Textbook notes on Enumerative Combinatorics.

David Cardozo

January 22, 2015

The following are notes based on the book *Enumerative Combinatorics* by Bóna.

Chapter 1

Basic Methods

1.0.1 When we add

Theorem 1. ***Addition Priniciple*** *If A and B are two disjoint finite sets, then:*

$$|A \cup B| = |A| + |B|$$

Proof. Both sides of the above relation count the elements of the same set, the set $A \cup B$. The left-hand side does this directly, while the right-hand side counts the elements of A and B separately. In either case, each element is counted exactly once (as A and B are disjoint), so the two sides are indeed equal □

Observe that the previous theorem was about two disjoint finite sets.

Theorem 2. ***Generalized Addition Principle*** *Let $A_1, A_2, \ldots A_n$ be finite sets that are pairwise disjoint. Then*

$$|A_1 \cup A_2 \cup \ldots A_n| = |A_1| + |A_2| + \ldots |A_n|$$

Proof. Again, both sides count the elements of the same set, the set $A_1 \cup A_2 \cup \ldots A_n$, therefore they have to be equal. □

1.1 When We Subtract

For this section we will use the following definition

Definition 1. ***Difference of two sets*** *If A and B are two sets, then $A - B$ is the set consisting of the elements of A that are not elements of B*

Although the difference is defined for $B \not\subset A$, we will only consider cases on which $B \subseteq A$.

Theorem 3. ***Subtraction Priniciple*** *Let A be a finite set, and let $B \subseteq A$. Then $|A - B| = |A| - |B|$*

Proof. On a more easy way, let us prove the equivalent relation:

$$|A - B| + |B| = |A|$$

This relation holds true by the Addition Principle. Indeed, $A - B$ and B are disjoint set that their union is A. $\square$

An important remark to denote here, is the fact that the hypothesis of $B \subseteq A$ is an important restriction.

The use of the Subtraction Principle is advisable in situations when it is easier to enumerate the elements of B ("bad guys") than the elements of $A - B$ (" good huys ")