DSA/ISE 5113 Advanced Analytics and Metaheuristics
Homework #2

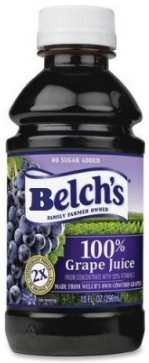
Instructor: Charles Nicholson

Due: See course website for due date

Requirement details

1. Homeworks should be submitted in a clean, clear, concise electronic format. You must show your logic, work, and/or code where appropriate in a PDF file – the PDF is your primary submission. Code files (e.g., AMPL model and/or data files) are also part of your required submission (turn in as separate files).
2. For mathematical programming problems, ensure you clearly define the following elements: (i) **necessary assumptions**, (ii) **decision variables**, (iii) **objective and objective function**, and (iv) **constraints**. Points may be deducted if proper definitions/documentation is missing.
3. Submit one PDF file and all code files. The code files may be zipped, however, do not zip the PDF file.

Question 1: Grapes of Wrath, Inc. (38 points)



Grapes of Wrath, Inc. packs, prepares, and distributes a variety of grape products, including raisins, jelly, and juice, under private brands in the US. The Vice-President of Operations, Ms. Camilla Azua, asked her executive team to meet with her to discuss the products to prepare for the upcoming season. The grape crop, which had been purchased during the previous accounting year, was beginning to arrive and appropriate operations would have to be started in the next two weeks.

Mr. Thomas, the Controller, and Ms. Bollman, the Sales Manager, were the first to arrive in Ms. Azua's office. Rick Grimes, the Production VP, came in a few minutes later and said that he had picked up the produce inspection's latest estimate of the quality of the incoming grapes. According to the report, 15% of the crop was Grade "A" quality and the remaining portion of the 6,200,000-pound crop was Grade "B."

Azua asked Bollman about the demand for grape products for the coming year. Bollman replied that they could sell all of the raisins they can produce. The expected demand for juice and jelly, on the other hand, was limited. The Sales Manager then passed around the latest demand forecast, which is shown in Table 1. She reminded the group that the selling prices had been set in light of the long-term marketing strategy of the company, and potential sales had been forecast at these prices.

Thomas, after looking at Bollman's estimates of demand, said that it looked like the company "should do quite well (on the grape crop) this year." With the new accounting system that had been set up, he had

been able to compute the contribution for each product, and according to his analysis the incremental profit on the raisins was greater than any other grape product. In May, after Grapes of Wrath, Inc. had signed contracts agreeing to purchase the grower's production at an average delivered price of 28 cents per pound, Thomas had computed the grape products' contributions (see Table 2).

Mr. Grimes brought to Thomas' attention that, although there was ample production capacity, it was impossible to produce all raisins as too small a portion of the crop was "A" quality. Grapes of Wrath used a numerical scale to record the quality of both raw produce and prepared products. This scale ran from zero to ten, the higher number representing better quality. Rating grape bunches according to this scale, "A" grapes averaged nine points per pound and "B" grapes averaged five points per pound. Rick noted that the minimum average input quality for raisins was eight and for juice it was six points per pound. Jelly could be made entirely from "B" grade grapes. This meant that raisin production is limited to using 1,240,00 pounds of raw produce.

Ms. Azua stated that this was not a real limitation. She has been recently solicited to purchase 300,000 pounds of Grade "A" grapes at 50 cents per pound and at that time had turned down the offer. She felt, however, that the grapes were still available.

Ms. Bollman, who had been doing some calculations, said that although she agreed that the company "should do quite well this year," it would not be by selling raisins. It seemed to her that the grape cost should be allocated on the basis of quality and quantity rather than by quantity only as Thomas had done. Therefore, she had recomputed the marginal profit on this basis (see Table 3), and from her results, no grapes should be sold as raisins!

(a) How does Grimes determine an upper bound for grapes that can be used for the raisins product?

(b) How does Bollman compute the fruit costs in Table 3?

(c) Ignoring for the moment the chance to buy additional A-grade grapes. Formulate the production question as a LP problem, solve with AMPL and answer the following questions:
   i. How much raisin, juice, and jelly product should be made?
   ii. What is the maximum contribution to profit?
   iii. Are there any grapes left over? If so, of what grade?
   iv. What is the average quality point count of the raisins, juice, and jelly?
   v. What would be the worth of one additional pound of A-grade grapes?
   vi. Should Grapes of Wrath, Inc. buy the extra 300,000 pounds of A-grade grapes at the offered price?
   vii. What is the maximize price that Grapes of Wrath would be willing to pay for an extra pound of A-grade product? for B-grade product?

(d) For the following questions, assume that the 300,000 pounds of A-grade is not in the formulation.
   i. What product mix would result if Thomas's contribution figures were used? Is Thomas's profit contribution more or less than the profit contribution to your (earlier) Part (c) solution? Using his figures, what is the maximum that Grapes of Wrath should pay per pound of additional A-grade grapes?
   ii. Alternatively, what product mix would result if Bollman's profit figures were used? What would the profit be? Is this more or less than you found in Part (c)? Using her figures, what is the maximum that Grapes of Wrath should pay per pound of additional A-grade grapes?
   iii. Ms. Azua is quite frustrated that her team is not giving her a straight answer – who is right and who is wrong (You, Thomas, and/or Bollman) and why? Make your case!

Table 1: Demand Forecasts

Product	Selling Price	Demand Forecast
2-pack, 13 oz. raisins	$8.29	unlimited
carton, 24, 10 oz grape juice	$16.20	190,000
6-pack, 32 oz grape jelly	$13.89	210,000

Table 2: Product Item Profitability

Product	2-pack 13 oz. raisins	24-carton 10 oz juice	6-pack 32 oz jelly
Selling Price ($)	8.29	16.20	13.89
Variable Costs ($)			
Direct Labor	1.98	3.40	2.68
Variable OHD.	0.24	0.42	0.25
Variable Selling	2.10	2.90	0.95
Packaging Material	0.75	3.10	2.62
Fruit*	1.82	3.92	5.04
Total Variable Costs	6.89	13.74	11.54
Contribution to profit	1.40	2.46	2.35
Less Allocated OHD	1.05	2.60	1.92
Net Profit	0.35	-0.14	0.43

*Product usage is given as follows:

Product	Pounds per product
2-pack, 13 oz. raisins	6.5
24-carton, 10 oz grape juice	14
6-pack, 32 oz grape jelly	18

Table 3: Bollman's marginal analysis of tomato products

1. Let z = cost per pound of A grapes in cents
2. Let y = cost per pound of B grapes in cents
3. (930,000 lb. $\times$ z) + (5,270,000 lb. $\times$ y) = (6,200,000 lb. $\times$ 28)
4. $\frac{z}{9} = \frac{y}{5}$
5. Therefore, $z = 45$ cents per pound; $y = 25$ cents per pound.

Product	2-pack 13 oz. raisins	24-carton 10 oz juice	6-pack 32 oz jelly
Selling Price	\$8.29	\$16.20	\$13.89
Variable Costs (excluding tomato costs)	5.07	9.82	6.50
	\$3.22	\$6.38	\$7.39
Fruit cost	2.60	4.20	4.50
Marginal profit	\$0.62	\$2.18	\$2.89

Question 2: Outdoor grilling (18 points)

"Immerse yourself in the pleasures of unique flavor creations and the gratification of outdoor cooking" says the salesman of a particular brand of high-end open-air culinary system. The proposition is simply too good, your passion has been ignited, and as visions of lazy Summer afternoons grilling delicious meats and veggies in your backyard with your friends, colleagues, and Advanced Analytics professor, you simply cannot resist purchasing this amazing piece of equipment. And thus, the following homework problem is motivated...

As equipment ages, it typically degrades, resulting in rising maintenance costs and decreasing salvage values. These conditions provide ample reasons to periodically replace such equipment.

Consider the specific case described above concerning a new gas griller you purchased to host parties at your home during the next five years. The cost of the grill is \$7,600. The cost of maintaining such a high-end item per year is significant if you wish to keep it in the best condition as possible (which, of course, you do). Assume the maintenance costs during a year depend on its age at the beginning of the year, as given in Table 4.

Table 4: Open-air culinary system costs and resale values

	Project	
Age (years)	Maintenance costs	eBay value
0	800	.
1	1250	5000
2	2000	4100
3	2900	1500
4	4800	950
5	.	0

To avoid the high maintenance costs associated with an older grill, you may sell the grill on eBay and purchase a new grill. The price you receive on eBay depends on the age of the grill at the time of sale (see Table 4). Assume that at any time, it costs \$7,600 to purchase a new grill.

The goal is to minimize the net cost (purchasing costs + maintenance costs - money received from eBay) for the next five years. Note: there is no option to simply sell the grill and not replace it: remember, your passion has been ignited and you must grill! Also, to simplify things, you will only make decisions at the beginning of each year..

For instance, when you first purchase the grill, it costs you \$7600. You will keep the grill for at least one year, so you would incur \$800 of maintenance costs. Now, at the beginning of the second year, you can either decide to keep it for another year (and incur the additional maintenance costs) or sell it on eBay for \$5,000 and buy a new grill for \$7,600. Then you would need to make the same decision the next year, etc.

Formulate this problem as a *shortest-path netfwork flow problem* and solve.

Question 3: Racecar tires (18 points)

An automobile association is organizing a series of car races that will last four days. The organizers know that $r_j \geq 0$ special race tires in good condition will be required on each of the four successive days, $j = 1, 2, 3, 4$. They can meet these needs either by buying new tires at P dollars apiece or by reshaping[1] used tires[2] after the day's race or some combination of both. Two kinds of reshaping service are available: *normal* service, which takes one full day at N dollars per tire, and *quick* service, which is an overnight service and costs Q dollars per tire. How should the association, which starts out with no special tires, meet the daily requirements at minimal cost?

(a) Formulate a network flow model to address the problem.

(b) Using the following values, solve an instance of problem: $P = \$600, N = \$95, Q = \$250$; and r_j for $j = 1, 2, 3, 4$ is 320, 240, 400, and 520, respectively.

Question 4: Dunder Mifflin Paper Company (26 points)

You have been hired as a director in the *Paper Products* division of Dunder Mifflin to support operations for a certain B2B[3] product line.

Specifically, you have two major business customers, Goggle and Faceblock, who have firm contracts for your product each month of 1000 and 600 units, respectively.

Dundler Mifflin has three different production plants that can produce the product, one in Scranton, PA; one in Utica, NY, and one in Stamford, CT. However, each of the plants is limited on how much they can produce during regular time. Additionally, each plant has a different cost (not including labor) to produce one unit of the product. The transportation costs to ship the product from each plant to each customer is also different. All of these details are provided in Table 5.

If overtime is used, then each plant has an additional capacity of 100 units per month. The cost for overtime (not including labor) is 1.5 times the regular time cost.

Given your previous success at determining workforce levels, Dundler Mifflin has also decided to ask you to deal with the labor options available to this operations group. There are two sources of labor: certified specialists (100 available in the area available at \$2000 per month) and generalists (200 available in the area at \$1700 per month). You do not have to hire all of the workers. The specialists are able to produce 12 units of product each month, whereas the generalists only produce 10 units per month.

[1] Reshaping is a technique by which the grooves on the tire are deepened, using a special profile-shaped tool.
[2] You can safely assume that after a day of racing any used tire is no longer in "good" condition.
[3] Business-to-business

Table 5: Plant informations and relevant costs

Plant	Max capacity (units)	Costs per unit ($)		
		Manufacture	Transportation	
			Goggle	Faceblock
Scranton	505	90	8	15
Utica	465	105	14	18
Stamford	570	115	24	20

Dundler Mifflin must provide transportation for all workers to the plants. The cost per month of transporting a worker to Scranton is $300, to Utica is $250, and to Stamford is $275.

Create a network flow formulation of this problem and solve to minimize costs while meeting demand. Hint: you may need to consider the *generalized* network flow model.