Comprehensive Final Exam

Adv. Analytics and Metaheuristics

Daniel Carpenter

May 2022

Contents

1 - *Question 1* (Version 1)

1.1 *Part 1:* Mathematical Formulation

1.1.1 Sets

$NewBuildTypes$: Set of new build types $b \in (Homes,\ Duplex,\ MiniPark)$

1.1.2 Parameters

Parameter	Description	Default Value
$budget$	Federal grant allocation to revitalize neighborhoods	\$15MM total budget
$maxBuildingDemod$	Max amount of buildings that can be demolished	300 total buildings
$demoCost$	Cost of demolishing a building	\$4,000 per building
$freedUpSpace$	Acreage generated from demolishing a building	0.25 per building
$newBuildSpace_b$	Amount of acreage that a new building ($b \in NewBuildTypes$) consumes	$Homes$: 0.2, $Duplex$: 0.4, $MiniPark$: 1.0
$newBuildTax_b$	Amount of tax dollars generated from a new building ($b \in NewBuildTypes$)	$Homes$: 1,500, $Duplex$: 2,750, $MiniPark$: 500
$newBuildCost_b$	Amount of dollars used to create a new building ($b \in NewBuildTypes$)	$Homes$: 150,000, $Duplex$: 190,000, $MiniPark$: 20,000
$newBuildPercShare_b$	Minimum required percentage share of new buildings ($b \in NewBuildTypes$) created	$Homes$: 20%, $Duplex$: 10%, $MiniPark$: 5%

1.1.3 Decision Variables

Variable	Description
$numOldBuildsDemod$	Number of old buildings to demolish
$numNewBuilds_b$	Number of new buildings ($b \in NewBuildTypes$) to produce
$newBuildTotalCost$	Variables to hold total cost of new builds ($b \in NewBuildTypes$). Calculation: $\sum_{b \in NewBuildTypes}(numNewBuilds_b \times newBuildCost_b)$
$oldDemoTotalCost$	Variables to hold total cost of old demolitions. Calculation: $numOldBuildsDemod \times demoCost$
$sumOfNewBuilds$	# Variable to hold the sum of all new build types over all New build types ($b \in NewBuildTypes$). Calculation: $\sum_{b \in NewBuildTypes}(numNewBuilds_b)$

1.1.4 Objective

Maximize the tax revenue from the projects

$$maximize\ taxRevenue : \sum_{b \in NewBuildTypes} (numNewBuilds_b \times newBuildTax_b)$$

1.1.5 Constraints

C1 Spend less than or equal to the federal budget (see variable definitions)

$$meetTheBudget : newBuildTotalCost + oldDemoTotalCost \leq budget$$

C2 Can only produce new builds using the demolished buildings land

$$useAvailLand : \sum_{b \in NewBuildTypes} (numNewBuilds_b \times newBuildSpace_b) \\ \leq numOldBuildsDemod \times freedUpSpace$$

C3 Can only clear a certain amount of old buildings

$$maxBuildingsCleared : numOldBuildsDemod \leq maxBuildingDemod$$

C4 For each new build type ($b \in NewBuildTypes$), the percentage share of the new build type must meet the minimum required (see variables)

$$share : numNewBuilds_b \geq newBuildPercShare_b \times sumOfNewBuilds, \\ \forall\ b \in Businesses$$

C5 Non-negativity and integer constraints

$$numOldBuildsDemod \in \mathbb{Z}, \geq 0$$
$$numNewBuilds_b \in \mathbb{Z}, \geq 0,\ \forall\ b \in NewBuildTypes$$

1.2 *Part 2*: AMPL Code & Output

1.2.1 AMPL Code

Data *problem1.dat*

```
data;

# SETS ===============================================================

# Set of new build types
set NewBuildTypes := Homes Duplex MiniPark;

# PARAMETERS =========================================================

param budget            := 15000000; # federal budget
param maxBuildingDemod := 300;      # max buildings can be demo'd
param demoCost          := 4000;    # Cost of each demolition
param freedUpSpace      := 0.25;    # Freed up space from demolition

# Amount of acreage that a new building (b in NewBuildTypes) consumes
param: newBuildSpace :=
       Homes    0.2
       Duplex   0.4
       MiniPark 1.0
       ;
# Amount of tax dollars generated from a new building (b in NewBuildTypes)
param: newBuildTax :=
       Homes    1500
       Duplex   2750
       MiniPark 500
       ;

# Amount of dollars used to create a new building (b in NewBuildTypes)
param: newBuildCost :=
       Homes    150000
       Duplex   190000
       MiniPark 20000
       ;

# Minimum required percentage share of new buildings (b in NewBuildTypes) created
param: newBuildPercShare :=
       Homes    0.20
       Duplex   0.10
       MiniPark 0.05
```

```
        ;
```

Model *problem1.mod*

```
reset;                    # Reset globals
options solver cplex;     # Using cplex for simplex alg

# SETS =========================================================
set NewBuildTypes; # Set of new build types

# PARAMETERS ===================================================
param budget             >= 0; # federal budget
param maxBuildingDemod >= 0; # max buildings can be demo'd
param demoCost           >= 0; # Cost of each demolition
param freedUpSpace       >= 0; # Freed up space from demolition

param newBuildSpace      {NewBuildTypes} >= 0; # new build acreage
param newBuildTax        {NewBuildTypes} >= 0; # n.b. tax generation
param newBuildCost       {NewBuildTypes} >= 0; # n.b. cost
param newBuildPercShare{NewBuildTypes} >= 0; # n.b. min % share

# DECISION VARIABLES ===========================================
var numOldBuildsDemod                  >= 0 integer; # Num old builds to demo
var numNewBuilds      {NewBuildTypes} >= 0 integer; # Num new builds to create

# Variables to hold total cost of new builds over all types
var newBuildTotalCost = sum{b in NewBuildTypes} ( (numNewBuilds[b] * newBuildCost[b]));

# Variables to hold total cost of old demolitions
var oldDemoTotalCost = (numOldBuildsDemod * demoCost) ;

# Variable to hold the sum of all new build types over all New build types
var sumOfNewBuilds =  sum{b in NewBuildTypes}( numNewBuilds[b] );

# OBJECTIVE FUNCTION ===========================================
maximize taxRevenue: sum{b in NewBuildTypes}( numNewBuilds[b] * newBuildTax[b] );

# CONSTRAINTS ==================================================

# C1  Spend less than or equal to the federal budget
s.t. meetTheBudget:
    newBuildTotalCost + oldDemoTotalCost <= budget ;

# C2 Can only produce new builds using the demolished buildings land
```

```
s.t. useAvailLand:
    sum{b in NewBuildTypes}( numNewBuilds[b] * newBuildSpace[b] )
    <= numOldBuildsDemod * freedUpSpace ;

# C3 Can only clear a certain amount of old buildings
s.t. maxBuildingsCleared: numOldBuildsDemod <= maxBuildingDemod ;

# C4 For each new build type (b in NewBuildTypes),
#    the percentage share of the new build type must meet the minimum required
s.t. share {b in NewBuildTypes}:
    numNewBuilds[b] >= newBuildPercShare[b] * sumOfNewBuilds ;

# CONTROLS ========================================================
    data problem1.dat; # retreive data file with sets/param. values
    solve;

    print;

    print "Number of old buildings to demolish and cost (dollars):";
    display numOldBuildsDemod, oldDemoTotalCost ;

    print "Number of new buildings produced and cost (dollars):";
    display numNewBuilds , newBuildTotalCost ;

    print "Total Budget Used (dollars):";
    display newBuildTotalCost + oldDemoTotalCost ;

    print "Part 3: Max Tax Revenue generated (dollars):";
    display taxRevenue ;
```

1.2.2 *Part 2/3:* Solve AMPL Model and Display Solution

```
CPLEX 20.1.0.0: optimal integer solution; objective 199000
6 MIP simplex iterations
0 branch-and-bound nodes

Number of old buildings to demolish and cost (dollars):
numOldBuildsDemod = 137
oldDemoTotalCost = 548000

Number of new buildings produced and cost (dollars):
numNewBuilds [*] :=
  Duplex  62
   Homes  17
MiniPark   6
;

newBuildTotalCost = 14450000

Total Budget Used (dollars):
newBuildTotalCost + oldDemoTotalCost = 14998000

Part 3: Max Tax Revenue generated (dollars):
taxRevenue = 199000
```

2 - *Question 2* (Version 6)

2.1 Code to get Root Node

Root Node is Node 1 in the diagram

2.1.1 Root Node AMPL Model *problem2.mod*

- Note code below also used/altered to test upper and lower bounds of child nodes.

```
reset;
option solver cplex; # Solver

var x >= 0; # Not integer because this is used to check the optimal when fixing a var
var y >= 0; # ""

# Original problem:
maximize theSolution: 2.5*x + 6*y;
    s.t. first:  3*x + 5*y <= 26;
    s.t. second:   x       >= 4;

# Used to check the BnB nodes
# Fixing x = 4 to solve for initial values of y
s.t. checkNode: x = 4;

# Solve and display
solve;
display theSolution, x,y;
```

2.1.2 Output of Root Node AMPL Model (fixing x=4 & solving for y)

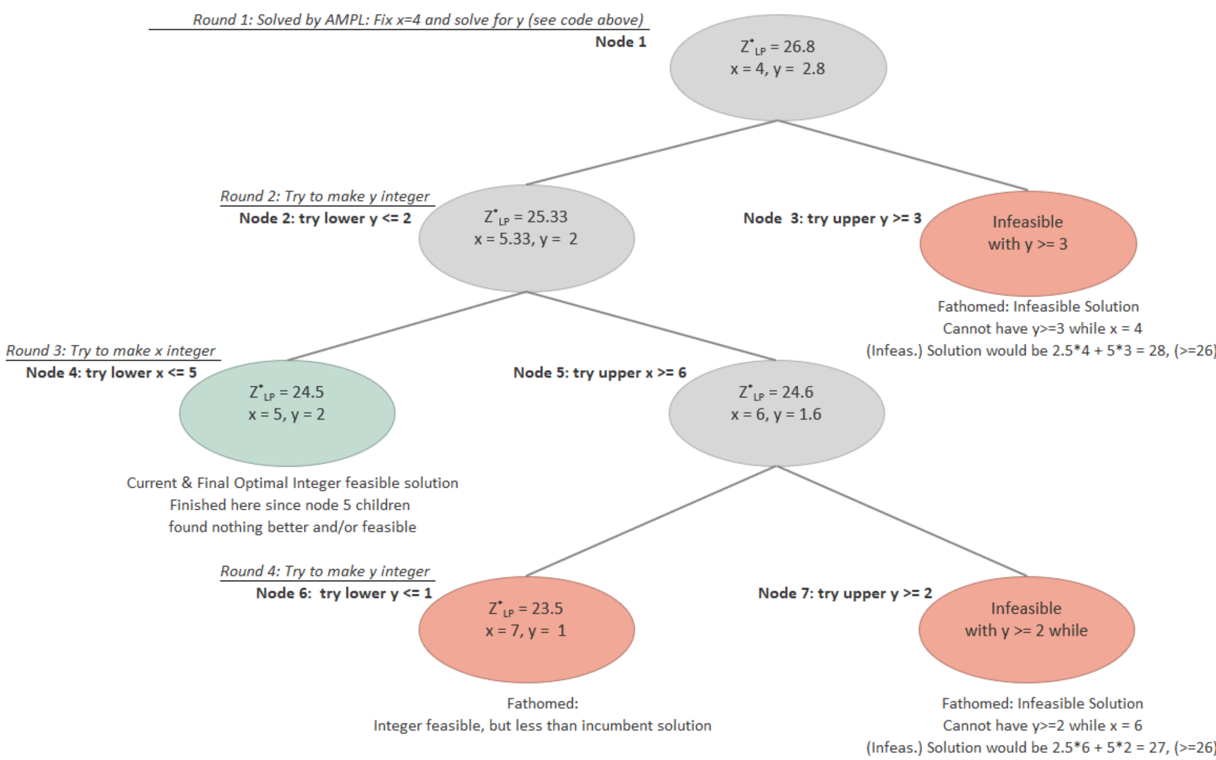
```
CPLEX 20.1.0.0: optimal solution; objective 26.8
0 dual simplex iterations (0 in phase I)
theSolution = 26.8
x = 4
y = 2.8
```

2.2 Branch and Bound Diagram

2.2.1 Summary of Diagram

- Optimal solution reached at node 4 with integer feasible values of $x = 5$, and $y = 2$, `optimal of 24.5`
- Checked 7 total nodes
- Fathomed 3 nodes (see description of "why" in diagram)
- Each "Round" label shows which variable is being checked and for what integer value (lower or upper bound of the parent node)
- Please note that the AMPL model above was used as a calculator to test upper and lower bounds.

2.2.2 Branch and Bound Diagram



3 - *Question 3* (Version 2)

3.1 *Part i*: Hill Climbing

- Please see the below snippet on rightmost comments about the algorithm
- Ended at iteration 3 with minimum value of -1.9991 due to no change in best neighbor

	x_1	x_2	Fitness Evaluation	Calculation Note / Comment
Starting Solution, S_0	2	2	**0.2432**	=x_1 *cos(x_1)*sin(x_2) + 0.5*x_2

	x_1	x_2	Fitness Evaluation	
Current Solution	2	2	**0.2432**	=x_1 *cos(x_1)*sin(x_2) + 0.5*x_2
Done?	FALSE			

Iteration 1				
Best Neighbor (current)	2	2	0.2432	current solution from past iteration

Neighbors of (2, 2)	x_1	x_2	**Fitness Evaluation**	
Neighbor 1 (x1 + 1, x2)	3	2	-1.7006	=x_1 *cos(x_1)*sin(x_2) + 0.5*x_2
Neighbor 2 (x1 - 1, x2)	1	2	1.4913	...
Neighbor 3 (x1, x2 + 1)	2	3	1.3825	...
Neighbor 4 (x1, x2 - 1)	2	1	-0.2004	...

(New) Best Neighbor	**3**	**2**	**-1.7006**	=min(neighbors, best_neighbor)
is Best Neighbor Same as Old?	FALSE			
Done?	FALSE			done if same

	x_1	x_2	Fitness Evaluation	
Current Solution	3	2	**-1.7006**	=x_1 *cos(x_1)*sin(x_2) + 0.5*x_2

Iteration 2				
Best Neighbor (current)	3	2	-1.7006	current solution from past iteration

Neighbors of (3, 2)	x_1	x_2	**Fitness Evaluation**	
Neighbor 1 (x1 + 1, x2)	4	2	-1.3774	=x_1 *cos(x_1)*sin(x_2) + 0.5*x_2
Neighbor 2 (x1 - 1, x2)	2	2	0.2432	...
Neighbor 3 (x1, x2 + 1)	3	3	1.0809	...
Neighbor 4 (x1, x2 - 1)	3	1	-1.9991	...

(New) Best Neighbor	**3**	**1**	**-1.9991**	=min(neighbors, best_neighbor)
is Best Neighbor Same as Old?	FALSE			
Done?	FALSE			done if same

	x_1	x_2	Fitness Evaluation	
Current Solution	3	1	**-1.9991**	=x_1 *cos(x_1)*sin(x_2) + 0.5*x_2

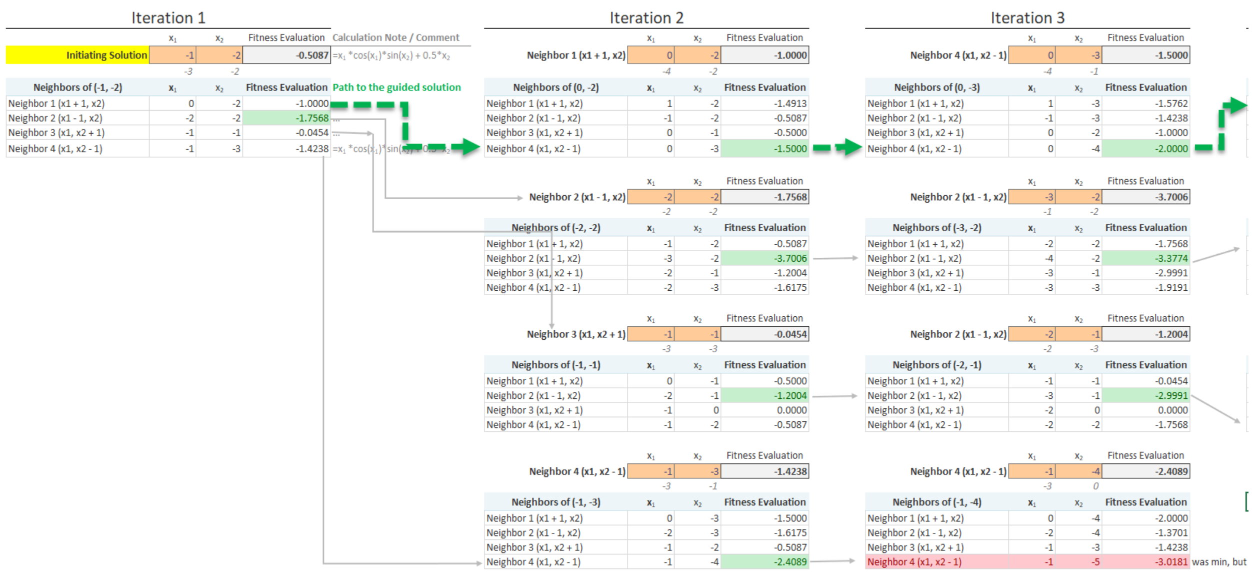
Iteration 3				
Best Neighbor (current)	3	1	-1.9991	current solution from past iteration

Neighbors of (3, 1)	x_1	x_2	**Fitness Evaluation**	
Neighbor 1 (x1 + 1, x2)	4	1	-1.7001	=x_1 *cos(x_1)*sin(x_2) + 0.5*x_2
Neighbor 2 (x1 - 1, x2)	2	1	-0.2004	...
Neighbor 3 (x1, x2 + 1)	3	2	-1.7006	...
Neighbor 4 (x1, x2 - 1)	3	0	0.0000	...

(New) Best Neighbor	**3**	**1**	**-1.9991**	=min(neighbors, best_neighbor)
is Best Neighbor Same as Old?	TRUE			
Done?	TRUE			done if same: end if end while

3.2 *Part ii*: Path Relinking

- Below shows two images with the path re-linking process iterations (from initiation to guided solution).
- Green arrow highlights the path to the guided solution
- **Logic overview:** Evaluates neighbors of initiation solution, then moves along path of each neighbor. Makes best move for each neighbor, then checks difference between current and guided solution. Repeats until finding the guided solution
- `Iteration 3` *duplicated for ease of viewing*



Iteration 1

	x_1	x_2	Fitness Evaluation	Calculation Note / Comment
Initiating Solution	-1	-2	-0.5087	$=x_1 * \cos(x_1) * \sin(x_2) + 0.5 * x_2$
	-3	-2		

Neighbors of (-1, -2)	x_1	x_2	Fitness Evaluation	Path to the guided solution
Neighbor 1 (x1 + 1, x2)	0	-2	-1.0000	
Neighbor 2 (x1 - 1, x2)	-2	-2	-1.7568	...
Neighbor 3 (x1, x2 + 1)	-1	-1	-0.0454	...
Neighbor 4 (x1, x2 - 1)	-1	-3	-1.4238	$=x_1 * \cos(x_1) * \sin(x_2) + 0.5 * x_2$

Iteration 2

	x_1	x_2	Fitness Evaluation
Neighbor 1 (x1 + 1, x2)	0	-2	-1.0000
	-4	-2	

Neighbors of (0, -2)	x_1	x_2	Fitness Evaluation
Neighbor 1 (x1 + 1, x2)	1	-2	-1.4913
Neighbor 2 (x1 - 1, x2)	-1	-2	-0.5087
Neighbor 3 (x1, x2 + 1)	0	-1	-0.5000
Neighbor 4 (x1, x2 - 1)	0	-3	-1.5000

	x_1	x_2	Fitness Evaluation
Neighbor 2 (x1 - 1, x2)	-2	-2	-1.7568
	-2	-2	

Neighbors of (-2, -2)	x_1	x_2	Fitness Evaluation
Neighbor 1 (x1 + 1, x2)	-1	-2	-0.5087
Neighbor 2 (x1 - 1, x2)	-3	-2	-3.7006
Neighbor 3 (x1, x2 + 1)	-2	-1	-1.2004
Neighbor 4 (x1, x2 - 1)	-2	-3	-1.6175

	x_1	x_2	Fitness Evaluation
Neighbor 3 (x1, x2 + 1)	-1	-1	-0.0454
	-3	-3	

Neighbors of (-1, -1)	x_1	x_2	Fitness Evaluation
Neighbor 1 (x1 + 1, x2)	0	-1	-0.5000
Neighbor 2 (x1 - 1, x2)	-2	-1	-1.2004
Neighbor 3 (x1, x2 + 1)	-1	0	0.0000
Neighbor 4 (x1, x2 - 1)	-1	-2	-0.5087

	x_1	x_2	Fitness Evaluation
Neighbor 4 (x1, x2 - 1)	-1	-3	-1.4238
	-3	-1	

Neighbors of (-1, -3)	x_1	x_2	Fitness Evaluation
Neighbor 1 (x1 + 1, x2)	0	-3	-1.5000
Neighbor 2 (x1 - 1, x2)	-2	-3	-1.6175
Neighbor 3 (x1, x2 + 1)	-1	-2	-0.5087
Neighbor 4 (x1, x2 - 1)	-1	-4	-2.4089

Iteration 3

	x_1	x_2	Fitness Evaluation
Neighbor 4 (x1, x2 - 1)	0	-3	-1.5000
	-4	-1	

Neighbors of (0, -3)	x_1	x_2	Fitness Evaluation
Neighbor 1 (x1 + 1, x2)	1	-3	-1.5762
Neighbor 2 (x1 - 1, x2)	-1	-3	-1.4238
Neighbor 3 (x1, x2 + 1)	0	-2	-1.0000
Neighbor 4 (x1, x2 - 1)	0	-4	-2.0000

	x_1	x_2	Fitness Evaluation
Neighbor 2 (x1 - 1, x2)	-3	-2	-3.7006
	-1	-2	

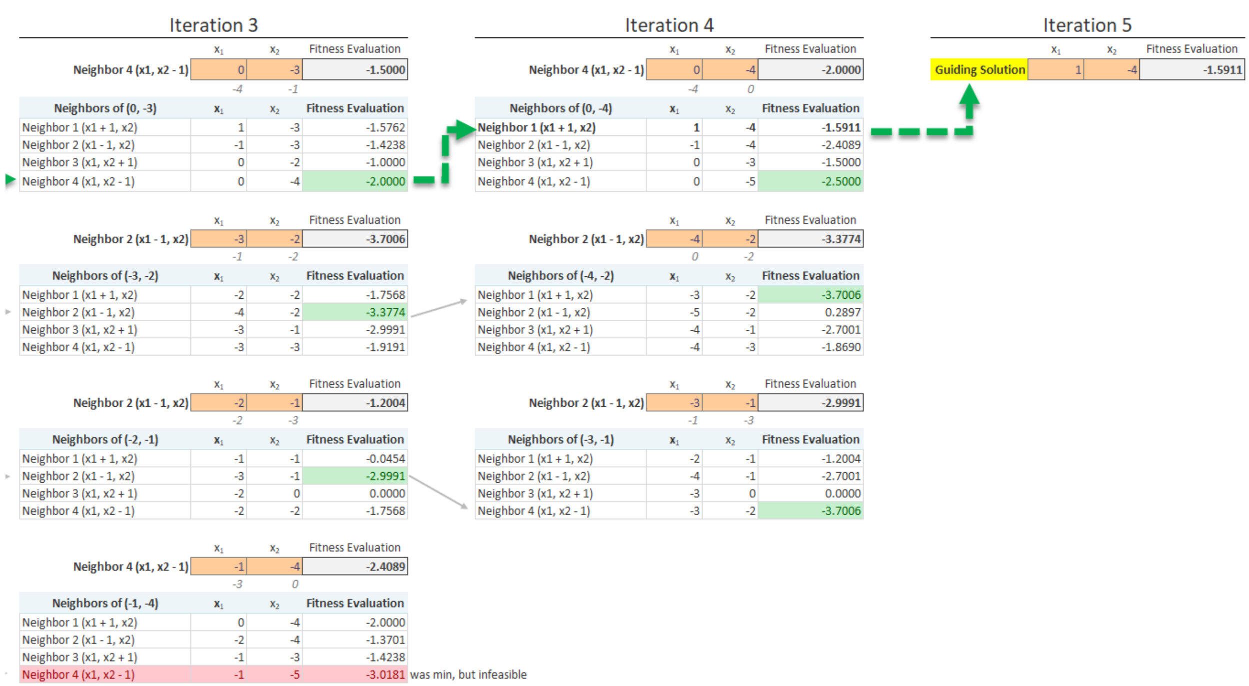
Neighbors of (-3, -2)	x_1	x_2	Fitness Evaluation
Neighbor 1 (x1 + 1, x2)	-2	-2	-1.7568
Neighbor 2 (x1 - 1, x2)	-4	-2	-3.3774
Neighbor 3 (x1, x2 + 1)	-3	-1	-2.9991
Neighbor 4 (x1, x2 - 1)	-3	-3	-1.9191

	x_1	x_2	Fitness Evaluation
Neighbor 2 (x1 - 1, x2)	-2	-1	-1.2004
	-2	-3	

Neighbors of (-2, -1)	x_1	x_2	Fitness Evaluation
Neighbor 1 (x1 + 1, x2)	-1	-1	-0.0454
Neighbor 2 (x1 - 1, x2)	-3	-1	-2.9991
Neighbor 3 (x1, x2 + 1)	-2	0	0.0000
Neighbor 4 (x1, x2 - 1)	-2	-2	-1.7568

	x_1	x_2	Fitness Evaluation
Neighbor 4 (x1, x2 - 1)	-1	-4	-2.4089
	-3	0	

Neighbors of (-1, -4)	x_1	x_2	Fitness Evaluation	
Neighbor 1 (x1 + 1, x2)	0	-4	-2.0000	
Neighbor 2 (x1 - 1, x2)	-2	-4	-1.3701	
Neighbor 3 (x1, x2 + 1)	-1	-3	-1.4238	
Neighbor 4 (x1, x2 - 1)	-1	-5	-3.0181	was min, but



Iteration 3

	x_1	x_2	Fitness Evaluation
Neighbor 4 (x1, x2 - 1)	0	-3	-1.5000
	-4	-1	

Neighbors of (0, -3)	x_1	x_2	Fitness Evaluation
Neighbor 1 (x1 + 1, x2)	1	-3	-1.5762
Neighbor 2 (x1 - 1, x2)	-1	-3	-1.4238
Neighbor 3 (x1, x2 + 1)	0	-2	-1.0000
Neighbor 4 (x1, x2 - 1)	0	-4	-2.0000

	x_1	x_2	Fitness Evaluation
Neighbor 2 (x1 - 1, x2)	-3	-2	-3.7006
	-1	-2	

Neighbors of (-3, -2)	x_1	x_2	Fitness Evaluation
Neighbor 1 (x1 + 1, x2)	-2	-2	-1.7568
Neighbor 2 (x1 - 1, x2)	-4	-2	-3.3774
Neighbor 3 (x1, x2 + 1)	-3	-1	-2.9991
Neighbor 4 (x1, x2 - 1)	-3	-3	-1.9191

	x_1	x_2	Fitness Evaluation
Neighbor 2 (x1 - 1, x2)	-2	-1	-1.2004
	-2	-3	

Neighbors of (-2, -1)	x_1	x_2	Fitness Evaluation
Neighbor 1 (x1 + 1, x2)	-1	-1	-0.0454
Neighbor 2 (x1 - 1, x2)	-3	-1	-2.9991
Neighbor 3 (x1, x2 + 1)	-2	0	0.0000
Neighbor 4 (x1, x2 - 1)	-2	-2	-1.7568

	x_1	x_2	Fitness Evaluation
Neighbor 4 (x1, x2 - 1)	-1	-4	-2.4089
	-3	0	

Neighbors of (-1, -4)	x_1	x_2	Fitness Evaluation	
Neighbor 1 (x1 + 1, x2)	0	-4	-2.0000	
Neighbor 2 (x1 - 1, x2)	-2	-4	-1.3701	
Neighbor 3 (x1, x2 + 1)	-1	-3	-1.4238	
Neighbor 4 (x1, x2 - 1)	-1	-5	-3.0181	was min, but infeasible

Iteration 4

	x_1	x_2	Fitness Evaluation
Neighbor 4 (x1, x2 - 1)	0	-4	-2.0000
	-4	0	

Neighbors of (0, -4)	x_1	x_2	Fitness Evaluation
Neighbor 1 (x1 + 1, x2)	**1**	**-4**	**-1.5911**
Neighbor 2 (x1 - 1, x2)	-1	-4	-2.4089
Neighbor 3 (x1, x2 + 1)	0	-3	-1.5000
Neighbor 4 (x1, x2 - 1)	0	-5	-2.5000

	x_1	x_2	Fitness Evaluation
Neighbor 2 (x1 - 1, x2)	-4	-2	-3.3774
	0	-2	

Neighbors of (-4, -2)	x_1	x_2	Fitness Evaluation
Neighbor 1 (x1 + 1, x2)	-3	-2	-3.7006
Neighbor 2 (x1 - 1, x2)	-5	-2	0.2897
Neighbor 3 (x1, x2 + 1)	-4	-1	-2.7001
Neighbor 4 (x1, x2 - 1)	-4	-3	-1.8690

	x_1	x_2	Fitness Evaluation
Neighbor 2 (x1 - 1, x2)	-3	-1	-2.9991
	-1	-3	

Neighbors of (-3, -1)	x_1	x_2	Fitness Evaluation
Neighbor 1 (x1 + 1, x2)	-2	-1	-1.2004
Neighbor 2 (x1 - 1, x2)	-4	-1	-2.7001
Neighbor 3 (x1, x2 + 1)	-3	0	0.0000
Neighbor 4 (x1, x2 - 1)	-3	-2	-3.7006

Iteration 5

	x_1	x_2	Fitness Evaluation
Guiding Solution	1	-4	-1.5911

3.3 *Part iii*: Simulated Annealing

Using the evaluation function for this question (note language below is R)

```r
# Create an Evaluation function to evaluate fitness
evaluateFitness <- function(x1, x2) { x1*cos(x1)*sin(x2) + 0.5*x2 }
```

The probability p of accepting a move uses the following formula: $p = e^{\frac{-(f(s_1)-f(s_2))}{T}}$.

Where $f(s_1)$ is the current solution, and $f(s_2)$ is the candidate solution since minimization. See calculation below (using evaluateFitness() function)

```r
# Uses the evaluation function to return the probability (see above formula)
calculateProb <- function(temperature, currentPosition, newPosition) {
  # Assuming minimization, evaluation of the Current and Candidate solutions:
  f_s1 = evaluateFitness(currentPosition[1], currentPosition[2]) # f(s[1]) current sol.
  f_s2 = evaluateFitness(newPosition[1],     newPosition[2]    ) # f(s[2]) candidate so

  # If Candidate evaluation is WORSE than the current, accept with prob. p
  if (f_s2 < f_s1) { # '<' since minimization
    p = exp( ( -(f_s1-f_s2) / temperature ) )

  } else { p = 1 } # Else accept move since better

  return( round(p, 3) ) # Round to 3 decimal places
}
```

Now calculate the probability p from the current to the candidate solutions

```r
# What is the probability of accepting a move from
# (4, 0) to candidate neighbor solution (4, 1)?
paste('At temp. 3, the probability from (4, 0) to (4, 1) is',
      calculateProb(temperature     = 3,
                    currentPosition = c(4, 0),
                    newPosition     = c(4, 1)) )
```

```
## [1] "At temp. 3, the probability from (4, 0) to (4, 1) is 0.567"
```

```r
# What is the probability of accepting a move from
# (4, 0) to candidate neighbor solution (4,-1)?
paste('At temp. 3, the probability from (4, 0) to (4, -1) is',
      calculateProb(temperature     = 3,
                    currentPosition = c(4, 0),
                    newPosition     = c(4, -1)) )
```

```
## [1] "At temp. 3, the probability from (4, 0) to (4, -1) is 1"
```

4 - *Question 4* (Version 3)

4.1 *Part (i)* - Roulette Probability

- Evaluates the fitness for each chromosome $c \in Chromosomes$ using $f(x) = x_1 + 2x_2 + 3x_3 + 4x_4 + 5x_5$ as evaluation function.
- Then computes the probability based on the following equation: $\frac{f_c}{\sum_{c \in Chromosomes} f_c}$
- See yellow highlighted cells for final roulette wheel probabilities for each chromosome

Part (i)

These are the evaluation function coefficients for x1..., x5

Eval Coefs	1	2	3	4	5
Eval Function	1*x1 +	2*x2 +	3*x3 +	4*x4 +	5*x5

Step 1: Calculate fitness of each, Then sum up mating pool

Chromosome	x1	x2	x3	x4	x5	Fitness	calculation: 1*x1 + 2*x2 + 3*x3 + 4*x4 + 5*x5
1	1	0	0	0	1	6	=SUMPRODUCT(C10:G10, C$5:G$5)
2	0	0	1	0	1	8	=SUMPRODUCT(C11:G11, C$5:G$5)
3	0	1	0	1	1	11	=SUMPRODUCT(C12:G12, C$5:G$5)
4	1	1	0	0	0	3	=SUMPRODUCT(C13:G13, C$5:G$5)
Sum Total Fitness Value of Pool						28	

Step 2: Calculate Probability of Selection

Chromosome	Fitness	Sum Total Fitness (Step 1)	(i) Prob. of Selection	calculation: Fitness$_{chromosome}$ / sum(FitnessOfAllChromosomes)
1	6	28	21.43%	=C18 / D18
2	8	28	28.57%	=C19 / D19
3	11	28	39.29%	=C20 / D20
4	3	28	10.71%	=C21 / D21

4.2 *Part (ii/iii)* - Breed Offspring

- Select the highest `(Chrom. 3)` and lowest `(Chrom. 4)` fitness valued chromosomes, then split at crossover point to produce offspring
- Offspring split between the second and third bit (highlighted for ease)
- Highest fitness valued offspring: `Offspring 2` with fitness value of `12`

Part (ii)

Assumes no use of the probability of selection from part (i)

Step 1: Find the Lowest and Highest Fitness Value Chromosomes (Note Crossover point between 2nd and 3rd bit - highlighted for ease)

Chromosome	x1	x2	x3	x4	x5	Fitness	*Note on selection*
3	0	1	0	1	1	11	Highest
4	1	1	0	0	0	3	Lowest

Step 2 - Offspring: Breed the above chromosomes at crossover point

Offspring	x1	x2	x3	x4	x5	Fitness	calculation: 1*x1 + 2*x2 + 3*x3 + 4*x4 + 5*x5
Offspring 1	0	1	0	0	0	2	=SUMPRODUCT(C35:G35, C$5:G$5)
Offspring 2	1	1	0	1	1	12	=SUMPRODUCT(C36:G36, C$5:G$5)

5 - *Question 5* (Version 2)

5.1 *Part (i)* - Global Best

5.1.1 Assumptions

- Uses all calculations and parameter values from the problem question. See header in picture as well for the calculation
- Parameters of interest highlighted in orange
- See `global` best logic in picture below

5.1.2 Solution

- Particle 1's Next `Velocity: (-1.90, 3.55, 3.70)`
- Particle 1's Next `Position: (12.10, 8.55, 5.70)`

5.1.3 Relevant Work

Part (i)

					Inertia			Cognitive					Social			
		Next Position	*Next Vel.*		*I. Weight*	*Curr. Vel.*		""	""	*P. Best*	*Curr. Pos.*		""	""	*G. Best*	*Curr. Pos.*
Particle 1's...:	**Position Index**	**X[t+1] = V[t+1] + X[t]**	**V[t+1]**	=	**w**	**V[t]**	+	**phi[1]**	**r[1]**	**(P[i]**	**- X[t])**	+	**phi[2]**	**r[2]**	**(P[g]**	**- X[t])**
	Position Idx. 1	**12.10**	**-1.90**		1	1		1	0.5	10	14		1	0.15	8	14
	Position Idx. 2	**8.55**	**3.55**		1	0		1	0.5	13	5		1	0.15	2	5
	Position Idx. 3	**5.70**	**3.70**		1	1		1	0.5	8	2		1	0.15	0	2

Particle 1's Next Velocity: (-1.90, 3.55, 3.70)
Particle 1's Next Position: (12.10, 8.55, 5.70)

Global Best Logic:
Highest personal best fitness value is 290, which is from particle 4
Use particle 4's personal best position for the global best

5.2 *Part (ii)* - Local Best w/Ring

5.2.1 Assumptions

- Local best paramaters highlighted in yellow
- See `local` best logic with `ring` structure in picture below

5.2.2 Solution

- Particle 1's Next `Velocity: (-0.40, 4.30, 4.45)`
- Particle 1's Next `Position: (13.60, 9.30, 6.45)`

5.2.3 Relevant Work

Part (ii)

					Inertia			Cognitive					Social			
		Next Position	*Next Vel.*		*I. Weight*	*Curr. Vel.*		""	""	*P. Best*	*Curr. Pos.*		""	""	*L. Best*	*Curr. Pos.*
Particle 1's...:	**Position Index**	**X[t+1] = V[t+1] + X[t]**	**V[t+1]**	=	**w**	**V[t]**	+	**phi[1]**	**r[1]**	**(P[i]**	**- X[t])**	+	**phi[2]**	**r[2]**	**(P[1]**	**- X[t])**
	Position Idx. 1	**13.60**	**-0.40**		1	1		1	0.5	10	14		1	0.15	18	14
	Position Idx. 2	**9.30**	**4.30**		1	0		1	0.5	13	5		1	0.15	7	5
	Position Idx. 3	**6.45**	**4.45**		1	1		1	0.5	8	2		1	0.15	5	2

Particle 1's Next Velocity: (-0.40, 4.30, 4.45)
Particle 1's Next Position: (13.60, 9.30, 6.45)

Local Best Logic:

Neighbors of particle 1 are:

Particle 2 Personal Best Fitness value: 120

Particle 5 Personal Best Fitness value: 150

Use 5's p. best position since it has the highest personal best