Structural Vector Autoregressions
Proxy Identification

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Overview

- Proxy Identification:
- **Mertens K., and Ravn M. O.**, 2013. "The Dynamic Effects of Personal and Corporate Income Tax Changes in the United States," *American Economic Review*, vol. 103(4), pages 1212-1247.
- **Gertler M., and Karadi P.**, 2015. "Monetary Policy Surprises, Credit Costs, and Economic Activity," *American Economic Journal: Macroeconomics*, vol. 7(1), pages 44-76.
- **Caldara D., and Kamps C.**, 2017. "The Analytics of SVARs: A Unified Framework to Measure Fiscal Multipliers," *Review of Economic Studies*, vol. 84(3), pages 1015-1040.
- **Caldara D., and Herbst E.**, 2018. "Monetary Policy, Real Activity, and Credit Spreads: Evidence from Bayesian Proxy SVARs," *American Economic Journal: Macroeconomics*, forthcoming.

Instrumental Variable Approach

- Refresh memory. Instrumental variable estimation:

$$y_t = \beta x_t + u_t,\ \mathbb{E}[x_t u_t] \neq 0$$

- Let w_t be an instrument for x_t satisfying:

$$\begin{aligned} \mathbb{E}[w_t x_t] &\neq 0 \qquad (\textit{Relevance}) \\ \mathbb{E}[w_t u_t] &= 0 \qquad (\textit{Exogeneity}) \end{aligned}$$

- Two stage least square (2SLS):
  1. First stage: Regress x_t on w_t and obtain $\hat{x}_t$.
  2. Second stage: Regress y_t on $\hat{x}_t$ to obtain consistent estimate of β.

Instrumental Variables and SVAR Identification

- Identifying restrictions generate 'instruments'.
- The elements of B_0 (or B_0^{-1}) can also be obtained by IV methods.
- Structural VAR:

$$B_0 y_t = B_1 y_{t-1} + \cdots + B_p y_{t-p} + w_t$$

- Reduced form VAR:

$$y_t = A_1 y_{t-1} + \cdots + A_p y_{t-p} + u_t$$

- Relationship between reduced form residuals and structural shocks:

$$B_0 u_t = w_t$$

Example: 3-Equation Monetary SVAR

- Assume VAR(0), so that $y_t = u_t$.

$$\begin{aligned} b_{11,0}\Delta x_t + b_{12,0}\pi_t + b_{13,0}i_t &= w_{1t} \\ b_{21,0}\Delta x_t + b_{22,0}\pi_t + b_{23,0}i_t &= w_{2t} \\ b_{31,0}\Delta x_t + b_{32,0}\pi_t + b_{33,0}i_t &= w_{3t} \end{aligned}$$

- Rewrite the system as:

$$\begin{aligned} \Delta x_t &= \psi_{12,0}\pi_t + \psi_{13,0}i_t + \sigma_x w_{x,t} \\ \pi_t &= \psi_{21,0}\Delta x_t + \psi_{23,0}i_t + \sigma_\pi w_{\pi,t} \\ i_t &= \psi_{31,0}\Delta x_t + \psi_{32,0}\pi_t + \sigma_{mp} w_{mp,t} \end{aligned}$$

Cholesky Identification as IV

- Assume this variable ordering: $[\Delta x_t, \pi_t, i_t]$:

$$\begin{aligned} \Delta x_t &= \sigma_x w_{x,t} \\ \pi_t &= \psi_{21,0}\Delta x_t + \sigma_\pi w_{\pi,t} \\ i_t &= \psi_{31,0}\Delta x_t + \psi_{32,0}\pi_t + \sigma_{mp} w_{i,t} \end{aligned}$$

- IV implementation:
  1. Obtain σ_x as the square root of Δx_t.
     - Calculate $w_{x,t} = \sigma_x^{-1}\Delta x_t$.
  2. Regress π_t on Δx_t using $w_{x,t}$ as an instrument to obtain $\psi_{21,0}$.
     - Obtain σ_π by calculating the standard deviation of the residual $\pi_t - \psi_{21,0}\Delta x_t$.
     - Calculate $w_{\pi,t} = \sigma_\pi^{-1}(\pi_t - \psi_{21,0}\Delta x_t)$.
  3. Regress i_t on Δx_t and π_t using $w_{x,t}$ and $w_{\pi,t}$ as instruments to obtain $\psi_{31,0}$ and $\psi_{32,0}$.
     - Obtain σ_{mp} by calculating the standard deviation of the residual $i_t - \psi_{31,0}\Delta x_t - \psi_{32,0}\pi_t$.

Proxy Identification

- Identification schemes (e.g. recursive) can be thought as generating 'instruments' that identify the system.
- What if we have actual instruments that can help identifying the structure of the model?
- Within the VAR literature, instruments are often referred to as:
  - External instruments: as they are constructed using information that is external to the model, as opposed to the internal instruments we can generate, say, through a Cholesky.
  - Proxies: as they are assumed to be imperfect measures (proxies) for the structural shocks we want to identify in the structral VAR.

Examples of Instruments

What are these instruments?

- **Event Studies:**
  - Tax reforms legislated for ideological (as opposed to economic) reasons.
    Romer and Romer (2010)
  - Changes in military spending.
    Ramey (2011)
- **Natural Experiments:**
  - Oil price changes due to wars & natural disasters.
    Kilian (2009); Caldara, Cavallo & Iacoviello (2018)
- **High Frequency Data:**
  - Surprise changes in monetary policy calculated within minutes of FOMC statements using high frequency data.
    Gürkaynak, Swanson, & Sack (2005); Gertler & Karadi (2014)

Proxy Identification

- Assume we have a proxy (instrument), denoted by m_t that measures surprise changes in monetary policy.

- Assume that proxy m_t satisfies the following conditions:

$$\begin{aligned} \mathbb{E}[m_t w_{mp,t}] &\neq 0 \qquad (\textit{Relevance}) \\ \mathbb{E}[m_t w_{-mp,t}] &= 0 \qquad (\textit{Exogeneity}) \end{aligned}$$

  where $w_{-mp,t}$ denote all non monetary policy shocks.

- Because the proxy sounds like a measure of the unobserved monetary policy structural shock, many folks think that proxy SVAR identification means estimating a VAR assuming that the shock is observed... THIS IS WRONG!

- Yet, it is correct that, if we only have a proxy for monetary policy shocks, we will identify only the monetary policy shock in the SVAR.

Proxy Identification

- The goal is to identify $w_{mp,t}$ without imposing any a-priori restriction (e.g. zeros).

$$\begin{aligned}\Delta x_t &= \psi_{12,0}\pi_t + \psi_{13,0} i_t + \sigma_x w_{x,t} \\ \pi_t &= \psi_{21,0}\Delta x_t + \psi_{23,0} i_t + \sigma_\pi w_{\pi,t}\end{aligned}$$

- Rewrite these two equations as:

$$\begin{aligned}\Delta x_t &= \zeta_1 i_t + \tilde{w}_{x,t} \\ \pi_t &= \zeta_2 i_t + \tilde{w}_{\pi,t}\end{aligned}$$

where

$$\zeta_1 = \frac{\psi_{12,0}\psi_{23,0} + \psi_{13,0}}{1 - \psi_{12,0}\psi_{21,0}},\ \zeta_2 = \psi_{21,0}\zeta_1 + \psi_{23,0},$$

$$\tilde{w}_{x,t} = \frac{1}{1 - \psi_{12,0}\psi_{21,0}}\left(\psi_{12,0}\sigma_\pi w_{\pi,t} + \sigma_x w_{x,t}\right)$$

$$\tilde{w}_{\pi,t} = \psi_{21,0}\tilde{w}_{x,t} + \sigma_\pi w_{\pi,t}$$

Proxy SVAR: Implementation

$$
\begin{aligned}
\Delta x_t &= \zeta_1 i_t + \tilde{w}_{x,t} \\
\pi_t &= \zeta_2 i_t + \tilde{w}_{\pi,t} \\
i_t &= \psi_{31,0} \Delta x_t + \psi_{32,0} \pi_t + \sigma_{mp} w_{i,t}
\end{aligned}
$$

- Proxy SVAR implementation:
  - 1a Regress Δx_t on i_t using m_t as an instrument to obtain ζ_1.
  - 1b Regress π_t on i_t using m_t as an instrument to obtain ζ_2.
  - 1c Obtain residuals $\tilde{w}_{x,t} = \Delta x_t - \zeta_1 i_t$ and $\tilde{w}_{\pi,t} = \pi_t - \zeta i_t$.
  - 2 Regress i_t on Δx_t and π_t using $\tilde{w}_{x,t}$ and $\tilde{w}_{\pi,t}$ as instruments to obtain $\psi_{31,0}$ and $\psi_{32,0}$.
  - 3 Obtain σ_{mp} by calculating the standard deviation of the residual $i_t - \psi_{31,0}\Delta x_t - \psi_{32,0}\pi_t$.

Proxy Identification: Some Considerations

- This is a SVAR identification strategy: we impose some restrictions on the structural coefficients.
- The restrictions are covariance restrictions coming from the relationship between the proxy and the model variables!
  - Possibly more appealing than zero (or even sign) restrictions...
  - Especially if we can use a reliable and exogenous proxy!
- Proxy identification identifies:
  - Shocks for which we have proxies available
  - The structural parameters of the equation associated with the identified shock $\rightarrow$ In our example: the systematic component of monetary policy.

Proxy Identification: Some Considerations

- The VAR is partially identified.
  - How do you see it?
  - What does it mean?

- The monetary policy equation is fully identified ($\psi_{31,0}$, $\psi_{32,0}, w_{mp,t}$).

- The other two equations are not. We only identify:
  - Non-linear functions of the structural parameters (ζ_1 and ζ_2);
  - Linear combinations of the 'true' structural shocks ($\tilde{w}_{x,t}$, $\tilde{w}_{\pi,t}$), where $corr(\tilde{w}_{x,t}, \tilde{w}_{\pi,t} \neq 0$.

- To fully identify the system, I should make additional assumptions. E.g. assume $\psi_{21,0} = 0$.

$$
\begin{aligned}
\zeta_1 = \psi_{12,0}\psi_{23,0} + \psi_{13,0},\ \zeta_2 = \psi_{23,0}, \\
\tilde{w}_{x,t} = (\psi_{12,0}\sigma_\pi w_{\pi,t} + \sigma_x w_{x,t}) \\
\tilde{w}_{\pi,t} = \sigma_\pi w_{\pi,t}
\end{aligned}
$$

Proxy Identification: Some Considerations

- From the proxy identification we have estimates of: $\psi_{31,0}, \psi_{32,0}, \zeta_1, \zeta_2$

- Then we can obtain:

$$
\begin{aligned}
\psi_{23,0} &= \zeta_2 \\
\psi_{12,0} &= \frac{\zeta_1 - \psi_{13,0}}{\psi_{23,0}} \\
w_{\pi,t} &= \sigma_\pi^{-1} \tilde{w}_{\pi,t} \\
w_{x,t} &= \sigma_x^{-1} (\tilde{w}_{x,t} - \psi_{12,0} \sigma_\pi w_{\pi,t})
\end{aligned}
$$

- NOTE!!!: The identification of the two non-policy shocks $w_{x,t}$ and $w_{\pi,t}$ does not affect the identification of the policy shock $w_{mp,t}$!

- The reverse is not true: the identification of the policy shock affects the identification of the non-policy shocks.