**Information and Coding Theory**

University of Chinese Academy of Sciences

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# Homework 1013

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1. 第一题

（1）由题可得：$p(y_1) = \frac{3}{4} \times 0.6 + \frac{1}{4} \times 0.4 = 0.55, p(y_2) = \frac{3}{4} \times 0.4 + \frac{1}{4} \times 0.6 = 0.45$
$H(X) = -(\frac{3}{4}\log\frac{3}{4} + \frac{1}{4}\log\frac{1}{4}) = 0.811\ bits/symbol$
$H(X|Y) = -(\frac{3}{4} \times 0.6 \log\frac{\frac{3}{4}\times 0.6}{0.55} + \frac{3}{4} \times 0.4 \log\frac{\frac{3}{4}\times 0.4}{0.45} + \frac{1}{4} \times 0.4 \log\frac{0.1}{0.55} + \frac{1}{4} \times 0.6 \log\frac{0.15}{0.45}) = 0.789\ bits/symbol$
$I(X;Y) = H(X) - H(X|Y) = 0.811 - 0.789 = 0.022\ bits/symbol$

（2）由题可得：$p(y_1) = 0.6p + 0.4(1-p) = 0.4 + 0.2p$, $p(y_2) = 0.4p + 0.6(1-p) = 0.6 - 0.2p$, $H(Y) = -(0.4+0.2p)\log(0.4+0.2p) - (0.6-0.2p)\log(0.6-0.2p)\ bits/symbol$
$H(Y|X) = -(0.6\log 0.6 + 0.4\log 0.4) = 0.970\ bits/symbol$
$C = I(X;Y) = H(Y) - H(Y|X) = -(0.4+0.2p)\log(0.4+0.2p) - (0.6-0.2p)\log(0.6-0.2p) - 0.970$

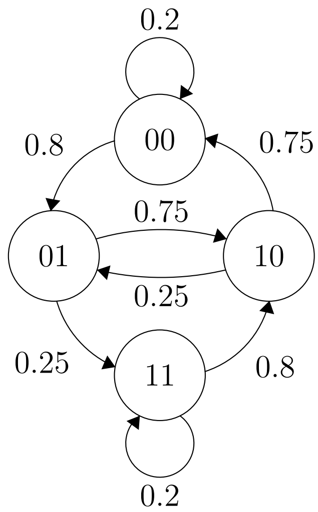
（3）由题可得：$C = \max\limits_{p(x)} I(X;Y) = \max\limits_{p(x)}[-(0.4+0.2p)\log(0.4+0.2p) - (0.6-0.2p)\log(0.6-0.2p) - 0.970]$，当 $p(x_1 = 0) = p(x_2 = 0) = \frac{1}{2}$ 时，$C_{max} = 0.03$

2. 第二题

（1）由题可得：$X, Y, Z$ 不独立，因此 $I(X;Y;Z) > 0$，故 $I(X;Y|Z) = I(X;Y) - I(X;Y;Z) < I(X;Y)$，证毕
（2）由题可得：因为 $X, Y$ 独立，故 $I(X;Y) = 0, H(X|Y,Z) = 0$，又因为 $X$ 与 $Z = X + Y$ 不独立，因此 $H(X|Z) > 0$，故 $I(X;Y|Z) = H(X|Z) - H(X|Y;Z) = H(X|Z) > 0 = I(X;Y)$，证毕

3. 第三题

由题可得:状态转移矩阵为 $\begin{pmatrix} 0.2 & 0.8 & 0 & 0 \\ 0 & 0 & 0.75 & 0.25 \\ 0.75 & 0.25 & 0 & 0 \\ 0 & 0 & 0.8 & 0.2 \end{pmatrix}$,状态空间 $S = \{00, 01, 10, 11\}$




可得方程组：$\begin{cases} p(s_1) = 0.2p(s_1) + 0.75p(s_3) \\ p(s_2) = 0.8p(s_1) + 0.25p(s_3) \\ p(s_3) = 0.75p(s_2) + 0.8p(s_4) \\ p(s_4) = 0.25p(s_2) + 0.2P(s_4) \end{cases}$，解得 $\begin{cases} p(s_1) = \frac{15}{52} \\ p(s_2) = \frac{4}{13} \\ p(s_3) = \frac{4}{13} \\ p(s_4) = \frac{5}{52} \end{cases}$

$H_\infty(X) = H_3(x) = -\sum_i \sum_j p(s_i)p(s_j|s_i) \log p(s_j|s_i) = 0.777$