Spatial autocorrelation in ecological modelling

Workshop: Symposium for European Freshwater Sciences 2015

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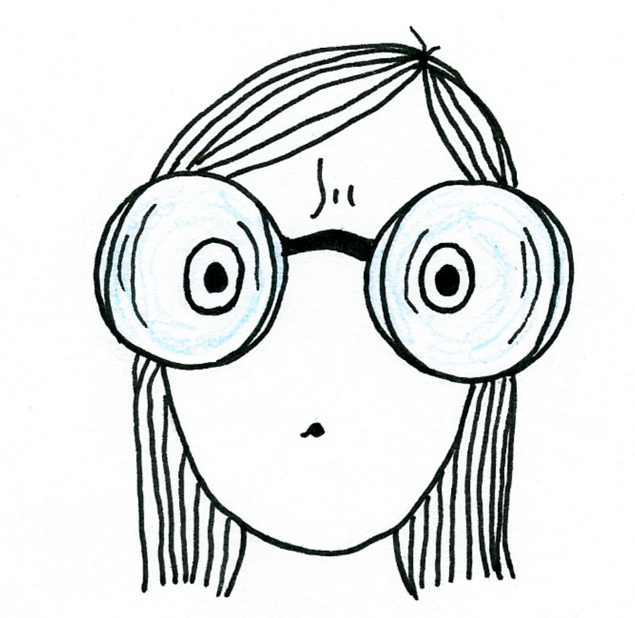
We need spatially explicit models. Why?

Wear the GI glasses



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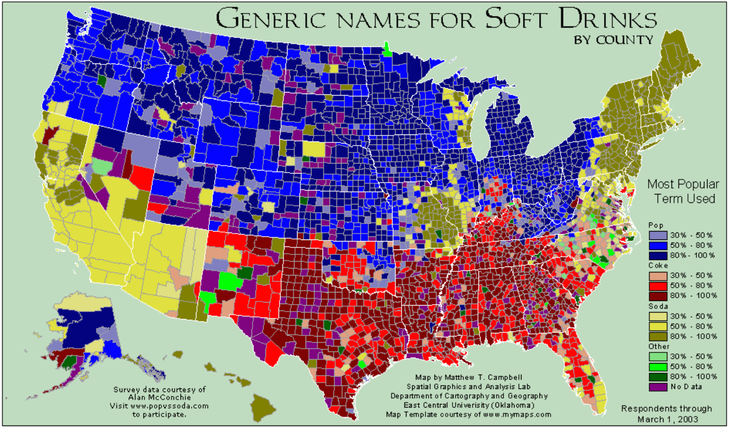

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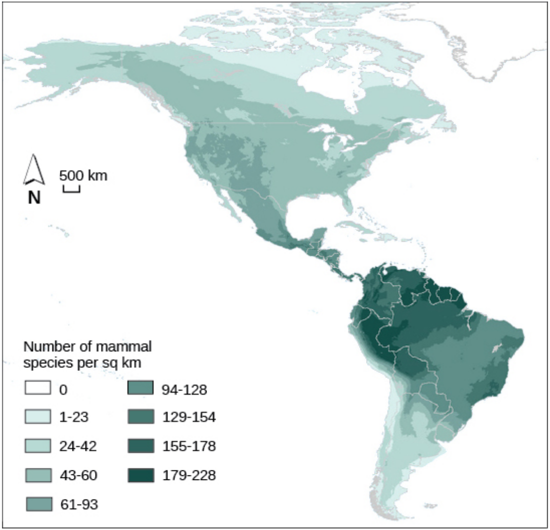

(NASA, CIESIN, Columbia University, 2010)

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“the property of random variables taking values, at pairs of locations a certain distance apart, that are more similar (positive autocorrelation) or less similar (negative autocorrelation) than expected for randomly associated pairs of observations”

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Fact (Fortin and Dale, 2005. Spatial Analysis)

“natural systems almost always have autocorrelation in the form of patchiness or gradientsover a wide range of spatial and temporal scales”

Two types of spatial autocorrelation

1. Endogenous
   - caused by biotic processes, e.g. dispersal
2. Exogenous
   - caused by functional dependence on spatially autocorrelated drivers, e.g. climate

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- Spatial autocorrelation is a nuisance that complicates statistical hypothesis testing
- Spatial autocorrelation is functionally important in many ecosystems, so we must revise our theories and models to incorporate spatial structure (Fortin and Dale, 2005. Spatial Analysis)

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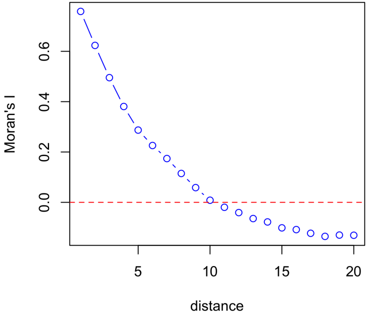
- Correlogram plot distance on X-axis against correlation coefficient on Y-axis
  - Mantel correlogram for multivariate response
  - Observe distance decay!



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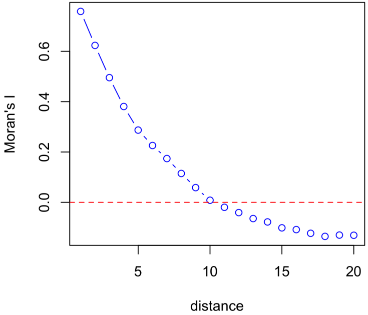

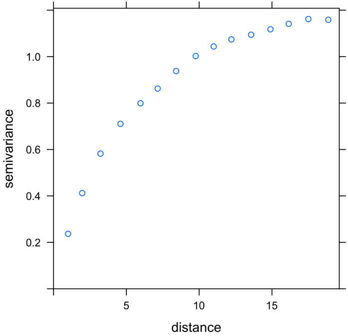

- Semi-variogram or variogram
  - Inverse correlogram!

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- Incorporate a variance-covariance matrix (C) that is affected by a proximity matrix (W) of neighbor weights

$$y = X\beta + \epsilon$$

$$y = X\beta + \rho W(y - X\beta) + \epsilon$$

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- I will show you two models:
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R package "nlme"

```
library(nlme)
?gls
?corClasses
```

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R package “MASS” and “lme4”

```
library(MASS)
?glmmPQL
library(lme4)
?glmer()
```

For more and on temporally autocorrelated data

Dormann et al. (2007)
Methods to account for spatial autocorrelation in the analysis of species distributional data: a review
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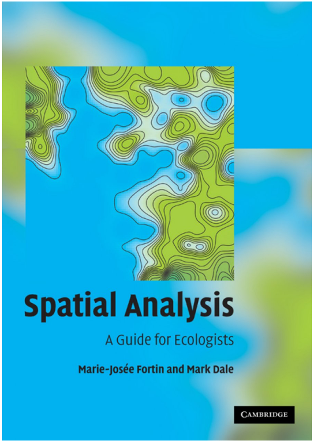

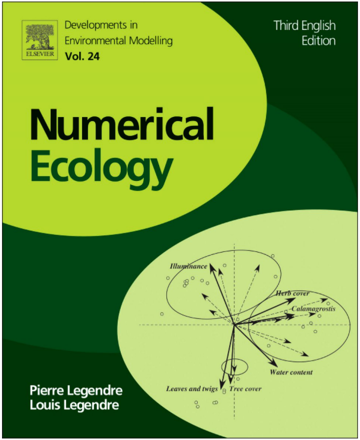

Snouters!

http://encyclopedia.thefreedictionary.com/Snouter

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- We will use Snouter data from Dormann et al., 2007. Ecogra provided in the GItHub repo
- Questions
  - Do Snouter abundance and presence exhibit spatial autocorrelation?
  - What are the explained variances in Snouter abundance and presence by precipitation and distance to jungle?

Let's do it together!