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Interrelation Between Safety Factors and Reliability

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Interrelation Between Safety Factors and Reliability: Part 1

Random Actual Stress & Deterministic Yield Stress

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"The factor of safety was a useful invention of the engineer a long time ago that served him well. But it now quite outlived its usefulness and has become a serious threat to real progress in design"

D. Faulkner

"Current structural safety design practices are considered inadequate for future launch vehicles and spacecraft."

V. Verderaime

"[Deterministic safety measures] ignore much information which may be available about uncertainties in structural strengths or applied loads."

R. Melchers

1. Introduction

Attempts at probabilistic interpretation of the deterministic safety factor have been made in the literature despite the fact that the "spirits" of these two approaches are entirely different. Before discussing them, it is instructive to quote some representative excerpts from popular textbooks concerning its definition:

(a) "To allow for accidental overloading of the structure, as well as for possible inaccuracies in the construction and possible unknown variables in the analysis of

the structure, a factor of safety is normally provided by choosing an allowable stress (or working stress) below the proportional limit".

(b)   "Although not commonly used, perhaps a better term for this ratio is factor of ignorance".

(c)   "The need for the safety margin is apparent for many reasons: stress itself is seldom uniform; materials lack the homogeneous properties theoretically assigned to the abnormal loads might occur; manufacturing processes often impart dangerous stresses within the component. These and other factors make it necessary to select working stresses substantially below those known to cause failure".

(d)   "A factor of safety is used in the design of structures to allow for (1) uncertainty of loading. (2) the statistical variation of material strengths, (3) inaccuracies in geometry and theory, and (4) the grave consequences of failure of some structures".

(e)   "Factor of safety (N_{FS}), where $N_{FS} > 1$, is the ratio of material strength (usually ultimate strength or yield point) to actual or calculated stress. Alternatively, factor of safety can be defined as the ratio of load at failure to actual or calculated load. The factor of safety provides a margin of safety to account for uncetainties such as errors in predicting loading of a part, variations in material properties, and differences between the ideal model and actual material behavior" (Wilson, 1997).

(f)   "Choosing the safety factor is often a confusing proposition for the beginning designer. The safety factor can be thought of as a measure of the designer's

uncertainty in the analytical models, failure theories, and material property data used, and should be chosen accordingly ... Nothing is absolute in engineering any more than in any other endeavor. The strength of materials may vary from sample to sample. The actual size of different examples of the "same" parts made in quantity may vary due to manufacturing tolerance. As a result, we should take the statistical distributuions into account in our calculations." (Norton, 2000).

Freudenthal remarks [20] "... it seems absurd to strive for more and more refinement of methods of stress-analysis if in order to determine the dimensions of the structural elements, its results are subsequently compared with so called working stress, derived in a rather crude manner by dividing the values of somewhat dubious material parameters obtained in conventional materials tests by still more dubious empirical numbers called safety factors".

Indeed, it appears to the present writer that in addition to its role as a "safety" parameter for the structure, it is intended as "personal insurance" factor of sorts for the design companies.

2. Four Different Probabilistic Definitions of a Safety Factor

Consider an element subjected to a stress σ. Let it be a random variable, denoted by capital letter Σ, whereas a lower case notation describes the possible values σ that the random variable Σ may take. The strength characteristics say the yield stress may also be designated by upper case notation Σ_y, with σ_y being the possible values Σ_y may take. The various possible definitions of the safety factor s are

$$s_1 = \frac{E(\Sigma_y)}{E(\Sigma)} \qquad (1)$$

which is referred to as the central safety; $E(\Sigma_y)$ denotes the mathematical expectation of Σ_y, while $E(\Sigma)$ is associated with the mathematical expectation of Σ.

On the other hand one can treat the ratio

$$Q = \frac{\Sigma_y}{\Sigma} \qquad (2)$$

as a random variable. Its mathematical expectation

$$s_2 = E(Q) = E\left(\frac{\Sigma_y}{\Sigma}\right) \qquad (3)$$

could be also interpreted as a safety factor. The third possible definition of the safety factor is

$$s_3 = E(\Sigma_y)E\left(\frac{1}{\Sigma}\right) \qquad (4)$$

In specific cases some of the above safety factors may coincide. For example, when the yield stress is random, but the stress is a deterministic quantity, $i.e.$ takes a single value σ with unity probability, we have

$$s_1 = s_2 = s_3 \qquad (5)$$

If stress is random, but yield stress is a deterministic quantity σ_y, then

$$s_2 = s_3 \qquad (6)$$

The fourth definition of the safety factor was proposed by Birger (1970). He considered the probability distribution function of the random variable Q

$$F_Q(q) = \text{Prob}\left(\frac{\Sigma_y}{\Sigma} \leq q\right) \tag{7}$$

Then he demands this function to equal some value p_0:

$$F_Q(q) = \text{Prob}\left(\frac{\Sigma_y}{\Sigma} \leq q\right) = p_0 \tag{8}$$

The value of $q = q_0$ that corresponds to the p_0th fractile of the distribution function $F_Q(q)$ is declared as the safety factor. This implies, that of $p_0 = 0.01$, and say $q_0 = 1$, that in about 99% of the realizations of the structure the deterministic safety factor will be not less than 13.

We are asking ourselves the following question: Can we express the safety factors by probabilistic characterization of the structural performance? The central idea of the probabilistic design of structures is reliability, *i.e.* probability that the structure will perform its mission adequately, as required. In our context the mission itself is defined deterministically, namely we are interested in the event

$$\Sigma < \Sigma_y \tag{9}$$

i.e. that the actual stress is less than the yield stress.

Since both Σ and Σ_y may take values from a finite or infinite range of values, the inequality (9) will not always take place. For some realizations of random variables Σ and Σ_y the inequality may be satisfied, whereas for the other ones it may be violated. Engineers are interested in the probability that the inequality (9) will hold. Such a probability is called *reliability*, denoted by R

$$R = \text{Prob}(\Sigma \leq \Sigma_y) \tag{10}$$

Its complement

$$P_f = 1 - R = \mathrm{Prob}(\Sigma \geq \Sigma_y) \tag{11}$$

is called the probability of failure. It is a probability that the stress will be equal to or will exceed the yield stress. It is understandable that engineers want to achieve a very high reliability, allowing, if at all, a extremely small probability of failure.

It appears, at the first glance, that the approaches, based on the deterministic allocation of the safety factors, or that based on reliability design are totally contradictory. We will pursue this subject in more detail. In this report and its companion (report #2), we discuss particular cases, whereas at a later stage (in our report #3) we will pursue the general case, in which *both* Σ and Σ_y will be treated as random variables.

3. Case 1: Stress Has an Uniform Probability Density, Strength Is Deterministic

Let the stress Σ be a random variable with the uniform probability density

$$f_\Sigma(\sigma) = \begin{cases} \dfrac{1}{\sigma_U - \sigma_L}, & \textit{for } \sigma_L < \sigma < \sigma_U \\ 0, & \textit{otherwise} \end{cases} \tag{12}$$

where σ_L is the lowest value that the stress may take, whereas σ_U is the greatest value the stress may assume. We treat the yield stress Σ_y to be a deterministic quantity, *i.e.* to take a single value σ_y with unity probability. The probability distribution function

$$F_\Sigma(\sigma) = \mathrm{Prob}(\Sigma \leq \sigma) = \int_{-\infty}^{\sigma} f_\Sigma(\alpha)d\alpha \tag{13}$$

reads

$$F_{\Sigma}(\sigma) = \begin{cases} 0, & for \quad \sigma < \sigma_L \\ \dfrac{\sigma - \sigma_L}{\sigma_U - \sigma_L}, & for \quad \sigma_L \leq \sigma < \sigma_U \\ 1, & for \quad \sigma_U \leq \sigma \end{cases} \qquad (14)$$

The reliability reads

$$R = \mathrm{Prob}(\Sigma \leq \Sigma_y) = \mathrm{Prob}(\Sigma \leq \sigma_y) \qquad (15)$$

or, in light of Eq. (13) we get

$$R = F_{\Sigma}(\sigma_y) \qquad (16)$$

In other words, the reliability equals the stress distribution function F_{Σ} evaluated at the yield stress (Fig. 1).
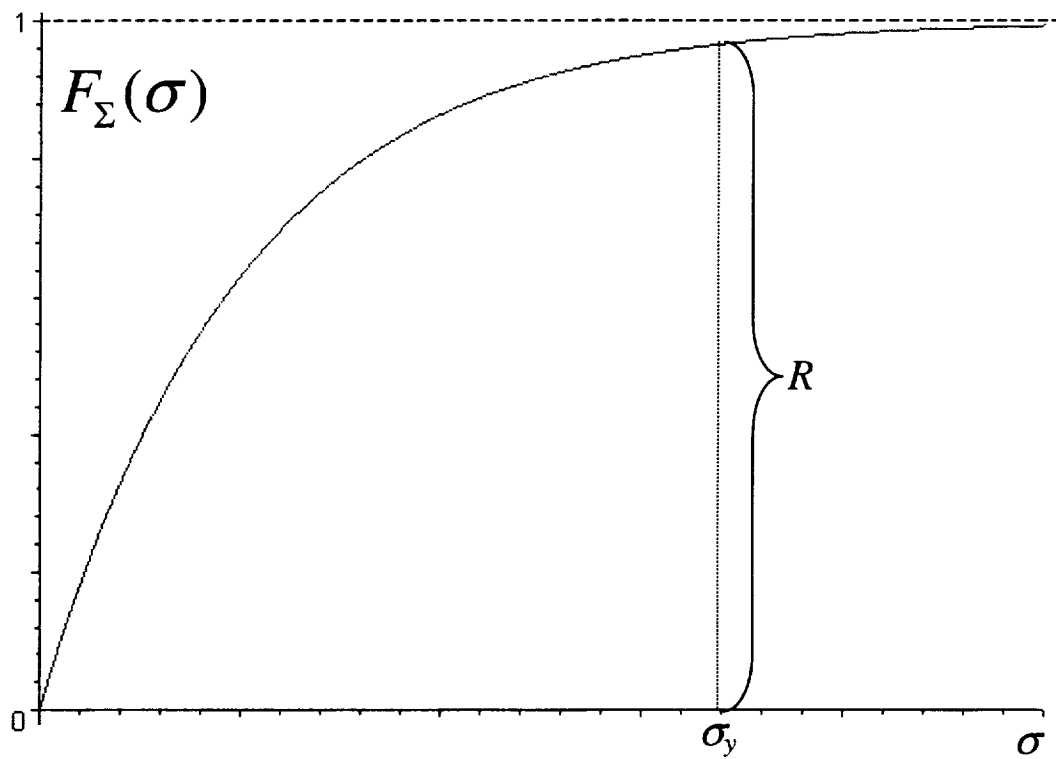
Fig. 1 Reliability equals the probability distribution function of the actual stress evaluated at the level of yield stress

Bearing in mind Eq. (14) we get

$$R = \begin{cases} 0, & for \quad \sigma_y < \sigma_L \\ \dfrac{\sigma_y - \sigma_L}{\sigma_U - \sigma_L}, & for \quad \sigma_L \leq \sigma_y < \sigma_U \\ 1, & for \quad \sigma_U \leq \sigma_y \end{cases} \tag{17}$$

This formula can be rewritten in more convenient form. We note that the mean value of the stress equals

$$E(\Sigma) = \tfrac{1}{2}(\sigma_L + \sigma_U) \tag{18}$$

whereas the variance of the stress equals

$$Var(\Sigma) = \tfrac{1}{12}(\sigma_U - \sigma_L)^2 \tag{19}$$

From these two equations we first express the denominator in Eq. (17)

$$\sigma_U - \sigma_L = \sqrt{12 Var(\Sigma)} \tag{20}$$

as well as the lowest possible value the stress can take

$$\sigma_L = \frac{2E(\Sigma) - \sqrt{12 Var(\Sigma)}}{2} \tag{21}$$
$$= E(\Sigma) - \sqrt{3 Var(\Sigma)}$$

Let

$$\sigma_L \leq \sigma < \sigma_U \tag{22}$$

then, in accordance with Eq. (17), we have

$$R = \frac{\sigma_y - E(\Sigma) - \sqrt{3 Var(\Sigma)}}{2\sqrt{3 Var(\Sigma)}} \tag{23}$$

By dividing both the numerator and the denominator by the mean stress $E(\Sigma)$, and introducing the coefficient of variation of the actual stress

$$v_\Sigma = \frac{\sqrt{Var(\Sigma)}}{E(\Sigma)} \qquad (24)$$

we get, instead of Eq. (23)

$$R = \frac{s_1 - 1 - \sqrt{3}v_\Sigma}{2\sqrt{3}v_\Sigma} \qquad (25)$$

As is seen reliability is *directly* expressed in terms of the central safety factor s_1 and the coefficient of variation of the involved random variable v_Σ. Thus, the reliability methods allow to *rigorously*, rather than *arbitrarily* introduce the safety factors. The safety factor s_1 is expressed from Eq. (25) as follows

$$s_1 = 1 + \sqrt{3}v_\Sigma(1 + 2R) \qquad (26)$$

Maximum value of the safety factor is achieved when the reliability tends to unity from below

$$s_{1,max} \approx 3\sqrt{3}v_\Sigma + 1 \qquad (27)$$

For example, for coefficient of variation 0.05 the safety factor assumes the value 1.26; for the coefficient of variation 0.1 the safety factor equals 1.52; for coefficient of variation 0.15 it takes a value 1.78 *etc*. We conclude that with greater variation of the involved random variable, the safety factor must be *increased*. This *qualitative* conclusion is in line with our anticipation. Yet, it is seen that the reliability context allows one to make *quantitative* judgements in terms of the *required reliability* and the *coefficient of variability*.

4. Case 2: Stress Has an Exponential Probability Density, Yield Stress Is Deterministic

Consider now that the stress has an exponential probability density

$$f_\Sigma(\sigma) = \begin{cases} 0, & \textit{for} \quad \sigma < 0 \\ a\exp(-a\sigma), & \textit{for} \quad \sigma \geq 0 \end{cases} \qquad (28)$$

The corresponding probability distribution function reads

$$F_\Sigma(\sigma) = \mathrm{Prob}(\Sigma \leq \sigma_y) = [1 - \exp(-a\sigma)]U(\sigma) \qquad (29)$$

where $U(\sigma)$ is the unit step function; it equals unity for positive y and vanishes otherwise. The parameter a is reciprocal to mean value of stress

$$E(\Sigma) = \frac{1}{a} \qquad (30)$$

Also, since parameter a is the only free parameter in the density (28) *all* probabilistic moments depend solely upon it. Thus, variance also is expressible in terms of a, as follows:

$$Var(\Sigma) = \frac{1}{a^2} \qquad (31)$$

Since the coefficient of variation

$$v_\Sigma = \frac{\sqrt{Var(\Sigma)}}{E(\Sigma)} = \frac{1/a}{1/a} = 1 \qquad (32)$$

is unity, or 100%, we must anticipate high levels of safety factor, in order to ensure the high level of required reliability. The latter equals, in view of Eq. (19)

$$R = \mathrm{Prob}(\Sigma \leq \sigma_y)$$
$$= [1 - \exp(-a\sigma_y)]U(\sigma_y) \qquad (33)$$

We first express a from Eq. (30) as

$$a = 1/E(\Sigma) \tag{34}$$

and substitute it into Eq. (33), to arrive at

$$R = \left[1 - \exp\left(-\frac{\sigma_y}{E(\Sigma)}\right)\right] U(\sigma_y) \tag{35}$$

In view of the central safety factor s_1, Eq. (35) is rewritten as

$$R = [1 - \exp(-s)] U(\sigma_y) \tag{36}$$

As is seen, a *direct* relationship is being established between the safety factor and the reliability. Once the required reliability is specified the associated safety factor equals

$$s = \ln\frac{1}{1-R} \tag{37}$$

For example, reliability of 0.9 leads to the safety factor 2.3; the reliability of 0.95 results in safety 3 *etc*. Such high values, as indicated above stem from the fact that the stress exponential probability density is associated with high, namely 100% variability. It is immediately seen, that one of the reasons for the high variations in this particular case is the fact that the stress can take any value on the positive axis.

5. Case 3: Stress Has a Rayleigh Probability Density, Yield Stress is Deterministic

Consider now the case in which the stress is characterized by a Rayleigh probability density:

$$f_\Sigma(\sigma) = \begin{cases} 0, & \text{for } \sigma < 0 \\ \dfrac{\sigma}{b^2}\exp(-\dfrac{\sigma^2}{2b^2}), & \text{for } \sigma \geq 0 \end{cases} \tag{38}$$

The approximate probability distribution function is

$$F_\Sigma(\sigma) = \text{Prob}(\Sigma \leq \sigma) = \int_{-\infty}^{\sigma} f_\Sigma(\alpha)d\alpha = \left[1 - \exp\left(-\frac{\sigma^2}{2b^2}\right)\right]U(\sigma) \tag{39}$$

The reliability evaluation reads:

$$R = \text{Prob}(\Sigma \leq \sigma_y) = F_\Sigma(\sigma_y) \tag{40}$$

Again, the reliability equals the probability distribution function of the *stress evaluated at the level of the yield stress*. Hence, in view of Eq. (39)

$$R = \left[1 - \exp\left(-\frac{\sigma_y^2}{2b^2}\right)\right]U(\sigma_y) \tag{41}$$

We would like now to express parameter b in Eqs. (38) and (39) through the probabilistic characterization of the stress:

$$E(\Sigma) = \int_{-\infty}^{\infty} \sigma f_\Sigma(\sigma)d\sigma = \frac{b\sqrt{\pi}}{\sqrt{2}} \approx 1.25b \tag{42}$$

$$Var(\Sigma) = \int_{-\infty}^{\infty} (\sigma - E(\Sigma))^2 f_\Sigma(\sigma)d\sigma$$
$$= \frac{4-\pi}{2}b^2 \approx 0.43b^2 \tag{43}$$

We express b from Eq. (42)

$$b \approx \frac{E(\Sigma)}{1.25} = 0.8E(\Sigma) \tag{44}$$

and substitute it into Eq. (41) to yield

$$R = \left\{1 - \exp\left(-\frac{\sigma_y^2}{2[0.8E(\Sigma)]^2}\right)\right\}U(\sigma_y)$$
$$= \left\{1 - \exp\left(-\frac{0.78125\sigma_y^2}{[E(\Sigma)]^2}\right)\right\}U(\sigma_y) \tag{45}$$

We take into account the definition of the central safety factor to get

$$R = [1 - \exp(-0.78125s_1^2)]U(\sigma_y) \qquad (46)$$

This formula allows to express the safety factor by the reliability

$$s \approx 1.13\sqrt{\ln\frac{1}{1-R}} \qquad (47)$$

Thus, the reliability $R = 0.9$ yields in central safety factor 1.71, the reliability of 0.95 results in safety factor 1.96. The required reliability of 0.99 is associated with safety factor 2.42 *etc*. Again, reason for these values is the high coefficient of variation. Indeed, Eqs. (42) and (43) suggest that the coefficient of variation equals:

$$v_\Sigma = \frac{\sqrt{Var(\Sigma)}}{E(\Sigma)} \approx \frac{\sqrt{0.43b^2}}{1.25b} = 0.52 \qquad (48)$$

Although this is a smaller variability than in the case of the stress with exponential probability density, still, hopefully, 52% variation is seldom encountered in practice.

6. Case 4: Stress Has a Normal Probability Density, Yield Stress Is Deterministic

We consider now the case in which the stress is characterized by a normal probability density

$$f_\Sigma(\sigma) = \frac{1}{b\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{\sigma-a}{b}\right)^2\right], \quad -\infty < \sigma < \infty \qquad (49)$$

The distribution function reads

$$F_\Sigma(\sigma) = \frac{1}{b\sqrt{2\pi}} \int_{-\infty}^{\sigma} \exp\left[-\frac{1}{2}\left(\frac{t-a}{b}\right)^2 \right] dt \tag{50a}$$

$$= \Phi\left(\frac{\sigma - a}{b}\right)$$

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left(-\frac{1}{2}t^2 \right) dt \tag{50b}$$

where a is the mean stress, and b is the mean square deviation,

$$E(\Sigma) = a \tag{51}$$

$$Var(\Sigma) = b^2 \tag{52}$$

The reliability equals

$$R = \text{Prob}(\Sigma < \sigma_y) = F_\Sigma(\sigma_y) = \Phi\left(\frac{\sigma_y - a}{b}\right) \tag{53}$$

or with Eq. (50) taken into account

$$R = \Phi\left(\frac{\sigma_y - E(\Sigma)}{\sqrt{Var(\Sigma)}}\right) \tag{54}$$

Dividing both the numerator and the denominator by $E(\Sigma)$ we rewrite Eq. (53) as follows:

$$R = \Phi\left(\frac{s_1 - 1}{v_\Sigma}\right) \tag{55}$$

where s_1 is the central safety factor, v_Σ is the coefficient variation of the stress. Eq. (54)

allows again to express the safety factor via the reliability

$$s_1 = 1 + v_\Sigma \Phi^{-1}(R) \tag{56}$$

where $\Phi^{-1}(R)$ is a function that is inverse to $\Phi(R)$.

We note the following values of the inverse normal probability function (Benjamin and

Cornell, 1970, p. 655)

$$\Phi^{-1}(0.9) = \Phi^{-1}(1-10^{-1}) = 1.28$$

$$\Phi^{-1}(0.99) = \Phi^{-1}(1-10^{-2}) = 2.32$$

$$\Phi^{-1}(0.999) = \Phi^{-1}(1-10^{-3}) = 3.09$$

$$\Phi^{-1}(0.9\ 999) = \Phi^{-1}(1-10^{-4}) = 3.72$$

$$\Phi^{-1}(0.99\ 999) = \Phi^{-1}(1-10^{-5}) = 4.27$$

$$\Phi^{-1}(0.999\ 999) = \Phi^{-1}(1-10^{-6}) = 4.75 \tag{57}$$

$$\Phi^{-1}(0.9\ 999\ 999) = \Phi^{-1}(1-10^{-7}) = 5.20$$

$$\Phi^{-1}(0.99\ 999\ 999) = \Phi^{-1}(1-10^{-8}) = 5.61$$

$$\Phi^{-1}(0.999\ 999\ 999) = \Phi^{-1}(1-10^{-9}) = 6.00$$

$$\Phi^{-1}(0.9\ 999\ 999\ 999) = \Phi^{-1}(1-10^{-10}) = 6.36$$

$$\Phi^{-1}(0.99\ 999\ 999\ 999) = \Phi^{-1}(1-10^{-11}) = 6.71$$

Thus, the safety factor becomes, for the coefficient of variation equal 0.05, respectively

$$s_1 = 0.05\Phi^{-1}(0.9) = 1.064, \quad for \quad R = 1-10^{-1}$$

$$s_1 = 0.05\Phi^{-1}(0.99) = 1.116, \quad for \quad R = 1-10^{-2}$$

$$s_1 = 0.05\Phi^{-1}(0.999) = 1.155, \quad for \quad R = 1-10^{-3}$$

$$s_1 = 0.05\Phi^{-1}(0.9\ 999) = 1.186, \quad for \quad R = 1-10^{-4}$$

$$s_1 = 0.05\Phi^{-1}(0.99\ 999) = 1.2135, \quad for \quad R = 1-10^{-5}$$

$$s_1 = 0.05\Phi^{-1}(0.999\ 999) = 1.2375, \quad for \quad R = 1-10^{-6} \tag{58}$$

$$s_1 = 0.05\Phi^{-1}(0.9\ 999\ 999) = 1.26, \quad for \quad R = 1-10^{-7}$$

$$s_1 = 0.05\Phi^{-1}(0.99\ 999\ 999) = 1.2805, \quad for \quad R = 1-10^{-8}$$

$$s_1 = 0.05\Phi^{-1}(0.999\ 999\ 999) = 1.3, \quad for \quad R = 1-10^{-9}$$

$$s_1 = 0.05\Phi^{-1}(0.9\ 999\ 999\ 999) = 1.318, \quad for \quad R = 1-10^{-10}$$

$$s_1 = 0.05\Phi^{-1}(0.99\ 999\ 999\ 999) = 1.3355, \quad for \quad R = 1-10^{-11}$$

As is een, there is direct relationship between the safety factor and required reliability. One can suggest asymptotic relationship between the safety factor and reliability. One observes from Eq. (57) that the knowledge of the coefficient of variation (the ratio

between the standard deviation and the mean) and required reliability directly yields the level of the required safety factor.

7. Case 5: Actual Stress Has a Log-Normal Probability Density, Yield Stress is Deterministic

Consider now the case in which the actual stress Σ is distributed log-normally, with the following probability density function:

$$f_\Sigma(\sigma) = \frac{1}{\sigma b_\Sigma \sqrt{2\pi}} \exp\left[-\frac{(\ln \sigma - a_\Sigma)^2}{2b_\Sigma^2} \right], \quad \sigma > 0 \tag{59}$$

and vanishes otherwise. The mean value of the stress equals

$$E(\Sigma) = \exp(a_\Sigma + \tfrac{1}{2} b_\Sigma^2) \tag{60}$$

whereas the variance reads:

$$Var(\Sigma) = \exp(2a_\Sigma + b_\Sigma^2)[\exp(b_\Sigma^2) - 1] \tag{61}$$

The reliability equals

$$R = \mathrm{Prob}(\Sigma \leq \sigma_y) = F_\Sigma(\sigma_y) \tag{62}$$

The probability distribution function for the log-normal variable Σ is

$$F_\Sigma(\sigma) = \int_0^\sigma \frac{1}{t b_\Sigma \sqrt{2\pi}} \exp\left[-\left(\frac{\ln t - a_\Sigma}{b_\Sigma}\right)^2 \right] dt \tag{63}$$

We make a substitution

$$\frac{\ln t - a_\Sigma}{b_\Sigma} = z \tag{64}$$

to obtain

$$F_{\Sigma}(\sigma) = \int_{0}^{\frac{\ln \sigma - a_{\Sigma}}{b_{\Sigma}}} \frac{1}{\sqrt{2\pi}} \exp\left(-\tfrac{1}{2}z^2\right) dt$$

$$= \Phi\left(\frac{\ln \sigma - a_{\Sigma}}{b_{\Sigma}}\right)$$

(65)

In view of Eq. (65), expression for the reliability

$$R = \Phi\left(\frac{\ln \sigma_y - a_{\Sigma}}{b_{\Sigma}}\right)$$

(66)

Central safety factor equals

$$s_1 = \frac{\sigma_y}{E(\Sigma)} = \frac{\sigma_y}{\exp(a_{\Sigma} + \tfrac{1}{2}b_{\Sigma}^2)}$$

(67)

Knowing parameters a_{Σ} and b_{Σ} determines both the central safety factor and the reliability.

On the other hand, if $E(\Sigma)$ and $Var(\Sigma)$ are given, one needs the formula of transformation from Eqs. (60) and (61). We substitute Eq. (67) into Eq. (66) to get

$$R = \Phi\left(\frac{\ln \sigma_y - \ln E(\Sigma) - \dfrac{1}{2}\left\{\ln[E^2(\Sigma) + Var(\Sigma)] - \ln[E^2(\Sigma)]\right\}}{\sqrt{\ln[E^2(\Sigma) + Var(\Sigma)] - \ln[E^2(\Sigma)]}}\right)$$

(68)

Thus, the find formula can be rewritten as:

$$R = \Phi\left(\frac{\ln s_1 - \tfrac{1}{2}[\ln(1 + v_{\Sigma}^2)]}{\sqrt{\ln(1 + v_{\Sigma}^2)}}\right)$$

(69)

We are unaware of the other derivation of this expression.

8. Case 6: Actual Stress Has a Weibull Probability Density, Strength is Deterministic

Let us study the case of the probability of the distribution function stress

$$F_\Sigma(\sigma) = \exp\left[-\exp\left(\frac{a_\Sigma - \sigma}{b_\Sigma}\right)\right] \tag{70}$$

The reliability, therefore, is given by

$$R = F_\Sigma(\sigma_y) = \exp\left[-\exp\left(\frac{a_\Sigma - \sigma_y}{b_\Sigma}\right)\right] \tag{71}$$

According to Haldar and Mahadevan (2000) who do not deal with the material in this section, but use the Weibull distribution, the mean value and the variance can be expressed via a_Σ and b_Σ analytically. In our setting their formulas read:

$$\frac{1}{b_\Sigma} = \frac{1}{\sqrt{\sigma}} \frac{\pi}{\sqrt{Var(\Sigma)}} \tag{72}$$

$$a_\Sigma = E(\Sigma) - 0.5772 b_\Sigma$$

Thus,

$$E(\Sigma) = a_\Sigma + 0.5772 b_\Sigma \tag{73}$$

$$Var(\Sigma) = \frac{\pi^2}{6} b_\Sigma{}^2 \tag{74}$$

Reliability becomes:

$$R = \exp\left[-\exp\left(\frac{E(\Sigma) - 0.5772\dfrac{\sqrt{6Var(\Sigma)}}{\pi} - \sigma_y}{\dfrac{\sqrt{6Var(\Sigma)}}{\pi}}\right)\right] \tag{75}$$

or, dividing both numerator and denominator by $E(\Sigma)$ and recalling definition of the central safety factor $s_1 = \sigma_y / E(\Sigma)$ and of the coefficient of variation of the actual stress $v_\Sigma = \sqrt{Var(\Sigma)} / E(\Sigma)$ we get

$$R = \exp\left[-\exp\left(\frac{1 - 0.45 v_\Sigma - s_1}{0.78 v_\Sigma} \right) \right] \qquad (76)$$

This formula too apparently is given for the first time. It connects the reliability with the central safety factor s_1 and the variability v_Σ. Conversely, if the required reliability is specified, one can directly determine the safety factor

$$s_1 = 1 - v_\Sigma\left[0.78 \ln\left(\ln\frac{1}{R} \right) - 0.45 \right] \qquad (77)$$

The following values are obtained for $v_\Sigma = 0.05$:

$$
\begin{aligned}
R &= 0.9, & s_1 &= 1.11 \\
R &= 0.95, & s_1 &= 1.14 \\
R &= 0.99, & s_1 &= 1.18 \\
R &= 0.999, & s_1 &= 1.29 \\
R &= 0.9\ 999, & s_1 &= 1.36 \\
R &= 0.99\ 999, & s_1 &= 1.45
\end{aligned} \qquad (78)
$$

For $v_\Sigma = 0.1$ we get

$$
\begin{aligned}
R &= 0.9, & s_1 &= 1.22 \\
R &= 0.95, & s_1 &= 1.28 \\
R &= 0.99, & s_1 &= 1.36 \\
R &= 0.999, & s_1 &= 1.58 \\
R &= 0.9\ 999, & s_1 &= 1.72 \\
R &= 0.99\ 999, & s_1 &= 1.90
\end{aligned} \qquad (79)
$$

9. Conclusion

As is observed from this report the use of the safety factor is not contradictory to the employment of the probabilistic methods. Moreover, in many cases the safety factors can be directly expressed by the required reliability levels. However, there is a major difference that must be emphasized: whereas the safety factors are allocated in an *ad hoc* manner, the probabilistic approach offers a unified mathematical framework. The establishment of the interrelation between the concepts opens an avenue for rational of safety factors, based on reliability.

If there are several forms of failure then the allocation of safety factors should be based on having the *same* reliability associated with each failure modes. This immediately suggests, that by the probabilistic methods the existing overdesign or underdesign can be eliminated.

This is done by calibration of the reliability levels with one of the safety factors that is already accepted. Thus, via such an approach, the other failure modes' safety factors can be established.

This is illustrated fig. 2, which shows that presently safety factor are assigned in an *ad hoc* manner to each failure mode, but there is no interrelation between them. Fig. 3 illustrates the consistent allocation of the safety factors can be performed.

The report No. 2 deals with the reverse case, namely when the actual stress is deterministic, but the yield stress is random. Report No. 3 will discuss the general case in which *both* the actual stress and the yield stress are treated as random quantities with the attendant interrelationship between the reliability and safety factors.
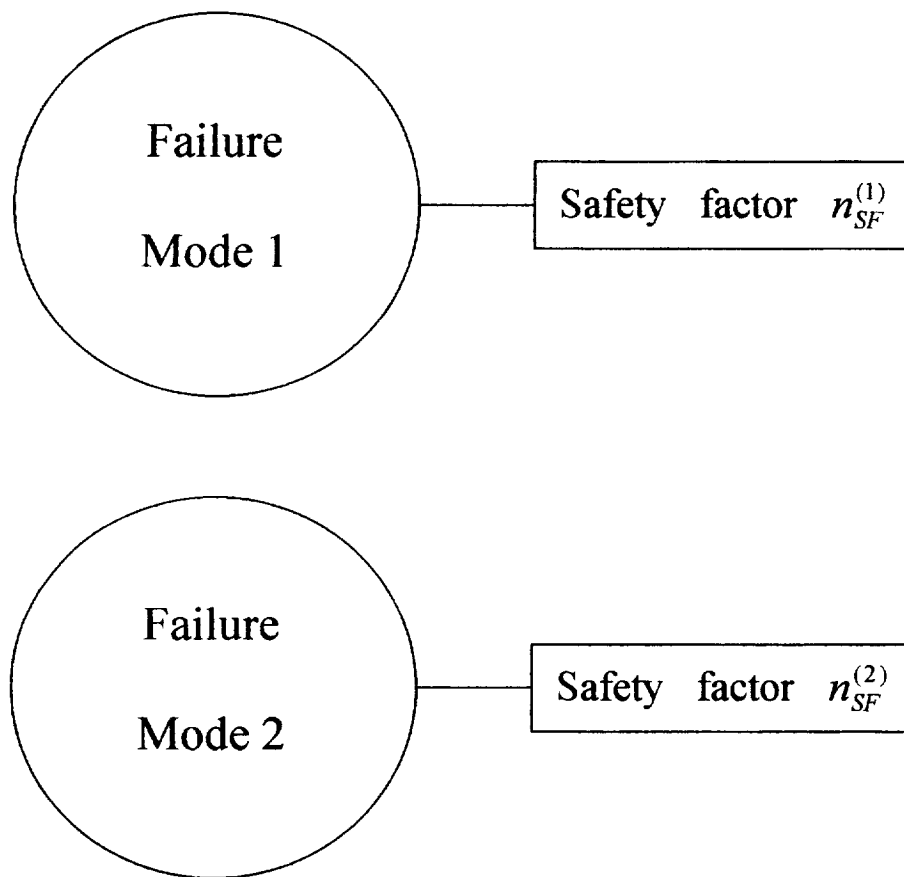
Fig. 2 Present Status: No Connection Between Safety Factors, Leading to Overdesign or Underdesign
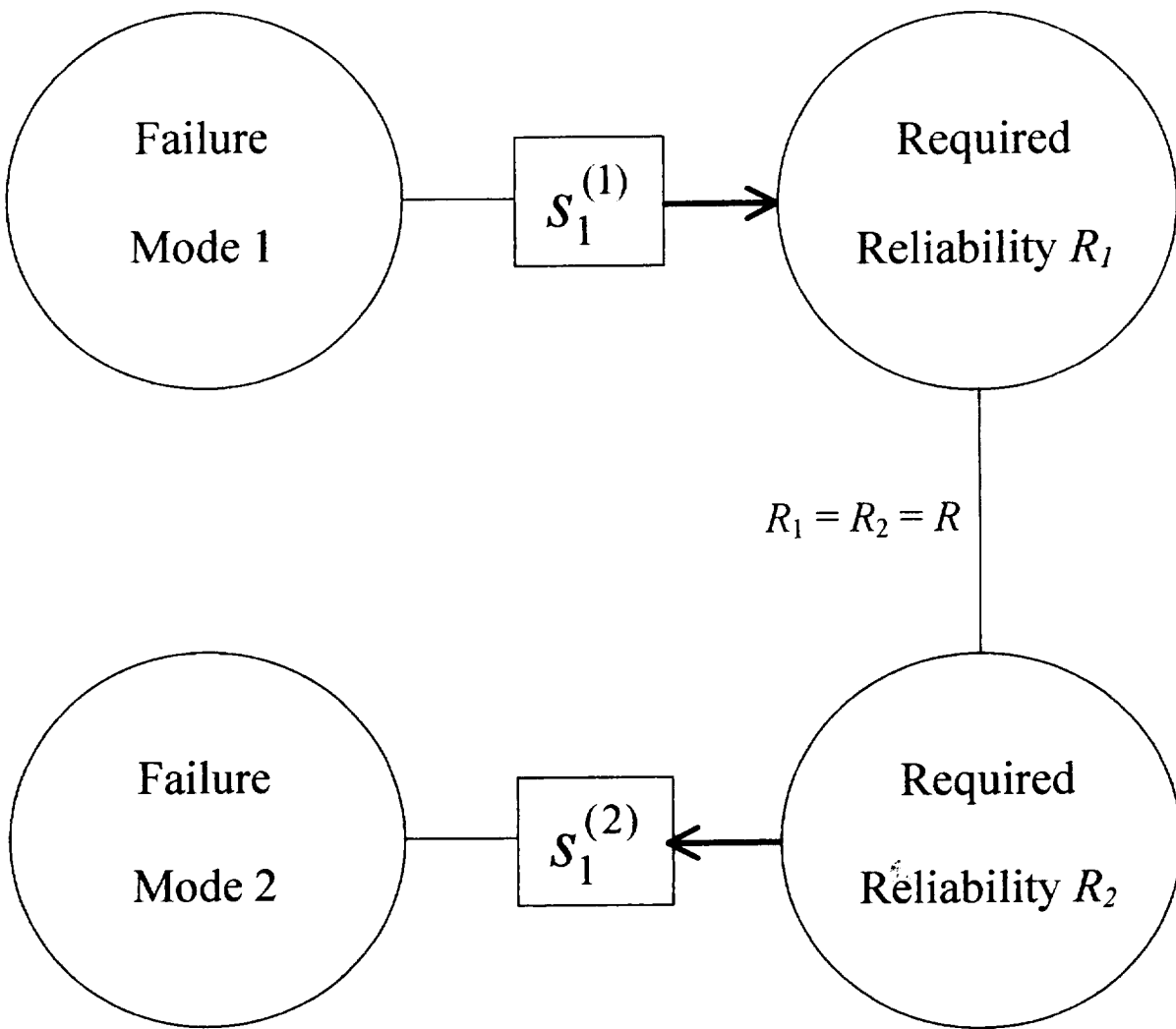
Fig. 3 Future Status: Equal Reliability Allocation May Connect Dafety Factors

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Interrelation Between Safety Factors and Reliability: Part 2

Deterministic Actual Stress & Random Yield Stress

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In the previous report we studied the case in which the actual stress was treated as a random variable, while the yield stress was considered as a deterministic quantity. In this report we investigate the reverse case, namely, when the actual stress is deterministic, while the yield stress is treated as a random variable. Various probability densities to model the actual behavior of the structural element in question are considered.

Case 1: Yields Stress Has an Uniform Probability Density, Actual Stress is Deterministic

Let the yield stress have an uniform probability density

$$f_{\Sigma_y}(\sigma_y) = \begin{cases} \dfrac{1}{\sigma_{y,U} - \sigma_{y,L}} & for \quad \sigma_{y,L} < \sigma < \sigma_{y,U} \\ 0, & otherwise \end{cases} \tag{1}$$

where $\sigma_{y,L}$ is the lower possible level the yield stress may take; $\sigma_{y,U}$ is the upper possible level the yield stress may assume.

The probability distribution function of the yield stress reads

$$F_{\Sigma_y}(\sigma_y) = \begin{cases} 0, & \text{for } \sigma_y < \sigma_{y,L} \\ \dfrac{\sigma_y - \sigma_{y,L}}{\sigma_{y,U} - \sigma_{y,L}} & \text{for } \sigma_{y,L} \le \sigma_y < \sigma_{y,U} \\ 1, & \text{for } \sigma_{y,U} \le \sigma_y \end{cases} \qquad (2)$$

The reliability equals

$$R = \mathrm{Prob}(\Sigma \le \Sigma_y) = \mathrm{Prob}(\sigma \le \Sigma_y)$$
$$= \mathrm{Prob}(\Sigma_y \ge \sigma) = 1 - \mathrm{Prob}(\Sigma_y \le \sigma) \qquad (3)$$

Thus, in view Eq. (2), we get

$$R = \begin{cases} 1, & \text{for } \sigma_y < \sigma_{y,L} \\ 1 - \dfrac{\sigma - \sigma_{y,L}}{\sigma_{y,U} - \sigma_{y,L}} & \text{for } \sigma_{y,L} \le \sigma_y < \sigma_{y,U} \\ 0, & \text{for } \sigma_{y,U} \le \sigma_y \end{cases} \qquad (4)$$

Consider the case in which the yield stress belongs to the interval $[\sigma_{y,L}, \sigma_{y,U}]$. In this case from Eq. (4) we have for the reliability

$$R = \frac{\sigma_{y,U} - \sigma}{\sigma_{y,U} - \sigma_{y,L}} \qquad (5)$$

We note that the mean yield stress equals

$$E(\Sigma_y) = \tfrac{1}{2}(\sigma_{y,L} + \sigma_{y,U}) \qquad (6)$$

whereas the variance of the yield stress reads

$$Var(\Sigma_y) = \tfrac{1}{12}(\sigma_{y,U} - \sigma_{y,L})^2 \qquad (7)$$

We express upper level of the yield stress $\sigma_{y,U}$ as follows, in terms of the mean yield stress $E(\Sigma_y)$ and variance of the yield stress $Var(\Sigma_y)$ via Eqs. (6) and (7):

$$\sigma_{y,U} = \frac{2E(\Sigma_y) + \sqrt{12Var(\Sigma_y)}}{2}$$

$$= E(\Sigma_y) + \sqrt{3Var(\Sigma_y)} \tag{8}$$

The denominator in Eq. (5) is directly expressible by the variance as $2\sqrt{3Var(\Sigma_y)}$ in Eq. (7). Thus, the reliability in Eq. (5) can be rewritten as

$$R = \frac{E(\Sigma_y) + \sqrt{3Var(\Sigma_y)} - \sigma}{2\sqrt{3Var(\Sigma_y)}} \tag{9}$$

We divide both the numerator and denominator by σ and express the ration

$$\frac{\sqrt{3Var(\Sigma_y)}}{\sigma} = \frac{\sqrt{3Var(\Sigma_y)}}{E(\Sigma_y)} \frac{E(\Sigma_y)}{\sigma}$$

$$= \sqrt{3} v_{\Sigma_y} s_1 \tag{10}$$

where

$$v_{\Sigma_y} = \sqrt{Var(\Sigma_y)} / E(\Sigma_y) \tag{11}$$

is the coefficient of variation of the yield stress,

$$s_1 = E(\Sigma_y) / \sigma \tag{12}$$

is the central safety factor:

$$R = \frac{s_1(1 + \sqrt{3} v_{\Sigma_y}) - 1}{2\sqrt{3} v_{\Sigma_y} s_1} \tag{13}$$

Eq. (13) allows to express the central safety factor as a function of the reliability:

$$s_1 = \frac{1}{1 + \sqrt{3} v_{\Sigma_y} (1 - 2R)} \tag{14}$$

This equation is remarkable for the required reliability R is *directly* connected with the central *safety factor* s_1. Thus, if the required reliability 0.9 is set, at the coefficient of

variation of the yield stress $v_{\Sigma_y} = 0.05$ we get the level of safety factor equal 1.07; for $R = 0.99$ we get $s_1 = 1.09$; reliability level 0.999 corresponds to $s_1 = 1.095$. At the greater coefficient of variation, namely, that comprising 10% we get

$$
\begin{aligned}
s_1 &= 1.16, \quad for \quad R = 0.9 \\
s_1 &= 1.20, \quad for \quad R = 0.99 \\
s_1 &= 1.21, \quad for \quad R = 0.999
\end{aligned}
\tag{15}
$$

When the variability constitutes 20%, we obtain

$$
\begin{aligned}
s_1 &= 1.38, \quad for \quad R = 0.9 \\
s_1 &= 1.51, \quad for \quad R = 0.99 \\
s_1 &= 1.53, \quad for \quad R = 0.999
\end{aligned}
\tag{16}
$$

etc. yielding greater needed safety factors with greater variability, if the demanded reliability level is fixed.

Case 2: Yield Stress Has an Exponential Probability Density, Actual Stress Is Deterministic

Consider now the case in which Σ_y is variable with exponential probability density but Σ is deterministic. Hence Σ takes only a single value σ with unity probability.

The probability density of Σ_y reads:

$$
f_{\Sigma_y}(\sigma_y) = \begin{cases} 0, & for \quad \sigma_y < 0 \\ a\exp(-a\sigma_y), & for \quad \sigma_y > 0 \end{cases}
\tag{17}
$$

Here f_{Σ_y} is the probability density of the yield stress.

The probability distribution function of Σ_y is defined as (Fig. 1)

$$
F_{\Sigma_y}(\sigma_y) = \mathrm{Prob}(\Sigma_y \le \sigma_y)
\tag{18}
$$

i.e. as a probability that Σ_y will take values that are not in excess of any pre-selected value σ_y.

According to the definition of the probability distribution

$$F_{\Sigma_y}(\sigma_y) = \int_{-\infty}^{\sigma_y} f_{\Sigma_y}(t)dt \tag{19}$$

we get

$$F_{\Sigma_y}(\sigma_y) = \begin{cases} 0, & \textit{for} \quad \sigma_y < 0 \\ 1 - \exp(-a\sigma_y), & \textit{for} \quad \sigma_y \geq 0 \end{cases} \tag{20}$$

The parameter a is the reciprocal of the mathematical expectation

$$E(\Sigma_y) = \frac{1}{a} \tag{21}$$

The reliability reads

$$\begin{aligned} R = \mathrm{Prob}(\Sigma \leq \Sigma_y) &= \mathrm{Prob}(\sigma \leq \Sigma_y) \\ &= \mathrm{Prob}(\Sigma_y \geq \sigma) = 1 - \mathrm{Prob}(\Sigma_y \leq \sigma) \end{aligned} \tag{22}$$

In the right side of the equation (20) we recognize that the quantity Prob($\Sigma_y \leq \sigma$) coincides with Eq. (16) when instead of σ_y in Eq. (16) we substitute σ. In other words Prob($\Sigma_y \leq \sigma$) equals the probability distribution function of the yield stress evaluated at the level of the actual stress (Fig. 2):

$$\mathrm{Prob}(\Sigma_y \leq \sigma) = F_{\Sigma_y}(\sigma) \tag{23}$$

Thus, bearing in mind Eq. (18) we get

$$\mathrm{Prob}(\Sigma_y \leq \sigma) = \begin{cases} 0, & \textit{for} \quad \sigma \leq 0 \\ 1 - \exp(-a\sigma), & \textit{for} \quad \sigma \geq 0 \end{cases} \tag{24}$$
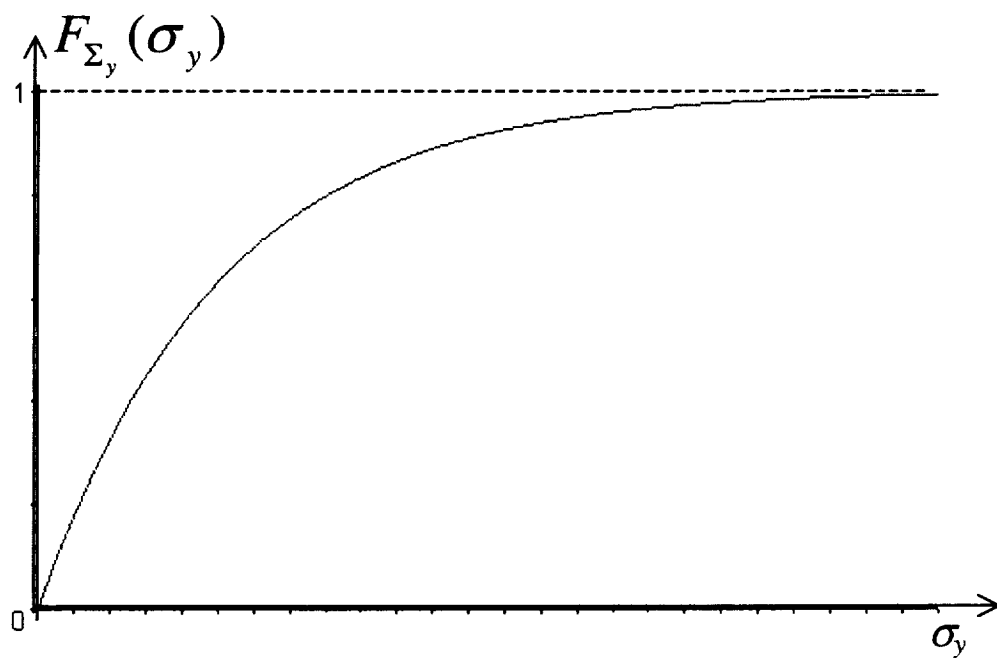
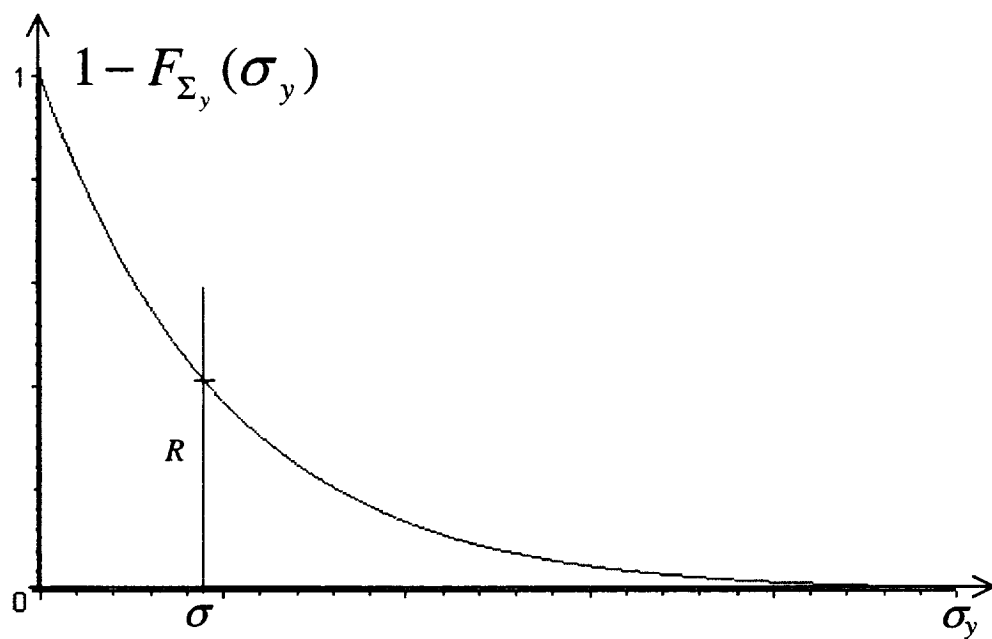**Fig. 1 Probability distribution of the yield stress**

Fig. 1 Reliability equals the function $1 - F_{\Sigma_y}(\sigma_y)$ **evaluated at the actual stress**

34

Hence, the reliability in Eq. (20) becomes

$$R = 1 - \text{Prob}(\Sigma_y \leq \sigma) = 1 - F_{\Sigma_y}(\sigma)$$
$$= 1 - [1 - \exp(-a\sigma)]U(\sigma)$$

(25)

where $U(\sigma)$ is the unit step function, *i.e.* $U(\sigma)$ equals unity for positive σ and vanishes otherwise. Taking into account the relationship (21) we can rewrite Eq. (20) in the following manner

$$R = \left\{ \exp\left[-\frac{\sigma}{E(\Sigma_y)} \right] \right\} U(\sigma)$$

(26)

We recognize the argument in Eq. (26) to be reciprocal of the safety factor. Due to Eq. (5), of the report 1, three safety factors coincide in this case. Hence we denote them by a single notation s. Thus we get the following relationship:

$$R = \exp(-1/s)U(\sigma)$$

(27)

As we see, reliability is intimately connected with the safety factor in the case under consideration. In fact, the safety factor can be expressed directly from Eq. (22) for $\sigma \geq 0$.

$$s = -\frac{1}{\ln(R)} = \frac{1}{\ln(\frac{1}{R})}$$

(28)

In this particular case if the required reliability equals 0.9 the safety factor $1/ln(0.9)$ is greater than 9!

The results in this case, although may seem to be very surprising, are quite understandable. The variance of the yield stress

$$Var(\Sigma_y) = \frac{1}{a^2}$$

(29)

The coefficient of variation in this case

$$c.o.v. = \frac{\sqrt{Var(\Sigma_y)}}{E(\Sigma_y)} = \frac{1/a}{1/a} = 1 \tag{30}$$

equals unity; *i.e.* there is a large variation around the mean value of the yield stress; hence, large safety factors are needed to achieve the required reliability levels.

Case 3: Strength Has a Rayleigh Probability Density, Actual Stress Is Deterministic

The probability density of the strength is given by

$$f_{\Sigma_y}(\sigma_y) = \begin{cases} 0, & \text{for } \sigma_y < 0 \\ \frac{\sigma_y}{b^2} \exp\left(-\frac{\sigma_y^2}{2b^2}\right), & \text{for } \sigma_y \geq 0 \end{cases} \tag{31}$$

with parameter b^2. The distribution function is

$$F_{\Sigma_y}(\sigma_y) = \left[1 - \exp\left(-\frac{\sigma_y^2}{2b^2}\right)\right]U(\sigma_y) \tag{32}$$

We also note that the mean strength is (Ref. 1, p75)

$$E(\Sigma_y) = \frac{b\sqrt{\pi}}{\sqrt{2}} \approx 1.25b \tag{33}$$

whereas the variance of the strength is (Ref. 1, p75)

$$Var(\Sigma_y) = \frac{(4-\pi)b^2}{2} \approx 0.43b^2 \tag{34}$$

Reliability is given by Eq. (3)

$$R = 1 - \Pr ob(\Sigma_y \leq \sigma) = 1 - F_{\Sigma_y}(\sigma) \tag{35}$$

Bearing in mind Eq. (27) we get:

$$R = \left[\exp\left(-\frac{\sigma^2}{2b^2} \right) \right] U(\sigma) \tag{36}$$

Now, taking into account Eq. (33) we can substitute instead of b,

$$b \approx \frac{E(\Sigma_y)}{1.25} = 0.8E(\Sigma_y) \tag{37}$$

to get

$$R = U(\sigma) \exp\left\{ -\frac{\sigma^2}{2[0.8E(\Sigma_y)]^2} \right\} \tag{38}$$

or, in terms of the safety factor

$$s = \frac{E(\Sigma_y)}{\sigma}, \tag{39}$$

we obtain

$$R = U(\sigma) \exp\left\{ -\frac{0.78125}{s^2} \right\} \tag{40}$$

Safety factor s can be expressed from the reliability

$$s = \sqrt{\frac{0.78125}{\ln(1/R)}} = \frac{0.8839}{\sqrt{\ln(1/R)}} \tag{41}$$

Let the reliability be set at $R = 0.99$. Eq. (40) yields safety factor 8.82. This result is again understandable since the coefficient of variation in this case too is quite large:

$$c.o.v. = \frac{\sqrt{Var(\Sigma_y)}}{E(\Sigma_y)} \approx \frac{\sqrt{0.43b^2}}{1.25b} = 0.52 \tag{42}$$

Case 4: Various Factors of Safety in Buckling

It is best to start with an engineering example, first in the deterministic setting. Consider an element that is simply supported at its ends. It is subjected to the compressive load P at the ends, as well as the concentrate bending moment. M. The section modulus of the cross section is denoted by S. Material's proportionality limit σ_{pl} as well as the yield stress σ_y are given. We are interested in determining the safety factor in 3 different regimes.

(a)   During Use of the Element Both M and P Increase Simultaneously
In this case we have

$$\sigma_{max} = \frac{M_{max}}{S} + \frac{P}{A} \qquad (43)$$

where M_{max} the maximum bending moment

$$M_{max} = \frac{M}{\cos\dfrac{kL}{2}} \qquad (44)$$

where

$$k = \sqrt{\frac{P}{EI}} \qquad (45)$$

A = cross sectional area.

Since the relationship between the stresses and the load P is nonlinear, the safety factor n_{SF} is determined as follows. We multiply the load by n_{SF} so as to achieve a level of stress equal to the yield stress. Thus, the deterministic safety factor is derived from

$$\sigma_y = \frac{n_{SF}M}{S\cos\dfrac{k_yL}{2}} + \frac{n_{SF}P}{A} \tag{46}$$

where

$$k_y = \sqrt{\frac{n_{SF}P}{EI}} \tag{47}$$

Consider now the probabilistic setting of the problem. Let σ_y be a random variable Σ_y.

Then the central safety factor s_1 is determined from the equation, in the manner, analogous to the deterministic setting, except that σ_y is replaced by $E(\Sigma_y)$, and n_{SF} is replaced by s_1.

$$E(\Sigma_y) = \frac{s_1M}{S\cos\dfrac{k_yL}{2}} + \frac{s_1P}{A} \tag{48}$$

where

$$k_y = \sqrt{\frac{s_1P}{EI}} \tag{49}$$

(b) During Use of the Element the Axial Force Remains Constant, Concentrated Moment Increases

Deterministic safety factor n_{SF} is found from the equation:

$$\sigma_y = \frac{n_{SF}M}{S\cos\dfrac{kL}{2}} + \frac{P}{A} \tag{50}$$

whereas the appropriate probabilistic central safety factor is determined from the equation:

$$E(\Sigma_y) = \frac{s_1 M}{S\cos\dfrac{kL}{2}} + \frac{P}{A} \tag{51}$$

where

$$k = \sqrt{\frac{P}{EI}} \tag{52}$$

(c) *During Use of Element the Concentrated Moment Remains Constant, Axial Force Increases*

The deterministic safety factor n_{SF} is determined from the equation

$$\sigma_y = \frac{M}{S\cos\dfrac{k_y L}{2}} + \frac{n_{SF} P}{A} \tag{53}$$

where

$$k_y^{(c)} = \sqrt{\frac{n_{SF} P}{EI}} \tag{54}$$

The probability analog of this equation reads:

$$E(\Sigma_y) = \frac{M}{S\cos\dfrac{k_y L}{2}} + \frac{s_1 P}{A} \tag{55}$$

where

$$k_y = \sqrt{\frac{s_1 P}{EI}} \tag{56}$$

For example, let $M = 2$ kN.m, $P = 100$ kN. The cross-sectional area is annular with mean diameter $D_m = 10$ cm; thickness $= 0.5$ cm, $L = 3$ m. Then the deterministic safety factors become, in three settings

Case (a): $n_{SF} = 1.85$

Case (b): $n_{SF} = 3.37$

Case (c): $n_{SF} = 2.52$

In probabilistic setting, if $E(\Sigma_y) = 300$ MPa, the same "central" safety factor is obtained. Yet, straightforward application of the definition $E(\Sigma)/\sigma$ would be incorrect, since the load P and the stress σ are not interrelated linearly.

In some cases, a simplified analysis can be performed: Consider the column that is simultaneously subjected to the uniform distributed load q and axial compressive load P. Then the exact analysis of the differential equation

$$EIw''+Pw = M_t \tag{58}$$

where w = displacement, M_t = bending moment due to the transverse load,

$$M_t = \frac{qL}{2}x - \frac{q}{2}x^2 \tag{59}$$

leads to the maximum bending moment

$$M_{max} = \frac{q}{k^2}\frac{1-\cos\dfrac{kL}{2}}{\cos\dfrac{kL}{2}} \tag{60}$$

where $k = \sqrt{P/EI}$. The approximate relationship, as is well known from the applied theory of elasticity reads

$$M_{max} \approx \frac{qL^2/8}{1-\dfrac{P}{P_{cr}}} \tag{61}$$

Thus the stresses would read

$$\sigma = \frac{qL^2/8}{S\left(1-\dfrac{P}{P_{cr}}\right)} + \frac{P}{A} \tag{62}$$

If both q and P increase simultaneously, the safety factor is found from equation

$$\sigma_y = \frac{n_{SF} q L^2 / 8}{S\left(1 - \frac{n_{SF} P}{P_{cr}}\right)} + \frac{n_{SF} P}{A} \tag{63}$$

The probabilistic "central" safety factor is found from the equation

$$E(\Sigma_y) = \frac{s_1 q L^2 / 8}{S\left(1 - \frac{s_1 P}{P_{cr}}\right)} + \frac{s_1 P}{A} \tag{64}$$

and not what would appear appropriate at the first glance:

$$s_1 = \frac{E(\Sigma_y)}{\frac{q L^2 / 8}{S\left(1 - \frac{P}{P_{cr}}\right)} + \frac{P}{A}} \tag{65}$$

The correct equation (64) leads to a quadratic equation for the central safety factor s_1:

$$s_1^2 \frac{SP^2}{AP_{cr}} - s_1 \left(\frac{SPE(\Sigma_y)}{P_{cr}} - \frac{SP}{A} \right) - \frac{q L^2}{8} + SE(\Sigma) = 0 \tag{66}$$

Case 5: Yield Stress Has a Weibull Probability Density, Actual Stress Is Deterministic

Consider now the case when the probability distribution function of the yield stress reads

$$F_{\Sigma_y}(\sigma_y) = \exp\left[-\exp\left(\frac{a_{\Sigma_y} - \sigma_y}{b_{\Sigma_y}} \right) \right] \tag{67}$$

where a_{Σ_y} and b_{Σ_y} are positive parameters.

Reliability becomes

$$R = Prob(\sigma \le \Sigma_y) = Prob(\Sigma_y \ge \sigma) = 1 - Prob(\Sigma_y \le \sigma)$$
$$= 1 - F_{\Sigma_y}(\sigma) \tag{68}$$

thus yielding the reliability as follows:

$$R = \exp\left[-\exp\left(\frac{a_{\Sigma_y} - \sigma}{b_{\Sigma_y}}\right)\right] \tag{69}$$

According to Haldar and Mahadevan (2000) (who do not deal with safety factors in the present context, but discuss the Weibull distribution) the average and variance of Σ_y are directly expressible via a_{Σ_y} and b_{Σ_y}:

$$\frac{1}{b_{\Sigma_y}} = \frac{1}{\sqrt{6}} \frac{\pi}{\sqrt{Var(\Sigma_y)}} \tag{70}$$

$$a_{\Sigma_y} = E(\Sigma_y) - 0.5772 b_{\Sigma_y} \tag{71}$$

Therefore,

$$E(\Sigma_y) = a_{\Sigma_y} + 0.5772 b_{\Sigma_y} \tag{72}$$

$$Var(\Sigma_y) = \frac{\pi^2}{6} b_{\Sigma_y}^2 \tag{73}$$

Substitution into Eq. (69) yields

$$R = \exp\left[-\exp\left(\frac{E(\Sigma_y) - 0.5772 \frac{\sqrt{6Var(\Sigma_y)}}{\pi} - \sigma}{\sqrt{6Var(\Sigma_y)}/\pi}\right)\right] \tag{74}$$

or

$$R = \exp\left[-\exp\left(\frac{E(\Sigma_y) - 0.45\sqrt{6Var(\Sigma_y)} - \sigma}{0.78\sqrt{6Var(\Sigma_y)}}\right)\right] \tag{75}$$

Dividing both the numerator and denominator in Eq. (75) by σ, and recalling the definition of central safety factor $s_1 = E(\Sigma_y)/\sigma$ and variability coefficient of the yield stress $v_{\Sigma_y} = \sqrt{Var(\Sigma_y)}/E(\Sigma_y)$ we get

$$R = \exp\left[-\exp\left(\frac{s_1 - 0.45v_{\Sigma_y} - 1}{0.78v_{\Sigma_y}s_1}\right)\right] \tag{76}$$

If the reliability is fixed, one can find the appropriate safety factor:

$$s_1 = \frac{1 + 0.45v_{\Sigma_y}}{1 - 0.78v_{\Sigma_y}\ln(\ln\frac{1}{R})} \tag{77}$$

Since this formula yields safety factors that are less than unity, the use of the Weibull distribution appears to be questionable in the case in question. This leads to an all-important lesson: Direct randomization of the deterministic problem not always may be advisible.

Conclusion

In this report we dealt with the case that is reverse to that discussed in report #1.

As Mischke (1970) notes, "there is disenchantment with the term factor of safety." Shigley (1970) writes: "One of the unfortunate facts of life is that there are almost no publications of data on the distribution of stress and strength."

We fell that the expert opinions on one hand, and the expert systems on the other, along with the use of accumulated data available in engineering firms, will close the gap between the present safety factor design and the probabilistic approach to its both justification and the rational allocation.

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Interrelation Between Safety Factor and Reliability: Part 3

Both Actual Stress and Yield Stress Are Random

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In this part we consider most realistic case when both the yield stress Σ_y and the actual stress Σ are represented as random variables. The reliability reads:

$$R = \mathrm{Prob}(\Sigma \leq \Sigma_y) \tag{1}$$

We denote by $f_{\Sigma\Sigma_y}(\sigma, \sigma_y)$ a joint probability density function of Σ and Σ_y. Then Eq. (1) becomes

$$R = \iint\limits_{\Sigma \leq \Sigma_y} f_{\Sigma\Sigma_y}(\sigma, \sigma_y)\, d\sigma d\sigma_y = \int_0^\infty \left[\int_\sigma^\infty f_{\Sigma\Sigma_y}(\sigma, \sigma_y)\, d\sigma_y \right] d\sigma \tag{2}$$

or, alternatively,

$$R = \iint\limits_{\Sigma \leq \Sigma_y} f_{\Sigma\Sigma_y}(\sigma, \sigma_y)\, d\sigma d\sigma_y = \int_0^\infty \left[\int_0^{\sigma_y} f_{\Sigma\Sigma_y}(\sigma, \sigma_y)\, d\sigma \right] d\sigma_y \tag{3}$$

where the integration domain extends over the region in which Σ and Σ_y to be independent random variables. We find two formulas, stemming from Eqs (2) and (3), respectively:

$$R = \int_0^\infty [1 - F_{\Sigma_y}(\sigma)] f_\Sigma(\sigma)\, d\sigma \tag{4}$$

$$R = \int_0^\infty F_\Sigma(\sigma_y) f_{\Sigma_y}(\sigma_y) d\sigma_y \tag{5}$$

We will use either of Eqs. (4) or (5) based on convenience of computation.

Case 1: Both Actual Stress and Yield Stress Have Normal Probability Density

Let

$$f_\Sigma(\sigma) = \frac{1}{b_\Sigma \sqrt{2\pi}} \exp\left[-\frac{1}{2} \left(\frac{\sigma - E(\Sigma)}{b_\Sigma} \right)^2 \right] \tag{6}$$

$$f_{\Sigma_y}(\sigma_y) = \frac{1}{b_{\Sigma_y} \sqrt{2\pi}} \exp\left[-\frac{1}{2} \left(\frac{\sigma_y - E(\Sigma_y)}{b_{\Sigma_y}} \right)^2 \right] \tag{7}$$

where

$E(\Sigma)$ = mean value of the actual stress

$b_\Sigma = \sqrt{Var(\Sigma)}$ = standard deviation of the actual stress

$E(\Sigma_y)$ = mean value of the yield stress

$b_{\Sigma_y} = \sqrt{Var(\Sigma_y)}$ = standard deviation of the yield stress

We introduce a new random variable

$$M = \Sigma_y - \Sigma \tag{8}$$

which is called the safety margin. Since Eq. (8) expresses linearly Σ and Σ_y, the safety margin, as a linear function of the normal variables, is also a normal variable with the mean value

$$E(M) = E(\Sigma_y) - E(\Sigma) \tag{9}$$

and standard deviation b_M found as follows

$$b_M = \sqrt{b_\Sigma^2 + b_{\Sigma_y}^2}$$

(10)

The reliability is then

$$R = \mathrm{Prob}(\Sigma \leq \Sigma_y) = \mathrm{Prob}(M \geq 0)$$

$$= \int_0^\infty \frac{1}{b_M \sqrt{2\pi}} \exp\left[-\left(\frac{t - E(M)}{b_M}\right)^2\right] dt$$

(11)

To perform an integration in Eq. (11), we introduce new variable

$$z = \frac{t - E(M)}{b_M}$$

(12)

Hence

$$dt = b_M dz$$

(13)

Also, when $t = 0$, the lower limit of z equals

$$z = \frac{0 - E(M)}{b_M} = -\frac{E(\Sigma_y) - E(\Sigma)}{\sqrt{b_\Sigma^2 + b_{\Sigma_y}^2}}$$

(14)

Hence,

$$R = \frac{1}{\sqrt{2\pi}} \int_{-\frac{E(\Sigma_y)-E(\Sigma)}{\sqrt{b_\Sigma^2 + b_{\Sigma_y}^2}}}^\infty \exp(-z^2/2) dz$$

(15)

Thus, reliability in Eq. (11) becomes

$$R = 1 - \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-\frac{E(\Sigma_y)-E(\Sigma)}{\sqrt{b_\Sigma^2 + b_{\Sigma_y}^2}}} \exp(-z^2/2) dz$$

$$= 1 - \Phi\left(-\frac{E(\Sigma_y) - E(\Sigma)}{\sqrt{b_\Sigma^2 + b_{\Sigma_y}^2}}\right)$$

(16)

This formula can be rewritten in several alternative ways. First of all we note that

$$\frac{E(\Sigma_y) - E(\Sigma)}{\sqrt{b_\Sigma^2 + b_{\Sigma_y}^2}} = \frac{1}{v_M} \tag{17}$$

where v_M is the coefficient of variation of the safety margin. Thus

$$R = 1 - \Phi\left(-\frac{1}{v_M}\right) \tag{18}$$

We also introduce coefficients of variation of the actual stress and the yield stress, respectively,

$$v_\Sigma = \frac{b_\Sigma}{E(\Sigma)} \tag{19}$$

$$v_{\Sigma_y} = \frac{b_{\Sigma_y}}{E(\Sigma_y)} \tag{20}$$

Then,

$$\sqrt{b_\Sigma^2 + b_{\Sigma_y}^2} = E(\Sigma)\sqrt{\frac{b_\Sigma^2}{E^2(\Sigma)} + \frac{b_{\Sigma_y}^2}{E^2(\Sigma)}} = E(\Sigma)\sqrt{v_\Sigma^2 + \frac{b_{\Sigma_y}^2}{E^2(\Sigma_y)}\frac{E^2(\Sigma_y)}{E^2(\Sigma)}}$$

$$= E(\Sigma)\sqrt{v_\Sigma^2 + v_{\Sigma_y}^2 s_1^2} \tag{21}$$

Hence, the reliability in Eq. (16) becomes

$$R = 1 - \Phi\left(-\frac{s_1 - 1}{\sqrt{v_\Sigma^2 + v_{\Sigma_y}^2 s_1}}\right) \tag{22}$$

As is seen the reliability R, the central safety factor s_1 and the variabilities v_Σ and $v_{\Sigma_y}^2$ are directly interrelated.

Case 2: Actual Stress Has an Exponential Density, Yield Stress Has a Normal Probability Density

For the titled case the probability densities of Σ and Σ_y read, respectively,

$$f_\Sigma(\sigma) = a\exp(-a\sigma), \quad for \quad \sigma \geq 0$$

$$f_{\Sigma_y}(\sigma_y) = \frac{1}{b_{\Sigma_y}\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{\sigma_y - E(\sigma_y)}{b_{\Sigma_y}}\right)^2\right], \quad for \quad -\infty \leq \sigma_y \leq \infty \tag{23}$$

We note that

$$E(\Sigma) = \frac{1}{a}, \quad Var(\Sigma) = \frac{1}{a^2} \tag{24}$$

$$E(\Sigma_y) = b_{\Sigma_y}, \quad Var(\Sigma_y) = b_{\Sigma_y}{}^2 \tag{25}$$

We evaluate reliability function as follows

$$R = \int_0^\infty f_{\Sigma_y}(\sigma_y)\left[\int_0^{\sigma_y} f_\Sigma(\sigma)d\sigma\right]d\sigma_y \tag{26}$$

Now, the inner integral equals:

$$\int_0^{\sigma_y} f_\Sigma(\sigma)d\sigma = \int_0^{\sigma_y} a\exp(-a\sigma)d\sigma$$

$$= 1 - \exp(-a\sigma_y)$$

This results in the following evaluation:

$$R = \int_0^\infty \frac{1}{b_{\Sigma_y} \sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\sigma_y - E(\Sigma_y)}{b_{\Sigma_y}}\right)^2\right](1 - e^{-a\sigma_y})d\sigma_y$$

$$= \frac{1}{b_{\Sigma_y} \sqrt{2\pi}} \int_0^\infty \exp\left[-\frac{1}{2}\left(\frac{\sigma_y - E(\Sigma_y)}{b_{\Sigma_y}}\right)^2\right]d\sigma_y \qquad (27)$$

$$- \frac{1}{b_{\Sigma_y} \sqrt{2\pi}} \int_0^\infty \exp\left[-\frac{1}{2}\left(\frac{\sigma_y - E(\Sigma_y)}{b_{\Sigma_y}}\right)^2\right]e^{-a\sigma_y}d\sigma_y$$

$$= 1 - \Phi\left(-\frac{E(\Sigma_y)}{b_{\Sigma_y}}\right) - \frac{1}{b_{\Sigma_y} \sqrt{2\pi}} \int_0^\infty \exp\left\{-\frac{1}{2b_{\Sigma_y}^2}\left[\sigma_y - E(\Sigma_y) + ab_{\Sigma_y}^2 + 2E(\Sigma_y)b_{\Sigma_y}^2 - a^2 b\right.\right.$$

We introduce the following variables

$$t = \frac{\sigma_y - E(\Sigma_y) + ab_{\Sigma_y}^2}{b_{\Sigma_y}} \qquad (28)$$

Hence

$$b_{\Sigma_y}\, dt = d\sigma_y \qquad (29)$$

The expression for the reliability reads

$$R = 1 - \Phi\left[-\frac{E(\Sigma_y)}{b_{\Sigma_y}}\right] - \frac{1}{\sqrt{2\pi}} \int_{\frac{E(\Sigma_y) - ab_{\Sigma_y}^2}{b_{\Sigma_y}}}^{\infty} I(t)dt \qquad (30)$$

where

$$I(t) = \exp\left(-\frac{t^2}{2}\right)\exp\left[-\frac{1}{2}\left(2E(\Sigma_y)a - a^2 b_{\Sigma_y}^2\right)\right] \qquad (31)$$

leading to the final expression

$$R = 1 - \Phi\left[-\frac{E(\Sigma_y)}{b_{\Sigma_y}}\right] - \exp\left[-\frac{1}{2}(2E(\Sigma_y)a - a^2 b_{\Sigma_y}^2\right]\left[1 - \Phi\left(-\frac{E(\Sigma_y) - ab_{\Sigma_y}^2}{b_{\Sigma_y}}\right)\right] \qquad (32)$$

This expression is rewritten as follows with notation:

$$v_{\Sigma_y} = \frac{b_{\Sigma_y}}{E(\Sigma_y)} \qquad (33)$$

we get

$$\exp\left[-\frac{1}{2}\left(2E(\Sigma_y)a - a^2 b_{\Sigma_y}^{\,2}\right)\right] = \exp\left[-\frac{1}{2}\left(\frac{E(\Sigma_y)}{E(\Sigma)} - \frac{Var(\Sigma_y)}{E^2(\Sigma)}\right)\right]$$

$$= \exp\left[-\frac{1}{2}\left(s_1 - \frac{Var(\Sigma_y)}{E^2(\Sigma_y)}\frac{E^2(\Sigma_y)}{E^2(\Sigma)}\right)\right] \qquad (34)$$

$$= \exp\left[-\frac{1}{2}\left(s_1 - v_{\Sigma_y}^{\,2} s_1^{\,2}\right)\right]$$

Also, the expression in Eq. (32) becomes

$$1 - \Phi\left[-\frac{E(\Sigma_y) - ab_{\Sigma_y}^{\,2}}{b_{\Sigma_y}}\right] = 1 - \Phi\left[-\frac{E(\Sigma_y)}{\sqrt{Var(\Sigma_y)}} - \frac{\sqrt{Var(\Sigma_y)}}{E(\Sigma)}\right]$$

$$= 1 - \Phi\left(-\frac{1}{v_{\Sigma_y}} - \frac{\sqrt{Var(\Sigma_y)}}{E(\Sigma_y)}\frac{E(\Sigma_y)}{E(\Sigma)}\right) \qquad (35)$$

$$= 1 - \Phi\left(-\frac{1}{v_{\Sigma_y}} - v_{\Sigma_y} s_1\right)$$

Thus, all parameters in Eq. (32) are expressed in terms of the coefficients of variation and the central safety factor s_1

$$R = 1 - \Phi\left(\frac{1}{v_{\Sigma_y}}\right) - \exp\left[-\frac{1}{2}(s_1 - v_{\Sigma_y}^{\,2} s_1^{\,2})\right]\left[1 - \Phi\left(-\frac{1}{v_{\Sigma_y}} - v_{\Sigma_y} s_1\right)\right] \qquad (36)$$

Consider an example. Let the yield stress has a normal probability density with mean yield stress equal $E(\Sigma_y) = 100$ MPa. The variance equals $Var(\Sigma_y) = 100(\text{MPa})^2$, or standard deviation equals $b_{\Sigma_y} = 10$ MPa, leading to the coefficient of variation to be

$$v_{\Sigma_y} = \frac{b_{\Sigma_y}}{E(\Sigma_y)} = \frac{10}{100} = 0.1 \tag{37}$$

The central safety factor is set at 2, *i.e.*

$$s_1 = \frac{E(\Sigma_y)}{E(\Sigma)} = 2 \tag{38}$$

leading to the value of the mean stress to be equal

$$E(\Sigma) = \frac{1}{2}E(\Sigma_y) = \frac{1}{2} \times 100 = 50 \quad \text{MPa} \tag{39}$$

i.e.

$$a = 1/E(\Sigma) = 1/50 \quad (\text{MPa})^{-1} \tag{40}$$

Calculations in accordance to the formula (36) yield the reliability $R = 0.86194$.

Case 3: Actual Stress Has a Normal Probability Density, Strength Has an Exponential Probability Density

In this case we get

$$R = \int_{-\infty}^{\infty} f_\Sigma(\sigma) \left[\int_{\sigma}^{\infty} f_{\Sigma_y}(\sigma_y) d\sigma_y \right] d\sigma \tag{41}$$

In new circumstances the probability densities read:

$$f_\Sigma(\sigma) = \frac{1}{\sqrt{2\pi Var(\Sigma)}} \exp\left[-\frac{1}{2}\left(\frac{\sigma - E(\Sigma)}{\sqrt{Var(\Sigma)}} \right)^2 \right]$$

$$f_{\Sigma_y}(\sigma_y) = \frac{1}{E(\Sigma_y)} \exp\left[-\frac{\sigma_y}{E(\Sigma_y)} \right], \quad \sigma_y > 0 \tag{42}$$

The reliability becomes:

$$R = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi Var(\Sigma)}} \exp\left[-\left(\frac{\sigma - E(\Sigma)}{2Var(\Sigma)} \right)^2 \right] \exp\left(-\frac{\sigma}{E(\Sigma_y)} \right) d\sigma \qquad (43)$$

The find expression is as follows:

$$R = \Phi\left(-\frac{E(\Sigma)}{\sqrt{Var(\Sigma)}} \right) + \exp\left[-\frac{1}{2}\left(\frac{2E(\Sigma)}{E(\Sigma_y)} - \frac{Var(\Sigma)}{E^2(\Sigma_y)} \right) \right] \left[1 - \Phi\left(-\frac{E(\Sigma) - Var(\Sigma)/E^2(\Sigma_y)}{\sqrt{Var(\Sigma)}} \right) \right] \quad (44)$$

In terms of coefficients of variation and the central safety factor this expression becomes:

$$R = \Phi\left(-\frac{1}{v_\Sigma} \right) + \exp\left[-\frac{1}{2}\left(\frac{2}{s_1} - \frac{v_\Sigma^2}{s_1^2} \right) \right] \left[1 - \Phi\left(-\frac{1}{v_\Sigma} - \frac{v_\Sigma}{s_1^2} \right) \right] \qquad (44)$$

Case 4: Both Actual Stress and Yield Stress Have Log-Normal Probability Densities

Let the actual stress have the following density

$$f_\Sigma(\sigma) = \frac{1}{\sigma b_\Sigma \sqrt{2\pi}} \exp\left[-\frac{(\ln\sigma - a_\Sigma)^2}{2b_\Sigma^2} \right], \quad \sigma > 0 \qquad (45)$$

where parameters a_Σ and b_Σ are related to the mean value $E(\Sigma)$ as follows:

$$E(\Sigma) = \exp(a_\Sigma + \tfrac{1}{2} b_\Sigma^2) \qquad (46)$$

The variance of the stress equals

$$Var(\Sigma) = \exp(2a_\Sigma + b_\Sigma^2)[\exp(b_\Sigma^2) - 1] \qquad (47)$$

The probability density of the yield stress reads:

$$f_{\Sigma_y}(\sigma_y) = \frac{1}{\sigma_y b_{\Sigma_y} \sqrt{2\pi}} \exp\left[-\frac{(\ln\sigma_y - a_{\Sigma_y})^2}{2b_{\Sigma_y}^2} \right], \quad \sigma_y > 0 \qquad (48)$$

The parameters a_{Σ_y} and b_{Σ_y} are related in the following way with the mean yield stress:

$$E(\Sigma_y) = \exp(a_{\Sigma_y} + \tfrac{1}{2}b_{\Sigma_y}{}^2) \tag{49}$$

Variance $Var(\Sigma_y)$ is expressed as

$$Var(\Sigma_y) = \exp(2a_{\Sigma_y} + b_{\Sigma_y}{}^2)[\exp(b_{\Sigma_y}{}^2) - 1] \tag{50}$$

The reliability reads

$$R = \mathrm{Pr\,ob}(\Sigma \le \Sigma_y) \tag{51}$$

Yet, it is easier to express reliability as follows:

$$R = \mathrm{Pr\,ob}\left(\frac{\Sigma}{\Sigma_y} \le 1\right) \tag{52}$$

Introducing a new variable Z,

$$Z = \frac{\Sigma}{\Sigma_y} \tag{53}$$

we get

$$R = \mathrm{Pr\,ob}(Z \le 1) \tag{54}$$

which can be rewritten as follows:

$$R = \mathrm{Pr\,ob}(\ln Z \le 0) \tag{55}$$

We note that

$$\ln Z = \ln \Sigma - \ln \Sigma_y \tag{56}$$

But $\ln \Sigma$ has a normal probability density with

$$E(\ln \Sigma) = a_\Sigma \tag{57}$$

$$Var(\ln \Sigma) = b_\Sigma{}^2 \tag{58}$$

Likewise $\ln \Sigma_y$ has a normal probability density with

$$E(\ln \Sigma_y) = a_{\Sigma_y}$$

$$Var(\ln \Sigma_y) = b_{\Sigma_y}^{~2}$$

(59)

Hence, the difference $\ln Z = \ln \Sigma - \ln \Sigma_y$ has a normal probability density with

$$E(\ln Z) = a_{\Sigma} - a_{\Sigma_y}$$

$$Var(\ln Z) = b_{\Sigma}^{~2} + b_{\Sigma_y}^{~2}$$

(60)

We also conclude that Z is log-normal random variable with

$$E(Z) = \exp[a_{\Sigma} - a_{\Sigma_y} + \tfrac{1}{2}(b_{\Sigma}^{~2} + b_{\Sigma_y}^{~2})]$$

(61)

$$Var(Z) = \exp[2(a_{\Sigma} - a_{\Sigma_y}) + b_{\Sigma}^{~2} + b_{\Sigma_y}^{~2}][\exp(b_{\Sigma}^{~2} + b_{\Sigma_y}^{~2}) - 1]$$

(62)

The reliability reads

$$R = \Phi\left[-\frac{a_{\Sigma} - a_{\Sigma_y}}{\sqrt{b_{\Sigma}^{~2} + b_{\Sigma_y}^{~2}}} \right]$$

(63)

The central safety factor reads

$$s_1 = \frac{E(\Sigma_y)}{E(\Sigma)} = \frac{\exp(a_{\Sigma} + b_{\Sigma}^{~2}/2)}{\exp(a_{\Sigma_y} + b_{\Sigma_y}^{~2}/2)}$$

(64)

Consider an example. Let

$$E(\Sigma) = 60,000 \quad kPa$$

$$\sqrt{Var(\Sigma)} = 20,000 \quad kPa$$

$$E(\Sigma_y) = 100,000 \quad kPa$$

$$\sqrt{Var(\Sigma_y)} = 10,000 \quad kPa$$

(65)

We know that the central safety factor equals

$$s_1 = \frac{E(\Sigma_y)}{E(\Sigma)} = \frac{100,000}{60,000} = 1.67$$

(66)

The reliability in this case turns out to be equal $R = 0.9495$.

Case 5: The Characteristic Safety Factor And the Design Safety Factor

The characteristic safety factor reads

$$\gamma = \frac{\Sigma_{y,0.05}}{\Sigma_{0.95}} \qquad (67)$$

where $\Sigma_{y,0.05}$ is 0.05 fractile of the probability distribution of yield stress, $\Sigma_{0.95}$ is 0.95 fractile of the probability distribution of stress.

When both random variables Σ and Σ_y are normal, then, according to Leporati [2]

$$\gamma = \frac{E(\Sigma_y)(1 - 1.645v_{\Sigma_y})}{E(\Sigma)(1 + 1.645v_{\Sigma})} = s_1 \frac{1 - 1.645v_{\Sigma_y}}{1 + 1.645v_{\Sigma}} \qquad (68)$$

If, for example, both coefficients of variation are set at 0.05 then characteristic safety factor equals

$$\gamma = \frac{0.91775}{1.08225} s_1 = 0.848 s_1 \qquad (69)$$

For the coefficients of variation set at 0.1 we get

$$\gamma = 0.316 s_1 \qquad (70)$$

The design safety factor, according to Leporati [2] equals

$$\gamma^* = \frac{\Sigma_{y,0.005}}{\Sigma_{0.95}} \qquad (71)$$

where $\Sigma_{y,0.005}$ is the 0.005 fractile of the yield stress. It equals

$$\gamma^* = s_1 \frac{1 - 2.576v_{\Sigma_y}}{1 + 1.645v_{\Sigma}} \qquad (72)$$

Case 6: Asymptotic Analysis

Eq. (16) can be put in the following form:

$$p_f = \Phi(\beta) \tag{73}$$

where β is referred to as a reliability index.

For $\beta > 5$, the probability of failure can be written via an asymptotic formula:

$$p_f \approx \frac{1}{\sqrt{2\pi}} \frac{\beta^2 - 1}{\beta^2} \exp\left(-\frac{\beta^2}{2}\right) \tag{74}$$

The reliability index itself is represented as follows, via Eq. (16)

$$\beta = \frac{E(\Sigma_y) - E(\Sigma)}{\sqrt{Var(\Sigma_y) + Var(\Sigma)}} \tag{75}$$

Dividing both numerator and denominator by $E(\Sigma)$ we get

$$\beta = \frac{s_1 - 1}{\sqrt{v_\Sigma^2 + s_1^2 v_{\Sigma_y}^2}} \tag{76}$$

From Eq. (76) we can find s_1 by solving the quadratic

$$\beta^2 v_\Sigma^2 + \beta^2 s_1^2 v_{\Sigma_y}^2 = (s_1 - 1)^2 \tag{77}$$

$$s_1^2 (\beta^2 v_{\Sigma_y}^2 - 1) + 2s_1 + \beta^2 v_\Sigma^2 - 1 = 0 \tag{78}$$

We get

$$s_1 = \frac{1 + \sqrt{1 - (\beta^2 v_\Sigma^2 - 1)(\beta^2 v_{\Sigma_y}^2 - 1)}}{1 - \beta^2 v_{\Sigma_y}^2}$$

$$= \frac{1 + \sqrt{\beta^2 v_{\Sigma_y}^2 + \beta^2 v_\Sigma^2 - \beta^4 v_{\Sigma_y}^2 v_\Sigma^2}}{1 - \beta^2 v_{\Sigma_y}^2} \tag{79}$$

As is easily seen, not for all coefficients of variability v_Σ and v_{Σ_y} one can find the

central factor of safety such that the demanded reliability index would be achieved. For

example for

$$(\beta^2 v_\Sigma^2 - 1)(\beta^2 v_{\Sigma_y}^2 - 1) > 1 \tag{80}$$

s_1 gets complex values.

Solving the inequality (80) we get

$$\beta^2(\beta^2 v_\Sigma^2 v_{\Sigma_y}^2 - v_{\Sigma_y}^2 - v_\Sigma^2) > 0 \tag{81}$$

This implies, that the following inequality must be met

$$\beta < \sqrt{\frac{1}{v_\Sigma^2} + \frac{1}{v_{\Sigma_y}^2}} \tag{82}$$

If the inequality (82) is violated, then the required reliability cannot be achieved by any

factor of safety.

From Eq. (76) we observe that when s_1 tends to infinity, the reliability index tends

to

$$\beta \to \frac{1}{v_\Sigma} \tag{83}$$

We can differentiate Eq (76) with respect to s_1:

$$\frac{\partial \beta}{\partial s_1} = \frac{v_\Sigma^2 + v_{\Sigma_y}^2}{(v_\Sigma^2 + s_1^2 v_{\Sigma_y}^2)^{3/2}} > 0 \tag{84}$$

This implies that when increasing s_1 from unity (when $\beta = 0$) to infinity, β varies from

zero to the value $1/v_{\Sigma_y}$.

If the variation of the stress is zero $v_\Sigma = 0$ Eq. (79) yields

$$s_1 = \frac{1}{1 - \beta v_{\Sigma_y}}$$ (85)

When we have a zero variability of the yield stress $v_{\Sigma_y} = 0$, we get

$$s_1 = 1 + \beta v_{\Sigma}$$ (86)

Case 7: Actual Stress and Yield Stress Are Correlated

"In most cases the correlation between the actual stress and the yield stresses is absent. Yet, when such a correlation exists, its expressing is ambiguous, and it is difficult to express it numerically, "as Rzhanitzin (1981) notes.

Positive correlation between Σ and Σ_y take place when the stronger elements take more load. Partially this takes place for statically indeterminate systems, in which greater strength is associated with greater stiffness, and hence with more loads. Safety margin

$$M = \Sigma_y - \Sigma$$ (87)

has a variance

$$Var(M) = Var(\Sigma_y) - 2Cov(\Sigma_y, \Sigma) + Var(\Sigma)$$ (88)

where $Cor(\Sigma_y, \Sigma)$ is the covariance between the actual stress and yield stress.

Then instead of Eq. (75) we get

$$\beta = \frac{E(\Sigma_y) - E(\Sigma)}{\sqrt{Var(\Sigma_y) - 2Cov(\Sigma_y, \Sigma) + Var(\Sigma)}}$$ (89)

Hence, in terms of central safety factor, we have

$$\beta = \frac{s_1 - 1}{\sqrt{v_{\Sigma_y}^2 - 2s_1 v_{\Sigma\Sigma_y}^2 + s_1^2 v_{\Sigma}^2}}$$ (90)

where $v_{\Sigma\Sigma_y}$ is the correlation coefficient

$$v_{\Sigma\Sigma_y} = \sqrt{\frac{Cov(\Sigma_y, \Sigma)}{E(\Sigma_y)E(\Sigma)}} \tag{91}$$

One can express s_1 via β. Eq. (90) becomes

$$(1 - v_{\Sigma_y}{}^2 \beta^2)s_1{}^2 - 2(1 - \beta^2 v_{\Sigma\Sigma_y}{}^2)s_1 + (1 - \beta^2 v_{\Sigma}{}^2) = 0 \tag{92}$$

yielding

$$s_1 = \frac{1 - \beta^2 v_{\Sigma_y}{}^2 + \sqrt{\beta^2 (v_{\Sigma_y}{}^2 - 2v_{\Sigma\Sigma_y}{}^2 + v_{\Sigma}{}^2) - \beta^4 (v_{\Sigma}{}^2 v_{\Sigma_y}{}^2 - v_{\Sigma\Sigma_y}{}^4)}}{1 - \beta^2 v_{\Sigma_y}{}^2} \tag{93}$$

When $v_{\Sigma\Sigma_y}$ vanishes Eq. (93) reduces to the case of uncorralated actual stress and yield stress.

Case 8: Both Actual Stress and Yield Stress Follow the Pearson Probability Densities

The random variable is said to have a Pearson probability density, if it has a form

$$f_X(x) = Ax^a e^{-bx} \quad for \quad x > 0 \tag{94}$$

where

$$A = \frac{b^{a+1}}{\Gamma(a+1)} \tag{95}$$

The mean value $E(X)$ reads

$$E(X) = \frac{a+1}{b} \tag{96}$$

whereas the variance equals

$$Var(X) = \frac{a+1}{b^2} \tag{97}$$

The coefficient of variation equals

$$v_X = \frac{1}{\sqrt{a+1}} \tag{98}$$

It is interesting that the coefficient of variation does not depend upon b. It is seen that for small values of a we get very high variability, whereas for small variability, of the order of 0.1, greater values of a are needed ($a \approx 100$).

Let the actual stress have the Pearson density

$$f_\Sigma(\sigma) = A_1 \sigma^{\alpha-1} \exp\left(-\frac{\alpha\sigma}{E(\Sigma)}\right) \tag{99}$$

The yield stress also has the Pearson density

$$f_{\Sigma_y}(\sigma_y) = A_2 \sigma_y^{\beta-1} \exp\left(-\frac{\beta\sigma_y}{E(\Sigma_y)}\right) \tag{100}$$

where

$$A_1 = \frac{\alpha^\alpha}{\Gamma(\alpha)E^\alpha(\Sigma)}, \quad A_2 = \frac{\beta^\beta}{\Gamma(\beta)E^\beta(\Sigma)} \tag{101}$$

where

$$\alpha = \frac{1}{v_\Sigma^2}, \quad \beta = \frac{1}{v_{\Sigma_y}^2} \tag{102}$$

The probability of failure becomes

$$P_f = \frac{B_\delta(\beta,\alpha)}{B(\beta,\alpha)} = J_\delta(\alpha,\beta) \tag{103}$$

where

$$B(\beta,\alpha) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha + \beta)} \qquad (104)$$

is the Euler function of the first kind, or beta-function, whereas

$$B_\delta(\beta,\alpha) = \int_0^\delta t^{\beta-1}(1-t)^{\alpha-1} dt \qquad (105)$$

is the Euler function of the second kind, or incomplete beta-function. The quantity δ in

Eq. (105) is defined as follows:

$$\delta = \frac{\beta E(\Sigma)}{\beta E(\Sigma) + \alpha E(\Sigma_y)} = \left(1 + \frac{\nu_{\Sigma_y}}{\Sigma_y^2} s_1\right)^{-1} \qquad (106)$$

The evaluation for the function $J_\delta(\alpha,\beta)$ can be done by the numerical evaluation of

integrals in Eqs. (105) and (106).

Conclusion: Reliability & Safety Factor Can Peacefully Coexist

At this junction we ask the most important question: Are the safety factor and the

reliability concepts contradictory or they can coexist peacefully? The Fig. 1-12 depict the

dependence of the reliability in Eq. (22) vs the central safety factor s_1. As is seen, for

various variabilities of the stress and the yield stress one can assign both reliability and

the central safety factor. Indeed, Fig. 1 shows that of the coefficient of variability of the

stress $\gamma_\Sigma = 0.04$ and the designer wants the safety factor to be set, say at 1.3, the demand

that the reliability is above 0.92 results in the choice of materials with $\gamma_{\Sigma y} \le 0.15$. We

immediately observe that both the central safety factor and the reliability requirements

can be combined. Likewise, for $\gamma_\Sigma = 0.05$ (Fig. 2), the central safety factor s=1.2 is

associated with reliabilites grater than or equal to 0.93. Thus, one can also impose the

reliability constraint. Analogous features are characteristic to figures 3-12.This leads to the conclusion that these 2 concepts can coexist peacefully.

But this coexistence cannot be done without some adjustments. Reliability concept provides the rigorous values of the factors, for otherwise, *i.e.* without the general context of reliability, the safety factor will remain as the factor of experience but still the factor of theoretical ignorance. Adopting reliability as the main concept, the allocated values of safety factor will naturally follow. In a sense probabilistic methods do not constitute a "revolution" but rather a natural "evolution".
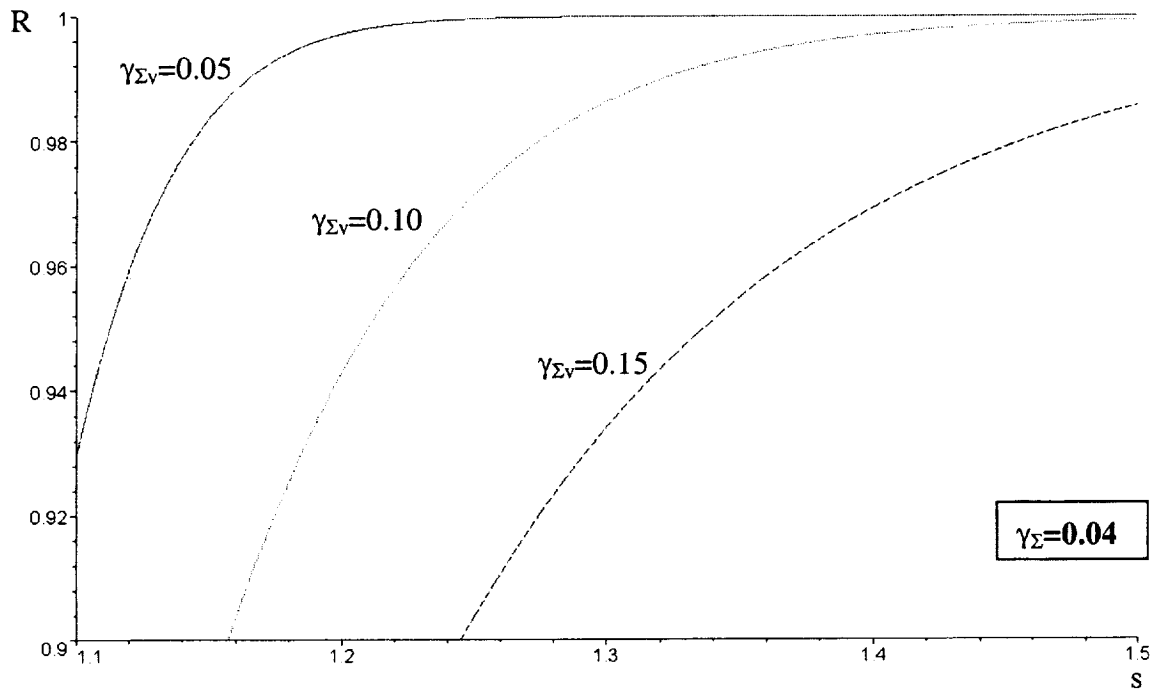
Fig 1: Reliability, R, versus central safety Factor, s
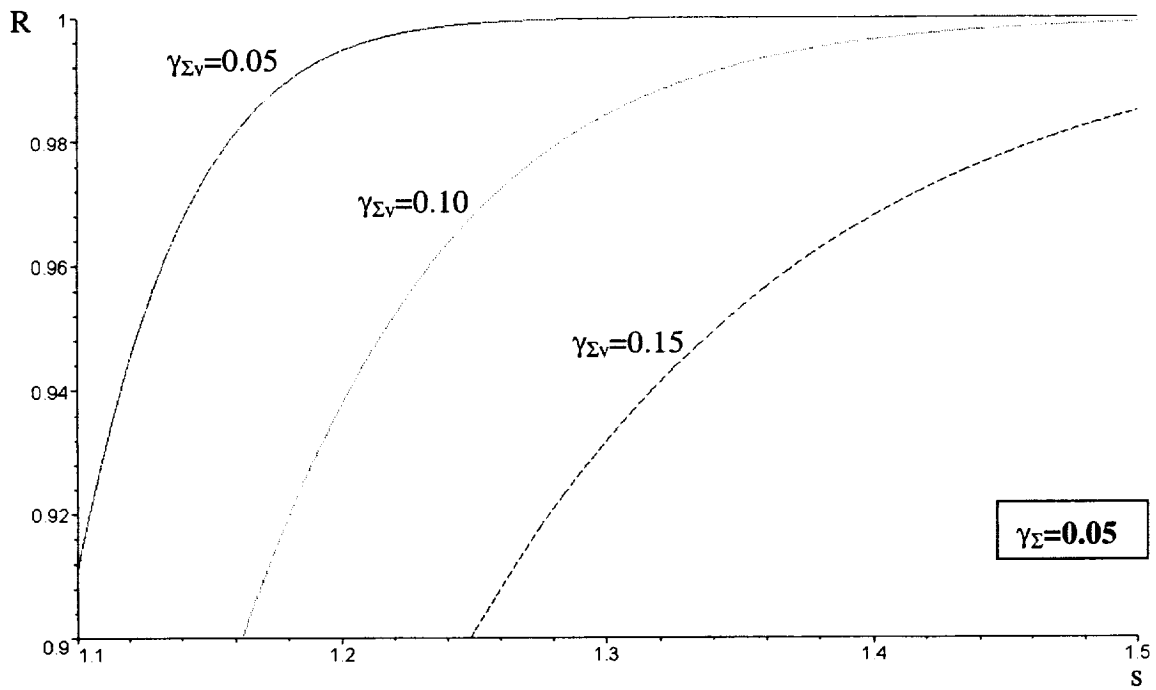
Fig 2: Reliability, R, versus Central Safety Factor, s
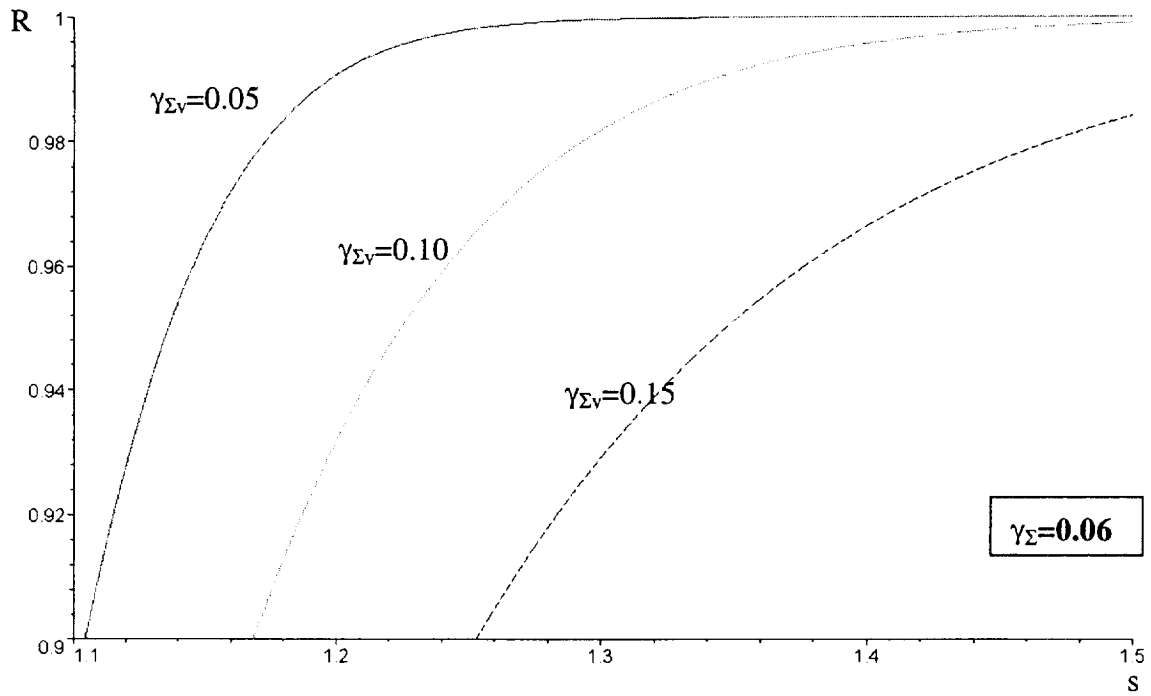
Fig 3: Reliability, R, versus Central Safety Factor, s
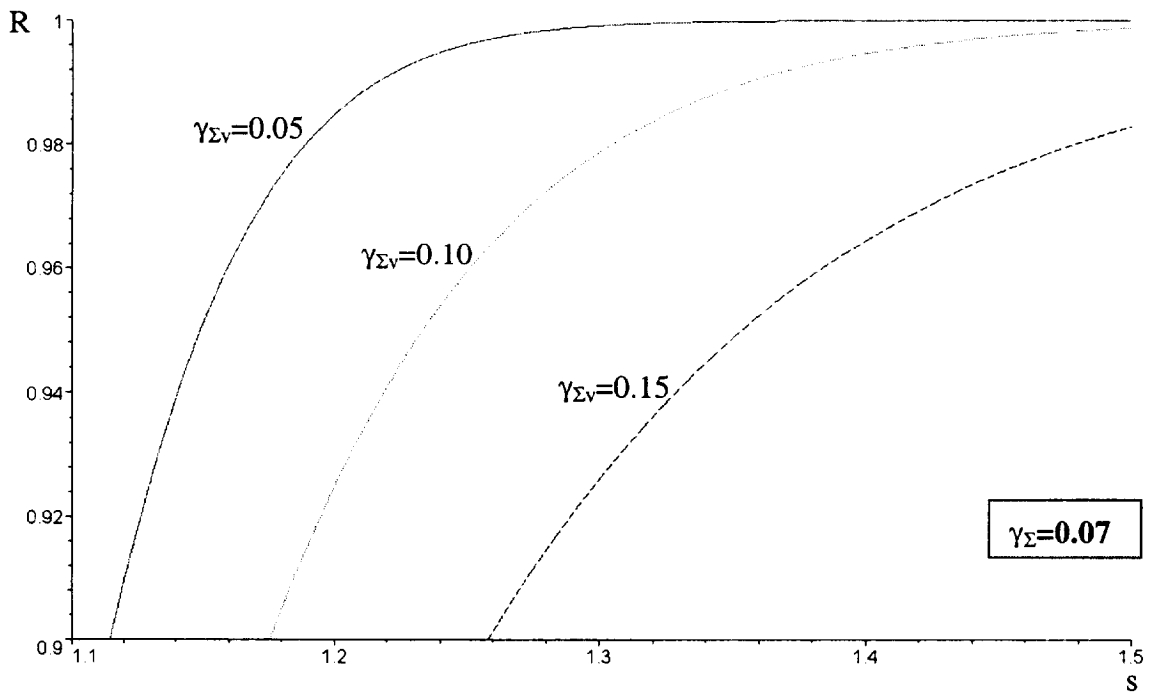
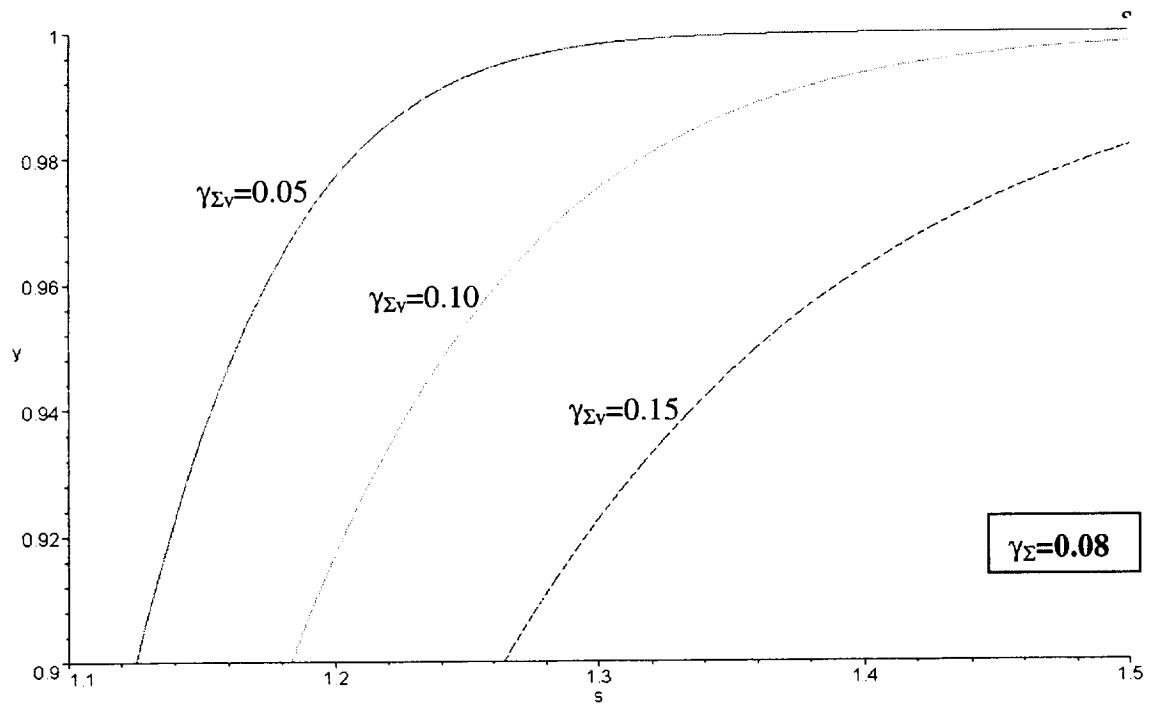**Fig 4**: Reliability, R, versus Central Safety Factor, s

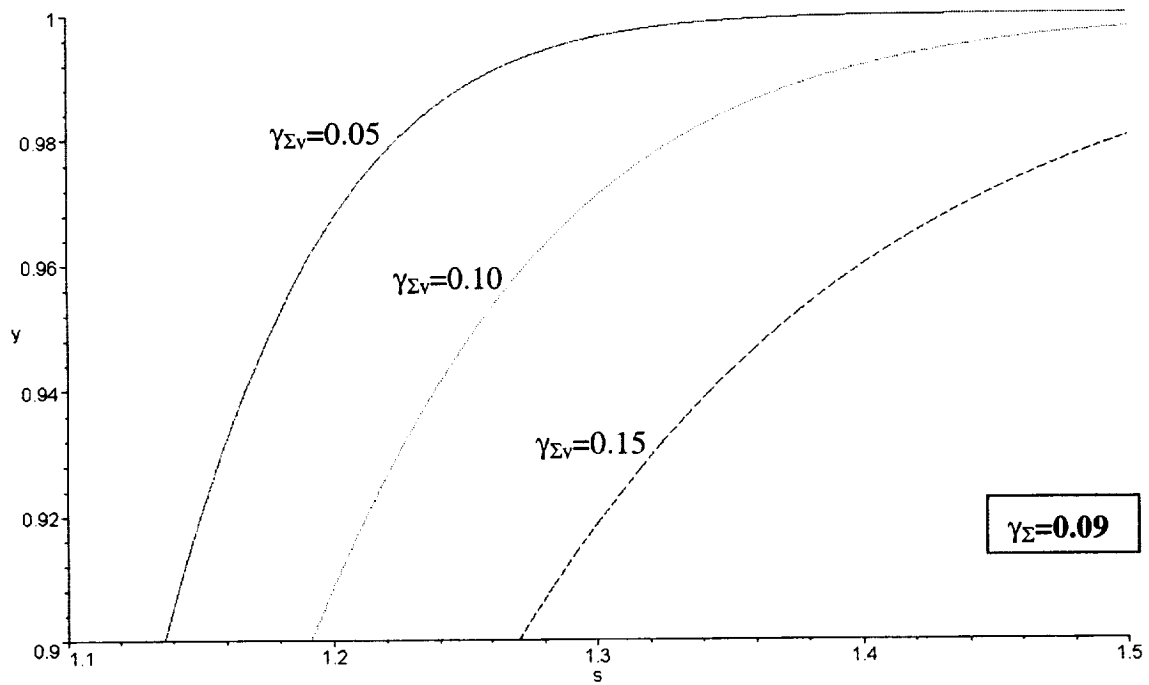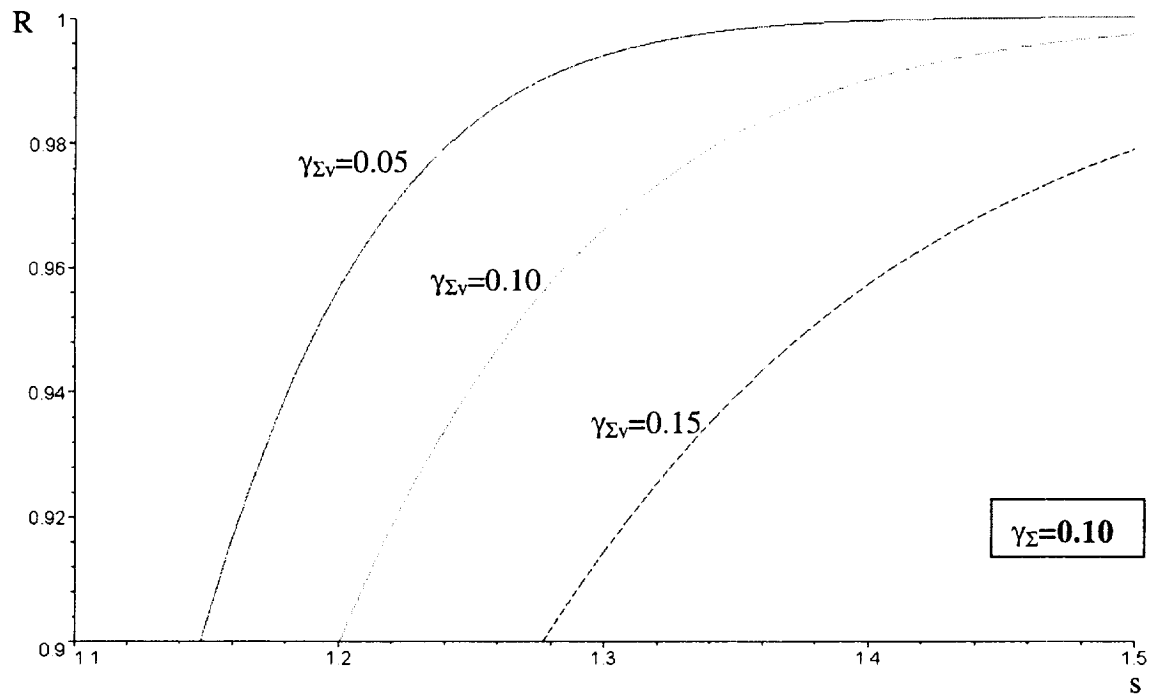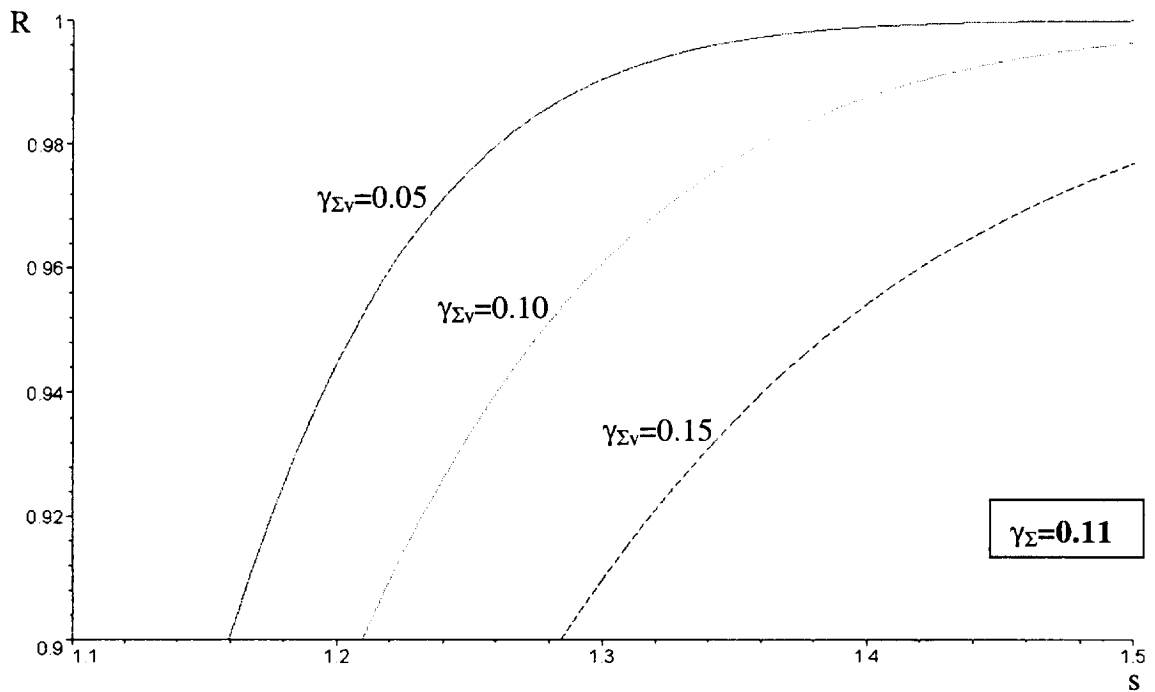**Fig 5**: Reliability, R, versus Central Safety Factor, s

Fig 6: Reliability, R, versus Central Safety Factor, s

Fig 7: Reliability, R, versus Central Safety Factor, s

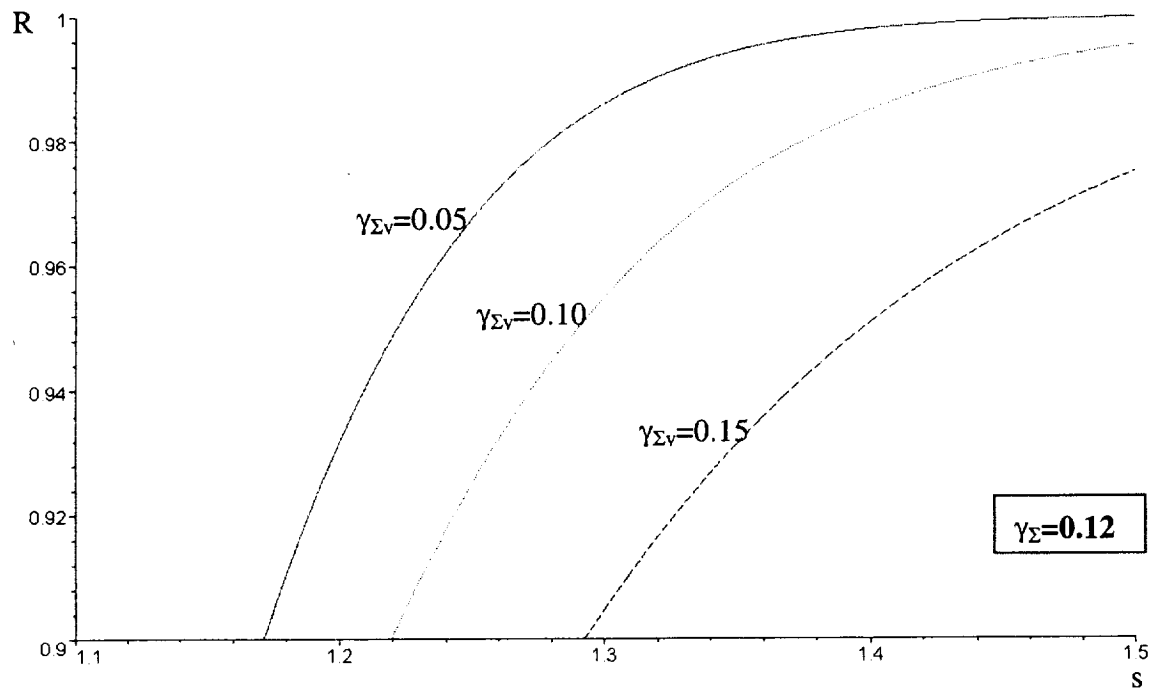**Fig 8**: Reliability, R, versus Central Safety Factor, s

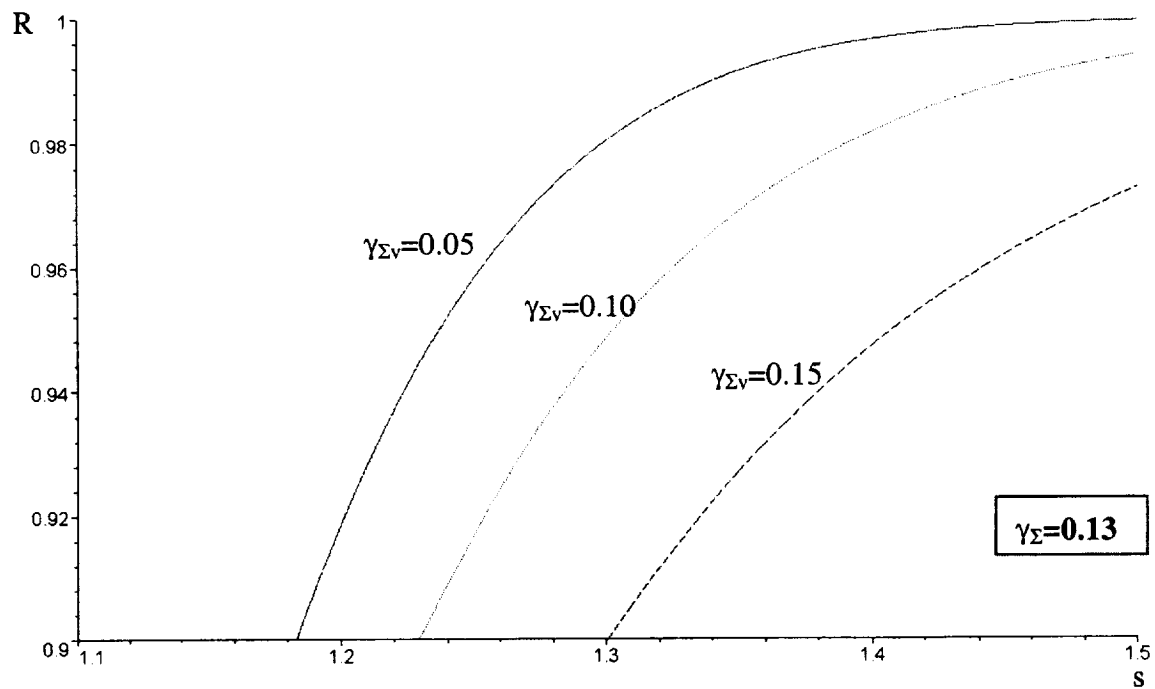**Fig 9**: Reliability, R, versus Central Safety Factor, s

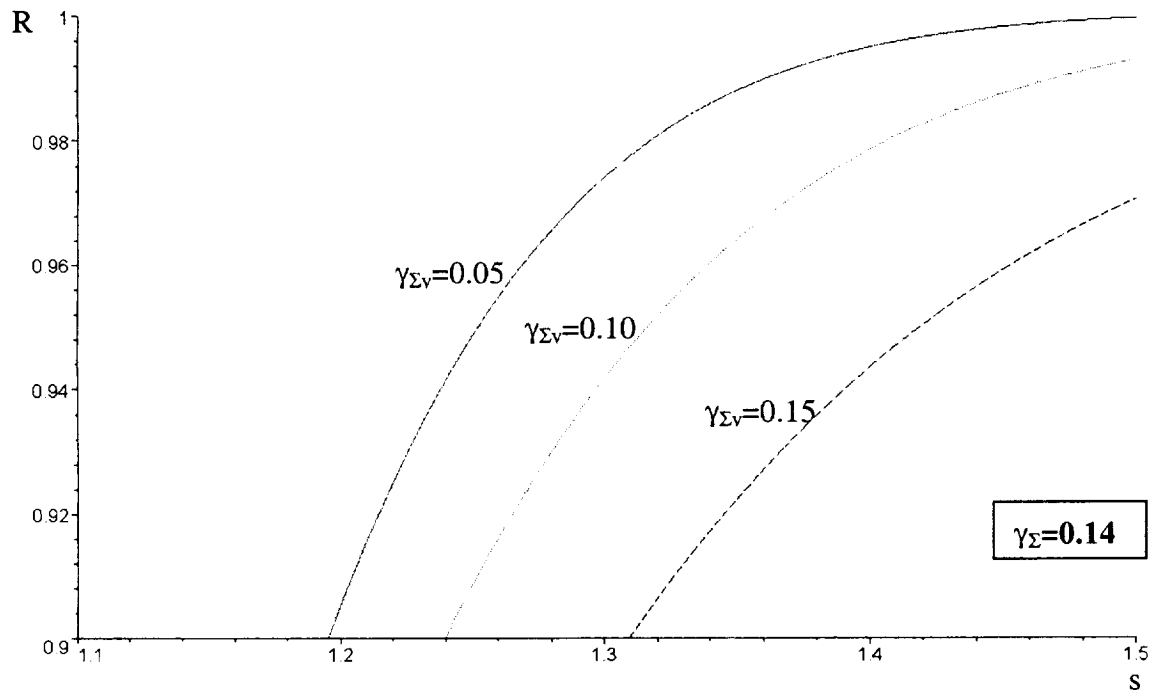**Fig 10**: Reliability, R, versus Central Safety Factor, s

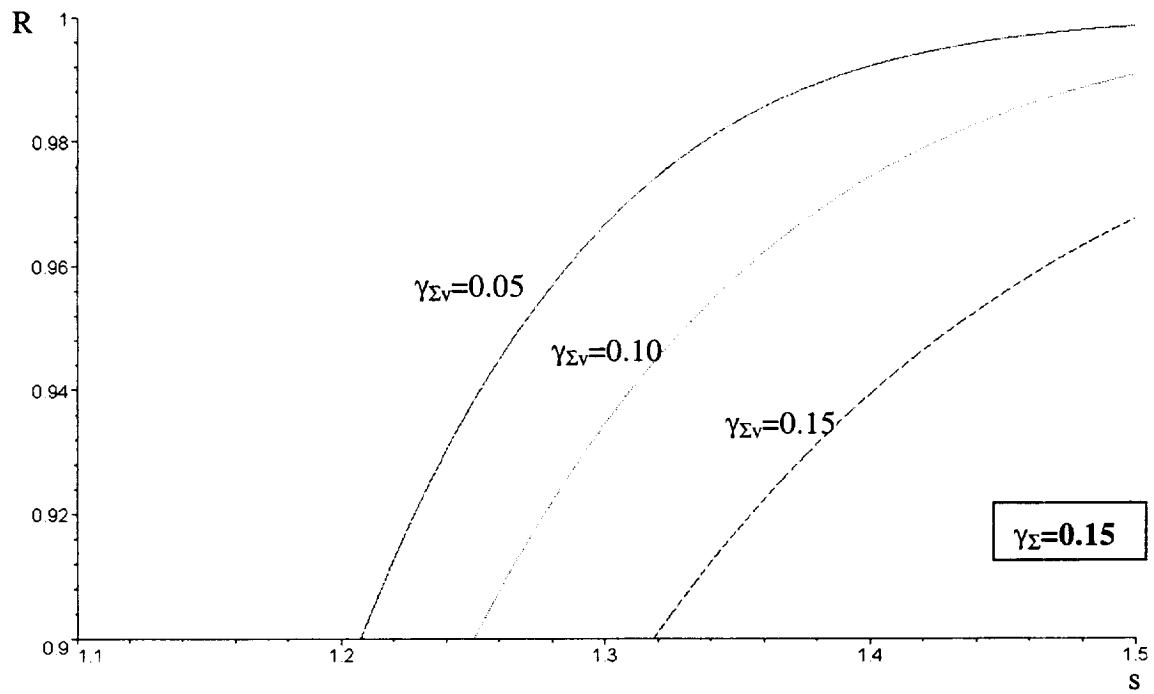**Fig 11**: Reliability, R, versus Central Safety Factor, s

Fig 12: Reliability, R, versus Central Safety Factor, s

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13. ABSTRACT (Maximum 200 words)

An evaluation was performed to establish relationships between safety factors and reliability relationships. Results obtained show that the use of the safety factor is not contradictory to the employment of the probabilistic methods. In many cases the safety factors can be directly expressed by the required reliability levels. However, there is a major difference that must be emphasized: whereas the safety factors are allocated in an *ad hoc* manner, the probabilistic approach offers a unified mathematical framework. The establishment of the interrelation between the concepts opens an avenue to specify safety factors based on reliability. In cases where there are several forms of failure, then the allocation of safety factors should be based on having the same reliability associated with each failure mode. This immediately suggests that by the probabilistic methods the existing over-design or under-design can be eliminated. The report includes three parts: Part 1–Random Actual Stress and Deterministic Yield Stress; Part 2–Deterministic Actual Stress and Random Yield Stress; Part 3–Both Actual Stress and Yield Stress Are Random.

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