

Homework 3 - Integer Programming

Adv. Analytics and Metaheuristics

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March 2022

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1 - Problem 1

1.1 Mathematical Formulation

1.1.1 Sets

Set Name	Description
<i>GENERATORS</i>	Set of generators i that can be used (A,B,C)
<i>PERIODS</i>	2 possible periods p (1, 2) in the production day

1.1.2 Parameters

Parameter Name	Description
S_i	Fixed cost to start a generator ($i \in GENERATORS$) in the entire day
F_i	Fixed cost to operate a generator ($i \in GENERATORS$) in any period
C_i	Variable cost per megawatt to operator a generator ($i \in GENERATORS$) in any period
U_i	Max. megawatts generated for generator ($i \in GENERATORS$) in any period
$demand_p$	Total demanded megawatts for period ($p \in PERIODS$)
M_i	Value to map watts used by each generator ($i \in GENERATORS$)

1.1.3 Decision Variables

Variable Name	Description
$watts_{i,p}$	<i>Integer variable:</i> Number of watts to produce per generator ($i \in GENERATORS$) per period ($p \in PERIODS$)
$x_{i,p}$	<i>Binary variable:</i> 1 if a generator ($i \in GENERATORS$) is in period p ($p \in PERIODS$), 0 if not turned on at all
y_i	<i>Binary variable:</i> 1 if a generator ($i \in GENERATORS$) is used, 0 if not turned on at all

1.1.4 Objective Function

$$\text{minimize cost : } \sum_{i \in GENERATORS} \left((\sum_{p \in PERIODS} (watts_{i,p}) \times C_i) + (F_i \times \sum_{p \in PERIODS} x_{i,p}) + (S_i \times y_i) \right)$$

1.1.5 Constraints

C1: For each period, meet the demanded megawatts

$$\text{requiredWatts : } \sum_{i \in GENERATORS} (watts_{i,p}) = demand_p, \forall p \in PERIODS$$

C2: For each generator, don't surpass the allowable megawatts

$$\text{upperBound : } \sum_{p \in PERIODS} (watts_{i,p}) \leq U_i, \forall i \in GENERATORS$$

C3: For each generator, map decision variables together to account for the fixed costs in a given day S_i

$$\text{mapVars : } \sum_{p \in PERIODS} (watts_{i,p}) \leq M_i \times y_i, \forall i \in GENERATORS$$

C4: For each generator and period, map decision variables y and $watts$ together to account for the fixed costs in a per period p

$$\text{mapVars2 : } watts_{i,p} \leq M_i \times x_{i,p}, \forall i \in GENERATORS, p \in PERIODS$$

1.2 Code and Output

1.2.1 Code

```

1 # Homework 3 - Integer Programming
2 # Adv. Analytics and Metaheuristics
3 # Daniel Carpenter and Iker Zarandona
4 # March 2022
5 # Problem 1
6
7 reset; # Reset globals
8 options solver cplex; # Using cplex for simplex alg
9
10 # SETS =====
11 set GENERATORS; # Set of generators to use
12 set PERIODS; # Periods in the day
13
14 # PARAMETERS =====
15 param S (GENERATORS) >= 0; # Fixed cost to start
16 param F (GENERATORS) >= 0; # Fixed cost to operate
17 param C (GENERATORS) >= 0; # Variable cost per megawatt
18 param U (GENERATORS) >= 0; # Upper bound on megawatts in a day
19 param M (GENERATORS) >= 0; # Map decision variables
20 param demand (PERIODS) >= 0; # Megawatts required per period
21
22 # DECISION VARIABLES =====
23 var watts {GENERATORS, PERIODS} >= 0 Integer; # Megawatts to use
24 var x (GENERATORS, PERIODS) binary; # Map to watts for fixed daily costs
25 var y (GENERATORS) binary; # Map to watts for fixed daily costs
26
27 # OBJECTIVE FUNCTION =====
28 minimize cost:
29 (sum(i in GENERATORS) (sum(p in PERIODS) watts[i,p])*C[i])
30 + (sum(i in GENERATORS) F[i]*sum(p in PERIODS)x[i,p])
31 + (sum(i in GENERATORS) S[i]*y[i]);
32
33 # CONSTRAINTS =====
34
35 # C1: For each period, meet the demanded megawatts
36 subject to requiredWatts (p in PERIODS):
37 (sum(i in GENERATORS) watts[i,p]) = demand[p];
38
39 # C2: For each generator, don't surpass the allowable megawatts
40 subject to upperBound (i in GENERATORS):
41 (sum(p in PERIODS) watts[i,p]) <= U[i];
42
43 # C3: For each generator, map decision variables together to account for the
44 fixed costs in a given day S1
45 subject to mapVars (i in GENERATORS):
46 (sum(p in PERIODS) watts[i,p]) <= M[i] * y[i];
47
48 # C4: For each generator and period, map decision variables y and watts together
49 to account for the fixed costs in a per period p
50 subject to mapVars2 (i in GENERATORS, p in PERIODS):
51 watts[i,p] <= M[i] * x[i,p];
52
53 # CONTROLS =====
54 data group23_HW3_p1.dat;
55 solve;
56
57 print;
58 print "Which generators are used?";
59 display y;
60
61 print "Which periods were the generators used?";
62 display x;
63
64 print "Optimal Amount of Megawatts for each generator and period:";
65 display watts;
66

```

```

1 # Homework 3 - Integer Programming
2 # Adv. Analytics and Metaheuristics
3 # Daniel Carpenter and Iker Zarandona
4 # March 2022
5 # Problem 1
6
7 # SETS =====
8 set GENERATORS := A B C; # Set of generators to use
9 set PERIODS := 1 2; # Periods in the day
10
11 # PARAMETERS =====
12
13 # S: Fixed cost to start a generator (i ∈ GENERATORS) in the entire day
14 # F: Fixed cost to operate a generator (i ∈ GENERATORS) in any period
15 # C: Variable cost per megawatt to operate a generator (i ∈ GENERATORS) in any period
16 # U: Max. megawatts generated for generator (i ∈ GENERATORS) in any period
17 # M: Value to map watts used by each generator (i ∈ GENERATORS)
18 # Set to be slightly over the max megawatts per day
19 param: S F C U M :=
20 A 3000 700 5.00 2100 2200
21 B 2000 500 4.00 1800 1900
22 C 1000 900 7.00 3000 3100
23 ;
24
25 # Total demanded megawatts for period (p ∈ PERIODS)
26 param demand :=
27 1 2900
28 2 3900
29 ;

```

1.2.2 Output

```
ampl: model 'C:\Users\daniel.carpenter\OneDrive - the Chickasaw
CPLEX 20.1.0.0: optimal integer solution; objective 46100
7 MIP simplex iterations
0 branch-and-bound nodes

which generators are used?
y [*] :=
A 1
B 1
C 1
;

which periods were the generators used?
x :=
A 1 0
A 2 1
B 1 0
B 2 1
C 1 1
C 2 0
;

Optimal Amount of Megawatts for each generator and period:
watts :=
A 1 0
A 2 2100
B 1 0
B 2 1800
C 1 2900
C 2 0
;
```

1.2.2.1 Analysis of the Output

- The minimized cost is \$46,100
- Generator A , B , and C run
- Generator C runs in period 1. Generator A and B run in period 2
- Generator A produces 2,100 megawatts in total
- Generator B produces 1,800 megawatts in total
- Generator C produces 2,900 megawatts in total