



# CS1010

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[https://github.com/DigiPie/cs1010\\_tut\\_c09](https://github.com/DigiPie/cs1010_tut_c09)



# Today's plan

- Unit 23: Binary Search
- Unit 24: Sorting
  - *Problem Set 24*
- Programming Exercise
- Consultation



# UNIT 23

## BINARY SEARCH

Recap.



# The most straightforward approach – Linear Search

```
1 long search(long n, const long list[n], long q) {  
2     for (long i = 0; i < n; i += 1) {  
3         if (list[i] == q) {  
4             return i;  
5         }  
6         return -1;  
7     }
```

- Time complexity:  $O(n)$ 
  - *Cannot be sure  $q$  does not exist till  $n$  elements are checked*

# The most straightforward approach – Linear Search

```
1 long search(long n, const long list[n], long q) {  
2     for (long i = 0; i < n; i += 1) {  
3         if (list[i] == q) {  
4             return i;  
5         }  
6     }  
7     return -1;  
}
```

- But what if the list is already sorted?

# What if the list is already sorted?

- If the list is sorted in increasing order
  - *Pick a random element  $x$  from the list.*
    - Any element to the left of  $x \leq x$
    - Any element to the right of  $x \geq x$ .
- If the list is sorted in decreasing order
  - *Pick a random element  $x$  from the list.*
    - Any element to the left of  $x \geq x$
    - Any element to the right of  $x \leq x$ .

# What if the list is already sorted?

- If the list is sorted in increasing order
  - *Pick a random element  $x$  from the list.*
    - Any element to the left of  $x \leq x$
    - Any element to the right of  $x \geq x$ .
- If looking for  $q$ ,
  - *If  $q == x$ , return position of  $x$*
  - *Else if  $q < x$ , search left-side*
  - *Else (given  $q > x$ ), search right-side*

# Binary Search

```
7  long search(const long list[], long i, long j, long q) {
8      if (i > j) {
9          return -1;
10     }
11     long mid = (i+j)/2;
12     if (list[mid] == q) {
13         return mid;
14     } else if (list[mid] > q) {
15         return search(list, i, mid-1, q);
16     }
17     return search(list, mid+1, j, q);
18 }
```



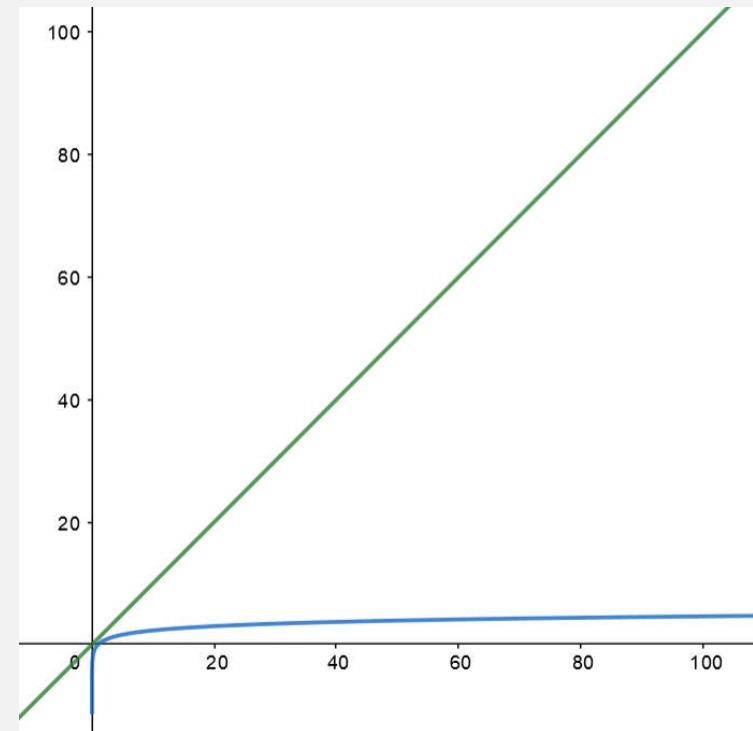
# Binary Search

```
7  long search(const long list[], long i, long j, long q) {
8      if (i > j) {
9          return -1;
10     }
11     long mid = (i+j)/2;
12     if (list[mid] == q) {
13         return mid;
14     } else if (list[mid] > q) {
15         return search(list, i, mid-1, q);
16     }
17     return search(list, mid+1, j, q);
18 }
```

- Time complexity:  $O(\log_2 n)$

# Linear Search vs Binary Search

- **Binary Search** is much more efficient than **Binary Search**.
  - *But only works if list is sorted.*





# UNIT 24

## SORTING

Recap. PS 24.1. PS 24.2. PS 24.3.



# Visualisation of sorting



- <https://visualgo.net/en/sorting>



# UNIT 24

## SORTING

Recap. [PS 24.1](#). PS 24.2. PS 24.3.



## Problem Set 24.1

```
1 void bubble_pass(long last, long a[])
2 {
3     for (long i = 0; i < last; i += 1) {
4         if (a[i] > a[i+1]) {
5             swap(a, i, i+1);
6         }
7     }
8 }
9
10 void bubble_sort(long n, long a[n]) {
11     for (long last = n - 1; last > 0; last -= 1) {
12         bubble_pass(last, a);
13     }
14 }
```

- In this implementation, we always make  $n-1$  passes. But it is possible to terminate early when a pass through the array does not lead to any swapping. Modify the code above to achieve this optimization.

## Problem Set 24.1

```
bool bubble_pass(long last, long a[])
{
    bool swapped = false;
    for (long i = 0; i < last; i += 1) {
        if (a[i] > a[i+1]) {
            swap(a, i, i+1);
            swapped = true;
        }
    }
    return swapped;
}

void bubble_sort(long n, long a[n]) {
    bool swapped = true;
    for (long last = len - 1; last > 0 && swapped; last -= 1) {
        swapped = bubble_pass(last, a);
    }
}
```

In this implementation, we always make  $n-1$  passes. But it is possible to terminate early when a pass through the array does not lead to any swapping. Modify the code above to achieve this optimization.



# UNIT 24

## SORTING

Recap. PS 24.1. [PS 24.2.](#) PS 24.3.





## Problem Set 24.2

a) Suppose the input list to insertion sort is already sorted. What is the running time of insertion sort?

```
1 void insert(long a[], long curr)
2 {
3     long i = curr - 1;
4     long temp = a[curr];
5     while (temp < a[i] && i >= 0) {
6         a[i+1] = a[i];
7         i -= 1;
8     }
9     a[i+1] = temp;
10 }
11
12 void insertion_sort(long n, long a[n]) {
13     for (long curr = 1; curr < n; curr += 1) {
14         insert(a, curr);
15     }
16 }
```

## Problem Set 24.2

a) Suppose the input list to insertion sort is already sorted. What is the running time of insertion sort?

- $O(n)$

```
1 void insert(long a[], long curr)
2 {
3     long i = curr - 1;
4     long temp = a[curr];
5     while (temp < a[i] && i >= 0) {
6         a[i+1] = a[i];
7         i -= 1;
8     }
9     a[i+1] = temp;
10 }
11
12 void insertion_sort(long n, long a[n]) {
13     for (long curr = 1; curr < n; curr += 1) {
14         insert(a, curr);
15     }
16 }
```

## Problem Set 24.2

b) Suppose the input list to insertion sort is inversely sorted. What is the running time of insertion sort?

```
1 void insert(long a[], long curr)
2 {
3     long i = curr - 1;
4     long temp = a[curr];
5     while (temp < a[i] && i >= 0) {
6         a[i+1] = a[i];
7         i -= 1;
8     }
9     a[i+1] = temp;
10 }
11
12 void insertion_sort(long n, long a[n]) {
13     for (long curr = 1; curr < n; curr += 1) {
14         insert(a, curr);
15     }
16 }
```

## Problem Set 24.2

b) Suppose the input list to insertion sort is inversely sorted. What is the running time of insertion sort?

- $O(n^2)$

```
1 void insert(long a[], long curr)
2 {
3     long i = curr - 1;
4     long temp = a[curr];
5     while (temp < a[i] && i >= 0) {
6         a[i+1] = a[i];
7         i -= 1;
8     }
9     a[i+1] = temp;
10 }
11
12 void insertion_sort(long n, long a[n]) {
13     for (long curr = 1; curr < n; curr += 1) {
14         insert(a, curr);
15     }
16 }
```

## Problem Set 24.2

### Insertion Sort

- Best time complexity:
  - $O(n)$
- Worst time complexity:
  - $O(n^2)$

```
1 void insert(long a[], long curr)
2 {
3     long i = curr - 1;
4     long temp = a[curr];
5     while (temp < a[i] && i >= 0) {
6         a[i+1] = a[i];
7         i -= 1;
8     }
9     a[i+1] = temp;
10 }
11
12 void insertion_sort(long n, long a[n]) {
13     for (long curr = 1; curr < n; curr += 1) {
14         insert(a, curr);
15     }
16 }
```



# UNIT 24

## SORTING

Recap. PS 24.1. PS 24.2. [PS 24.3.](#)



## Problem Set 24.3

- What is the loop invariant for the loop in the function insert?

```
1 void insert(long a[], long curr)
2 {
3     long i = curr - 1;
4     long temp = a[curr];
5     while (temp < a[i] && i >= 0) {
6         a[i+1] = a[i];
7         i -= 1;
8     }
9     a[i+1] = temp;
10 }
11
12 void insertion_sort(long n, long a[n]) {
13     for (long curr = 1; curr < n; curr += 1) {
14         insert(a, curr);
15     }
16 }
```

## Problem Set 24.3

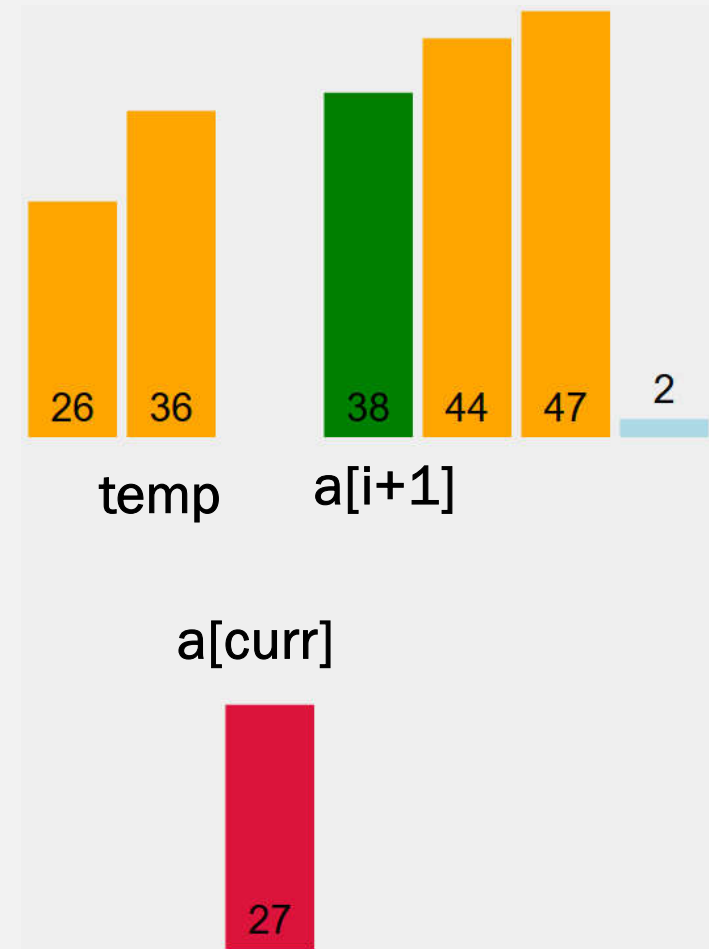
- What is the loop invariant for the loop in the function insert?
- $\text{temp} \leq a[i+1]..a[\text{curr}]$ .

```
1 void insert(long a[], long curr)
2 {
3     long i = curr - 1;
4     long temp = a[curr];
5     while (temp < a[i] && i >= 0) {
6         a[i+1] = a[i];
7         i -= 1;
8     }
9     a[i+1] = temp;
10 }
11
12 void insertion_sort(long n, long a[n]) {
13     for (long curr = 1; curr < n; curr += 1) {
14         insert(a, curr);
15     }
16 }
```



## Problem Set 24.3

- What is the loop invariant for the loop in the function insert?
- $\text{temp} \leq a[i+1]..a[\text{curr}]$ .





# IN-CLASS EXERCISES

1. Implement Binary Search with Loop
2. Modify binary search so that it returns a position  $k$  such that  $a[k] \leq q \leq a[k+1]$ 
  - $-1$  if  $q < a[0]$
  - $n-1$  if  $q > a[n-1]$ .
    - Basically, this is the position that we should insert  $k$  in to keep the array sorted.
3. Combine insertion sort with binary search

```
repeat
  take the first element X from the unsorted pile
  use binary search to find the correct position to insert X
  insert X into the right place
until the unsorted pile is empty
```



# THE END

[https://github.com/DigiPie/cs1010\\_tut\\_c09](https://github.com/DigiPie/cs1010_tut_c09)