# 1 Fundamental

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## 1.1 Intervals

Intervals are basically subsets of $\mathbb{R}$ and are commonly used in solving inequalities or in finding domains. If there are two numbers $a, b \in \mathbb{R}$ such that $a < b$, there are four types of intervals:

- Open interval: $(a, b) = \{x : a < x < b\}$ i.e end points are not included. Symbols: () or ][
- Closed interval: $[a, b] = \{x : a \leq x \leq b\}$ i.e end points are also included. Symbol: [ ]
- Open-Closed interval: $(a, b) = \{x : a < x \leq b\}$. Symbols: ( ] or ] ]
- Closed-Open interval: $(a, b) = \{x : a \leq x < b\}$. Symbols: [ ) or [ [

**Infinite Intervals**

- $(a, \infty) = \{x : x > a\}$
- $[a, \infty) = \{x : x \geq a\}$
- $(-\infty, b) = \{x : x < b\}$
- $(\infty, b] = \{x : x \leq b\}$
- $(-\infty, \infty) = \{x : x \in \mathbb{R}\}$

## 1.2 Sets and Relations

### 1.2.1 Set

A collection of any kind of objects. The objects that make up a set are called *elements* or *members*. The statement '$a$ is an element of set $A$' can be written as $a \in A$ and set containing elements $a, b$ and $c$ is denoted by $\{a, b, c\}$. A *empty* or *null* set is denoted by $\varnothing$, which is the set that contains no elements.

**Union(join,sum):** The union of two sets $A$ and $B$, denoted by $A \cup B$, consists of those elements that belong to $A$ or to $B$:

$$A \cup B = \{x : (x \in A) \vee (x \in B)\}$$

For example, if $A$ is $\{1, 2, 3, 4\}$ and $B$ is $\{1, 4, 5, 6\}$ then $A \cup B$ is $\{1, 2, 3, 4, 5, 6\}$.

**Intersection(meet,product):** The intersection of two sets $A$ and $B$, denoted by $A \cap B$, consists of those elements that belong to both $A$ and $B$:

$$A \cap B = \{x : (x \in A) \wedge (x \in B)\}$$

For example, if $A$ is $\{1, 2, 3, 4, 5, 6\}$ and $B$ is $\{1, 4, 5, 6, 7, 8\}$ then $A \cap B$ is $\{1, 4, 5, 6\}$.

**Complement:** The complement of a set $A$, denoted by $A'$ or $A^{\complement}$, consists of all those elements that are not members of $A$:

$$A' = \{x : x \notin A\}$$

For example, in the domain of natural numbers, if $A$ is set of even numbers the its complement $A'$ is the set of odd numbers.

**Universal Set:** Relative to a particular domain, the universal set, denoted by $\mathbf{U}$, is set of all objects of that domain:

$$\mathbf{U} = \{x : x = x\}$$

#### 1.2.2 Properties of sets

**Commutative law:**

- $(A \cup B) = B \cup A$
- $(A \cap B) = B \cap A$

**Associative law:**

- $(A \cup B) \cup C = A \cup (B \cup C)$
- $(A \cap B) \cap C = A \cap (B \cap C)$

**Distributive law:**

- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

**De Morgan's law:**

- $(A \cup B)' = A' \cap B'$
- $(A \cap B)' = A' \cup B'$

**Identity law:**

- $A \cup \mathbf{U} = A$
- $A \cup \varnothing = A$

**Complement law:**

- $A \cup A' = \mathbf{U}$
- $A \cap A' = \varnothing$
- $(A')' = A$

**Idempotent law:**

- $A \cap A = A$
- $A \cup A = A$

### 1.2.3 Results on number of elements in sets

If $A, B, C$ are finite sets and $\mathbf{U}$ be Universal finite set then:

- $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- $n(A - B) = n(A) - n(A \cap B)$
- $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$