Information and Coding Theory

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Homework 1109

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1. Let C and D be linear codes over F_q of the same length. Define:

$$C + D = \{c + d : c \in C, d \in D\}$$

Show that $C + D$ is a linear code and that $(C + D)^{\perp} = C^{\perp} \cap D^{\perp}$.

SOLUTION

(i) 1> obviously, $0 \in C, 0 \in D$, thus $0 + 0 = 0 \in C + D$

2> Suppose $\forall c_1, c_2 \in C, \forall d_1, d_2 \in D$

so $c_1 + d_1 \in C + D, c_2 + d_2 \in C + D$

Since $c_1 + c_2 \in C, d_1 + d_2 \in D$

Thus $(c_1 + d_1) + (c_2 + d_2) = (c_1 + c_2) + (d_1 + d_2) \in C + D$

Summarizing: $C + D$ is a linear code

(ii) Suppose $\forall e_1 \in (C + D)^{\perp}, \forall c \in C, \forall d \in D$

Since $0 \in C, 0 \in D$

So $e_1 \cdot (c + 0) = 0 = e_1 \cdot c, e_1 \in C^{\perp}$

$e_1 \cdot (d + 0) = 0 = e_1 \cdot d, e_1 \in D^{\perp}$

Thus $\forall e_1, e_1 \in C^{\perp} \cap D^{\perp}$

Summarizing: $(C + D)^{\perp} = C^{\perp} \cap D^{\perp}$

Q.E.D

2. *Construct a binary code C of length 6 as follows: for every $(x1, x2, x3) \in F_2^3$, construct a 6-bit word $(x1, x2, x3, x4, x5, x6) \in C$, where $x4 = x1 + x2 + x3, x5 = x1 + x3, x6 = x2 + x3$.*

SOLUTION

(i) Show that C is a linear code.

(ii) Find a generator matrix and a parity-check matrix for C

(i) 1> Let $(x_1, x_2, x_3) = (0, 0, 0) \in F_2^3$

Thus $(x_1, x_2, x_3, x_4, x_5, x_6) = (0, 0, 0, 0, 0, 0) \in C$

2> Let $C = (C_1|C_2, C_1 = (x_1, x_2, x_3), C_2 = (x_4, x_5, x_6)$

Obviously C_1 *is a linear code since* $(x1, x2, x3) \in F_2^3$

Since $x4 = x1 + x2 + x3, x5 = x1 + x3, x6 = x2 + x3$, *thus* $C_2 = \{000, 001, 010, 100, 101, 110, 011, 111\}$, *thus* C_2 *is a linear code*

Suppose $a_1, a_2 \in C_1, b_1, b_2 \in C_2$, *so* $(a_1|b_1)+(a_2|b_2) = (a_1+a_2|b_1+b_2) \in (C_1|C_2) = C$

Summarizing: C *is a linear code*

3. *Let* C *be the binary linear code with parity-check matrix*

$$H = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$$

(i) Write down a generator matrix for C *and list all the codewords in* C.

(ii) Decode the following words:(a) 110110, (b) 011011, (c) 101010

SOLUTION

(i) $G = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}, H = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$

$C = \{000000, 001011, 010101, 100110, 110011, 101101, 011110, 111000\}$

(ii)

Coset Leader	Syndrome
000000	000
000001	001
000010	010
000100	100
001000	011
010000	101
100000	110
100001	111

$e_1 = w_1 \cdot H^T = 101, v_1 = w_1 - e_1 = 100110$

$e_2 = w_2 \cdot H^T = 101, v_2 = w_2 - e_2 = 001011$

$e_3 = w_3 \cdot H^T = 111, v_3 = w_3 - e_3 = 001011$