Homework 1 - Truth Tables & Linear Programming with AMPL

Daniel Carpenter and Effouehi Moody

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3 Problem 3

3.1 Task a

3.1.1 Decision Variables

`bondA`: dollars $\in \mathbb{R}$ to invest in bond A
`bondB`: dollars $\in \mathbb{R}$ to invest in bond B
`bondC`: dollars $\in \mathbb{R}$ to invest in bond C
`bondD`: dollars $\in \mathbb{R}$ to invest in bond D
`bondE`: dollars $\in \mathbb{R}$ to invest in bond E

3.1.2 Objective Function

- Maximize the Expected Earnings of the portfolio

$$Maximize\ Z = (0.043 \times bondA) + (0.027 \times bondB) + (0.025 \times bondC) + (0.022 \times bondD) + (0.045 \times bondE)$$

3.1.3 Constraints

C1: Budget to invest is \$10 MM or less

$$budget : bondA + bondB + bondC + bondD + bondE \leq 10$$

C2: At least \$4 million must be invested in government and agency bonds

$$govtAndAgency : bondB + bondC + bondD \geq 4$$

C3: Average Quality of the Portfolio must not exceed 1.4

$$avgQuality : (0.6 \times bondA) + (0.6 \times bondB) - (0.4 \times bondC) - (0.4 \times bondD) + (3.6 \times bondE) \leq 0$$

C4: The Average Maturity must not Exceed Five Years

$$avgMaturity : (4 \times bondA) + (10 \times bondB) - (1 \times bondC) - (2 \times bondD) - (3 \times bondE) \leq 0$$

3.1.4 Code

3.1.5 Output

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4 Problem 4

4.1 Task a

4.1.1 Decision Variables

`tv` = the number of minutes $\in \mathbb{R}$ to air advertising on the *television* medium
`magazine` = the number of pages $\in \mathbb{I}$ to to advertise on the *magazine* medium

4.1.2 Objective Function

- Maximize the total audience reach

$$Maximize\ Z = (1.8 \times tv) + (1.0 \times magazine)$$

4.1.3 Constraints

C1: Must not Exceed Budget of 1 Million dollars

$$budget : (20,000 \times tv) + (10,000 \times magazine) \leq 1,000,000$$

C2: Must have at least 10 minutes of air time on the TV medium

$$minTimeTV : tv \geq 10$$

4.1.4 Code

4.1.5 Output

4.1.6 Solving Problem `4(a)` Graphically by Hand

4.2 Task `b`

4.2.1 Additional Constraint: Labor Time

C3: Only 100 person weeks available, given it takes three weeks and one week to create a `tv` and `magazine` minute for advertisement, respectively.

$$personWeeks : (3 \times tv) + (1 \times magazine) \leq 100$$

4.2.2 Code

4.2.3 Output

4.3 Task `c`

4.3.1 Additional Constraint: Radio Advertising Medium

4.3.2 Decision Variables

`tv` = the number of minutes $\in \mathbb{R}$ to air advertising on the *television* medium
`magazine` = the number of pages $\in \mathbb{I}$ to to advertise on the *magazine* medium `radio` = the number of minutes $\in \mathbb{R}$ to air advertising on the *radio* medium

4.3.3 Objective Function

- Maximize the total audience reach

$$Maximize\ Z = (1.80 \times tv) + (1.00 \times magazine) + (0.25 \times radio)$$

4.3.4 New Constraints

C1: Must not Exceed Budget of 1 Million dollars

$$budget : (20,000 \times tv) + (10,000 \times magazine) + (2,000 \times radio) \leq 1,000,000$$

C2: Must have at least 10 minutes of air time on the TV medium

$$minTimeTV : tv \geq 10$$

C3: Only 100 person weeks available, given it takes three weeks and one week to create a `tv` and `magazine` minute for advertisement, respectively. It only takes one day for `radio`.

$$personWeeks : (3 \times tv) + (1 \times magazine) + (\frac{1}{7} \times radio) \leq 100$$

4.3.5 Code

4.3.6 Output

4.4 Task `d`

4.4.1 Additional Constraints: Miminum Magazine and Maximum Radio Requirements

C4: Must sign up for at least 2 magazine pages

$$minMagazines : magazine \geq 2$$

C5: Must to exceed 120 minutes of radio

$$maxRadio : radio \leq 120$$

4.4.2 Code

4.4.3 Output

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5 Problem 5

5.1 Base Mathematical Formulation and Code

- *Each task shows a separate change to the base model. Therefore, each change should not accumulate.*

5.1.1 Mathematical Formulation

5.1.2 Code for Model `.mod` and Input Data `.dat`

5.2 Task `a`

5.2.1 Changed Constraint for Total Hours

- *Change the constraints so that total hours used by all products must equal the total hours available for each stage*

5.2.2 Code

5.2.3 Output

There is no difference in the optimal solution because the range of Time before there is a change in optimal remains the same, and the hours available have not changed.

5.3 Task `b`

5.3.1 New Constraint for Max Weight

- *Restrict the total weight of all products to be less than a new parameter, max_weight = 6,500*

$$totalWeight: \sum_{p \ \in \ PROD} Make_p \leq max\_weight$$

5.3.2 Code

5.3.3 Output

The total number of tons has reduced from 7,000 to 6,500 per week

5.4 Task `c`

5.4.1 Changed Objective Function

- *Change the objective function to maximize total tons*

$$maximize \ Total\_Tons = \sum_{p \ \in \ PROD} Make_p$$

5.4.2 Code

5.4.3 Output

The data file does not make a diference in the optimal (assuming that is what the question is asking). Please note that the total number of tons produced are the same as in the `base` model; however, the allocation of tons have shifted among each of the products.

5.5 Task `d`

5.5.1 New Constraint

- *Minimum Share of Tons for each Product*

$$Share\_of\_Products: Make_j \geq share_j \times \sum_{k \ \in \ PROD} Make_k, \ \ \forall \ j \ \in \ PROD$$

5.5.2 Code (Part I)

5.5.3 Output (Part I)

Note that bands represent ~49.99%, coils: 40%, and plates: 10%

5.5.4 Code (Part II)

5.5.5 Output (Part II)

Profit is zero because it is impossible for bands to reach 50% of the share.

5.6 Task `e`

5.6.1 Changing Input Data via `.dat` File

Simply add the new item within the set called `finishing`, then add the its the associate values to the `rate` and `avail` parameters.

5.6.2 Output

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6 Problem `6`

6.1 Task `a - c`

6.1.1 Decision Variables

`bondA`: dollars $\in \mathbb{R}$ to invest in bond A
`bondB`: dollars $\in \mathbb{R}$ to invest in bond B
`bondC`: dollars $\in \mathbb{R}$ to invest in bond C
`bondD`: dollars $\in \mathbb{R}$ to invest in bond D
`bondE`: dollars $\in \mathbb{R}$ to invest in bond E

6.1.2 Objective Function

- Maximize the Expected Earnings of the portfolio

$$Maximize\ Z = (0.043 \times bondA) + (0.027 \times bondB) + (0.025 \times bondC) + (0.022 \times bondD) + (0.045 \times bondE)$$

6.1.3 Constraints

C1: Budget to invest is \$10 MM or less

$$budget : bondA + bondB + bondC + bondD + bondE \leq 10$$

C2: At least \$4 million must be invested in government and agency bonds

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C3: Average Quality of the Portfolio must not exceed 1.4

$$avgQuality : (0.6 \times bondA) + (0.6 \times bondB) - (0.4 \times bondC) - (0.4 \times bondD) + (3.6 \times bondE) \leq 0$$

C4: The Average Maturity must not Exceed Five Years

$$avgMaturity : (4 \times bondA) + (10 \times bondB) - (1 \times bondC) - (2 \times bondD) - (3 \times bondE) \leq 0$$

C5: Only select Bonds A and D (Don't select B, C, or E)

$$onlyAandB : bondB + bondC + bondE = 0;$$

C6: Municipal Bonds must be less than or equal to \$3 MM

$$municipal : bondA \leq 3;$$

6.1.4 Code

6.1.5 Output

6.2 Task `d`:

You may not borrow more than 2.83%, since that is the expected yield to maturity (30% of bondA * 4.3%) + (70% of bondD * 2.2%)

6.3 Task `e`:

If you borrowed at a rate greater than the expected YTM, then the venture would not be profitable.