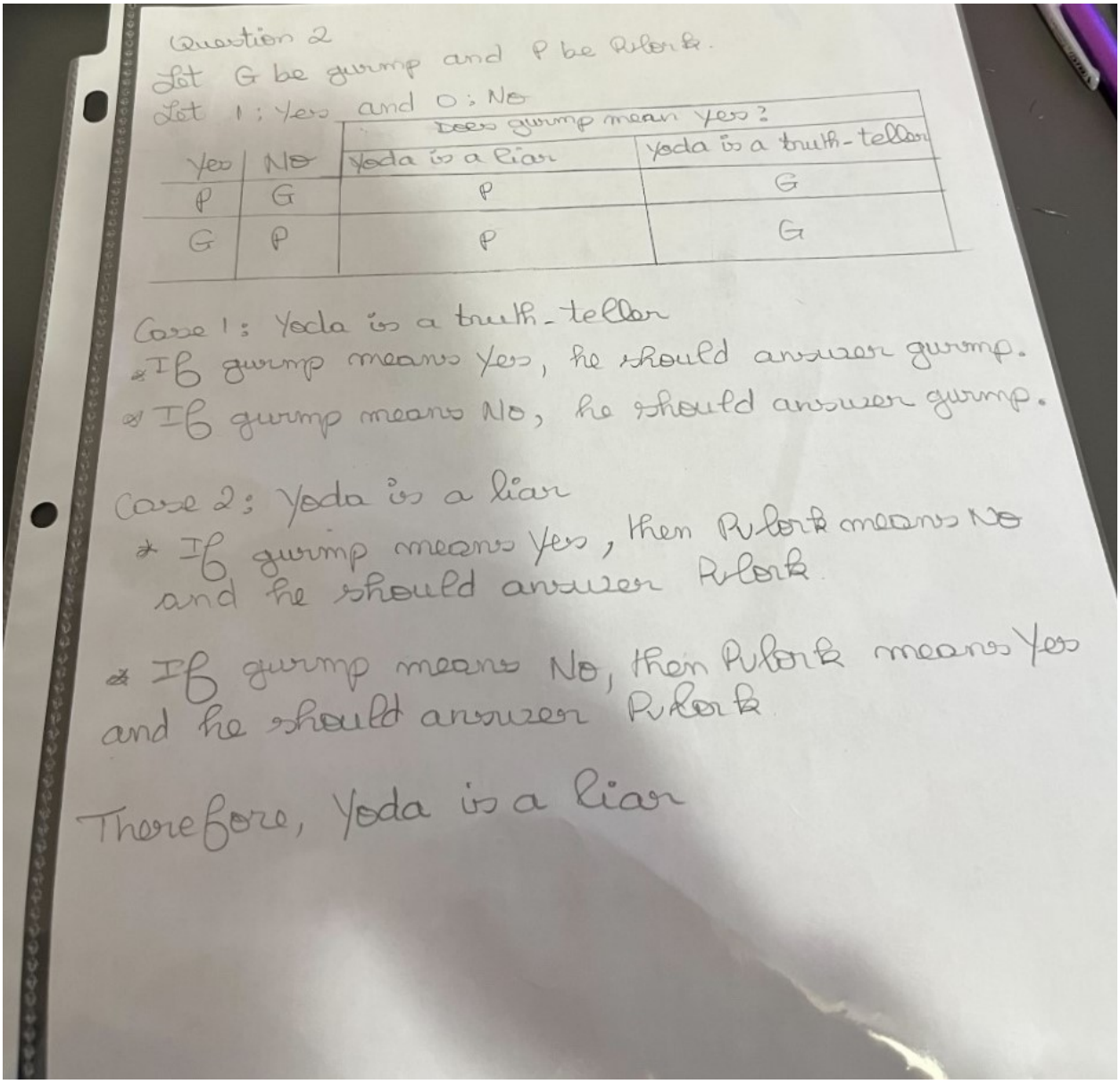
Question 1 (continued)

Case 7: Gregor is a liar. This means not exactly 1 person is truth-teller but Tyxlin is a liar and Catelyn remain the only truth-tellers. This case is also impossible.

Case 3: Tyxlin is a liar, that means Gregor didn't say "one of us is truth-teller", he probably said "2 of us are truth-tellers", then Gregor and Catelyn are truth-tellers and Tyxlin is a liar. This case is correct.

Therefore we can say that Gregor and Catelyn are truth-tellers and Tyxlin is a liar.

2 Problem 2



Question 2

Let G be gurmp and P be Pulorfk.

Let 1 : Yes and 0 : No

Yes	No	Does gurmp mean Yes?	
		Yoda is a liar	Yoda is a truth-teller
P	G	P	G
G	P	P	G

Case 1: Yoda is a truth-teller

* If gurmp means Yes, he should answer gurmp.
* If gurmp means No, he should answer gurmp.

Case 2: Yoda is a liar

* If gurmp means Yes, then Pulorfk means No and he should answer Pulorfk.
* If gurmp means No, then Pulorfk means Yes and he should answer Pulorfk.

Therefore, Yoda is a liar

3 Problem 3

3.1 Task a

3.1.1 Decision Variables

`bondA`: dollars $\in \mathbb{R}$ to invest in bond A
`bondB`: dollars $\in \mathbb{R}$ to invest in bond B
`bondC`: dollars $\in \mathbb{R}$ to invest in bond C
`bondD`: dollars $\in \mathbb{R}$ to invest in bond D
`bondE`: dollars $\in \mathbb{R}$ to invest in bond E

3.1.2 Objective Function

- Maximize the Expected Earnings of the portfolio

$$Maximize\ Z = (0.043 \times bondA) + (0.027 \times bondB) + (0.025 \times bondC) + (0.022 \times bondD) + (0.045 \times bondE)$$

3.1.3 Constraints

C1: Budget to invest is \$10 MM or less

$$budget : bondA + bondB + bondC + bondD + bondE \leq 10$$

C2: At least \$4 million must be invested in government and agency bonds

$$govtAndAgency : bondB + bondC + bondD \geq 4$$

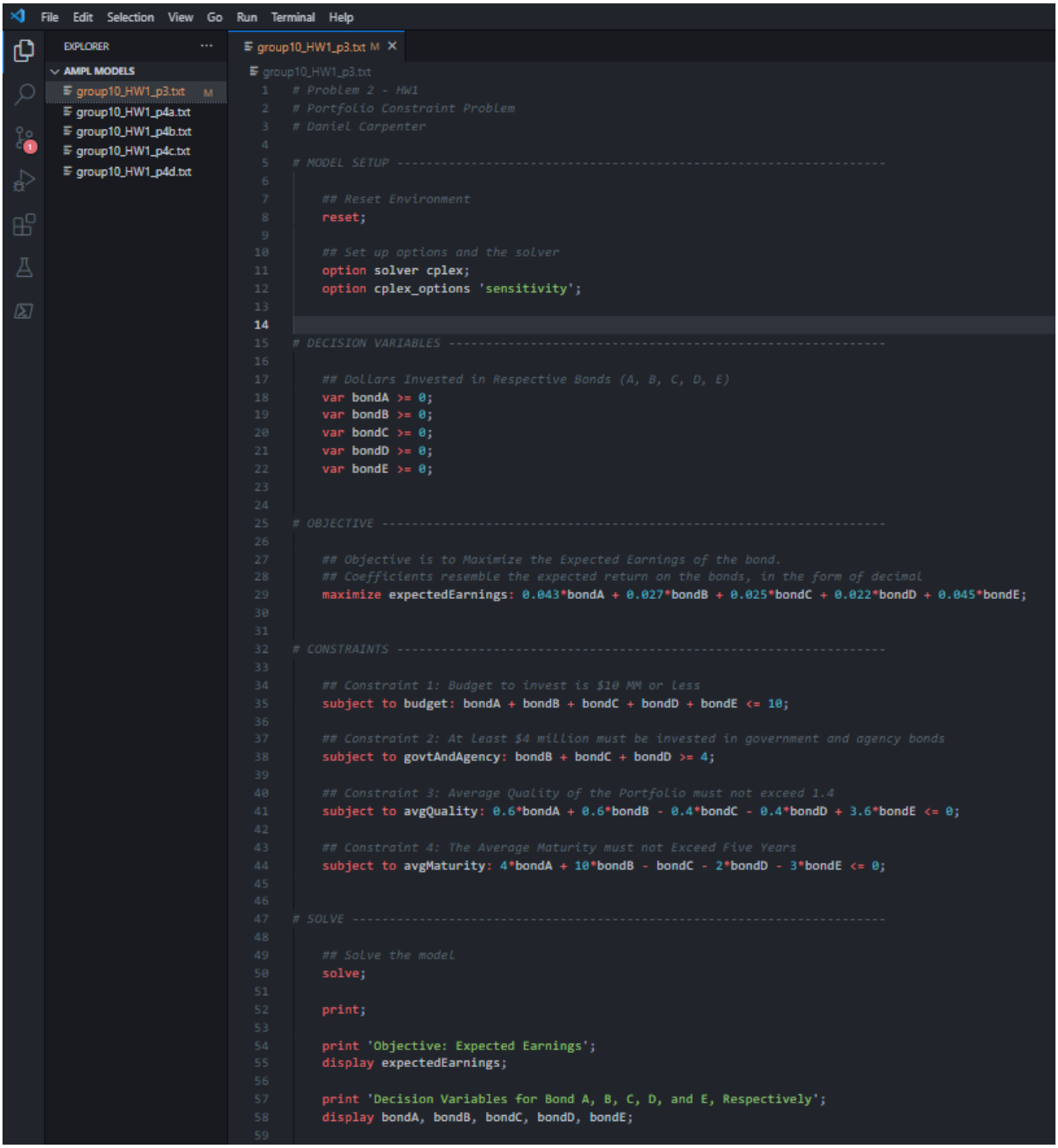
C3: Average Quality of the Portfolio must not exceed 1.4

$$avgQuality : (0.6 \times bondA) + (0.6 \times bondB) - (0.4 \times bondC) - (0.4 \times bondD) + (3.6 \times bondE) \leq 0$$

C4: The Average Maturity must not Exceed Five Years

$$avgMaturity : (4 \times bondA) + (10 \times bondB) - (1 \times bondC) - (2 \times bondD) - (3 \times bondE) \leq 0$$

3.1.4 Code


File Edit Selection View Go Run Terminal Help
EXPLORER
AMPL MODELS
group10_HW1_p3.txt M
group10_HW1_p4a.txt
group10_HW1_p4b.txt
group10_HW1_p4c.txt
group10_HW1_p4d.txt
group10_HW1_p3.txt M
group10_HW1_p3.txt
Problem 2 - HW1
Portfolio Constraint Problem
Daniel Carpenter
MODEL SETUP
Reset Environment
reset;
Set up options and the solver
option solver cplex;
option cplex_options 'sensitivity';
DECISION VARIABLES
Dollars Invested in Respective Bonds (A, B, C, D, E)
var bondA >= 0;
var bondB >= 0;
var bondC >= 0;
var bondD >= 0;
var bondE >= 0;
# OBJECTIVE
Objective is to Maximize the Expected Earnings of the bond.
Coefficients resemble the expected return on the bonds, in the form of decimal
maximize expectedEarnings: 0.043*bondA + 0.027*bondB + 0.025*bondC + 0.022*bondD + 0.045*bondE;
CONSTRAINTS
Constraint 1: Budget to invest is $10 MM or less
subject to budget: bondA + bondB + bondC + bondD + bondE <= 10;
Constraint 2: At least $4 million must be invested in government and agency bonds
subject to govtAndAgency: bondB + bondC + bondD >= 4;
Constraint 3: Average Quality of the Portfolio must not exceed 1.4
subject to avgQuality: 0.6*bondA + 0.6*bondB - 0.4*bondC - 0.4*bondD + 3.6*bondE <= 0;
Constraint 4: The Average Maturity must not Exceed Five Years
subject to avgMaturity: 4*bondA + 10*bondB - bondC - 2*bondD - 3*bondE <= 0;
# SOLVE
Solve the model
solve;
print;
print 'Objective: Expected Earnings';
display expectedEarnings;
print 'Decision Variables for Bond A, B, C, D, and E, Respectively';
display bondA, bondB, bondC, bondD, bondE;

3.1.5 Output

```
ampl: model group10_HW1_p3.txt
CPLEX 20.1.0.0: sensitivity
CPLEX 20.1.0.0: optimal solution; objective 0.2983636364
3 dual simplex iterations (1 in phase I)

suffix up OUT;
suffix down OUT;
suffix current OUT;

Objective: Expected Earnings
expectedEarnings = 0.298364

Decision Variables for Bond A, B, C, D, and E, Respectively
bondA = 2.18182
bondB = 0
bondC = 7.36364
bondD = 0
bondE = 0.454545
```

4 Problem `4`

4.1 Task `a`

4.1.1 Decision Variables

`tv` = the number of minutes $\in \mathbb{R}$ to air advertising on the *television* medium
`magazine` = the number of pages $\in \mathbb{I}$ to to advertise on the *magazine* medium

4.1.2 Objective Function

- Maximize the total audience reach

$$Maximize\ Z = (1.8 \times tv) + (1.0 \times magazine)$$

4.1.3 Constraints

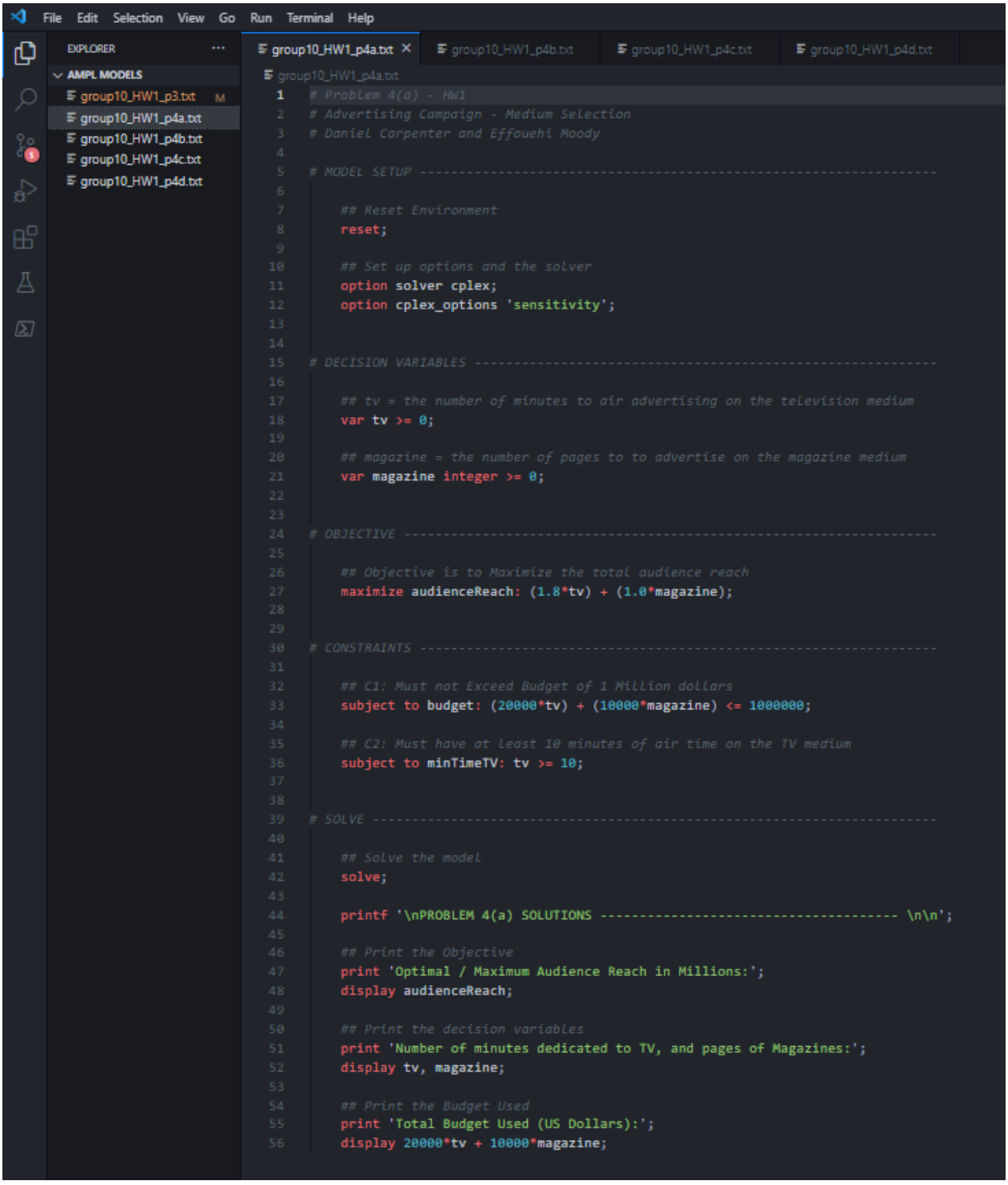
C1: Must not Exceed Budget of 1 Million dollars

$$budget : (20,000 \times tv) + (10,000 \times magazine) \leq 1,000,000$$

C2: Must have at least 10 minutes of air time on the TV medium

$$minTimeTV : tv \geq 10$$

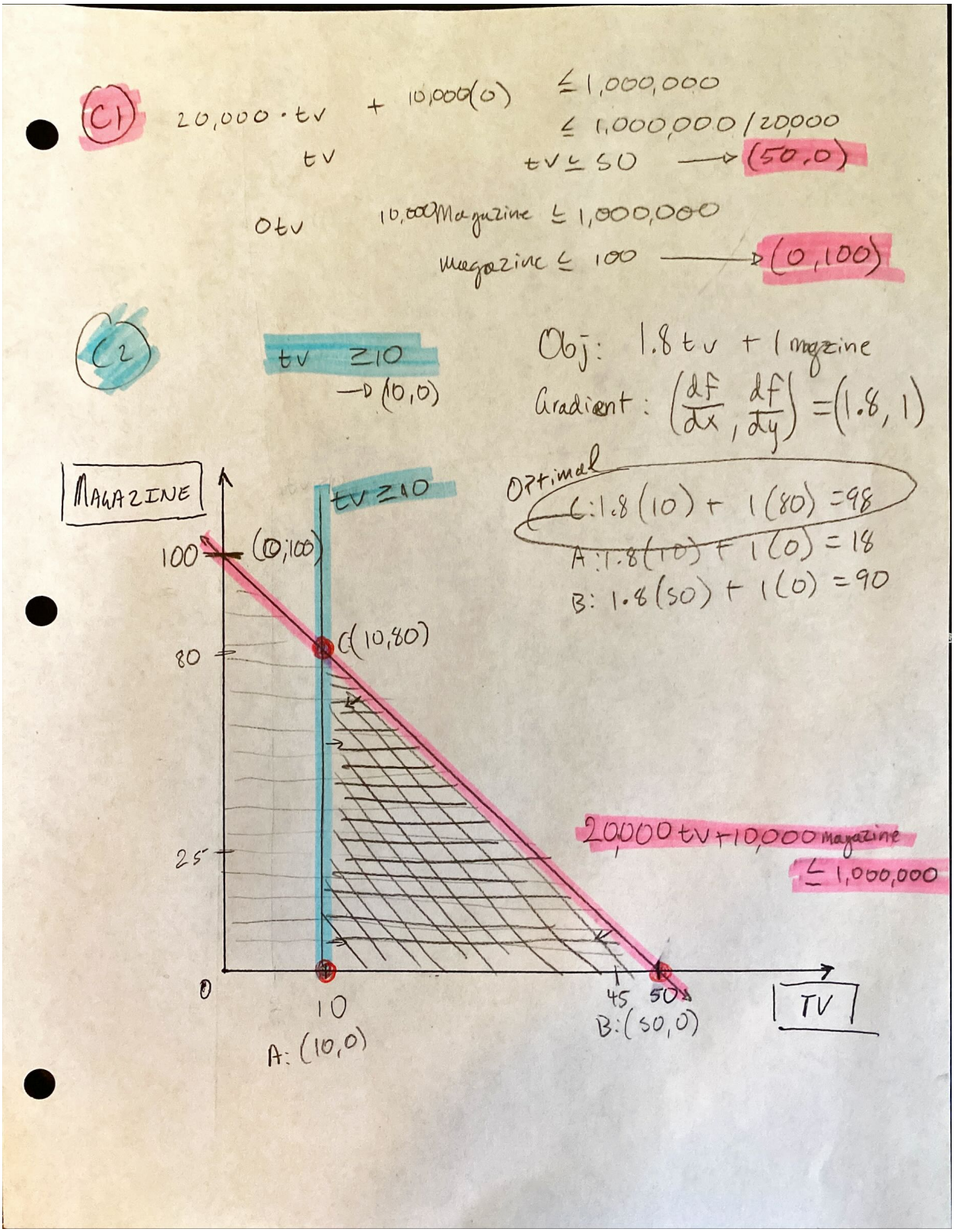
4.1.4 Code


File Edit Selection View Go Run Terminal Help
EXPLORER
AMPL MODELS
group10_HW1_p3.txt M
group10_HW1_p4a.txt
group10_HW1_p4b.txt
group10_HW1_p4c.txt
group10_HW1_p4d.txt
group10_HW1_p4a.txt
group10_HW1_p4b.txt
group10_HW1_p4c.txt
group10_HW1_p4d.txt
Problem 4(a) - HW1
Advertising Campaign - Medium Selection
Daniel Carpenter and Effouehi Moody
MODEL SETUP
Reset Environment
reset;
Set up options and the solver
option solver cplex;
option cplex_options 'sensitivity';
DECISION VARIABLES
tv = the number of minutes to air advertising on the television medium
var tv >= 0;
magazine = the number of pages to to advertise on the magazine medium
var magazine integer >= 0;
# OBJECTIVE
Objective is to Maximize the total audience reach
maximize audienceReach: (1.8*tv) + (1.0*magazine);
CONSTRAINTS
C1: Must not Exceed Budget of 1 Million dollars
subject to budget: (20000*tv) + (10000*magazine) <= 1000000;
C2: Must have at least 10 minutes of air time on the TV medium
subject to minTimeTV: tv >= 10;
# SOLVE
Solve the model
solve;
printf '\nPROBLEM 4(a) SOLUTIONS ---------- \n\n';
Print the Objective
print 'Optimal / Maximum Audience Reach in Millions:';
display audienceReach;
Print the decision variables
print 'Number of minutes dedicated to TV, and pages of Magazines:';
display tv, magazine;
Print the Budget Used
print 'Total Budget Used (US Dollars):';
display 20000*tv + 10000*magazine;

4.1.5 Output

```
ampl: model group10_HW1_p4a.txt
CPLEX 20.1.0.0: sensitivity
CPLEX 20.1.0.0: optimal solution; objective 98
0 dual simplex iterations (0 in phase I)

suffix up OUT;
suffix down OUT;
suffix current OUT;

PROBLEM 4(a) SOLUTIONS ---------------------------------------

Optimal / Maximum Audience Reach in Millions:
audienceReach = 98

Number of minutes dedicated to TV, and pages of Magazines:
tv = 10
magazine = 80

Total Budget Used (US Dollars):
20000*tv + 10000*magazine = 1e+06
```

4.1.6 Solving Problem 4(a) Graphically by Hand



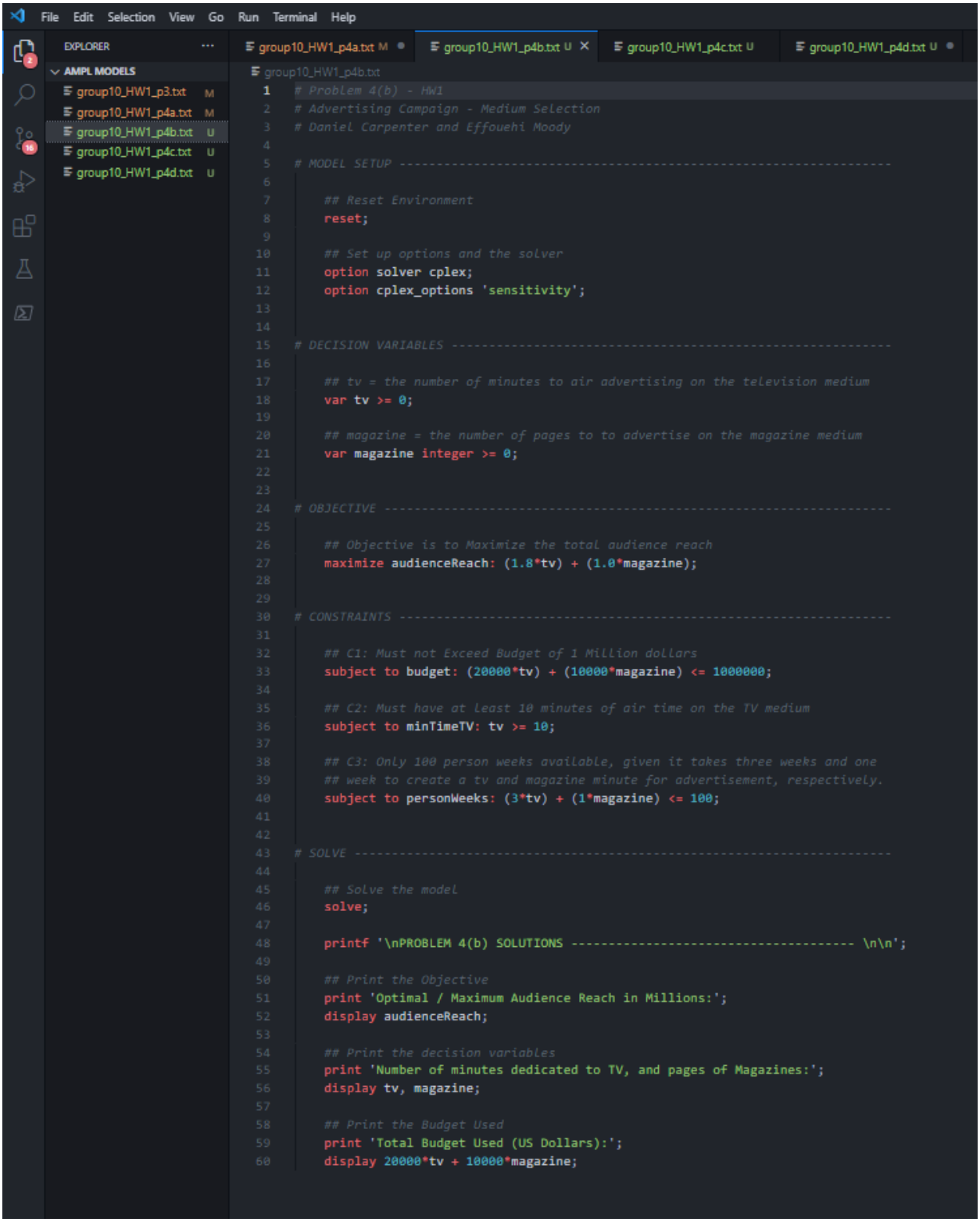
4.2 Task `b`

4.2.1 Additional Constraint: Labor Time

C3: Only 100 person weeks available, given it takes three weeks and one week to create a `tv` and `magazine` minute for advertisement, respectively.

$$personWeeks : (3 \times tv) + (1 \times magazine) \leq 100$$

4.2.2 Code


File Edit Selection View Go Run Terminal Help
EXPLORER
AMPL MODELS
group10_HW1_p3.txt M
group10_HW1_p4a.txt M
group10_HW1_p4b.txt U
group10_HW1_p4c.txt U
group10_HW1_p4d.txt U
group10_HW1_p4a.txt M
group10_HW1_p4b.txt U
group10_HW1_p4c.txt U
group10_HW1_p4d.txt U
group10_HW1_p4b.txt
Problem 4(b) - HW1
Advertising Campaign - Medium Selection
Daniel Carpenter and Effouehi Moody
MODEL SETUP
Reset Environment
reset;
Set up options and the solver
option solver cplex;
option cplex_options 'sensitivity';
DECISION VARIABLES
tv = the number of minutes to air advertising on the television medium
var tv >= 0;
magazine = the number of pages to to advertise on the magazine medium
var magazine integer >= 0;
# OBJECTIVE
Objective is to Maximize the total audience reach
maximize audienceReach: (1.8*tv) + (1.0*magazine);
CONSTRAINTS
C1: Must not Exceed Budget of 1 Million dollars
subject to budget: (20000*tv) + (10000*magazine) <= 1000000;
C2: Must have at least 10 minutes of air time on the TV medium
subject to minTimeTV: tv >= 10;
C3: Only 100 person weeks available, given it takes three weeks and one
week to create a tv and magazine minute for advertisement, respectively.
subject to personWeeks: (3*tv) + (1*magazine) <= 100;
# SOLVE
Solve the model
solve;
printf '\nPROBLEM 4(b) SOLUTIONS ------------------------------------ \n\n';
Print the Objective
print 'Optimal / Maximum Audience Reach in Millions:';
display audienceReach;
Print the decision variables
print 'Number of minutes dedicated to TV, and pages of Magazines:';
display tv, magazine;
Print the Budget Used
print 'Total Budget Used (US Dollars):';
display 20000*tv + 10000*magazine;

4.2.3 Output

```
ampl: model group10_HW1_p4b.txt
CPLEX 20.1.0.0: sensitivity
CPLEX 20.1.0.0: optimal solution; objective 88
1 dual simplex iterations (1 in phase I)

suffix up OUT;
suffix down OUT;
suffix current OUT;

PROBLEM 4(b) SOLUTIONS ---------------------------------------

Optimal / Maximum Audience Reach in Millions:
audienceReach = 88

Number of minutes dedicated to TV, and pages of Magazines:
tv = 10
magazine = 70

Total Budget Used (US Dollars):
20000*tv + 10000*magazine = 9e+05
```

4.3 Task `c`

4.3.1 Additional Constraint: Radio Advertising Medium

4.3.2 Decision Variables

`tv` = the number of minutes $\in \mathbb{R}$ to air advertising on the *television* medium
`magazine` = the number of pages $\in \mathbb{I}$ to to advertise on the *magazine* medium `radio` = the number of minutes $\in \mathbb{R}$ to air advertising on the *radio* medium

4.3.3 Objective Function

- Maximize the total audience reach

$$Maximize\ Z = (1.80 \times tv) + (1.00 \times magazine) + (0.25 \times radio)$$

4.3.4 New Constraints

C1: Must not Exceed Budget of 1 Million dollars

$$budget : (20,000 \times tv) + (10,000 \times magazine) + (2,000 \times radio) \leq 1,000,000$$

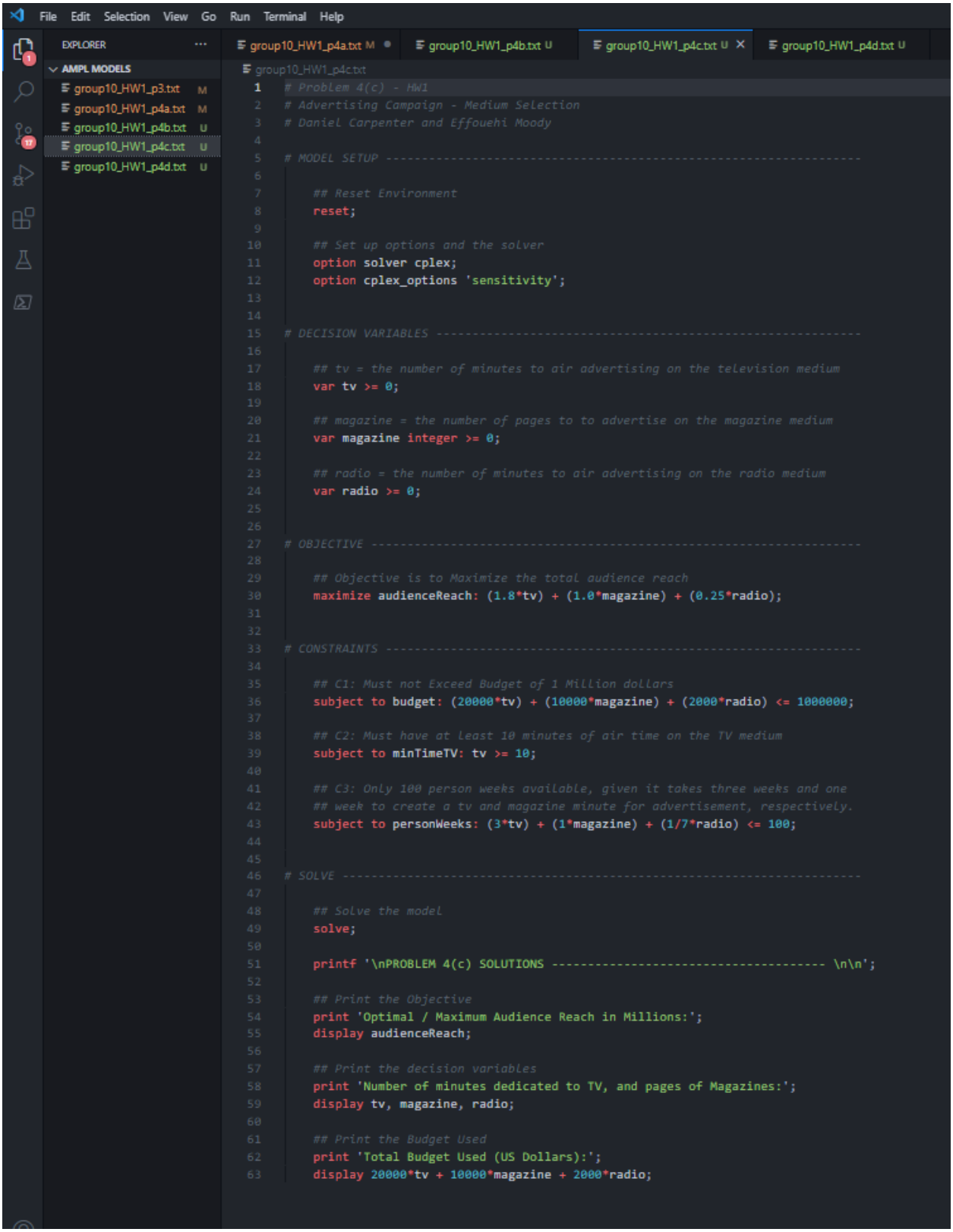
C2: Must have at least 10 minutes of air time on the TV medium

$$minTimeTV : tv \geq 10$$

C3: Only 100 person weeks available, given it takes three weeks and one week to create a `tv` and `magazine` minute for advertisement, respectively. It only takes one day for `radio`.

$$personWeeks : (3 \times tv) + (1 \times magazine) + (\frac{1}{7} \times radio) \leq 100$$

4.3.5 Code


File Edit Selection View Go Run Terminal Help
EXPLORER
AMPL MODELS
group10_HW1_p3.txt M
group10_HW1_p4a.txt M
group10_HW1_p4b.txt U
group10_HW1_p4c.txt U
group10_HW1_p4d.txt U
group10_HW1_p4a.txt M
group10_HW1_p4b.txt U
group10_HW1_p4c.txt U
group10_HW1_p4d.txt U
group10_HW1_p4c.txt
Problem 4(c) - HW1
Advertising Campaign - Medium Selection
Daniel Carpenter and Effouehi Moody
MODEL SETUP
Reset Environment
reset;
Set up options and the solver
option solver cplex;
option cplex_options 'sensitivity';
DECISION VARIABLES
tv = the number of minutes to air advertising on the television medium
var tv >= 0;
magazine = the number of pages to to advertise on the magazine medium
var magazine integer >= 0;
radio = the number of minutes to air advertising on the radio medium
var radio >= 0;
# OBJECTIVE
Objective is to Maximize the total audience reach
maximize audienceReach: (1.8*tv) + (1.0*magazine) + (0.25*radio);
CONSTRAINTS
C1: Must not Exceed Budget of 1 Million dollars
subject to budget: (20000*tv) + (10000*magazine) + (2000*radio) <= 1000000;
C2: Must have at least 10 minutes of air time on the TV medium
subject to minTimeTV: tv >= 10;
C3: Only 100 person weeks available, given it takes three weeks and one
week to create a tv and magazine minute for advertisement, respectively.
subject to personWeeks: (3*tv) + (1*magazine) + (1/7*radio) <= 100;
# SOLVE
Solve the model
solve;
printf '\nPROBLEM 4(c) SOLUTIONS ---------- \n\n';
Print the Objective
print 'Optimal / Maximum Audience Reach in Millions:';
display audienceReach;
Print the decision variables
print 'Number of minutes dedicated to TV, and pages of Magazines:';
display tv, magazine, radio;
Print the Budget Used
print 'Total Budget Used (US Dollars):';
display 20000*tv + 10000*magazine + 2000*radio;

4.3.6 Output

```
ampl: model group10_HW1_p4c.txt
CPLEX 20.1.0.0: sensitivity
CPLEX 20.1.0.0: optimal solution; objective 118
1 dual simplex iterations (1 in phase I)

suffix up OUT;
suffix down OUT;
suffix current OUT;

PROBLEM 4(c) SOLUTIONS ---------------------------------------

Optimal / Maximum Audience Reach in Millions:
audienceReach = 118

Number of minutes dedicated to TV, and pages of Magazines:
tv = 10
magazine = 0
radio = 400

Total Budget Used (US Dollars):
20000*tv + 10000*magazine + 2000*radio = 1e+06
```

4.4 Task `d`

4.4.1 Additional Constraints: Miminum Magazine and Maximum Radio Requirements

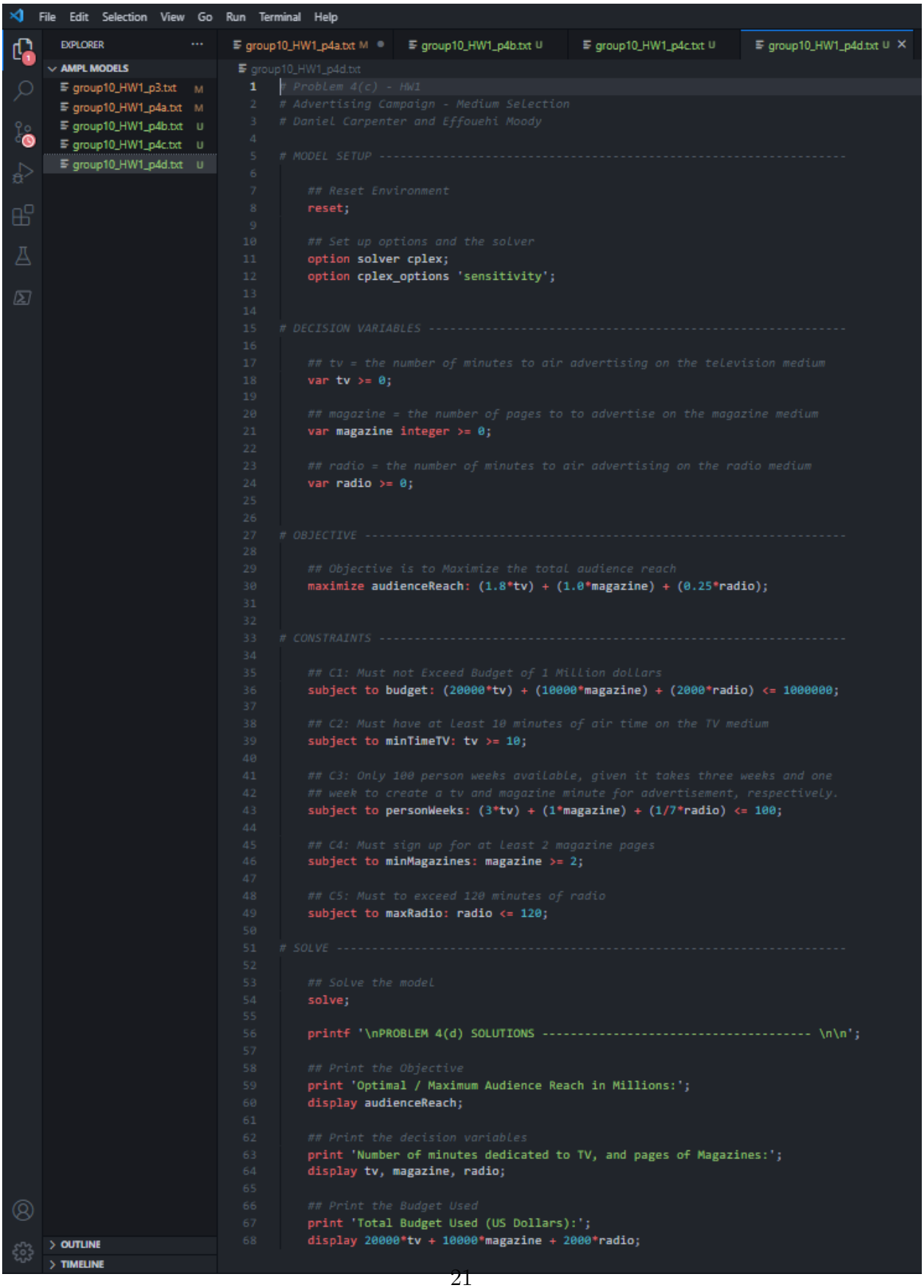
C4: Must sign up for at least 2 magazine pages

$$minMagazines : magazine \geq 2$$

C5: Must to exceed 120 minutes of radio

$$maxRadio : radio \leq 120$$

4.4.2 Code


File Edit Selection View Go Run Terminal Help
EXPLORER
AMPL MODELS
group10_HW1_p3.txt M
group10_HW1_p4a.txt M
group10_HW1_p4b.txt U
group10_HW1_p4c.txt U
group10_HW1_p4d.txt U
group10_HW1_p4a.txt M
group10_HW1_p4b.txt U
group10_HW1_p4c.txt U
group10_HW1_p4d.txt U
group10_HW1_p4d.txt
Problem 4(c) - HW1
Advertising Campaign - Medium Selection
Daniel Carpenter and Effouehi Moody
MODEL SETUP
Reset Environment
reset;
Set up options and the solver
option solver cplex;
option cplex_options 'sensitivity';
DECISION VARIABLES
tv = the number of minutes to air advertising on the television medium
var tv >= 0;
magazine = the number of pages to to advertise on the magazine medium
var magazine integer >= 0;
radio = the number of minutes to air advertising on the radio medium
var radio >= 0;
# OBJECTIVE
Objective is to Maximize the total audience reach
maximize audienceReach: (1.8*tv) + (1.0*magazine) + (0.25*radio);
CONSTRAINTS
C1: Must not Exceed Budget of 1 Million dollars
subject to budget: (20000*tv) + (10000*magazine) + (2000*radio) <= 1000000;
C2: Must have at least 10 minutes of air time on the TV medium
subject to minTimeTV: tv >= 10;
C3: Only 100 person weeks available, given it takes three weeks and one
week to create a tv and magazine minute for advertisement, respectively.
subject to personWeeks: (3*tv) + (1*magazine) + (1/7*radio) <= 100;
C4: Must sign up for at least 2 magazine pages
subject to minMagazines: magazine >= 2;
C5: Must to exceed 120 minutes of radio
subject to maxRadio: radio <= 120;
# SOLVE
Solve the model
solve;
printf '\nPROBLEM 4(d) SOLUTIONS ------------------------------------ \n\n';
Print the Objective
print 'Optimal / Maximum Audience Reach in Millions:';
display audienceReach;
Print the decision variables
print 'Number of minutes dedicated to TV, and pages of Magazines:';
display tv, magazine, radio;
Print the Budget Used
print 'Total Budget Used (US Dollars):';
display 20000*tv + 10000*magazine + 2000*radio;
OUTLINE
TIMELINE

4.4.3 Output

```
ampl: model group10_HW1_p4d.txt
CPLEX 20.1.0.0: sensitivity
CPLEX 20.1.0.0: optimal integer solution; objective 100.75
1 MIP simplex iterations
0 branch-and-bound nodes

suffix up OUT;
suffix down OUT;
suffix current OUT;

PROBLEM 4(d) SOLUTIONS ---------------------------------------

Optimal / Maximum Audience Reach in Millions:
audienceReach = 100.75

Number of minutes dedicated to TV, and pages of Magazines:
tv = 10
magazine = 53
radio = 119

Total Budget Used (US Dollars):
20000*tv + 10000*magazine + 2000*radio = 968000
```

5 Problem 5

5.1 Base Mathematical Formulation and Code

- *Each task shows a separate change to the base model. Therefore, each change should not accumulate.*

5.1.1 Mathematical Formulation

Given: P, a set of products
a_j = tons per hour of product j, for each $j \in P$
b = hours available at the mill
c_j = profit per ton of product j, for each $j \in P$
u_j = maximum tons of product j, for each $j \in P$

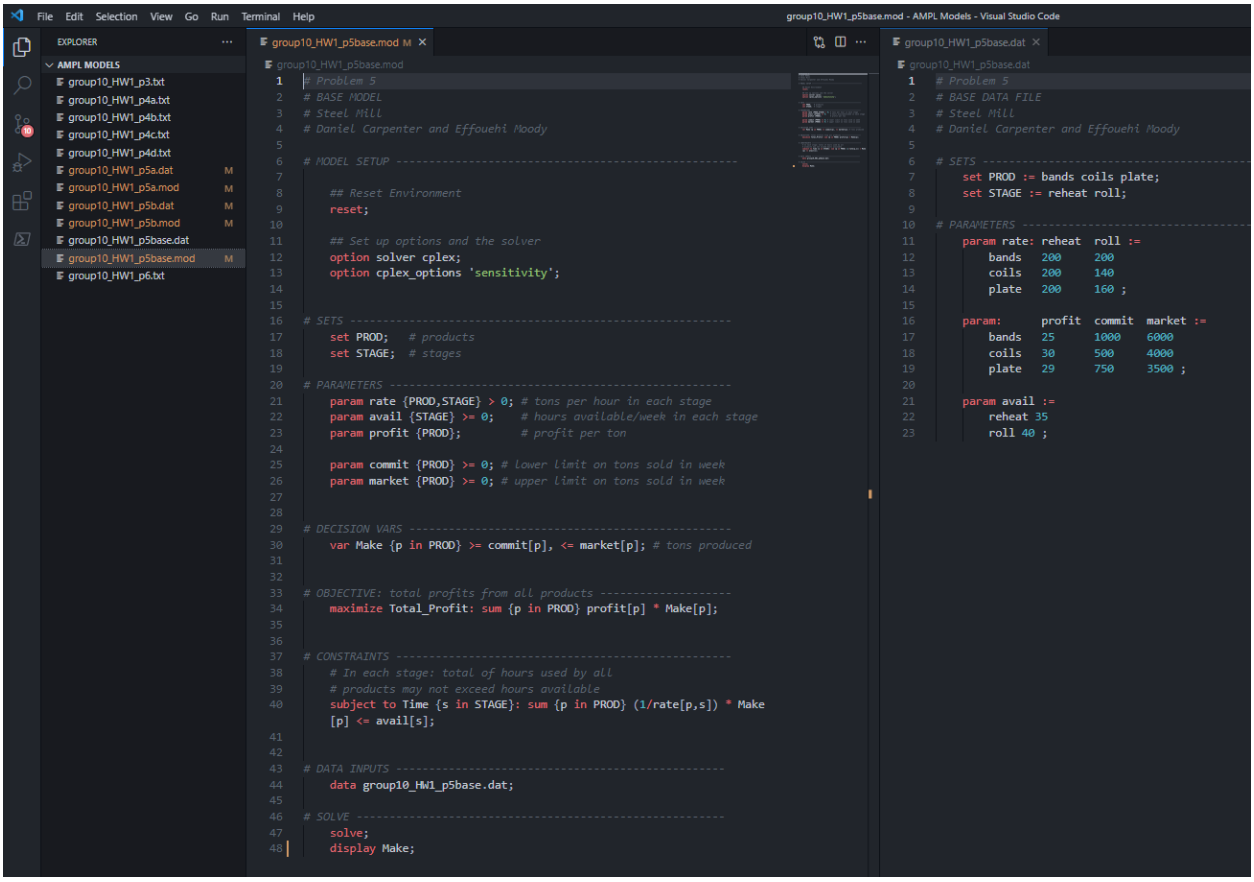
Define variables: X_j = tons of product j to be made, for each $j \in P$

Maximize: $\sum_{j \in P} c_j X_j$

Subject to: $\sum_{j \in P} (1/a_j) X_j \leq b$

$0 \leq X_j \leq u_j$, for each $j \in P$

5.1.2 Code for Model `.mod` and Input Data `.dat`

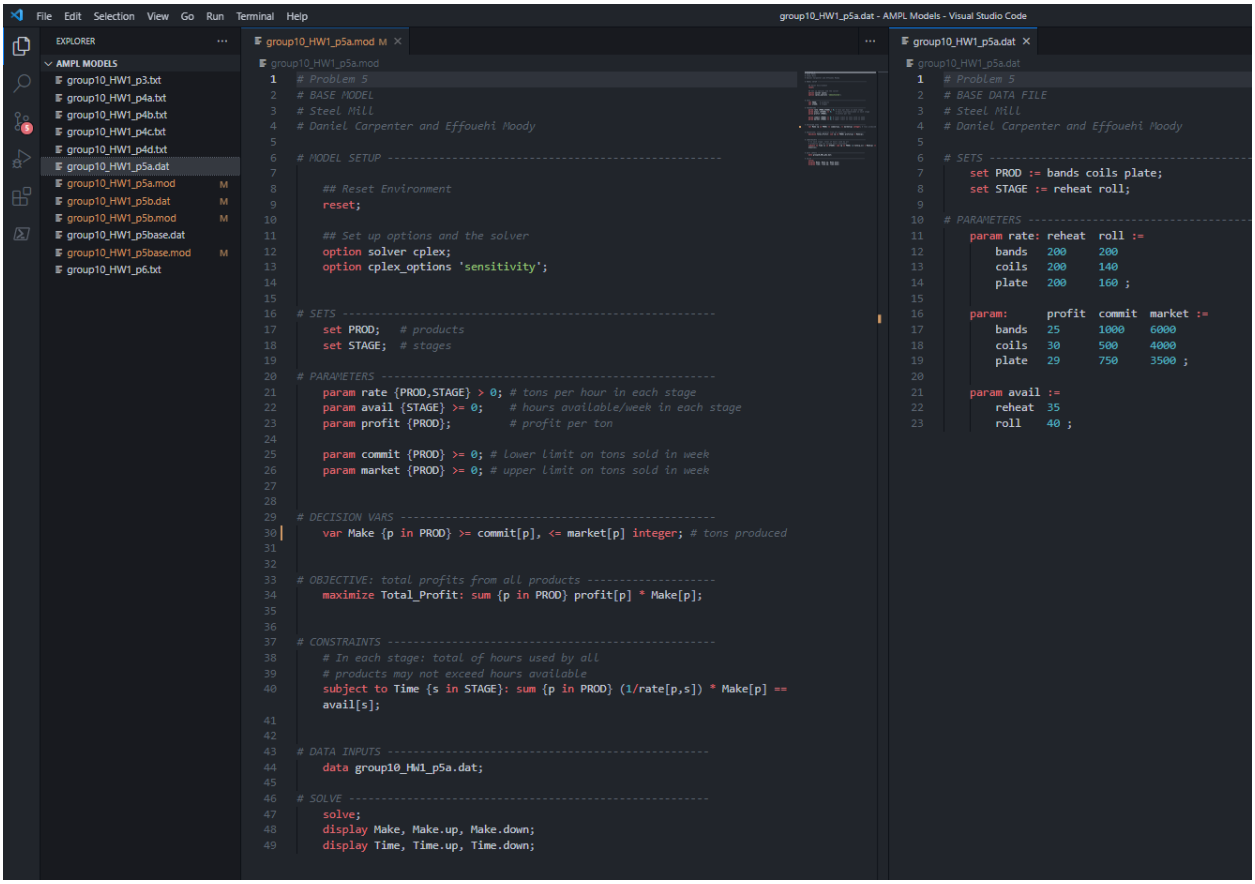


5.2 Task a

5.2.1 Changed Constraint for Total Hours

- *Change the constraints so that total hours used by all products must equal the total hours available for each stage*

5.2.2 Code



5.2.3 Output

There is no difference in the optimal solution because the range of Time before there is a change in optimal remains the same, and the hours available have not changed.

```
ampl: model group10_HW1_p5base.mod
CPLEX 20.1.0.0: sensitivity
CPLEX 20.1.0.0: optimal solution; objective 190071.4286
2 dual simplex iterations (0 in phase I)

suffix up OUT;
suffix down OUT;
suffix current OUT;
ampl: model group10_HW1_p5a.mod
CPLEX 20.1.0.0: sensitivity
CPLEX 20.1.0.0: optimal solution; objective 190071.4286
2 dual simplex iterations (0 in phase I)

suffix up OUT;
suffix down OUT;
suffix current OUT;
:         Make       Make.up       Make.down      :=
bands    3357.14       27.6        -1e+20
coils     500          31.8571     -1e+20
plate    3142.86    1e+20              27.9167
;

:         Time    Time.up  Time.down    :=
reheat    1800    37.6429    34.5536
roll      3200    40.4464    37.0089
;
```

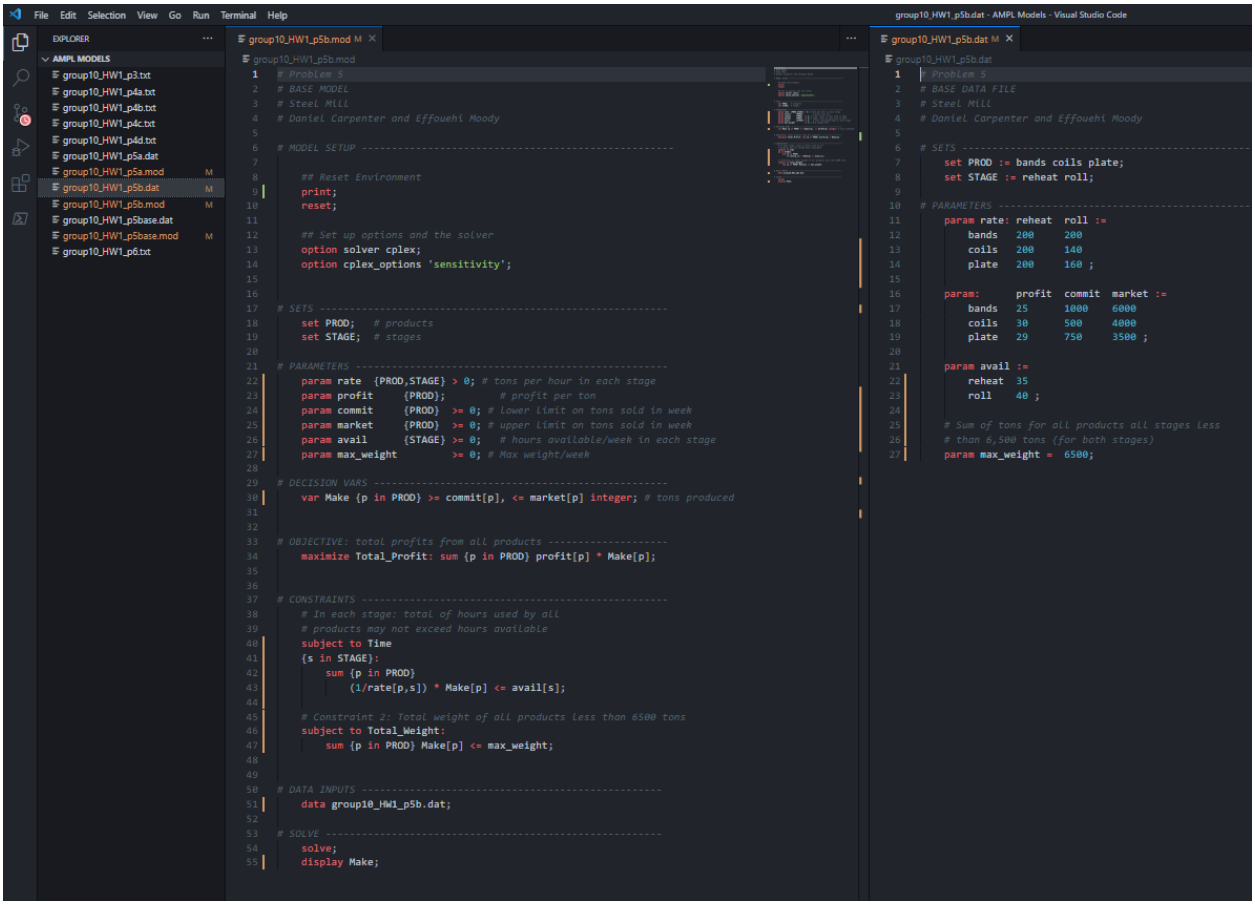
5.3 Task b

5.3.1 New Constraint for Max Weight

- *Restrict the total weight of all products to be less than a new parameter, max_weight = 6,500*

$$totalWeight : \sum_{p \in PROD} Make_p \leq max\_weight$$

5.3.2 Code



5.3.3 Output

The total number of tons has reduced from 7,000 to 6,500 per week

```
ampl: model group10_HW1_p5base.mod
CPLEX 20.1.0.0: sensitivity
CPLEX 20.1.0.0: optimal solution; objective 190071.4286
2 dual simplex iterations (0 in phase I)

suffix up OUT;
suffix down OUT;
suffix current OUT;
Make [*] :=
bands  3357.14
coils   500
plate  3142.86
;

ampl: model group10_HW1_p5b.mod

CPLEX 20.1.0.0: sensitivity
CPLEX 20.1.0.0: optimal solution; objective 183791.6667
3 dual simplex iterations (0 in phase I)

suffix up OUT;
suffix down OUT;
suffix current OUT;
Make [*] :=
bands  1541.67
coils  1458.33
plate  3500
;
```

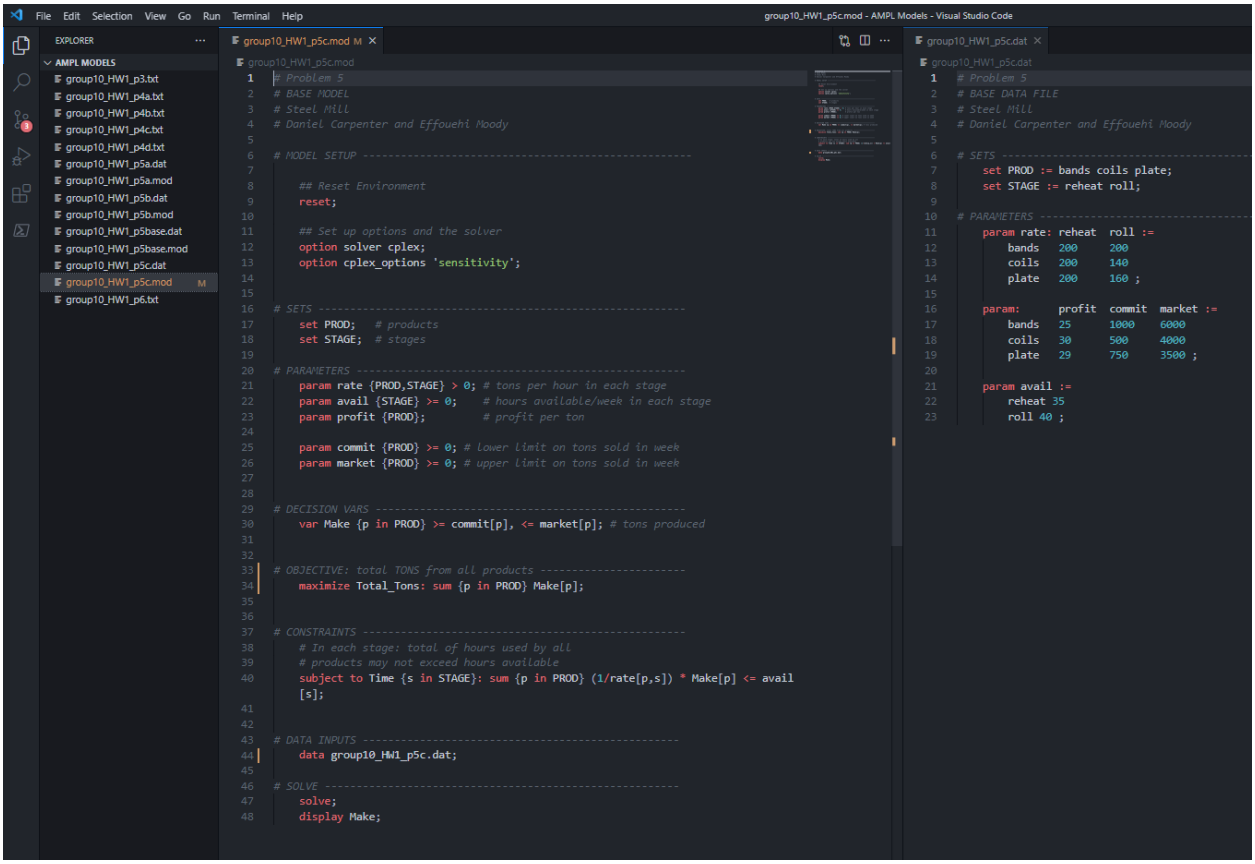
5.4 Task c

5.4.1 Changed Objective Function

- *Change the objective function to maximize total tons*

$$maximize\ Total\_Tons = \sum_{p \in PROD} Make_p$$

5.4.2 Code



5.4.3 Output

The data file does not make a diference in the optimal (assuming that is what the question is asking). Please note that the total number of tons produced are the same as in the `base` model; however, the allocation of tons have shifted among each of the products.

```
ampl: model group10_HW1_p5base.mod
CPLEX 20.1.0.0: sensitivity
CPLEX 20.1.0.0: optimal solution; objective 190071.4286
2 dual simplex iterations (0 in phase I)

suffix up OUT;
suffix down OUT;
suffix current OUT;
Make [*] :=
bands  3357.14
coils   500
plate  3142.86
;

ampl: model group10_HW1_p5c.mod
CPLEX 20.1.0.0: sensitivity
CPLEX 20.1.0.0: optimal solution; objective 7000
1 dual simplex iterations (0 in phase I)

suffix up OUT;
suffix down OUT;
suffix current OUT;
Make [*] :=
bands  5750
coils   500
plate   750
;
```

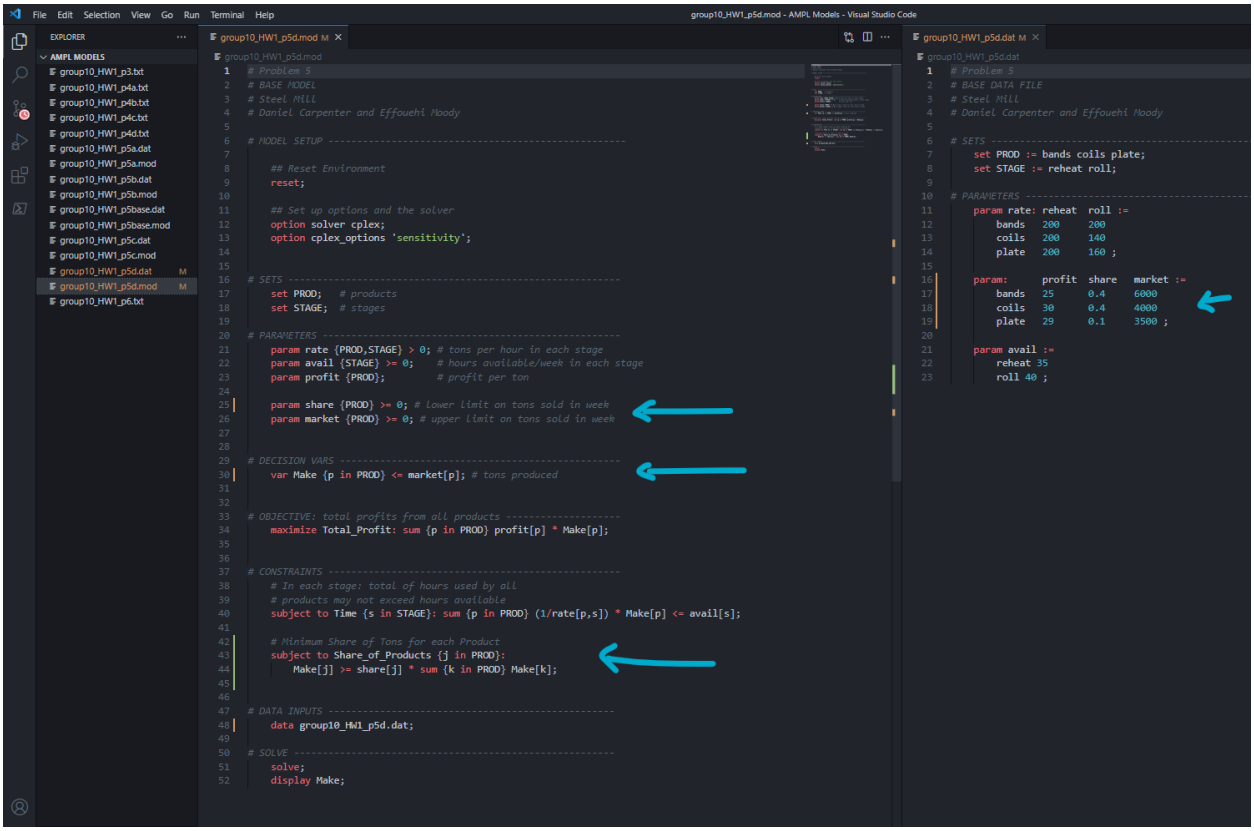
5.5 Task d

5.5.1 New Constraint

- *Minimum Share of Tons for each Product*

$$Share\_of\_Products : Make_j \geq share_j \times \sum_{k \in PROD} Make_k, \quad \forall\, j \in PROD$$

5.5.2 Code (Part I)

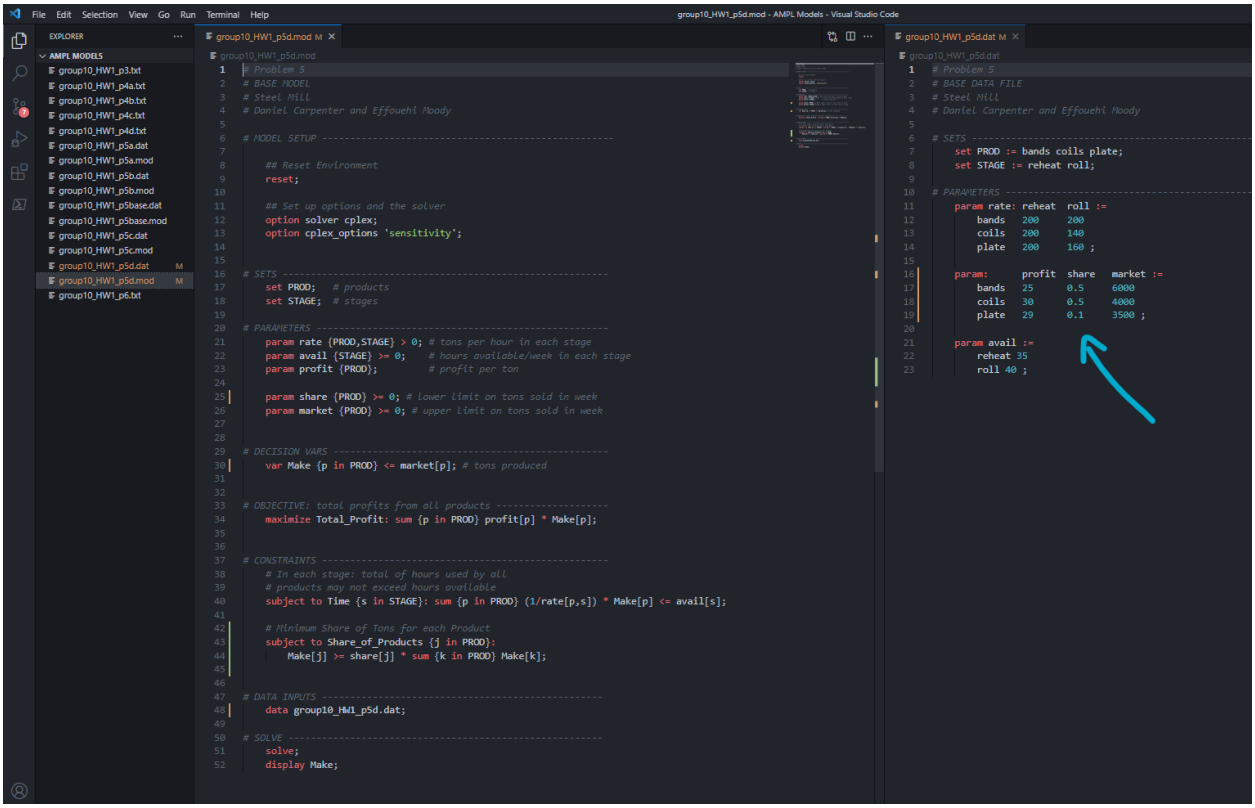


5.5.3 Output (Part I)

Note that bands represent ~49.99%, coils: 40%, and plates: 10%

```
ampl: model group10_HW1_p5d.mod
CPLEX 20.1.0.0: sensitivity
CPLEX 20.1.0.0: optimal solution; objective 183211.9403
5 dual simplex iterations (0 in phase I)

suffix up OUT;
suffix down OUT;
suffix current OUT;
Make [*] :=
bands  3343.28
coils  2674.63
plate   668.657
;
```

5.5.4 Code (Part II)



5.5.5 Output (Part II)

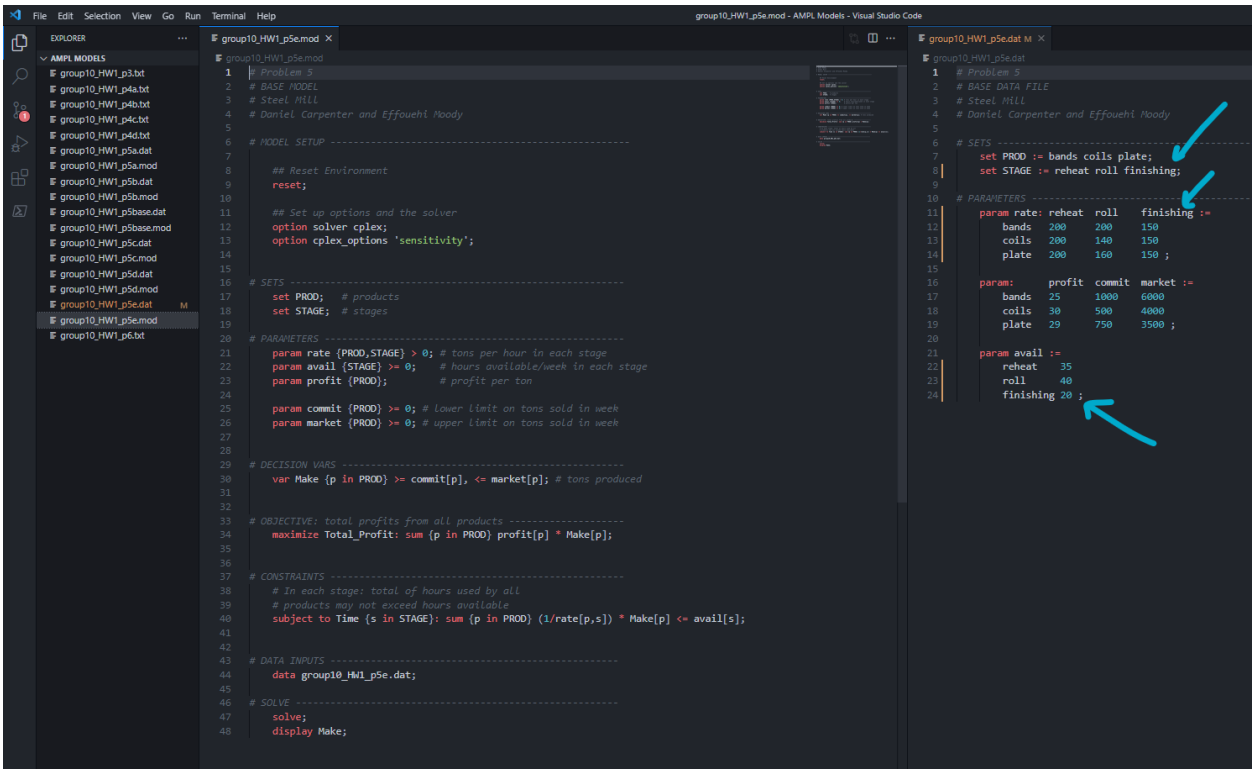
Profit is zero because it is impossible for bands to reach 50% of the share.

```
ampl: model group10_HW1_p5d.mod
CPLEX 20.1.0.0: sensitivity
CPLEX 20.1.0.0: optimal solution; objective 0
4 dual simplex iterations (0 in phase I)

suffix up OUT;
suffix down OUT;
suffix current OUT;
Make [*] :=
bands  0
coils  0
plate  0
;
```

5.6 Task `e`

5.6.1 Changing Input Data via `.dat` File

Simply add the new item within the set called `finishing`, then add the its the associate values to the `rate` and `avail` parameters.



5.6.2 Output

```
ampl: model group10_HW1_p5e.mod
CPLEX 20.1.0.0: sensitivity
CPLEX 20.1.0.0: optimal solution; objective 84250
0 dual simplex iterations (0 in phase I)

suffix up OUT;
suffix down OUT;
suffix current OUT;
Make [*] :=
bands  1000
coils  1250
plate   750
;
```

6 Problem 6

6.1 Task a - c

6.1.1 Decision Variables

bondA: dollars $\in \mathbb{R}$ to invest in bond A
bondB: dollars $\in \mathbb{R}$ to invest in bond B
bondC: dollars $\in \mathbb{R}$ to invest in bond C
bondD: dollars $\in \mathbb{R}$ to invest in bond D
bondE: dollars $\in \mathbb{R}$ to invest in bond E

6.1.2 Objective Function

- Maximize the Expected Earnings of the portfolio

$$Maximize\ Z = (0.043 \times bondA) + (0.027 \times bondB) + (0.025 \times bondC) + (0.022 \times bondD) + (0.045 \times bondE)$$

6.1.3 Constraints

C1: Budget to invest is \$10 MM or less

$$budget : bondA + bondB + bondC + bondD + bondE \leq 10$$

C2: At least \$4 million must be invested in government and agency bonds

$$govtAndAgency : bondB + bondC + bondD \geq 4$$

C3: Average Quality of the Portfolio must not exceed 1.4

$$avgQuality : (0.6 \times bondA) + (0.6 \times bondB) - (0.4 \times bondC) - (0.4 \times bondD) + (3.6 \times bondE) \leq 0$$

C4: The Average Maturity must not Exceed Five Years

$$avgMaturity : (4 \times bondA) + (10 \times bondB) - (1 \times bondC) - (2 \times bondD) - (3 \times bondE) \leq 0$$

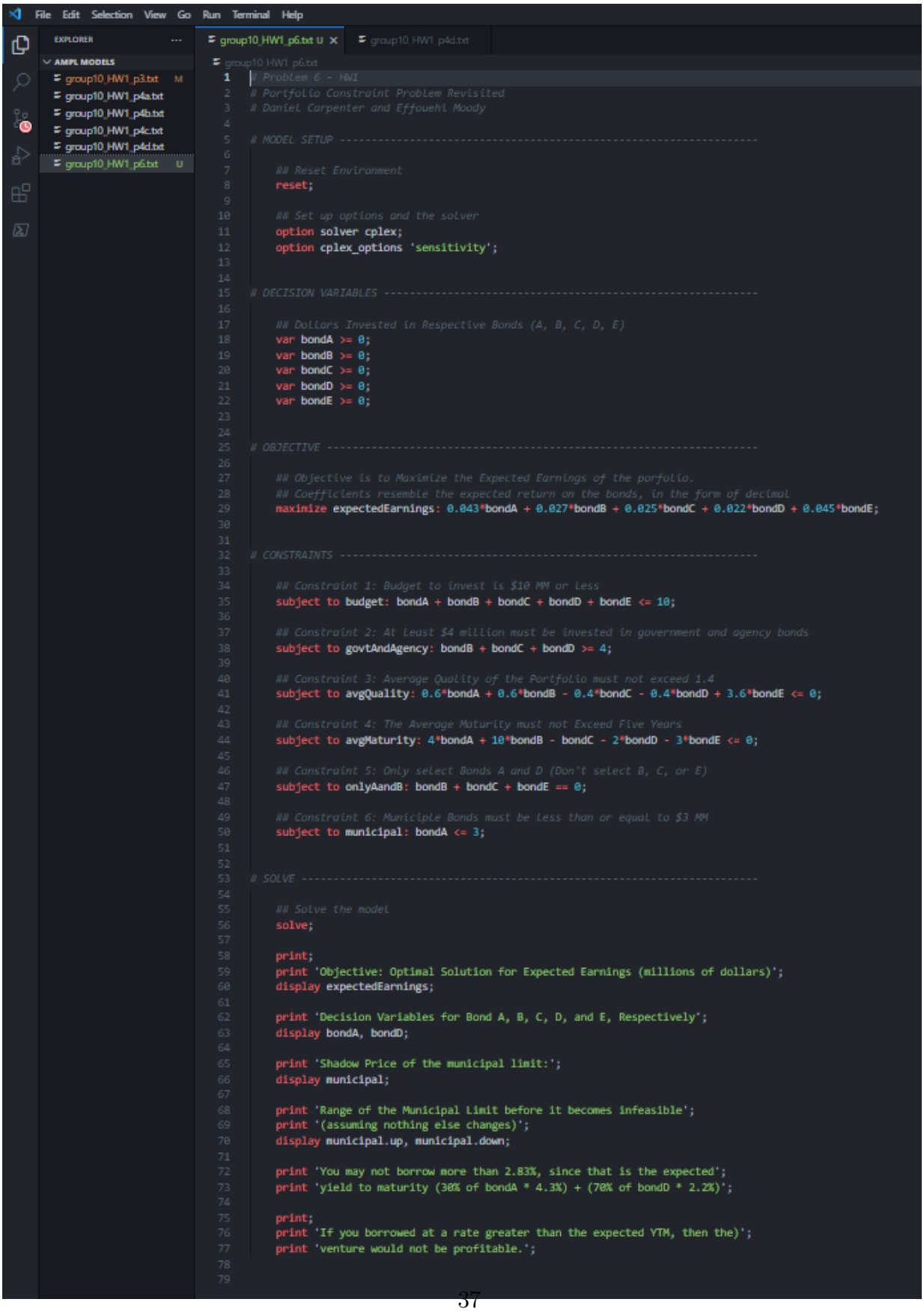
C5: Only select Bonds A and D (Don't select B, C, or E)

$$onlyAandB : bondB + bondC + bondE = 0;$$

C6: Municipal Bonds must be less than or equal to \$3 MM

$$municipal : bondA \leq 3;$$

6.1.4 Code



```
# Problem 6 - HW1
# Portfolio Constraint Problem Revisited
# Daniel Carpenter and Effouehi Moody

# MODEL SETUP ------------------------------------------------------------------

    ## Reset Environment
    reset;

    ## Set up options and the solver
    option solver cplex;
    option cplex_options 'sensitivity';


# DECISION VARIABLES -----------------------------------------------------------

    ## Dollars Invested in Respective Bonds (A, B, C, D, E)
    var bondA >= 0;
    var bondB >= 0;
    var bondC >= 0;
    var bondD >= 0;
    var bondE >= 0;


# OBJECTIVE --------------------------------------------------------------------

    ## Objective is to Maximize the Expected Earnings of the porfolio.
    ## Coefficients resemble the expected return on the bonds, in the form of decimal
    maximize expectedEarnings: 0.043*bondA + 0.027*bondB + 0.025*bondC + 0.022*bondD + 0.045*bondE;


# CONSTRAINTS ------------------------------------------------------------------

    ## Constraint 1: Budget to invest is $10 MM or less
    subject to budget: bondA + bondB + bondC + bondD + bondE <= 10;

    ## Constraint 2: At least $4 million must be invested in government and agency bonds
    subject to govtAndAgency: bondB + bondC + bondD >= 4;

    ## Constraint 3: Average Quality of the Portfolio must not exceed 1.4
    subject to avgQuality: 0.6*bondA + 0.6*bondB - 0.4*bondC - 0.4*bondD + 3.6*bondE <= 0;

    ## Constraint 4: The Average Maturity must not Exceed Five Years
    subject to avgMaturity: 4*bondA + 10*bondB - bondC - 2*bondD - 3*bondE <= 0;

    ## Constraint 5: Only select Bonds A and D (Don't select B, C, or E)
    subject to onlyAandB: bondB + bondC + bondE == 0;

    ## Constraint 6: Municiple Bonds must be less than or equal to $3 MM
    subject to municipal: bondA <= 3;


# SOLVE ------------------------------------------------------------------------

    ## Solve the model
    solve;

    print;
    print 'Objective: Optimal Solution for Expected Earnings (millions of dollars)';
    display expectedEarnings;

    print 'Decision Variables for Bond A, B, C, D, and E, Respectively';
    display bondA, bondD;

    print 'Shadow Price of the municipal limit:';
    display municipal;

    print 'Range of the Municipal Limit before it becomes infeasible';
    print '(assuming nothing else changes)';
    display municipal.up, municipal.down;

    print 'You may not borrow more than 2.83%, since that is the expected';
    print 'yield to maturity (30% of bondA * 4.3%) + (70% of bondD * 2.2%)';

    print;
    print 'If you borrowed at a rate greater than the expected YTM, then the)';
    print 'venture would not be profitable.';
```

6.1.5 Output

```
ampl: model group10_HW1_p6.txt
CPLEX 20.1.0.0: sensitivity
CPLEX 20.1.0.0: optimal solution; objective 0.283
0 dual simplex iterations (0 in phase I)

suffix up OUT;
suffix down OUT;
suffix current OUT;

Objective: Optimal Solution for Expected Earnings (millions of dollars)
expectedEarnings = 0.283

Decision Variables for Bond A, B, C, D, and E, Respectively
bondA = 3
bondD = 7

Shadow Price of the municipal limit:
municipal = 0.021

Range of the Municipal Limit before it becomes infeasible
(assuming nothing else changes)
municipal.up = 0
municipal.down = 0
```

6.2 Task `d`:

You may not borrow more than 2.83%, since that is the expected yield to maturity (30% of bondA * 4.3%) + (70% of bondD * 2.2%)

6.3 Task `e`:

If you borrowed at a rate greater than the expected YTM, then the venture would not be profitable.