GEOPHYSICS, VOL. 72, NO. 6 (NOVEMBER-DECEMBER 2007); P. R109–R114, 3 FIGS.
10.1190/1.2766467

Seismic absorption compensation: A least squares inverse scheme

Changjun Zhang[1] and Tadeusz J. Ulrych[2]

ABSTRACT

The problem of instability plagues conventional inverse Q filtering. We formulate the deabsorption problem as an inverse problem in terms of least squares and impose regularization by means of Bayes' theorem. The solution is iterative and nonparametric and returns a reflectivity that has been constrained to be sparse. The inverse scheme is tested on both synthetic and real data and the results obtained demonstrate the viability of the approach.

INTRODUCTION

The aim of seismic processing is to obtain a high-resolution image of the subsurface. Many steps are involved in the attempt to achieve this objective. One such step consists, or most certainly should consist, of the compensation for the ubiquitous absorption of seismic energy in the earth, a step referred to as deabsorption (McGinn and Duijndam, 1988). To a first approximation, the earth is a linear attenuator, the effect of which is to attenuate the higher frequencies of the seismic signal. The mechanisms and the relevant mathematical details have been well described in the literature and we will not deal with these issues here. We would like, however, to refer the interested reader to a few of the classical papers in this field. Futterman (1962), Kjartansson (1992), and Aki and Richards (2002) in our view, present particularly insightful views. An excellent summary and overview of different Q models may be found in Wang and Guo (2004). For the sheer pleasure of reading related work by a great mind, we recommend Brillouin (1960).

The most serious setback to the application of deabsorption, is the inherent instability of deconvolution filters which, unless very carefully designed, unavoidably amplify high-frequency noise. Many authors have considered this problem (Robinson, 1979; Hale, 1991; Hargreaves and Calvert, 1991; Bickel, 1993; Varela et al., 1993; Wang, 2002, 2003, 2006; Margrave et al., 2003; Taner and Treitel, 2003). Although, as a result of the effort of the aforementioned authors, a variety of algorithms are now available to the industry, it is probably safe to say that all are very sensitive to the presence of additive noise. Our approach to this noise-associated problem is by means of the application of least squares (LSQ) inversion. This approach attempts to obtain a solution to the inverse problem of deabsorption by minimizing the misfit between observed data and theoretically modeled data. Thus, the instability of applying an inverse operator to the random noise in the data, is avoided.

The LSQ objective function may be constructed in various ways, but the manner which we prefer is based on a probabilistic rational (Tarantola, 1987; Ulrych and Sacchi, 2005), where we consider the measured and modeled data (which become the solution of the inverse problem) as realizations of random processes.

There are two main advantages to the LSQ approach proposed here, as compared to previously published methods. One advantage is that, because we formulate the problem as under determined, we have control over the type of solution that is desired. Another advantage is that the LSQ approach uses only the forward operator and does not require the unstable inverse operator. In other words, the formulation of deabsorption as an inverse problem, allows a stable solution to be achieved. As pointed out by Claerbout (1992), earth absorption might be compensated for by amplifying high-frequency energy during downward continuation, shifting a wavefield using an absorption operator. This very interesting topic of migration with absorption compensation is, however, outside of the scope of the present paper which deals with single trace absorption compensation.

This paper is arranged as follows: First, we discuss, very briefly, critical issues in deabsorption, which include the Q model of seismic absorption and the velocity dispersion associated with amplitude attenuation. Second, we describe our deabsorption approach using a LSQ optimization scheme based on probabilistic inverse theory. Finally, we discuss the results of both synthetic and real data examples using the LSQ algorithm.

A BRIEF SUMMARY CONCERNING Q

The theory that is used is linear. As so succinctly put by Futterman (1962), the obvious advantages of a linear theory are given up only

Manuscript received by the Editor 7 January 2007; revised manuscript received 10 May 2007; published online 5 September 2007.
[1]3DGeo Inc., Houston, Texas. E-mail: changjun.zhang@gmail.com.
[2]University of British Columbia, Department of Earth and Ocean Science, Vancouver, British Columbia, Canada. E-mail: ulrych@geop.ubc.ca.

for sufficient cause and in the Q case, there appears not to be sufficient cause. We present here, very briefly and following Futterman (1962), the essence of Q and the associated dispersion relations that may not be as familiar to the reader as is Q itself.

Consider a single-frequency component of a displacement, $u(t)$, such as

$$u(t) = Ae^{-\gamma}\cos(\omega + \theta),$$

where the symbols define themselves. Defining Δ as the logarithmic increment, the amplitude in one period is reduced by

$$e^{-2\pi\gamma/\omega} = e^{-\Delta}.$$

If we define the energy loss/cycle to the maximum energy stored as $\Delta W/W$ then, because the energy is proportional (for a linear system) to amplitude squared, we obtain

$$\frac{\Delta W}{W} = 1 - e^{-2\Delta}.$$

Defining the energy loss in term of Q, obtains

$$Q^{-1} = \frac{1}{2\pi}(1 - e^{-2\Delta}).$$

Dispersion relations express the fact that, because of the causality of wave propagation in the earth, attenuation of a traveling pulse must always be accompanied by dispersion. Dispersion describes the fact that waves of different frequencies travel at different velocities. Causality enters in the following manner. Suppose that a delta-like pulse, $\delta(t - x/c)$, is propagated through a medium of velocity c. If there is no dispersion, the frequencies which make up the pulse all travel at the same velocity, and the pulse is broadened into a symmetric function centered at $x = c/t$. Theoretically, the tails of this pulse extend to infinity and we encounter the situation that energy is recorded even before the explosion has generated the initial pulse. This is acausality, of course, and because we insist on the causal character of the medium, the low frequency tails must travel at a slower velocity than the higher frequency components. This is dispersion. In fact, attenuation and dispersion are bonded together by the Hilbert transform which is central in causal systems.

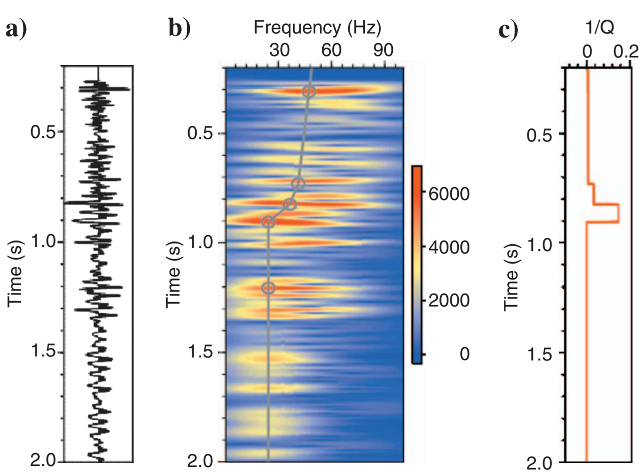

Figure 1. Q estimation. (a) A real seismic trace. (b) The windowed time-variant spectrum. The picked peak frequency points are marked by circles and connected by straight lines. (c) The inverse $(1/Q)$ of estimated quality factors.

ESTIMATING Q

We are concerned with poststack trace-by-trace deabsorption processing. In reality, a Q curve is required for each trace and, consequently, a Q section is the ultimate desideratum. Many techniques for Q estimation exist (Tonn, 1991) such as the spectral ratio method, rise time method, and many others.

Zhang and Ulrych (2002) proposed an analytical method to estimate Q values from the peak frequency shift of a seismic wavelet. The assumption in this approach is that the amplitude spectrum of the seismic wavelet is akin to that of a Ricker wavelet, i.e., it rises and decays smoothly and exhibits a single maximum. The method is applicable to Q estimation from CMP gathers, as well as from single traces. For Q estimation from a single trace, the method first computes the windowed time-variant spectrum (WTVS) along a trace, and then picks the peak frequencies of the amplitude spectrum at main event locations. Layer Q values are then computed from the shift of peak frequencies. Figure 1 shows a field seismic trace, its WTVS, and the estimated Qs.

TIME-VARIANT DECONVOLUTION AS AN INVERSE PROBLEM

As mentioned before, inverse Q filtering is inherently unstable in that the inverse operator will boost high-frequency noise. To ensure that noise is not unnecessarily amplified, it is important to design the inverse operator appropriately at high frequencies. One way to avoid the instability is to use a band-limited version of the inverse operator i.e., to replace the amplitude compensation operator $A^{-1}(\omega)$ by $W(\omega)A^{-1}(\omega)$, where $W(\omega)$ represents a low-pass filter. Of course, such a low-pass operation causes an undesirable loss of resolution.

Noise and deconvolution

The instability issue is not unique to the absorption compensation problem. It is, in fact, ubiquitous in the application of deconvolution in general. In seismic data processing, the recorded data in the 1D case can be modeled as

$$x(t) = w(t) * r(t) + n(t), \tag{1}$$

where $x(t)$, $w(t)$, and $n(t)$ represent the recorded trace, a seismic wavelet, the earth's reflectivity, and random noise, respectively. Seismic data deconvolution aims at obtaining the reflectivity from the observed trace. The estimated reflectivity will be adversely affected by the random noise term, whether or not $w(t)$ is band limited. Defining $f(t)$ as the inverse of $w(t)$, assumed known [i.e. $f(t)w(t) = \delta(t)$], the deconvolution result is

$$\hat{r}(t) = r(t) + f(t) * n(t).$$

Depending on the signal-to-noise ratio, the effect of the term $f(t)n(t)$ on the output reflectivity can be very severe and for which it must be compensated. The required regularization always leads to a loss of resolution.

To better handle the problem of additive noise, the inverse filter is often designed using a LSQ approach rather than by directly inverting with the known or estimated seismic wavelet. The resulting filters are called optimum Wiener filters and are well elaborated by Robinson and Treitel (2002). We take a probabilistic approach to the LSQ solution here, an approach that we believe has a flexibility which leads to improved results. Specifically, we formulate deabsorption as an inverse problem and use a Cauchy-Gauss objective function (Sacchi et al., 1998) to arrive, iteratively, at the desired solution.

Bayes probabilistic inference

Probabilistic inference is a very powerful approach to the ubiquitous inverse problem (Tarantola, 1987; Ulrych and Sacchi, 2005). It is common to consider the measured data $\mathbf{d}$ (where we indicate vectors by bold symbols) as uncertain. That is, true data do exist, but are unknown. Measured data can then be considered as random variables whose expectation, in the ensemble sense, is the true value. Traditionally, the assumption is made that the observed data are random variables with a Gaussian distribution. This leads to the well-known χ^2 test for goodness of fit and to a l_2 norm solution.

Let us represent the solution to the inverse problem by the model vector $\mathbf{m}$. The model vector $\mathbf{m}$ is not unique, in the sense that an infinity of such models may be found that fit the data. The reason for this, of course, is that the data, even if noise free, can never describe the continuous model completely. Our problem is under determined. We may indeed pose the problem as an overdetermined one, if we so choose. Wiener inversion is one such possibility. However, we believe, having been well-schooled by the late Edwin Jaynes, that an inverse problem posed in such manner, is incorrectly posed (Jaynes, 2004). A discretized inverse problem can never be unique. The overdetermined solution returns the smallest model, i.e., that model whose energy is most distributed in model space. When estimating deconvolution models describing reflectivities of the earth, such models do not appear to be reasonable.

An infinity of models is not a useful set. We must impose some a priori knowledge, if such exists, feeling, or at least hope, associated with our search. The manner of so doing lies at the heart of Bayes' theorem, and at the heart of the regularization thus imposed. This is hardly the place for a discussion of the underlying logic, but we feel that a few remarks are in order (for a fairly full discussion vis-à-vis Bayes, inversion, and a priori information, see Ulrych and Sacchi [2005]). The central point in the application of Bayesian logic is in how one handles prior information. We begin with Bayes' theorem, which states that

$$p(\mathbf{m}|\mathbf{d}) = \frac{p(\mathbf{d}|\mathbf{m})p(\mathbf{m})}{p(\mathbf{d})}, \tag{2}$$

where $p(\mathbf{d}|\mathbf{m})$ is the conditional probability density function (pdf) of $\mathbf{d}$, the data, given that $\mathbf{m}$ has occurred. It is called the likelihood and is the function that is maximized in l_2 norm problems when in Gaussian form. The Gaussian pdf for a random variable x, that we will have occasion to use is

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2}, \tag{3}$$

where σ^2 is the variance and μ the mean of the pdf, respectively.

Returning to equation 2, $p(\mathbf{d})$ is the pdf of the observed data and often is not a factor in the inversion. In as much as we consider $\mathbf{d}$ to be a random vector, we treat $\mathbf{m}$ in equivalent manner. The pdf that we wish to obtain is $p(\mathbf{m}|\mathbf{d})$. It is the probability of obtaining $\mathbf{m}$ given the data. Maximizing this pdf will obtain an estimated model that is referred to as the maximum a posteriori solution. If $p(\mathbf{m}) = 1$, i.e., a uniform pdf, the formulation of the problem is exactly least squares. Now, however, comes a point of much discord, $p(\mathbf{m})$. In the opinion of a group of statisticians known as frequentists, prior probabilities cannot be assigned, they must be measured. Edwin Jaynes, like Laplace before him, fought passionately against this philosophy, and most certainly emerged the victor. The pdf that we wish to associate with our expected model is $p(\mathbf{m})$. It is our choice based on our belief. It could certainly be inappropriate, but who is to say that the l_2 model, the one most distributed in time and/or space, is any more so? What pdf should be chosen when dealing with deabsorption? We follow the logic of Sacchi et al. (1998) and use the Cauchy distribution, given by

$$p(x) = \frac{1}{1 + \dfrac{x^2}{2\sigma^2}}, \tag{4}$$

where σ^2 governs the spread of the distribution (the Cauchy pdf being one that does not possess a theoretical variance).

The reason behind this choice is, very basically, that this pdf exhibits long tails and, consequently, is a candidate for the pdf that characterizes sparse reflectivities. Sparseness is of much relevance these days (Herrmann, 2005; Candès et al. 2005). It is not a concept applicable in all cases, of course. One would not model a Beethoven concerto in this manner, but layers in the earth are, one hopes, sparsely distributed.

Absorption compensation and inversion

We now formulate our problem in light of the above discussion. Assume the observed data to be contaminated by noise that is normally distributed as $N(0,\sigma_{\mathbf{n}}^2)$, where $\mathbf{n}$ represents the noise vector. The Gaussian assumption stems from the principle of maximum entropy (see Ulrych and Sacchi [2005] for a discussion and references) which, informally speaking, states that if nothing is known, use the simplest hypothesis. In this case, the Gaussian pdf follows from the central limit theorem.

The conditional distribution of the data is given by (equation 2)

$$p(\mathbf{m}|\mathbf{d},\sigma_{\mathbf{n}}) = \left(\frac{1}{2\pi\sigma_{\mathbf{n}}^2}\right)^{(M-1)/2} e^{-(1/2\sigma_{\mathbf{n}}^2)\|\mathbf{d}-\mathbf{Gm})\|_2^2}, \tag{5}$$

where M is the length of the data vector and, for our linear system

$$\mathbf{n} = \mathbf{d} - \mathbf{Gm},$$

with $\mathbf{G}$ the coefficient or kernel matrix.

Let $p(\mathbf{m}|\sigma_{\mathbf{m}})$ indicate a prior distribution for $\mathbf{m}$ conditional on a parameter $\sigma_{\mathbf{m}}$. From Bayes' theorem, equation 2, we have

$$p(\mathbf{m}|\mathbf{d},\sigma_{\mathbf{m}},\sigma_{\mathbf{n}}) = \frac{p(\mathbf{d}|\mathbf{m},\sigma_{\mathbf{n}})p(\mathbf{m}|\sigma_{\mathbf{m}})}{p(\mathbf{d})}. \tag{6}$$

The model vector $\mathbf{m}$ is the reflectivity function, and we use a sparseness constraint in the inversion scheme.

Cauchy-Gauss model

In the following, we will assume that a seismic wavelet is available. It is either obtained from check-shot survey, or from a seismic trace in some manner. We postulate, as reasoned above, that the model, i.e., the reflectivity $\mathbf{m}$, with elements m_k of $\mathbf{m}$ iid with equal variance $\sigma_{\mathbf{m}}^2$ (a standard hypothesis). The joint pdf of the m_k is

$$p(\mathbf{m}|\sigma_{\mathbf{m}}) = \prod_k p(m_k|\sigma_{\mathbf{m}}),$$

where $p(m_k|\sigma_{\mathbf{m}})$ satisfies a Cauchy distribution given by equation 4. Substitution yields

$$p(\mathbf{m}|\sigma_{\mathbf{m}}) = \prod_k \left(1 + \frac{m_k^2}{2\sigma_{\mathbf{m}}^2}\right)^{-1}.$$

By inserting this Cauchy prior, and the data likelihood (equation 3) into equation 6, and taking logarithms of both sides, we obtain

$$\ln[p(\mathbf{m}|\mathbf{d},\sigma_{\mathbf{m}},\sigma_{\mathbf{n}})] = -c(\mathbf{m}) - \frac{1}{2\sigma_{\mathbf{n}}^2}(\mathbf{d} - \mathbf{Gm})^T \times(\mathbf{d} - \mathbf{Gm}), \tag{7}$$

where $c(\mathbf{m})$ is a constraint imposed by the Cauchy distribution

$$c(\mathbf{m}) = \sum_{k=0}^{M-1} \ln\left(1 + \frac{m_k^2}{2\sigma_{\mathbf{m}}^2}\right).$$

Furthermore, denoting $\phi_{cg}(\mathbf{m}) = -\ln[p(\mathbf{m}|\mathbf{d},\sigma_{\mathbf{m}},\sigma_{\mathbf{n}})]$, we obtain the cost function for the Cauchy-Gauss model as

$$\phi_{cg}(\mathbf{m}) = c(\mathbf{m}) + \frac{1}{2\sigma_{\mathbf{n}}^2}(\mathbf{d} - \mathbf{Gm})^T(\mathbf{d} - \mathbf{Gm}), \tag{8}$$

where $\mathbf{G}$ is composed of time-variant wavelets, $b_\tau(t-\tau)$. Both t and τ are in the range of the length of a trace and

$$\mathbf{G} = \begin{bmatrix} b_0(t-0) \\ b_1(t-1) \\ \vdots \\ b_M(t-M) \end{bmatrix}.$$

In this manner, 1D absorption compensation is formulated as an inverse problem. The model which is related to the minimum of the cost function is the sparse reflectivity function we desire. The Cauchy-Gauss model has also been used in acoustic impedance inversion, signal interpolation, and extrapolation (Sacchi et al., 1998).

Implementation

The solution of the inverse problem, formulated above, follows the algorithm outlined by Sacchi et al. (1998). The objective function that will be minimized is $\phi_{cg}(\mathbf{m})$ in equation 8. Taking the derivative of $\phi_{cg}(\mathbf{m})$ with respect to $\mathbf{m}$, we obtain

$$\frac{\partial \phi_{cg}(\mathbf{m})}{\partial \mathbf{m}} = \frac{\partial c(\mathbf{m})}{\partial \mathbf{m}} + \frac{1}{2\sigma_{\mathbf{n}}^2}\mathbf{G}^T(\mathbf{d} - \mathbf{Gm}) \tag{9}$$

and

$$\frac{\partial c(\mathbf{m})}{\partial \mathbf{m}} = \frac{1}{\sigma_{\mathbf{m}}^2}\mathbf{S}^{-1}\mathbf{m},$$

where $\mathbf{S}$ is a $M\times M$ diagonal matrix with elements $s_{kk} = 1 + m_k^2/\sigma_{\mathbf{m}}$, $(k = 0,1,\ldots,M-1)$ and is $\mathbf{m}$ dependent. Equating the derivative in equation 9 to zero, yields

$$\mathbf{m} = (\lambda\mathbf{S}^{-1} + \mathbf{G}^T\mathbf{G})\mathbf{G}^T\mathbf{d}, \tag{10}$$

where $\lambda = 2\sigma_{\mathbf{n}}^2/\mathbf{m}^2$. This equation is nonlinear and must be solved iteratively. To construct an iterative algorithm, equation 10 is written as

$$\mathbf{m} = \mathbf{SG}^T(\lambda\mathbf{I}_N + \mathbf{GSG}^T)^{-1}\mathbf{d} = \mathbf{SG}^T\mathbf{p}. \tag{11}$$

The auxiliary vector $\mathbf{p}$ is obtained from the solution of the system

$$(\lambda\mathbf{I}_N + \mathbf{GSG}^T)\mathbf{p} = \mathbf{d}. \tag{12}$$

The iterative computation is initiated by setting the observed seismic data as the initial model, $\mathbf{m}^0$, which is also used to generate the matrix $\mathbf{S}^0$. In each iteration, k, we first compute

$$\mathbf{p}^{(k-1)} = [\lambda\mathbf{I}_N + \mathbf{GS}^{(k-1)}\mathbf{G}^T]^{-1}\mathbf{m},$$

and then update the model as

$$\mathbf{m}^{(k)} = \mathbf{S}^{(k-1)}\mathbf{G}^T\mathbf{p}^{(k-1)}.$$

The algorithm, although iterative, is computationally efficient because the coefficient matrix in equation 12 is Hermitian-Toeplitz and is inverted using the Levinson recursion.

The processing flow for absorption compensation on a stacked seismic section is implemented in the following five steps:

1) extract Q values by means of the WTVS approach;
2) apply the required dispersive phase correction;
3) use a given wavelet or extract a minimum phase wavelet from the early part of the trace;
4) construct the kernel matrix $\mathbf{G}$;
5) solve the inverse problem to obtain the reflectivity.

Wavelet and Q estimation need not necessarily be performed at every CDP location. Indeed, Q values need only be estimated for selected CDPs and interpolated to form a Q profile.

Computations required, other than those associated with the iterative algorithm, are trivial if both Q and the seismic wavelet do not change dramatically along the CDP direction, and the resulting LSQ scheme for seismic absorption compensation is computationally practical.

Numerical experiments

This section illustrates results obtained using the LSQ approach on both synthetic and real data. The first test is based on the assumption of known wavelet and Q. A 20% random noise has been added to the trace. Figure 2 shows the convergence of the inversion. Figure 2e is the input trace, and Figure 2a-d are the deabsorption results after one, two, three, and four iterations, respectively. After two or three iterations, the inverted reflectivities are very close to those in the actual model, both as far as locations and amplitudes are concerned. This result shows that the Cauchy-Gauss objective function is a very viable a priori model for obtaining accurate reflectivity inversion.

Figure 3a illustrates a stacked seismic section. The structures are not complex and consist of several flat layers. Processing of the section is initiated by first estimating the Q curve from a trace in the section using the WTVS method. Second, using these extracted Qs, the dispersive phase correction is computed and is followed by the estimation of the minimum-phase wavelet from the early times of a trace. The next step involves computations yielding the kernel matrix **G**. The inversion begins by setting a value for the parameter λ in equation 12 and the result is updated iteratively. Following inversion, the wavelet is convolved with the extracted reflectivities. The final results are shown in Figure 3b. A number of reflections, which are not separated in the section shown in Figure 3a can be observed around time $t = 920$ ms after deabsorption and the lateral continuity can be tracked from trace to trace. The improvement in resolution is clear. The deabsorption result would certainly facilitate the interpretation of this seismic section.

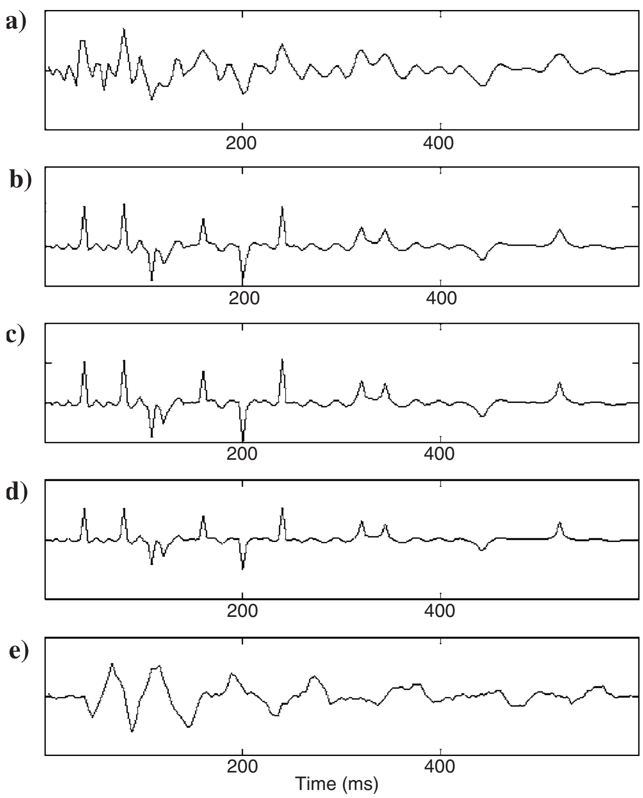

Figure 2. Convergence process of the inverse scheme. (e) A synthetic trace. (a-d) are the inverse results after iteration one, two, three, and four, respectively.

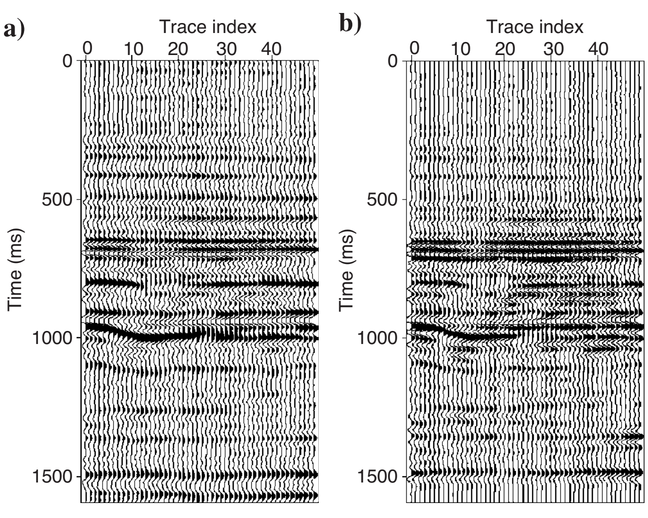

Figure 3. Q compensation for a real seismic section. (a) A real seismic section. (b) The output of absorption compensation.

CONCLUSIONS

The LSQ approach to deabsorption that is presented here differs quite radically from the deconvolution techniques in customary use. The main difference is that the inverse filter is designed using a Bayesian inference approach and is robust with respect to additive noise. The technique described by us has very general application. Specifically, because robust Q compensation provides more accurate information concerning both the amplitude and location of the earth's reflectivity, hydrocarbon reservoir characterization is one obvious target for our method. In particular, the resulting improved resolution may have important consequences in 4D processing where the objective is to delineate changes in reservoir properties.

Results that we have obtained on synthetic and real data confirm, we believe, both the sparse assumption that we have made, and the viability of the Cauchy-Gauss prior model that is used to impose it. By taking the advantage of the Hermitian-Toeplitz property of the kernel matrix that describes the forward problem, stable inverse solutions are achieved economically. The synthetic and real data results demonstrate the viability of the approach and the pivotal role of deabsorption in improving the resolution of the processed sections.

Our LSQ Q compensation is implemented in a trace by trace manner and does not take into account lateral continuity or the actual raypaths that are involved. Multichannel Q compensation and poststack time migration with Q compensation are interesting and important topics which are under investigation.

ACKNOWLEDGMENTS

We thank the editors and the reviewers for their opinions to improve this paper and the related research. We thank Tury Taner for being Tury Taner and aiding us so. We also thank NSERC and the sponsors of CDSST for support and funding. The work presented in this paper was carried out in Zhang's Ph.D. program at the University of British Columbia.

REFERENCES

Aki, K., and P. G. Richards, 2002, Quantitative seismology: University Science Books.
Bickel, S. H., 1993, Similarity and the inverse Q filter: The pareto-levy stretch: Geophysics, **58**, 1629–1633.
Brillouin, L., 1960, Wave propagation and group velocity: Academic Press.
Candès, E., J. Romberg, and T. Tao, 2005, Stable signal recovery from incomplete and inaccurate measurements: Communications on Pure and Applied Mathematics, **59**, 1207–1223.
Claerbout, J. F., 1992, Imaging the earth's interior: Blackwell Scientific.
Futterman, W. I., 1962, Dispersive body waves: Journal of Geophysical Research, **67**, 5279–5291.
Hale, D., 1991, Stable explicit depth extrapolation of seismic wavefields: Geophysics, **56**, 1770–1777.
Hargreaves, N. D., and A. J. Calvert, 1991, Inverse Q-filtering by fourier transform: Geophysics, **56**, 519–527.
Herrmann, F. J., 2005, Robust curvelet-domain data continuation with sparseness constraints, 67th Annual Conference and Exhibition, EAGE, Extended Abstracts, 4248–4251.
Jaynes, E. T., 2004, Probability theory: The logic of science: Cambridge University Press.
Kjartansson, E., 1992, Constant Q wave propagation and attenuation: Journal of Geophysical Research, **84**, 4737–4748.
Margrave, G. F., D. C. Henley, D. C. Lamoureux, M. P. Iliescu, and J. P. Grossman, 2003, Gabor deconvolution revisited: 73rd Annual International Meeting, SEG, Expanded Abstracts, 714–717.

McGinn, A., and B. Duijndam, 1998, Land seismic data quality improvements: The Leading Edge, **17**, 1570–1577.
Robinson, E. A., and S. Treitel, 2002, Geophysical signal analysis: SEG.
Robinson, J. C., 1979, A technique for the continuous representation of dispersion in seismic data: Geophysics, **44**, 1345–1351.
Sacchi, M. D., T. J. Ulrych, and C. J. Walker, 1998, Fourier transform estimation and data extrapolation using prior information: IEEE Transactions on Signal Processing, **46**, 31–38.
Taner, T., and S. Treital, 2003, A robust method for Q estimation: 73rd Annual International Meeting, SEG, Expanded Abstracts, 710–713.
Tarantola, A., 1987, Inverse problem theory: Methods for data fitting and model parameter estimation: Elsevier Science Publ. Co., Inc.
Tonn, R., 1991, The determination of the seismic quality factor *Q* from VSP data: A comparison of different computational methods: Geophysical Prospecting, **39**, 1–27.
Ulrych, T. J., and M. D. Sacchi, 2005, Information-based inversion and processing with applications: Elsevier Science Publ. Co., Inc.
Varela, C. L., A. L. Romanelli, and T. J. Ulrych, 1993, Modeling of attenuation and dispersion: Geophysics, **58**, 1167–1173.
Wang, Y., 2002, A stable and efficient approach to inverse *Q* filtering: Geophysics, **67**, 657–663.
———, 2003, Quantifying the effectiveness of stabilized inverse *Q* filtering: Geophysics, **68**, 337–345.
———, 2006, Inverse *Q*-filter for seismic resolution enhancement: Geophysics, **71**, no. 3, V51–V60.
Wang, Y., and J. Guo, 2004, Modified kolsky model for seismic attenuation and dispersion: Journal of Geophysics and Engineering, **1**, 187–196.
Zhang, C., and T. J. Ulrych, 2002, Estimation of quality factors from CMP records: Geophysics, **67**, 1542–1547.