Structural Vector Autoregressions
Intro & Estimation

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Structural Vector Autoregressions

- Structural vector autoregressions (VAR): A system of dynamic simultaneous equations.

- Matrix notation:

$$B_0 y_t = B_1 y_{t-1} + \cdots + B_p y_{t-p} + w_t$$

  where

- y_t is a $(K \times 1)$ vector of endogenous variables;

- B_i for $i = 0, \ldots, p$ are $(K \times K)$ matrices of structural parameters.

- w_t is a $(K \times 1)$ vector of structural shocks:
  - $\mathbb{E}[w_t w_t'] \equiv \Sigma_w = I_K$
  - $w_t \sim (0, I_K)$

A Monetary Structural VAR

- EXAMPLE: A simple 3-equation monetary policy VAR:

$$\begin{aligned}
b_{11,0}\Delta x_t + b_{12,0}\pi_t + b_{13,0}i_t &= b_{11,1}\Delta x_{t-1} + b_{12,1}\pi_{t-1} + b_{13,1}i_{t-1} + w_{1t} \\
b_{21,0}\Delta x_t + b_{22,0}\pi_t + b_{23,0}i_t &= b_{21,1}\Delta x_{t-1} + b_{22,1}\pi_{t-1} + b_{23,1}i_{t-1} + w_{2t} \\
b_{31,0}\Delta x_t + b_{32,0}\pi_t + b_{33,0}i_t &= b_{31,1}\Delta x_{t-1} + b_{32,1}\pi_{t-1} + b_{33,1}i_{t-1} + w_{3t}
\end{aligned}$$

- In this example:

$$y_t = \begin{bmatrix} \Delta x_t \\ \pi_t \\ i_t \end{bmatrix};\ B_i = \begin{bmatrix} b_{11,i} & b_{12,i} & b_{13,i} \\ b_{21,i} & b_{22,i} & b_{23,i} \\ b_{31,i} & b_{32,i} & b_{33,i} \end{bmatrix}\ i = 0, 1;\ w_t = \begin{bmatrix} w_{1t} \\ w_{2t} \\ w_{3t} \end{bmatrix}.$$

- x_t: indicator of economic activity (e.g. GDP, GNP, IP);
- π_t: inflation;
- i_t: monetary policy instrument (e.g. federal funds rate).

A Monetary Structural VAR

- Rearrange equations (step 1):

$$
\begin{aligned}
b_{11,0}\Delta x_t &= -b_{12,0}\pi_t - b_{13,0}i_t + b_{11,1}\Delta x_{t-1} + b_{12,1}\pi_{t-1} + b_{13,1}i_{t-1} + w_{1t} \\
b_{22,0}\pi_t &= -b_{21,0}\Delta x_t - b_{23,0}i_t + b_{21,1}\Delta x_{t-1} + b_{22,1}\pi_{t-1} + b_{23,1}i_{t-1} + w_{2t} \\
b_{33,0}i_t &= -b_{31,0}\Delta x_t - b_{32,0}\pi_t + b_{31,1}\Delta x_{t-1} + b_{32,1}\pi_{t-1} + b_{33,1}i_{t-1} + w_{3t}
\end{aligned}
$$

- Rearrange equations (step 2):

$$
\begin{aligned}
\Delta x_t &= \psi_{12,0}\pi_t + \psi_{13,0}i_t + \psi_{11,1}\Delta x_{t-1} + \psi_{12,1}\pi_{t-1} + \psi_{13,1}i_{t-1} + \sigma_1 w_{1t} \\
\pi_t &= \psi_{21,0}\Delta x_t + \psi_{23,0}i_t + \psi_{21,1}\Delta x_{t-1} + \psi_{22,1}\pi_{t-1} + \psi_{23,1}i_{t-1} + \sigma_2 w_{2t} \\
i_t &= \psi_{31,0}\Delta x_t + \psi_{32,0}\pi_t + \psi_{31,1}\Delta x_{t-1} + \psi_{32,1}\pi_{t-1} + \psi_{33,1}i_{t-1} + \sigma_3 w_{3t}
\end{aligned}
$$

  - $\psi_{kj,0} = -b_{kj,0}/b_{kk,0}$;
  - $\psi_{kj,i} = b_{kj,i}/b_{kk,0}$, with $b_{kj,i}$ denoting the kj-th element of B_i, for $i = 1, ..., p$.
  - $\sigma_k = 1/b_{kk,0}$.

Economic Interpretation of Structural VARs

- "*To use this mathematical structure for economic policy analysis, one has to identify it - give its elements economic interpretations.*" (Leeper, Sims, and Zha, 1996)

- Let's do that!

- How we are going to proceed:

  1. Define and understand the structural shocks w_t.
  2. Define and understand the model equations.
  3. Identification problem: SVAR model representation is not unique.

Economic Interpretation: Structural Shocks

- What is a structural shock w_{it}?
- Example:

$$i_t = \psi_{31,0}\Delta x_t + \psi_{32,0}\pi_t + \psi_{31,1}\Delta x_{t-1} + \psi_{32,1}\pi_{t-1} + \psi_{33,1} i_{t-1} + \sigma_3 w_{3t}$$

- Why can we call w_{3t} a monetary policy shock?
- Structural VAR models are probability models of the data.
- By using these models, we subscribe to the view that policy has a random component.
- Why could policy be a random variable?
  - Outcome of an extremely complicated decision process.
  - Policy unpredictable by the public.
  - Change in policymakers or in their decision process.

Economic Interpretation: Structural Shocks

"*Some would say that one cannot contemplate improving policy as if one could choose it rationally and, at the same time, think of policy as a random variable. This notion is simply incorrect.*"

(Leeper, Sims, and Zha, 1996)

- Going back to our example:

$$i_t = \psi_{31,0}\Delta x_t + \psi_{32,0}\pi_t + \psi_{31,1}\Delta x_{t-1} + \psi_{32,1}\pi_{t-1} + \psi_{33,1}i_{t-1} + \sigma_3 w_{3t}$$

- Ramey (2016): A structural shock
  - is exogenous to other current and lagged endogenous variables in the model;
  - is uncorrelated with other structural shocks;
  - represents unanticipated movements or news about future movements in a variable.
- What are the implications of this definition?

Economic Interpretation: Equations

$$i_t = \psi_{31,0}\Delta x_t + \psi_{32,0}\pi_t + \psi_{31,1}\Delta x_{t-1} + \psi_{32,1}\pi_{t-1} + \psi_{33,1}i_{t-1} + \sigma_3 w_{3t}$$

- Let us assume that w_{3t} represents monetary policy shocks.
- The equation associated with w_{3t} characterizes monetary policy behaviour.
- That is, the equation above is a monetary policy rule.
- Furthermore, since we assume that the structural shocks are orthogonal, we know that the remaining equations describe the nonpolicy part of the economy.
- The associated shocks are nonpolicy sources of variation in the economy.

The Systematic Component of Monetary Policy

$$i_t = \psi_{31,0}\Delta x_t + \psi_{32,0}\pi_t + \psi_{31,1}\Delta x_{t-1} + \psi_{32,1}\pi_{t-1} + \psi_{33,1} i_{t-1} + \sigma_3 w_{3t}$$

- Terms in red represent the systematic component of monetary policy.

March 21, 2018

Federal Reserve issues FOMC statement

For release at 2:00 p.m. EDT

Share

Information received since the Federal Open Market Committee met in January indicates that the labor market has continued to strengthen and that economic activity has been rising at a moderate rate. Job gains have been strong in recent months, and the unemployment rate has stayed low. Recent data suggest that growth rates of household spending and business fixed investment have moderated from their strong fourth-quarter readings. On a 12-month basis, both overall inflation and inflation for items other than food and energy have continued to run below 2 percent. Market-based measures of inflation compensation have increased in recent months but remain low; survey-based measures of longer-term inflation expectations are little changed, on balance.

In view of realized and expected labor market conditions and inflation, the Committee decided to raise the target range for the federal funds rate to 1-1/2 to 1-3/4 percent. The stance of monetary policy remains accommodative, thereby supporting strong labor market conditions and a sustained return to 2 percent inflation.

Monetary Policy: Systematic Component and Shock

$$i_t = \psi_{31,0}\Delta x_t + \psi_{32,0}\pi_t + \psi_{31,1}\Delta x_{t-1} + \psi_{32,1}\pi_{t-1} + \psi_{33,1}i_{t-1} + \sigma_3 w_{3t}$$

- Remember: Identifying a monetary policy shock is equivalent to identifying the systematic component of monetary policy.
- This intuition is exploited in many papers in the literature:
  - Leeper, Sims and Zha (1996); Bernanke and Mihov (1998); Blanchard and Perotti (2002); Baumeister and Hamilton (2016).
  - Caldara and Kamps (2017); Arias, Caldara and Rubio-Ramirez (2018); Caldara and Herbst (2018).

Structural VAR: Identification

$$i_t = \psi_{31,0}\Delta x_t + \psi_{32,0}\pi_t + \psi_{31,1}\Delta x_{t-1} + \psi_{32,1}\pi_{t-1} + \psi_{33,1} i_{t-1} + \sigma_3 w_{3t}$$

- This looks pretty easy... just get an estimate of the coefficients $\psi_{kj,i}$!
- Unfortunately it is not that easy...
- Data alone cannot be used to estimate the structural parameters.
- Intuition: there are many structural models that are compatible with the data.

From Structural Form to Reduced Form

- Structural VAR in matrix notation:

$$B_0 y_t = B_1 y_{t-1} + \cdots + B_p y_{t-p} + w_t$$

- Reduced form:

$$\begin{aligned} y_t &= B_0^{-1} B_1 y_{t-1} + \cdots + B_0^{-1} B_p y_{t-p} + B_0^{-1} w_t \\ y_t &= A_1 y_{t-1} + \cdots + A_p y_{t-p} + u_t \end{aligned}$$

- A_i is the matrix of reduced-form parameters.
- u_t is the vector of reduced-form innovations.
- u_t is white noise (or i.i.d, stronger assumption);
- $u_t \sim (0, \Sigma_u)$.
- $\mathbb{E}[u_t u_t'] \equiv \Sigma_u = B_0^{-1} B_0^{-1'}$

From Structural Form to Reduced Form

- Going from reduced form to structural representation requires knowledge of the matrix B_0.

$$u_t = B_0^{-1} w_t$$

- B_0^{-1} maps the (uncorrelated) structural shocks into the (correlated) reduced-form innovations we estimate from the data.

- Identification problem:
  - Use data to estimate reduced form VAR
  - Use B_0^{-1} to construct the structural VAR.
  - BUT B_0^{-1} is not unique!!!

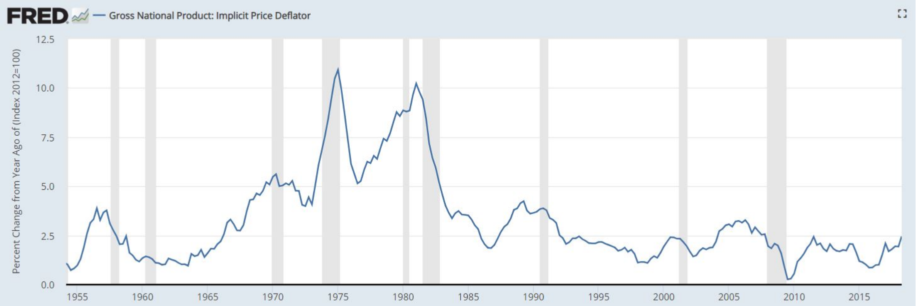
- I will extensively discuss identification throughout the course.

- Today: A simple MATLAB example

Data

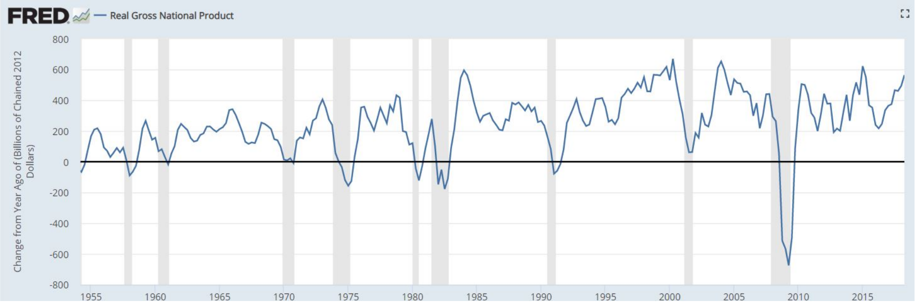

FRED
Effective Federal Funds Rate
Percent
20.0
17.5
15.0
12.5
10.0
7.5
5.0
2.5
0.0
1955
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005
2010
2015

Data

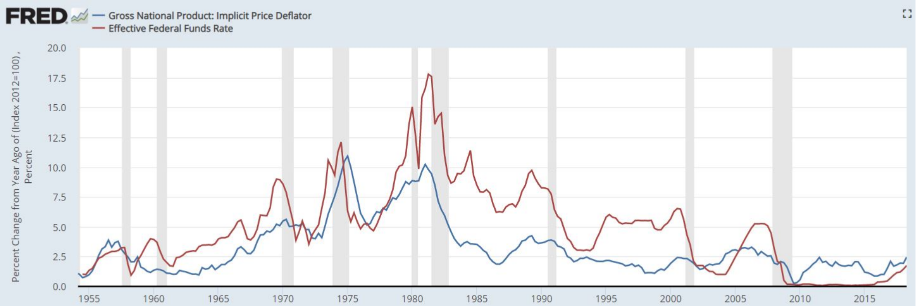


FRED
Gross National Product: Implicit Price Deflator
Percent Change from Year Ago of (Index 2012=100)
12.5
10.0
7.5
5.0
2.5
0.0
1955
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005
2010
2015

Data


FRED
Real Gross National Product
Change from Year Ago of (Billions of Chained 2012 Dollars)
800
600
400
200
0
-200
-400
-600
-800
1955
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005
2010
2015

Data



FRED
Gross National Product: Implicit Price Deflator
Effective Federal Funds Rate
Percent Change from Year Ago of (Index 2012=100) ,
Percent
20.0
17.5
15.0
12.5
10.0
7.5
5.0
2.5
0.0
1955
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005
2010
2015