Homework 3 - Integer Programming

Adv. Analytics and Metaheuristics

Daniel Carpenter and Iker Zarandona

March 2022

Contents

1 - Problem 1

1.1 Mathematical Formulation

1.1.1 Sets

Set Name	Description
$GENERATORS$	Set of generators i that can be used (`A`,`B`,`C`)
$PERIODS$	2 possible periods p (`1`, `2`) in the production day

1.1.2 Parameters

Parameter Name	Description
S_i	Fixed cost to start a generator ($i \in GENERATORS$) in the entire day
F_i	Fixed cost to operate a generator ($i \in GENERATORS$) in any period
C_i	Variable cost per megawatt to operator a generator ($i \in GENERATORS$) in any period
U_i	Max. megawatts generated for generator ($i \in GENERATORS$) in any period
$demand_p$	Total demanded megawatts for period ($p \in PERIODS$)
M	Large constant to map watts used by each generator ($i \in GENERATORS$)

1.1.3 Decision Variables

Variable Name	Description
$watts_{i,p}$	*Integer variable*: Number of watts to produce per generator ($i \in GENERATORS$) per period ($p \in PERIODS$)
$x_{i,p}$	*Binary variable*: `1` if a generator ($i \in GENERATORS$) is in period p ($p \in PERIODS$), `0` if not turned on at all
y_i	*Binary variable*: `1` if a generator ($i \in GENERATORS$) is used, `0` if not turned on at all

1.1.4 Objective Function

$$minimize\,cost: \sum_{i \in GENERATORS} \Big((\textstyle\sum_{p \in PERIODS} (watts_{i,p}) \times C_i) + (F_i \times \textstyle\sum_{p \in PERIODS} x_{i,p}) + (S_i \times y_i) \Big)$$

1.1.5 Constraints

C1: For each period, meet the demanded megawatts

$$requiredWatts: \sum_{i \in GENERATORS} (watts_{i,p}) = demand_p, \forall\ p \in PERIODS$$

C2: For each generator, don't surpass the allowable megawatts

$$upperBound: \sum_{p \in PERIODS} (watts_{i,p}) \leq U_i, \forall\ i \in GENERATORS$$

C3: For each generator, map decision variables together to account for the fixed costs in a given day S_i

$$mapVars: \sum_{p \in PERIODS} (watts_{i,p}) \leq M_i \times y_i, \forall\ i \in GENERATORS$$

C4: For each generator and period, map decision variables y and $watts$ together to account for the fixed costs in a per period p

$$mapVars2: watts_{i,p} \leq M_i \times x_{i,p}, \forall\ i \in GENERATORS,\ p \in PERIODS$$

C5 Non-negativity or Binary restraints of decision variables

$$watts_{i,p} \geq 0$$

$$x_{i,p},\ y_i \in (0,1)$$

1.2 Code and Output

1.2.1 Code

group23_HW3_p1.mod

C: > Users > daniel.carpenter > OneDrive - the Chickasaw Nation > Documents > GitHub > OU-DSA > Metaheuristics > 03 - Homework > HW 03 > AMPL Mo

```
# Homework 3 - Integer Programming
# Adv. Analytics and Metaheuristics
# Daniel Carpenter and Iker Zarandona
# March 2022
# Problem 1

reset;                  # Reset globals
options solver cplex;   # Using cplex for simplex alg

# SETS ============================================================
    set GENERATORS; # Set of generators to use
    set PERIODS;    # Periods in the day

# PARAMETERS ========================================================
    param S {GENERATORS}   >= 0; # Fixed cost to start
    param F {GENERATORS}   >= 0; # Fixed cost to operate
    param C {GENERATORS}   >= 0; # Variable cost per megawatt
    param U {GENERATORS}   >= 0; # Upper bound on megawatts in a day
    param M {GENERATORS}   >= 0; # Map decision variables
    param demand {PERIODS} >= 0; # Megawatts required per period

# DECISION VARIABLES ================================================
    var watts {GENERATORS, PERIODS} >= 0 integer; # Megawatts to use
    var x {GENERATORS, PERIODS} binary; # Map to watts for fixed daily costs
    var y {GENERATORS} binary; # Map to watts for fixed daily costs

# OBJECTIVE FUNCTION ================================================
  minimize cost:
      (sum{i in GENERATORS} (sum{p in PERIODS} watts[i,p])*C[i])
    + (sum{i in GENERATORS} F[i]*sum{p in PERIODS}x[i,p])
    + (sum{i in GENERATORS} S[i]*y[i]);

# CONSTRAINTS =======================================================

    # C1: For each period, meet the demanded megawatta
    subject to requiredWatts {p in PERIODS}:
        (sum{i in GENERATORS} watts[i,p]) = demand[p];

    # C2: For each generator, don't surpass the allowable megawatts
    subject to upperBound {i in GENERATORS}:
        (sum{p in PERIODS} watts[i,p]) <= U[i];

    # C3: For each generator, map decision variables together to account for the
    fixed costs in a given day Si
    subject to mapVars {i in GENERATORS}:
        (sum{p in PERIODS} watts[i,p]) <= M[i] * y[i];

    # C4: For each generator and period, map decision variables y and watts together
    to account for the fixed costs in a per period p
    subject to mapVars2 {i in GENERATORS, p in PERIODS}:
        watts[i,p] <= M[i] * x[i,p];

# CONTROLS ==========================================================
    data group23_HW3_p1.dat;
    solve;

    print;
    print "Which generators are used?";
    display y;

    print "Which periods were the generators used?";
    display x;

    print "Optimal Amount of Megawatts for each generator and period:";
    display watts;
```

group23_HW3_p1.dat

C: > Users > daniel.carpenter > OneDrive - the Chickasaw Nation > Documents > GitHub > OU-DSA > Metaheuristics > 03 - Homework > HV

```
# Homework 3 - Integer Programming
# Adv. Analytics and Metaheuristics
# Daniel Carpenter and Iker Zarandona
# March 2022
# Problem 1

# SETS ============================================================
    set GENERATORS  := A B C;   # Set of generators to use
    set PERIODS     := 1 2;     # Periods in the day


# PARAMETERS ========================================================

    # S: Fixed cost to start a generator (i ∈ GENERATORS) in the entire day
    # F: Fixed cost to operate a generator (i ∈ GENERATORS) in any period
    # C: Variable cost per megawatt to operator a generator (i ∈ GENERATORS) in any period
    # U: Max. megawatts generated for generator (i ∈ GENERATORS) in any period
    # M: Value to map watts used by each generator (i ∈ GENERATORS)
    #    Set to be slighlty over the max megawatts per day
    param: S    F    C     U    M :=
        A  3000 700  5.00  2100 2200
        B  2000 500  4.00  1800 1900
        C  1000 900  7.00  3000 3100
        ;

    # Total demanded megawatts for period (p ∈ PERIODS)
    param demand :=
        1   2900
        2   3900
        ;
```

1.2.2 Output

```
ampl: model 'C:\Users\daniel.carpenter\OneDrive - the Chickasaw
CPLEX 20.1.0.0: optimal integer solution; objective 46100
7 MIP simplex iterations
0 branch-and-bound nodes

Which generators are used?
y [*] :=
A  1
B  1
C  1
;

Which periods were the generators used?
x :=
A 1   0
A 2   1
B 1   0
B 2   1
C 1   1
C 2   0
;

Optimal Amount of Megawatts for each generator and period:
watts :=
A 1      0
A 2   2100
B 1      0
B 2   1800
C 1   2900
C 2      0
;
```

1.2.2.1 Analysis of the Output

- The minimized cost is $\$46,100$
- Generator A, B, and C run
- Generator C runs in period 1. Generator A and B run in period 2
- Generator A produces $2,100$ megawatts in total
- Generator B produces $1,800$ megawatts in total
- Generator C produces $2,900$ megawatts in total

2 - Problem 2

2.1 Mathematical Formulation (Part a)

2.1.1 Sets

Set Name	Description
$PRODUCTS$	5 types of landscaping and construction products (e.g., cement, sand, etc.) labeled product (p) $A, B, C, D,$ and E
$SILOS$	8 different silos s that each product must be stored in $(1, 2, \ldots, 8)$

2.1.2 Parameters

Parameter Name	Description
$cost_{s,p}$	Cost of storing *one ton* of product $p \in PRODUCTS$ in silo $s \in SILOS$
$supply_p$	Total supply *in tons* available of product $p \in PRODUCTS$
$capacity_s$	Total capacity *in tons* of silo $s \in SILOS$. Can store products.
M	Variable to map *decision variable* $tonsOfProduct_{p,s}$ to $isStored_{p,s}$. Uses big M method.

2.1.3 Decision Variables

Variable Name	Description
$tonsOfProduct_{p,s}$	*Tons* of product $p \in PRODUCTS$ to store in silo $s \in SILOS$. Non-negative.
$isStored_{p,s}$	*Binary variable* indicating if product $p \in PRODUCTS$ is stored in silo $s \in SILOS$.

2.1.4 Objective Function

$$minimize\ costOfStorage : \sum_{p \in PRODUCTS} \sum_{s \in SILOS} tonsOfProduct_{p,s} \times cost_{p,s}$$

2.1.5 Constraints

C1: For each silo s, the *tons* of the supplied product p must be less than or equal to the capacity limit of silo s

$$meetCapacity: \sum_{p \in PRODUCTS} tonsOfProduct_{p,s} \leq capacity_s, \ \forall \ s \in SILOS$$

C2: For each product p, must use all of the total product that is available

$$useAllProduct: \sum_{s \in SILOS} tonsOfProduct_{p,s} = supply_p, \ \forall \ p \in PRODUCTS$$

C3: For each silo s and product p,

$$oneProductInSilo: \sum_{pinPRODUCTS} isStored_{p,s} = 1, \ \forall \ s \in SILOS$$

C4: Map the decision variables together using the Big M method

$$mapVars: tonsOfProduct_{p,s} \leq M \times isStored_{p,s}, \ \forall \ p \in PRODUCTS, \ \forall \ s \in SILOS$$

C5 Non-negativity or Binary restraints of decision variables

$$tonsOfProduct_{p,s} \geq 0$$

$$isStored_{p,s} \in (0, 1)$$

2.2 Code and Output (Part a)

2.2.1 Code

group23_HW3_p2.mod

```
# Homework 3 - Integer Programming
# Adv. Analytics and Metaheuristics
# Daniel Carpenter and Iker Zarandona
# March 2022
# Problem X

reset;                  # Reset globals
options solver cplex;   # Using cplex for simplex alg

# SETS ============================================================
    set PRODUCTS;     # The 5 products
    set SILOS;        # The 8 silos for storage

# PARAMETERS ======================================================
    param cost     {PRODUCTS, SILOS}; # Cost of storing product in silo
    param supply   {PRODUCTS};        # Supply of products
    param capacity {SILOS};           # Capacity of the silos
    param M;                          # Map decision variables together

# DECISION VARIABLES ==============================================
    var tonsOfProduct {PRODUCTS, SILOS} >= 0;  # Amount of each product p to store in silo s
    var isStored      {PRODUCTS, SILOS} binary; # If a product is stored in a silo or not

# OBJECTIVE FUNCTION ==============================================

    minimize costOfStorage:
        sum{p in PRODUCTS, s in SILOS} tonsOfProduct[p,s] * cost[p,s];

# CONSTRAINTS =====================================================

    # C1: For each silo s, the tons of the supplied product p must be less than or equal to
    the capacity limit of silo s
    subject to meetCapacity {s in SILOS}:
        (sum{p in PRODUCTS} tonsOfProduct[p,s]) <= capacity[s];

    # C2: For each product p, must use all of the total product that is available
    subject to useAllProduct {p in PRODUCTS}:
        (sum{s in SILOS} tonsOfProduct[p,s]) == supply[p];

    # C3: Only one product can be in a silo
    subject to oneProductInSilo {s in SILOS}:
        sum{p in PRODUCTS} isStored[p,s] == 1;

    # C4: Map decision variables together
    subject to mapVars {p in PRODUCTS, s in SILOS}:
        tonsOfProduct[p,s] <= M * isStored[p,s];

# CONTROLS ========================================================
    data group23_HW3_p2.dat;
    solve;

    print;
    print "Which silo(s) stores what product?";
    display isStored;

    print "Optimal tons of product allocated to each silo:";
    display tonsOfProduct;
```

group23_HW3_p2.dat

```
# Homework 3 - Integer Programming
# Adv. Analytics and Metaheuristics
# Daniel Carpenter and Iker Zarandona
# March 2022
# Problem X

# SETS ============================================================

    set PRODUCTS := A B C D E;      # 5 types of products
    set SILOS    := 1 2 3 4 5 6 7 8; # 8 different silos to store product p

# PARAMETERS ======================================================

    # Cost of storing one ton of product p ∈ PRODUCTS in silo s ∈ SILOS
    param cost:
            1   2   3   4   5   6   7   8 :=
        A   1   2   2   1   4   4   5   3
        B   2   3   3   3   1   4   5   2
        C   3   4   1   2   1   4   5   1
        D   1   1   2   2   3   4   5   2
        E   1   1   1   1   1   1   5   5
        ;

    # Supply of each product that is available
    param supply :=
        A   75
        B   50
        C   25
        D   80
        E   20
        ;

    # Capacity of each silo
    param capacity :=
        1   25
        2   25
        3   30
        4   60
        5   80
        6   85
        7   100
        8   50
        ;

    # Variable to map decision variable tonsOfProduct p,s to
    # isStored p,s . Value is slightly more than the capacity of each silo.
    param M := 200;
```

2.2.2 Output (Part a)

```
ampl: model 'C:\Users\daniel.carpenter\OneDrive - the Chickasaw Nation\D
CPLEX 20.1.0.0: optimal integer solution; objective 320
48 MIP simplex iterations
0 branch-and-bound nodes

Which silo(s) stores what product?
isStored [*,*] (tr)
:   A   B   C   D   E    :=
1   1   0   0   0   0
2   0   0   0   1   0
3   0   0   1   0   0
4   1   0   0   0   0
5   0   1   0   0   0
6   0   0   0   0   1
7   0   0   0   1   0
8   0   0   0   1   0
;

Optimal tons of product allocated to each silo:
tonsOfProduct [*,*] (tr)
:    A    B    C    D    E     :=
1   25    0    0    0    0
2    0    0    0   25    0
3    0    0   25    0    0
4   50    0    0    0    0
5    0   50    0    0    0
6    0    0    0    0   20
7    0    0    0    5    0
8    0    0    0   50    0
;
```

2.2.2.1 Analysis of the Output

- Minimized loading cost for 250 tons of 5 products over the 8 silos is 320 (problem does not state cost units).
- Product A stores 25 $tons$ in $silo$ 1 and 50 $tons$ in $silo$ 4
- Product B stores 50 $tons$ in $silo$ 5
- Product C stores 25 $tons$ in $silo$ 3
- Product D stores 25 $tons$ in $silo$ 2, 5$tons$ in $silo$ 7, and and 50 $tons$ in $silo$ 8
- Product E stores 20 $tons$ in $silo$ 6

2.3 Problem 2 b

- Create a new objective that also minimizes the distance between capacity and stored tons of product
- *For each silo, minimize the variance between the total capacity and the tons of product*

$$minimize\ capacityActualVariance : capacity_s - \sum_{p \in PRODUCTS} tonsOfProduct_{p,s},\ \forall s \in SILOS$$

2.4 Code and Output (Part b)

2.4.1 Code

group23_HW3_p2.2.mod

C: > Users > daniel.carpenter > OneDrive - the Chickasaw Nation > Documents > GitHub > OU-DSA > Metaheuristics > 03 - Homework > HW 03 > AMPL Models > group

```
# Homework 3 - Integer Programming
# Adv. Analytics and Metaheuristics
# Daniel Carpenter and Iker Zarandona
# March 2022
# Problem X

reset;                  # Reset globals
options solver cplex;   # Using cplex for simplex alg

# SETS ==========================================================
    set PRODUCTS;     # The 5 products
    set SILOS;        # The 8 silos for storage

# PARAMETERS ====================================================
    param cost     {PRODUCTS, SILOS}; # Cost of storing product in silo
    param supply   {PRODUCTS};        # Supply of products
    param capacity {SILOS};           # Capacity of the silos
    param M;                          # Map decision variables together

# DECISION VARIABLES ============================================
    var tonsOfProduct {PRODUCTS, SILOS} >= 0;  # Amount of each product p to store in silo s
    var isStored      {PRODUCTS, SILOS} binary; # If a product is stored in a silo or not

# OBJECTIVE FUNCTION ============================================

    # Minimize the cost of the storage
    minimize costOfStorage:
        sum{p in PRODUCTS, s in SILOS} tonsOfProduct[p,s] * cost[p,s];

    # For each silo, minimuze the variance between the total capacity and the tons of product
    minimize capacityActualVariance{s in SILOS}:
        capacity[s] - sum{p in PRODUCTS} tonsOfProduct[p,s];


# CONSTRAINTS ===================================================

    # C1: For each silo s, the tons of the supplied product p must be less than or equal to
    the capacity limit of silo s
    subject to meetCapacity {s in SILOS}:
        (sum{p in PRODUCTS} tonsOfProduct[p,s]) <= capacity[s];

    # C2: For each product p, must use all of the total product that is available
    subject to useAllProduct {p in PRODUCTS}:
        (sum{s in SILOS} tonsOfProduct[p,s]) == supply[p];

    # C3: Only one product can be in a silo
    subject to oneProductInSilo {s in SILOS}:
        sum{p in PRODUCTS} isStored[p,s] == 1;

    # C4: Map decision variables together
    subject to mapVars {p in PRODUCTS, s in SILOS}:
        tonsOfProduct[p,s] <= M * isStored[p,s];

# CONTROLS ======================================================
    data group23_HW3_p2.2.dat;
    solve;

    print;
    print "Which silo(s) stores what product?";
    display isStored;

    print "Optimal tons of product allocated to each silo:";
    display tonsOfProduct;
```

group23_HW3_p2.2.dat

C: > Users > daniel.carpenter > OneDrive - the Chickasaw Nation > Documents > GitHub > OU-DSA > Metaheuristics >

```
# Homework 3 - Integer Programming
# Adv. Analytics and Metaheuristics
# Daniel Carpenter and Iker Zarandona
# March 2022
# Problem X

# SETS ==========================================================

    set PRODUCTS := A B C D E;      # 5 types of products
    set SILOS    := 1 2 3 4 5 6 7 8; # 8 different silos to store product p

# PARAMETERS ====================================================

    # Cost of storing one ton of product p ∈ PRODUCTS in silo s ∈ SILOS
    param cost:
              1   2   3   4   5   6   7   8 :=
        A     1   2   2   1   4   4   5   3
        B     2   3   3   3   1   4   5   2
        C     3   4   1   2   1   4   5   1
        D     1   1   2   2   3   4   5   2
        E     1   1   1   1   1   1   5   5
        ;

    # Supply of each product that is available
    param supply :=
        A    75
        B    50
        C    25
        D    80
        E    20
        ;

    # Capacity of each silo
    param capacity :=
        1    25
        2    25
        3    30
        4    60
        5    80
        6    85
        7    100
        8    50
        ;

    # Big M Scaler for mapping decision variables in binary prog.
    param M := 1000;
```

2.4.2 Output (Part `b`)

```
ampl: model 'C:\Users\daniel.carpenter\OneDrive - the Chickasaw
CPLEX 20.1.0.0: optimal integer solution; objective 320
48 MIP simplex iterations
0 branch-and-bound nodes
Objective = costOfStorage

Which silo(s) stores what product?
isStored [*,*] (tr)
:   A   B   C   D   E    :=
1   1   0   0   0   0
2   0   0   0   1   0
3   0   0   1   0   0
4   1   0   0   0   0
5   0   1   0   0   0
6   0   0   0   0   1
7   0   0   0   1   0
8   0   0   0   1   0
;

Optimal tons of product allocated to each silo:
tonsOfProduct [*,*] (tr)
:   A    B    C    D    E     :=
1   25    0    0    0    0
2    0    0    0   25    0
3    0    0   25    0    0
4   50    0    0    0    0
5    0   50    0    0    0
6    0    0    0    0   20
7    0    0    0    5    0
8    0    0    0   50    0
;
```

2.4.2.1 Analysis of the Output

- The optimal cost actually stays the same, but the amount of iterations to get to that solution is much more.
- The values of the decision variables are the same.

3 - Problem 3

3.1 Mathematical Formulation

3.1.1 Parameters

Parameter Name	Description
theDemand	The demanded amount of products
M	Large scaler that is not inf, used for logical constraints via Big M Method
mcWII	Marginal cost component of WII. Set to $4.95
availWII	Amount of WII that is available. Set to 18,000
mcWRS	Marginal cost component of WRS. Set to $2.30
fixWRS	Fixed Cost component of WRS. Set to 20,000
availWRS	Amount of WRS that is available. Set to 14,000
mcWE1	If we buy from WRS, then Marg. cost for WE set to $3.95
mcWE2	If we do not buy from WRS, then Marg. cost for WE set to $4.10
availWE	Amount of WE that is available. Set to 7,000
mcWU	Marginal cost component of WU. Set to 4.25
availWU	Amount of WU that is available. Set to 22,000
minBuyAmt	Must buy at least 15k of WU. Set to 15,000
mcWOW1	Marginal cost of 9.50 for WOW 3000 upper bound
mcWOW1Upper	3000 upper bound
mcWOW2	Marginal cost of 4.90 for WOW 3000 + 6000 = 9000 upper bound
mcWOW2Upper	3000 + 6000 = 9000 upper bound
mcWOW3	Marginal cost of 2.75 for WOW Cannot exceed 25000 due to supply

mcWOW3Upper Cannot exceed 25000 due to supply 25000

3.1.2 Decision Variables

3.1.2.1 Main Decision Variables:

Variable Name	Description
WII	Amount of product WOW to produce
WRS	Amount of product WRS to produce
WU	Amount of product WU to produce
WE	Amount of product WE to produce
WE1	Decision variable associated with 3.95 marginal cost for WE
WE2	Decision variable associated with 4.10 marginal cost for WE
WOW	Amount of product WOW to produce
d1WOW	Piece wise component 1 of var WOW
d2WOW	Piece wise component 2 of var WOW
d3WOW	Piece wise component 3 of var WOW

3.1.2.2 Binary Helper Decision Variables:

Variable Name	Description
yWRS1	Used for fixed WRS cost if used
yWRS	Used if WRS is selected
yWII	Used if WII is selected
yWE1	Used if $WE1$ is selected
yWE2	Used if $WE2$ is selected
z	Used to activate only one constraint for WE (below)
yWU	Used for fixed cost if WU used
y1WOW	To model piecewise cost for var WOW
y2WOW	To model piecewise cost for var WOW

3.1.3 Objective Function

$minimizecost:$

$$
\begin{aligned}
&\quad mcWII * WII \\
&+ fixWRS * yWRS1 + mcWRS * WRS \\
&+ mcWE1 * WE1 + mcWE2 * WE2 \\
&+ mcWU * WU \\
&+ mcWOW1 * d1WOW + mcWOW2 * d2WOW + mcWOW3 * d3WOW
\end{aligned}
$$

3.1.4 Constraints

Description	Constraint
Upper bound on WII production	$upperBoundWII : WII \leq availWII$
Upper bound on WU production	$upperBoundWU : WU \leq availWU$
Upper bound on WE production	$upperBoundWE : WE \leq availWE$
Upper bound and map to WRS via Big M	$map\_yWRS1 : WRS \leq availWRS \times yWRS1$
Map the W vars to the y binary	$mapWE1 : WE1 \leq M \times yWE1$
""	$mapWE2 : WE2 \leq M \times yWE2$
Map the WRS vars to the y binary	$mapWRS : WRS \leq M \times yWRS$
Map the WII vars to the y binary	$mapWII : WII \leq M \times yWII$
WE: If buy from WRS, then can do WE1. (Use of Mz to choose one constraint)	$ifWRS\_ThenWE1 : yWRS \leq yWE1 + M \times z$
WE: If $WE2$, cannot do WII. (Use of Mz to choose one constraint)	$ifWRS\_thenNotWII : yWE2 + yWII \leq 1 + M \times (1 - z)$
WE: If $WE1$, then cannot do $WE2$, Must choose one	$only1WE : yWE1 + yWE2 \leq 1$
WE: Finally, set WE to the sum of WE1 and WE2 for the final output	$setWE : WE = WE1 + WE2$
WU: Buy at least min amount	$buyAtLeastMin : WU \leq availWU \times yWU$
WU: Under the upper bound	$map\_yWU : WU \geq minBuyAmt \times yWU$
WOW: connect WOW with d1WOW, d2WOW, and d3WOW;	$X_WOW : WOW = d1WOW + d2WOW + d3WOW$
WOW: ensure that the piece wise costs are used correctly,	$piece1a : mcWOW1Upper \times y1WOW \leq d1WOW$
WOW: First Piece (Between 0 and Upper)	$piece1b : d1WOW \leq mcWOW1Upper$
WOW: Second Piece (Between last piece and Upper)	$piece2a : mcWOW2Upper \times y2WOW \leq d2WOW$
WOW: Second Piece (Between last piece and Upper)	$piece2b : d2WOW \leq mcWOW2Upper \times y1WOW$
WOW: Third Piece (Between last piece and Upper)	$piece3 : d3WOW \leq mcWOW3Upper \times y2WOW$
WOW: Cannot go over upper	$upperBoundWOW : WOW \leq mcWOW3Upper$
Meet the total demand	$meetTheDemand : WII + WRS + WE + WU + WOW \geq theDemand$

CX: Non-negatitvity of Decision Vars

3.2 Code and Output

3.2.1 Code

group23_HW3_p3.mod

C: > Users > daniel.carpenter > OneDrive - the Chickasaw Nation > Documents > GitHub > OU-DSA > Metaheuristics > 03 - Homewor

```
# Homework 3 - Integer Programming
# Adv. Analytics and Metaheuristics
# Daniel Carpenter and Iker Zarandona
# March 2022
# Problem 3

reset;                  # Reset globals
options solver cplex;   # Using cplex for simplex alg

# GLOBAL PARAMETERS ==============================================
param theDemand := 55000;     # The demanded amount of products
param M         := 10000000; # Large scaler that is not inf

# WII - Basic Marginal Cost Model ===============================

    # PARAMETERS -----------------------------------------------
    param mcWII    := 4.95;   # Marginal cost compnent of WII
    param availWII := 18000;  # Amount of WII that is available

    # DECISION VARIABLES ---------------------------------------
    var WII >= 0;   #amt of product WOW to produce

    # CONSTRAINTS ----------------------------------------------
    s.t. upperBoundWII: WII <= availWII;

# END OF WII - Basic Marginal Cost Model =========================


# WRS - Marginal Cost + Fixed Cost Model =========================

    # PARAMETERS -----------------------------------------------
    param mcWRS    := 2.30;  # Marginal cost compnent of WRS
    param fixWRS   := 20000; # Fixed Cost component of WRS
    param availWRS := 14000; # Amount of WRS that is available

    # DECISION VARIABLES ---------------------------------------
    var WRS >= 0;    # amt of product WRS to produce
    var yWRS1 binary; # Binary used for fixed cost if used

    # CONSTRAINTS ----------------------------------------------
    s.t. map_yWRS1: WRS <= availWRS * yWRS1; # Upper bound and map

# END OF WRS - Basic Marginal Cost Model =========================


# WE - Basic Marginal Cost Model ================================

    # PARAMETERS -----------------------------------------------
    param mcWE1   := 3.95;  # If buy from WRS, m. cost for WE
    param mcWE2   := 4.10;  # Else m. cost for WE
    param availWE := 7000;  # Amount of WE that is available

    # DECISION VARIABLES ---------------------------------------
        # WE decision vars
        var WE1 >= 0;   # Decision variable associated with $3.95 marginal cost
        var WE2 >= 0;   # Decision variable associated with $4.10 marginal cost
        var WE  >= 0;   # Decision variable for final output

        # Binary Vars to see what product is selected
        var yWRS  binary;   # If WRS is selected
        var yWII  binary;   # If WII is selected
        var yWE1  binary;   # If WE  is selected
        var yWE2  binary;   # If WE  is selected
        var z     binary;   # Octivates only one constraint
```

group23_HW3_p3.mod ×

C: > Users > daniel.carpenter > OneDrive - the Chickasaw Nation > Documents > GitHub > OU-DSA > Metaheuristics > 03 - Homework > HW

```
    # CONSTRAINTS ---------------------------------------------------
        # Map binary variables to show selection of products
        s.t. mapWE1: WE1  <= M * yWE1; # Map the W vars to the y binary
        s.t. mapWE2: WE2  <= M * yWE2; # ""
        s.t. mapWRS: WRS  <= M * yWRS; # ""
        s.t. mapWII: WII  <= M * yWII; # ""

        # Logical Constraints
            # If buy from WRS, then can do WE1. (Use of Mz to choose one constraint)
            s.t. ifWRS_ThenWE1:    yWRS <= yWE1 + M*z;

            # If WE2, cannot do WII. (Use of Mz to choose one constraint)
            s.t. ifWRS_thenNotWII: yWE2 +  yWII <= 1 + M*(1-z);

            # If WE1, then cannot do WE2, Must choose one
            s.t. only1WE:   yWE1 + yWE2 <= 1;

            # Finally, set WE to the sum of WE1 and WE2 for the final output
            s.t. setWE: WE == WE1 + WE2;

        s.t. upperBoundWE: WE <= availWE; # Meet the upper bound limit

# END OF WE - Basic Marginal Cost Model ========================


# WU - Marginal Cost + Fixed Cost Model ==============================

    # PARAMETERS ----------------------------------------------------
    param mcWU       := 4.25;  # Marginal cost compnent of WU
    param availWU    := 22000; # Amount of WU that is available
    param minBuyAmt := 15000; # Must buy at least 15k

    # DECISION VARIABLES ----------------------------------------
    var WU >= 0;     # amt of product WU to produce
    var yWU binary; # Binary used for fixed cost if used

    # CONSTRAINTS ---------------------------------------------------
    s.t. buyAtLeastMin: WU <= availWU   * yWU;    # Buy at least min amount
    s.t. map_yWU:       WU >= minBuyAmt * yWU;  # Under the upper bound

# END OF WU - Basic Marginal Cost Model ========================


# WOW - Piecewise Linear Cost Model ==============================

    # PARAMETERS ---------------------------------------------------

    #assume the linear costs for decision variable WOW are as follows
    #cost = 9.50 for     0 <=WOW < 3000
    #cost = 4.90 for  3000 <=WOW < 9000
    #cost = 2.75 for  9000 <=WOW < INFINITY

    param mcWOW1 := 9.50; param mcWOW1Upper := 3000;  # 3000 upper bound
    param mcWOW2 := 4.90; param mcWOW2Upper := 6000;  # 3000 + 6000 = 9000 upper bound
    param mcWOW3 := 2.75; param mcWOW3Upper := 25000; # Cannot exceed 25000 due to supply
    # DECISION VARIABLES ----------------------------------------
    var WOW >= 0;   #amt of product WOW to produce

    var d1WOW >=0;  # piecewise component 1 of var WOW
    var d2WOW >=0;  # piecewise component 2 of var WOW
    var d3WOW >=0;  # piecewise component 3 of var WOW

    var y1WOW binary; #to model piecewise cost for var WOW
    var y2WOW binary; #to model piecewise cost for var WOW
```

group23_HW3_p3.mod ×

C: > Users > daniel.carpenter > OneDrive - the Chickasaw Nation > Documents > GitHub > OU-DSA > Metaheuristics > 03 - Homework > HW

```

    # CONSTRAINTS --------------------------------------------------

        #connect WOW with d1WOW, d2WOW, and d3WOW;
        s.t. X_WOW: WOW = d1WOW + d2WOW + d3WOW;

        #ensure that the piece wise costs are used correctly,
        #i.e., you have to use all of d1WOW before you use d2WOW,...
        # First Piece (Between 0 and Upper)
        s.t. piece1a: mcWOW1Upper*y1WOW <= d1WOW;
        s.t. piece1b: d1WOW <= mcWOW1Upper;

        # Second Piece (Between last piece and Upper)
        s.t. piece2a: mcWOW2Upper*y2WOW <= d2WOW;
        s.t. piece2b: d2WOW <= mcWOW2Upper*y1WOW;

        # Third Piece (Between last piece and Upper)
        s.t. piece3: d3WOW <= mcWOW3Upper*y2WOW;

        # Cannot go over upper
        s.t. upperBoundWOW: WOW <= mcWOW3Upper;

# END OF WOW - Piecewise Linear Cost Model ==========================

# Last Constraint: Must meet the demand
s.t. meetTheDemand: WII + WRS + WE + WU + WOW >= theDemand;


# ==================================================================
# OBJECTIVE FUNCTION
# ==================================================================

minimize cost:    mcWII*WII                  # WII: Variable cost only
                + fixWRS*yWRS1 + mcWRS*WRS  # WRS: Fixed plus variable
                + mcWE1*WE1    + mcWE2*WE2  # WE: Continguint mc based on scenario
                + mcWU*WU                   # WU: Restricted range to over 15k
                + mcWOW1*d1WOW + mcWOW2*d2WOW + mcWOW3*d3WOW # WOW: Piecewise
                ;


# CONTROLS =========================================================

solve;

print;
printf  "Demand\t| WII\t| WRS\t| WE\t|  WU\t|  WOW\t| Total Cost";
printf "\n%s\t %s\t %s\t %s\t %s\t %s\t %f", theDemand, WII, WRS, WE, WU, WOW, cost;
print;

```

3.2.2 Output

Summary table of Output

Demand	WII	WRS	WE	WU	WOW	Total Cost
5000	0	0	5000	0	0	19750.000000
10000	3000	0	7000	0	0	42500.000000
25000	4000	14000	7000	0	0	99650.000000
35000	0	14000	6000	15000	0	139650.000000
45000	0	14000	6000	0	25000	177800.000000
50000	4000	14000	7000	0	25000	201550.000000
55000	0	14000	1000	15000	25000	221800.000000

Snapshots of Compilation

```
ampl: model 'C:\Users\daniel.carpenter\OneDrive - the Chickasaw N
CPLEX 20.1.0.0: optimal integer solution; objective 19750
3 MIP simplex iterations
0 branch-and-bound nodes

Demand    | WII     | WRS     | WE      |  WU     |  WOW    | Total Cost
5000        0         0         5000      0         0         19750.000000
```

```
Demand    | WII     | WRS     | WE      |  WU     |  WOW    | Total Cost
10000       3000      0         7000      0         0         42500.000000
```

```
Demand    | WII     | WRS     | WE      |  WU     |  WOW    | Total Cost
25000       4000      14000     7000      0         0         99650.000000
```

```
Demand    | WII     | WRS     | WE      |  WU     |  WOW    | Total Cost
35000       0         14000     6000      15000     0         139650.000000
```

```
Demand    | WII     | WRS     | WE      |  WU     |  WOW    | Total Cost
45000       0         14000     6000      0         25000     177800.000000
```

```
Demand    | WII     | WRS     | WE      |  WU     |  WOW    | Total Cost
50000       4000      14000     7000      0         25000     201550.000000
```

```
Demand    | WII     | WRS     | WE      |  WU     |  WOW    | Total Cost
55000       0         14000     1000      15000     25000     221800.000000
```