Graphs Connecting Pairs that Sum to Squares

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Characteristic Polynomials of A:

The reason the characteristic polynomials were generated is that they provide a lot of information about the graphs from the adjacency matrix. For example since the constant of the polynomial (corresponds to $\det(A)$) for most of the graphs is equal to 0, then most of the adjacency matrices are equal to 0 and at least one eigenvalue must be 0 too.

$$x^{2}$$

$$-x^{3} + x$$

$$x^{4} - x^{2}$$

$$-x^{5} + 2x^{3} - x$$

$$x^{6} - 3x^{4} + 2x^{2}$$

$$-x^{7} + 4x^{5} - 5x^{3} + 2x$$

$$x^{8} - 5x^{6} + 8x^{4} - 5x^{2} + 1$$

$$-x^{9} + 6x^{7} - 12x^{5} + 9x^{3} - 2x$$

$$x^{10} - 7x^{8} + 17x^{6} - 17x^{4} + 6x^{2}$$

$$-x^{11} + 8x^{9} - 23x^{7} + 28x^{5} - 12x^{3}$$

$$x^{12} - 9x^{10} + 30x^{8} - 45x^{6} + 29x^{4} - 6x^{2}$$

$$-x^{13} + 11x^{11} - 45x^{9} + 86x^{7} - 78x^{5} + 29x^{3} - 2x$$

$$x^{14} - 13x^{12} + 65x^{10} - 158x^{8} + 195x^{6} - 116x^{4} + 27x^{2} - 1$$

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-x^{15} + 15x^{13} - 88x^{11} + 2x^{10} + 257x^9 - 16x^8 - 392x^7 + 42x^6 + 297x^5 - 40x^4 - 94x^3 + 10x^2 + 8x
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$$x^{16} - 16x^{14} + 102x^{12} - 2x^{11} - 332x^{10} + 18x^9 + 586x^8 - 56x^7 - 547x^6 + 70x^5 + 241x^4 - 30x^3 - 38x^2 + 2x + 1$$

$$-x^{17} + 17x^{15} - 117x^{13} + 2x^{12} + 421x^{11} - 20x^{10} - 852x^9 + 74x^8 + 968x^7 - 126x^6 - 578x^5 + 100x^4 + 153x^3 - 32x^2 - 11x + 2$$

$$x^{18} - 18x^{16} + 132x^{14} - 2x^{13} - 510x^{12} + 22x^{11} + 1122x^{10} - 90x^9 - 1419x^8 + 172x^7 + 992x^6 - 160x^5 - 347x^4 + 68x^3 + 48x^2 - 10x - 1$$

$$-x^{19} + 20x^{17} - 165x^{15} + 2x^{14} + 731x^{13} - 22x^{12} - 1895x^{11} + 88x^{10} + 2935x^{9} - 154x^{8} - 2660x^{7} + 106x^{6} + 1325x^{5} - 6x^{4} - 319x^{3} - 14x^{2} + 27x + 2$$

$$x^{20} - 22x^{18} + 202x^{16} - 2x^{15} - 1009x^{14} + 26x^{13} + 2994x^{12} - 126x^{11} - 5399x^{10} + 280x^9 + 5791x^8 - 270x^7 - 3440x^6 + 56x^5 + 970x^4 + 44x^3 - 92x^2 - 8x$$

$$-x^{21} + 24x^{19} - 242x^{17} + 2x^{16} + 1339x^{15} - 26x^{14} - 4452x^{13} + 122x^{12} + 9142x^{11} - 236x^{10} - 11435x^9 + 94x^8 + 8209x^7 + 256x^6 - 2942x^5 - 268x^4 + 370x^3 + 40x^2 - 4x$$

$$x^{22} - 26x^{20} + 285x^{18} - 2x^{17} - 1723x^{16} + 28x^{15} + 6302x^{14} - 144x^{13} - 14367x^{12} + 312x^{11} + 20226x^{10} - 152x^9 - 16745x^8 - 408x^7 + 7314x^6 + 526x^5 - 1344x^4 - 130x^3 + 67x^2 + 8x$$

$$-x^{23} + 28x^{21} - 333x^{19} + 2x^{18} + 2207x^{17} - 32x^{16} - 8980x^{15} + 196x^{14} + 23269x^{13} - 560x^{12} - 38498x^{11} + 658x^{10} + 39616x^{9} + 114x^{8} - 23850x^{7} - 890x^{6} + 7454x^{5} + 600x^{4} - 933x^{3} - 110x^{2} + 17x + 2$$

$$x^{24} - 30x^{22} + 384x^{20} - 4x^{19} - 2758x^{18} + 72x^{17} + 12283x^{16} - 530x^{15} - 35343x^{14} + 2064x^{13} + 66326x^{12} - 4600x^{11} - 79966x^{10} + 5940x^9 + 59424x^8 - 4298x^7 - 25142x^6 + 1564x^5 + 5213x^4 - 176x^3 - 369x^2 - 24x + 1$$

$$-x^{25} + 32x^{23} - 440x^{21} + 4x^{20} + 3425x^{19} - 76x^{18} - 16718x^{17} + 594x^{16} + 53482x^{15} - 2460x^{14} - 113676x^{13} + 5774x^{12} + 159112x^{11} - 7538x^{10} - 142058x^9 + 4798x^8 + 75964x^7 - 754x^6 - 21620x^5 - 400x^4 + 2525x^3 + 92x^2 - 43x - 2$$

$$x^{26} - 34x^{24} + 500x^{22} - 4x^{21} - 4195x^{20} + 80x^{19} + 22275x^{18} - 662x^{17} - 78394x^{16} + 2926x^{15} + 185886x^{14} - 7408x^{13} - 295380x^{12} + 10560x^{11} + 305982x^{10} - 7366x^{9} - 194894x^{8} + 1040x^{7} + 68136x^{6} + 1006x^{5} - 10315x^{4} - 110x^{3} + 388x^{2} + 2x - 4$$

$$-x^{27} + 36x^{25} - 564x^{23} + 4x^{22} + 5077x^{21} - 88x^{20} - 29173x^{19} + 812x^{18} + 112307x^{17} - 4078x^{16} - 295422x^{15} + 12052x^{14} + 530966x^{13} - 20924x^{12} - 640167x^{11} + 19330x^{10} + 497046x^9 - 5572x^8 - 230393x^7 - 4270x^6 + 55152x^5 + 3022x^4 - 4908x^3 - 360x^2 + 92x + 8$$

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x^{28} - 38x^{26} + 632x^{24} - 6x^{23} - 6077x^{22} + 150x^{21} + 37554x^{20} - 1604x^{19} - 156664x^{18} + 9606x^{17} + 450477x^{16} - 35408x^{15} - 894282x^{14} + 83054x^{13} + 1206662x^{12} - 123694x^{11} - 1067810x^{10} + 113152x^9 + 580975x^8 - 59366x^7 - 173385x^6 + 15952x^5 + 22843x^4 - 1762x^3 - 876x^2 - 10x + 4
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 $\begin{array}{l} -x^{29} + 40x^{27} - 703x^{25} + 8x^{24} + 7171x^{23} - 222x^{22} - 47210x^{21} + 2642x^{20} + \\ 210836x^{19} - 17700x^{18} - 652809x^{17} + 73586x^{16} + 1405750x^{15} - 197112x^{14} - \\ 2077226x^{13} + 341596x^{12} + 2039283x^{11} - 374170x^{10} - 1253764x^9 + 245468x^8 + \\ 434938x^7 - 87724x^6 - 70352x^5 + 14480x^4 + 3858x^3 - 572x^2 - 75x \end{array}$

 $x^{30} - 42x^{28} - 2x^{27} + 778x^{26} + 64x^{25} - 8399x^{24} - 892x^{23} + 58801x^{22} + 7130x^{21} - 280912x^{20} - 36194x^{19} + 937635x^{18} + 121926x^{17} - 2199498x^{16} - 276164x^{15} + 3593314x^{14} + 416002x^{13} - 3986673x^{12} - 400866x^{11} + 2867708x^{10} + 226566x^{9} - 1236905x^{8} - 60702x^{7} + 281998x^{6} + 2636x^{5} - 28782x^{4} + 432x^{3} + 936x^{2}$

 $-x^{31} + 44x^{29} + 2x^{28} - 858x^{27} - 68x^{26} + 9808x^{25} + 1016x^{24} - 73244x^{23} - 8812x^{22} + 376793x^{21} + 49332x^{20} - 1371278x^{19} - 187334x^{18} + 3566577x^{17} + 492968x^{16} - 6610858x^{15} - 900584x^{14} + 8595970x^{13} + 1122906x^{12} - 7595396x^{11} - 916154x^{10} + 4314292x^9 + 448306x^8 - 1431091x^7 - 108010x^6 + 233457x^5 + 6304x^4 - 13994x^3 + 264x^2 + 36x$

 $x^{32} - 46x^{30} - 2x^{29} + 941x^{28} + 72x^{27} - 11330x^{26} - 1148x^{25} + 89546x^{24} + 10722x^{23} - 490375x^{22} - 65324x^{21} + 1913687x^{20} + 273332x^{19} - 5388050x^{18} - 804324x^{17} + 10950304x^{16} + 1673462x^{15} - 15895540x^{14} - 2435626x^{13} + 16102964x^{12} + 2408838x^{11} - 10931270x^{10} - 1530018x^9 + 4640296x^8 + 559038x^7 - 1086977x^6 - 90268x^5 + 107537x^4 + 1212x^3 - 1856x^2 - 72x$

 $-x^{33} + 49x^{31} + 2x^{30} - 1071x^{29} - 76x^{28} + 13831x^{27} + 1286x^{26} - 117788x^{25} - 12824x^{24} + 698881x^{23} + 84008x^{22} - 2974551x^{21} - 381158x^{20} + 9207529x^{19} + 1229086x^{18} - 20782906x^{17} - 2840758x^{16} + 33960724x^{15} + 4679590x^{14} - 39473170x^{13} - 5384512x^{12} + 31652811x^{11} + 4162990x^{10} - 16673244x^9 - 2018538x^8 + 5329910x^7 + 536378x^6 - 900454x^5 - 53674x^4 + 59756x^3 - 1542x^2 - 428x - 8$

 $x^{34} - 52x^{32} - 2x^{31} + 1210x^{30} + 78x^{29} - 16699x^{28} - 1354x^{27} + 152667x^{26} + 13846x^{25} - 977708x^{24} - 93054x^{23} + 4521069x^{22} + 434328x^{21} - 15327779x^{20} - 1451424x^{19} + 38276102x^{18} + 3531126x^{17} - 70085516x^{16} - 6302294x^{15} + 92802786x^{14} + 8250566x^{13} - 86654323x^{12} - 7838874x^{11} + 54763383x^{10} + 5245774x^9 - 21914669x^8 - 2309612x^7 + 4950252x^6 + 577360x^5 - 501588x^4 - 57152x^3 + 11676x^2 + 416x - 20$

Characteristic Polynomials of A^n :

$$\begin{array}{l} x^2 \\ -x^3+x \\ x^4-2x^3+x^2 \\ -x^5+2x^3-x \\ x^6-18x^5+97x^4-144x^3+64x^2 \\ -x^7+130x^5-257x^3+128x \\ x^8-98x^7+2593x^6-9506x^5+14020x^4-9506x^3+2593x^2-98x+1 \\ -x^9+6291x^7-2964627x^5+2958849x^3-512x \end{array}$$

(Further polynomials were omitted due to simplification issues)