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BM 2101 - ANALYSIS OF PHYSIOLOGICAL SYSTEMS

Branched Cylinders: Dendritic Tree Approximations

This is submitted as a partial fulfillment for the module
BM 2101 - Analysis of Physiological Systems

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1 Question 1

First differentiate the given expressions by X.

$$\frac{dV_1}{dX} = -A_1 e^{-X} + B_1 e^X$$

Then substitute the given them to the nodal and boundary conditions. Finally the expressions can be obtained.

2 Question 2

The given linear transformation is as follows.

$$Ax = b$$

By applying the definition of matrix multiplication.

$$\sum_{k=1}^6 A_{ik} x_{k1} = b_{i1}$$

Here $i = 1, 2, 3, \dots, 6$.

For instance considering $k = 1$;

$$A_1 - B_1 = (r_i \lambda_i) I_{app}$$

In the same manner, the remaining rows can be proved.

3 Question 3

According to the code in Listing[1] in Appendix[A], the coefficients are given from the variable x in line 46. The values are as follows.

$$A_1 = 7.3698 \times 10^{-4}$$

$$B_1 = 1.6723 \times 10^{-5}$$

$$A_{21} A_{22} = 0.0011$$

$$B_{21} B_{22} = -2.7987 \times 10^{-6}$$

4 Question 4

The voltage profiles are the same in both the daughter branches. Therefore the lines have overlapped.

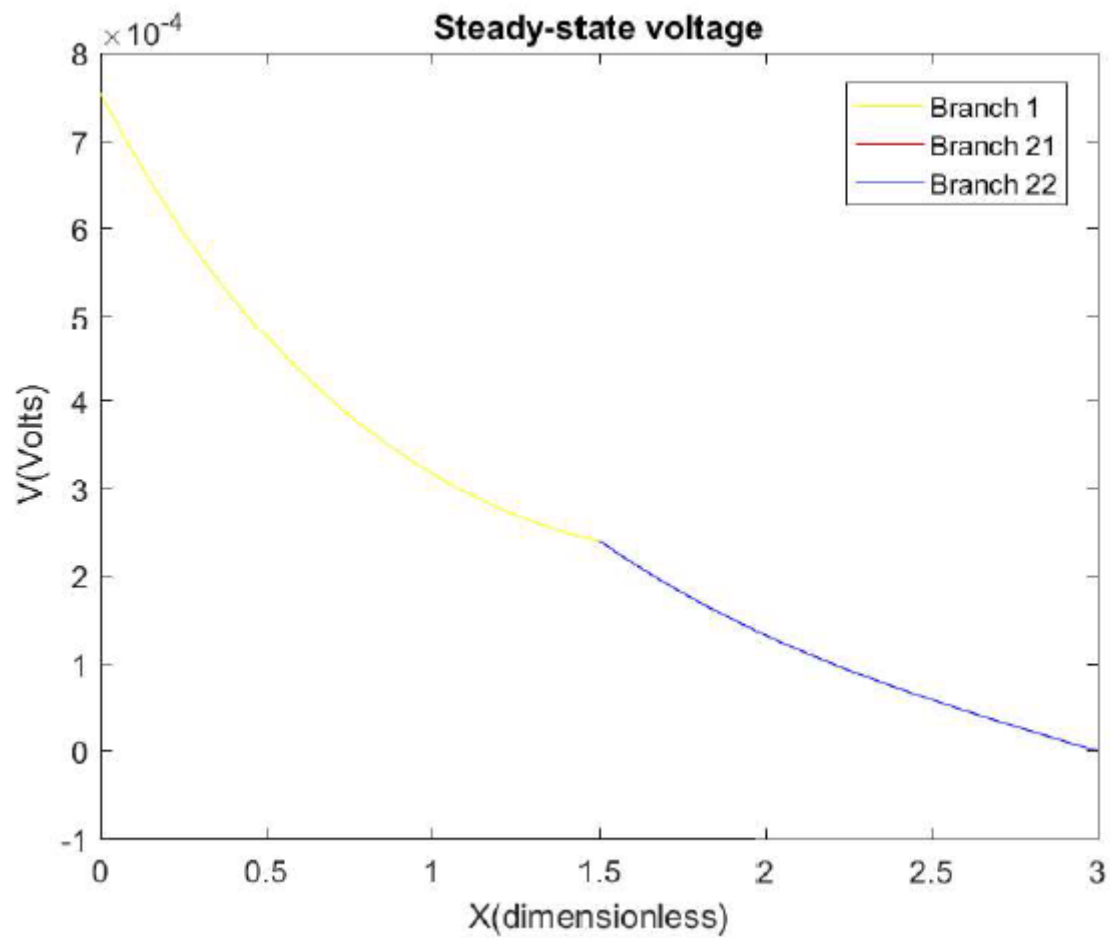


Figure 1: The voltage profile of the whole system

5 Question 5

The $\frac{dV_{21}}{dX}$ and $\frac{dV_{22}}{dX}$ shows the change of voltage along with distance.

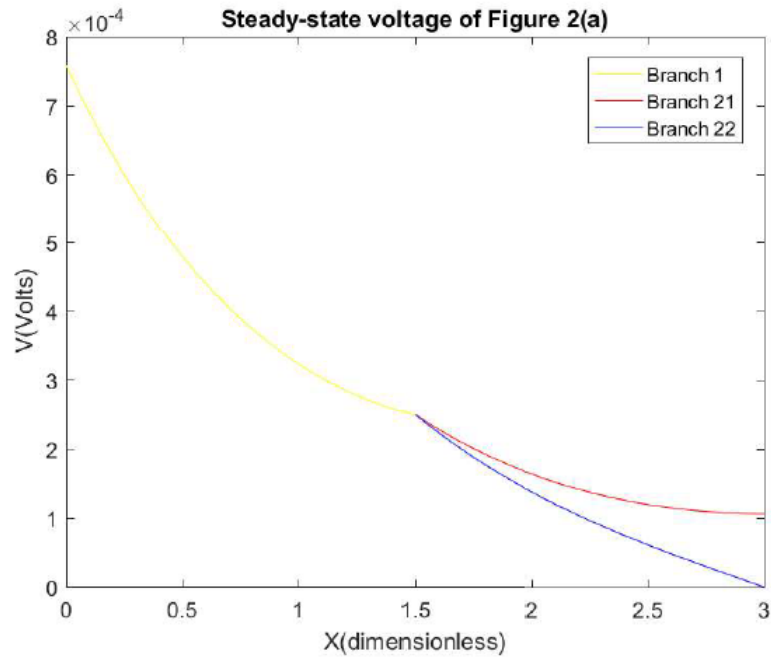


Figure 2: The voltage profile of the system 2(a)

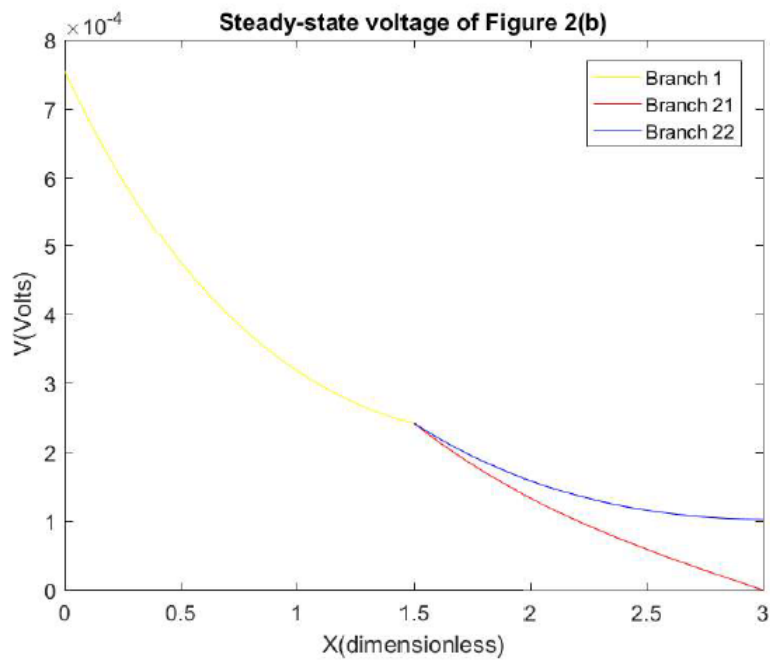


Figure 3: The voltage profile of the system 2(b)

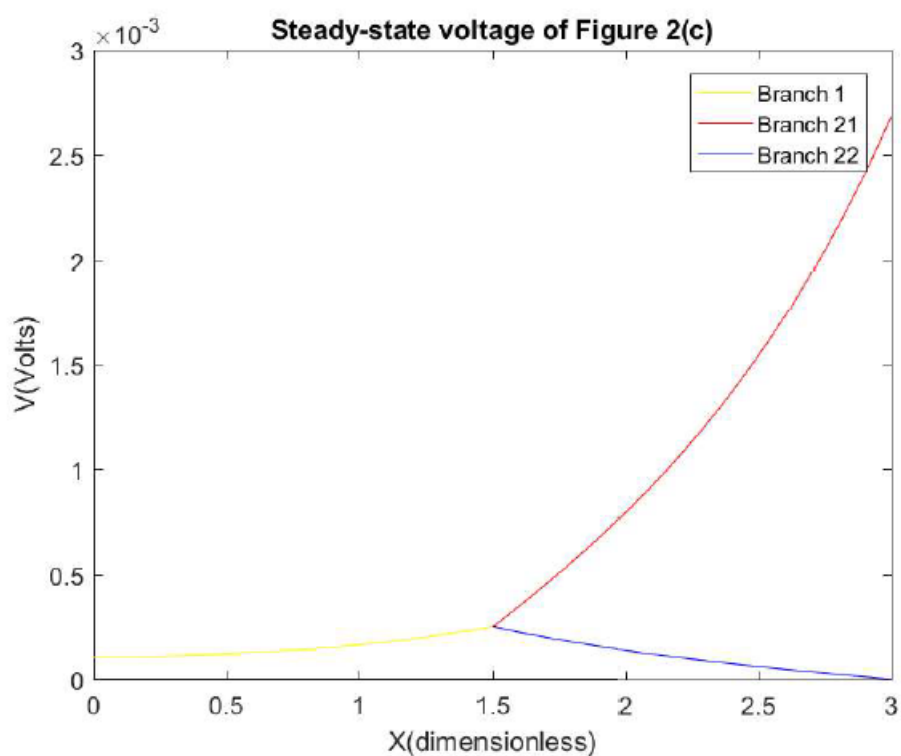


Figure 4: The voltage profile of the system 2(c)

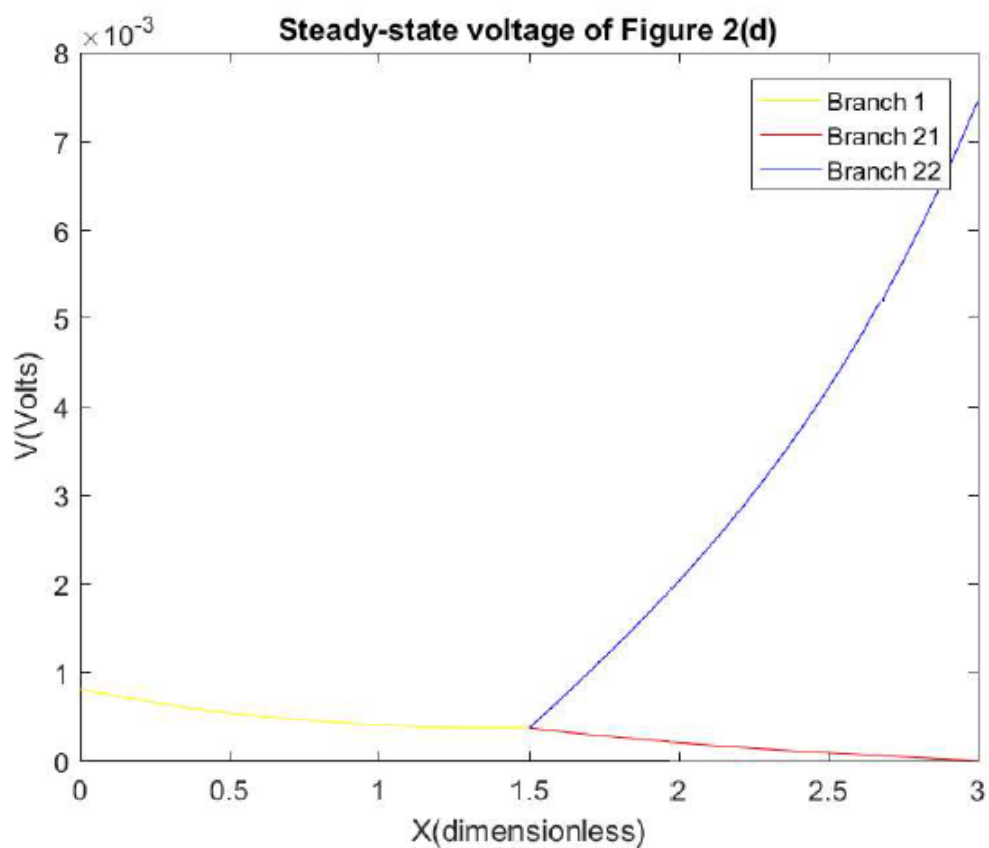


Figure 5: The voltage profile of the system 2(d)

6 Question 6

When we compare the Figure[3] and the Figure[6], the decrease of voltage per unit length is lesser in the daughter branches when $d_{21} = d_{22}$.

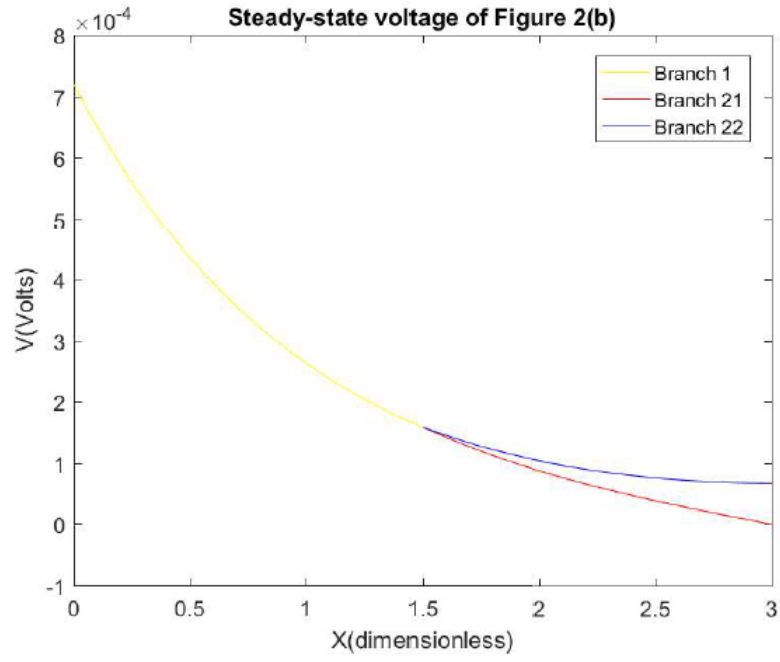


Figure 6: The voltage profile of the system 2(b)

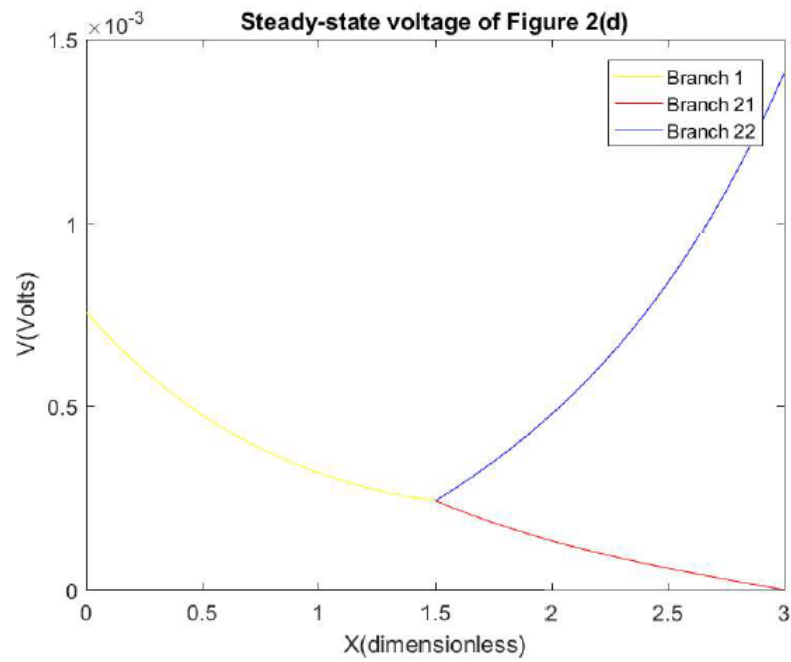


Figure 7: The voltage profile of the system 2(d)

When we compare the Figure[5] and the Figure[7], the decrease of voltage per unit length in the daughter branch

22 is lesser when $d_{21} = d_{22}$.

A MATLAB code

Listing 1: The Matlab code

```

1 % electrical constants and derived quantities for typical
2 % mammalian dendrite
3
4 % Dimensions of compartments
5
6 d1 = 75e-4;      % cm
7 d21 = 30e-4;     % cm
8 d22 = 15e-4;     % cm
9
10 % Commented till Question 6
11 % d21 = 47.2470e-4; % E9 cm
12 % d22 = d21;       % E9 cm
13 %%%%%%%%%
14 l1 = 1.5;        % dimensionless
15 l21 = 3.0;       % dimensionless
16 l22 = 3.0;       % dimensionless
17
18 % Electrical properties of compartments
19
20 Rm = 6e3;        % Ohms cm^2
21 Rc = 90;         % Ohms cm
22 Rs = 1e6;        % Ohms
23
24 c1 = 2*(Rc*Rm)^(1/2)/pi;
25
26 rl1 = c1*d1^(-3/2); % Ohms
27 rl21 = c1*d21^(-3/2); % Ohms
28 rl22 = c1*d22^(-3/2); % Ohms
29
30
31 % Applied current
32
33 iapp = 1e-9;     % Amps
34
35 % Coefficient matrices
36
37 A = [1 -1 0 0 0 0;
38      0 0 exp(-l21) exp(l21) 0 0;
39      0 0 0 0 exp(-l22) exp(l22);
40      exp(-l1) exp(l1) -exp(-l1) -exp(l1) 0 0;
41      0 0 exp(-l1) exp(l1) -exp(-l1) -exp(l1);
42      -exp(-l1) exp(l1) rl1*exp(-l1)/rl21 -rl1*exp(l1)/rl21 rl1*exp(-l1)/rl22
43      -rl1*exp(-l1)/rl22];
44
45 b = [rl1*iapp; 0 ;0; 0 ;0; 0];
46 x = A\b;%Calculating the value of x
47 %Plotting the graph

```

```

48 y1 = linspace(0,l1,20);
49 y21 = linspace(l1,l21,20);
50 y22 = linspace(l1,l22,20);
51 v1 = x(1)*exp(-y1)+x(2)*exp(y1);
52 v21 = x(3)*exp(-y21)+x(4)*exp(y21);
53 v22 = x(5)*exp(-y22)+x(6)*exp(y22);
54 figure;
55 plot(y1,v1,'y-');
56 hold on;
57 plot(y21,v21,'r-');
58 hold on;
59 plot(y22,v22,'b-');
60 hold off;
61 xlabel('X(dimensionless)');
62 ylabel('V(Volts)');
63 title('Steady-state voltage');
64 legend('Branch 1','Branch 21','Branch 22');
65 %Changing A according to Figure 2a
66 A1 = A;
67 A1(2,:) = [0 0 -exp(-l21) exp(l21) 0 0];
68 x1 = A1\b;
69 v21 = x1(1)*exp(-y1)+x1(2)*exp(y1);
70 v221 = x1(3)*exp(-y21)+x1(4)*exp(y21);
71 v222 = x1(5)*exp(-y22)+x1(6)*exp(y22);
72 figure;
73 plot(y1,v21,'y-');
74 hold on;
75 plot(y21,v221,'r-');
76 hold on;
77 plot(y22,v222,'b-');
78 hold off;
79 xlabel('X(dimensionless)');
80 ylabel('V(Volts)');
81 title('Steady-state voltage of Figure 2(a)');
82 legend('Branch 1','Branch 21','Branch 22');
83 %Changing A according to Figure 2b
84 A2 = A;
85 A2(3,:) = [0 0 0 0 -exp(-l22) exp(l22)];
86 x2 = A2\b;
87 v31 = x2(1)*exp(-y1)+x2(2)*exp(y1);
88 v321 = x2(3)*exp(-y21)+x2(4)*exp(y21);
89 v322 = x2(5)*exp(-y22)+x2(6)*exp(y22);
90 figure;
91 plot(y1,v31,'y-');
92 hold on;
93 plot(y21,v321,'r-');
94 hold on;
95 plot(y22,v322,'b-');
96 hold off;
97 xlabel('X(dimensionless)');
98 ylabel('V(Volts)');

```

```

99 title('Steady-state voltage of Figure 2(b)');
100 legend('Branch 1','Branch 21','Branch 22');
101 %Changing b according to Figure 2c
102 b1 = b;
103 b1(1) = 0;
104 b1(2) = rl21*iapp;
105 x3 = A1\b1;
106 v41 = x3(1)*exp(-y1)+x3(2)*exp(y1);
107 v421 = x3(3)*exp(-y21)+x3(4)*exp(y21);
108 v422 = x3(5)*exp(-y22)+x3(6)*exp(y22);
109 figure;
110 plot(y1,v41,'y-');
111 hold on;
112 plot(y21,v421,'r-');
113 hold on;
114 plot(y22,v422,'b-');
115 hold off;
116 xlabel('X(dimensionless)');
117 ylabel('V(Volts)');
118 title('Steady-state voltage of Figure 2(c)');
119 legend('Branch 1','Branch 21','Branch 22');
120 %Changing b according to Figure 2d
121 b2 = b;
122 b2(3) = rl22*iapp;
123 x4 = A2\b2;
124 v51 = x4(1)*exp(-y1)+x4(2)*exp(y1);
125 v521 = x4(3)*exp(-y21)+x4(4)*exp(y21);
126 v522 = x4(5)*exp(-y22)+x4(6)*exp(y22);
127 figure;
128 plot(y1,v51,'y-');
129 hold on;
130 plot(y21,v521,'r-');
131 hold on;
132 plot(y22,v522,'b-');
133 hold off;
134 xlabel('X(dimensionless)');
135 ylabel('V(Volts)');
136 title('Steady-state voltage of Figure 2(d)');
137 legend('Branch 1','Branch 21','Branch 22');

```