Q1

→ Time Complexity of finding the middle of the linked list size n is $O(n)$. this is because the slower pointer is at the middle node once we pan through the entire linked list of n element. Another way to think of it is the linked list is traversed for the entire linked list and second till the middle of the linked list. Big O of traversing the entire list is $O(n)$ and for the middle is $O(n/2)$ adding $O(n)$ to $O(n/2)$ gives $O(n)$.

Q2

→ 2.1)

The time complexity of the given code is $O(n\log n)$. The outer loop has $\log(n)$ iterations as (i) is doubled in each iteration. In the inner loop, j starts from 1 and goes up to (i) so the number of iterations of the inner loop depends on the value of i in each iteration of the outter loop.

Inner loop iterations is $1+2+4+\ldots+n$ which represents a geometric series and hence can be calculated as n.

2.2) Time complexity of outter loop is $\log_2 n$. Each iteration of i is doublod in this loop. for the inner loop, in each iteration, the value of j is doubled, and when j becomes greater than i, the loop will terminate. This gives time complexity of this code: $O(\log^2 n)$

2.3) The time complexity of outer loop is $O(\log n)$ as the value of i increases with a factor of 3 in each iteration. For the inner loop, time complexity is $O(\log n)$ as the value of j increases with a factor of 2 in each iteration

overall time complexity is $O(\log^2 n)$

2.4) In the outer loop, the time complexity of this loop is $O(n)$

In the inner loop, time complexity is $O(\log n)$.

Total time complexity of the code is $O(n\log n)$

Q3

→ 3.1]

1) $f_A(n) = 4n^3 + 3n^2 + 2n - 5$

$4n^3 \leq 4n^3 + 3n^2 + 2n - 5 \leq 4n^3 + 3n^3 + 2n^3 - 5n^3$

$4n^3 \leq f_A(n) \leq (4+3+2-5)n^3$

$4n^3 \leq f_A(n) \leq 4n^3$

$4g(n) \leq f_A(n)\ 4g(n)$

$c_1 = 4$

$c_2 = 4$

$n_0 = 1$

$= f(n) + \cancel{\theta}\ (n^3)$

$= O(n^3)$

2) $n\log n + 3n - 15$

$f(n) + \theta(n\log n)$

$g(n) = n\log n$

$c_1 gn \leq f(n) \leq c_2\ g(n)$

$c_1\ n\log n \leq n\log n + 3n - 15 \leq c_2\ n\log n$

$n\log n \leq n\log n + 3n - 15 \leq -11\ n\log n$

$c_1 = 1$

$c_2 = -11$

$n_0 = 1$

$f(n) \in \theta(n\log n)$

big $\theta = \theta(n\log n)$

3.2]

1) $n^3\log(3n) + 2n^4 + 3n^2 + 127$

$= O(n^4)$

2) $\sqrt{2n} + 30\log(4n)^2 + 27n - 3$

$= O(\sqrt{n})$

3) $(n+1)! + 2^n$

$= O(n!)$

4) $\sqrt[3]{n^2} + 3n\ \log^2(2n) + 4n^{\frac{6}{7}}$

$= O(n^{6/7})$