1 Fundamental

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1.1 Intervals

Intervals are basically subsets of $\mathbb{R}$ and are commonly used in solving inequalities or in finding domains. If there are two numbers $a, b \in \mathbb{R}$ such that $a < b$, there are four types of intervals:

- Open interval: $(a, b) = \{x : a < x < b\}$ i.e end points are not included. Symbols: () or][
- Closed interval: $[a, b] = \{x : a \leq x \leq b\}$ i.e end points are also included. Symbol: [ ]
- Open-Closed interval: $(a, b) = \{x : a < x \leq b\}$. Symbols: ( ] or]]
- Closed-Open interval: $(a, b) = \{x : a \leq x < b\}$. Symbols: [ ) or [[

Infinite Intervals

- $(a, \infty) = \{x : x > a\}$
- $[a, \infty) = \{x : x \geq a\}$
- $(-\infty, b) = \{x : x < b\}$
- $(\infty, b] = \{x : x \leq b\}$
- $(-\infty, \infty) = \{x : x \in \mathbb{R}\}$

1.2 Sets and Relations

1.2.1 Set

A collection of any kind of objects. The objects that make up a set are called *elements* or *members*. The statement 'a is an element of set A' can be written as $a \in A$ and set containing elements a, b and c is denoted by $\{a, b, c\}$. A *empty* or *null* set is denoted by $\varnothing$, which is the set that contains no elements.

Union(join,sum): The union of two sets A and B, denoted by $A \cup B$, consists of those elements that belong to A or to B:

$$A \cup B = \{x : (x \in A) \vee (x \in B)\}$$

For example, if A is $\{1, 2, 3, 4\}$ and B is $\{1, 4, 5, 6\}$ then $A \cup B$ is $\{1, 2, 3, 4, 5, 6\}$.

Intersection(meet,product): The intersection of two sets A and B, denoted by $A \cap B$, consists of those elements that belong to both A and B:

$$A \cap B = \{x : (x \in A) \wedge (x \in B)\}$$

For example, if A is $\{1, 2, 3, 4, 5, 6\}$ and B is $\{1, 4, 5, 6, 7, 8\}$ then $A \cap B$ is $\{1, 4, 5, 6\}$.

Complement: The complement of a set A, denoted by A' or A^{C}, consists of all those elements that are not members of A:

$$A' = \{x : x \notin A\}$$

For example, in the domain of natural numbers, if A is set of even numbers the its complement A' is the set of odd numbers.

Universal Set: Relative to a particular domain, the universal set, denoted by $\mathbf{U}$, is set of all objects of that domain:

$$\mathbf{U} = \{x : x = x\}$$

1.2.2 Properties of sets

Commutative law:

- $(A \cup B) = B \cup A$
- $(A \cap B) = B \cap A$

Associative law:

- $(A \cup B) \cup C = A \cup (B \cup C)$
- $(A \cap B) \cap C = A \cap (B \cap C)$

Distributive law:

- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

De Morgan's law:

- $(A \cup B)' = A' \cap B'$
- $(A \cap B)' = A' \cup B'$

Identity law:

- $A \cup \mathbf{U} = A$
- $A \cup \varnothing = A$

Complement law:

- $A \cup A' = \mathbf{U}$
- $A \cap A' = \varnothing$
- $(A')' = A$

Idempotent law:

- $A \cap A = A$
- $A \cup A = A$

1.2.3 Results on number of elements in sets

If A, B, C are finite sets and $\mathbf{U}$ be Universal finite set then:

- $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- $n(A - B) = n(A) - n(A \cap B)$
- $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

1.2.4 Relations

An association between, or property of, two or more objects. Thus ($x = y$) and (a lies between b and c) are relations, but (N is a prime) is not.

1.2.5 Types of Relations

Void Relation: Let A be a set. Then $\phi \subseteq A \times A$ and so it is a relation on A. This relation is also called empty relation on A.
Universal Relation: Let A be a set. Then $A \times A \subseteq A \times A$ and so it is a relation on A. This relation is called the universal relation on A.
Identity relation: Let A be a set. Then the relation $I_A = \{(a, a) : a \in A\}$is called the identity relation on A. In other words, a relation I_A on A is called the identity relation if every element of A is related to itself only.
Reflexive Relation: A relation R on a set A is said to be reflexive if every element of A is related to itself. Thus R on set A is not reflexive if there exits an element $a \in A$ such that $(a, a) \notin R$.
Symmetric Relation: A Relation R on set A is said to be symmetric if

$$(a, b) \in R \implies (b, a) \in R \,\forall\, a, b \in A$$

i.e

$$a\, R\, b \implies b\, R\, a \,\forall\, a, b \in A$$

Transitive Relation: A Relation R on set A is said to be transitive if $(a, b) \in R$ and $(b, c) \in R \implies (a, c) \in R \,\forall\, a, b, c \in A$.
Equivalence Relation: A Relation is said to be equivalence if it satisfies $reflexive, symmetric$ and $transitive$ relations.
Equivalence Class: If R is an equivalence relation defined on set A then the equivalence class of any element $x \in A$, denoted by $[x]$, is the set of elements to which x is related by equivalence relation R:

$$[x] = \{y : x\, \mathbf{R}\, y\}$$

For example, if R is the equivalence relation (the same height as), then the equivalence class of the element $x \in A$ consists of all elements of A with same height as x.

2 Quadratic Equations

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2.1 Equation and Basic Results

An equation of the form

$$ax^2 + bx + c = 0$$

where $a \neq 0$ and $a, b, c \in \mathbb{R}$ is called a quadratic equation. The roots of quadratic equation are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The quantity D ($D = b^2 - 4ac$) is known as discriminant of equation.

2.1.1 Results

1. Let α and β be two roots of given quadratic equation. Then

- $\alpha + \beta = -\frac{b}{a}$
- $\alpha\beta = \frac{c}{a}$

2. A quadratic equation, whose roots are α and β can be written as

$$(x - \alpha)(x - \beta) = 0$$

i.e

$$ax^2 + bx + c \equiv a(x - \alpha)(x - \beta)$$

3. If the quadratic equation is satisfied by more than two distinct numbers (real or complex), then it becomes an identity, i.e

$$a = b = c = 0$$

4. The quadratic equation has real and equal roots if and only if $D = 0$ i.e $b^2 - 4ac = 0$