CS 170 Homework 1

Due **Monday 9/5/2022, at 10:00 pm (grace period until 11:59pm)**

1   Study Group

List the names and SIDs of the members in your study group. If you have no collaborators, you must explicitly write "none".

2   Course Policies

(a) What dates and times are the exams for CS170 this semester? Are there planned alternate exams?

Note: We will make accommodations for students in faraway timezones.

(b) Homework is due Mondays at 10:00pm, with a late deadline at 11:59pm. At what time do we recommend you have your homework finished?

(c) We provide 2 homework drops for cases of emergency or technical issues that may arise due to homework submission. If you miss the Gradescope late deadline (even by a few minutes) and need to submit the homework, what should you do?

(d) What is the primary source of communication for CS170 to reach students? We will send out all important deadlines through this medium, and you are responsible for checking your emails and reading each announcement fully.

(e) Please read all of the following:

   (i) **Syllabus and Policies:** https://cs170.org/syllabus/
  (ii) **Homework Guidelines:** https://cs170.org/resources/homework-guidelines/
 (iii) **Regrade Etiquette:** https://cs170.org/resources/regrade-etiquette/
 (iv) **Forum Etiquette:** https://cs170.org/resources/forum-etiquette/

Once you have read them, copy and sign the following sentence on your homework submission.

"I have read and understood the course syllabus and policies."

3   Understanding Academic Dishonesty

Before you answer any of the following questions, make sure you have read over the syllabus and course policies (https://cs170.org/syllabus/) carefully. For each statement below, write *OK* if it is allowed by the course policies and *Not OK* otherwise.

(a) You ask a friend who took CS 170 previously for their homework solutions, some of which overlap with this semester's problem sets. You look at their solutions, then later write them down in your own words.

(b) You had 5 midterms on the same day and are behind on your homework. You decide to ask your classmate, who's already done the homework, for help. They tell you how to do the first three problems.

(c) You look up a homework problem online and find the exact solution. You then write it in your words and cite the source.

(d) You were looking up Dijkstra's on the internet, and inadvertently run into a website with a problem very similar to one on your homework. You read it, including the solution, and then you close the website, write up your solution, and cite the website URL in your homework writeup.

4   Recurrence Relations

For each part, find the asymptotic order of growth of T; that is, find a function g such that $T(n) = \Theta(g(n))$. In all subparts, you may ignore any issues arising from whether a number is an integer.

(a) $T(n) = 4T(n/4) + 32n$

(b) $T(n) = 4T(n/3) + n^2$

(c) $T(n) = T(3n/5) + T(4n/5)$ (We have $T(1) = 1$)

5   In Between Functions

Find a function $f(n) \geq 0$ such that:

- For all $c > 0$, $f = \Omega(n^c)$

- For all $\alpha > 1$, $f = \mathcal{O}(\alpha^n)$

Give a proof for why it satisfies both these properties.

6   Sequences

Suppose we have a sequence of integers A_n, where $A_0, \ldots, A_{k-1} < 50$ are given, and each subsequent term in the sequence is given by some integer linear combination of the k previous terms: $A_i = A_{i-1}b_1 + A_{i-2}b_2 + \cdots + A_{i-k}b_k$. You are given as inputs A_0 through A_{k-1} and the coefficients b_1 through b_k.

(a) Devise an algorithm which computes A_n **mod** 50 in $\mathcal{O}(\log n)$ time (**Hint:** use the matrix multiplication technique from class). You should treat k as a constant, and you may assume that all arithmetic operations involving numbers of $\mathcal{O}(\log n)$ bits take constant time. [1]

   Give a 3-part solution as described in the homework guidelines.

(b) Devise an even faster algorithm which doesn't use matrix multiplication at all. Once again, you should still treat k as a constant.

   Hint: Exploit the fact that we only want the answer mod a constant (here 50).

   Give a 3-part solution as described in the homework guidelines.

7   Decimal to Binary

Given the n-digit decimal representation of a number, converting it into binary in the natural way takes $O(n^2)$ steps. Give a divide and conquer algorithm to do the conversion and show that it does not take much more time than Karatsuba's algorithm for integer multiplication.

   Just state the main idea behind your algorithm and its runtime analysis; no proof of correctness is needed as long as your main idea is clear.

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[1] A similar assumption – that arithmetic operations involving numbers of $\mathcal{O}(\log N)$ bits take constant time, where N is the number of bits needed to describe the entire input – is known as the *transdichotomous word RAM model* and it is typically at least implicitly assumed in the study of algorithms. Indeed, in an input of size N it is standard to assume that we can index into the input in constant time (do we not typically assume that indexing into an input array takes constant time?!). Implicitly this is assuming that the register size on our computer is at least $\log N$ bits, which means it is natural to assume that we can do all standard machine operations on $\log N$ bits in constant time.