

Seismic attenuation compensation by Bayesian inversion



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ARTICLE INFO

Article history:

Received 20 January 2014

Accepted 16 October 2014

Available online 25 October 2014

Keywords:

Attenuation compensation

Bayesian inversion

Q estimation

Prior information

ABSTRACT

As an effective method to improve seismic data resolution, attenuation compensation has been paid great attention. The popular method inverse Q-filter performs effectively in phase correction. But its energy compensation part is at an extraordinary discount because of its instability. By contrast, the compensation method based on inversion has a great advantage in algorithm stability. In this paper, the inversion process is combined with Bayesian principle so that the prior information we learned about the actual model can be used sufficiently. Here, we have an assumption that it is more reasonable to describe the reflectivities with sparse distribution. This information, in general, can be transferred to a sparse constraint of the object function. And compared with Tikhonov regularization method, it is proved to perform better in seismic resolution improvement. Meanwhile, it is insensitive to the error of Q value. Example of real data shows the validity of the method.

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1. Introduction

Seismic wave experiences energy attenuation and velocity dispersion while propagating through the subsurface medium. These properties are proved to be two fundamental factors of seismic resolution reducing. To represent the absorption issue mathematically, Futterman (1962) derived a dispersion relationship by assuming that the Q model is frequency independent. This model obtained widespread applications in the researches of seismic compensation. And it is used as the attenuation model in this paper.

To improve the resolution of seismic data, attenuation compensation must be taken into consideration. Many experts engaged themselves in compensation problems. Hargreaves et al. (1991) propose an approach similar to Stolt migration by regarding the compensation process as a kind of inverse Q-filter. The revise for energy absorption and phase distortion can be performed with the theory of wave-field downward continuation. With this method, the phase distortion can be corrected efficiently, but amplitude compensation is ignored because of its instability. Wang (2002, 2003) points out that stability and efficiency are two general concerns of compensation problem. He introduces a stable factor into inverse Q-filter to control algorithm stability. And considering the computational efficiency, the compensation process is achieved in two steps. The wave-field of the surface record is first extrapolated to the top of the current layer. And then constant Q inverse filtering was applied in each layer. Based on this method, Zhang et al. (2007) extract the gain-constraint frequencies from the Gabor spectrum to deduce the gain-constraint

amplitude. Yan and Liu (2009) implemented the inverse Q-filtering on pre-stack common shot PP- and PS-waves along the ray path. In these methods, however, the amplitude compensation is suppressed by the stable factor. Absolutely, the record of deep reflection cannot be compensated well. Zhang and Ulrych (2007) regard the deabsorption process as a time-variant deconvolution and performed it with least squares inversion which can compensate the seismic amplitude better. This method was implemented in time domain. Based on exploding reflector idea, Wang (2011) reduces the compensation problem to an inversion problem and achieves it by Tikhonov regularization. As the compensation process is performed in frequency domain, we can choose the frequency band that involved in the inversion. So this method is useful for the data with high frequency or monofrequent noises. Tests on this method show its stability in amplitude compensation. Based on this theory, we have further research and get some improvement on the method.

As inversion problems are always undetermined, we need a rule to help us to choose a proper answer from the large amount of solutions. Tikhonov regularization added a smoothness constraint to the objective function to improve the inversion stability. However, it will reduce the seismic resolution, especially for the data with high dominant frequency and low signal noise ratio (SNR). Here, we choose Bayesian method to solve this inversion problem. The Bayesian theory provides us a framework to combine the priori model information with the information contained in the data and build a more refined inversion function. To describe the sparseness of the reflectivities, we suggest that it follows Laplace distribution. Then with Bayesian inversion method, the seismic resolution can be improved sufficiently, and this method is insensitive to the error of the Q value.

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2. The theory of compensation based on inversion

Wang (2011) proposed a compensation method which attributes the deabsorption problem to an inversion issue. He states the thought and deductive procedure in detail. Now, we just have the key steps being discussed.

Suppose that the seismic wavelet is $w(t)$:

$$w(t) = \int_{-\infty}^{\infty} \hat{w}(\omega) e^{i\omega t} d\omega. \quad (1)$$

$\hat{w}(\omega)$ is the frequency spectrum. This equation means that the seismic wavelet can be regarded as the superposition of a series of simple harmonic waves $\hat{w}(\omega) e^{i\omega t}$, in which ω is the frequency of the harmonic wave.

With the theory of exploding reflector, the zero-offset synthetic record can be expressed as:

$$s(t) = \int d\omega \int r(t') \hat{w}(\omega) e^{i\omega t} e^{-i\omega t'} dt', \quad (2)$$

in which $r(t')$ is the reflector coefficient of the model.

Then, using Futterman model, Wang (2011) gets the absorbed record:

$$s(t) = \int d\omega \int r(t') \hat{w}(\omega) e^{i\omega t} e^{-i\omega t' \left| \frac{\omega_0}{\omega} \right|^\gamma} e^{-\omega t' \left| \frac{\omega_0}{\omega} \right|^\gamma \frac{1}{2Q(r')}} dt'. \quad (3)$$

where $\gamma \approx \frac{1}{\pi Q}$, $Q(t')$ is the Q-factor of the medium, and ω_0 is the reference frequency which is always regarded as the dominant frequency.

However, the derivation by Wang (2011) doesn't take random noises into consideration. To close to the real data, we represent the data as:

$$s(t) = \int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} r(t') \hat{w}(\omega) e^{i\omega t} e^{-i\omega t' \left| \frac{\omega_0}{\omega} \right|^\gamma} e^{-\omega t' \left| \frac{\omega_0}{\omega} \right|^\gamma \frac{1}{2Q(r')}} dt' + n(t) \quad (4)$$

in which $n(t)$ is the environment noises.

By Fourier transformation, $n(t)$ can be rewritten as:

$$n(t) = \int_{-\infty}^{+\infty} n(\omega) e^{i\omega t} d\omega, \quad (5)$$

where $n(\omega)$ is the frequency spectrum of the environment noises.

Replace $n(t)$ in Eq. (4) with (5):

$$s(t) = \int_{-\infty}^{\infty} e^{i\omega t} \left(\int_{-\infty}^{\infty} r(t') \hat{w}(\omega) e^{-i\omega t' \left| \frac{\omega_0}{\omega} \right|^\gamma} e^{-\omega t' \left| \frac{\omega_0}{\omega} \right|^\gamma \frac{1}{2Q(r')}} dt' + n(\omega) \right) d\omega \quad (6)$$

Then the spectrum of the attenuated data is like:

$$\hat{s}(\omega) = \hat{w}(\omega) \int_{-\infty}^{\infty} r(t') e^{-i\omega t' \left| \frac{\omega_0}{\omega} \right|^\gamma} e^{-\omega t' \left| \frac{\omega_0}{\omega} \right|^\gamma \frac{1}{2Q(r')}} dt' + \hat{n}(\omega). \quad (7)$$

Suppose the data for compensation is deconvolved, the frequency spectrum can be reduced to:

$$\hat{s}(\omega) = \int_{-\infty}^{\infty} r(t') e^{-i\omega t' \left| \frac{\omega_0}{\omega} \right|^\gamma} e^{-\omega t' \left| \frac{\omega_0}{\omega} \right|^\gamma \frac{1}{2Q(r')}} dt' + \hat{n}(\omega) \quad (8)$$

where $\hat{n}(\omega) = \frac{n(\omega)}{\hat{w}(\omega)}$.

Based on Eq. (8), we define

$$g(\omega, t') = e^{-i\omega t' \left| \frac{\omega_0}{\omega} \right|^\gamma} e^{-\omega t' \left| \frac{\omega_0}{\omega} \right|^\gamma \frac{1}{2Q(r')}}. \quad (9)$$

Then Eq. (8) can be rewritten as:

$$\hat{s}(\omega) = \int g(\omega, t') r(t') dt' + \hat{n}(\omega) \quad (10)$$

For each separated frequency ω_i , we rewrite the equation in discrete form:

$$\hat{s}(\omega_i) = \sum_j g(\omega_i, t'_j) r(t'_j) \Delta t' + \hat{n}(\omega_i); \quad i = 1, 2, 3 \dots N \quad (11)$$

$t'_j (j = 1 : M)$ is the sampling time of the model and $\Delta t'$ indicates the time sampling interval.

The process of attenuation compensation is to obtain the reflector coefficient $r(t')$ from the frequency spectrum $\hat{s}(\omega)$. It is an inversion problem in some extent, and the kernel matrix is $\mathbf{G}_{ij} = g(\omega_i, t'_j) \Delta t'$.

Then we can find something in common between the theory mentioned above and the deabsorption method proposed by Zhang and Ulrych (2007). The similarity is that both methods concludes the compensation issue to an inversion problem while they have a big difference. Zhang did the seismic compensation in time domain and its kernel matrix \mathbf{G} is composed of time-variant wavelets with amplitude attenuation only. And the required dispersive phase correction is applied on the traces before amplitude compensation. In our method, the kernel matrix \mathbf{G} takes both energy absorption and frequency dispersion into account. Phase correction and energy compensation are done in one step. In addition, it is implemented in frequency domain which is convenient for us to choose the frequency band with higher SNR.

3. The Bayesian inversion

In order to discuss the inversion problem, we represent the solution (the sparse reflectivities) with vector \mathbf{m} and the observed data with \mathbf{d} . The data \mathbf{d} is not the right response of vector \mathbf{m} because the seismic data is mixed with noises. Meanwhile, the observed data which is discrete can never describe the continuous model completely. All these factors lead to the fact that infinity models can be found to fit the data.

As the discrete frequency spectrum of absorbed data is like:

$$\hat{s}(\omega_i) = \sum_j g(\omega_i, t'_j) r(t'_j) \Delta t' + \hat{n}(\omega_i) \quad (12)$$

Let $g_{ij} = g(\omega_i, t'_j) \Delta t'$, Eq. (12) can be rewritten as:

$$\begin{pmatrix} \hat{s}_1 \\ \hat{s}_2 \\ \vdots \\ \hat{s}_N \end{pmatrix} = \begin{pmatrix} g_{1,1} & g_{1,2} & \dots & g_{1,M} \\ g_{2,1} & g_{2,2} & \dots & g_{2,M} \\ \vdots & \vdots & \ddots & \vdots \\ g_{N,1} & g_{N,2} & \dots & g_{N,M} \end{pmatrix} \begin{pmatrix} r_1 \\ r_2 \\ \vdots \\ r_M \end{pmatrix} + \begin{pmatrix} \hat{n}_1 \\ \hat{n}_2 \\ \vdots \\ \hat{n}_N \end{pmatrix} \quad (13)$$

$\mathbf{d} = (\hat{s}_1, \hat{s}_2, \dots, \hat{s}_N)^T$ is the observed data, $\mathbf{m} = (r_1, r_2, \dots, r_M)^T$ is the solution of the inverse problem, and $\hat{\mathbf{n}} = (\hat{n}_1, \hat{n}_2, \dots, \hat{n}_N)^T$ represents the recorded environment noises, in which \cdot^T indicates the transposition of vector \cdot . $\mathbf{G} = (g_{ij})_{N \times M}$ is the kernel matrix.

Zhang (2009) points out the noises \mathbf{n} is normally distributed as $N(\mu = 0, \sigma_n^2)$, where σ_n^2 is the variance and μ is the mean of the probability density function (PDF) respectively.

The likelihood of the data is given by:

$$p(\mathbf{d}|\mathbf{m}, \sigma_n) = \left(\frac{1}{2\pi\sigma_n^2} \right)^{\frac{N}{2}} e^{-\frac{1}{2\sigma_n^2} \|\mathbf{d} - \mathbf{Gm}\|_2^2}, \quad (14)$$

where N is the length of the data.

Consider now that a prior distribution $p(m)$ is known. According to Bayes's rule, the posterior distribution is given by:

$$p(m|d) = \frac{p(d|m)p(m)}{p(d)} \quad (15)$$

It provides us a frame to combine the priori information with the information contained in the data to produce a more refined model distribution.

$p(d)$, the probability \mathbf{d} has been observed, is a constant which can be ignored in the inversion process. Then the Bayesian equation is:

$$P(m|d) \propto P(d|m) * P(m) \quad (16)$$

The value $p(m)$ deserves special attention. It is the priori information we introduced into the inversion process to reduce the multiplicity of the solutions. For compensation issue, $p(m)$ is the priori probability of the model reflectivities.

Our purpose of data processing is to get the information of the strong reflecting interface. And sparseness means to indicate the data with the least non-zero numbers. Therefore, it is more advisable to describe the model reflectivities with sparse distribution. So in this part, our choice for $p(m)$ is the Laplace distribution which exhibits long tails and can describe the sparseness. During real data processing, we can get the distribution characteristic of reflectivities from the log data and seismic data. This distribution is also applicative in our method.

The PDF of Laplace distribution is

$$p(x) = \frac{1}{\sqrt{2}\sigma} e^{-\frac{\sqrt{2}}{\sigma}|x-\mu|}. \quad (17)$$

Then, we have the priori information like

$$p(\mathbf{m}, \sigma_m) = \left(\frac{1}{2\sigma_m^2}\right)^{\frac{M}{2}} e^{-\frac{\sqrt{2}}{\sigma_m}\|\mathbf{m}\|_1}. \quad (18)$$

M is the dimension of \mathbf{m} . Finally, $p(m|d)$ is the PDF expected. It is the model distribution given that data \mathbf{d} has occurred. It means the probability that each model is correct. With Eq. (16), the posterior distribution is

$$p(\mathbf{m}|\mathbf{d}) \propto e^{-\left(\frac{\|\mathbf{d}-\mathbf{Gm}\|_2^2}{2\sigma_n^2} + \frac{\sqrt{2}\|\mathbf{m}\|_1}{\sigma_m}\right)}. \quad (19)$$

Taking logarithms of both sides, we obtain

$$\ln[p(\mathbf{m}|\mathbf{d})] \propto -\left(\frac{1}{2\sigma_n^2}\|\mathbf{d}-\mathbf{Gm}\|_2^2 + \frac{\sqrt{2}}{\sigma_m}\|\mathbf{m}\|_1\right) \quad (20)$$

$$-2\sigma_n^2 \ln[p(\mathbf{m}|\mathbf{d})] \propto \|\mathbf{d}-\mathbf{Gm}\|_2^2 + \frac{2\sqrt{2}\sigma_n^2}{\sigma_m}\|\mathbf{m}\|_1 \quad (21)$$

Furthermore, denoting the cost function of posterior distribution with $L(\mathbf{m})$:

$$L(\mathbf{m}) = \|\mathbf{d}-\mathbf{Gm}\|_2^2 + \frac{2\sqrt{2}\sigma_n^2}{\sigma_m}\|\mathbf{m}\|_1. \quad (22)$$

Now, re-examining the compensation problem, we intend to get the reflectivities that fit the data best. Absolutely, the best \mathbf{m} means that the one has maximum posterior probability $p(\mathbf{m}|\mathbf{d})$, and also that the one brings $L(\mathbf{m})$ to the least.

4. Implementation

Following the algorithm outlined by Sacchi, (2010), we give the solution of the inverse problem mentioned above.

First, evaluating $\nabla_{\mathbf{m}} L = 0$ leads to

$$\frac{\partial L(\mathbf{m})}{\partial \mathbf{m}} = \mathbf{G}^T(\mathbf{d}-\mathbf{Gm}) - \mu\mathbf{\Omega}(\mathbf{m}) \cdot \mathbf{m} = 0, \quad (23)$$

and $\lambda = \frac{2\sqrt{2}\sigma_n^2}{\sigma_m}$. $\mathbf{\Omega}(\mathbf{m})$ is a matrix that depends on \mathbf{m} with elements $\Omega_{kk} = \frac{1}{|m_k|}$, ($k = 0, 1, \dots, N-1$) and others all zero.

Therefore, we obtain

$$\mathbf{m} = (\mathbf{G}^T\mathbf{G} + \lambda\mathbf{\Omega}(\mathbf{m}))^{-1}\mathbf{G}^T\mathbf{d}. \quad (24)$$

We can use the iterative algorithm to solve this non-linear problem. Starting with $\mathbf{m}^0 = \mathbf{0}$, we can calculate out the model by:

$$\mathbf{m}^k = (\mathbf{G}^T\mathbf{G} + \lambda\mathbf{\Omega}(\mathbf{m}^{k-1}))^{-1}\mathbf{G}^T\mathbf{d}, \quad (25)$$

until $\|\mathbf{d} - \mathbf{Gm}^k\|_2$ is smaller than a given constant ε . The factor λ is variable with the degree of attenuation and the SNR of seismic data.

To sum up, the attenuation compensation process can be implemented as follows: construct the attenuation matrix \mathbf{G} first and then solve the inversion problem with Eq. (25).

Q value is essential to construct matrix \mathbf{G} . When not given, it can be estimated from the post-stack seismic data by frequency shifting method (Zhang and Ulrych (2002)). First, we get the time-frequency spectrum of the chosen trace by Wavelet Transform and then pick up the peak frequency of each energy group. Finally, according to the changing of peak frequency, we can work out the Q value of each layer. For the model with slight lateral variation, the Q value can be extracted from one standard trace and applied to the whole seismic section.

5. Numerical example

This section illustrates the compensation results obtained using Bayesian inversion both on synthetic and real data.

Theoretically, after deconvolution, we can obtain the sparse like reflectivities of the geologic model. However, the deconvoluted traces always contain the information of wavelet and random noises. So we compose the synthetic traces with Rick wavelet (45 Hz) to simulate the deconvoluted ones in the modeling tests. And to be more practical, we add some random noises into the traces.

To demonstrate the effectiveness of this algorithm, we consider the signal to consist of a sequence of Ricker wavelets (45 Hz) with $t = 100, 400, 700, 1000, 1300, 1600$, and 1900 ms respectively. Fig. 1a shows four synthetic traces with different Q values ($Q = 400, 200, 100, 50$) constant with depth in each case, and the SNR of the data is 5. We can see amplitude absorption and phase distortion clearly in the traces. And the smaller the Q value, the more serious the attenuation is. The mission of compensation is to revise these variations so that the seismic traces can describe the model accurately. In this model, the Q value is given, so compensated data can be worked out following the two steps mentioned above. The compensated traces with Bayesian inversion are shown in Fig. 1b. For comparison, we solve the inversion problem by Tikhonov regularization method (Fig. 1c). For traces with $Q = 400$ and 200 , the process restores the wavelet with correct phase and amplitude. However, the amplitude in Fig. 1b is recovered better as the Q value decreases and imaging time increases. Besides, Fig. 1b indicates that the sparse constraint is robust to the random noises.

To contrast the resolution of the compensated data, we calculated the amplitude spectrum of the 4th trace ($Q = 50$) from 1.8 s to 2 s. In Fig. 2, the red line is the spectrum of inversion result by Bayesian inversion and the blue one is the spectrum of the result by Tikhonov

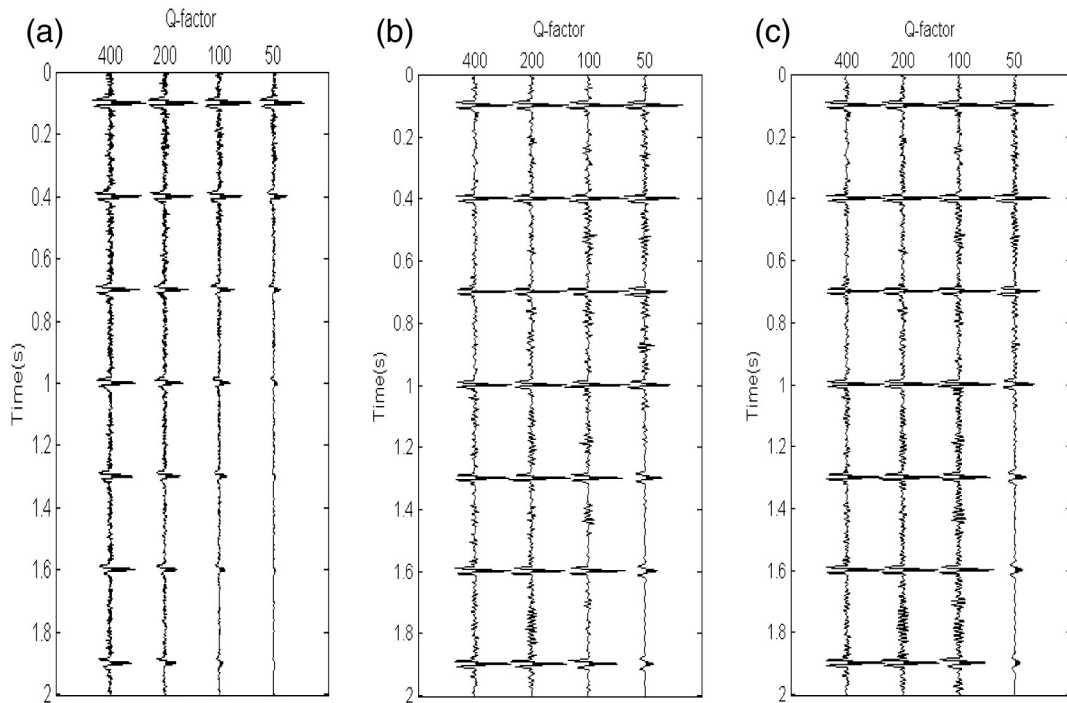


Fig. 1. The earth Q attenuation and the compensation results: (a) synthetic traces show the effect of the earth attenuation with $Q = 400, 200, 100$, and 50 and $SNR = 5$; (b) the compensated data with Bayesian inversion, which clearly indicates the numerical stability; (c) the inversion result by Tikhonov regularization.

regularization inversion. The former one has higher dominant frequency and wider frequency band. It means that the compensation method based on Bayesian inversion performs better in seismic resolution improvement.

Parameter choosing is of great importance for this method. Parameter ε chosen is 10^{-3} for the data in Fig. 1a. To prevent the inversion process from dropping into endless circles, we put an interaction number in it. This number depends on the size of the imaging problem. For this data, the results reached converge after 3 times, so the interaction number we choose is 5.

In the inversion process, parameter λ indicates the weight of priori information. To learn the impact of λ on the inversion result, we have a test on the 3rd trace ($Q = 100$) in Fig. 1a. Given different λ (various from 5 to 0.005), the compensated results are shown in Fig. 3. The larger λ , the higher SNR of the compensated data. But the amplitude of the deep layer cannot be compensated well.

For seismic compensation, our criterion for λ choosing is to ensure fine compensation effect as well as high SNR of the deabsorbed result. The λ values we choose for the data in Fig. 1a are 0.005, 0.01, 0.05, and 0.1 respectively and the compensated trace is shown in Fig. 1b. The λ value is rising with the Q decrease. Besides, λ is also influenced by the SNR of data. Similarity, for the 3rd trace ($Q = 100$) in Fig. 1a, the SNR of the data is 5. We can change the noise level in this trace to

get seismic data with different SNR. The SNR of the traces shown in Fig. 4a is 8, 5, 3, and 2 respectively. The proper λ for this data is 0.001, 0.05, 0.1, and 0.5. The compensated traces are shown in Fig. 4b. For the data with low SNR, we should choose larger λ to control the noise level of the result. Meanwhile, amplitude compensation is suppressed in some extent.

To visualize the resolution improvement in time domain, we test this method by thin interbed model. It has two separated interfaces at 0.4 s and 1.0 s and three adjacent interfaces at 1.566 s, 1.600 s, and 1.634 s respectively. The reflectivities of the five interface are 1, -1 , 0.7, and -1 , 0.7 and the Q value of this model is 50. Fig. 5b is the synthetic data without absorption. The absorbed trace is shown in Fig. 5a ($SNR = 5$). The reflection wavelets interfere with each other between 1.5 s and 1.65 s. Fig. 5c is the compensation result with Bayesian inversion in which the adjacent reflectivities can be separated clearly. For comparison, the inversion result by Tikhonov regularization cannot reflect the right location of the interfaces. This test indicates that the compensation result with Bayesian inversion has higher resolution.

In seismic data processing, the Q value we have known is always inaccurate. So it is important to test the algorithm sensitivity to the error of Q factor. In the following part, we design a model with five different reflectivities ($r = [0.1, 0.3, -0.4, 0.2, -0.3]$) and layered Q value

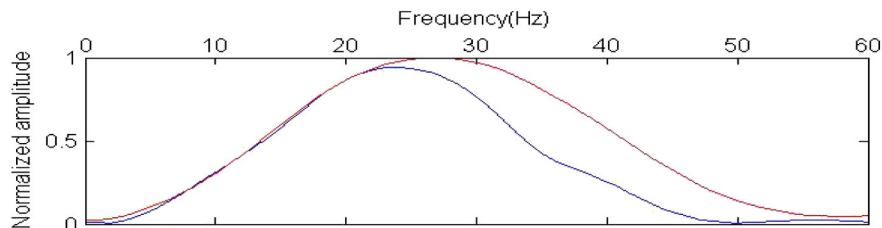


Fig. 2. The amplitude spectrum of the seismic data from 1.8 s to 2.0 s with a Q value 50: the spectrum of Bayesian inversion result is in red and the one of Tikhonov regularization in blue.

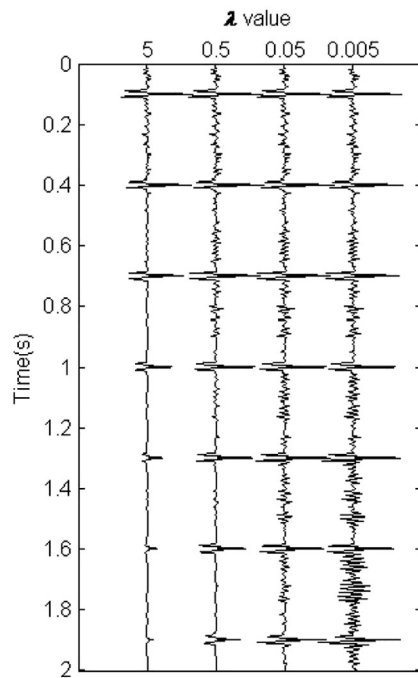


Fig. 3. The compensation result with various λ (5–0.005): the amplitude compensation will be suppressed when λ is large enough and the smaller λ will result in low SNR of the compensated trace.

($Q = [70, 100, 30, 120, 150, 150]$). Fig. 6a shows the synthetic record of the given model with SNR 5. Fig. 6b is the wavelet spectrum of the trace from which the peak frequency shifting can be extracted. Then with peak frequency shifting method, we can calculate the Q value of the layered model (the red line in Fig. 6c) and the real one is in blue. Although the extracted Q is imprecise, we find that our method is insensitive to its error. Fig. 7a is the attenuated data of the layered model. And the

compensated data by Bayesian inversion and Tikhonov regularization method is shown in Fig. 7b and c respectively. For comparison, the data without attenuation is displayed in Fig. 7d. Apparently, the seismic amplitude in Fig. 7b can describe the reflectance well although the Q value is not precise. This test indicates that the method is insensitive to the error of the Q value.

At last, we apply the compensation method in real data to verify its validity. Fig. 9a illustrates a post-stack seismic section. First we begin the compensation process by estimating Q value from the 10th trace (Fig. 8a). Then the spectrum of the trace and the extracted Q value is shown in Fig. 8b and c respectively. The compensation result with the Q value extracted is shown in Fig. 9b. The comparison of two sections indicates a clear improvement in seismic resolution of the compensated section. The events at 0.2 s and 0.5 s are clearer and more continuous from trace to trace. Besides, the reflections between 0.6 s and 0.65 s are separated, and it is the same for the events between 0.8 s and 0.9 s. Absolutely, the deabsorption result will have a great facilitation to the seismic interpretation.

6. Conclusion

The compensation method based on inversion differs quite radically from other methods. It attributes the compensation issue to an inversion problem and performs well in amplitude compensation. But ambiguity and instability would be two major problems in inversion process. To solve them, we choose Bayesian inversion, because it improves the inversion stability by the use of priori information. Here, assumption is made that the reflectivities follow sparse distribution. Compared to Tikhonov regularization, this method shows great advantage in amplitude compensation and is robust with the respect to random noise. Meanwhile, the tests indicate its advantage in improving seismic resolution. For the choice of parameter λ , our criterion is to ensure fine compensation effect as well as high SNR. The value of λ is influenced by the SNR of the input data. It is always larger for the data with lower SNR. The example of layered Q model demonstrates that this method is insensitive to the error of Q value. This point indicates the potential applications of this method in complex subsurface structures, because

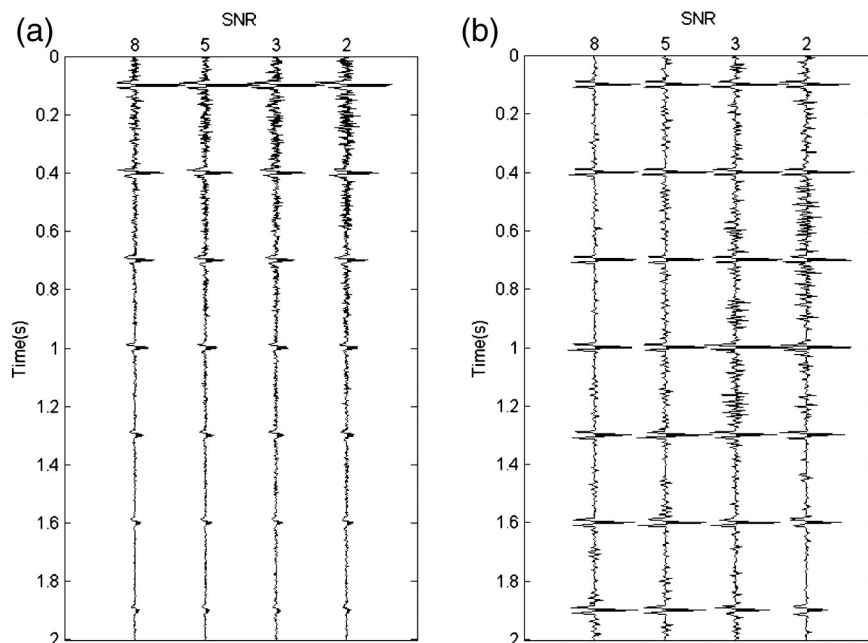


Fig. 4. The attenuated data with different SNR and the compensated results: (a) attenuated traces ($Q = 100$) show the effect of the random noises with SNR = 8, 5, 3, 2; (b) the compensated result with proper λ value for each trace.

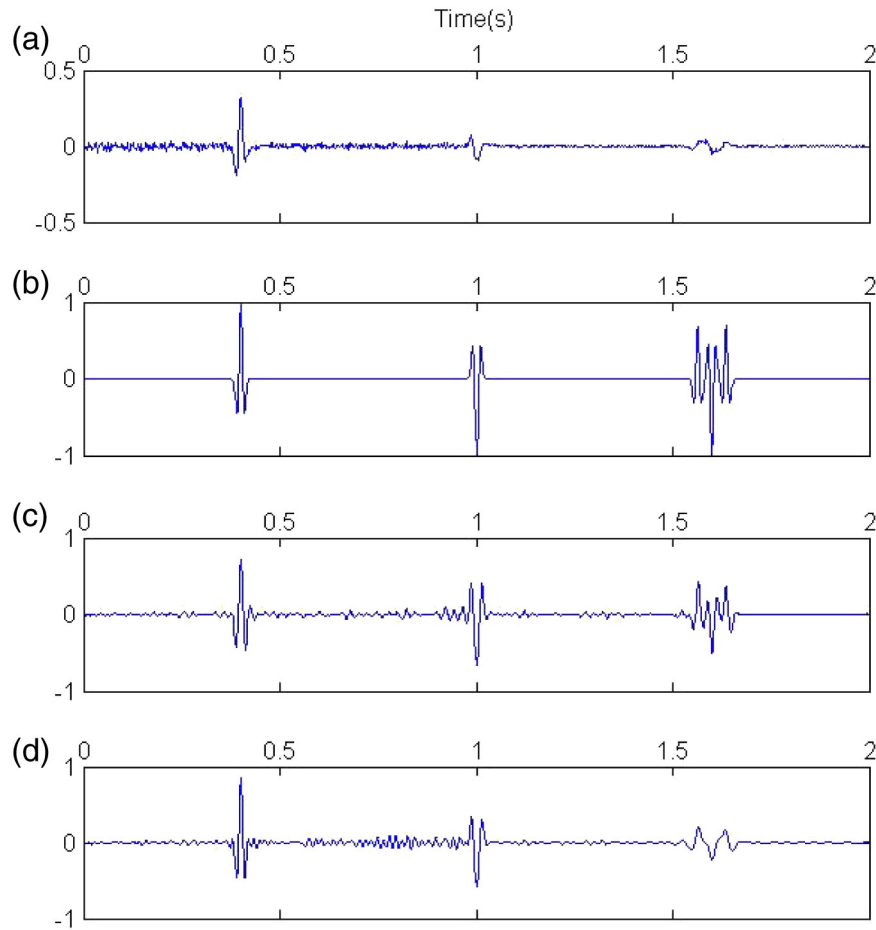


Fig. 5. Compensation method test on thin interbed model: (a) synthetic traces show the effect of the earth attenuation ($Q = 50$) on adjacent reflectivities which interfere with each other; (b) the trace without absorption; (c) compensated trace with Bayesian inversion in which the adjacent reflectivities can be separated clearly; (d) compensation result with Tikhonov regularization method.

the extracted Q value is always imprecise. Results obtained on real data confirm the usability of the proposed method. Since the inversion process is computationally expensive, this compensation method has

more calculation than the method based on wave-field extrapolation. Besides, it indicates that attenuation compensation plays a significant role in seismic resolution improvement.

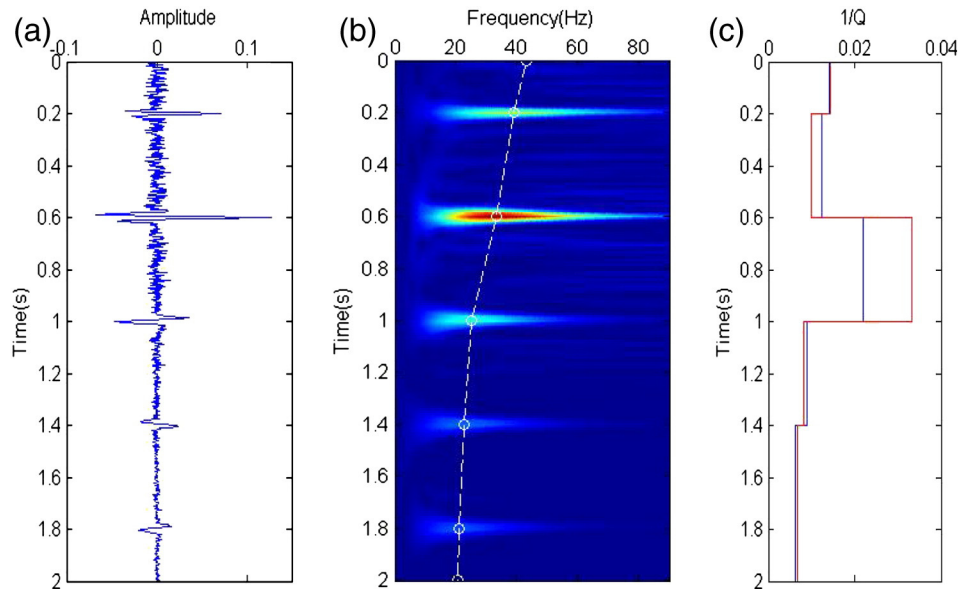


Fig. 6. Q estimation by peak frequency shifting method: (a) attenuated synthetic trace with layered Q model and the SNR of the data is 5; (b) wavelet spectrum of the trace which indicates the peak frequency shifting; (c) the comparison between the estimated Q (red) and the real one (blue).

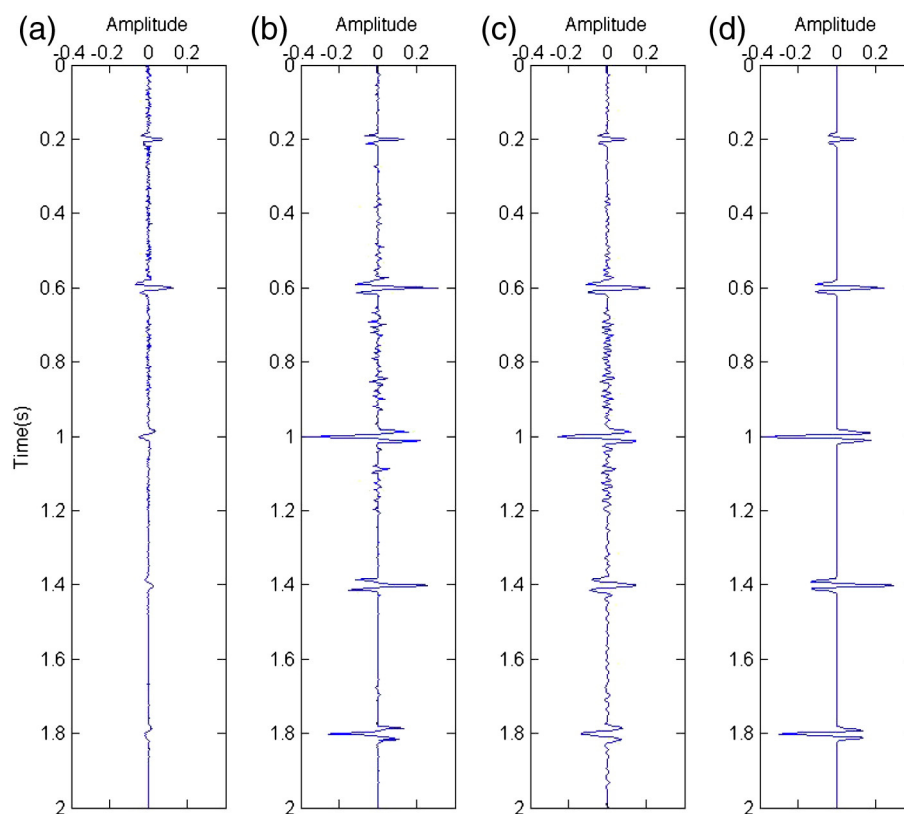


Fig. 7. Test of algorithm sensitivity to the error of Q factor: (a) synthetic traces with layered Q attenuation (SNR = 5); (b) compensated data with Bayesian inversion using the Q value extracted and this result indicates that the method is insensitive to the error of the Q value; (c) inversion result with Tikhonov regularization; (d) seismic data without attenuation and random noise.

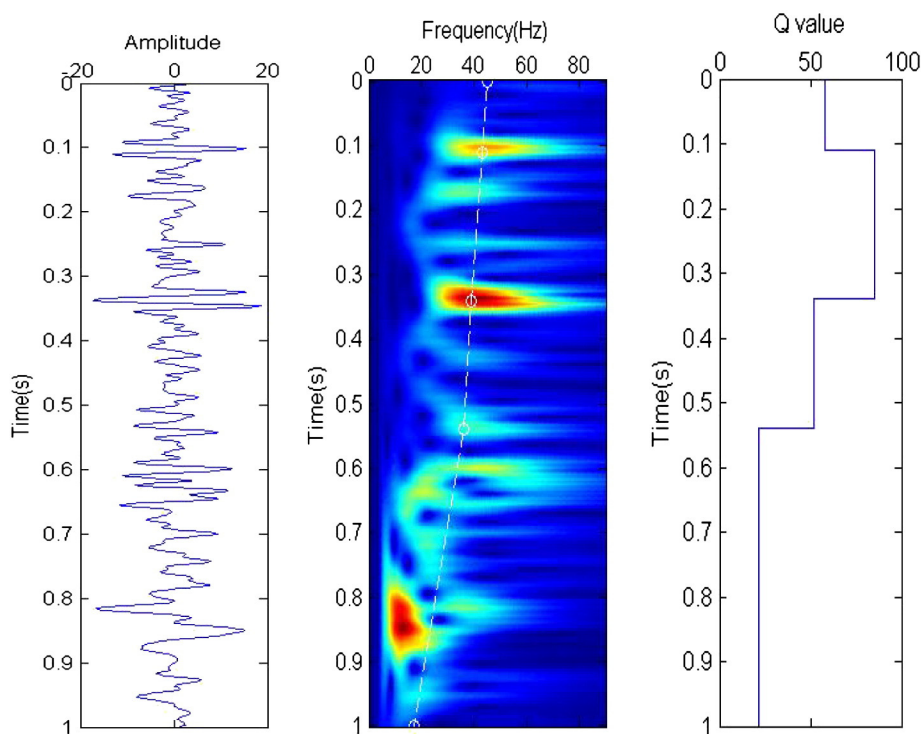


Fig. 8. Q estimation from the real data: (a) the 10th trace of the real data which is used for Q extraction; (b) the wavelet spectrum of the trace from which we can extract the changing peak frequency; (c) the extracted Q value.

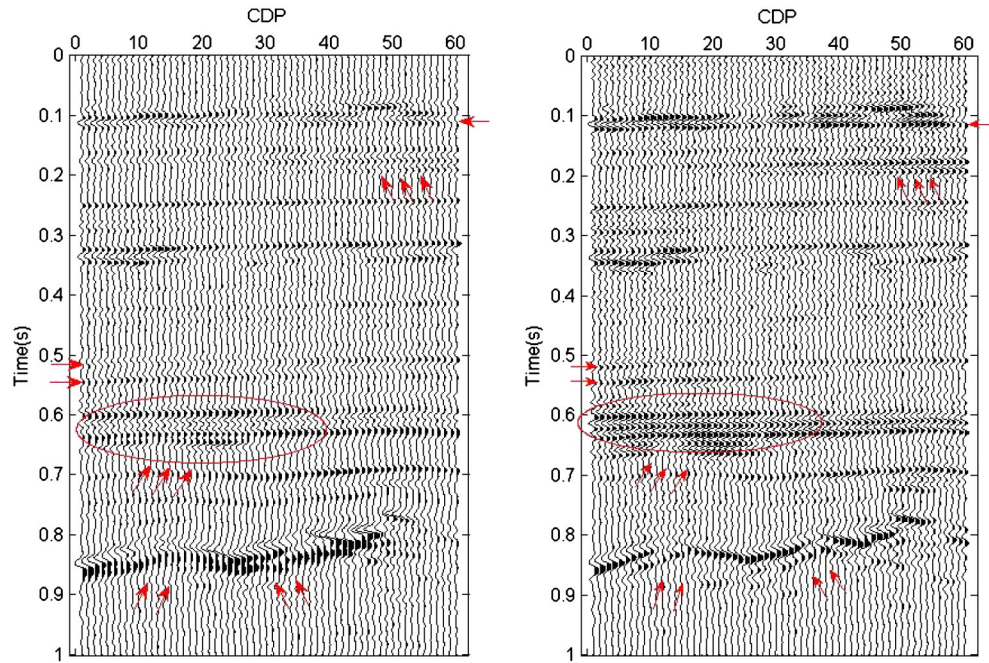


Fig. 9. The application of the compensation method by Bayesian inversion on real data: (a) the real data with 60 traces; (b) compensated section of the real data (a) which clearly indicates the seismic resolution improvement.

Acknowledgment

The work is financially supported by the National Science and Technology Major Project of China (2011ZX05023-005-005) and the Natural Science Foundation of China (41274137).

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