CS1010

Evan Tay | *evantay@comp.nus.edu.sg*

https://github.com/DigiPie/cs1010_tut_c09

Today's plan

- **Unit 25:** Tower of Hanoi
- **Unit 26:** Permutations
- **Practical Exam 1**
  - ***Q2:*** *Newton*
  - ***Q5:*** *Square*

TOWER OF HANOI

Unit 25

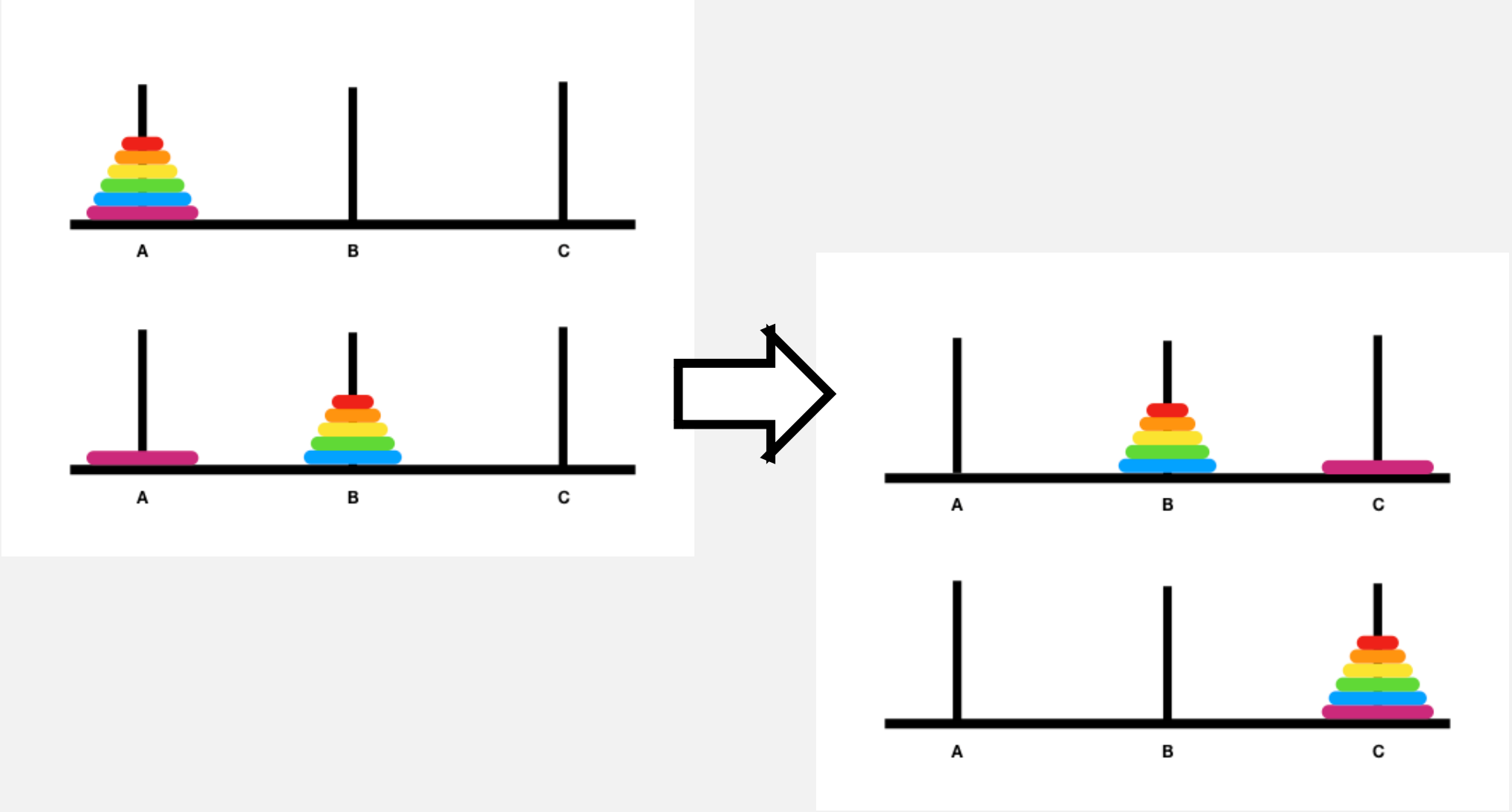
Tower of Hanoi

There are *3* pegs and *n* disks of various size that we can slide into any of the pegs. There are a few rules that we have to obey:

- we can only move one disk at a time;
- we can only move the topmost disk from one peg and place the disk on another peg;
- no disk can be placed on top of a smaller disk.

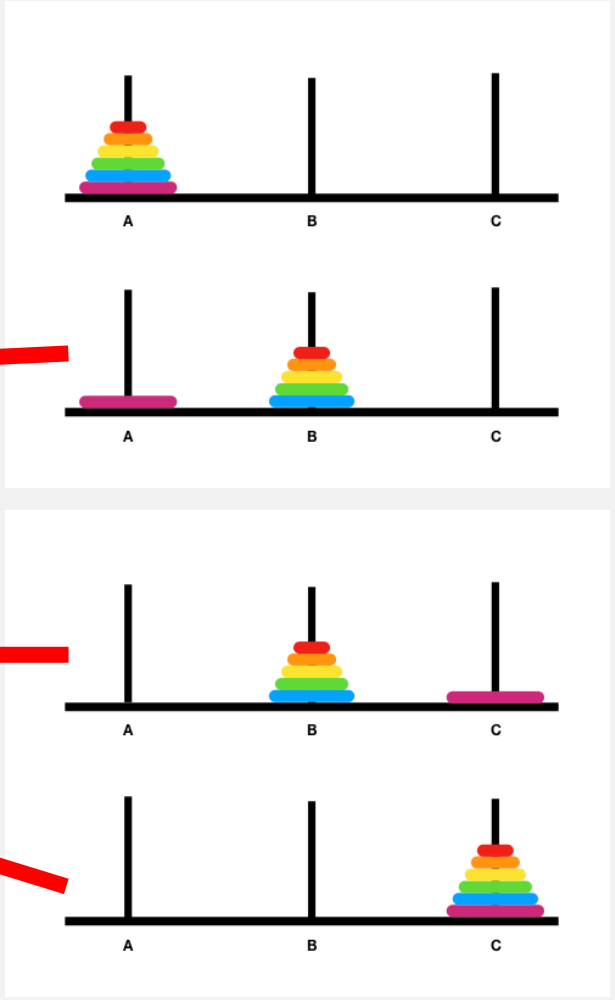
evantay@comp.nus.edu.sg

Tower of Hanoi



Tower of Hanoi

```
void solve(long k, long source, long dest, long placeholder) {
  if (k == 1) {
    print(k, source, dest);
  }
  else {
    solve(k - 1, source, placeholder, dest);
    print(k, source, dest);
    solve(k - 1, placeholder, dest, source);
  }
}
```



Tower of Hanoi

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    solve(k - 1, placeholder, dest, source);
  }
}
```

$$T(k) = 2^{k-1} + 2^{k-2} + \ldots + 4 + 2 + 1 = 2^k - 1$$

Problem Set 25.1

- Suppose that we add a new restriction to the Tower of Hanoi puzzle. Let's say that the disks are on Peg A (or Peg 1) to begin with, and we want to move the disk to Peg C (or Peg 3). We are only allowed to move a disk either to Peg B from another peg or from Peg B to another peg. In other words, we **cannot move the disks between Peg A and Peg C directly.**

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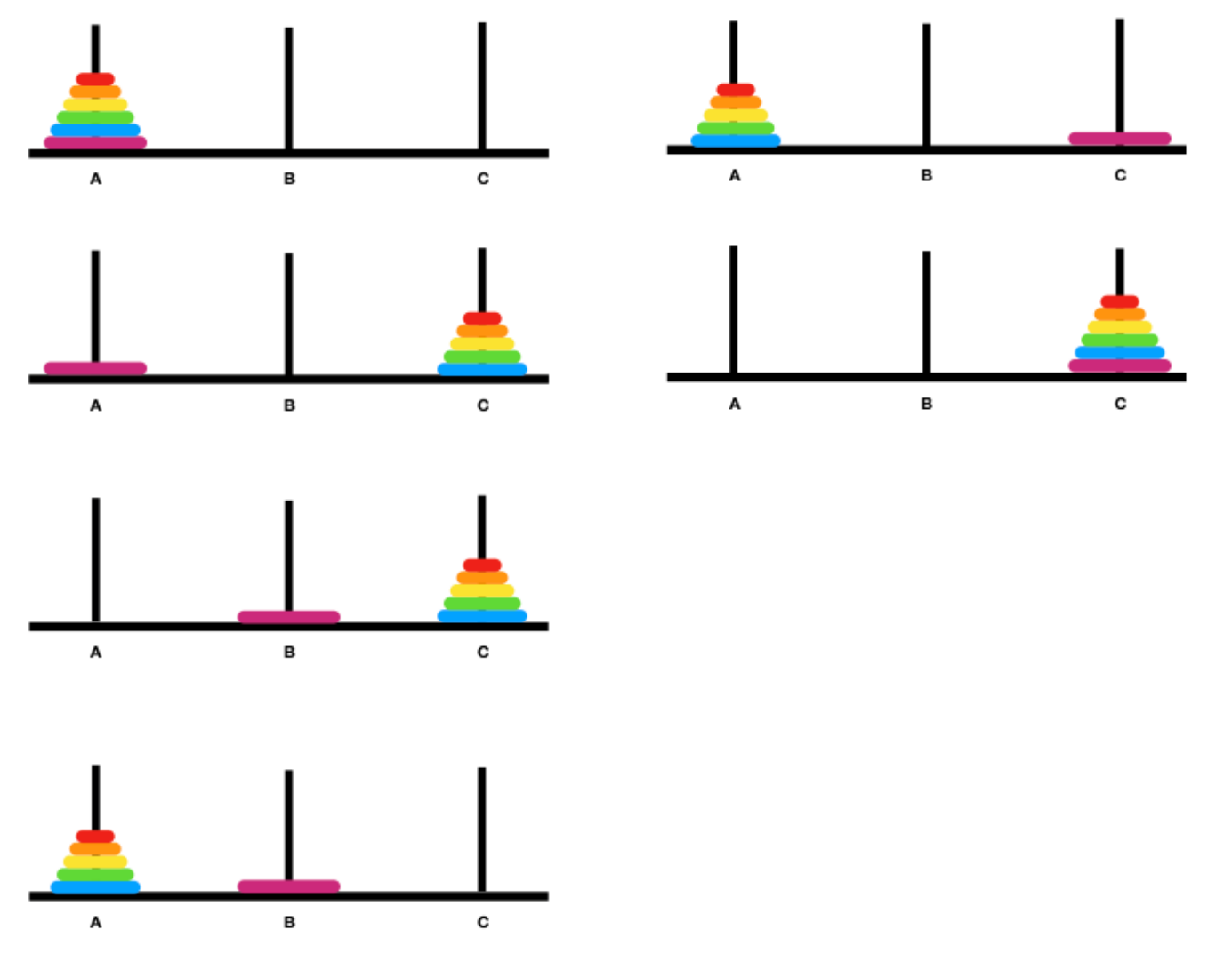
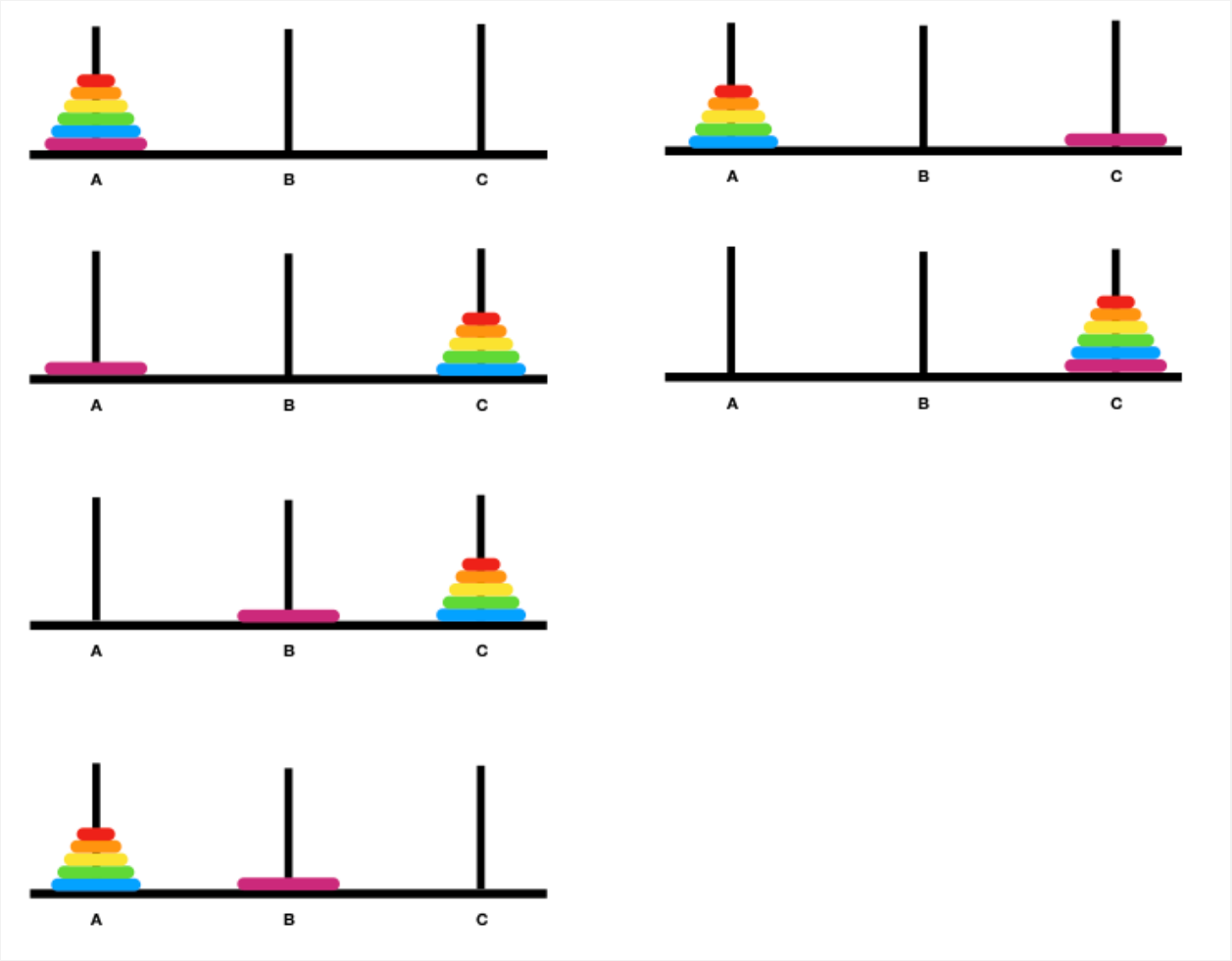
  - *Can only move pegs between Peg A and B, and Peg B and C*

Problem Set 25.1



Problem Set 25.1

```
void solve(long k, long source, long dest, long placeholder) {
  if (k == 1) {
    move(k, source, placeholder);
    move(k, placeholder, dest);
    return;
  }
  solve(k - 1, source, dest, placeholder);
  move(k, source, placeholder);
  solve(k - 1, dest, source, placeholder);
  move(k, placeholder, dest);
  solve(k - 1, source, dest, placeholder);
}
```



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  solve(k - 1, dest, source, placeholder);
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  solve(k - 1, source, dest, placeholder);
}
```

How many steps (use big O notation) are needed now?

From O(2^k) to O(3^k)

T(k) = 3T(k-1) + 2 = 9T(k-2) + 6 + 2 + ... = O(3^k)

Problem Set 25.1

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  move(k, placeholder, dest);
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}
```

How many steps (use big O notation) are needed now?

From O(2^k) to O(3^k)

T(k) = 3T(k-1) + 2 = 9T(k-2) + 6 + 2 + ... = O(3^k)

Tower of Hanoi

- Original Tower of Hanoi code: https://github.com/DigiPie/cs1010_tut_c09/blob/master/Tutorial_10/tower_of_hanoi.c
- Problem Set 25.1 solution: https://github.com/DigiPie/cs1010_tut_c09/blob/master/Tutorial_10/problem25_1.c

PERMUTATIONS

Unit 26

Permutations

- Consider a string length 3, abc. We start with a as the first character, and generate all the permutations of the string bc. We get two permutations abc and acb. The next character is b. We generate all permutations of the string ac. We get bac and bca. Similarly, we get the permutations cab and cba by considering c as the first character and permutating ba.

Permutations

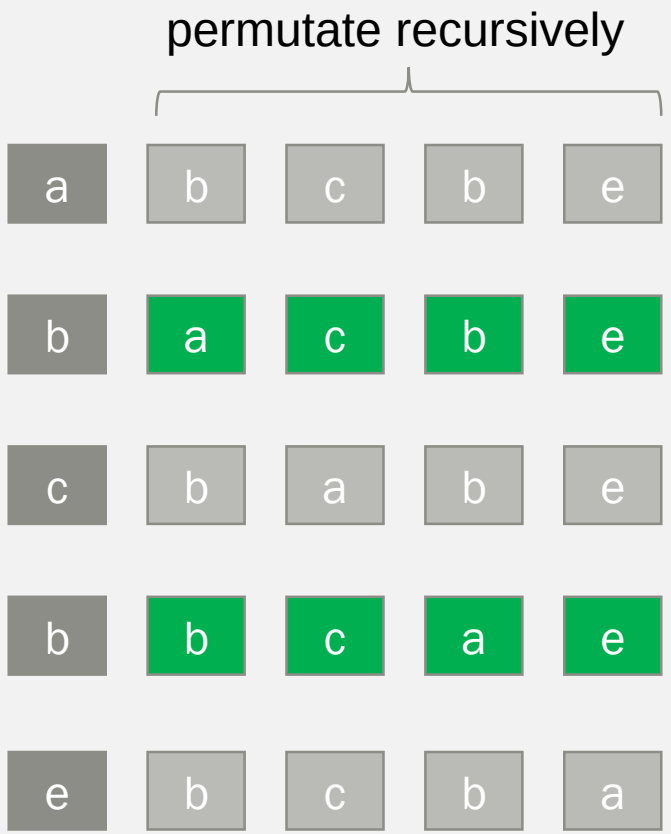
```
void permute(char a[], long len, long curr) {
 // permute characters a[curr]..a[len-1] and print out a for each permutation.
 if (curr == len-1) {
   cs1010_println_string(a);
   return;
 }

 permute(a, len, curr + 1);
 for (long i = curr + 1; i < len; i += 1) {
    swap(a, curr, i);
    permute(a, len, curr + 1);
    swap(a, i, curr);
 }
}
```

Problem Set 26.1

- Write a boolean function that we can call in Line A to check if we should continue to permute the rest of the string, and therefore avoid generating duplicate permutations when the input string contains duplicate characters.
  - *For instance, if the input is **aaa**, the existing code would print **aaa** six times. Print one time only instead.*

Problem Set 26.1



Duplicate work!

Problem Set 26.1

```
void permute(char a[], long len, long curr) {
 if (curr == len-1) {
  cs1010_println_string(a);
  return;
 }

 permute(a, len, curr + 1);
 for (long i = curr + 1; i < len; i += 1) {
    if (...) { // Line A
       swap(a, curr, i);
       permute(a, len, curr + 1);
       swap(a, i, curr);
    }
 }
}
```

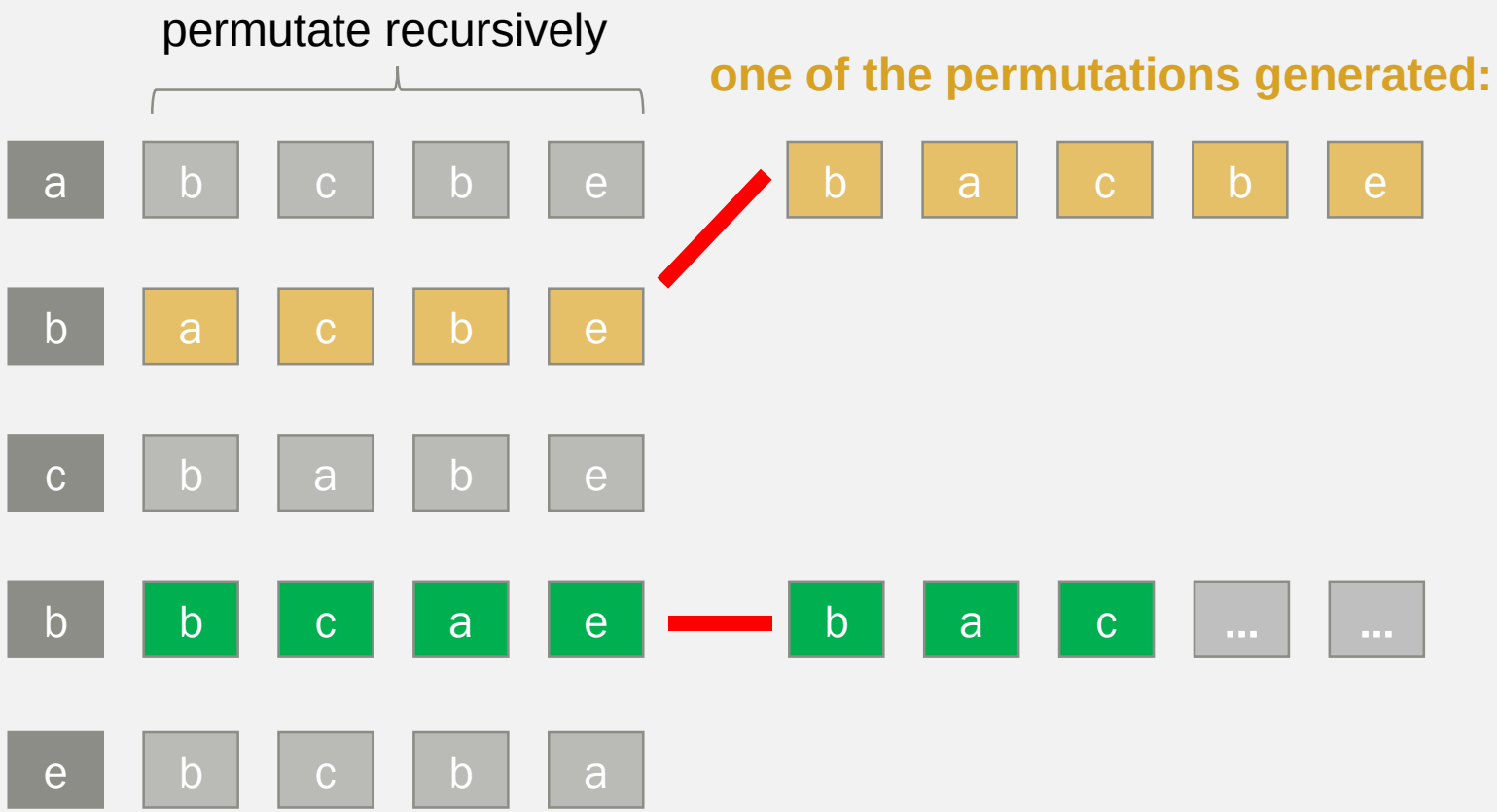
Problem Set 26.1

```
void permute(char a[], long len, long curr) {
 if (curr == len-1) {
  cs1010_println_string(a);
  return;
 }

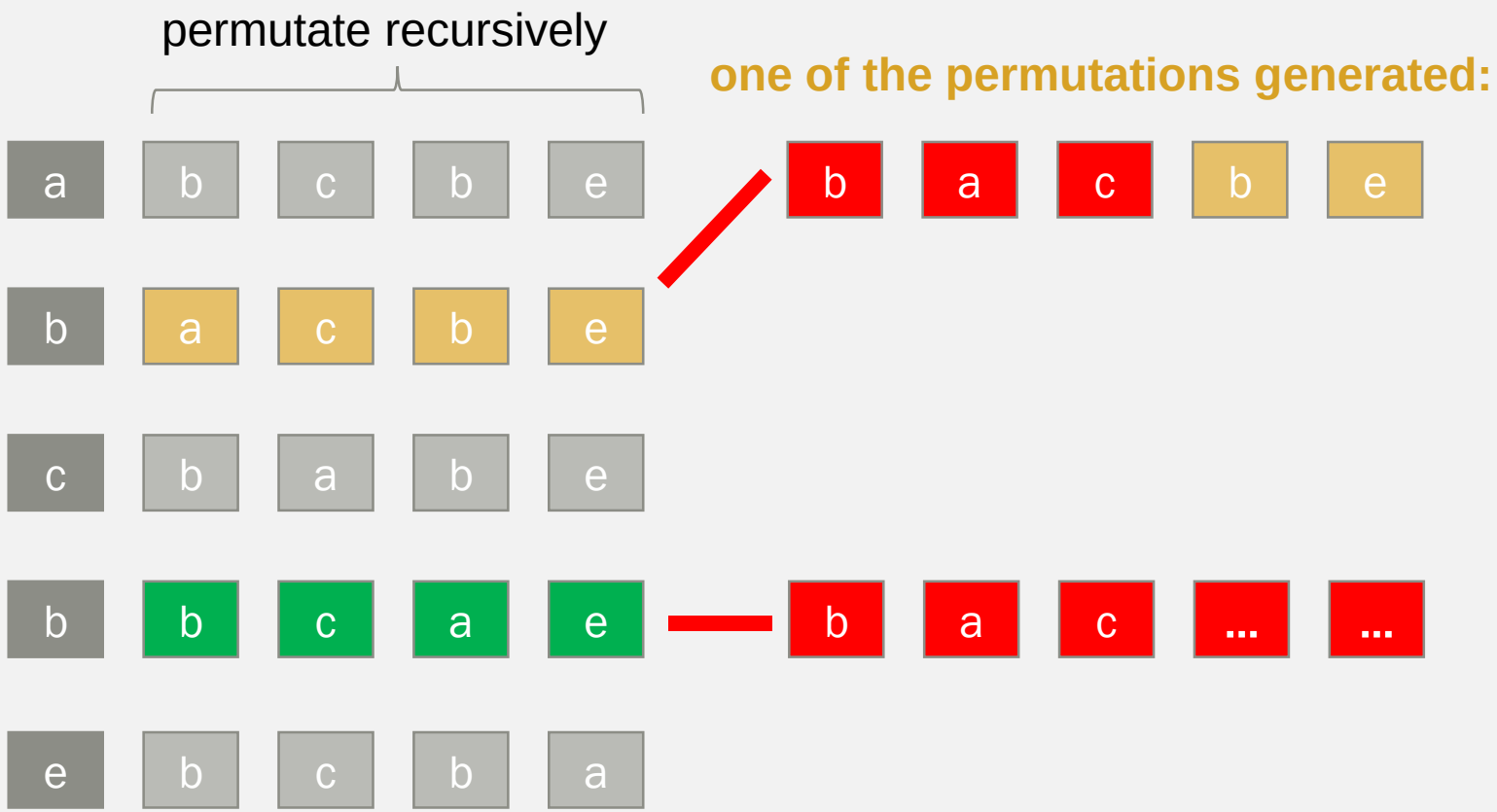
 permute(a, len, curr + 1);
 for (long i = curr + 1; i < len; i += 1) {
    if (!appear_before(a, curr, i)) { // If appeared before, skip further permutation
       swap(a, curr, i);
       permute(a, len, curr + 1);
       swap(a, i, curr);
    }
 }
}
```

evantay@comp.nus.edu.sg

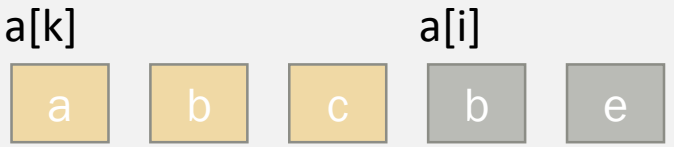
Problem Set 26.1



Problem Set 26.1



Problem Set 26.1




```
bool appear_before(char a[], long k, long i) {
  for (int j = k; j < i; j += 1) {
    if (a[j] == a[i]) {
      return true;
    }
  }
  return false;
}
```

Permutations

- Original Permutations code: https://github.com/DigiPie/cs1010_tut_c09/blob/master/Tutorial_10/permutations.c
- Problem Set 26.1 solution: https://github.com/DigiPie/cs1010_tut_c09/blob/master/Tutorial_10/problem26_1.c

PRACTICAL EXAM 1

Q2 Newton. Q5 Square

THE END

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