# Homework 2 - Advanced LP & Network Flow Models Adv. Analytics and Metaheuristics

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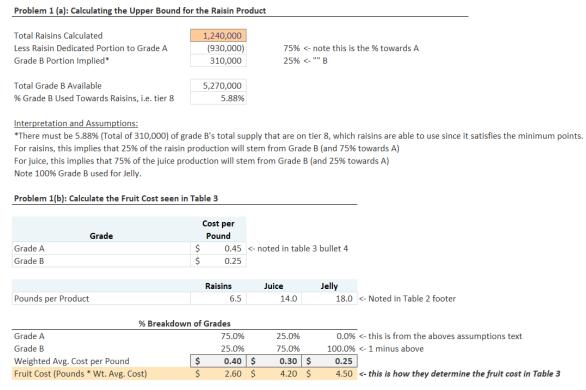
# February 2022

# Contents

1	- Pı	roblem 1	2
	1.1	Problems a and b	2
	1.2	Model Assumptions and Overview	2
	1.3	Mathematical Formulation	3
	1.4	Code and Output	5
2	- Pı	roblem 2	6
3	- Pı	roblem 3	7
	3.1	Model Overview	7
	3.2	Mathematical Formulation	8
	3.3	Code and Output	9
4	- Pı	roblem 4	11
	4.1	Model Overview	11
	4.2	Mathematical Formulation	13
	4.3	Code and Output	14

#### 1.1 Problems a and b

• Below shows how we get to the answer of Problems a and b



# 1.2 Model Assumptions and Overview

• This model calculates the maximum **net profit** for a set of products and inputs grades (See below sets).

# 1.3 Mathematical Formulation

# 1.3.1 Sets

Set Name	Description
$\overline{FRUIT}$	Fruit grade levels of grapes, which are Grade A and
PRODUCTS	Grade B grapes Types of products to be sold, which are Raisins, Juice, and Jelly

#### 1.3.2 Parameters

Parameter Name	Description
$amountOfFruit_f$	Pounds of fruit grade $(f \in FRUIT)$ available to use
$avgGradeOfFruit_f$	Avg. point quality of fruit grade $(f \in FRUIT)$
$productLimit_p$	Upper bound of number of products $(p \in PRODUCTS)$ to produce
$poundsPerProduct_p$	The amount of pounds associated with a single unit of product $(p \in PRODUCTS)$
$contrToProfit_p$	The contribution to profit of product $(p \in PRODUCTS)$
$netProfit_p$	The net profit (net of OH allocation) of product $(p \in PRODUCTS)$
$productGradeLimit_p$	Requirement of mean point quality of a product $(p \in PRODUCTS)$

#### 1.3.3 Decision Variables

Variable Name	Description
$produce_{f,p}$	Number (as integer) of products $(p \in PRODUCTS)$ to be produced by fruit grade $(f \in FRUIT)$

# 1.3.4 Objective Function

$$Maximize\ Net\_Profit: \sum_{f \in FRUIT, p \in PRODUCTS} produce_{f,p} \times netProfit_p$$

#### 1.3.5 Constraints

C1: For each fruit grade  $(f \in FRUIT)$ , the number of products produced  $(p \in PRODUCTS)$  must be equal to numbers of fruit provided

 $maxWeight: \sum_{p \in PRODUCTS} (produce_{f,p} \times numbersPerProduct_p) = amountOfFruit_f, \ \forall \ f \in FRUIT$ 

C2: For each product produced  $(p \in PRODUCTS)$ , limit the number of products p produced to  $\leq$  to the demanded (product numbers)

$$demand: \sum_{f \in FRUIT} produce_{f,p} \leq productLimit_p, \ \forall \ p \in PRODUCTS$$

C3: For each product  $(p \in PRODUCTS)$ , the grade of fruit  $(f \in FRUIT)$  is greater than the minimum required grade of the product  $(p \in PRODUCTS)$ . minAvgGrade:

$$\sum_{f \in FRUIT} (produce_{f,p} \times avgGradeOfFruit_f) \geq productGradeLimit_p \times \sum_{f \in FRUIT} (produce_{f,p}),$$

$$\forall \ p \in PRODUCTS$$

C4: Non-negativity constraints

produce 
$$f,p \ge 0$$
,  $\forall f \in FRUIT$ ,  $\forall p \in PRODUCTS$ 

# 1.4 Code and Output

#### 1.4.1 Code

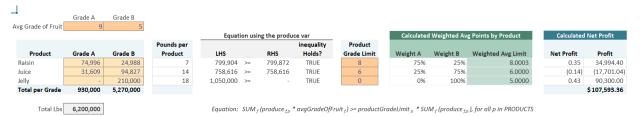
```
# Proposition (Accordance to Continue the Continue them is Continue than 10 continue that 1
```

#### **1.4.2** Output

```
CPLEX 20.1.0.0: optimal integer solution within mipgap or absmipgap; objective 107593.36
14 MIP simplex iterations
0 branch-and-bound nodes
absmipgap = 2.60826, relmipgap = 2.42418e-05

At maximum profit, the number of products to produce (by fruit grade):
produce :=
gradeA raisins 74996
gradeA juice 31609
gradeA juice 31609
gradeB raisins 24988
gradeB juice 94827
gradeB jelly 210000
;
```

Optimal profit is \$107,600. 74,996 Raisins produced using Grade A, 24,988 of Grade B, and so on for each product. See validation of the weighted-average point constraint below.



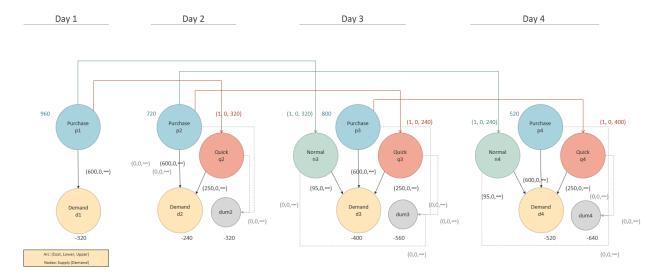
#### 3.1 Model Overview

#### 3.1.1 Assumptions and Calculations for Network Flow Diagram

- Below shows how we decided to balance the network with supply and dummy nodes
- In order to obtain a balanced network (i.e. supply equals demand), we must allow for all possible routes to have enough supply.
- Excess supply allowed on certain days is captured by dummy nodes so that the business does not actually produce the tires.

Determine Supply and Dummy Allocation							
		Incoming	Potential Supply	Dummy Allocation			
Day	Demand (d)	Arcs (in)	in*d	$d_{day-1} + d_{day-2}$			
1	-320	3	960	0			
2	-240	3	720	-320			
3	-400	2	800	-560			
4	-520	1	520	-640			

#### 3.1.2 Network Flow Diagram



#### 3.2 Mathematical Formulation

#### 3.2.1 Sets, Parameters, Decision Vars

Set Name Description	
$\overline{NODES}$	Set of all nodes in above network flow diagram:
p1 p2 p3 p4	Number of tires to purchase on day $\in (1-4)$
n1 n2 n3 n4	Number of tires to reshape using the <i>normal</i> service on day $\in$ (1-4)
q1 q2 q3 q4	Number of tires to reshape using the $quick$ service on day $\in (1-4)$
d1 d2 d3 d4	Demand of tires on each day $\in (1-4)$
dum2 dum3 dum4	Dummy nodes to balance excess supply for days 2 3 and $4 \in (2-4)$ . Do not need one for day 1 since purchasing

The set A is a set of arcs, e.g. (i,j) for  $i \in N, j \in N$  each of which may carry flow of a commodity

Decision variable:  $x_{ij}$  determines the units of flow on arc (i, j)

Arc(i,j)

- cost  $c_{ii}$  per unit of flow on arc (i, j)
- ullet upper bound on flow of  $u_{ij}$  (capacity)
- lower bound on flow of  $\ell_{ij}$  (usually 0)

#### 3.2.2 Objective, and Constraints

$$\begin{array}{l} \text{minimize } \sum_{(i,j)\in A} c_{ij} x_{ij} \\ \text{subject to } \sum_{j:(i,j)\in A} x_{ij} - \sum_{j:(j,i)\in A} x_{ji} = b_i \quad \forall i\in N \\ \\ l_{ij} \leq x_{ij} \leq u_{ij} \qquad \qquad \forall (i,j)\in A \end{array}$$

• Upper and lower bounds use to direct the flow of tires from *purchasing* to *quick* or *normal* service

# 3.3 Code and Output

#### 3.3.1 Model: group12\_HW2\_p3.mod

- Used mcnfp.txt from course website and renamed to group12\_HW2\_p3.mod.
- Added data group12\_HW2\_p3.dat; solve; and display x;

#### 3.3.2 Data: group12\_HW2\_p3.dat

#### Data Continued:

```
[p1, d1] 600
[p2, d2] 600
[p3, d3] 600
[q2, d2]
[q3, d3] 250
[q4, d4] 250
[n3, d3] 95
[n4, d4]
[p1, q2]
[p1, n3]
[p2, q3]
[p2, n4]
[p3, q4] 1
[p2, dum2]
[p3, dum3]
[p4, dum4]
[q2, dum2]
[q3, dum3]
[q4, dum4] 0
[n3, dum3] 0
[n4, dum4] 0
```

#### 3.3.3 Output

- Total minimized cost: 396,720
- Interpretation of the tires purchased on each day:
  - 1. 320 tires purchased
  - 2. 240 tires reshaped with Quick Service from previous day
  - 3. 80 Reshaped with quick service from previous day. 320 tires used from reshaping via Normal service from day 1.
  - 4. 280 Reshaped with quick service from previous day. 240 tires used from reshaping via Normal service from day 2.

```
ampl: model 'C:\Users\daniel.carpenter\OneDrive - the Chickasaw Na CPLEX 20.1.0.0: optimal solution; objective 396720 1 dual simplex iterations (0 in phase I) x [*,*] (tr) : n3 n4 p1 p2 p3 p4 q2 q3 q4 :=
                                       p1
320
d1
                                                        0
                                                                                         240
d2
d3
d4
                                                                    0
                                                                                                       80
               320
                                                                                                                  280
                           240
                                                    240
                                                                                           80
dum2
                                                                400
dum3
                   0
                                                                                                     160
                                                                             520
                                                                                                                  120
dum4
                                        320
n3
n4
                                                    240
                                        320
                                                    240
                                                                400
```

### 4.1 Model Overview

#### 4.1.1 Assumptions and Calculations for Network Flow Diagram

- Goal of below tables are to put all data on a per unit of product basis
- Need to be on per unit basis so that we can effectively minimize the cost
- Color of tables correspond to the network nodes on the next page

Labor, Manufacturing, and Transportation Cost Calculations for Arcs

Labor (Cost per Unit Output and Total Supply Available)							
Cost per Unit Output Total Labo						<b>Total Labor</b>	TTL Product
Туре	P	erson	per Person	Co	st / Unit	Avail	Supply
Specialist	\$	2,000	12	\$	166.67	100	1,200
Generalist	\$	1,700	10	\$	170.00	200	2,000

Cost of Transportation						
Scranton, PA Utica, NY Stamford, CT						
Per Person		300		250		275
Per Unit of Pro	duct	(trans. Co	st/	unit outpu	it by	type)
Specialist	\$	25.00	\$	20.83	\$	22.92
Generalist	\$	30.00	\$	25.00	\$	27.50

*Cost of Transportation + Labor per Unit of Output*						
	Scr	anton, PA	U	tica, NY	Sta	mford, CT
Specialist	\$	191.67	\$	187.50	\$	189.58
Generalist	\$	200.00	\$	195.00	\$	197.50

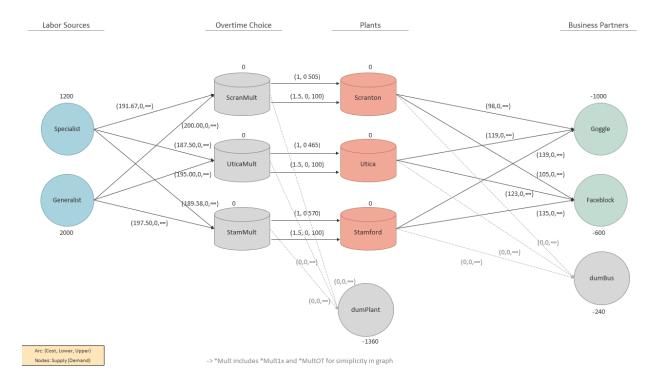
icranton, PA Utica, NY Stamford, CT 191.67,0,-) (187.50,0,-) (189.58,0,-) 200.00,0,-) (195.00,0,-) (197.50,0,-)

Plant Production Limits								
Base OT OT Mult								
Scranton	505	100	1.5					
Utica	465	100	1.5					
Stamford	570	100	1.5					

Manufacturing and Transportation Costs							
	Manu	ıfacture		Goggle	Fac	ceblock	
Scranton	\$	90	\$	8	\$	15	
Utica	\$	105	\$	14	\$	18	
Stamford	\$	115	\$	24	\$	20	

Man. + Trans Cost							
Go	oggle	Fac	eblock				
\$	98	\$	105				
\$	119	\$	123				
\$	139	\$	135				

# 4.1.2 Network Flow Diagram



#### 4.2 Mathematical Formulation

#### 4.2.1 Sets, Parameters, Decision Vars

Set Name	Description
$\overline{NODES}$	Set of all nodes in above network flow diagram:
Specialist, Generalist	The two types of Supply of Labor
ScranMult1x, UticaMult1x,	Passed through if did not use overtime
StamMult1x	
ScranMultOT, UticaMultOT,	Passed through if $did$ use overtime
StamMultOT	
Scranton, Utica, Stamford	Transshipment nodes which are the plants
dumPlant, dumBus	Dummy nodes that account for excess supply from
	unbalanced supply from labor nodes

The set A is a set of arcs, e.g. (i, j) for  $i \in N, j \in N$  each of which may carry flow of a commodity

Decision variable:  $x_{ij}$  determines the units of flow on arc (i, j)

Arc(i,j)

- cost  $c_{ij}$  per unit of flow on arc (i, j)
- upper bound on flow of  $u_{ij}$  (capacity)
- lower bound on flow of  $\ell_{ij}$  (usually 0)

#### 4.2.2 Objective, and Constraints

$$\begin{aligned} & \text{minimize } \sum_{(i,j) \in A} c_{ij} x_{ij} \\ & \text{subject to } \sum_{j:(i,j) \in A} x_{ij} - \sum_{j:(j,i) \in A} x_{ji} = b_i \quad \forall i \in N \\ & l_{ij} \leq x_{ij} \leq u_{ij} \qquad \forall (i,j) \in A \end{aligned}$$

• Upper and lower bounds use to direct the flow of the product

# 4.3 Code and Output

#### 4.3.1 Model: group12\_HW2\_p4.mod

- Used mcnfp.txt from course website and renamed to group12\_HW2\_p4.mod.
- Added data group12\_HW2\_p4.dat; solve; and display x;

#### 4.3.2 Data: group12\_HW2\_p4.dat

```
# Set of Nodes containing all sources of Labor, Plants, and Business Partners
set NODES := Specialist Generalist # Labor

ScranMultIx UticaMultIx StamMultIx # Multiplier dumBus 1x no OT

ScranMultOT UticaMultOT StamMultOT # Multiplier dumBus 1.5x for OT

dumPlant # Dummy to limit supply before plants

Scranton Utica Stamford # Plants

Goggle Faceblock dumBus # Business Partners + dumBus for unbal
              set ARCS :=
                                    (Specialist, *) ScranMult1x UticaMult1x StamMult1x
(Generalist, *) ScranMult1x UticaMult1x StamMult1x
(ScranMult1x, *) Scranton dumPlant
(UticaMult1x, *) Utica dumPlant
(StamMult1x, *) Stamford dumPlant
                               (Specialist, *) ScranMultOT UticaMultOT StamMultOT (Generalist, *) ScranMultOT UticaMultOT StamMultOT (ScranMultOT,*) Scranton dumPlant (UticaMultOT,*) Utica dumPlant (StamMultOT, *) Stamford dumPlant
                                    # Plants to demanders and dumBus for un
(Scranton, *) Goggle Faceblock dumBus
(Utica, *) Goggle Faceblock dumBus
(Stamford, *) Goggle Faceblock dumBus
                    # Transshippment (Plants or OT Multiplier)
ScranMult1x 0 # Not using OT
                     UticaMult1x 0
                      StamMult1x 0
                      ScranMultOT 0 # Using OT
                     UticaMultOT 0
                     StamMultOT 0
                      Scranton 0 # Plant Arrival
                     Utica
                      Stamford 0
```

#### Data Continued:

```
F group12_HW2_p4.mod M
                                  F group12_HW2_p4.dat M ●
              dumPlant
  58
              Goggle
                              -1000
              Faceblock
 61
              dumBus
                                                             1 u:=
              [Specialist, ScranMult1x] 191.67 . . # Supply -> Mult
             [Specialist, UticaMult1x] 187.5 . . . [Specialist, StamMult1x] 189.58 . . . [Generalist, ScranMult1x] 200 . . . [Generalist, UticaMult1x] 195 . . . [Generalist, StamMult1x] 197.5 . .
              [ScranMult1x, Scranton]
              [UticaMult1x, Utica]
              [StamMult1x, Stamford]
              [ScranMult1x, dumPlant]
              [UticaMult1x, dumPlant]
 79
              [StamMult1x, dumPlant]
              [Specialist, ScranMultOT] 191.67 . . # Supply -> Mult
[Specialist, UticaMultOT] 187.5 . .
[Specialist, StamMultOT] 189.58 . .
[Generalist, ScranMultOT] 200 . .
[Generalist, UticaMultOT] 195 . .
[Generalist, StamMultOT] 197.5 . .
              [UticaMultOT, Utica]
                                                    1.5
                                                                   100 # Mult -> Plant
              [ScranMultOT, Scranton]
              [StamMultOT, Stamford]
 91
              [ScranMultOT, dumPlant]
              [UticaMultOT, dumPlant]
 93
              [StamMultOT, dumPlant]
              [Scranton,
                             Goggle]
              [Scranton,
                              Faceblock]
 98
              [Scranton,
                              dumBus]
              [Utica,
                               Goggle]
                              Faceblock]
              [Utica,
101
              [Utica,
                              dumBus]
              [Stamford, Goggle]
              [Stamford,
                               Faceblock]
104
              [Stamford,
                             dumBus]
```

#### **4.3.3** Output

- Total minimized cost: \$806,192.95
- Scranton, Utica, and Stamford produce 0, 430, and 170 units of product for **Faceblock**, respectively.
- Scranton, Utica, and Stamford produce 605, 0, and 395 units of product for **Goggle**, respectively.
- All possible products produced (using a portion of the available regular and overtime hours). 200 products produced by Specialists using OT, and 100 from generalists using OT.

```
ampl: model 'C:\Users\daniel.carpenter\OneDrive - the Chickasaw Nation\Documents\GitHu
amp1: model C:\Users\daniel.carpenter\OneDrive - the
CPLEX 20.1.0.0: optimal solution; objective 806192.95
13 dual simplex iterations (0 in phase I)
x [*,*] (tr)
# $1 = Generalist
# $2 = ScranMult1x
# $3 = ScranMultOT
# $4 - Scranton
   $4 = Scranton
   $5 = Specialist
   $6 = StamMult1x
   $7 = StamMultOT
# $8 = Stamford
# $10 = UticaMult1x
# $11 = UticaMultOT
                       $1
                                $2
                                          $3
                                                   $4
                                                            $5
                                                                     $6
                                                                              $7
                                                                                        $8
                                                                                             Utica
                                                                                                          $10
                                                                                                                    $11 :=
                                                      0
                                                                                       430
Faceblock 

                                                                                                 170
Goggle
ScranMult1x
                                                   605
                                                                                          0
                                                                                                 395
                                                            505
                          0
ScranMult0T
                          0
                                                            100
                                         100
                                505
Scranton
                                                            495
StamMult1x
StamMult0T
                          0
                                                            100
                                                                     570
                                                                              100
Stamford
                                                                                                           465
Utica
                                                                                                                     100
                                                               0
UticaMult1x
                      1825
                       100
UticaMult0T
                                                               0
                                                      0
                                                                                        240
                                                                                                   0
dumBus
                                                                        0
                                                                                                                       0
dumPlant
                                   0
                                            0
                                                                                 0
                                                                                                          1360
```