Comprehensive Final Exam

Adv. Analytics and Metaheuristics

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Contents

1 - *Question 1* (Version 1)

1.1 *Part 1:* Mathematical Formulation

1.1.1 Sets

$NewBuildTypes$: Set of new build types $b \in (Homes,\ Duplex,\ MiniPark)$

1.1.2 Parameters

Parameter	Description	Default Value
$budget$	Federal grant allocation to revitalize neighborhoods	\$15MM total budget
$maxBuildingDemod$	Max amount of buildings that can be demolished	300 total buildings
$demoCost$	Cost of demolishing a building	\$4,000 per building
$freedUpSpace$	Acreage generated from demolishing a building	0.25 per building
$newBuildSpace_b$	Amount of acreage that a new building ($b \in NewBuildTypes$) consumes	$Homes$: 0.2, $Duplex$: 0.4, $MiniPark$: 1.0
$newBuildTax_b$	Amount of tax dollars generated from a new building ($b \in NewBuildTypes$)	$Homes$: 1,500, $Duplex$: 2,750, $MiniPark$: 500
$newBuildCost_b$	Amount of dollars used to create a new building ($b \in NewBuildTypes$)	$Homes$: 150,000, $Duplex$: 190,000, $MiniPark$: 20,000
$newBuildPercShare_b$	Minimum required percentage share of new buildings ($b \in NewBuildTypes$) created	$Homes$: 20%, $Duplex$: 10%, $MiniPark$: 5%

1.1.3 Decision Variables

Variable	Description
$numOldBuildsDemod$	Number of old buildings to demolish
$numNewBuilds_b$	Number of new buildings ($b \in NewBuildTypes$) to produce
$newBuildTotalCost$	Variables to hold total cost of new builds ($b \in NewBuildTypes$). Calculation: $\sum_{b \in NewBuildTypes}(numNewBuilds_b \times newBuildCost_b)$
$oldDemoTotalCost$	Variables to hold total cost of old demolitions. Calculation: $numOldBuildsDemod \times demoCost$
$sumOfNewBuilds$	# Variable to hold the sum of all new build types over all New build types ($b \in NewBuildTypes$). Calculation: $\sum_{b \in NewBuildTypes}(numNewBuilds_b)$

1.1.4 Objective

Maximize the tax revenue from the projects

$$maximize\ taxRevenue : \sum_{b \in NewBuildTypes}(numNewBuilds_b \times newBuildTax_b)$$

1.1.5 Constraints

C1 Spend less than or equal to the federal budget (see variable definitions)

$$meetTheBudget : newBuildTotalCost + oldDemoTotalCost \leq budget$$

C2 Can only produce new builds using the demolished buildings land

$$useAvailLand : \sum_{b \in NewBuildTypes}(numNewBuilds_b \times newBuildSpace_b)$$
$$\leq numOldBuildsDemod \times freedUpSpace$$

C3 Can only clear a certain amount of old buildings

$$maxBuildingsCleared : numOldBuildsDemod \leq maxBuildingDemod$$

C4 For each new build type ($b \in NewBuildTypes$), the percentage share of the new build type must meet the minimum required (see variables)

$$share : numNewBuilds_b \geq newBuildPercShare_b \times sumOfNewBuilds,$$
$$\forall\ b \in Businesses$$

C5 Non-negativity and integer constraints

$$numOldBuildsDemod \in \mathbb{Z}, \geq 0$$
$$numNewBuilds_b \in \mathbb{Z}, \geq 0,\ \forall\ b \in NewBuildTypes$$

1.2 *Part 2*: AMPL Code & Output

1.2.1 AMPL Code

Data *problem1.dat*

```
data;

# SETS ==============================================================

# Set of new build types
set NewBuildTypes := Homes Duplex MiniPark;

# PARAMETERS ========================================================

param budget           := 15000000; # federal budget
param maxBuildingDemod := 300;      # max buildings can be demo'd
param demoCost         := 4000;     # Cost of each demolition
param freedUpSpace     := 0.25;     # Freed up space from demolition

# Amount of acreage that a new building (b in NewBuildTypes) consumes
param: newBuildSpace :=
       Homes    0.2
       Duplex   0.4
       MiniPark 1.0
       ;
# Amount of tax dollars generated from a new building (b in NewBuildTypes)
param: newBuildTax :=
       Homes    1500
       Duplex   2750
       MiniPark 500
       ;

# Amount of dollars used to create a new building (b in NewBuildTypes)
param: newBuildCost :=
       Homes    150000
       Duplex   190000
       MiniPark 20000
       ;

# Minimum required percentage share of new buildings (b in NewBuildTypes) created
param: newBuildPercShare :=
       Homes    0.20
       Duplex   0.10
       MiniPark 0.05
```

```
      ;
```

Model *problem1.mod*

```
reset;                    # Reset globals
options solver cplex;     # Using cplex for simplex alg

# SETS =========================================================
set NewBuildTypes; # Set of new build types

# PARAMETERS ====================================================
param budget             >= 0; # federal budget
param maxBuildingDemod >= 0; # max buildings can be demo'd
param demoCost           >= 0; # Cost of each demolition
param freedUpSpace       >= 0; # Freed up space from demolition

param newBuildSpace     {NewBuildTypes} >= 0; # new build acreage
param newBuildTax       {NewBuildTypes} >= 0; # n.b. tax generation
param newBuildCost      {NewBuildTypes} >= 0; # n.b. cost
param newBuildPercShare{NewBuildTypes} >= 0; # n.b. min % share

# DECISION VARIABLES ============================================
var numOldBuildsDemod                 >= 0 integer; # Num old builds to demo
var numNewBuilds      {NewBuildTypes} >= 0 integer; # Num new builds to create

# Variables to hold total cost of new builds over all types
var newBuildTotalCost = sum{b in NewBuildTypes} ( (numNewBuilds[b] * newBuildCost[b]));

# Variables to hold total cost of old demolitions
var oldDemoTotalCost = (numOldBuildsDemod * demoCost) ;

# Variable to hold the sum of all new build types over all New build types
var sumOfNewBuilds =  sum{b in NewBuildTypes}( numNewBuilds[b] );

# OBJECTIVE FUNCTION ============================================
maximize taxRevenue: sum{b in NewBuildTypes}( numNewBuilds[b] * newBuildTax[b] );

# CONSTRAINTS ===================================================

# C1  Spend less than or equal to the federal budget
s.t. meetTheBudget:
    newBuildTotalCost + oldDemoTotalCost <= budget ;

# C2 Can only produce new builds using the demolished buildings land
```

```
s.t. useAvailLand:
    sum{b in NewBuildTypes}( numNewBuilds[b] * newBuildSpace[b] )
    <= numOldBuildsDemod * freedUpSpace ;

# C3 Can only clear a certain amount of old buildings
s.t. maxBuildingsCleared: numOldBuildsDemod <= maxBuildingDemod ;

# C4 For each new build type (b in NewBuildTypes),
#    the percentage share of the new build type must meet the minimum required
s.t. share {b in NewBuildTypes}:
    numNewBuilds[b] >= newBuildPercShare[b] * sumOfNewBuilds ;

# CONTROLS ========================================================
    data problem1.dat; # retreive data file with sets/param. values
    solve;

    print;

    print "Number of old buildings to demolish and cost (dollars):";
    display numOldBuildsDemod, oldDemoTotalCost ;

    print "Number of new buildings produced and cost (dollars):";
    display numNewBuilds , newBuildTotalCost ;

    print "Total Budget Used (dollars):";
    display newBuildTotalCost + oldDemoTotalCost ;

    print "Part 3: Max Tax Revenue generated (dollars):";
    display taxRevenue ;
```

1.2.2 *Part 2/3:* Solve AMPL Model and Display Solution

```
CPLEX 20.1.0.0: optimal integer solution; objective 199000
6 MIP simplex iterations
0 branch-and-bound nodes

Number of old buildings to demolish and cost (dollars):
numOldBuildsDemod = 137
oldDemoTotalCost = 548000

Number of new buildings produced and cost (dollars):
numNewBuilds [*] :=
  Duplex  62
   Homes  17
MiniPark   6
;

newBuildTotalCost = 14450000

Total Budget Used (dollars):
newBuildTotalCost + oldDemoTotalCost = 14998000

Part 3: Max Tax Revenue generated (dollars):
taxRevenue = 199000
```

2 - *Question 2* (Version 6)

2.1 Code to get Root Node

Root Node is Node 1 in the diagram

2.1.1 Root Node AMPL Model *problem2.mod*

```
reset;
option solver cplex; # Solver

var x >= 0; # Not integer because this is used to check the optimal when fixing a var
var y >= 0; # ""

# Original problem:
maximize theSolution: 2.5*x + 6*y;
    s.t. first:  3*x + 5*y <= 26;
    s.t. second:   x       >= 4;

# Used to check the BnB nodes
# Fixing x = 4 to solve for initial values of y
s.t. checkNode: x = 4;

# Solve and display
solve;
display theSolution, x,y;
```

2.1.2 Output of Root Node AMPL Model (fixing x=4 & solving for y)

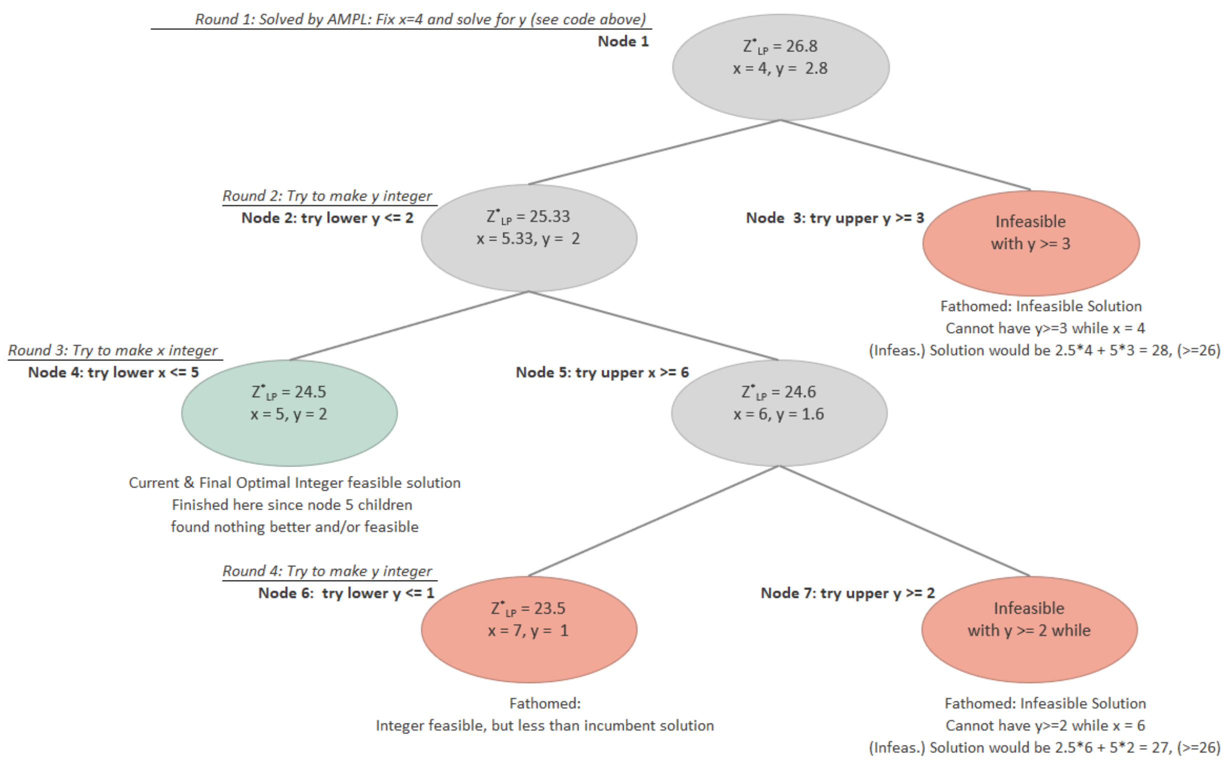
```
CPLEX 20.1.0.0: optimal solution; objective 26.8
0 dual simplex iterations (0 in phase I)
theSolution = 26.8
x = 4
y = 2.8
```

2.2 Branch and Bound Diagram

2.2.1 Summary of Diagram

- Optimal solution reached at node 4 with integer feasible values of $x = 5$, and $y = 2$
- Checked 7 total nodes
- Fathomed 3 nodes (see description of "why" in diagram)
- Each "Round" label shows which variable is being checked and for what integer value (lower or upper bound of the parent node)

2.2.2 Branch and Bound Diagram



3 - *Question 3* (Version 2)

3.1 *Part i*: Hill Climbing

- Please see the below snippet on rightmost comments about the algorithm
- Ended at iteration 3 with minimum value of -1.9991 due to no change in best neighbor

	x_1	x_2	Fitness Evaluation	Calculation Note / Comment
Starting Solution, S_0	2	2	**0.2432**	$=x_1 *\cos(x_1)*\sin(x_2) + 0.5*x_2$
	x_1	x_2	Fitness Evaluation	
Current Solution	2	2	**0.2432**	$=x_1 *\cos(x_1)*\sin(x_2) + 0.5*x_2$
Done?	FALSE			
Iteration 1				
Best Neighbor (current)	2	2	0.2432	current solution from past iteration
Neighbors of (2, 2)	x_1	x_2	**Fitness Evaluation**	
Neighbor 1 (x1 + 1, x2)	3	2	-1.7006	$=x_1 *\cos(x_1)*\sin(x_2) + 0.5*x_2$
Neighbor 2 (x1 - 1, x2)	1	2	1.4913	...
Neighbor 3 (x1, x2 + 1)	2	3	1.3825	...
Neighbor 4 (x1, x2 - 1)	2	1	-0.2004	...
(New) Best Neighbor	**3**	**2**	**-1.7006**	=min(neighbors, best_neighbor)
is Best Neighbor Same as Old?	FALSE			
Done?	FALSE			done if same
	x_1	x_2	Fitness Evaluation	
Current Solution	3	2	**-1.7006**	$=x_1 *\cos(x_1)*\sin(x_2) + 0.5*x_2$
Iteration 2				
Best Neighbor (current)	3	2	(1.7006)	current solution from past iteration
Neighbors of (3, 2)	x_1	x_2	**Fitness Evaluation**	
Neighbor 1 (x1 + 1, x2)	4	2	-1.3774	$=x_1 *\cos(x_1)*\sin(x_2) + 0.5*x_2$
Neighbor 2 (x1 - 1, x2)	2	2	0.2432	...
Neighbor 3 (x1, x2 + 1)	3	3	1.0809	...
Neighbor 4 (x1, x2 - 1)	3	1	-1.9991	...
(New) Best Neighbor	**3**	**1**	**-1.9991**	=min(neighbors, best_neighbor)
is Best Neighbor Same as Old?	FALSE			
Done?	FALSE			done if same
	x_1	x_2	Fitness Evaluation	
Current Solution	3	1	**-1.9991**	$=x_1 *\cos(x_1)*\sin(x_2) + 0.5*x_2$
Iteration 3				
Best Neighbor (current)	3	1	(1.9991)	current solution from past iteration
Neighbors of (3, 1)	x_1	x_2	**Fitness Evaluation**	
Neighbor 1 (x1 + 1, x2)	4	1	-1.7001	$=x_1 *\cos(x_1)*\sin(x_2) + 0.5*x_2$
Neighbor 2 (x1 - 1, x2)	2	1	-0.2004	...
Neighbor 3 (x1, x2 + 1)	3	2	-1.7006	...
Neighbor 4 (x1, x2 - 1)	3	0	0.0000	...
(New) Best Neighbor	**4**	**1**	**-1.9991**	=min(neighbors, best_neighbor)
is Best Neighbor Same as Old?	TRUE			
Done?	TRUE			done if same: end if end while

3.2 *Part ii*: Path Relinking

- Below shows the path re-linking overview.
- Shows each iterations that occurred in order to find the guided neighbor
- Green arrow highlights the, best solutions found along the path
- **Logic overview:** Evaluates neighbors from initiation solution, then makes the best move. Updates the difference between current and guided solution. Repeats until finding the guided solution
- `Iteration 3` *duplicated for ease of viewing*

Iteration 1

	x_1	x_2	Fitness Evaluation	Calculation Note / Comment
Initiating Solution	-1	-2	-0.5087	$=x_1 * \cos(x_1) * \sin(x_2) + 0.5 * x_2$
	-3	-2		

Neighbors of (-1, -2)	x_1	x_2	Fitness Evaluation	Path to the guided solution
Neighbor 1 (x1 + 1, x2)	0	-2	-1.0000	
Neighbor 2 (x1 - 1, x2)	-2	-2	-1.7568	...
Neighbor 3 (x1, x2 + 1)	-1	-1	-0.0454	...
Neighbor 4 (x1, x2 - 1)	-1	-3	-1.4238	$=x_1 * \cos(x_1) * \sin(x_2) + 0.5 * x_2$

Iteration 2

	x_1	x_2	Fitness Evaluation
Neighbor 1 (x1 + 1, x2)	0	-2	-1.0000
	-4	-2	

Neighbors of (0, -2)	x_1	x_2	Fitness Evaluation
Neighbor 1 (x1 + 1, x2)	1	-2	-1.4913
Neighbor 2 (x1 - 1, x2)	-1	-2	-0.5087
Neighbor 3 (x1, x2 + 1)	0	-1	-0.5000
Neighbor 4 (x1, x2 - 1)	0	-3	-1.5000

	x_1	x_2	Fitness Evaluation
Neighbor 2 (x1 - 1, x2)	-2	-2	-1.7568
	-2	-2	

Neighbors of (-2, -2)	x_1	x_2	Fitness Evaluation
Neighbor 1 (x1 + 1, x2)	-1	-2	-0.5087
Neighbor 2 (x1 - 1, x2)	-3	-2	-3.7006
Neighbor 3 (x1, x2 + 1)	-2	-1	-1.2004
Neighbor 4 (x1, x2 - 1)	-2	-3	-1.6175

	x_1	x_2	Fitness Evaluation
Neighbor 3 (x1, x2 + 1)	-1	-1	-0.0454
	-3	-3	

Neighbors of (-1, -1)	x_1	x_2	Fitness Evaluation
Neighbor 1 (x1 + 1, x2)	0	-1	-0.5000
Neighbor 2 (x1 - 1, x2)	-2	-1	-1.2004
Neighbor 3 (x1, x2 + 1)	-1	0	0.0000
Neighbor 4 (x1, x2 - 1)	-1	-2	-0.5087

	x_1	x_2	Fitness Evaluation
Neighbor 4 (x1, x2 - 1)	-1	-3	-1.4238
	-3	-1	

Neighbors of (-1, -3)	x_1	x_2	Fitness Evaluation
Neighbor 1 (x1 + 1, x2)	0	-3	-1.5000
Neighbor 2 (x1 - 1, x2)	-2	-3	-1.6175
Neighbor 3 (x1, x2 + 1)	-1	-2	-0.5087
Neighbor 4 (x1, x2 - 1)	-1	-4	-2.4089

Iteration 3

	x_1	x_2	Fitness Evaluation
Neighbor 4 (x1, x2 - 1)	0	-3	-1.5000
	-4	-1	

Neighbors of (0, -3)	x_1	x_2	Fitness Evaluation
Neighbor 1 (x1 + 1, x2)	1	-3	-1.5762
Neighbor 2 (x1 - 1, x2)	-1	-3	-1.4238
Neighbor 3 (x1, x2 + 1)	0	-2	-1.0000
Neighbor 4 (x1, x2 - 1)	0	-4	-2.0000

	x_1	x_2	Fitness Evaluation
Neighbor 2 (x1 - 1, x2)	-3	-2	-3.7006
	-1	-2	

Neighbors of (-3, -2)	x_1	x_2	Fitness Evaluation
Neighbor 1 (x1 + 1, x2)	-2	-2	-1.7568
Neighbor 2 (x1 - 1, x2)	-4	-2	-3.3774
Neighbor 3 (x1, x2 + 1)	-3	-1	-2.9991
Neighbor 4 (x1, x2 - 1)	-3	-3	-1.9191

	x_1	x_2	Fitness Evaluation
Neighbor 2 (x1 - 1, x2)	-2	-1	-1.2004
	-2	-3	

Neighbors of (-2, -1)	x_1	x_2	Fitness Evaluation
Neighbor 1 (x1 + 1, x2)	-1	-1	-0.0454
Neighbor 2 (x1 - 1, x2)	-3	-1	-2.9991
Neighbor 3 (x1, x2 + 1)	-2	0	0.0000
Neighbor 4 (x1, x2 - 1)	-2	-2	-1.7568

	x_1	x_2	Fitness Evaluation
Neighbor 4 (x1, x2 - 1)	-1	-4	-2.4089
	-3	0	

Neighbors of (-1, -4)	x_1	x_2	Fitness Evaluation	
Neighbor 1 (x1 + 1, x2)	0	-4	-2.0000	
Neighbor 2 (x1 - 1, x2)	-2	-4	-1.3701	
Neighbor 3 (x1, x2 + 1)	-1	-3	-1.4238	
Neighbor 4 (x1, x2 - 1)	-1	-5	-3.0181	was min, but

Iteration 3

	x_1	x_2	Fitness Evaluation
Neighbor 4 (x1, x2 - 1)	0	-3	-1.5000
	-4	-1	

Neighbors of (0, -3)	x_1	x_2	Fitness Evaluation
Neighbor 1 (x1 + 1, x2)	1	-3	-1.5762
Neighbor 2 (x1 - 1, x2)	-1	-3	-1.4238
Neighbor 3 (x1, x2 + 1)	0	-2	-1.0000
Neighbor 4 (x1, x2 - 1)	0	-4	-2.0000

	x_1	x_2	Fitness Evaluation
Neighbor 2 (x1 - 1, x2)	-3	-2	-3.7006
	-1	-2	

Neighbors of (-3, -2)	x_1	x_2	Fitness Evaluation
Neighbor 1 (x1 + 1, x2)	-2	-2	-1.7568
Neighbor 2 (x1 - 1, x2)	-4	-2	-3.3774
Neighbor 3 (x1, x2 + 1)	-3	-1	-2.9991
Neighbor 4 (x1, x2 - 1)	-3	-3	-1.9191

	x_1	x_2	Fitness Evaluation
Neighbor 2 (x1 - 1, x2)	-2	-1	-1.2004
	-2	-3	

Neighbors of (-2, -1)	x_1	x_2	Fitness Evaluation
Neighbor 1 (x1 + 1, x2)	-1	-1	-0.0454
Neighbor 2 (x1 - 1, x2)	-3	-1	-2.9991
Neighbor 3 (x1, x2 + 1)	-2	0	0.0000
Neighbor 4 (x1, x2 - 1)	-2	-2	-1.7568

	x_1	x_2	Fitness Evaluation
Neighbor 4 (x1, x2 - 1)	-1	-4	-2.4089
	-3	0	

Neighbors of (-1, -4)	x_1	x_2	Fitness Evaluation	
Neighbor 1 (x1 + 1, x2)	0	-4	-2.0000	
Neighbor 2 (x1 - 1, x2)	-2	-4	-1.3701	
Neighbor 3 (x1, x2 + 1)	-1	-3	-1.4238	
Neighbor 4 (x1, x2 - 1)	-1	-5	-3.0181	was min, but infeasible

Iteration 4

	x_1	x_2	Fitness Evaluation
Neighbor 4 (x1, x2 - 1)	0	-4	-2.0000
	-4	0	

Neighbors of (0, -4)	x_1	x_2	Fitness Evaluation
Neighbor 1 (x1 + 1, x2)	**1**	**-4**	**-1.5911**
Neighbor 2 (x1 - 1, x2)	-1	-4	-2.4089
Neighbor 3 (x1, x2 + 1)	0	-3	-1.5000
Neighbor 4 (x1, x2 - 1)	0	-5	-2.5000

	x_1	x_2	Fitness Evaluation
Neighbor 2 (x1 - 1, x2)	-4	-2	-3.3774
	0	-2	

Neighbors of (-4, -2)	x_1	x_2	Fitness Evaluation
Neighbor 1 (x1 + 1, x2)	-3	-2	-3.7006
Neighbor 2 (x1 - 1, x2)	-5	-2	0.2897
Neighbor 3 (x1, x2 + 1)	-4	-1	-2.7001
Neighbor 4 (x1, x2 - 1)	-4	-3	-1.8690

	x_1	x_2	Fitness Evaluation
Neighbor 2 (x1 - 1, x2)	-3	-1	-2.9991
	-1	-3	

Neighbors of (-3, -1)	x_1	x_2	Fitness Evaluation
Neighbor 1 (x1 + 1, x2)	-2	-1	-1.2004
Neighbor 2 (x1 - 1, x2)	-4	-1	-2.7001
Neighbor 3 (x1, x2 + 1)	-3	0	0.0000
Neighbor 4 (x1, x2 - 1)	-3	-2	-3.7006

Iteration 5

	x_1	x_2	Fitness Evaluation
Guiding Solution	1	-4	-1.5911

3.3 *Part iii*: Simulated Annealing

- Current temperature = 3

Using the evaluation function for this questions (note language below is `R`)

```
# Evaluation function
evaluateFitness <- function(x1, x2) {
  x1*cos(x1)*sin(x2) + 0.5*x2
}
```

- The probability fomula is the following: $p = e^{\frac{-(f(s_1)-f(s_2))}{T}}$
- Where $f(s_1)$ is the current solution, and $f(s_2)$ is the candidate solution since minimization.

```
# Uses the evaluation function to return the probability (see above formula)
calculateProb <- function(temperature, currentPosition, newPosition) {
  f_s1 = evaluateFitness(currentPosition[1], currentPosition[2]) # f(s[1]) current
  f_s2 = evaluateFitness(newPosition[1],     newPosition[2]    ) # f(s[2]) candidate

  # Probability using minimization
  p = exp(1)^( -(f_s1-f_s2) / temperature )

  return(round(p, 3))
}
```

Now calculate the probability from the current to the candidate solutions

```
# What is the probability of accepting a move from
# (4, 0) to candidate neighbor solution (4, 1)?
paste('At temp. 3, the probability from (4, 0) to (4, 1) is',
      calculateProb(temperature     = 3,
                    currentPosition = c(4, 0),
                    newPosition     = c(4, 1)) )
```

```
## [1] "At temp. 3, the probability from (4, 0) to (4, 1) is 0.567"
```

```
# What is the probability of accepting a move from
# (4, 0) to candidate neighbor solution (4,-1)?
paste('At temp. 3, the probability from (4, 0) to (4, -1) is',
      calculateProb(temperature     = 3,
                    currentPosition = c(4, 0),
                    newPosition     = c(4, -1)) )
```

```
## [1] "At temp. 3, the probability from (4, 0) to (4, -1) is 1.762"
```

4 - *Question 4* (Version 3)

4.1 *Part (i)* - Roulette Probability

- Evaluates the fitness for each chromosome $c \in Chromosomes$ using $f(x) = x_1 + 2x_2 + 3x_3 + 4x_4 + 5x_5$ as evaluation function.

- Then computes the probability based on the following equation: $\frac{f_c}{\sum_{c \in Chromosomes} f_c}$

- See yellow highlighted cells for final roulette wheel probabilities for each chromosome

Part (i)

These are the evaluation function coefficients for x1..., x5

Eval Coefs	1	2	3	4	5
Eval Function	x1*1 +	x2*2 +	x3*3 +	x4*4 +	x5*5

Step 1: Calculate fitness of each, Then sum up mating pool

Chromosome	x1	x2	x3	x4	x5	Fitness	*calculation: x1*1 + x2*2 + x3*3 + x4*4 + x5*5*
1	1	0	0	0	1	6	=SUMPRODUCT(C10:G10, C$5:G$5)
2	0	0	1	0	1	8	=SUMPRODUCT(C11:G11, C$5:G$5)
3	0	1	0	1	1	11	=SUMPRODUCT(C12:G12, C$5:G$5)
4	1	1	0	0	0	3	=SUMPRODUCT(C13:G13, C$5:G$5)
Sum Total Fitness Value of Pool						28	

Step 2: Calculate Probability of Selection

Chromosome	Fitness	Sum Total Fitness (Step 1)	(i) Prob. of Selection	*calculation: Fitness $_{chromosome}$ / sum(FitnessOfAllChromosomes)*
1	6	28	21.43%	=C18 / D18
2	8	28	28.57%	=C19 / D19
3	11	28	39.29%	=C20 / D20
4	3	28	10.71%	=C21 / D21

4.2 *Part (ii/iii)* - Breed Offspring

- Select the highest and lowest fitness valued chromosomes and produce offspring
- Offspring split between the second and third bit (highlighted for ease)
- Highest fitness valued offspring: `Offspring 2` with fitness value of `12`

Part (ii)

Assumes no use of the probability of selection from part (i)

Step 1: Find the Lowest and Highest Fitness Value Chromosomes
(Note Crossover point between 2nd and 3rd bit - highlighted for ease)

Chromosome	x1	x2	x3	x4	x5	Fitness	*calculation: x1*1 + x2*2 + x3*3 + x4*4 + x5*5*
2	0	1	0	1	1	11	Highest
3	1	1	0	0	0	3	Lowest

Step 2 - Offspring: Breed the above chromosomes at crossover point

Offspring	x1	x2	x3	x4	x5	Fitness	*calculation: x1*1 + x2*2 + x3*3 + x4*4 + x5*5*
Offspring 1	0	1	0	0	0	2	=SUMPRODUCT(C35:G35, C$5:G$5)
Offspring 2	1	1	0	1	1	12	=SUMPRODUCT(C36:G36, C$5:G$5)

5 - *Question 5* (Version 2)

5.1 *Part (i)* - Global Best

5.1.1 Assumptions

- Uses all calculations and parameter values from the problem question. See header in picture as well for the calculation
- Parameters of interest highlighted in orange
- See `global` best logic in picture below

5.1.2 Solution

- Particle 1's Next `Velocity: (-1.90, 3.55, 3.70)`
- Particle 1's Next `Position: (12.10, 8.55, 5.70)`

5.1.3 Relevant Work

Part (i)

					Inertia			Cognitive					Social			
		Next Position	*Next Vel.*		*I. Weight*	*Curr. Vel.*		""	""	*P. Best*	*Curr. Pos.*		""	""	*G. Best*	*Curr. Pos.*
Particle 1's...:	**Position Index**	**X[t+1] = V[t+1] + X[t]**	**V[t+1]**	=	**w**	**V[t]**	+	**phi[1]**	**r[1]**	**(P[i]**	**- X[t])**	+	**phi[2]**	**r[2]**	**(P[g]**	**- X[t])**
	Position Idx. 1	**12.10**	**-1.90**		1	1		1	0.5	10	14		1	0.15	8	14
	Position Idx. 2	**8.55**	**3.55**		1	0		1	0.5	13	5		1	0.15	2	5
	Position Idx. 3	**5.70**	**3.70**		1	1		1	0.5	8	2		1	0.15	0	2

Particle 1's Next Velocity: (-1.90, 3.55, 3.70)
Particle 1's Next Position: (12.10, 8.55, 5.70)

Global Best Logic:
Highest personal best fitness value is 290, which is from particle 4
Use particle 4's personal best position for the global best

5.2 *Part (ii)* - Local Best w/Ring

5.2.1 Assumptions

- Local best paramaters highlighted in yellow
- See `local` best logic with `ring` structure in picture below

5.2.2 Solution

- Particle 1's Next `Velocity: (-0.40, 4.30, 4.45)`
- Particle 1's Next `Position: (13.60, 9.30, 6.45)`

5.2.3 Relevant Work

Part (ii)

				Inertia			Cognitive					Social			
	Next Position	Next Vel.		I. Weight	Curr. Vel.		""	""	P. Best	Curr. Pos.		""	""	L. Best	Curr. Pos.
Particle 1's...: **Position Index**	**X[t+1] = V[t+1] + X[t]**	**V[t+1]**	=	**w**	**V[t]**	+	**phi[1]**	**r[1]**	**(P[i]**	**- X[t])**	+	**phi[2]**	**r[2]**	**(P[l]**	**- X[t])**
Position Idx. 1	**13.60**	**-0.40**		1	1		1	0.5	10	14		1	0.15	18	14
Position Idx. 2	**9.30**	**4.30**		1	0		1	0.5	13	5		1	0.15	7	5
Position Idx. 3	**6.45**	**4.45**		1	1		1	0.5	8	2		1	0.15	5	2

Particle 1's Next Velocity: (-0.40, 4.30, 4.45)
Particle 1's Next Position: (13.60, 9.30, 6.45)

Local Best Logic:
Neighbors of particle 1 are:
Particle 2 Personal Best Fitness value: 120
Particle 5 Personal Best Fitness value: 150
Use 5's p. best position since it has the highest personal best