A Markov Process-Based Approach for Reliability Evaluation of the Propulsion System in Multi-rotor Drones

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Abstract: Autonomous multirotors as a popular type of Unmanned Aerial Vehicles (UAVs) have a tremendous potential to facilitate activities such as logistics, emergency response, recording video, capturing special events, and traffic management. Despite the potential benefits the possibility of harming people during operation should be considered. This paper focuses on modeling multirotors' propulsion system with Markov chains. The validity of the models is proven with a combination of controllability theory and Monte Carlo simulations. Using the proposed model, both reliability and Mean Time To Failure (MTTF) of the propulsion system are evaluated. This study proposes a fault detection and recovery system based on a Markov Model for mission control of multirotors. Concretely, the proposed system aims to reduce potential injuries by increasing safety.

Keywords: Markov models, Mission planning and decision making, Mission planning and decision making, Reliability evaluation, Flight control, UAV.

1. INTRODUCTION

Nowadays, multirotors and small unmanned aircraft systems (sUAS) are being developed for consumer and commercial applications in urban areas that encompass logistics, emergency response, filming, traffic monitoring, agriculture monitoring, search and rescue, railway surveillance, and corrosion inspection in bridges (Belcastro, Klyde, Logan, Newman, & Foster, 2017). Many applications entail operating in populated areas where a failure of the multirotor can endanger humans. Therefore, reliability and safe landing of multirotors are important and challenging because of their dynamic behavior and complex models (Sadeghzadeh, Mehta, & Zhang, 2011). Reliability can be defined as the probability of multirotor functioning correctly during a given timespan. Safety, on the other hand, is the probability that a multirotor either functions correctly or stops its operation in a safe manner without causing injuries. To design a system that is safe and reliable, multirotor faults and failures need to be studied. Furthermore, performance modelling of the multirotor in the presence of faults and failures needs also to be done. By modeling multirotors for performance and functionality evaluation, bottlenecks and weak points of the system can be determined and strengthened. Among the performance evaluation methods, such as “Reliability Block Diagrams”, “Fault Trees” and “Petri-net”, the “Markov chain” is chosen because of its capabilities to model dynamic behaviors, spare systems, priorities, and dependencies. Having the Markov model of the multirotor is not only beneficial for reliability evaluation. But also, for proposing a mission control and diagnostics system to reduce possible injuries caused by faults and failures in multirotors’ systems and subsystems. As failures in the propulsion system can potentially cause a complete loss of control of the UAV it was chosen as the focus of this paper. In the following paragraphs, a brief literature survey on the research of faults and failures in UAVs is provided. Reliability improvement of UAVs in the design review procedures through Failure Modes and Effects Analysis (FMEA) and Fault Tree Analysis (FTA) has been studied by (Juliana de Oliveira Martins Franco & Carlos Sandoval Góes, 2007). In (Murtha, 2009) an FTA based procedure is proposed for reliability improvement in a cost-effective way. Dempster-Shafer Theory is also used to reduce the uncertainty in failure data. However, while aforementioned references have used FTA that is limited to modelling static characteristics of the systems we propose to cover the dynamic behavior of the multirotors by leveraging on a Markov model. Furthermore, while in (Olson & Atkins, 2013), only the qualitative failure analysis for the Michigan UAV has been addressed we will focus on the quantitative failure analysis. In this paper, the remaining useful life (RUL) of the UAV is estimated through fault tree analysis and has been used for health-based task allocation (Shi, Yang, & Quan, 2016), based on the controllability degree, the reliability of multirotor with different configurations has been studied. For casualty estimation in case of ground collision of vehicles and mid-air collision accidents (Barr, et al., 2017) proposed a preliminary risk analysis approach based on a probabilistic model-based for small sUAS. In (Belcastro, Klyde, Logan, Newman, & Foster, 2017) experimental flight test techniques with different possible sets of hazard-based test scenarios are proposed in order to evaluate the safety of sUAS operations. In spite of intensive research conducted so far, there is no published work that we are aware of on Markov modelling for fault detection and recovery for multirotor robots. This paper introduces the Markov model of a hexacopter and evaluates its reliability and

MTTF validating the results through controllability theory and Monte Carlo simulation. It also proposes an idea for building a Markov-based fault detection and recovery system. This systematic approach and its solution enable multirotor robot designers to have a deeper view of reliability behavior of these systems. The organization of this paper is as follows. Section II introduces Markov modeling of the propulsion system in multirotors and section III illustrates numerical results obtained from introduced Markov models. In section IV, a brief discussion on the idea of Markov-based fault detection and recovery is briefly explained. The paper ends with some concluding remarks and possible future avenues.

2. MARKOV MODELING OF THE UAVs' PROPULSION SYSTEMS

In this section, the proposed Markov model and its solution will be provided for reliability and MTTF calculation. In addition, the simplification of Markov model for multirotors will be introduced. The following assumptions are considered for the Markov modeling and evaluation:

- At the beginning, system is always operational;
- There is no common cause failure in the system;
- During the mission repair is not possible;
- The failure rates of the components obey an exponential distribution.

2.1 Simple Markov Model and Its Continuous-Time Solution

Fig. 1 shows the Markov reliability model of a simple system with two operational and failure states. To create the model, it is assumed that are no repairs. States and transition states are first recognized, and the rates of transitions are allocated. With exponential failure rate distribution function, the reliability of a module after time is then expressed by (1), and its unreliability (failure probability) is given by (2).

$$e^{-n\lambda t} = 1 - n\lambda\Delta t + n^2\lambda^2\Delta t^2 - \dots \quad (1)$$

$$1 - e^{-n\lambda t} = 1 - \left(1 - n\lambda\Delta t + n^2\lambda^2\Delta t^2 - \dots\right) \rightarrow n\lambda\Delta t \quad (2)$$

Thus, the probability of the module at the same state after elapsing Δt time is given by: $1-\lambda\Delta t$.

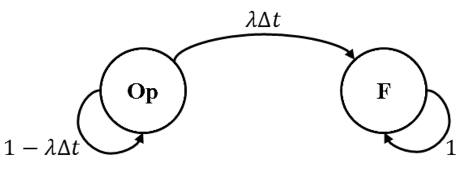

Fig. 1. Reliability Markov model of a simple system

2.1.1 Reliability Evaluation

Based on the Markov model illustrated in Figure 1 and according to the Markov theorem, (3) can be written as follows (Dubrova, 2013).

$$P\left(t+\Delta t\right) = M.P\left(t\right) \quad (3)$$

where P is the "probability states vector" (4) and M is the discrete state transition matrix (5).

$$P\left(t\right) = \left[P_{OP}\left(t\right), P_F\left(t\right)\right]^T \quad (4)$$

$$M = \begin{bmatrix} 1-\lambda\Delta t & \lambda\Delta t \\ 0 & 1 \end{bmatrix} \quad (5)$$

Equation (3) can be recursively solved if as an initial probability vector is known. The result for different times is given by (6).

$$P\left(n\Delta t\right) = P^n.P\left(0\right) \quad (6)$$

Equation (7) is the continuous form of (3).

$$\dot{P}\left(t\right) = A.P\left(0\right) \quad (7)$$

where A is the continuous Markov transition matrix in the form of (8).

$$A = \frac{\partial M^T}{\partial \Delta t} = \begin{bmatrix} -\lambda & 0 \\ \lambda & 0 \end{bmatrix} \quad (8)$$

Solving (7) gives the probability of system states at any time t.

$$P_{OP}(t) = e^{-\lambda t} \quad (9\text{-a})$$

$$P_F\left(t\right) = 1 - e^{-\lambda t} \quad (10\text{-b})$$

The reliability of a system can be obtained from the probability of states in Markov model those which represent the operational condition of the system. Hence, the reliability of a simple system is now calculated as equations and (11).

$$R_{Quadcopter}\left(t\right) = P_{OP}\left(t\right) = e^{-\lambda t} \quad (11)$$

Consider a simple quadcopter with rotors labeled 'a', 'b', 'c' and 'd'. A failure on a rotor can cause failure of the system. Hence, the reliability Markov model of quadcopter can be illustrated as in Fig. 2.

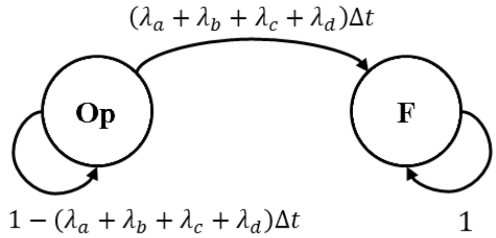

Fig. 2. Reliability Markov model of a Quadcopter

Having the Markov model for a quadcopter, the reliability expression of the system can be obtained easily by (12).

$$R_{Quadcopter}\left(t\right) = P_{OP}\left(t\right) = e^{-\left(\lambda_a t + \lambda_b t + \lambda_c t + \lambda_d t\right)} \quad (12)$$

2.1.2 MTTF Calculation

Same Markov model for MTTF calculation can be used as in Figure 1. This Markov model is an arbitrary absorbing Markov chain which is described by a transition matrix P. If there are r absorbing states and t transient states, the transition matrix will have the following canonical form. The MTTF Markov model has only one absorbing state and so in this case (I = 1).

$$P = \left[\begin{array}{c|c} Q & R \\ \hline 0 & I \end{array}\right] \tag{13}$$

Consider $N = (I - Q)^{-1}$ as the fundamental matrix of P and Let t_i be the expected number of steps before the chain is absorbed (goes to state A), given that the chain starts in state s_i and let t be the column vector whose i^{th} entry is t_i. The column vector of t can be written as follows:

$$t = [t(NA_1) \quad t(NA_2) \quad \dots \quad t(NA_n)]^T = NC \tag{14}$$

where C is a column vector all of whose entries are one. Once the system starts from state Op (Operational), the expected number of steps to be in the failure state (F) can be achieved by (15).

$$MTTF = t(Op) \tag{15}$$

2.2 Simple Markov Model and Its Continuous-Time Solution

Fig. 3 illustrates a simple hexacopter configuration with rotors labeled from a to f. In this configuration, motors are PNPNPN. P stands for positive clockwise direction and N stands for negative anti-clockwise direction (Shi, Yang, & Quan, 2016).

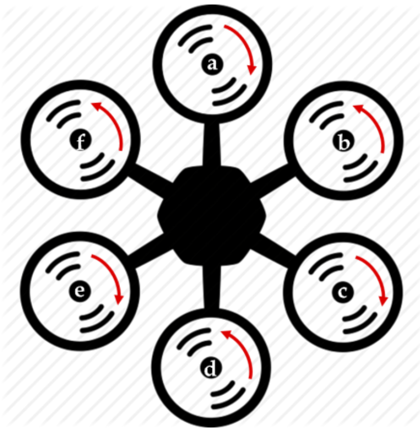

Fig. 3. A simple hexacopter with rotors labeled from a to f (PNPNPN configuration).

Based on the hexacopter illustrated in Fig. 3, the Markov model of Fig. 4 can be obtained. In this model, 19 states are considered. The first state in this model is "a b c d e f" where all rotors are fully operational. If during the mission one of the rotors fails (for example the one labeled "a"), then with the failure rate of $\lambda_a \Delta t$ the system goes to the "b c d e f" state. In the state: b c d e f", if the rotor "f" or "b" fails, then the systems goes to the "Failure" state. With all these possible failures in the hexacopter, the following model will be constructed. (For a more information about the operational and failure modes based on the controllability theory please refer to (Shi, Yang, & Quan, 2016)).

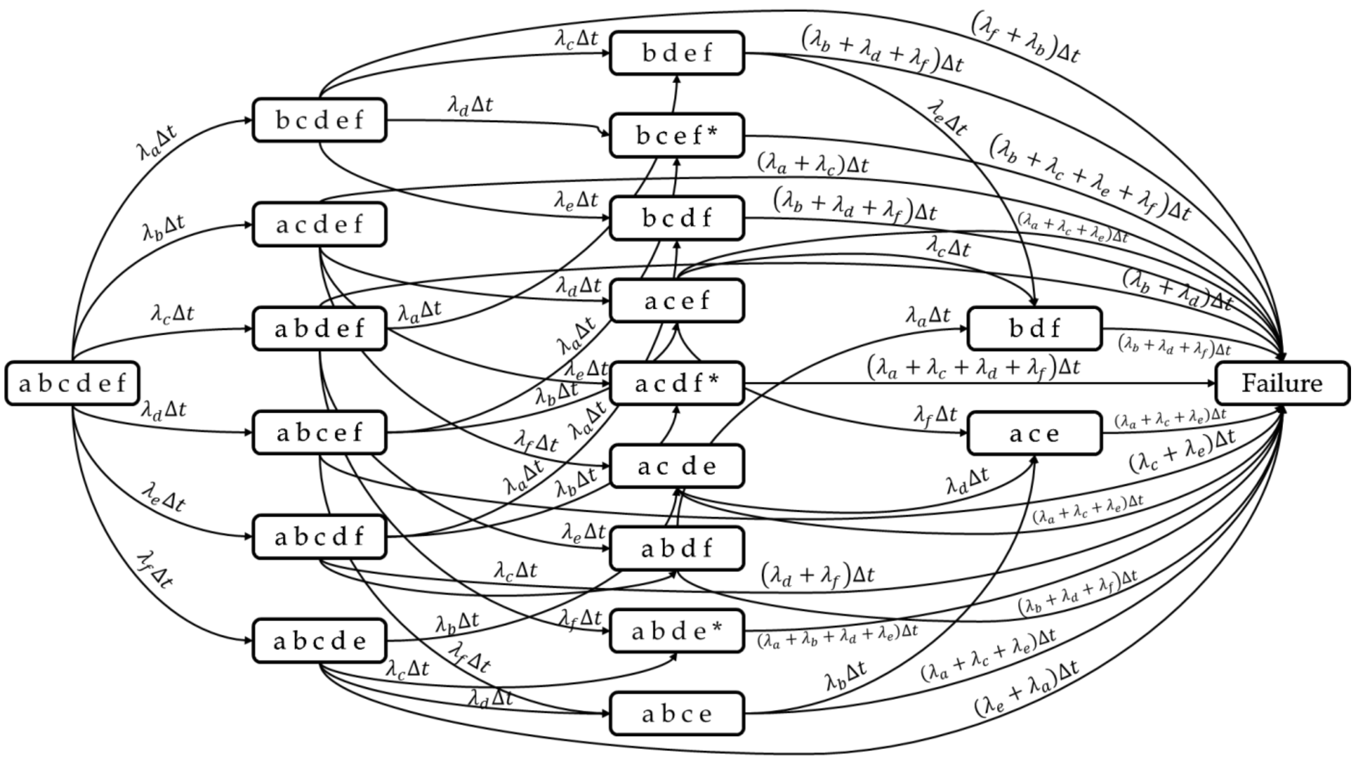

Fig. 4. Reliability Markov model of a hexacopter with PNPNPN configuration.

While the constructed reliability Markov model in Fig. 4 is for the PNPNPN configuration it is also possible to construct a reliability Markov model for PPNNPN configuration as Fig. 5. Consider the hexacopter with PPNNPN configuration as shown in Fig. 6. This kind of configuration is like quadcopter with redundancy in half the rotors. In this figure, rotors ‘a' and ‘b' are redundant. Also, rotors ‘c' and ‘d' are redundant with opposite rotation. In a quadcopter failure of a rotor causes a multirotor failure. Hence, failure of rotors ‘e' or ‘f' of a hexacopter can lead to a failure. Meanwhile, failure of a single rotor in each rotation block (green blocks) can be overcome. In comparison with a PNPNPN configuration, the PPNNPN configuration cannot tolerate the failure of three rotors.

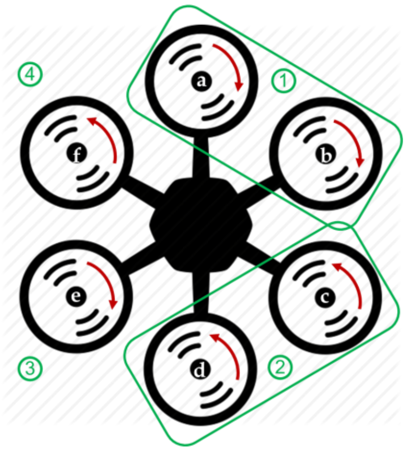

Fig. 5. A simple hexacopter with rotors labeled from a to f (PPNNPN configuration).

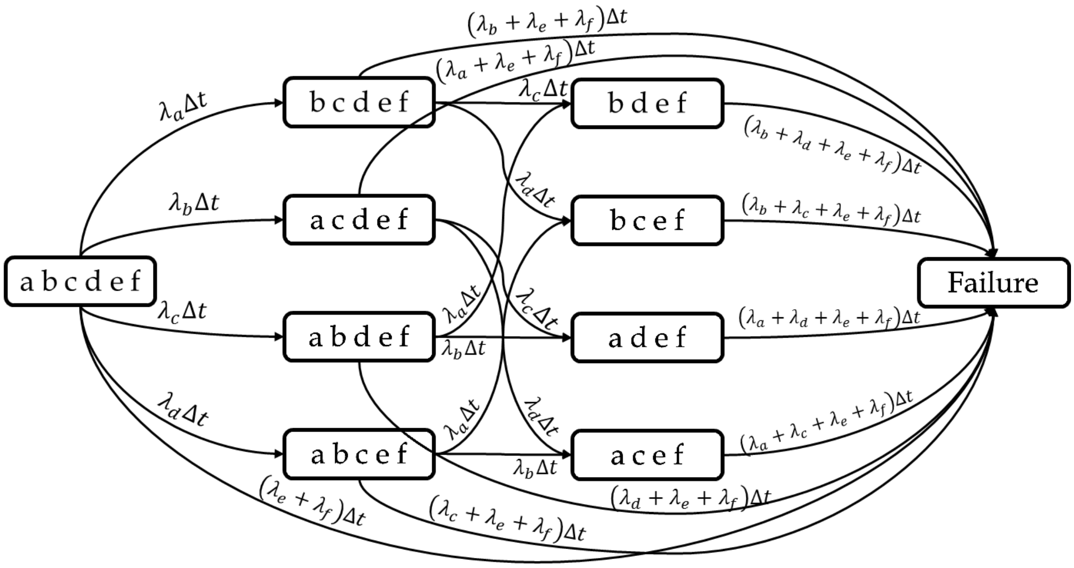

Fig. 6. Reliability Markov model of Hexacopter with PNNPPN configuration.

2.3 Model Simplification

Assuming that “the failure rates of all rotors are the same” the model presented in Fig. 5 can be simplified into the one depicted in Fig. 7 The first state in this model labeled with "6" and "M" means that in this state all six rotors are operational. In case of having only four rotors operational, two states can be considered. The star state demonstrates the system with operational rotors in a quadcopter configuration while the second one covers the other possible configurations. With the assumption of having same failure rates of all rotors, the 19 states Markov model is simplified to an understandable and less complicated 6 states Markov model. Similarly, the Markov model is presented in Fig 6. can be simplified as depicted in Fig 8. In this model, we have only four Markov states (three operational states and one failure state).

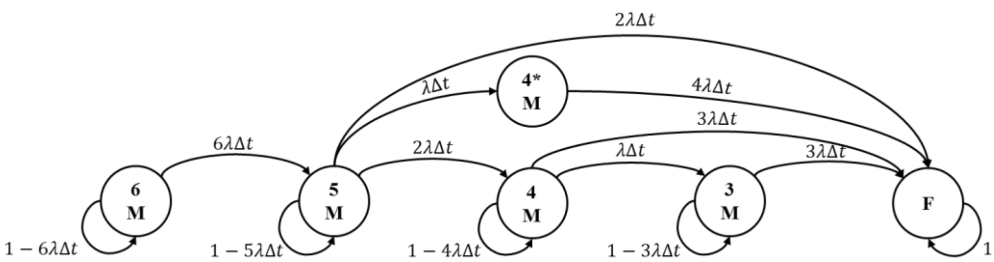

Fig. 7. Simplified reliability Markov model of Hexacopter with PNPNPN configuration.

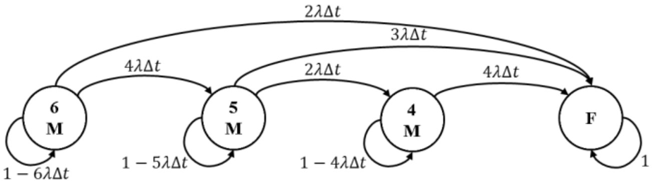

Fig. 8. Simplified reliability Markov model of Hexacopter with PPNNPN configuration.

2.4 Validation through Monte Carlo Simulation

To validate the proposed Markov model, consider the Hexacopter which is illustrated in Fig. 7 with PNPNPN propulsion system's configuration and same probabilistic failure characteristic as (16).

$$F(t) = 1 - e^{-\lambda t} \tag{16}$$

Equation (17) is used to obtain the time to failure from failure distribution function for each rotor.

$$t = -\frac{Log(1 - rand)}{\lambda} \tag{17}$$

Based on defined mission time and obtained failure time from (17), the failed rotors can be determined. Having determined failed rotors, by the use of controllability theory provided in (Shi, Yang, & Quan, 2016), the status of the hexacopter (failed or operational) is obtained. In a Monte Carlo simulation 1e06 independent iterations with the above-mentioned conditions are generated and based on the number of failures, the reliability of the hexacopter can be evaluated as it is shown in Table 1.

Table 1. Comparison between Monte Carlo simulation and Analytical solution results

Reliability (Mission Time)	Monte Carlo (1e06 Iteration)		Markov Solution
	Mean	Variance	
R(5)	0.841404	1.32e-3	0.840721
R(10)	0.578312	1.32e-3	0.577502
R(15)	0.357543	1.32e-3	0.358676
R(20)	0.211128	1.32e-3	0.210288
R(25)	0.117153	1.32e-3	0.119051
R(30)	0.066433	1.32e-3	0.065957
R(35)	0.036925	1.32e-3	0.036063

☐ As can be seen, the results of Monte Carlo simulation consistent with theoretical values.

3. NUMERICAL RESULTS

For understanding the system's behavior in terms of reliability and MTTF, numerical results are provided in this section. Fig. 9 shows the evaluation of the reliability of a quadcopter and hexacopter with two kinds of rotors configuration *versus* time. In this figure, the failure rate of each rotor is assumed as 0.04 failures per hour. The reliability of hexacopter with PNPNPN configuration is higher than the reliability of hexacopter with PPNNPN configuration and both are higher than the reliability of quadcopter. In (Shi, Yang, & Quan, 2016), the reliability of hexacopter with both configurations and without yaw control has been considered with the same expression. However, the result achieved from Markov model shows that they have different values of reliability vs. time in PPNNPN configuration. By investigating the reliability Markov model of hexacopter with PPNNPN and the equation (9) in (Shi, Yang, & Quan, 2016), the reason for difference can be obtained. In the equation (9) of (Shi, Yang, & Quan, 2016), after two motors failure, they consider three operational situations but as can be seen in Fig. 6 there are four operational situations.

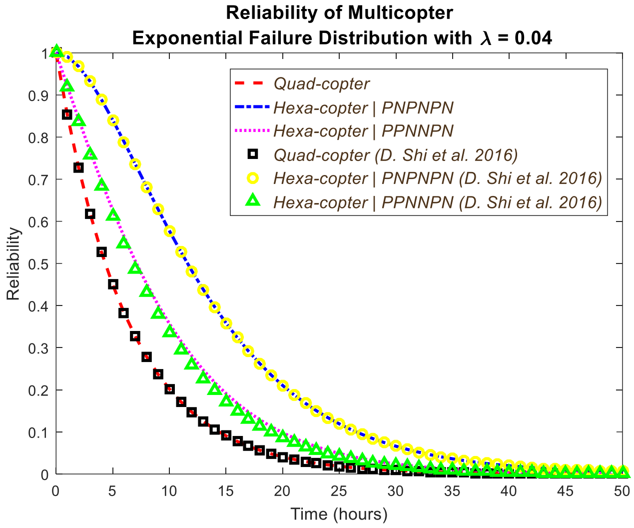

Fig. 9. Reliability evaluation of Multirotors (Quadcopter and Hex-copter) vs. time with failure rate of 0.04 failure/hour

Fig 10 delineates the reliability of quadcopter and two configurations of hexacopter vs failure rate of each rotor at two hours of the mission (t = 2). The lower rate of failure in each rotor means that rotor is more reliable. According to this figure, increasing the number of the rotor with same failure rate will increase the reliability of multirotor and the highest values of reliability can be achieved by hexacopter with PNPNPN configuration.

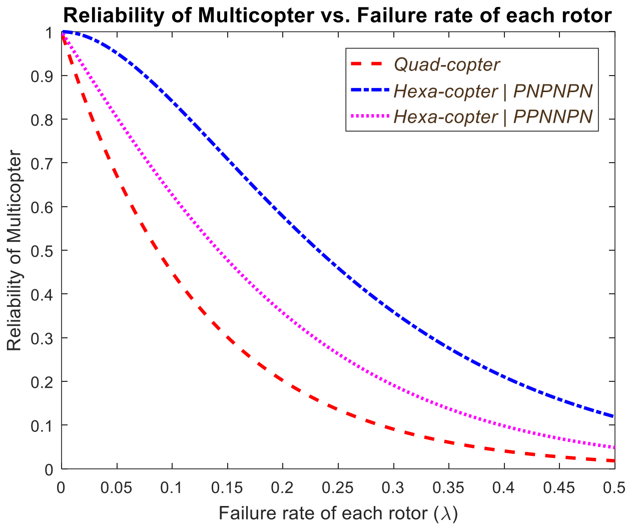

Fig. 10. Reliability evaluation of Multirotors (Quadcopter and Hexacopter) vs. failure rate of each rotor at two hours of mission

The calculated MTTF values from each system's states to failure state are depicted in Table 2. As can be seen, with the condition of two motors failing (4 M* or 4 M), the mean time to failure of "4 M" is more than "4 M*". Also, it seems that in hexacopter with PNPNPN configuration, mean time to failure of state '4 M*' is less than the states '4 M' and '3 M'. For the hexacopter with PPNNPN configuration, in state "6 M" when all six rotors of hexacopter are operational, the mean time to failure will be 9.17 hours that is 4.58 hours less than the equivalent state in PNPNPN configuration.

Table 2. Mean time to failure analysis of quadrotor from each system's states with failure rate of 0.04 failure/hour

	Failure Situation				
	6 M	**5 M**	**4 M***	**4 M**	**3 M**
MTTF	13.75	9.58	6.25	8.33	8.33
	6 M	**5 M**	--	**4 M**	--
MTTF	9.17	7.50	--	6.25	0

Fig. 11 shows the mean time to failure of state "6 M" of hexacopter in both PNPNPN and PPNNPN configurations. The MTTF of the hexacopter will decrease when the failure rate of each rotor increases and the MTTF of the hexacopter with PNPNPN configuration is always higher than the hexacopter with other configuration.

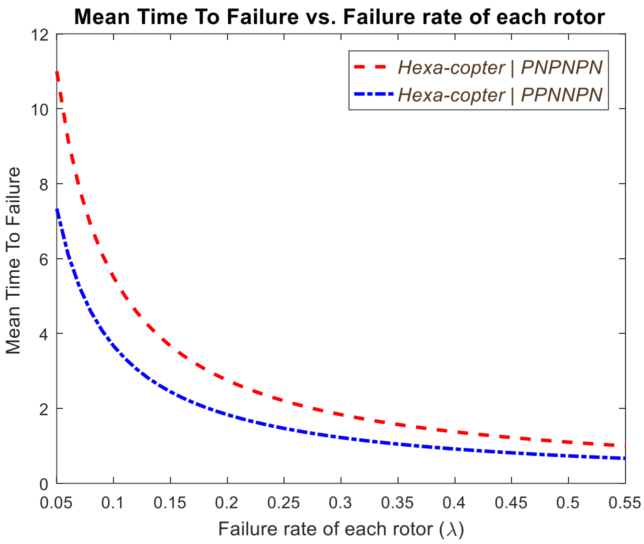

Fig. 11. Mean time to failure of hexacopter vs. failure rate of each rotor

4. DISCUSSION

Markov modeling of multirotors is the first step in designing the fault detection and recovery system. A fault diagnosis module will be needed to specify the current state of the multirotor in its Markov model. Hence, it is assumed that there is a fault detection and diagnosis module that can detect the failure of each rotor. Based on the proposed Markov model and detection of a failed rotor, the mean time to failure and the probability of failure from a related state can be estimated. If both probabilities of failure and MTTF are less than desired value, the robot will continue the mission. However, if the probability of failure or MTTF is more than the desired value the robot will abort the mission or perform a safe landing. Fig. 12 delineates the flowchart of the proposed Markov-based fault detection and recovery system for multirotors.

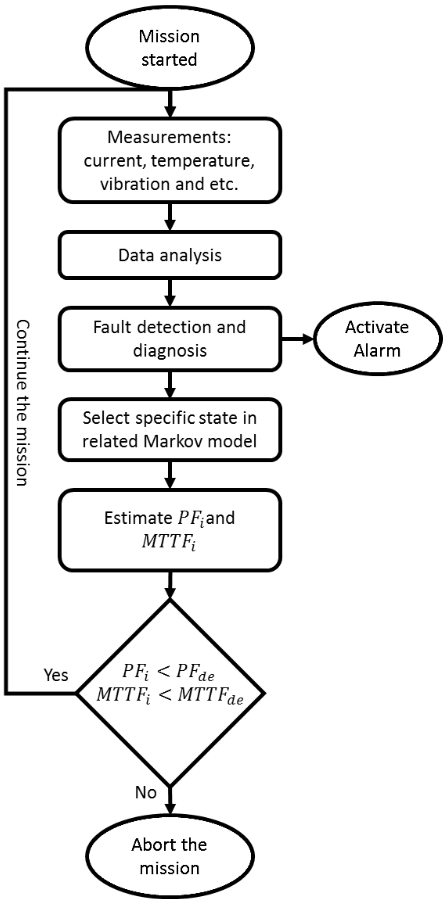

Fig. 12. Flowchart of the proposed Markov-based fault detection and recovery system for multirotors.

5. CONCLUSIONS

The higher manoeuvrability of multi-rotors compared with other types of UAVs makes them very popular. However, safety when operating multi-rotors in populated areas remain one of the most important challenges. In this sense, a Markov model was herein proposed as a fault detection and recovery module for the propulsion system of multi-rotors and was validated through Monte Carlo simulations. Reliability and MTTF of the robot were also evaluated via the developed models. In particular, it was shown that the reliability of a hexacopter configuration is higher than a quadcopter. Moreover, between the two hexacopter configurations studied, the PNPNPN configuration has higher reliability and MTTF when compared to the PPNNPN.

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