Exam 1

Adv. Analytics and Metaheuristics

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1 - Problem 1

1.1 Mathematical Formulation

1.1.1 Sets

Set Name	Description
P	The three types of products, High-Gloss, Semi-Gloss, and Flat

1.1.2 Parameters

Parameter Name	Description
$rawA_p$	The amount of raw ingredient A needed to produce product $p \in P$
$rawB_p$	The amount of raw ingredient B needed to produce product $p \in P$
$demand_p$	The minimum demand to be met for product $p \in P$
$profit_p$	The associated profit for product $p \in P$

1.1.3 Decision Variables

Variable Name	Description
$amtToProduce_p$	The amount of product $p \in P$ to produce and is ion the set of integers

1.1.4 Objective Function

$$maximize\ theProfit : \sum_p amtToProduce_p \times profit_p$$

1.1.5 Constraints

C1: Meet the minimum demand for each product

$$meetMinDemand : amtToProduce_p \geq demand_p,\ \forall\ p \in P$$

C2: Cannot exceed the supply of Raw Material A

$$rawSupplyA : \sum_p amtToProduce_p \times rawA_p \leq 4,000$$

C3: Cannot exceed the supply of Raw Material B

$$rawSupplyB : \sum_p amtToProduce_p \times rawB_p \leq 6,000$$

C4: Ratio of 3:2 for High and Semi Gloss, respectively

- Since $\frac{3}{2} = 1.5$, the amount of high gloss produced must always be $1.5\times$ semi gloss

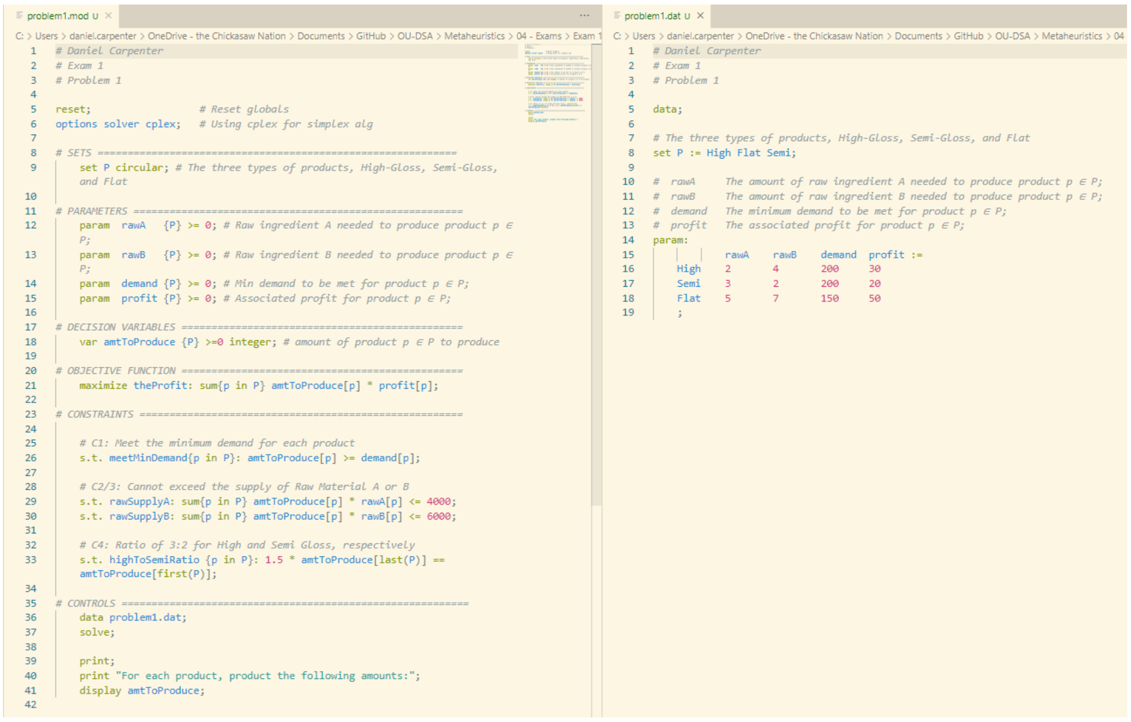
$$highToSemiRatio : 1.5 \times amtToProduce_{Semi \in P} = amtToProduce_{High \in P}$$

C5: Non-Negativity Constraints and is Integer

$$amtToProduce \geq 0, \in \mathbb{Z}$$

1.2 Code and Output

1.2.1 Code



problem1.mod U ×

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```
# Daniel Carpenter
# Exam 1
# Problem 1

reset;                  # Reset globals
options solver cplex;   # Using cplex for simplex alg

# SETS ============================================================
    set P circular; # The three types of products, High-Gloss, Semi-Gloss, and Flat

# PARAMETERS ======================================================
    param  rawA   {P} >= 0; # Raw ingredient A needed to produce product p ∈ P;
    param  rawB   {P} >= 0; # Raw ingredient B needed to produce product p ∈ P;
    param  demand {P} >= 0; # Min demand to be met for product p ∈ P;
    param  profit {P} >= 0; # Associated profit for product p ∈ P;

# DECISION VARIABLES ==============================================
    var amtToProduce {P} >=0 integer; # amount of product p ∈ P to produce

# OBJECTIVE FUNCTION ==============================================
    maximize theProfit: sum{p in P} amtToProduce[p] * profit[p];

# CONSTRAINTS =====================================================

    # C1: Meet the minimum demand for each product
    s.t. meetMinDemand{p in P}: amtToProduce[p] >= demand[p];

    # C2/3: Cannot exceed the supply of Raw Material A or B
    s.t. rawSupplyA: sum{p in P} amtToProduce[p] * rawA[p] <= 4000;
    s.t. rawSupplyB: sum{p in P} amtToProduce[p] * rawB[p] <= 6000;

    # C4: Ratio of 3:2 for High and Semi Gloss, respectively
    s.t. highToSemiRatio {p in P}: 1.5 * amtToProduce[last(P)] == amtToProduce[first(P)];

# CONTROLS ========================================================
    data problem1.dat;
    solve;

    print;
    print "For each product, product the following amounts:";
    display amtToProduce;

```

problem1.dat U ×

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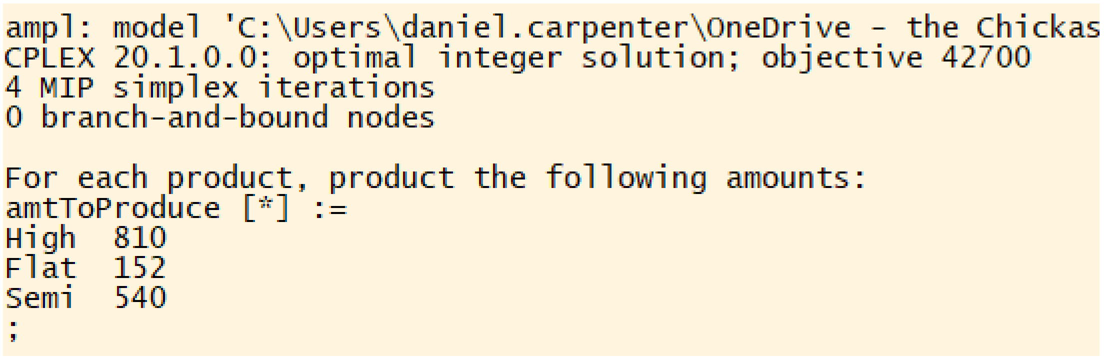
```
# Daniel Carpenter
# Exam 1
# Problem 1

data;

# The three types of products, High-Gloss, Semi-Gloss, and Flat
set P := High Flat Semi;

#  rawA     The amount of raw ingredient A needed to produce product p ∈ P;
#  rawB     The amount of raw ingredient B needed to produce product p ∈ P;
#  demand   The minimum demand to be met for product p ∈ P;
#  profit   The associated profit for product p ∈ P;
param:
            rawA    rawB    demand  profit :=
    High    2       4       200     30
    Semi    3       2       200     20
    Flat    5       7       150     50
    ;
```

1.2.2 Output

- Not High to Semi is a 3:2 ratio and all demand and supply constraints are satisfied.



```
ampl: model 'C:\Users\daniel.carpenter\OneDrive - the Chickas
CPLEX 20.1.0.0: optimal integer solution; objective 42700
4 MIP simplex iterations
0 branch-and-bound nodes

For each product, product the following amounts:
amtToProduce [*] :=
High   810
Flat   152
Semi   540
;
```

2 - Problem 2

2.1 Additions to Model

Please assume that the mathematical formulation in the course videos are present as well (see code), just named objects are named differently but hopefully are convienent to interpret

2.1.1 Parameters

Name	Description
M	"Big M," which is a large scaler used to help model disjunctive constraints

2.1.2 New Decision Vars

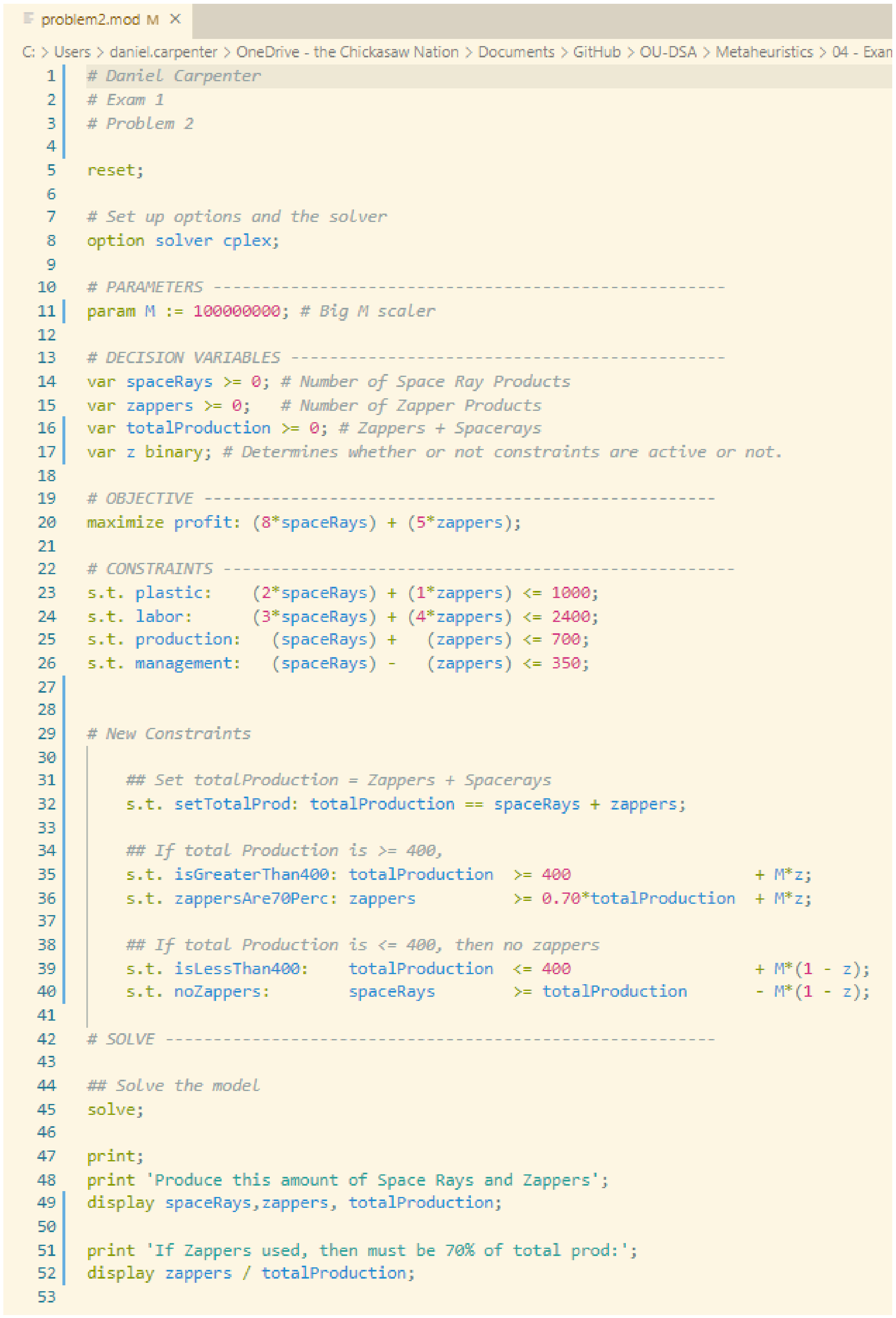
Name	Description
z	Determines which constraint to activate.
$totalProduction$	Simple variable that is Zappers + Spacerays

2.1.3 New Constraints

Description	Constraint
Set Total Production to Zappers + Spacerays	$setTotalProd : totalProduction = spaceRays + zappers$
If total production is ≥ 400	$isGreaterThan400 : totalProduction \geq 400 + M \times z$
then zappers must be 70% of total	$zappersAre70Perc : zappers \geq 0.70 \times totalProduction + M \times z$
If total Production is ≤ 400	$isLessThan400 : totalProduction \leq 400 + M \times (1 - z)$
then no zappers at all	$noZappers : spaceRays \geq totalProduction - M \times (1 - z)$

2.2 Code and Output

2.2.1 Code


problem2.mod M ×
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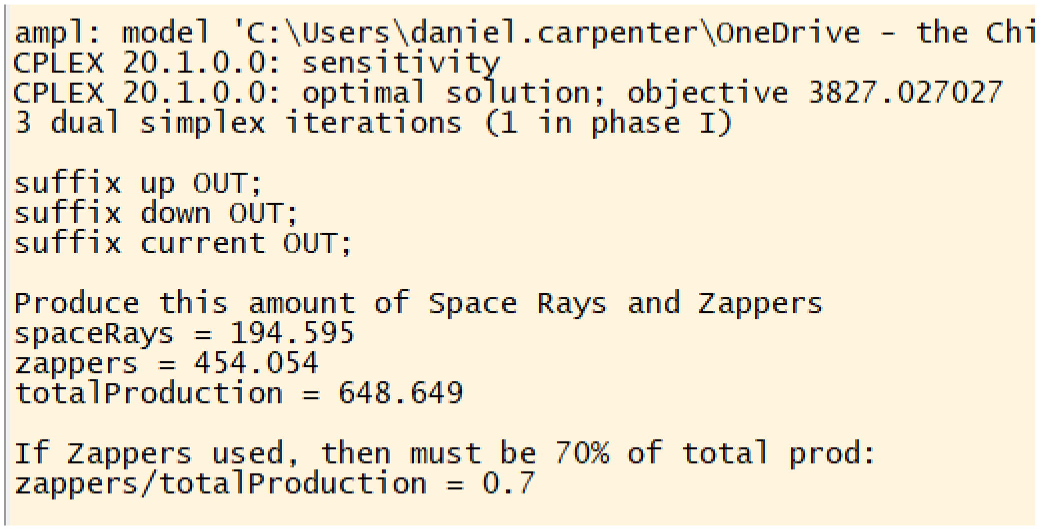
```
# Daniel Carpenter
# Exam 1
# Problem 2

reset;

# Set up options and the solver
option solver cplex;

# PARAMETERS -----------------------------------------------------
param M := 100000000; # Big M scaler

# DECISION VARIABLES ---------------------------------------------
var spaceRays >= 0; # Number of Space Ray Products
var zappers >= 0;   # Number of Zapper Products
var totalProduction >= 0; # Zappers + Spacerays
var z binary; # Determines whether or not constraints are active or not.

# OBJECTIVE -----------------------------------------------------
maximize profit: (8*spaceRays) + (5*zappers);

# CONSTRAINTS ----------------------------------------------------
s.t. plastic:    (2*spaceRays) + (1*zappers) <= 1000;
s.t. labor:      (3*spaceRays) + (4*zappers) <= 2400;
s.t. production:   (spaceRays) +   (zappers) <= 700;
s.t. management:   (spaceRays) -   (zappers) <= 350;


# New Constraints

    ## Set totalProduction = Zappers + Spacerays
    s.t. setTotalProd: totalProduction == spaceRays + zappers;

    ## If total Production is >= 400,
    s.t. isGreaterThan400: totalProduction  >= 400                  + M*z;
    s.t. zappersAre70Perc: zappers          >= 0.70*totalProduction + M*z;

    ## If total Production is <= 400, then no zappers
    s.t. isLessThan400:    totalProduction  <= 400                  + M*(1 - z);
    s.t. noZappers:        spaceRays        >= totalProduction      - M*(1 - z);

# SOLVE ---------------------------------------------------------

## Solve the model
solve;

print;
print 'Produce this amount of Space Rays and Zappers';
display spaceRays,zappers, totalProduction;

print 'If Zappers used, then must be 70% of total prod:';
display zappers / totalProduction;
```

2.2.2 Output

- Optimal solution included more than 400, so zappers are 70% of production



```
ampl: model 'C:\Users\daniel.carpenter\OneDrive - the Chi
CPLEX 20.1.0.0: sensitivity
CPLEX 20.1.0.0: optimal solution; objective 3827.027027
3 dual simplex iterations (1 in phase I)

suffix up OUT;
suffix down OUT;
suffix current OUT;

Produce this amount of Space Rays and Zappers
spaceRays = 194.595
zappers = 454.054
totalProduction = 648.649

If Zappers used, then must be 70% of total prod:
zappers/totalProduction = 0.7
```

3 - Problem 3

3.1 Model Overview

3.1.1 Assumptions and Calculations for Network Flow Diagram

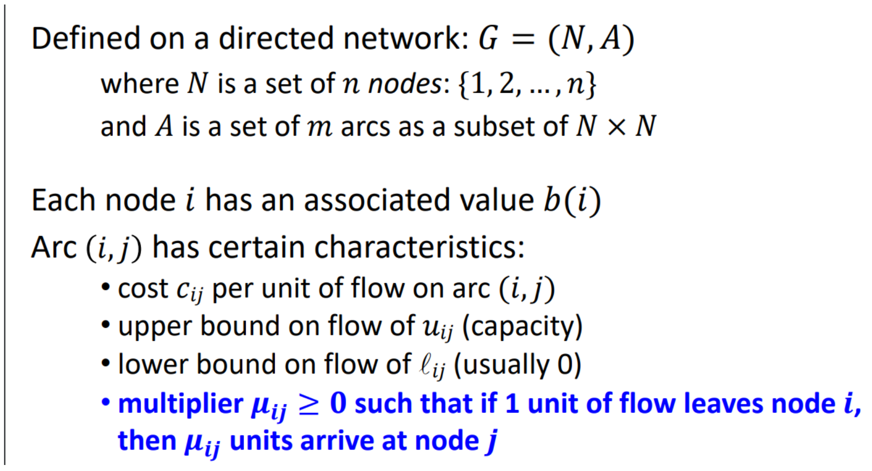
- Starting/Ending inventory is 2,000
- In period 1, can only use inventory to meet demand since it takes a period to produce the chemicals
- μ used to either show:
  - The 85:100 ratio when converting raw materials in production (seen as 0.85)
  - Converting the final product into dollars when selling, or
  - Showing the 8% degradation when storing the final product in inventory that did not sell (seen as 0.92, which is 1 minus 8%)

3.1.2 Network Flow Diagram

3.2 Mathematical Formulation

3.2.1 Sets, Parameters, Decision Vars

Set Name	Description
$NODES$	Set of all nodes in above network flow diagram:



Defined on a directed network: $G = (N, A)$

where N is a set of n *nodes*: $\{1, 2, \ldots, n\}$

and A is a set of m arcs as a subset of $N \times N$

Each node i has an associated value $b(i)$

Arc (i, j) has certain characteristics:

- cost c_{ij} per unit of flow on arc (i, j)
- upper bound on flow of u_{ij} (capacity)
- lower bound on flow of ℓ_{ij} (usually 0)
- **multiplier $\mu_{ij} \geq 0$ such that if 1 unit of flow leaves node i, then μ_{ij} units arrive at node j**

3.2.2 Objective, and Constraints

$$
\begin{aligned}
\text{minimize} \quad & \sum_{(i,j)\in A} c_{ij}x_{ij} \\
\text{subject to} \quad & \sum_{j:(i,j)\in A} x_{ij} - \sum_{j:(j,i)\in A} \overset{\text{Multiplier}}{\mu_{ji}} x_{ji} = b_i \quad \forall i \in N \\
& l_{ij} \leq x_{ij} \leq u_{ij} \qquad\qquad\qquad\qquad \forall (i,j) \in A
\end{aligned}
$$

- Upper and lower bounds use to direct the flow of the product

3.3 Code and Output

3.3.1 Model: `Problem3.mod`

- Used `gmcnfp.txt` from course website and renamed to `Problem3.mod`.
- Added `Problem3.dat; solve;` and `display x;`

3.3.2 Data: `Problem3.dat`