An Algebraic Introduction to Mathematical Logic Chapter 3 Properties of the Propositional Calculus Section 1 Introduction Exercises

David L. Meretzky

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Definition 1. A *logic* $\mathscr{L}$ is a system consisting of a set P of elements (called propositions), a set $\mathscr{V}$ of functions (called valuations) from P to some value set W, and, for each subset A of P, a set of finite sequences of elements of P (called proofs from the assumptions A).

Example 1. The logic called The Propositional Calculus on the set X, denoted $Prop(X)$, consists of the set $\mathscr{V}$ of all homomorphisms of $P(X)$ onto $\mathbb{Z}_2$, and the set of proofs defined as in section 4 of chapter 2.

Definition 2. A logic $\mathscr{L}$ is *sound* if $A \vdash p$ implies that $A \models p$.

Definition 3. A logic $\mathscr{L}$ is *consistent* if F is not a theorem.

Definition 4. A logic $\mathscr{L}$ is *adequate* if $A \models p$ implies that $A \implies p$.

Definition 5. A proposition is *valid* or *tautological* in a logic if for every valuation $v \in \mathscr{V}$, $v(p) = 1$ where $W = \mathbb{Z}_2$ and 1 captures our intuitive notion of truth.

Definition 6. A logic $\mathscr{L}$ is *decidable for validity* if there exists an algorithm which determines for every proposition p, in a finite number of steps, whether or not p is valid.

Definition 7. A logic $\mathscr{L}$ is *decidable for provability* if there exists an algorithm which determines for every proposition p, in a finite number of steps, whether or not p is a theorem.