example IP: basic facility location

Keys:
- Choose between a few options.
- Constraints for at least, at most,
- Contingencies like: can't do this unless this already is happening, etc.

A company is thinking about building new facilities in LA and SF.

	capital needed	expected profit
1. factory in LA	$6M	$9M
2. factory in SF	$3M	$5M
3. warehouse in LA	$5M	$6M
4. warehouse in SF	$2M	$4M

Total capital available for investment: $10M

Which facilities should be built to maximize the total profit?

example IP: basic facility location

- Define decision variables (i = 1, 2, 3, 4):

$$x_i = \begin{cases} 1 & \text{if facility } i \text{ is built} \\ 0 & \text{if not} \end{cases}$$

- Total expected benefit: $9x_1 + 5x_2 + 6x_3 + 4x_4$
- Total capital needed: $6x_1 + 3x_2 + 5x_3 + 2x_4$

➢ IP model:

$$\max 9x_1 + 5x_2 + 6x_3 + 4x_4$$

$$\text{s. t. } 6x_1 + 3x_2 + 5x_3 + 2x_4 \leq 10$$

$$x_1, x_2, x_3, x_4 \in \{0,1\}$$

adding new requirements

- Extra requirement:

  build *at most* *one* of the two warehouses

  $$x_3 + x_4 \leq 1$$

- Extra requirement:

  build *at least* *one* of the two factories

  $$x_1 + x_2 \geq 1$$

contingent decisions

- *Additional requirement:* Can't build a warehouse u*nless* there is a factory in the city:

$$x_3 \leq x_1 \text{ (LA)}$$
$$x_4 \leq x_2 \text{ (SF)}$$

- *Additional requirement:* Can't select option 3 *unless* at least one of options 1 and 2 is selected.

$$x_3 \leq x_1 + x_2$$

contingent decisions

- *Additional requirement:*

  Can't select option 4 *unless* at least two of options 1, 2 and 3 are selected.

$$2x_4 \leq x_1 + x_2 + x_3$$

$$\max 9x_1 + 5x_2 + 6x_3 + 4x_4$$

$$\begin{aligned}
\text{s. t.}\quad 6x_1 + 3x_2 + 5x_3 + 2x_4 &\leq 10 \\
x_3 + x_4 &\leq 1 \\
x_1 + x_2 &\geq 1 \\
x_3 - x_1 &\leq 0 \\
x_4 - x_2 &\leq 0 \\
x_3 - (x_1 + x_2) &\leq 0 \\
2x_4 - (x_1 + x_2 + x_3) &\leq 0 \\
x_1, x_2, x_3, x_4 &\in \{0,1\}
\end{aligned}$$