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BM 2101 - ANALYSIS OF PHYSIOLOGICAL SYSTEMS

Branched Cylinders: Dendritic Tree Approximations

This is submitted as a partial fulfillment for the module
BM 2101 - Analysis of Physiological Systems

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Contents

1 Question 1

First differentiate the given expressions by X.

$$\frac{dV_1}{dX} = -A_1 e^{-X} + B_1 e^{X}$$

Then substitute the given them to the nodal and boundary conditions. Finally the expressions can be obtained.

2 Question 2

The given linear transformation is as follows.

$$Ax = b$$

By applying the definition of matrix multiplication.

$$\sum_{k=1}^{6} A_{ik} x_{k1} = b_{i1}$$

Here $i = 1, 2, 3, .., 6$.
For instance considering k = 1;

$$A_1 - B_1 = (r_i \lambda_i) I_{app}$$

In the same manner, the remaining rows can be proved.

3 Question 3

According to the code in Listing[1] in Appendix[A], the coefficients are given from the variable x in line 46. The values are as follows.

$$A_1 = 7.3698 \times 10^{-4}$$

$$B_1 = 1.6723 \times 10^{-5}$$

$$A_{21} = A_{22} = 0.0011$$

$$B_{21} = B_{22} = -2.7987 \times 10^{-6}$$

4 Question 4

The voltage profiles are the same in both the daughter branches. Therefore the lines have overlapped.

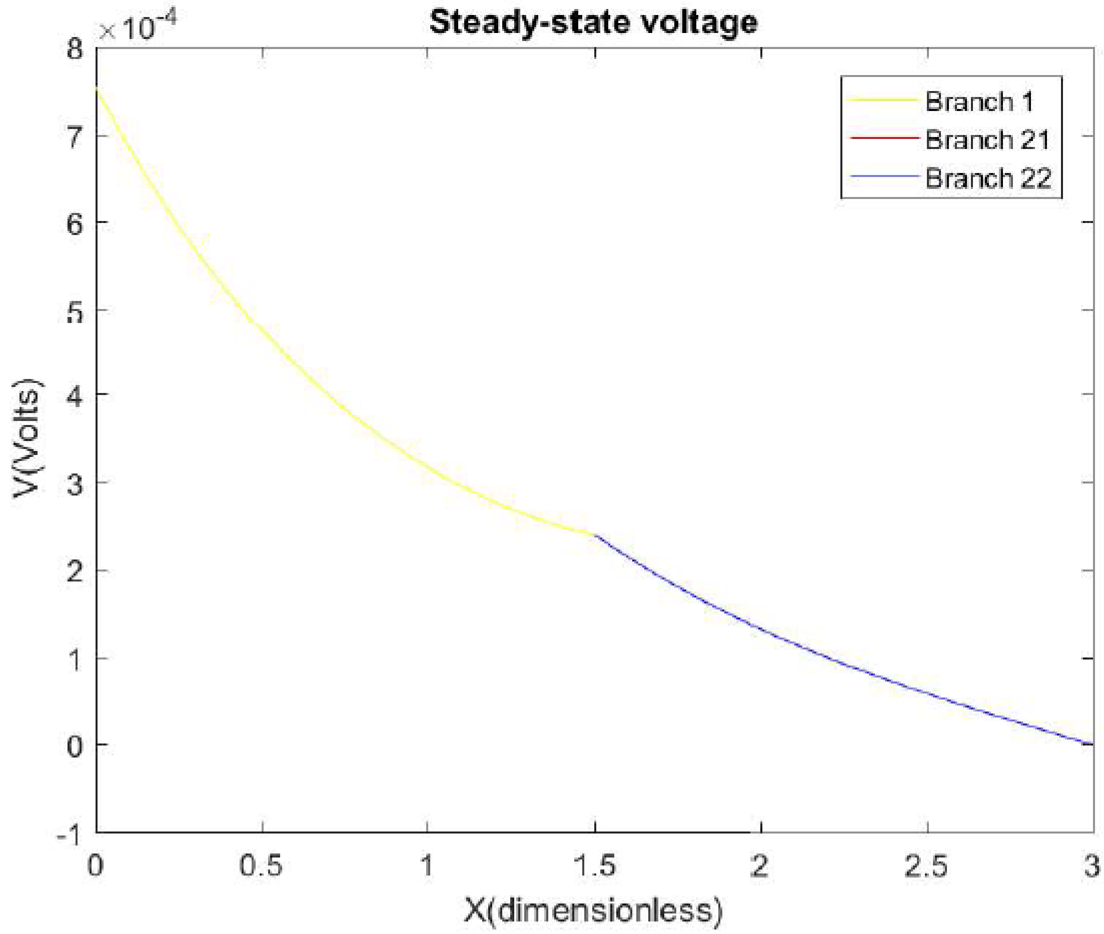

Figure 1: The voltage profile of the whole system

5 Question 5

The $\frac{dV_{21}}{dX}$ and $\frac{dV_{22}}{dX}$ shows the change of voltage along with distance.

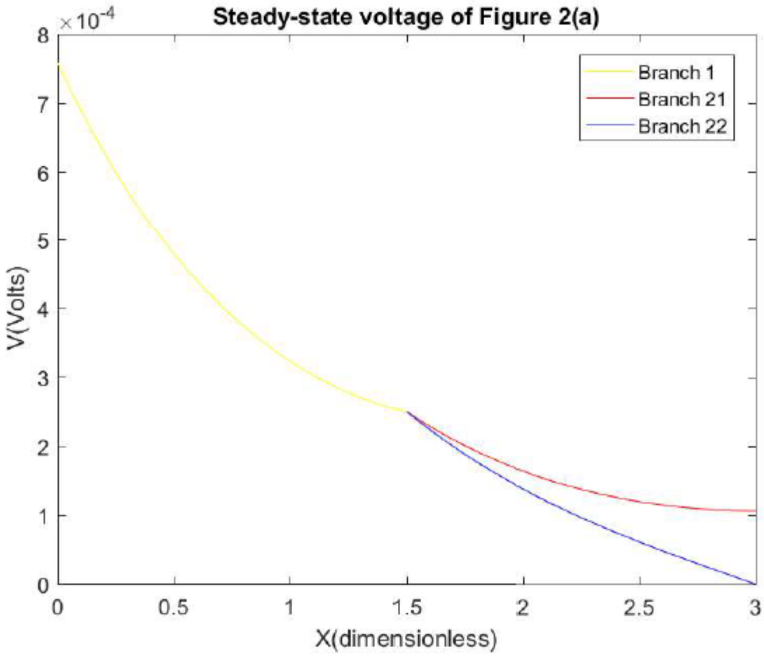

Figure 2: The voltage profile of the system 2(a)

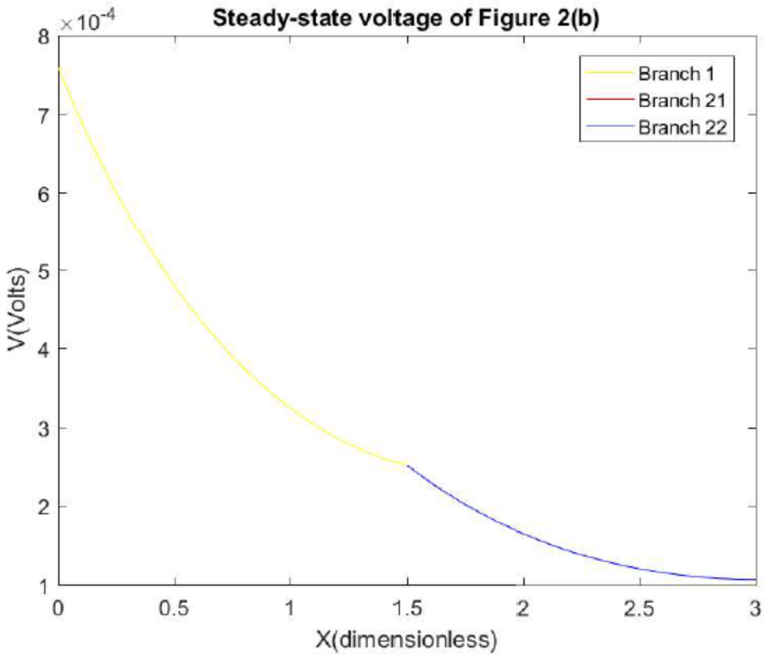

Figure 3: The voltage profile of the system 2(b)

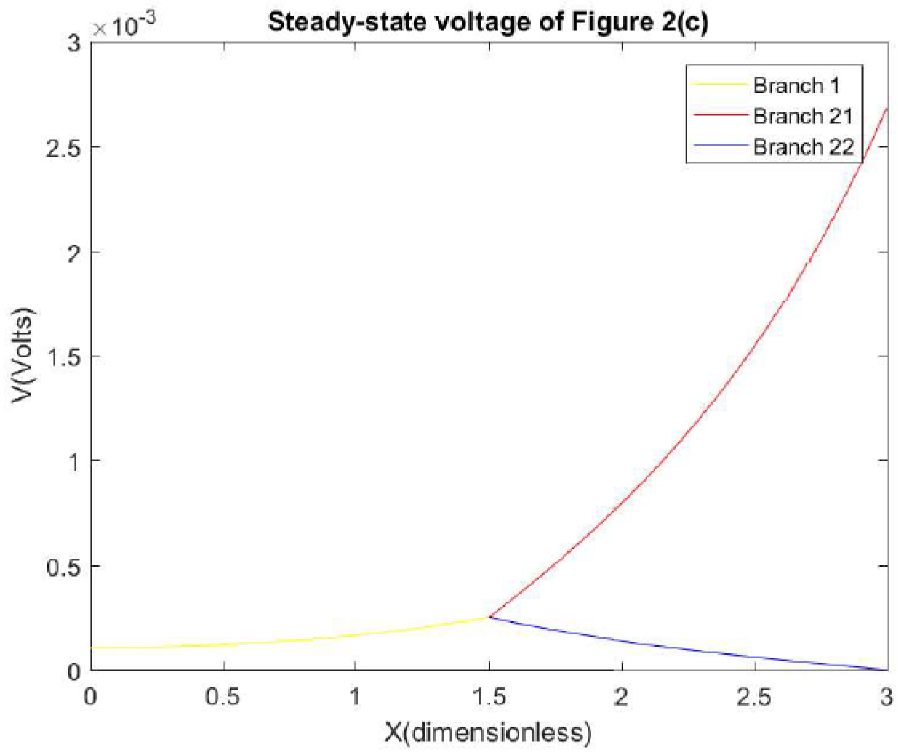

Figure 4: The voltage profile of the system 2(c)

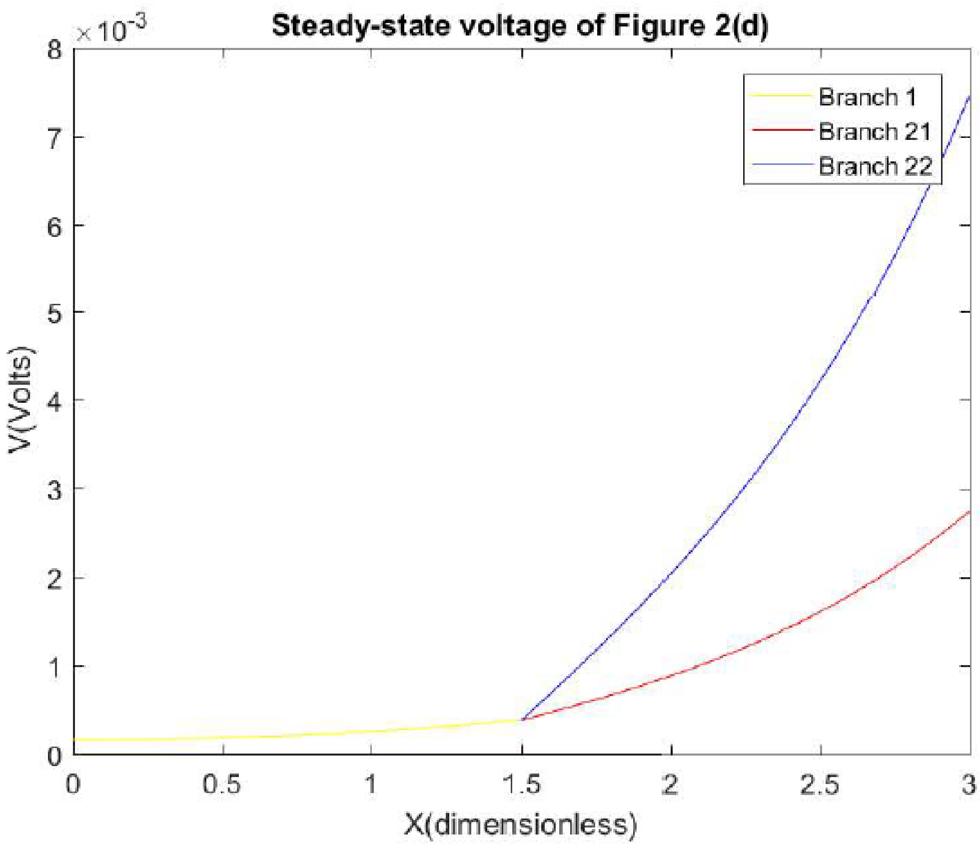

Figure 5: The voltage profile of the system 2(d)

6 Question 6

When we compare the Figure[3] and the Figure[6], the decrease of voltage per unit length is lesser in the daughter branches when $d_{21} = d_{22}$.

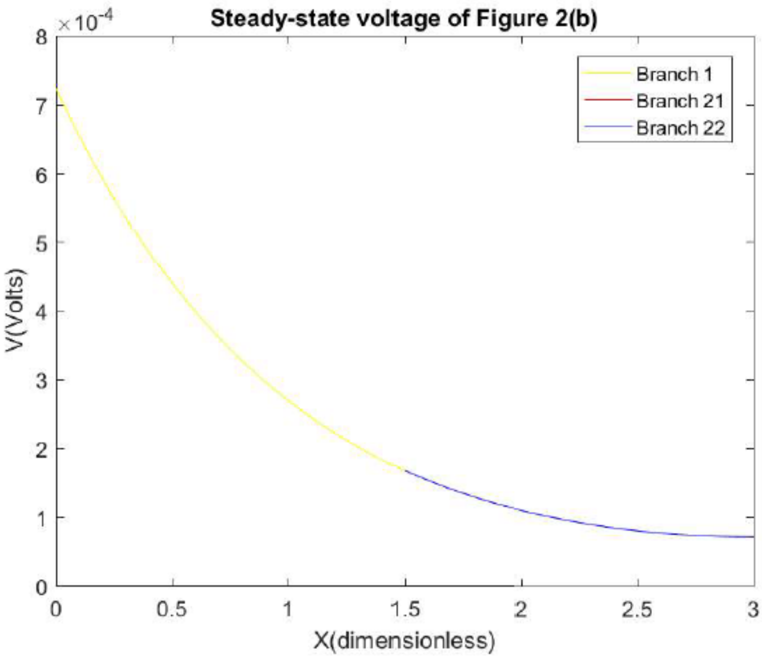

Figure 6: The voltage profile of the system 2(b)

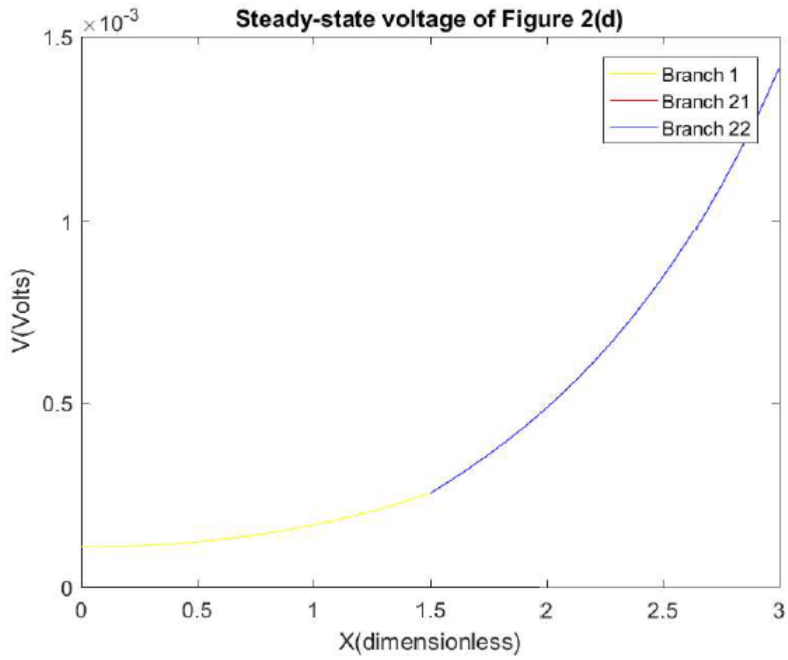

Figure 7: The voltage profile of the system 2(d)

When we compare the Figure[5] and the Figure[7], the decrease of voltage per unit length in the daughter branch 22 is lesser when $d_{21} = d_{22}$. In the latter, the daughter branches have the same voltage profile.

The new coefficients are as follows;

$$
\begin{aligned}
A_1 &= 7.1888 \times 10^{-4} \\
B_1 &= -1.3729 \times 10^{-6} \\
A_{21} = A_{22} &= 7.2748 \times 10^{-4} \\
B_{21} = B_{22} &= -1.8032 \times 10^{-6}
\end{aligned}
$$

A MATLAB code

Listing 1: The Matlab code

```
% electrical constants and derived quantities for typical
% mammalian dendrite

% Dimensions of compartments

% d1 = 75e-4;    % cm
% d21 = 30e-4;    % cm
% d22 = 15e-4;    % cm

% Commented till Question 6
d21 = 47.2470e-4; % E9 cm
d22 = d21;         % E9 cm
%%%%%%%
l1 = 1.5;     % dimensionless
l21 = 3.0;    % dimensionless
l22 = 3.0;    % dimensionless

% Electrical properties of compartments

Rm = 6e3;     % Ohms cm^2
Rc = 90;    % Ohms cm
Rs = 1e6;     % Ohms

c1 = 2*(Rc*Rm)^(1/2)/pi;

rl1 = c1*d1^(-3/2);  % Ohms
rl21 = c1*d21^(-3/2);  % Ohms
rl22 = c1*d22^(-3/2);  % Ohms


% Applied current

iapp = 1e-9;  % Amps

% Coefficient matrices

A = [1 -1 0 0 0 0;
    0 0 exp(-l21) exp(l21) 0 0;
    0 0 0 0 exp(-l22) exp(l22);
    exp(-l1) exp(l1) -exp(-l1) -exp(l1) 0 0;
    0 0 exp(-l1) exp(l1) -exp(-l1) -exp(l1);
    -exp(-l1) exp(l1) rl1*exp(-l1)/rl21 -rl1*exp(l1)/rl21 rl1*exp(-l1)/rl22
       -rl1*exp(-l1)/rl22];

b = [rl1*iapp; 0 ;0; 0 ;0; 0];
x = A\b;%Calculating the value of x
x
%Plotting the graph
```

```
y1 = linspace(0,l1,20);
y21 = linspace(l1,l21,20);
y22 = linspace(l1,l22,20);
v1 = x(1)*exp(-y1)+x(2)*exp(y1);
v21 = x(3)*exp(-y21)+x(4)*exp(y21);
v22 = x(5)*exp(-y22)+x(6)*exp(y22);
figure;
plot(y1,v1,'y-');
hold on;
plot(y21,v21,'r-');
hold on;
plot(y22,v22,'b-');
hold off;
xlabel('X(dimensionless)');
ylabel('V(Volts)');
title('Steady-state voltage');
legend('Branch 1','Branch 21','Branch 22');
%Changing A according to Figure 2a
A1 = A;
A1(2,:) = [0 0 -exp(-l21) exp(l21) 0 0];
x1 = A1\b;
v21 = x1(1)*exp(-y1)+x1(2)*exp(y1);
v221 = x1(3)*exp(-y21)+x1(4)*exp(y21);
v222 = x1(5)*exp(-y22)+x1(6)*exp(y22);
figure;
plot(y1,v21,'y-');
hold on;
plot(y21,v221,'r-');
hold on;
plot(y22,v222,'b-');
hold off;
xlabel('X(dimensionless)');
ylabel('V(Volts)');
title('Steady-state voltage of Figure 2(a)');
legend('Branch 1','Branch 21','Branch 22');
%Changing A according to Figure 2b
A2 = A1;
A2(3,:) = [0 0 0 0 -exp(-l22) exp(l22)];
x2 = A2\b;
v31 = x2(1)*exp(-y1)+x2(2)*exp(y1);
v321 = x2(3)*exp(-y21)+x2(4)*exp(y21);
v322 = x2(5)*exp(-y22)+x2(6)*exp(y22);
figure;
plot(y1,v31,'y-');
hold on;
plot(y21,v321,'r-');
hold on;
plot(y22,v322,'b-');
hold off;
xlabel('X(dimensionless)');
ylabel('V(Volts)');
```

```
title('Steady-state voltage of Figure 2(b)');
legend('Branch 1','Branch 21','Branch 22');
%Changing b according to Figure 2c
b1 = b;
b1(1) = 0;
b1(2) = rl21*iapp;
x3 = A1\b1;
v41 = x3(1)*exp(-y1)+x3(2)*exp(y1);
v421 = x3(3)*exp(-y21)+x3(4)*exp(y21);
v422 = x3(5)*exp(-y22)+x3(6)*exp(y22);
figure;
plot(y1,v41,'y-');
hold on;
plot(y21,v421,'r-');
hold on;
plot(y22,v422,'b-');
hold off;
xlabel('X(dimensionless)');
ylabel('V(Volts)');
title('Steady-state voltage of Figure 2(c)');
legend('Branch 1','Branch 21','Branch 22');
%Changing b according to Figure 2d
b2 = b1;
b2(3) = rl22*iapp;
x4 = A2\b2;
v51 = x4(1)*exp(-y1)+x4(2)*exp(y1);
v521 = x4(3)*exp(-y21)+x4(4)*exp(y21);
v522 = x4(5)*exp(-y22)+x4(6)*exp(y22);
figure;
plot(y1,v51,'y-');
hold on;
plot(y21,v521,'r-');
hold on;
plot(y22,v522,'b-');
hold off;
xlabel('X(dimensionless)');
ylabel('V(Volts)');
title('Steady-state voltage of Figure 2(d)');
legend('Branch 1','Branch 21','Branch 22');
```