

1 Fundamental

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1.1 Intervals

Intervals are basically subsets of \mathbb{R} and are commonly used in solving inequalities or in finding domains. If there are two numbers $a, b \in \mathbb{R}$ such that $a < b$, there are four types of intervals:

- Open interval: $(a, b) = \{x : a < x < b\}$ i.e end points are not included. Symbols: $()$ or $] [$
- Closed interval: $[a, b] = \{x : a \leq x \leq b\}$ i.e end points are also included. Symbol: $[]$
- Open-Closed interval: $(a, b] = \{x : a < x \leq b\}$. Symbols: $()]$ or $] [$
- Closed-Open interval: $[a, b) = \{x : a \leq x < b\}$. Symbols: $[])$ or $] [$

Infinite Intervals

- $(a, \infty) = \{x : x > a\}$
- $[a, \infty) = \{x : x \geq a\}$
- $(-\infty, b) = \{x : x < b\}$
- $(-\infty, b] = \{x : x \leq b\}$
- $(-\infty, \infty) = \{x : x \in \mathbb{R}\}$

1.2 Sets and Relations

1.2.1 Set

A collection of any kind of objects. The objects that make up a set are called *elements* or *members*. The statement ' a is an element of set A ' can be written as $a \in A$ and set containing elements a, b and c is denoted by $\{a, b, c\}$. A *empty* or *null* set is denoted by \emptyset , which is the set that contains no elements.

Union(join, sum): The union of two sets A and B , denoted by $A \cup B$, consists of those elements that belong to A or to B :

$$A \cup B = \{x : (x \in A) \vee (x \in B)\}$$

For example, if A is $\{1, 2, 3, 4\}$ and B is $\{1, 4, 5, 6\}$ then $A \cup B$ is $\{1, 2, 3, 4, 5, 6\}$.

Intersection(meet, product): The intersection of two sets A and B , denoted by $A \cap B$, consists of those elements that belong to both A and B :

$$A \cap B = \{x : (x \in A) \wedge (x \in B)\}$$

For example, if A is $\{1, 2, 3, 4, 5, 6\}$ and B is $\{1, 4, 5, 6, 7, 8\}$ then $A \cap B$ is $\{1, 4, 5, 6\}$.

Complement: The complement of a set A , denoted by A' or A^c , consists of all those elements that are not members of A :

$$A' = \{x : x \notin A\}$$

For example, in the domain of natural numbers, if A is set of even numbers the its complement A' is the set of odd numbers.

Universal Set: Relative to a particular domain, the universal set, denoted by U , is set of all objects of that domain:

$$U = \{x : x = x\}$$

1.2.2 Properties of sets

Commutative law:

- $(A \cup B) = B \cup A$
- $(A \cap B) = B \cap A$

Associative law:

- $(A \cup B) \cup C = A \cup (B \cup C)$
- $(A \cap B) \cap C = A \cap (B \cap C)$

Distributive law:

- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

De Morgan's law:

- $(A \cup B)' = A' \cap B'$
- $(A \cap B)' = A' \cup B'$

Identity law:

- $A \cup \mathbf{U} = A$

- $A \cup \emptyset = A$

Complement law:

- $A \cup A' = \mathbf{U}$

- $A \cap A' = \emptyset$

- $(A')' = A$

Idempotent law:

- $A \cap A = A$

- $A \cup A = A$

1.2.3 Results on number of elements in sets