AN IMPROVED ANT COLONY OPTIMIZATION ALGORITHM FOR MULTI-DEPOT SPLIT-DELIVERY VEHICLE ROUTING PROBLEM WITH TIME WINDOWS IN CHANGEABLE SPEEDS

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Abstract

This paper presents a new method for determining routes and schedules for the multi-depot split-delivery vehicle routing problem with time windows (MDSDVRPTW), considering the total difference to due date and the number of visited locations for transportation, as well as changeable speed, and load factor constraints, according to a predetermined planning horizon. The problem is also a fleet size and mix vehicle-routing problem (FSMVRP). Considering the case of a hard time window, in which, a vehicle arriving at a location ahead of schedule will wait. We take into account the trade-off between two objective function, and propose an improved ant colony optimization algorithm (IACO) to resolve the problem.

Keywords: MDSDVRPTW, FSMVRP, Load Factors, Changeable Speeds, IACO

Introduction

The purpose of this paper is to present and solve a new, important planning problem faced by many transport companies. In most of the literature on routing and scheduling problems cargo cannot be transported by more than one vehicle. By introducing split delivery this restriction is removed and each customer can be visited more than once. However, possible economic loss on split delivery such as inventory cost at depots is ignored in this paper.

We minimize the total difference to due date for a fleet that tries to transport all needed cargos to the assigned customers on time. If a vehicle starts unloading at an assigned customer at the earliest possible time on the first day of its time window, the difference to due date will become zero. The total difference to due date is associated with the satisfaction of customers.

Ray et al. [1] presented a centralized model and a heuristic algorithm for solving the MDSDVRP.

Zhang et al. [2] presented a new method for determining routes and schedules considering the total difference to due date and total CO_2 emissions for transportation, as well as split delivery, changeable speed, load factor, and time window constraints, according to a predetermined planning horizon.

In Chen[3], a multi-objective vehicle route program based on IPSO was proposed to solveVRP. The multi-objective model was established based on the objective cost, waiting time cost and penalty cost to select the search success rate, average travel cost and search time as the evaluation index.

Assumptions and Notations

Assumptions

Our model is based on the following assumptions.

(1) There is only one type of cargo but many types of vehicles.

(2) There are multi depots, multi customers and multi vehicles.

(3) Somevehicles may be not available at the beginning of the planning horizon.

(4) Onevehicle can transport cargo from a depot to different customers on a visit.

(5) Onevehicle can begin its transportation from the depots more than once during the planning horizon.

(6) Eachcustomer can be visited more than once
during the planning horizon if necessary.

(7) Loading/Unloading times of cargos are considered.

(8) Waiting is allowed for deliveries arriving ahead of schedule.

(9) Each customer has a time window within which it must be visited.

Notations

Let us define the relevant notations as follows.

Constant

S : Set of depots

D : Set of customers

$N = S \cup D$: Set of locations

$O = \{0\}$: The virtual node

NO : Set of nodes

V : Set of vehicles to be scheduled

U^v : Set of possible visits by vehicle v

H^v : Set of possible speed numbers of vehicle v

$u^v_{\max}$: Maximum possible number of visits by vehicle v

Q_i : The demand/supply of customer/depot i

C^v : Carrying capacity of vehicle v

E^v_i : The earliest possible time to start loading at depot i on the first visit by vehicle v

$l^{v,h}$: The h^{-th} possible speed of vehicle v

${l^{v,h}}_{\min}$:Minimum speed of vehicle v

Y_i : Set of days during which loading/unloading is available at location i

$[a_{i,r}, b_{i,r}]$: Time window of location i on the r^{-th} day

$q_{i,r}$: Accumulated time it is not available for loading/unloading before the earliest possible time to start on the r^{-th} day at location i

T : The predetermined planning horizon

t_1 : Loading time per unit

t_2 : Unloading time per unit

B : A large positive number

λ: A parameter that represents the required load factor, with $0 \le \lambda \le 1$

G_{ij} : Distance between node i and j

Variables

$t^{u,v}_i$: Start time for loading/unloading at location i by vehicle v on its u^{-th} visit

$w^{u,v}_i$: The number of cargos unloaded at customer i by vehicle v on its u^{-th} visit

$x^{u,v}_i$ ($i \in D$) : The number of cargos transported by vehicle v to the assigned customers before reaching customer i or returning to the virtual node on its u^{-th} visit

$x^{u,v}_s$ $(s \in S)$:The number of cargos loaded by vehicle $v$at depot s on its u^{-th} visit

$$I^{1,u,v}_{i,r} = \begin{cases} 1 & \text{if vehicle v starts loading/unloading at location i on the } r^{-th} \text{ day on its } u^{-th} \text{ visit} \\ 0 & \text{otherwise} \end{cases}$$

$$I^{2,u,v}_{i,s} = \begin{cases} 1 & \text{if vehicle v finishes loading/unlaoding at location i on the } s^{-th} \text{ day on its } u^{-th} \text{ visit} \\ 0 & \text{otherwise} \end{cases}$$

$$y^{u,v.h}_{ij} = \begin{cases} 1 & \text{if vehicle v visits node j directly from node i at the } h^{-th} \text{ speed on its } u^{-th} \text{ visit} \\ 0 & \text{otherwise} \end{cases}$$

Objective Functions

Generally, minimization of differences to due date is considered in scheduling and routing problem solving. Therefore, we set an objective function tominimize the total difference to due date for a fleet that tries to transport all needed cargos to the assigned customers on time.

$$F_1 = \sum_{v \in V} \sum_{u \in U^v} \sum_{i \in D} t^{u,v}_i - \sum_{v \in V} \sum_{u \in U^v} \sum_{i \in D} \sum_{r \in Y_i} a_{i,1} I^{1,u,v}_{i,r} \tag{1}$$

In many real-life situations, we need to change speeds to satisfy time window constraints. Lately, speed reduction has become a very popular operational measure to reduce fuel consumption and can obviously be used to curb CO2 emissions. Therefore, we propose an objective function to minimize speeds for transportation.

$$F_2 = \sum_{v \in V} \sum_{u \in U^v} \sum_{h \in H^v} \sum_{i \in NO} \sum_{j \in NO} \{ y^{u,v,h}_{ij} \times (l^{v,h} - {l^{v,h}}_{\min}) \} \tag{2}$$

To solve the above problem, we set up a 0-1 mixed-integer programming model and solve it on Lingo language. Also, we propose IACO to solve this problem more efficiently.

Alogrithm

Selection mechanism

There aremany groups of ants. Each group only includes one kind of vehicle. For instance, in group $K = \{1, 2, \cdots, k, \cdots\}$ all ants share the same pheromone.

Toselect the next ant k, we use the following equation.

$$p_k = \frac{(\omega_k)^\sigma \times \max\{(\tau_{i,j,k})^\alpha\}}{\sum_{c=1}^{m} \{(\omega_c)^\sigma \times \max\{(\tau_{i,j,c})^\alpha\}\}} \quad (k \in K) \tag{3}$$

Where m represents the number of ants. ω_k represents the capacity of ant k, constant $\sigma(\sigma > 0)$ establishes the importance of capacity.

Where$\tau_{i,j,k}$ is defined as pheromone which is shared by ant k on the arc between customer i and j. Constant $\alpha(\alpha > 0)$ establishes the importance of the pheromone.

To select thenext node$j$, ant $k$uses the following equation.

$$\begin{cases} \arg\max\{(\tau_{i,u,k})^\alpha \times (\eta_{i,u})^\beta \times (\sigma_{i,u})^\xi\} (u \notin Tabu) & q < q \\ p_{i,u} & q \geqslant q \end{cases} \tag{4}$$

q is a uniform random number in the range [0, 1], and $q_0$is a constant. $\eta_{i,u}$is defined as the inverse of the waiting time between customer i and u. $\sigma_{i,u}$ is defined as the inverse of the speed penalty. Parameter $\beta(\beta > 1)$establishes the importance of the speed penalty. Parameter $\xi(\xi > 0)$ establishes the importance of the speed penalty.

Where

$$p_{i,u} = \begin{cases} \dfrac{(\tau_{i,u,k})^\alpha \times (\eta_{i,u})^\beta \times (\sigma_{i,u})^\xi}{\sum_{j \notin tabu} \{(\tau_{i,j,k})^\alpha \times (\eta_{i,j})^\beta \times (\sigma_{i,u})^\xi\}} & u \notin tabu \\ 0 & otherwise \end{cases} \tag{5}$$

Updating Information

When ant k visits an arc, the pheromone on the arc is updated with the following equation.

$$\tau_{i,j,k} = (1-\mu)\tau_{i,j,k} + \mu\tau_0 \qquad (6)$$

Where μ ($0<\mu<1$) is a constant that decides the speed of evaporation, and τ_0 is defined as an initial pheromone value assigned to all arcs.

Considering time windows, we update the pheromone on the arc (i,j) that is deleted due to time windows as follows:

$$\tau_{i,j,k} = (1-\mu)\tau_{i,j,k} \qquad (7)$$

After a predetermined number of ants construct a feasible route, global trail updating is performed by adding pheromone to the best visit.

$$\tau_{i,j,k} = (1-\gamma)\tau_{i,j,k} + \gamma L^{-1} \qquad (8)$$

Where γ ($0<\gamma<1$) is a constant that controls the speed of evaporation. L is equal to the value of objective function F_1 or F_2.

Table 1 IACO to solve the problem

```
/ * initialization * /
initthe best solution
For every edge (i,j) and group K do
  τ_{i,j,k} = τ_0
End For
/ * Main Loop * /
While the aggregate demand is not satisfied
    Choose the next ant k according to equation (3)
    Choose the nextnode u according to equation (4)-(5)
    If ant k visits an arc.
        the pheromone on the arc is updated with equation (6)
    End if
End while
If the solution is not feasible
    Update the pheromone of the arcs on the route, according to equation (7)
End if
If the feasible solution is better than the best solution
Record it as the best solution
    For every arc (i,j) do
        Update pheromone trails according to equation (8)
    End for
End if
```

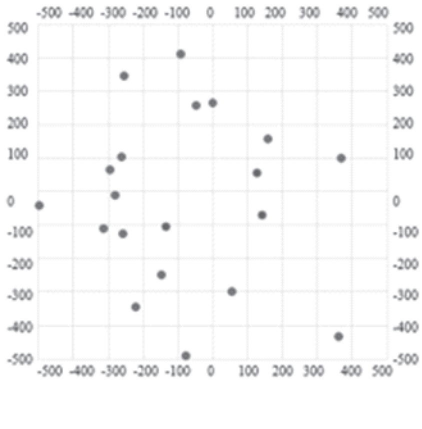
NumericalExample

Input Data

In this case, there are 3 depots, 17 customers and 5 vehicles.

Coordinates of the locations are shown in Table 2.

Table 2 Coordinates of the locations(km)



In Table 2, red dots represent depots while blackdots represent customer nodes. The planning horizon lasts for 10 days, starting from Monday.

$S=\{1,2,3\}$, $D=\{1,2,\cdots,17\}$, $V=\{1,2,\cdots,5\}$, $S_1=1200(unit)$, $S_2=1200(unit)$

$S_3=1250(unit)$, $D_1=200(unit)$, $D_2=250(unit)$

$D_3=400(unit)$, $D_4=250(unit)$, $D_5=300(unit)$

$D_6=200(unit)$, $D_7=300(unit)$, $D_8=200(unit)$

$D_9=150(unit)$, $D_{10}=100(unit)$, $D_{11}=300(unit)$

$D_{12}=200(unit)$, $D_{13}=100(unit)$, $D_{14}=200(unit)$

$D_{15}=300(unit)$, $D_{16}=100(unit)$, $D_{17}=100(unit)$

$E_s^v=8(s=1,2,3;v=1,2,\cdots,5)$

$l^{1,1}=80(km/h)$, $l^{1,2}=90(km/h)$, $l^{1,3}=100$

(km/h)

$l^{2,1}=90(km/h)$, $l^{2,2}=100(km/h)$, $l^{2,3}=110(km/h)$

$l^{3,1}=70(km/h)$, $l^{3,2}=80(km/h)$, $l^{3,3}=90(km/h)$

$l^{4,1}=70(km/h)$, $l^{4,2}=80(km/h)$, $l^{4,3}=90(km/h)$

$l^{5,1}=80(km/h)$, $l^{5,2}=90(km/h)$, $l^{5,3}=100(km/h)$

$T=2400(hr)=10(day)$, $t_1=t_2=0.0075(hr)$

$C^1=300(unit)$, $C^2=400(unit)$, $C^3=450(unit)$

$C^4=350(unit)$, $C^5=300(unit)$

$B=9999$, $\lambda=0.1$, $u^1_{\max}=u^2_{\max}=u^3_{\max}=u^4_{\max}=u^5_{\max}=4$

The time window of depots are as follows:

Loading at depot 1 is available between 8 a. m. and 8 p. m. from Monday to Friday, between 8 a. m. and 2 p. m. on Saturday, and not available on Sunday. In this paper, we can set working times and days off by assigning time windows.

$[a_{1,1},b_{1,1}]=[8,20]$, $[a_{1,2},b_{1,2}]=[32,44]$

$[a_{1,3},b_{1,3}]=[56,68]$, $[a_{1,4},b_{1,4}]=[80,92]$

$[a_{1,5},b_{1,5}]=[104,116]$, $[a_{1,6},b_{1,6}]=[128,134]$

$[a_{1,7},b_{1,7}]=[176,188]$, $[a_{1,8},b_{1,8}]=[200,212]$

$[a_{1,9},b_{1,9}]=[224,236]$

The time window of depot 2, 3 is the same as that of depot 1.

The time window ofcustomers is as follows:

Unloading is available at eachcustomer for two days. The required cargos will be accepted and unloaded within the two-day specified time window of the customer. If a vehicle arrives at the customer ahead of its schedule, it will wait until the customer is ready to unload it.

It is necessary to take a systems approach tosupply chain management (SCM) when planning transport of cargo. Therefore, we must consider requirements of customers when we set working times and days off. For instance, the unloading service is available between 8 a. m. and 8 p. m. at customer 14 on Sunday.

$[a_{1,1},b_{1,1}]=[128,140]$, $[a_{1,2},b_{1,2}]=[152,164]$

$[a_{2,1},b_{2,1}]=[32,44]$, $[a_{2,2},b_{2,2}]=[56,68]$

$[a_{3,1},b_{3,1}]=[32,44]$, $[a_{3,2},b_{3,2}]=[56,68]$

$[a_{4,1},b_{4,1}]=[200,212]$, $[a_{4,2},b_{4,2}]=[224,236]$

$[a_{5,1},b_{5,1}]=[8,20]$, $[a_{5,2},b_{5,2}]=[32,44]$

$[a_{6,1},b_{6,1}]=[80,92]$, $[a_{6,2},b_{6,2}]=[104,116]$

$[a_{7,1},b_{7,1}]=[104,116]$, $[a_{7,2},b_{7,2}]=[128,140]$

$[a_{8,1},b_{8,1}]=[128,140]$, $[a_{8,2},b_{8,2}]=[152,164]$

$[a_{9,1},b_{9,1}]=[104,116]$, $[a_{9,2},b_{9,2}]=[128,140]$

$[a_{10,1},b_{10,1}]=[176,188]$, $[a_{10,2},b_{10,2}]=[200,212]$

$[a_{11,1},b_{11,1}]=[32,44]$, $[a_{7,2},b_{7,2}]=[56,68]$

$[a_{12,1},b_{12,1}]=[104,116]$, $[a_{12,2},b_{12,2}]=[128,140]$

$[a_{13,1},b_{13,1}]=[200,212]$, $[a_{13,2},b_{13,2}]=[224,236]$

$[a_{14,1},b_{14,1}]=[152,164]$, $[a_{14,2},b_{14,2}]=[176,188]$

$[a_{15,1},b_{15,1}]=[176,188]$, $[a_{15,2},b_{15,2}]=[200,212]$

$[a_{16,1},b_{16,1}]=[32,44]$, $[a_{16,2},b_{16,2}]=[56,68]$

$[a_{17,1},b_{17,1}]=[200,212]$, $[a_{17,2},b_{17,2}]=[224,236]$

Output Data

We can obtain the following Pareto-optimal solutions using IACO. We implement the algorithm on a personal computer with Intel Core is i5-5200U CPU @ 2.20GHz 2.19GHz and Windows 10 operating system.

Wecan obtain Pareto-optimal solutions for a similar problem with 2 depots, 7 customers and 3 vehicles, using Lingo. However, no feasible solution has been found for this problem after using Lingo for 120 hours.

The time for calculationsin Table 3 demonstrates that we can make enough Pareto-optimal solutions to make decisions on schedule.

The efficiency of ouralgorithm proposed to determine the transport routes and schedules of transportation with time windows and split delivery in changeable speeds considering multi-objective functions is shown in Table 3.

Table 3 Characteristics of the Pareto-optimal solutions

Algorithm	Steps	The total difference to due date(F_1)	The speed penalty(F_2)	SD	CS	Time of calculation(sec)
IACO	Step1	$F_1^{\min}=45.41$	$F'_2=260(F_1\leqslant F_1^{\min})$	○	○	102.60
		$F'_1=149.02(F_2\leqslant F_2^{\min})$	$F_2^{\min}=190$	○	○	84.81
	Step2	97.10	$230(F_1\leqslant 97.2133)$	○	○	85.41
		$134.03(F_2\leqslant 225)$	220	○	○	100.23

○: Indicates that split delivery (SD) or changeable speeds (CS) happened in the solution.

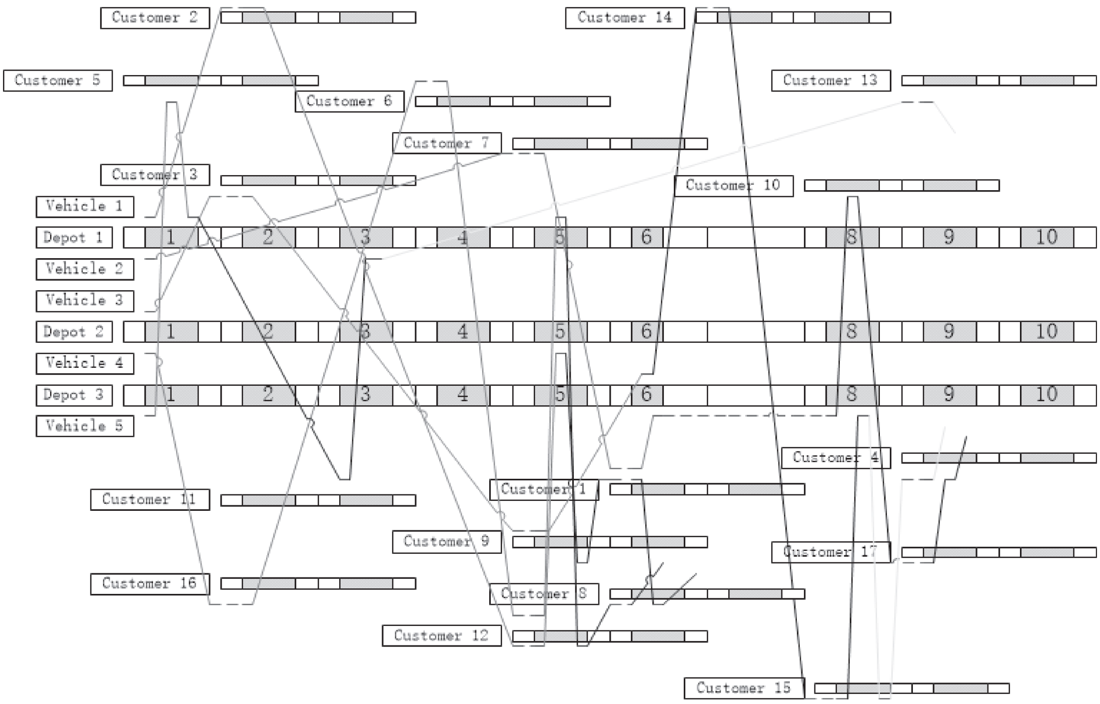


Figure 1 Pareto-optimal transport routes and schedules with minimum total difference to due date

Red lines represent loading/unloading time while blue lines represent waiting time. As depicted in Figure 1, when we calculate the time at which a vehicle is available to start unloading at a customer, we take into account the vehicle arrival time, the unloading time for the required cargos, and the customer's time window. If a vehicle arrives at the assigned customer ahead of schedule, it will wait until the customer is ready to unload it.

In this case, vehicle 5 starts its 1st visit from depot 3, then visits customer 5 at 100km/h to satisfy time windows. On its 2nd and 3rd visits, vehicle 5 travels at minimum speed to reduce fuel consumption. Green lines represent 1st visit, black lines represent 2nd visit, while yellow lines represent 3rd visit of any vehicle in Figure 1.

Customer1,4,8,9,12 and 15 are visited with split loads. If no split loads are allowed in this example, the minimum value of total difference to due date will become 50.64. The minimum value of the speed penalty will become 280. It is clear that introducing split loads is a very effective way to reduce the time penalty and total difference to due date.

The problem is also FSMVRP. Knowledge about split delivery, multi-depot, and time windows can be utilized in VRP and other related problems. Further research is still needed to solve the large-scale scheduling and routing problem more efficiently.

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