



Loop 5	<b>for</b> $j_c=0 : n-1$ <b>steps of</b> $n_c$
	$\mathcal{J}_c = j_c : j_c + n_c - 1$
Loop 4	<b>for</b> $p_c=0 : k-1$ <b>steps of</b> $k_c$
	$\mathcal{P}_c = p_c : p_c + k_c - 1$
	$B(\mathcal{P}_c, \mathcal{J}_c) \rightarrow B_c$ // Pack into $B_c$
Loop 3	<b>for</b> $i_c=0 : m-1$ <b>steps of</b> $m_c$
	$\mathcal{I}_c = i_c : i_c + m_c - 1$
	$A(\mathcal{I}_c, \mathcal{P}_c) \rightarrow A_c$ // Pack into $A_c$
	// Macro-kernel
Loop 2	<b>for</b> $j_r=0 : n_c-1$ <b>steps of</b> $n_r$
	$\mathcal{J}_r = j_r : j_r + n_r - 1$
Loop 1	<b>for</b> $i_r=0 : m_c-1$ <b>steps of</b> $m_r$
	$\mathcal{I}_r = i_r : i_r + m_r - 1$
	// Micro-kernel
Loop 0	<b>for</b> $k_r=0 : k_c-1$
	$C_c(\mathcal{I}_r, \mathcal{J}_r)$
	$\quad\quad\quad += A_c(\mathcal{I}_r, k_r) \quad B_c(k_r, \mathcal{J}_r)$
	<b>endfor</b>
	<b>endfor</b>
	<b>endfor</b>
	<b>endfor</b>

Figure 1: Left: The Goto algorithm for matrix-matrix multiplication as refactored in BLIS. Right: the same algorithm, but expressed as loops.

## An Exercise in Optimizing Matrix-Matrix Multiplication

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Draft  
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## Abstract

Abstract.

## 1 Introduction

## 2 The Goto Approach to Implementing GEMM

$$\begin{bmatrix} 1 \\ 5 \\ 4 \\ 2 \\ 3 \end{bmatrix}$$

### 3 Conclusion

Conclusion.

#### Additional information

For additional information on FLAME visit

<http://www.cs.utexas.edu/users/flame/>.

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#### References

- [1] Kazushige Goto and Robert A. van de Geijn. Anatomy of a high-performance matrix multiplication. *ACM Trans. Math. Soft.*, 34(3):12, May 2008. Article 12, 25 pages.
- [2] Tyler M. Smith, Robert van de Geijn, Mikhail Smelyanskiy, Jeff R. Hammond, , and Field G. Van Zee. Anatomy of high-performance many-threaded matrix multiplication. In *International Parallel and Distributed Processing Symposium 2014*, 2014.
- [3] Tze Meng Low, Francisco D. Igual, Tyler M. Smith, and Enrique S. Quintana-Ortí. Analytical modeling is enough for high performance blis. *ACM Transactions on Mathematical Software*. in review.
- [4] Field G. Van Zee, Tyler Smith, Bryan Marker, Tze Meng Low, Robert A. van de Geijn, Francisco D. Igual, Mikhail Smelyanskiy, Xianyi Zhang, Michael Kistler, Vernon Austel, John Gunnels, and Lee Killough. The blis framework: Experiments in portability. *ACM Transactions on Mathematical Software*. to appear.
- [5] Field G. Van Zee and Robert A. van de Geijn. BLIS: A framework for rapidly instantiating blas functionality (replicated computational results certified). *ACM Trans. Math. Soft.*, 41(3):14:1–14:33, June 2015.