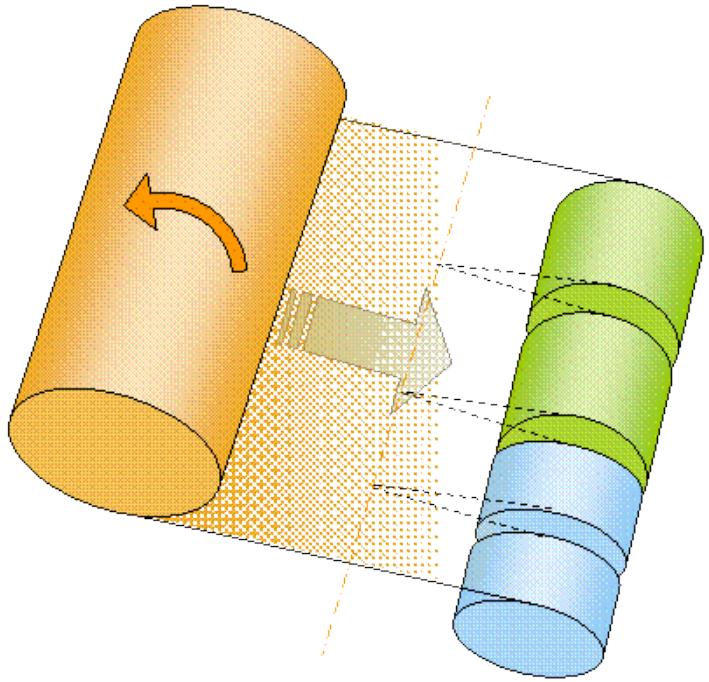
LINEAR PROGRAMMING MODELING EXAMPLES:

Cutting Stock (Trim Loss) Problem

trim-loss or cutting stock problem

- Given paper rolls of fixed width and a set of orders for rolls of smaller widths, the objective is to determine how to cut the rolls into smaller widths to fulfill the orders in such a way as to minimize the amount of scrap.
- Classic (hard) integer programming problem, but we will start with an LP relaxation.

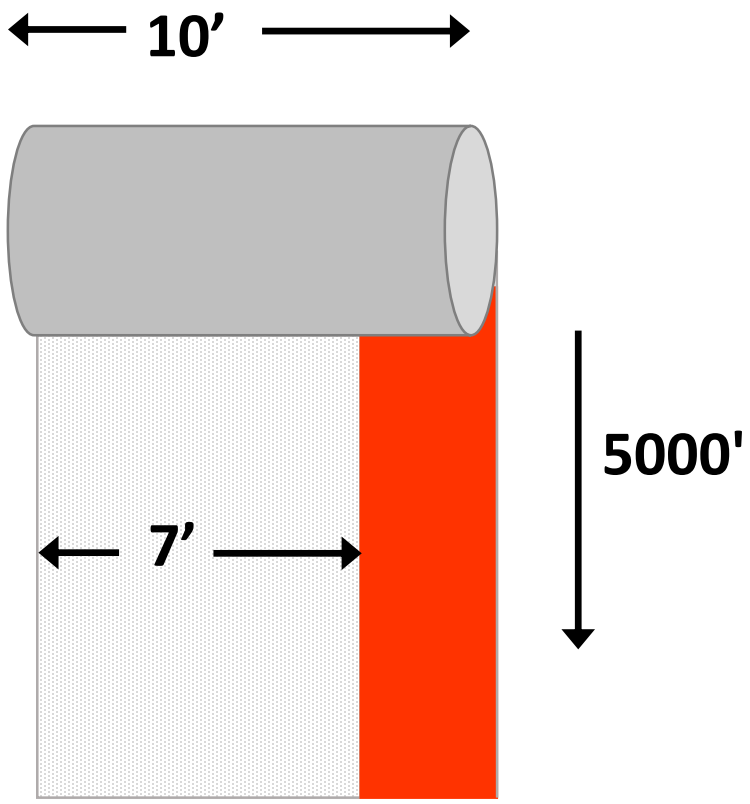


trim-loss or cutting stock problem

Three special orders for rolls of paper have been placed at a paper mill.

The orders are to be cut from standard rolls (5000’ in length) of 10’ and 20’ widths.

Problem: What is trim-loss?



trim-loss or cutting stock problem

Three special orders for rolls of paper have been placed at a paper mill. The orders are to be cut from standard rolls of 10’ and 20’ widths.

Order	Width	Length
1	5’	10,000’
2	7’	30,000’
3	9’	20,000’

Assumption: Lengthwise strips can be taped together

Goal: Throw away as little as possible

What are the decision variables for this problem?

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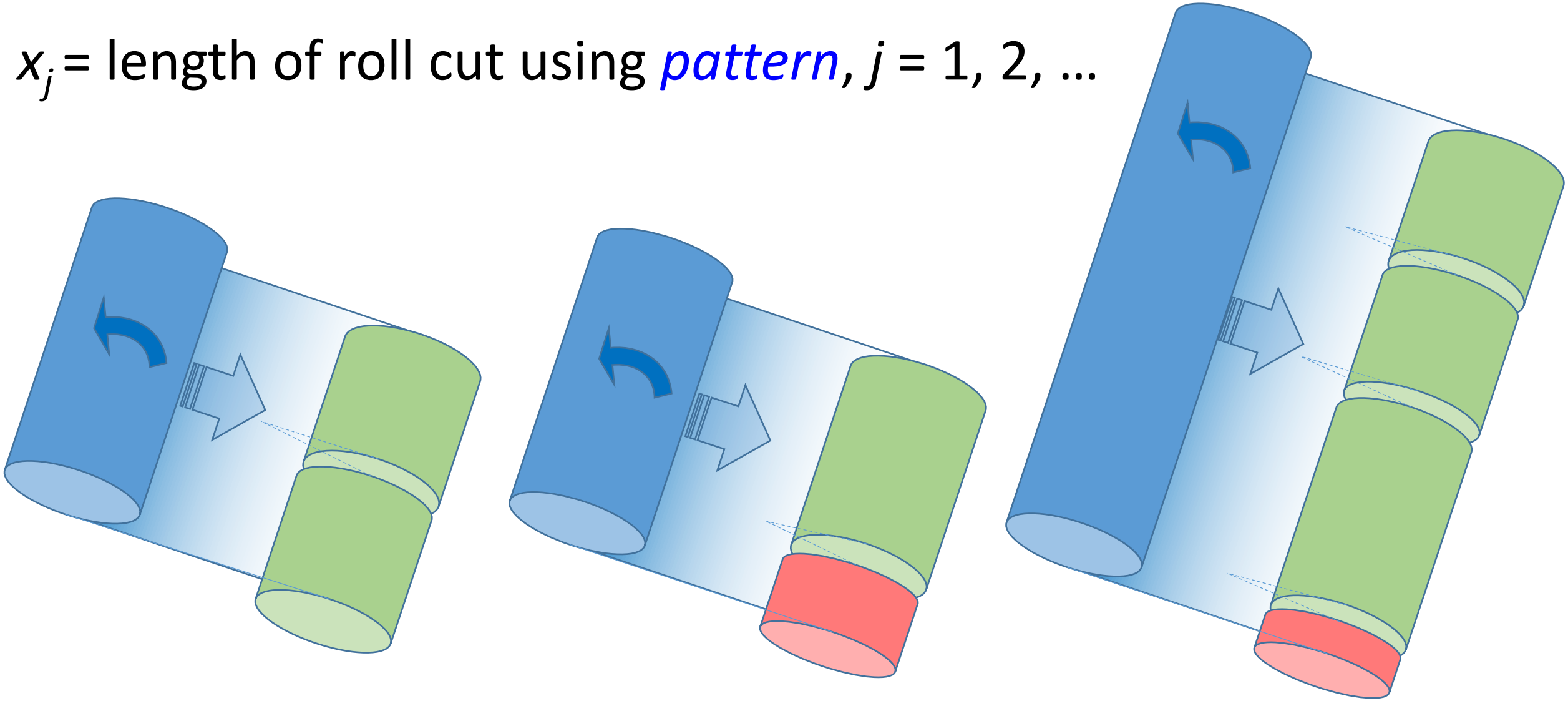
Decision variables:

x_j = length of roll cut using *pattern*, j = 1, 2, …



10’ roll: two 5’ cuts

10’ roll: one 7’ cut

20’ roll: two 5’ cuts and one 9’ cut

Possible Patterns

Order	Width
1	5’
2	7’
3	9’

	10’ roll			20’ roll					
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
5’	2	0	0	4	2	2	1	0	0
7’	0	1	0	0	1	0	2	1	0
9’	0	0	1	0	0	1	0	1	2
Trim loss:	0	3	1	0	3	1	1	4	2

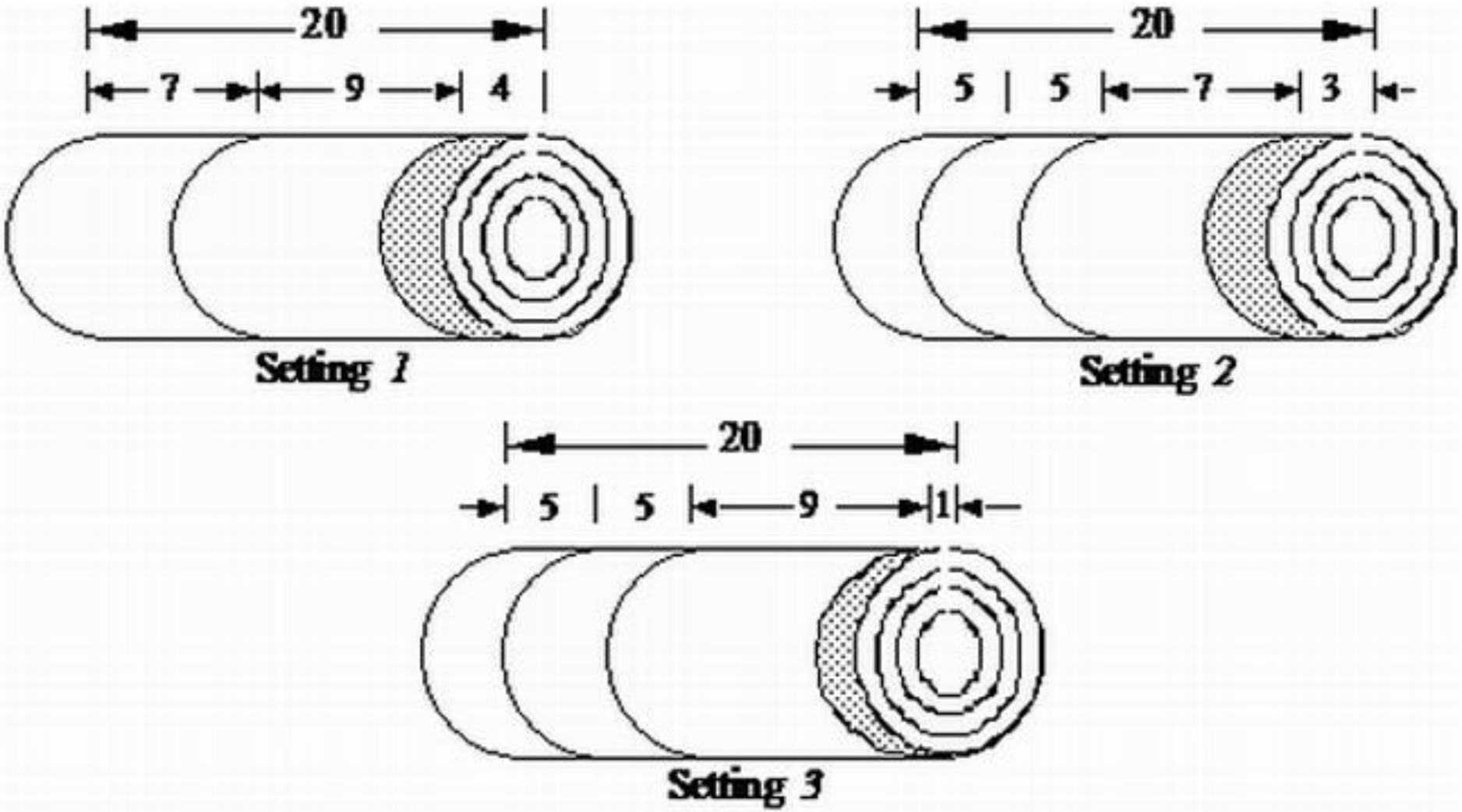
$$\min \quad 3x_2 + x_3 + 3x_5 + x_6 + x_7 + 4x_8 + 2x_9$$

	10' roll			20' roll					
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
5'	2	0	0	4	2	2	1	0	0
7'	0	1	0	0	1	0	2	1	0
9'	0	0	1	0	0	1	0	1	2
Trim loss:	0	3	1	0	3	1	1	4	2

Order	Width
1	5'
2	7'
3	9'

$$2x_1 + 4x_4 + 2x_5 + 2x_6 + x_7 \geq 10000$$

$$
\begin{aligned}
\min \quad & 3x_2 + x_3 + 3x_5 + x_6 + x_7 + 4x_8 + 2x_9 \\
\text{s.t.} \quad & 2x_1 + 4x_4 + 2x_5 + 2x_6 + x_7 = 10000 \\
& x_2 + x_5 + 2x_7 + x_8 = 30000 \\
& x_3 + x_6 + x_8 + 2x_9 = 20000 \\
& x_j \geq 0 \quad j = 1, \ldots, 9
\end{aligned}
$$


20
7
9
4
Setting 1
20
5
5
7
3
Setting 2
20
5
5
9
1
Setting 3