FACILITY LOCATION EXAMPLE (AGAIN!)

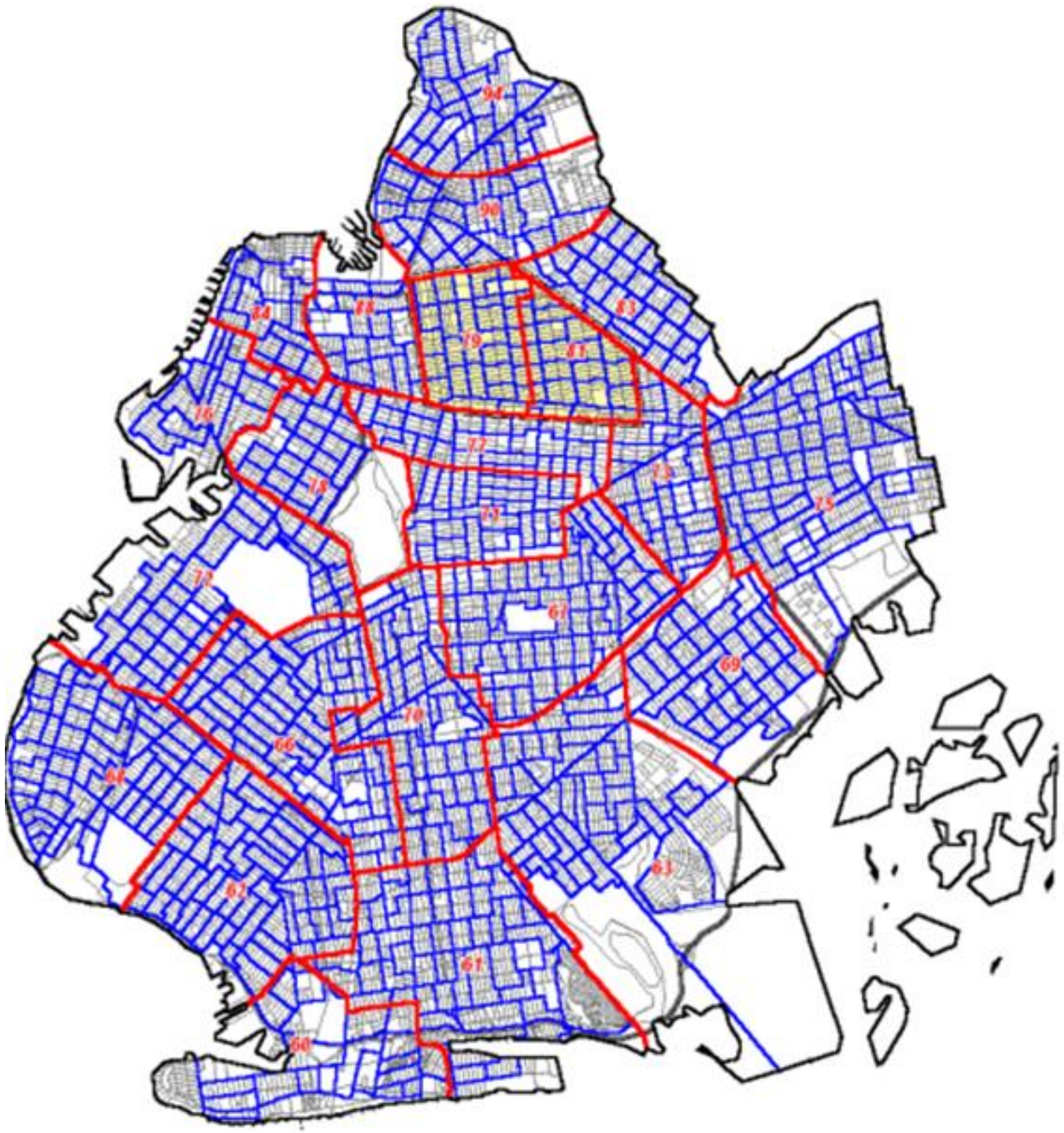
facility location example: this time with an attitude…

- Assume that the population is concentrated in I districts within the city and that district $i = 1, \ldots, I$ contains p_i people.
- Preliminary analysis (land surveys, politics, and so forth) has limited the potential location of firehouses to J sites.
- Let $d_{ij} \geq 0$ be the distance from the center of district i to site j.

facility location example

Census tracts

Police Precincts



Brooklyn, NY

facility location example

Census tracts

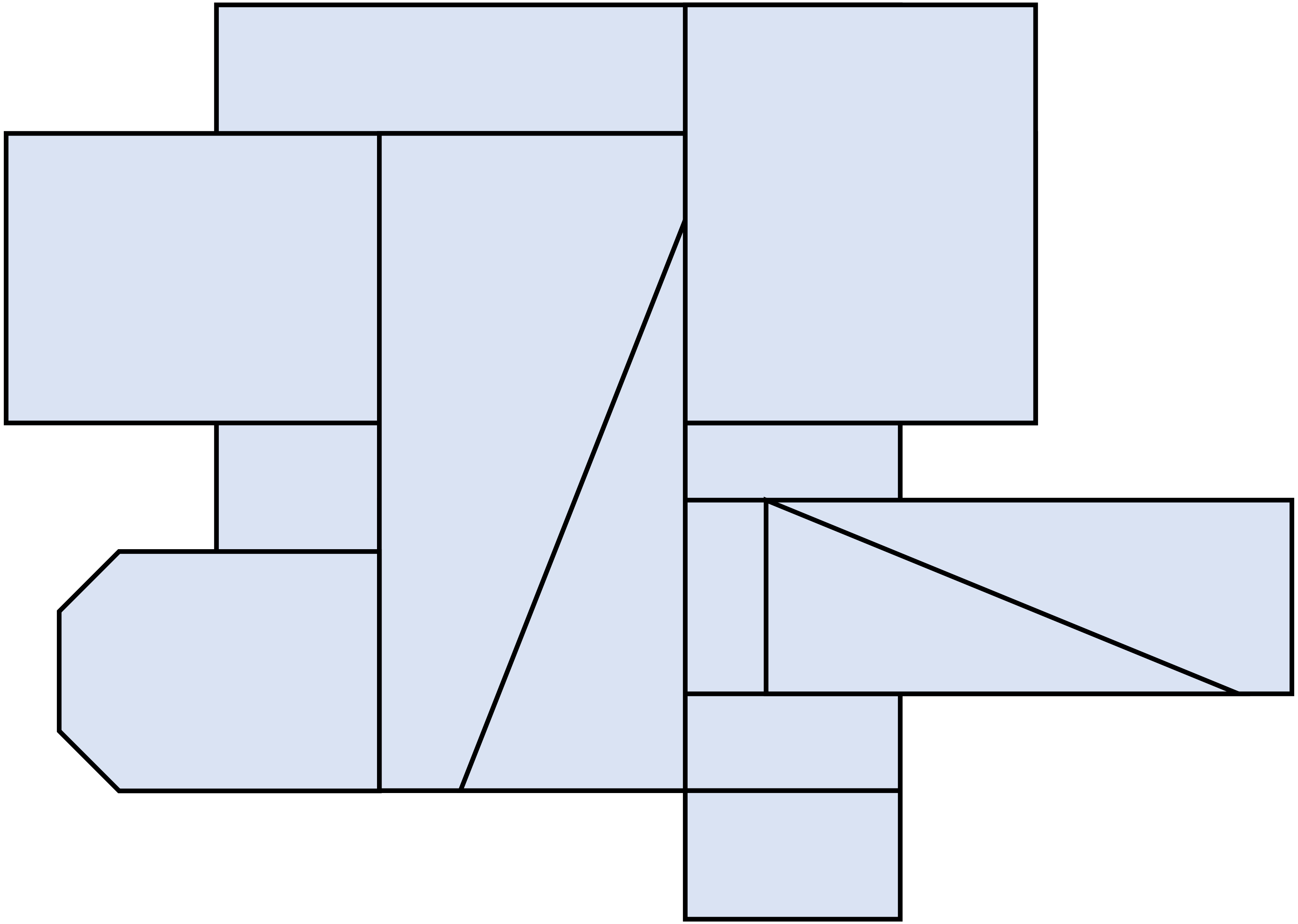
Police Precincts

facility location example: this time with an attitude…

- Every district should be assigned to exactly one firehouse
- No district should be assigned to an “unused” site
- There is a fixed cost f_j to build a firehouse on site j; and a variable cost c_j for servicing the assigned population
- Additionally: Assume it is important that either at least sites 1 and 2 *or* sites 3 and 4 are selected.

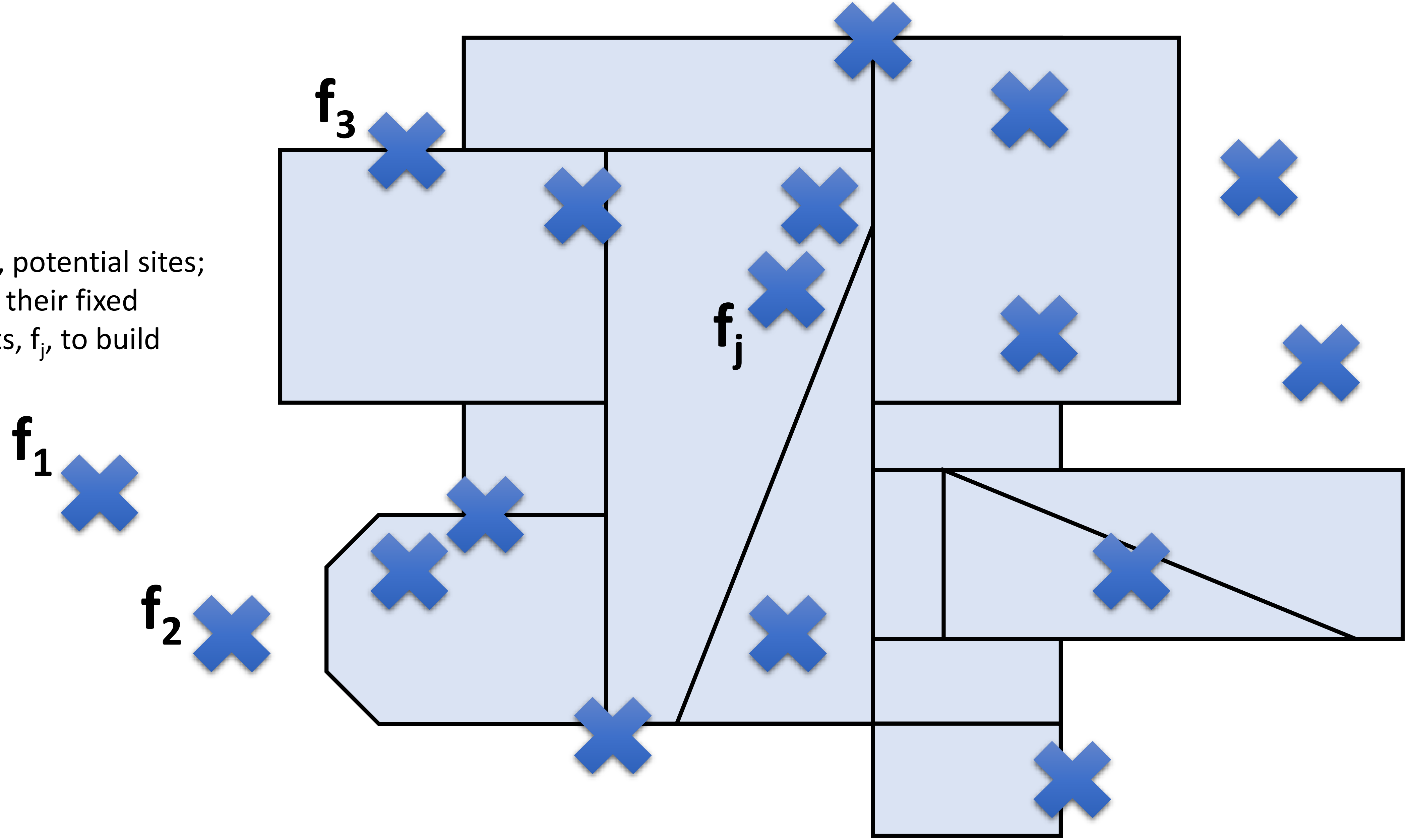
facility location example districts

e.g., districts in city



facility location example potential sites

e.g., potential sites; and their fixed costs, f_j, to build

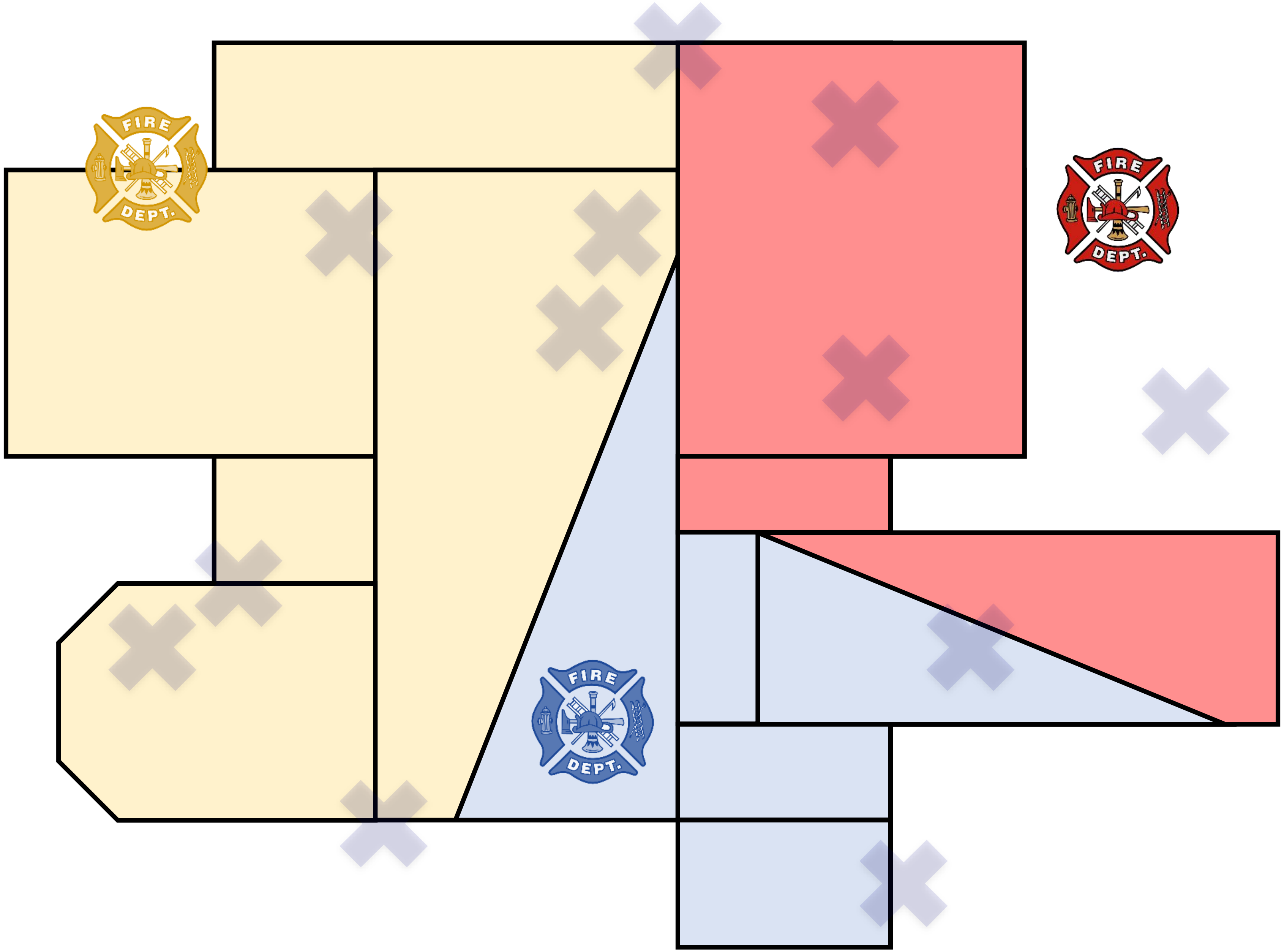


facility location example solution

e.g., possible solution with 3 sites chosen

Note:

- Each district assigned to a station, and only one station
- Variable cost for a station is related to total population in the districts it is assigned to; e.g., the yellow station is assigned to 5 districts; so the operating cost the “per person” rate times the total population from those 5 districts



facility location example: this time with an attitude…

What might be objectives for this problem?

There are several possible objectives!

<u>Our objective will be:</u>

- To determine the *best* site selection and assignment of districts to firehouses given a limited budget B.
- Where *best* means: minimize the “worst-case” distances, *a.k.a. the “p-center” problem*

Decision variables

$$y_j = \begin{cases} 1 & \text{if site } j \text{ is selected} \\ 0 & \text{otherwise} \end{cases}$$

$$x_{ij} = \begin{cases} 1 & \text{if district } i \text{ is assigned to site } j \\ 0 & \text{otherwise} \end{cases}$$

Every district should be assigned to exactly one firehouse

$$\sum_{j=1}^{J} x_{ij} = 1 \quad (i = 1, 2, \ldots, I)$$

No district should be assigned to an “unused” site

$$\sum_{i=1}^{I} x_{ij} \leq y_j I \quad (j = 1, 2, \ldots, J)$$

It is important that either at least sites 1 and 2 or sites 3 and 4 are selected.

$$y_1 + y_2 \geq 2$$

or

$$y_3 + y_4 \geq 2$$

It is important that either at least sites 1 and 2 or sites 3 and 4 are selected.

$$y_1 + y_2 \geq 2y$$

$$y_3 + y_4 \geq 2(1 - y)$$

There is a fixed cost to build a firehouse on a site; as well as a variable cost for servicing the population in that district.

For each site, create a new variable to denote the population that would be served by that site:

$$s_j = \sum_{i=1}^{I} p_i x_{ij}$$

There is a fixed cost to build a firehouse on a site; as well as a variable cost for servicing the population in that district.

Let f_j denote the fixed cost of building a firehouse at location j

$$c_j s_j + f_j y_j$$

There is a fixed cost to build a firehouse on a site; as well as a variable cost for servicing the population in that district.

Ensure total costs are less than available budget

$$\sum_{j=1}^{J} (c_j s_j + f_j y_j) \leq B$$

Need to minimize the "worst-case" distances (i.e. minimize the max travel time)

For each district: travel distance given all locations chosen and assignments

$$\sum_{j=1}^{J} d_{ij} x_{ij}$$

Would like to ensure that the no district has to travel too far... We can "minimize the maximum" distance per district...

Need to minimize the "worst-case" distances

That is, we'd like to:

$$\min \max_{i=1,2,\ldots,I} \sum_{j=1}^{J} d_{ij} x_{ij}$$

Need to minimize the "worst-case" distances

That is, we'd like to:

$$\min \quad \max \quad \begin{array}{l} \sum_{j=1}^{J} d_{1j}x_{1j} \\ \sum_{j=1}^{J} d_{2j}x_{2j}, \\ \vdots \\ \sum_{j=1}^{J} d_{Ij}x_{Ij} \end{array}$$

Need to minimize the “worst-case” distances

Create new real-valued decision variable D to denote the maximum value of each:

$$D = \max \left\{ \sum_{j=1}^{J} d_{1j} x_{1j}, \sum_{j=1}^{J} d_{2j} x_{2j}, \ldots, \sum_{j=1}^{J} d_{Ij} x_{Ij} \right\}$$

Need to minimize the "worst-case" distances

$$D \geq \sum_{j=1}^{J} d_{1j} x_{1j}$$

Which obviously means:

$$D \geq \sum_{j=1}^{J} d_{2j} x_{2j},$$

$$\vdots$$

$$D \geq \sum_{j=1}^{J} d_{Ij} x_{Ij}$$

And each of these is linear!! So yay!

Need to minimize the "worst-case" distances

$$D \geq \sum_{j=1}^{J} d_{ij} x_{ij} \qquad (i = 1, 2, \ldots, I)$$

Need to minimize the “worst-case” distances

$$\text{minimize } D$$

$$\text{minimize } D$$

s.t.

$$D \geq \sum_{j=1}^{J} d_{ij} x_{ij} \quad (i = 1, 2, \ldots, I)$$

$$\sum_{j=1}^{J} x_{ij} = 1 \quad (i = 1, 2, \ldots, I)$$

$$\sum_{i=1}^{I} x_{ij} \leq y_j I \quad (j = 1, 2, \ldots, J)$$

$$s_j = \sum_{i=1}^{I} p_i x_{ij}$$

$$\sum_{j=1}^{J} (c_j s_j + f_j y_j) \leq B$$

$$y_1 + y_2 \geq 2y$$

$$y_3 + y_4 \geq 2(1 - y)$$

$$x_{ij}, y_j, y \ \text{ binary}$$