Information and Coding Theory

University of Chinese Academy of Sciences

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Homework 2-2

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1. 第一题

   由题可得：$p(y_1)=\frac{1}{2}\times 0.98+\frac{1}{2}\times 0.2=0.59, p(y_2)=\frac{1}{2}\times 0.02+\frac{1}{2}\times 0.8=0.41$，故 Y 的信道模型为：$\begin{pmatrix} Y \\ P(Y) \end{pmatrix}=\begin{pmatrix} y_1 & y_2 \\ 0.59 & 0.41 \end{pmatrix}$

   $H(X)=-\frac{1}{2}\log\frac{1}{2}-\frac{1}{2}\log\frac{1}{2}=1\ bits/symbol$

   (1) $H(X|Y)=-\frac{1}{2}\times 0.98\times\log\frac{\frac{1}{2}\times 0.98}{0.59}-\frac{1}{2}\times 0.02\times\log\frac{\frac{1}{2}\times 0.02}{0.41}-\frac{1}{2}\times 0.2\times\log\frac{\frac{1}{2}\times 0.2}{0.59}-\frac{1}{2}\times 0.8\times\log\frac{\frac{1}{2}\times 0.8}{0.41}=0.455\ bits/symbol$

   (2) $I(X;Y)=H(X)-H(X|Y)=1-0.455=0.545\ bits/symbol$

   (3) $H(Y|X)=-\frac{1}{2}\times 0.98\times\log 0.98-\frac{1}{2}\times 0.02\times\log 0.02-\frac{1}{2}\times 0.2\times\log 0.2-\frac{1}{2}\times 0.8\times\log 0.8=0.432\ bits/symbol$

   (4) $H(XY)=-\frac{1}{2}\times 0.98\times\log 0.49-\frac{1}{2}\times 0.02\times\log 0.01-\frac{1}{2}\times 0.2\times\log 0.1-\frac{1}{2}\times 0.8\times\log 0.4=1.431\ bits/symbol$

2. 第二题

   由题可得：X 为离散平稳无记忆信源，因此 $\forall i,j\in 1,2,3, p(x_ix_j)=p(x_i)p(x_j)$
   故 X 的二次拓展信源 X^2 的概率空间为：
   $\begin{pmatrix} X^2 \\ P(X^2) \end{pmatrix}=\begin{pmatrix} x_1x_1 & x_1x_2 & x_1x_3 & x_2x_1 & x_2x_2 & x_2x_3 & x_3x_1 & x_3x_2 & x_3x_3 \\ \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{16} & \frac{1}{16} & \frac{1}{8} & \frac{1}{16} & \frac{1}{16} \end{pmatrix}$
   $H(X^2)=2H(X)=2\times(-\frac{1}{2}\log\frac{1}{2}-\frac{1}{4}\log\frac{1}{4}-\frac{1}{4}\log\frac{1}{4})=3\ bits/symbol$

3. 第三题

   (1) 由题可得：$\begin{pmatrix} M \\ P(M) \end{pmatrix}=\begin{pmatrix} M_1 & M_2 & M_3 & M_4 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$，因此，输入 0 和 1 的概率相等，故 $\begin{pmatrix} X \\ P(X) \end{pmatrix}=\begin{pmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$，$\begin{pmatrix} Y \\ P(Y) \end{pmatrix}=\begin{pmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$
   $I(M_1;y_1=0)=\log\frac{p(y_1=0|M_1)}{p(y_1)}=\log(1-p)+1\ bits/symbol$

$I(M_2;y_1=0)=\log\frac{p(y_1=0|M_2)}{p(y_1)}=\log(1-p)+1\ bits/symbol$
$I(M_3;y_1=0)=\log\frac{p(y_1=0|M_3)}{p(y_1)}=\log p+1\ bits/symbol$
$I(M_4;y_1=0)=\log\frac{p(y_1=0|M_4)}{p(y_1)}=\log p+1\ bits/symbol$

（2） 由题可得： $I(M_1;y=00)=\log\frac{p(y=00|M_1)}{p(y=00)}=2\log(1-p)+2\ bits/symbol$
$I(M_2;y=00)=\log\frac{p(y=00|M_2)}{p(y=00)}=\log(1-p)p+2\ bits/symbol$
$I(M_3;y=00)=\log\frac{p(y=00|M_1)}{p(y=00)}=\log(1-p)p+2\ bits/symbol$
$I(M_4;y=00)=\log\frac{p(y=00|M_1)}{p(y=00)}=2\log p+2\ bits/symbol$

4. 第四题

由题可得:$H(X)=-(0.31\log 0.31+0.45\log 0.45+0.24\log 0.24)=1.515\ bits/symbol$
$H(X_2|X_1)=-(0.25\log\frac{0.25}{0.31}+0.06\log\frac{0.06}{0.31}+0.06\log\frac{0.06}{0.45}+0.33\log\frac{0.33}{0.45}+0.06\log\frac{0.06}{0.45}+0.06\log\frac{0.06}{0.24}+0.18\log\frac{0.18}{0.24})=0.911\ bits/symbol$
$H_N(X)=\frac{1}{2}H(X^2)=\frac{1}{2}(H(X)+H(X_2|X_1))=1.213\ bits/symbol$
$H_\infty(X)=-(0.25\log\frac{0.25}{0.31}+0.06\log\frac{0.06}{0.31}+0.06\log\frac{0.06}{0.45}+0.33\log\frac{0.33}{0.45}+0.06\log\frac{0.06}{0.45}+0.06\log\frac{0.06}{0.24}+0.18\log\frac{0.18}{0.24})=0.911\ bits/symbol$