Information and Coding Theory

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Homework 1116

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1. Write down a parity-check matrix H for a binary Hamming code of length 15, where the jth column of H is the binary representation of j. Then use H to construct a syndrome look-up table and use it to decode the following words:

   (a) 01010 01010 01000

   (b) 11100 01110 00111

   (c) 11001 11001 11000

   SOLUTION

   The parity-check matrix H for a binary Hamming code of length 15 is

$$H = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$$

   The syndrome look-up table is Table 1, decoding process is as follows:

   (a) $e_1 = w_1 \cdot H^T = 0100, v_1 = w_1 - e_1 = 010000101001000$

   (b) $e_2 = w_2 \cdot H^T = 1010, v_2 = w_2 - e_2 = 111000111100111$

   (c) $e_3 = w_3 \cdot H^T = 1010, v_3 = w_3 - e_3 = 110011100011000$

Table 1: **Syndrome Look-up Table**

Coset Leader	Syndrome
000000000000000	0000
100000000000000	0001
010000000000000	0010
001000000000000	0011
000100000000000	0100
000010000000000	0101
000001000000000	0110
000000100000000	0111
000000010000000	1000
000000001000000	1001
000000000100000	1010
000000000010000	1011
000000000001000	1100
000000000000100	1101
000000000000010	1110
000000000000001	1111

2. *Let C be the code over $\mathbf{F}_4 = \{0, 1, \alpha, \alpha^2\}$ with generator matrix:*

$$H = \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & \alpha & \alpha^2 \end{pmatrix}$$

(i) Show that C is an MDS code.

(ii) Write down a generator matrix for the dual $C^\perp$.

(iii) Show that $C^\perp$ is an MDS code.

SOLUTION

(i) Since G is a 2×4 matrix, thus C is a $[4, 2, d]$ codes

Since any 2 columns of generator matrix G is linear independent, any 3 columns of generator matrix G is linear dependent, thus $d = 3$, C is a $[4, 2, 3]$ codes

Obviously, code C satisfies $n + 1 = 5 = k + d$, thus C is an MDS code.

Q.E.D

(ii) Since the parity-check matrix for the C is a generator matrix for the dual code

$C^{\perp}$, *thus* $G_{C^{\perp}} = H_C = \begin{pmatrix} 1 & \alpha & 1 & 0 \\ 1 & \alpha^2 & 0 & 1 \end{pmatrix}$

(iii) Given the generator matrix by (ii), we knew that $C^{\perp}$ *is also a* $[4,2,d]$ *codes, satisfies* $n+1=5=k+d$*, thus* $C^{\perp}$ *is an MDS code.*