Information and Coding Theory

University of Chinese Academy of Sciences

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Homework 1013

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1. 第一题

（1）由题可得：$p(y_1)=\frac{3}{4}\times 0.6+\frac{1}{4}\times 0.4=0.55, p(y_2)=\frac{3}{4}\times 0.4+\frac{1}{4}\times 0.6=0.45$
$H(X)=-(\frac{3}{4}\log\frac{3}{4}+\frac{1}{4}\log\frac{1}{4})=0.811\ bits/symbol$
$H(X|Y)=-(\frac{3}{4}\times 0.6\log\frac{\frac{3}{4}\times 0.6}{0.55}+\frac{3}{4}\times 0.4\log\frac{\frac{3}{4}\times 0.4}{0.45}+\frac{1}{4}\times 0.4\log\frac{0.1}{0.55}+\frac{1}{4}\times 0.6\log\frac{0.15}{0.45})=$ $0.789\ bits/symbol$
$I(X;Y)=H(X)-H(X|Y)=0.811-0.789=0.022\ bits/symbol$

（2）由题可得：$p(y_1)=0.6p+0.4(1-p)=0.4+0.2p$, $p(y_2)=0.4p+0.6(1-p)=0.6-0.2p$, $H(Y)=-(0.4+0.2p)\log(0.4+0.2p)-(0.6-0.2p)\log(0.6-0.2p)\ bits/symbol$
$H(Y|X)=-(0.6\log 0.6+0.4\log 0.4)=0.970\ bits/symbol$
$C=\max I(X;Y)=\max\{H(Y)\}-H(Y|X)=-(0.4+0.2p)\log(0.4+0.2p)-(0.6-0.2p)\log(0.6-0.2p)-0.971$，当 $p=0.5$ 时，$\max I(X;Y)=1-0.971=0.029\ bits/symbol$，故 $C=0.029\ bits/symbol$

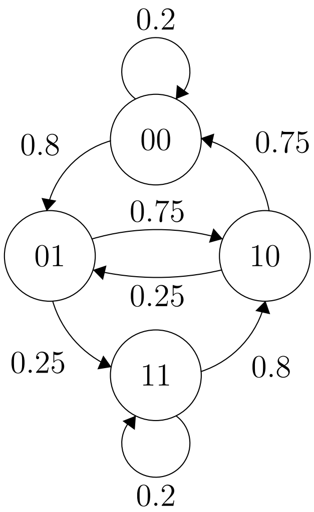
（3）由题可得：$C=\max_{p(x)} I(X;Y)=\max_{p(x)}[-(0.4+0.2p)\log(0.4+0.2p)-(0.6-0.2p)\log(0.6-0.2p)-0.970]$，当 $p(x_1=0)=p(x_2=0)=\frac{1}{2}$ 时，$C_{max}=0.03$

2. 第二题

（1）由题可得：X,Y,Z 不独立，因此 $I(X;Y;Z)>0$，故 $I(X;Y|Z)=I(X;Y)-I(X;Y;Z)<I(X;Y)$，证毕
（2）由题可得：因为 X,Y 独立，故 $I(X;Y)=0, H(X|Y,Z)=0$，又因为 X 与 $Z=X+Y$ 不独立，因此 $H(X|Z)>0$，故 $I(X;Y|Z)=H(X|Z)-H(X|Y;Z)=H(X|Z)>0=I(X;Y)$，证毕

3. 第三题

由题可得:状态转移矩阵为 $\begin{pmatrix} 0.2 & 0.8 & 0 & 0 \\ 0 & 0 & 0.75 & 0.25 \\ 0.75 & 0.25 & 0 & 0 \\ 0 & 0 & 0.8 & 0.2 \end{pmatrix}$,状态空间 $S=\{00,01,10,11\}$



可得方程组：$\begin{cases} p(s_1) = 0.2p(s_1) + 0.75p(s_3) \\ p(s_2) = 0.8p(s_1) + 0.25p(s_3) \\ p(s_3) = 0.75p(s_2) + 0.8p(s_4) \\ p(s_4) = 0.25p(s_2) + 0.2P(s_4) \end{cases}$ ，解得 $\begin{cases} p(s_1) = \frac{15}{52} \\ p(s_2) = \frac{4}{13} \\ p(s_3) = \frac{4}{13} \\ p(s_4) = \frac{5}{52} \end{cases}$

$H_\infty(X) = H_3(x) = -\sum_i \sum_j p(s_i) p(s_j|s_i) \log p(s_j|s_i) = 0.777$