

Information and Coding Theory

University of Chinese Academy of Sciences

Fall 2023

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Homework 1019

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2023.10.22

1. 第一题

(1) 由题可得: $\bar{D} = E[d(x_i, x_j)] = p \cdot 0 \cdot 1 + (1-p) \cdot p \cdot 1 + (1-p) \cdot (1-q) \cdot 0 = (1-p)q$

(2) 当 D_{\min} 时, $R(D)$ 取最大值, 此时 $q = 0, \bar{D} = 0$

$$R(D)_{\max} = H(X) = -p \log p - (1-p) \log(1-p)$$

(3) 当 D_{\max} 时, $R(D)$ 取最小值, 此时 $q = 1, \bar{D} = (1-p)$

$$R(D)_{\min} = H(Y) - H(Y|X) = 0$$

2. 第二题

$$D_{\min} = \sum_i^2 p(x_i) \min_j d(x_i, x_j) = 0$$

最小允许失真度的试验信道为: $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

3. 第三题

$$(1) \text{ 由题可得: } D_{\min} = \sum_i^2 p(x_i) \min_j d(x_i, x_j) = 0$$

信道转移概率矩阵为: $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

$$(2) \text{ 最小允许失真度的试验信道为: } P_D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$R(D_{\min}) = H(X) = -(0.4 \log 0.4 + 0.6 \log 0.6) = 0.970$$

$$(3) D_{\max} = \min\{0.4d(x_1|y_1) + 0.6p(x_2|y_1), 0.4d(x_1|y_2) + 0.6p(x_2|y_2), 0.4d(x_1|y_3) + 0.6p(x_2|y_3)\} = \max\{0.4, 0.6, 0.5\} = 0.6, \text{ 此时信道转移概率矩阵为: } \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$