LINEAR PROGRAMMING MODELING EXAMPLES: Transportation Problem

transportation problem

- Special class of LP problems that deal with shipping a commodity from *sources* (e.g. factories) to *destinations* (e.g. warehouses)
- Objective is to find a shipping plan that minimizes total shipping cost while satisfying supply and demand limits.
- The transportation problem can be extended to many types of applications – outside of transportation – such as inventory control, employment scheduling, and personnel assignment, etc.

transportation problem example

A mobile-home manufacturer in Indiana channels its mobile-home units through distribution centers located in Elkhart, Ind., Albany, N.Y., Camden, N.J., and Petersburg, Va.

An examination of their shipping department records indicates that, in the upcoming quarter, the distribution centers will have in inventory 30, 75, 60, and 35 mobile homes, respectively.

transportation problem example

Quarterly orders submitted by dealerships serviced by the distribution centers require the following mobile home units for the next quarter:

Dealer	A	B	C	D	E	F
Units	25	40	15	25	50	45

Transportation costs (in dollars per unit) between each distribution center and the dealerships are:

	A	B	C	D	E	F
Elkhart	75	65	175	90	110	150
Albany	90	30	45	50	105	130
Camden	40	55	35	80	70	75
Petersburg	95	150	100	115	55	55

Last chance to model an LP problem!!

PAUSE THE VIDEO

***n* distribution centers**

***m* dealerships**

$$\text{minimize} \quad \sum_{i=1}^{n} \sum_{j=1}^{m} c_{ij} x_{ij}$$

$$\text{subject to} \quad \sum_{j=1}^{m} x_{ij} = s_i \quad \text{for } i = 1, \ldots, n$$

i in Distributors (S)

j in Dealers (D)

$$\sum_{i=1}^{n} x_{ij} = d_j \quad \text{for } j = 1, \ldots, m$$

$$\mathbf{x} \geq 0$$

One logical requirement for the “transportation problem”…

Flow Balance Constraint (Supply EXACTLY meets Demand:

i in Distributors (S)

j in Dealers (D)

$$\sum_{i=1}^{n} s_i = \sum_{j=1}^{m} d_j$$

Integrality?

$$
\begin{aligned}
\text{minimize } & \mathbf{c}^T \mathbf{x} \\
\text{subject to } & \mathbf{A}\mathbf{x} = \mathbf{b} \\
& \mathbf{x} \geq 0
\end{aligned}
$$

$$\left[\begin{array}{cccccccccccccccccccccccc}
1&1&1&1&1&1&&&&&&&&&&&&&&&&&& \\
&&&&&&1&1&1&1&1&1&&&&&&&&&&&& \\
&&&&&&&&&&&&1&1&1&1&1&1&&&&&& \\
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1&&&&&&1&&&&&&1&&&&&&1&&&&& \\
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&&&&&1&&&&&&1&&&&&&1&&&&&&1
\end{array}\right]
\left[\begin{array}{c}
x_{1A} \\ x_{1B} \\ x_{1C} \\ x_{1D} \\ x_{1E} \\ x_{1F} \\ x_{2A} \\ x_{2B} \\ x_{2C} \\ x_{2D} \\ \cdot \\ \cdot \\ \cdot \\ x_{4F}
\end{array}\right]
=
\left[\begin{array}{c}
30 \\ 75 \\ 60 \\ 35 \\ 25 \\ 40 \\ 15 \\ 25 \\ 50 \\ 45
\end{array}\right]$$

For transportation problems where every s_i and d_j has an integer value, an optimal solution will always be integer!

LINEAR PROGRAMMING MODELING EXAMPLES: Assignment Problem

assignment problem applications

- Assigning teachers to time slots
- Assigning airplanes to flights
- Assigning project members to tasks
- Determining positions on a team
- Assigning medical residents to hospital training programs
- Assigning brides to grooms (once called the *marriage problem*

example assignment problem

A department has hired 11 new employees and needs to assign them offices. Each employee will have one office, and each office will only have one employee.

The employees have each ranked the offices that they would prefer from 1 (desired) to 11 (not desirable).

$$
\begin{aligned}
\text{minimize} \quad & \sum_{i=1}^{n} \sum_{j=1}^{m} c_{ij} x_{ij} \\
\text{subject to} \quad & \sum_{j=1}^{m} x_{ij} = s_i \quad \text{for } i = 1, \ldots, n \\
& \sum_{i=1}^{n} x_{ij} = d_j \quad \text{for } j = 1, \ldots, m \\
& \mathbf{x} \geq 0
\end{aligned}
$$

$$\text{minimize} \quad \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij}$$

$$\text{subject to} \quad \sum_{j=1}^{n} x_{ij} = 1 \quad \text{for } i = 1, \ldots, n$$

$$\sum_{i=1}^{n} x_{ij} = 1 \quad \text{for } j = 1, \ldots, n$$

$$\mathbf{x} \geq 0$$

Assignment problem is a special case of the transportation problem. As such, the LP solution will result in an optimal integer solution.