CS170–Fall 2022 — Homework 10 Solutions

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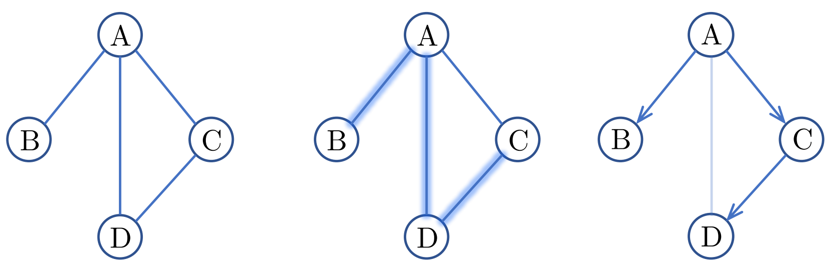
Collaborators: NONE

2 A Reduction Warm-up

The reduction is incorrect. This is because even if the longest path in this DAG has less than $(|V|-1)$ edges, it is still possible to find a Rudrata path in G.

For instance, in Figure 1, graph G has 4 vertices. But the DAG given by a DFS in G only has path with at most 2 edges. So, we cannot get the conclusion that a graph has no Rudrata path from the fact that the longest path in this DAG has less than $(|V|-1)$ edges.



(a) An undirected graph G (b) A Rudrata path in G (c) The DAG given by DFS in G

Figure 1: Graph G has a Rudrata path $B-A-D-C$. However, if we start depth first search at A, the longest path in the DAG we get has only 2 edges.

3 Reduction to 3-Coloring

(a) Let's label the vertices by A∼I, as shown in Figure.

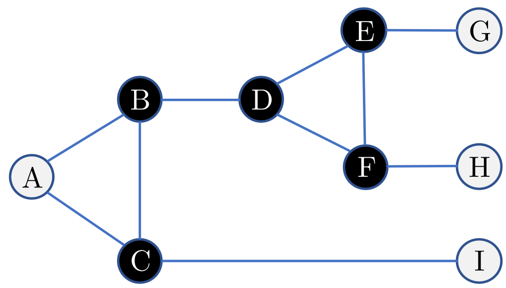


Figure 2: We label the vertices in the "gadget" by A∼I.

Suppose vertex G, H and I are all assigned the color 0. The colors of three vertices of triangle DEF are different. Since E cannot be assigned the color 0, and F cannot be assigned the color 0 either, D must be assigned the color 0. Similarly, A must be assigned the color 0, which contradicts the fact that vertex A is assigned the color 1.

(b) We can add 3 edges: (x_i, Base), ($\overline{x_i}$, Base) and (x_i, $\overline{x_i}$).

(c) Without loss of generality, assume that vertex True, False and Base are assigned the color 1, 0 and 2, respectively. Also, we assume that each clause has 3 literals.

Each clause is represented by a gadget, and each variable is represented by a pair of vertices. We also add three edges for each pair of vertices as (b). True vertex is the common gray vertex on the left of each gadget.

For clause $(x \vee y \vee z)$, we set x as vertex G, y as vertex H and z as vertex I. So, in each clause, G, H and I are assigned either color 0 (the same color as vertex False) or color 1 (the same color as vertex True). By (a), if all these three vertices are all assigned the color 0, then no valid coloring for the black vertices in the gadget exists. By the observation (ii), if one of these three vertex is assigned the color 1, then there is a valid coloring.

As a result, if a valid 3-coloring exists, at least one of the gray vertices on the right of every gadget is assigned the color 1. The corresponding assignment in 3SAT problem is valid. If a 3-SAT formula is satisfied, we can also find a valid coloring that satisfies the 3-coloring problem.