

# GENERALIZED NETWORK FLOWS

# elements of the Generalized Network Flow Problem

Defined on a directed network:  $G = (N, A)$

where  $N$  is a set of  $n$  nodes:  $\{1, 2, \dots, n\}$

and  $A$  is a set of  $m$  arcs as a subset of  $N \times N$

Each node  $i$  has an associated value  $b(i)$

Arc  $(i, j)$  has certain characteristics:

- cost  $c_{ij}$  per unit of flow on arc  $(i, j)$
- upper bound on flow of  $u_{ij}$  (capacity)
- lower bound on flow of  $\ell_{ij}$  (usually 0)
- **multiplier  $\mu_{ij} \geq 0$  such that if 1 unit of flow leaves node  $i$ , then  $\mu_{ij}$  units arrive at node  $j$**

# Generalized network flow examples\*

- **Financial networks** where nodes represent equities (e.g., stocks, bonds, current deposits, Treasury bills, etc.); and the arcs represent various investment alternatives that convert one type of equity into another. The arc multiplier represents the gain associated with the corresponding investment.
- **Mineral networks** where nodes represent mines, purification plants, refineries, ports, and final markets; arcs represent processing opportunities or flow of material through intermediate junctions. The multipliers represent loss associated with the corresponding process.
- **Energy networks** where nodes represent raw materials (e.g., crude oil, coal, uranium), and various outputs (e.g., electricity, domestic oil, gas); and the arcs represent the transformation of one raw material into an energy output; the multiplier represents the efficiency of this transformation.

# Generalized network formulation

$$\text{minimize} \quad \sum_{(i,j) \in A} c_{ij} x_{ij}$$

$$\text{subject to} \quad \sum_{j:(i,j) \in A} x_{ij} - \sum_{j:(j,i) \in A} \mu_{ji} x_{ji} = b_i \quad \forall i \in N$$

$$l_{ij} \leq x_{ij} \leq u_{ij} \quad \forall (i,j) \in A$$

# Generalized network AMPL model

```
set NODES;                # nodes in the network
set ARCS within {NODES, NODES}; # arcs in the network

param b {NODES} default 0;    # supply/demand for node i
param c {ARCS}  default 0;    # cost of one of flow on arc(i,j)
param l {ARCS}  default 0;    # lower bound on flow on arc(i,j)
param u {ARCS}  default Infinity; # upper bound on flow on arc(i,j)
param mu {ARCS} default 1;    # multiplier on arc(i,j)
                                # i.e., if one unit leaves i, mu[i,j] units arrive at j

var x {ARCS};                # flow on arc (i,j)

minimize cost: sum{(i,j) in ARCS} c[i,j] * x[i,j]; #objective: minimize arc flow cost

# Flow Out(i) - Flow In(i) = b(i)

subject to flow_balance {i in NODES}:
sum{j in NODES: (i,j) in ARCS} x[i,j] - sum{j in NODES: (j,i) in ARCS} mu[j,i] * x[j,i] = b[i];

subject to capacity {(i,j) in ARCS}: l[i,j] <= x[i,j] <= u[i,j];
```

# Generalized network flow example

- An entrepreneur is starting a new business in buying/selling political action figures. The prices for the figures are quite volatile, and she is trying to determine how to manage her purchases, sales, and storage patterns.
- She has \$8500 of cash on-hand; and 750 units of her preferred political figurine. Inventory capacity each period: 1,500 units. Inventory holding costs per unit per period: \$0.50
- She has good estimates of the price of the items over the next several periods: \$10, \$40, \$80, and \$50.
- Each month she can choose to put her money in the bank and earn interest (0.25%) per period.
- Due to a rodent problem, she assumes a spoilage rate of inventory of 1% each period.



# Generalized network flow example

Using a generalized network flow formulation, please help the entrepreneur determine a plan for cash management, purchases, sales, and storage patterns over the next four periods, in such a way to maximize the cash-on-hand in period 5.

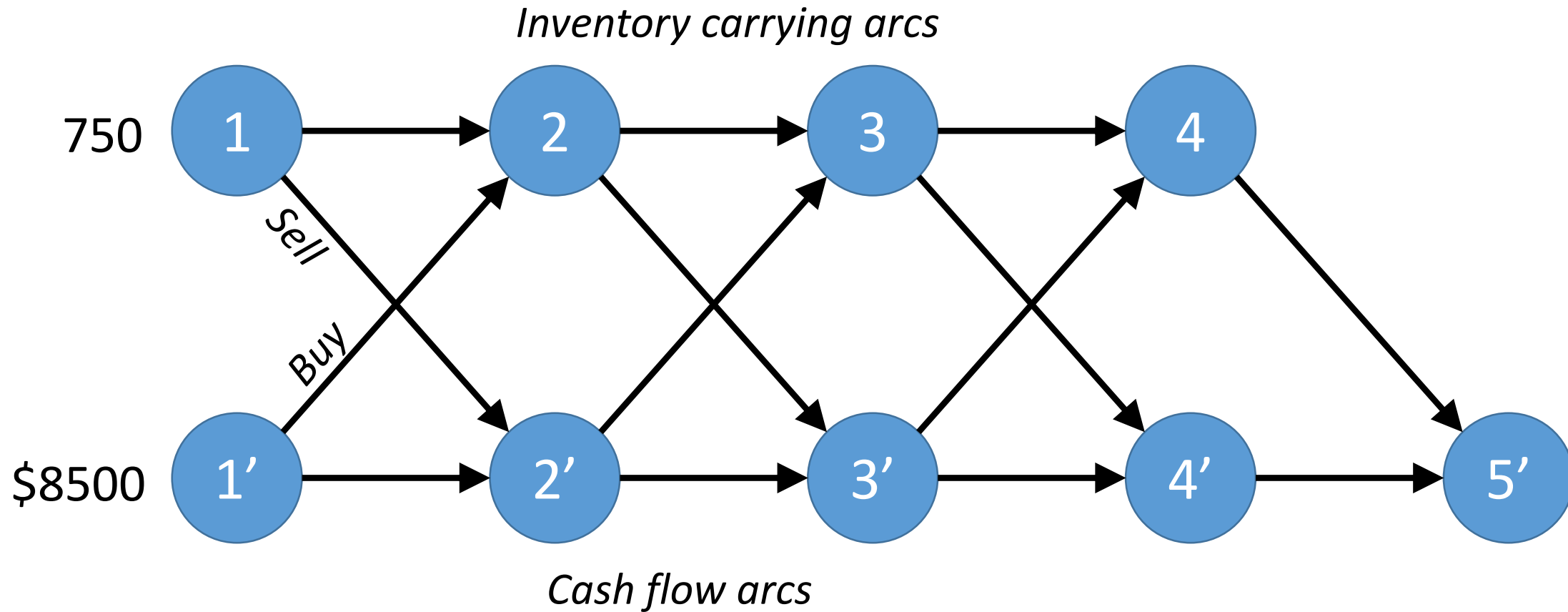
Assume any product on-hand during period  $k$  can be sold; the cash is available in period  $k + 1$ .

Similarly, assume any amount of product can be purchased in period  $k$  and is available in period  $k + 1$ .



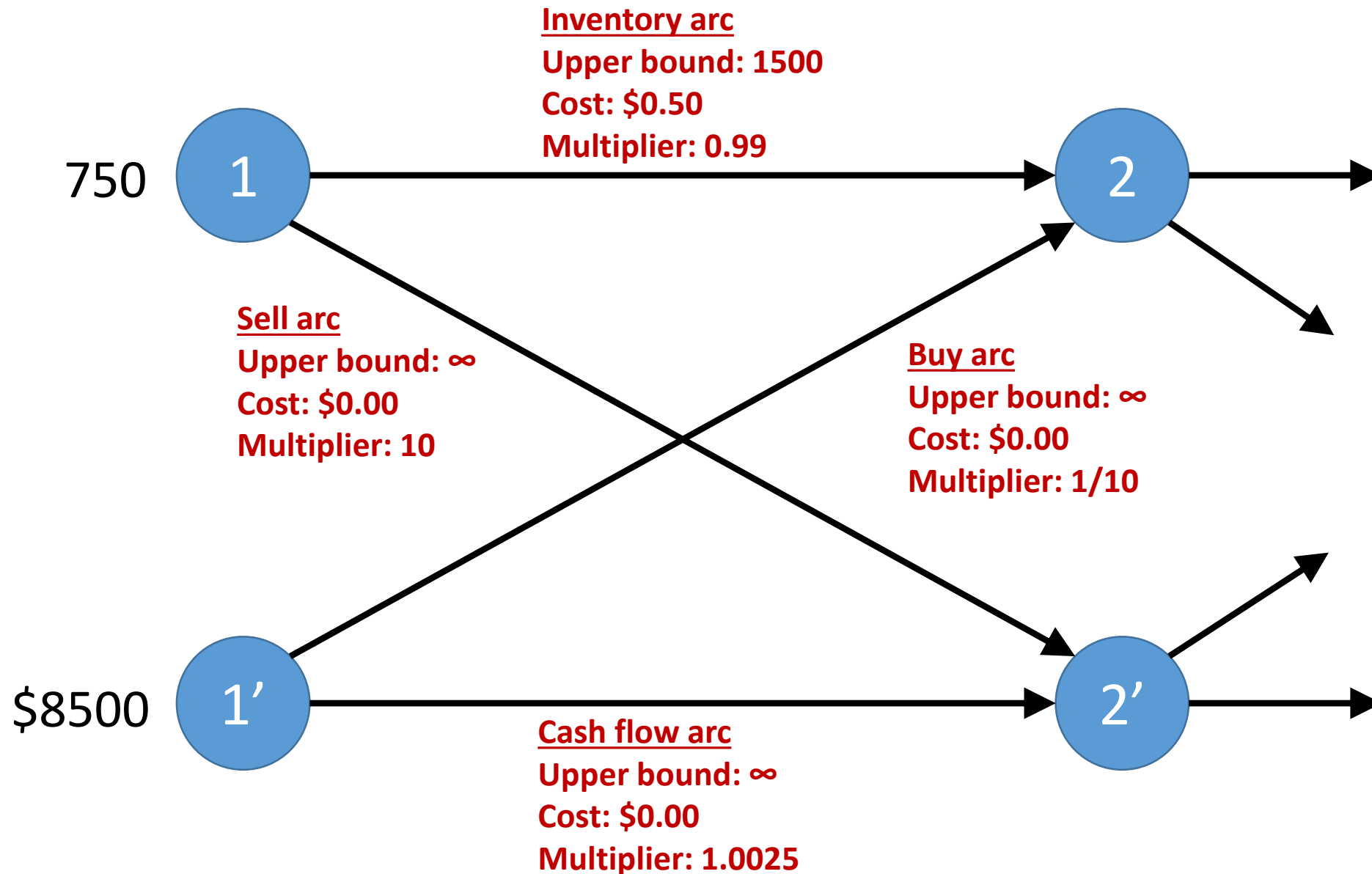
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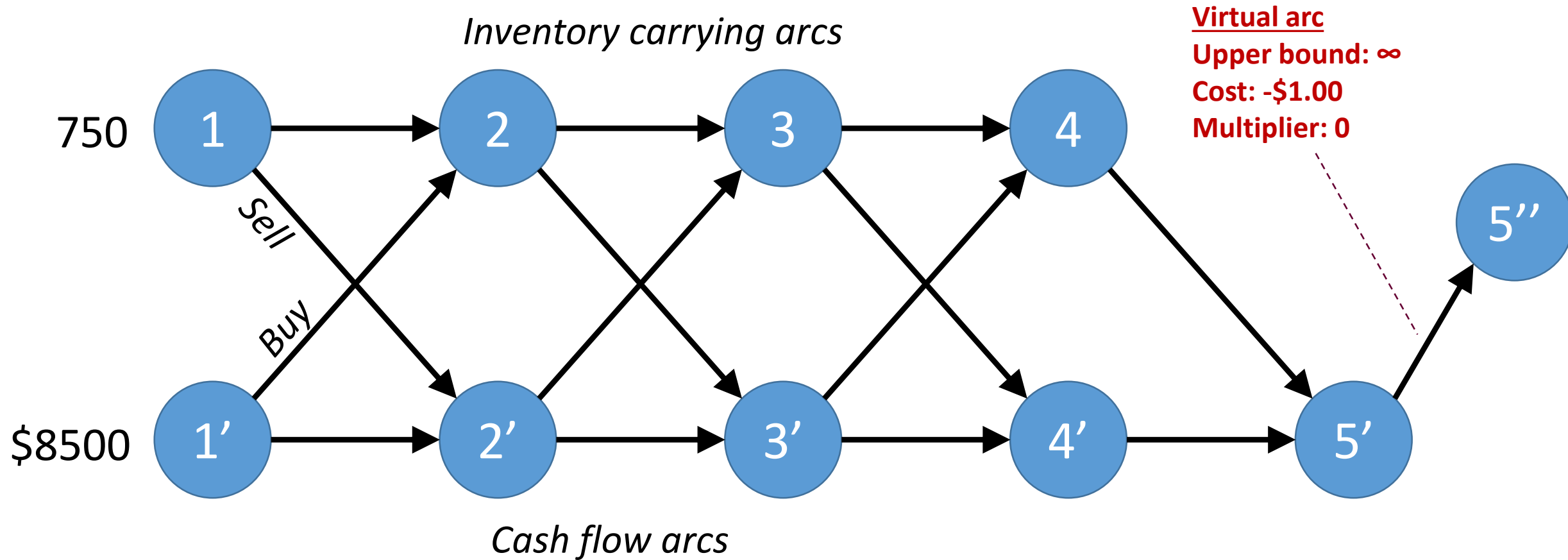


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