An Algebraic Introduction to Mathematical Logic
Chapter 2 Propositional Calculus
Section 3 Truth in Propositional Calculus
Exercises

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Problem 1 (3.6)**.** *Show that* $\{F\} \models p$ *for all* $p \in P(x)$

3.6 Solution. $p = \sim\sim p$ Therefore, given any valuation v with the property that $v(F) = 1$ we have $v(\sim\sim p) = v(1 + \sim p(1 + F)) = v(1 + \sim p(1 + 1)) = v(1 + \sim p * 0) = 1$. Therefore $\{F\}$ semantically implies p for all $p \in P(X)$. □

Problem 2 (3.7 a)**.** *Show that* $\{p, p \Rightarrow q\} \models q$ *for all* $p, q \in P(x)$.

Solution 3.7 a. We have, $1 = v(1 + p(1 + q))$ and $v(p) = 1$. Therefore, since v is a homomorphism, $1 = v(1 + 1(1 + q))$ this means that $1 = 1 + v(1 + q)$ so $0 = 1 + v(q)$ therefore we have $v(q) = 1$. □

Problem 3 (3.7 b)**.** *Show that* $\{p, \sim q \Rightarrow \sim p\} \models q$ *for all* $p, q \in P(x)$

Solution 3.7 b. We have, $1 = v(1 + \sim q(1 + \sim p))$ and $v(p) = 1$. Therefore, $v(\sim p) = 0$. Therefore,$1 = v(1 + \sim q(1 + \sim p)) = v(1 + \sim q(1 + 0)) = v(1 + \sim q)$. We finally obtain $0 = v(\sim q)$ which implies, $v(q) = 1$. □

Problem 4 (3.8)**.** *Show that* $p \Rightarrow (p \Rightarrow q)$ *is a tautology.*

Solution 3.8. $v(p \Rightarrow (p \Rightarrow q)) = v(1+p(1+p \Rightarrow q)) = v(1+p(1+1+p(1+q))) = v(1+p(q(1+p)) = 1$ When $p = 0$ this expression values to $v(1+0(q(1+p))) = 1$. Similarly, when $p = 1$ this expression values to $v(1+1(q(1+1))) = v(1+(q*0)) = v(1) = 1$. Therefore, regardless of what q is, this expression is always true. □