

# Textbook notes on Enumerative Combinatorics.

David Cardozo

January 19, 2015

The following are notes based on the book *Enumerative Combinatorics* by Bóna.

# Chapter 1

## Basic Methods

### 1.0.1 When we add

**Theorem 1. Addition Principle** *If  $A$  and  $B$  are two disjoint finite sets, then:*

$$|A \cup B| = |A| + |B|$$

*Proof.* Both sides of the above relation count the elements of the same set, the set  $A \cup B$ . The left-hand side does this directly, while the right-hand side counts the elements of  $A$  and  $B$  separately. In either case, each element is counted exactly once (as  $A$  and  $B$  are disjoint), so the two sides are indeed equal  $\square$

Observe that the previous theorem was about two disjoint finite sets.

**Theorem 2. Generalized Addition Principle** *Let  $A_1, A_2, \dots, A_n$  be finite sets that are pairwise disjoint. Then*

$$|A_1 \cup A_2 \cup \dots \cup A_n| = |A_1| + |A_2| + \dots + |A_n|$$

*Proof.* Again, both sides count the elements of the same set, the set  $A_1 \cup A_2 \cup \dots \cup A_n$ , therefore they have to be equal.  $\square$

### 1.1 When We Subtract

For this section we will use the following definition

**Definition 1. Difference of two sets** *If  $A$  and  $B$  are two sets, then  $A - B$  is the set consisting of the elements of  $A$  that are not elements of  $B$*

Although the difference is defined for  $B \not\subseteq A$ , we will only consider cases on which  $B \subseteq A$