

Comprehensive Final Exam

Adv. Analytics and Metaheuristics

Daniel Carpenter

May 2022

Contents

1 - <i>Question 1</i> (Version 1)	2
1.1 <i>Part 1</i> : Mathematical Formulation	2
1.2 <i>Part 2</i> : AMPL Code & Output	4
2 - <i>Question 2</i> (Version 6)	7
2.1 Code to get Root Node	7
2.2 Branch and Bound Diagram	8
3 - <i>Question 3</i> (Version 2)	9
3.1 <i>Part i</i> : Hill Climbing	9
3.2 <i>Part ii</i> : Path Relinking	10
3.3 <i>Part iii</i> : Simulated Annealing	11
4 - <i>Question 4</i> (Version 3)	12
4.1 <i>Part (i)</i> - Roulette Probability	12
4.2 <i>Part (ii/iii)</i> - Breed Offspring	12
5 - <i>Question 5</i> (Version 2)	13
5.1 <i>Part (i)</i> - Global Best	13
5.2 <i>Part (ii)</i> - Local Best w/Ring	14

1 - Question 1 (Version 1)

1.1 Part 1: Mathematical Formulation

1.1.1 Sets

NewBuildTypes: Set of new build types $b \in (\text{Homes}, \text{Duplex}, \text{MiniPark})$

1.1.2 Parameters

Parameter	Description	Default Value
<i>budget</i>	Federal grant allocation to revitalize neighborhoods	\$15MM total budget
<i>maxBuildingDemod</i>	Max amount of buildings that can be demolished	300 total buildings
<i>demoCost</i>	Cost of demolishing a building	\$4,000 per building
<i>freedUpSpace</i>	Acreage generated from demolishing a building	0.25 per building
<i>newBuildSpace_b</i>	Amount of acreage that a new building ($b \in \text{NewBuildTypes}$) consumes	<i>Homes</i> : 0.2, <i>Duplex</i> : 0.4, <i>MiniPark</i> : 1.0
<i>newBuildTax_b</i>	Amount of tax dollars generated from a new building ($b \in \text{NewBuildTypes}$)	<i>Homes</i> : 1,500, <i>Duplex</i> : 2,750, <i>MiniPark</i> : 500
<i>newBuildCost_b</i>	Amount of dollars used to create a new building ($b \in \text{NewBuildTypes}$)	<i>Homes</i> : 150,000, <i>Duplex</i> : 190,000, <i>MiniPark</i> : 20,000
<i>newBuildPercShare_b</i>	Minimum required percentage share of new buildings ($b \in \text{NewBuildTypes}$) created	<i>Homes</i> : 20%, <i>Duplex</i> : 10%, <i>MiniPark</i> : 5%

1.1.3 Decision Variables

Variable	Description
$numOldBuildsDemod$	Number of old buildings to demolish
$numNewBuilds_b$	Number of new buildings ($b \in NewBuildTypes$) to produce
$newBuildTotalCost$	Variables to hold total cost of new builds ($b \in NewBuildTypes$). Calculation: $\sum_{b \in NewBuildTypes} (numNewBuilds_b \times newBuildCost_b)$
$oldDemoTotalCost$	Variables to hold total cost of old demolitions. Calculation: $numOldBuildsDemod \times demoCost$
$sumOfNewBuilds$	# Variable to hold the sum of all new build types over all New build types ($b \in NewBuildTypes$). Calculation: $\sum_{b \in NewBuildTypes} (numNewBuilds_b)$

1.1.4 Objective

Maximize the tax revenue from the projects

$$maximize \text{ taxRevenue} : \sum_{b \in NewBuildTypes} (numNewBuilds_b \times newBuildTax_b)$$

1.1.5 Constraints

C1 Spend less than or equal to the federal budget (see variable definitions)

$$meetTheBudget : newBuildTotalCost + oldDemoTotalCost \leq budget$$

C2 Can only produce new builds using the demolished buildings land

$$useAvailLand : \sum_{b \in NewBuildTypes} (numNewBuilds_b \times newBuildSpace_b) \\ \leq numOldBuildsDemod \times freedUpSpace$$

C3 Can only clear a certain amount of old buildings

$$maxBuildingsCleared : numOldBuildsDemod \leq maxBuildingDemod$$

C4 For each new build type ($b \in NewBuildTypes$), the percentage share of the new build type must meet the minimum required (see variables)

$$share : numNewBuilds_b \geq newBuildPercShare_b \times sumOfNewBuilds, \\ \forall b \in Businesses$$

C5 Non-negativity and integer constraints

$$numOldBuildsDemod \in \mathbb{Z}, \geq 0 \\ numNewBuilds_b \in \mathbb{Z}, \geq 0, \forall b \in NewBuildTypes$$

1.2 Part 2: AMPL Code & Output

1.2.1 AMPL Code

Data *problem1.dat*

```
data;

# SETS =====

# Set of new build types
set NewBuildTypes := Homes Duplex MiniPark;

# PARAMETERS =====

param budget          := 15000000; # federal budget
param maxBuildingDemod := 300;      # max buildings can be demo'd
param demoCost         := 4000;     # Cost of each demolition
param freedUpSpace     := 0.25;     # Freed up space from demolition

# Amount of acreage that a new building (b in NewBuildTypes) consumes
param: newBuildSpace :=
    Homes    0.2
    Duplex   0.4
    MiniPark 1.0
    ;

# Amount of tax dollars generated from a new building (b in NewBuildTypes)
param: newBuildTax :=
    Homes    1500
    Duplex   2750
    MiniPark 500
    ;

# Amount of dollars used to create a new building (b in NewBuildTypes)
param: newBuildCost :=
    Homes    150000
    Duplex   190000
    MiniPark 20000
    ;

# Minimum required percentage share of new buildings (b in NewBuildTypes) created
param: newBuildPercShare :=
    Homes    0.20
    Duplex   0.10
    MiniPark 0.05
```

;

Model *problem1.mod*

```
reset;                # Reset globals
options solver cplex; # Using cplex for simplex alg

# SETS =====
set NewBuildTypes; # Set of new build types

# PARAMETERS =====
param budget          >= 0; # federal budget
param maxBuildingDemod >= 0; # max buildings can be demo'd
param demoCost        >= 0; # Cost of each demolition
param freedUpSpace    >= 0; # Freed up space from demolition

param newBuildSpace    {NewBuildTypes} >= 0; # new build acreage
param newBuildTax      {NewBuildTypes} >= 0; # n.b. tax generation
param newBuildCost     {NewBuildTypes} >= 0; # n.b. cost
param newBuildPercShare {NewBuildTypes} >= 0; # n.b. min % share

# DECISION VARIABLES =====
var numOldBuildsDemod          >= 0 integer; # Num old builds to demo
var numNewBuilds               {NewBuildTypes} >= 0 integer; # Num new builds to create

# Variables to hold total cost of new builds over all types
var newBuildTotalCost = sum{b in NewBuildTypes} ( (numNewBuilds[b] * newBuildCost[b]));

# Variables to hold total cost of old demolitions
var oldDemoTotalCost = (numOldBuildsDemod * demoCost) ;

# Variable to hold the sum of all new build types over all New build types
var sumOfNewBuilds = sum{b in NewBuildTypes}( numNewBuilds[b] );

# OBJECTIVE FUNCTION =====
maximize taxRevenue: sum{b in NewBuildTypes}( numNewBuilds[b] * newBuildTax[b] );

# CONSTRAINTS =====

# C1 Spend less than or equal to the federal budget
s.t. meetTheBudget:
    newBuildTotalCost + oldDemoTotalCost <= budget ;

# C2 Can only produce new builds using the demolished buildings land
```

```

s.t. useAvailLand:
    sum{b in NewBuildTypes}( numNewBuilds[b] * newBuildSpace[b] )
    <= numOldBuildsDemod * freedUpSpace ;

# C3 Can only clear a certain amount of old buildings
s.t. maxBuildingsCleared: numOldBuildsDemod <= maxBuildingDemod ;

# C4 For each new build type (b in NewBuildTypes),
# the percentage share of the new build type must meet the minimum required
s.t. share {b in NewBuildTypes}:
    numNewBuilds[b] >= newBuildPercShare[b] * sumOfNewBuilds ;

# CONTROLS =====
data problem1.dat; # retrieve data file with sets/param. values
solve;

print;

print "Number of old buildings to demolish and cost (dollars):";
display numOldBuildsDemod, oldDemoTotalCost ;

print "Number of new buildings produced and cost (dollars):";
display numNewBuilds , newBuildTotalCost ;

print "Total Budget Used (dollars):";
display newBuildTotalCost + oldDemoTotalCost ;

print "Part 3: Max Tax Revenue generated (dollars):";
display taxRevenue ;

```

1.2.2 Part 2/3: Solve AMPL Model and Display Solution

```

CPLEX 20.1.0.0: optimal integer solution; objective 199000
6 MIP simplex iterations
0 branch-and-bound nodes

Number of old buildings to demolish and cost (dollars):
numOldBuildsDemod = 137
oldDemoTotalCost = 548000

Number of new buildings produced and cost (dollars):
numNewBuilds [*] :=
    Duplex 62
    Homes  17
    MiniPark 6
;

newBuildTotalCost = 14450000

Total Budget Used (dollars):
newBuildTotalCost + oldDemoTotalCost = 14998000

Part 3: Max Tax Revenue generated (dollars):
taxRevenue = 199000

```

2 - Question 2 (Version 6)

2.1 Code to get Root Node

Root Node is Node 1 in the diagram

2.1.1 Root Node AMPL Model *problem2.mod*

```
reset;
option solver cplex; # Solver

var x >= 0; # Not integer because this is used to check the optimal when fixing a var
var y >= 0; # ""

# Original problem:
maximize theSolution: 2.5*x + 6*y;
    s.t. first: 3*x + 5*y <= 26;
    s.t. second: x >= 4;

# Used to check the BnB nodes
# Fixing x = 4 to solve for initial values of y
s.t. checkNode: x = 4;

# Solve and display
solve;
display theSolution, x,y;
```

2.1.2 Output of Root Node AMPL Model (fixing x=4 & solving for y)

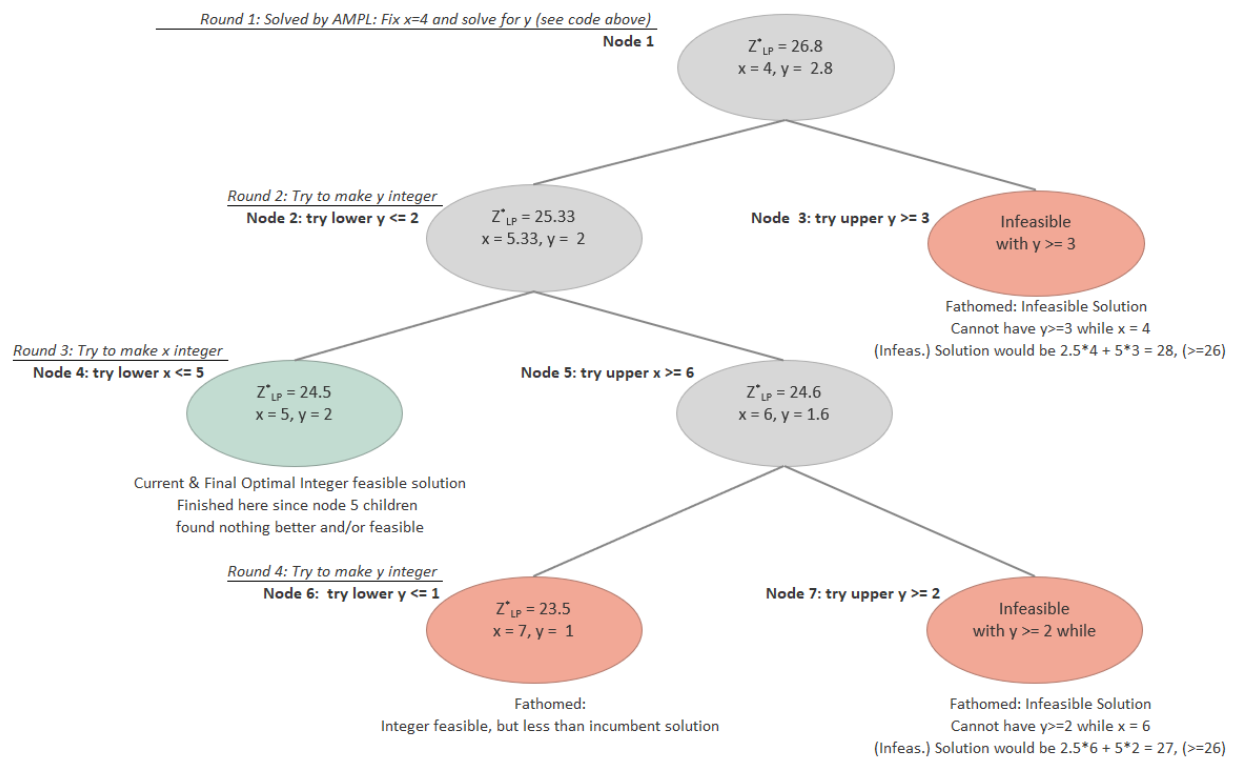
```
CPLEX 20.1.0.0: optimal solution; objective 26.8
0 dual simplex iterations (0 in phase I)
theSolution = 26.8
x = 4
y = 2.8
```

2.2 Branch and Bound Diagram

2.2.1 Summary of Diagram

- Optimal solution reached at node 4 with integer feasible values of $x = 5$, and $y = 2$
- Checked 7 total nodes
- Fathomed 3 nodes (see description of “why” in diagram)
- Each “Round” label shows which variable is being checked and for what integer value (lower or upper bound of the parent node)

2.2.2 Branch and Bound Diagram



3 - Question 3 (Version 2)

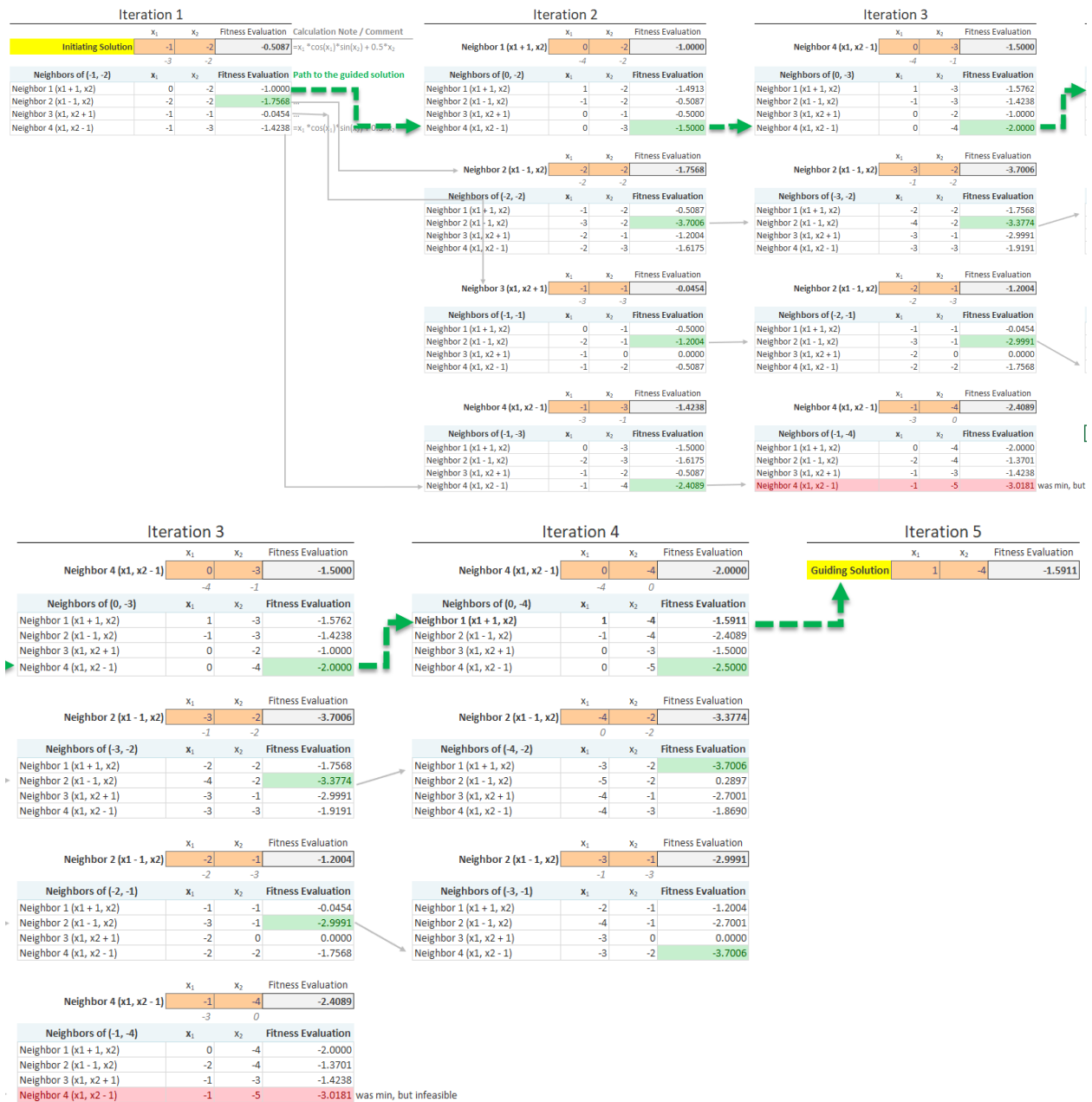
3.1 Part i: Hill Climbing

- Please see the below snippet on rightmost comments about the algorithm
- Ended at iteration 3 with minimum value of -1.9991 due to no change in best neighbor

	x_1	x_2	Fitness Evaluation	Calculation Note / Comment
Starting Solution, S_0	2	2	0.2432	$=x_1 * \cos(x_1) * \sin(x_2) + 0.5 * x_2$
Current Solution	2	2	0.2432	$=x_1 * \cos(x_1) * \sin(x_2) + 0.5 * x_2$
Done?	FALSE			
Iteration 1				
Best Neighbor (current)	2	2	0.2432	current solution from past iteration
Neighbors of (2, 2)				
Neighbor 1 ($x_1 + 1, x_2$)	3	2	-1.7006	$=x_1 * \cos(x_1) * \sin(x_2) + 0.5 * x_2$
Neighbor 2 ($x_1 - 1, x_2$)	1	2	1.4913	...
Neighbor 3 ($x_1, x_2 + 1$)	2	3	1.3825	...
Neighbor 4 ($x_1, x_2 - 1$)	2	1	-0.2004	...
(New) Best Neighbor	3	2	-1.7006	$=\min(\text{neighbors}, \text{best_neighbor})$
is Best Neighbor Same as Old?	FALSE			
Done?	FALSE			done if same
Current Solution	3	2	-1.7006	$=x_1 * \cos(x_1) * \sin(x_2) + 0.5 * x_2$
Iteration 2				
Best Neighbor (current)	3	2	-1.7006	current solution from past iteration
Neighbors of (3, 2)				
Neighbor 1 ($x_1 + 1, x_2$)	4	2	-1.3774	$=x_1 * \cos(x_1) * \sin(x_2) + 0.5 * x_2$
Neighbor 2 ($x_1 - 1, x_2$)	2	2	0.2432	...
Neighbor 3 ($x_1, x_2 + 1$)	3	3	1.0809	...
Neighbor 4 ($x_1, x_2 - 1$)	3	1	-1.9991	...
(New) Best Neighbor	3	1	-1.9991	$=\min(\text{neighbors}, \text{best_neighbor})$
is Best Neighbor Same as Old?	FALSE			
Done?	FALSE			done if same
Current Solution	3	1	-1.9991	$=x_1 * \cos(x_1) * \sin(x_2) + 0.5 * x_2$
Iteration 3				
Best Neighbor (current)	3	1	-1.9991	current solution from past iteration
Neighbors of (3, 1)				
Neighbor 1 ($x_1 + 1, x_2$)	4	1	-1.7001	$=x_1 * \cos(x_1) * \sin(x_2) + 0.5 * x_2$
Neighbor 2 ($x_1 - 1, x_2$)	2	1	-0.2004	...
Neighbor 3 ($x_1, x_2 + 1$)	3	2	-1.7006	...
Neighbor 4 ($x_1, x_2 - 1$)	3	0	0.0000	...
(New) Best Neighbor	3	1	-1.9991	$=\min(\text{neighbors}, \text{best_neighbor})$
is Best Neighbor Same as Old?	TRUE			
Done?	TRUE			done if same: end if end while

3.2 Part ii: Path Relinking

- Below shows two images with the path re-linking process iterations (from initiation to guided solution).
- Green arrow highlights the path to the guided solution
- Logic overview:** Evaluates neighbors of initiation solution, then moves along path of each neighbor. Makes best move for each neighbor, then checks difference between current and guided solution. Repeats until finding the guided solution
- Iteration 3 duplicated for ease of viewing*



3.3 Part iii: Simulated Annealing

- Current temperature = 3

Using the evaluation function for this questions (note language below is R)

```
# Evaluation function
evaluateFitness <- function(x1, x2) {
  x1*cos(x1)*sin(x2) + 0.5*x2
}
```

- The probability p of accepting a move uses the following formula: $p = e^{\frac{-(f(s_1)-f(s_2))}{T}}$
- Where $f(s_1)$ is the current solution, and $f(s_2)$ is the candidate solution since minimization. See calculation below (using evaluate function)

```
# Uses the evaluation function to return the probability (see above formula)
calculateProb <- function(temperature, currentPosition, newPosition) {
  f_s1 = evaluateFitness(currentPosition[1], currentPosition[2]) # f(s[1]) current
  f_s2 = evaluateFitness(newPosition[1],      newPosition[2]      ) # f(s[2]) candidate

  # Probability using minimization
  p = exp(1)^(- (f_s1-f_s2) / temperature )

  return(round(p, 3))
}
```

Now calculate the probability p from the current to the candidate solutions

```
# What is the probability of accepting a move from
# (4, 0) to candidate neighbor solution (4, 1)?
paste('At temp. 3, the probability from (4, 0) to (4, 1) is',
      calculateProb(temperature = 3,
                    currentPosition = c(4, 0),
                    newPosition = c(4, 1)) )
```

```
## [1] "At temp. 3, the probability from (4, 0) to (4, 1) is 0.567"
```

```
# What is the probability of accepting a move from
# (4, 0) to candidate neighbor solution (4,-1)?
paste('At temp. 3, the probability from (4, 0) to (4, -1) is',
      calculateProb(temperature = 3,
                    currentPosition = c(4, 0),
                    newPosition = c(4, -1)) )
```

```
## [1] "At temp. 3, the probability from (4, 0) to (4, -1) is 1.762"
```

4 - Question 4 (Version 3)

4.1 Part (i) - Roulette Probability

- Evaluates the fitness for each chromosome $c \in Chromosomes$ using $f(x) = x_1 + 2x_2 + 3x_3 + 4x_4 + 5x_5$ as evaluation function.
- Then computes the probability based on the following equation: $\frac{f_c}{\sum_{c \in Chromosomes} f_c}$
- See yellow highlighted cells for final roulette wheel probabilities for each chromosome

	A	B	C	D	E	F	G	H	I	J
1										
2										
3										
4										
5										
6										
7										
8										
9										
10										
11										
12										
13										
14										
15										
16										
17										
18										
19										
20										
21										
22										

Part (i)

These are the evaluation function coefficients for x1..., x5

Eval Coefs	1	2	3	4	5
Eval Function	1*x1 +	2*x2 +	3*x3 +	4*x4 +	5*x5

Step 1: Calculate fitness of each, Then sum up mating pool

Chromosome	x1	x2	x3	x4	x5	Fitness	
1	1	0	0	0	1	6	calculation: 1*x1 + 2*x2 + 3*x3 + 4*x4 + 5*x5
2	0	0	1	0	1	8	=SUMPRODUCT(C10:G10, C\$5:G\$5)
3	0	1	0	1	1	11	=SUMPRODUCT(C11:G11, C\$5:G\$5)
4	1	1	0	0	0	3	=SUMPRODUCT(C13:G13, C\$5:G\$5)
Sum Total Fitness Value of Pool						28	

Step 2: Calculate Probability of Selection

Chromosome	Fitness	Sum Total Fitness (Step 1)	(i) Prob. of Selection	
1	6	28	21.43%	=C18 / D18
2	8	28	28.57%	=C19 / D19
3	11	28	39.29%	=C20 / D20
4	3	28	10.71%	=C21 / D21

4.2 Part (ii/iii) - Breed Offspring

- Select the highest (Chrom. 3) and lowest (Chrom. 4) fitness valued chromosomes, then split at crossover point to produce offspring
- Offspring split between the second and third bit (highlighted for ease)
- Highest fitness valued offspring: Offspring 2 with fitness value of 12

A

B

C

D

E

F

G

H

I

J

22

23

24

25

26

27

28

29

30

31

32

33

34

35

36

37

Part (ii)

Assumes no use of the probability of selection from part (i)

Step 1: Find the Lowest and Highest Fitness Value Chromosomes

(Note Crossover point between 2nd and 3rd bit - highlighted for ease)

Chromosome	x1	x2	x3	x4	x5	Fitness	Note on selection
3	0	1	0	1	1	11	Highest
4	1	1	0	0	0	3	Lowest

Step 2 - Offspring: Breed the above chromosomes at crossover point

Offspring	x1	x2	x3	x4	x5	Fitness	calculation: 1*x1 + 2*x2 + 3*x3 + 4*x4 + 5*x5
Offspring 1	0	1	0	0	0	2	=SUMPRODUCT(C35:G35, C\$5:G\$5)
Offspring 2	1	1	0	1	1	12	=SUMPRODUCT(C36:G36, C\$5:G\$5)

5 - Question 5 (Version 2)

5.1 Part (i) - Global Best

5.1.1 Assumptions

- Uses all calculations and parameter values from the problem question. See header in picture as well for the calculation
- Parameters of interest highlighted in orange
- See global best logic in picture below

5.1.2 Solution

- Particle 1's Next Velocity: (-1.90, 3.55, 3.70)
- Particle 1's Next Position: (12.10, 8.55, 5.70)

5.1.3 Relevant Work

Part (i)

		Next Position		Next Vel.		Inertia		Cognitive				Social					
						I. Weight	Curr. Vel.			P. Best	Curr. Pos.			G. Best	Curr. Pos.		
Particle 1's...	Position Index	$X[t+1] = V[t+1] + X[t]$	$V[t+1]$	=		w	$V[t]$	+	phi[1]	r[1]	(P[i]	- X[t])	+	phi[2]	r[2]	(P[g]	- X[t])
	Position Idx. 1	12.10	-1.90			1	1		1	0.5	10	14		1	0.15	8	14
	Position Idx. 2	8.55	3.55			1	0		1	0.5	13	5		1	0.15	2	5
	Position Idx. 3	5.70	3.70			1	1		1	0.5	8	2		1	0.15	0	2

Particle 1's Next Velocity: (-1.90, 3.55, 3.70)

Particle 1's Next Position: (12.10, 8.55, 5.70)

Global Best Logic:

Highest personal best fitness value is 290, which is from particle 4

Use particle 4's personal best position for the global best

5.2 Part (ii) - Local Best w/Ring

5.2.1 Assumptions

- Local best paramaters highlighted in yellow
- See local best logic with ring structure in picture below

5.2.2 Solution

- Particle 1's Next Velocity: (-0.40, 4.30, 4.45)
- Particle 1's Next Position: (13.60, 9.30, 6.45)

5.2.3 Relevant Work

Part (ii)

				Inertia		Cognitive				Social							
Next Position				Next Vel.		I. Weight		Curr. Vel.		P. Best		Curr. Pos.					
Particle 1's....	Position Index	$X[t+1] = V[t+1] + X[t]$		$V[t+1]$	=	w	$V[t]$	+	phi[1]	r[1]	(P[i])	- X[t]	+	phi[2]	r[2]	(P[1]	- X[t])
	Position Idx. 1	13.60		-0.40		1	1		1	0.5	10	14		1	0.15	18	14
	Position Idx. 2	9.30		4.30		1	0		1	0.5	13	5		1	0.15	7	5
	Position Idx. 3	6.45		4.45		1	1		1	0.5	8	2		1	0.15	5	2

Particle 1's Next Velocity: (-0.40, 4.30, 4.45)

Particle 1's Next Position: (13.60, 9.30, 6.45)

Local Best Logic:

Neighbors of particle 1 are:

Particle 2

Personal Best Fitness value: 120

Particle 5

Personal Best Fitness value: 150

Use 5's p. best position since it has the highest personal best