Information and Coding Theory

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Fall 2023

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Homework 1123

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2023.11.23

1. Determine whether the following polynomials are generator polynomials of cyclic codes of given lengths:
   (a) $g(x) = 1 + x + x^2 + x^3 + x^4$ for a binary cyclic code of length 7;
   (b) $g(x) = 2 + 2x^2 + x^3$ for a ternary cyclic code of length 8;
   (c) $g(x) = 2 + 2x + x^3$ for a ternary cyclic code of length 13.

   SOLUTION

   (a) Let the binary cyclic code of length 7 as C_1. Since $n_{C_1} = 7$, now divides $x^7 - 1$, obviously, $x^7 - 1 = (x + 1)(x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)$, then calculate:

$$
\begin{array}{r|l}
 & \phantom{x^6 + x^5 + x^4 + x^3 + {}} x^2 \\
\cline{2-2}
x^4 + x^3 + x^2 + x + 1 & x^6 + x^5 + x^4 + x^3 + x^2 + x + 1 \\
 & x^6 + x^5 + x^4 + x^3 + x^2 \\
\cline{2-2}
 & \phantom{x^6 + x^5 + x^4 + x^3 + x^2 + {}} x + 1
\end{array}
$$

   Thus $1+x+x^2+x^3+x^4$ is not a monic polynomial of x^7-1, $g(x) = 1+x+x^2+x^3+x^4$ is not the generator polynomial of C_1

   (b) Let the ternary cyclic code of length 8 as C_2. Since $n_{C_2} = 8$, now divides $x^8 - 1$, obviously, $x^8 - 1 = (x + 2)(x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)$, then calculate:

$$
\begin{array}{r|r}
 & x^4 + 2x^3 + 2x + 2 \\ \hline
x^3 + 2x^2 + 2 & x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1 \\
 & x^7 + 2x^6 + 2x^4 \qquad\qquad\qquad\qquad\qquad \\ \hline
 & 2x^6 + x^5 + 2x^4 + x^3 + x^2 + x + 1 \\
 & 2x^6 + x^5 + x^3 \qquad\qquad\qquad \\ \hline
 & 2x^4 + x^2 + x + 1 \\
 & 2x^4 + x^3 + x \qquad \\ \hline
 & 2x^3 + x^2 + 1 \\
 & 2x^3 + x^2 + 1 \\ \hline
 & 0
\end{array}
$$

Thus $2 + 2x^2 + x^3$ is a monic polynomial of $x^8 - 1$, $g(x) = 2 + 2x^2 + x^3$ is the generator polynomial of C_2

(c) Let the ternary cyclic code of length 13 as C_3. Since $n_{C_3} = 13$, now divides $x^{13} - 1$, then calculate:

$$
\begin{array}{r|r}
 & x^9 + x^8 + 2x^7 + x^5 + 2x^3 + 2x^2 + 2 \\ \hline
x^3 + 2x + 2 & x^{12} + x^{11} + x^{10} + x^9 + x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1 \\
 & x^{12} + 2x^{10} + 2x^9 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \\ \hline
 & x^{11} + 2x^{10} + 2x^9 + x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1 \\
 & x^{11} + 2x^9 + 2x^8 \qquad\qquad\qquad\qquad\qquad\qquad\qquad \\ \hline
 & 2x^{10} + 2x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1 \\
 & 2x^{10} + x^8 + x^7 \qquad\qquad\qquad\qquad\qquad\qquad \\ \hline
 & x^8 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1 \\
 & x^8 + 2x^6 + 2x^5 \qquad\qquad\qquad\qquad \\ \hline
 & 2x^6 + 2x^5 + x^4 + x^3 + x^2 + x + 1 \\
 & 2x^6 + x^4 + x^3 \qquad\qquad\quad \\ \hline
 & 2x^5 + x^2 + x + 1 \\
 & 2x^5 + x^3 + x^2 \qquad \\ \hline
 & 2x^3 + x + 1 \\
 & 2x^3 + x + 1 \\ \hline
 & 0
\end{array}
$$

Thus $2+2x+x^3$ is a monic polynomial of $x^{13}-1$, $g(x) = 2+2x+x^3$ is the generator polynomial of C_3