

example IP: basic facility location

A company is thinking about building new facilities in LA and SF.

	capital needed	expected profit
1. factory in LA	\$6M	\$9M
2. factory in SF	\$3M	\$5M
3. warehouse in LA	\$5M	\$6M
4. warehouse in SF	\$2M	\$4M

Total capital available for investment: \$10M

Which facilities should be built to maximize the total profit?

example IP: basic facility location

- Define decision variables ($i = 1, 2, 3, 4$):

$$x_i = \begin{cases} 1 & \text{if facility } i \text{ is built} \\ 0 & \text{if not} \end{cases}$$

- Total expected benefit: $9x_1 + 5x_2 + 6x_3 + 4x_4$
- Total capital needed: $6x_1 + 3x_2 + 5x_3 + 2x_4$

➤ IP model:

$$\begin{aligned} \max & 9x_1 + 5x_2 + 6x_3 + 4x_4 \\ \text{s. t.} & 6x_1 + 3x_2 + 5x_3 + 2x_4 \leq 10 \\ & x_1, x_2, x_3, x_4 \in \{0,1\} \end{aligned}$$

adding new requirements

- Extra requirement:

build *at most one* of the two warehouses

$$x_3 + x_4 \leq 1$$

- Extra requirement:

build *at least one* of the two factories

$$x_1 + x_2 \geq 1$$

contingent decisions

- *Additional requirement:* Can't build a warehouse *unless* there is a factory in the city:

$$x_3 \leq x_1 \text{ (LA)}$$

$$x_4 \leq x_2 \text{ (SF)}$$

- *Additional requirement:* Can't select option 3 *unless* at least one of options 1 and 2 is selected.

$$x_3 \leq x_1 + x_2$$

contingent decisions

- *Additional requirement:*

Can't select option 4 *unless* at least two of options 1, 2 and 3 are selected.

$$2x_4 \leq x_1 + x_2 + x_3$$

$$\max 9x_1 + 5x_2 + 6x_3 + 4x_4$$

$$\text{s. t. } 6x_1 + 3x_2 + 5x_3 + 2x_4 \leq 10$$

$$x_3 + x_4 \leq 1$$

$$x_1 + x_2 \geq 1$$

$$x_3 - x_1 \leq 0$$

$$x_4 - x_2 \leq 0$$

$$x_3 - (x_1 + x_2) \leq 0$$

$$2x_4 - (x_1 + x_2 + x_3) \leq 0$$

$$x_1, x_2, x_3, x_4 \in \{0,1\}$$