Information and Coding Theory

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Homework 1105

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1. Design a channel coding scheme to correct one error for the message source 000, 100, 010, 001, 110, 101, 011, 111.
   Can you find one of the best schemes in terms of information transmission speed?

   if the coding scheme has the ability of correcting one error, the redundancy of codes must satisfy: $\lfloor \frac{d(c)-1}{2} \rfloor \geq 1 \Rightarrow d(c) \geq 3$, thus here gives the following code scheme which has the fastest information transmission speed:

$$\begin{pmatrix} 000 & 100 & 010 & 001 & 110 & 101 & 011 & 111 \\ 000000 & 100011 & 010111 & 001110 & 110100 & 101101 & 011001 & 111010 \end{pmatrix}$$

2. *Let* $C = \{001, 01$ *be a binary code.*
   a. Suppose we have a memoryless binary channel with following probabilities:
   $P(0received|0sent) = 0.1 and P(1received|1sent) = 0.5$
   Use the maximum likihood decoding rule to decode the received word 000.
   b. Use the minimum distance decoding rule to decode 000.

   a.$P(000|001) = P(0|0)P(0|0)P(0|1) = 0.1 \times 0.1 \times 0.5 = 0.005$
   $P(000|011) = P(0|0)P(0|1)P(0|1) = 0.1 \times 0.5 \times 0.5 = 0.025$
   the maximum likihood codes to decode the received word 000 is 011
   b.the distance matrix between received code 000 and source is: $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

   thus the minimum distance codes to decode the received word 000 is 001