Team Notebook

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1 DP

1.1 Knuth Optimization

```
for (int s = 0: s <= k: s++)
                                          //s - length(size)
    of substring
   for (int L = 0; L+s<=k; L++) {</pre>
                                            //L - left point
     int R = L + s:
                                            //R - right point
     if (s < 2) {
       res[L][R] = 0;
                                            //DP base -
            nothing to break
       mid[L][R] = 1;
                                            //mid is equal to
             left border
       continue:
     int mleft = mid[L][R-1];
                                            //Knuth's trick:
          getting bounds on M
     int mright = mid[L+1][R];
     res[L][R] = 1000000000000000000LL:
     for (int M = mleft; M<=mright; M++) { //iterating for M</pre>
           in the bounds only
       int64 tres = res[L][M] + res[M][R] + (x[R]-x[L]):
       if (res[L][R] > tres) {
                                            //relax current
            solution
         res[L][R] = tres;
         mid[L][R] = M;
 int64 answer = res[0][k]:
```

2 Data Structures

2.1 Fenwick Tree

```
#define LSOne(S) (S & (-S))
vector<int> fen;
void ft_create(int n) { fen.assign(n + 1, 0); }
// initially n + 1 zeroes
int ft_rsq(int b)
{ // returns RSQ(1, b)
    int sum = 0;
    for (; b; b -= LSOne(b)) sum += fen[b];
    return sum;
}
int ft_rsq(int a, int b)
{ // returns RSQ(a, b)
```

```
return ft_rsq(b) - (a == 1 ? 0 : ft_rsq(a - 1));
}
// adjusts value of the k-th element by v (v can be +ve/inc
    or -ve/dec).
void ft_adjust(int k, int v)
{
    for (; k < (int)fen.size(); k += LSOne(k)) fen[k] += v;
}</pre>
```

2.2 Ranged Fenwick Tree

```
//not tested
#define LSOne(S) (S & (-S))
vector<ll> fen[2]:
void ft_create(int n) { fen[0].assign(n + 1, 0), fen[1].
    assign(n + 1, 0); }
11 ft_rsq(int b, bool bl)
11 \text{ sum} = 0:
for (; b; b -= LSOne(b)) sum += fen[bl][b];
return sum:
void ft_adjust(ll k, ll v, bool bl)
for (; k <= (int)fen[bl].size(); k += LSOne(k)) fen[bl][k]</pre>
void range_adjust(ll l, ll r, ll v)
ft_adjust(1, v, 0), ft_adjust(r+1, -v, 0);
ft_adjust(1, v*(1-1), 1), ft_adjust(r+1, -v*r, 1);
ll range_rsq(ll l, ll r)
11 x=0, a, b;
if(1)
 a=ft_rsq(l-1, 0), b=ft_rsq(l-1, 1);
 x=(1-1)*a-b;
a=ft_rsq(r, 0), b=ft_rsq(r, 1);
11 y=r*a-b;
return y-x;
```

2.3 Sparse Table

```
int n, arr[MAXN], table[MAXN][20], table2[MAXN][20];
int Log[MAXN];
vector<int> tmp. best:
int GCD(int a, int b)
if(b==0) return a;
return GCD(b, a%b);
void init(int n)
int mx=0, l=0, r=n+1, lim=2, lg=0;
Rep(i, MAXN)
 if(i==lim)
  lg++:
  lim*=2;
 Log[i]=lg;
Rep(i, n) table[i][0]=table2[i][0]=arr[i]:
For(i, 1, Log[n]+1) for(int j=0; j+(1<<i)-1< n; j++)
 table[j][i]=GCD(table[j][i-1], table[j+(1<<(i-1))][i-1]);
 table2[j][i]=min(table2[j][i-1], table2[j+(1<<(i-1))][i
int get_gcd(int 1, int r)
int lg=Log[r-l+1];
return GCD(table[l][lg], table[r-(1<<lg)+1][lg]);</pre>
```

3 Games

3.1 Declare Winner

```
void declareWinner(int whoseTurn, int piles[], int Grundy[],
    int n) {
    int xorValue = Grundy[piles[0]];
    for (int i=1; i<=n-1; i++)
        xorValue = xorValue ^ Grundy[piles[i]];
    if (xorValue != 0) {
        if (whoseTurn == PLAYER1)
            printf("Player 1 will win\n");
        else</pre>
```

```
3
```

```
printf("Player 2 will win\n");
}
else {
    if (whoseTurn == PLAYER1)
        printf("Player 2 will win\n");
    else
        printf("Player 1 will win\n");
}
return;
```

3.2 Grundy

```
// A function to Compute Grundy Number of 'n'
// Only this function varies according to the game
int calculateGrundy(int n) {
   if (n == 0)
        return (0);
   unordered_set<int> Set; // A Hash Table
   for (int i=0; i<=n-1; i++)
        Set.insert(calculateGrundy(i));
   return (calculateMex(Set));
}</pre>
```

3.3 Mex

```
int calculateMex(unordered_set<int> Set) {
   int Mex = 0;
   while (Set.find(Mex) != Set.end())
        Mex++;
   return (Mex);
}
```

4 Geometry

4.1 3D Rotation

```
tXZ-sY tYZ+sX tZ^2+c 0
0 0 0 1
2. Right handed about arbitrary axis:
tX^2+c tXY+sZ tXZ-sY 0
tXY-sZ tY^2+c tYZ+sX 0
tXZ+sY tYZ-sX tZ^2+c 0
0 0 0 1
3. About X Axis
1 0 0 0
0 c -s 0
0 s c 0
0 0 0 1
4. About Y Axis
c 0 s 0
0 1 0 0
-s 0 c 0
0 0 0 1
5. About Z Axis
c -s 0 0
s c 0 0
0 0 1 0
0 0 0 1
```

4.2 Angle Bisector

```
// angle bisector
int bcenter( PT p1, PT p2, PT p3, PT& r ){
   if( triarea( p1, p2, p3 ) < EPS ) return -1;
   double s1, s2, s3;
   s1 = dist( p2, p3 );
   s2 = dist( p1, p3 );
   s3 = dist( p1, p2 );
   double rt = s2/(s2+s3);
PT a1,a2;
   a1 = p2*rt+p3*(1.0-rt);
   rt = s1/(s1+s3);
   a2 = p1*rt+p3*(1.0-rt);
   intersection( a1,p1, a2,p2, r );
   return 0;
}</pre>
```

4.3 Circle Circle Intersection

```
// rotate a point CCW or CW around the origin
PT RotateCCW90(PT p) { return PT(-p.y,p.x); }
PT RotateCW90(PT p) { return PT(p.y,-p.x); }
PT RotateCCW(PT p, double t) {
return PT(p.x*cos(t)-p.y*sin(t), p.x*sin(t)+p.y*cos(t));
// compute intersection of circle centered at a with radius
// with circle centered at b with radius R
vector<PT> CircleCircleIntersection(PT a, PT b, double r,
    double R) {
 vector<PT> ret:
 double d = sqrt(dist2(a, b));
 if (d > r + R \mid | d + min(r, R) < max(r, R)) return ret;
 double x = (d * d - R * R + r * r) / (2 * d):
 double y = sqrt(r * r - x * x);
 PT v = (b - a) / d:
 ret.push_back(a + v * x + RotateCCW90(v) * y);
 if (v > 0)
   ret.push_back(a + v * x - RotateCCW90(v) * y);
 return ret:
```

4.4 Circle Line Intersection

```
// compute intersection of line through points a and b with
// circle centered at c with radius r > 0
vector<PT> CircleLineIntersection(PT a, PT b, PT c, double r
    ) {
 vector<PT> ret:
 b = b-a;
 a = a-c:
 double A = dot(b, b);
 double B = dot(a, b);
 double C = dot(a, a) - r*r;
 double D = B*B - A*C;
 if (D < -EPS) return ret:</pre>
 ret.push back(c+a+b*(-B+sgrt(D+EPS))/A):
 if (D > EPS)
   ret.push_back(c+a+b*(-B-sqrt(D))/A);
 return ret;
```

4.5 Circle from Three Points

```
4
```

```
Point center_from(double bx, double by, double cx, double cy
    ) {
    double B=bx*bx+by*by, C=cx*cx+cy*cy, D=bx*cy-by*cx;
    return Point((cy*B-by*C)/(2*D), (bx*C-cx*B)/(2*D));
}

Point circle_from(Point A, Point B, Point C) {
    Point I = center_from(B.X-A.X, B.Y-A.Y, C.X-A.X, C.Y-A.Y);
    return Point(I.X + A.X, I.Y + A.Y);
}
```

4.6 Closest Point on Line

```
//From In 1010101 We Trust cheatsheet:
//the closest point on the line p1->p2 to p3
void closestpt( PT p1, PT p2, PT p3, PT &r ){
   if(fabs(triarea(p1, p2, p3)) < EPS){ r = p3; return; }
   PT v = p2-p1; v.normalize();
   double pr; // inner product
   pr = (p3.y-p1.y)*v.y + (p3.x-p1.x)*v.x;
   r = p1+v*pr;
}</pre>
```

4.7 Delaunay Triangulation

```
// Slow but simple Delaunay triangulation. Does not handle
// degenerate cases (from O'Rourke, Computational Geometry
    in C)
//
// Running time: O(n^4)
// INPUT: x = x-coordinates
           v[] = v-coordinates
// OUTPUT: triples = a vector containing m triples of
     indices
                     corresponding to triangle vertices
typedef double T;
struct triple {
   int i. i. k:
   triple() {}
   triple(int i, int j, int k) : i(i), j(j), k(k) {}
};
```

```
vector<triple> delaunavTriangulation(vector<T>& x. vector<T</pre>
int n = x.size();
vector<T> z(n):
 vector<triple> ret;
 for (int i = 0; i < n; i++)</pre>
    z[i] = x[i] * x[i] + y[i] * y[i];
for (int i = 0; i < n-2; i++) {</pre>
    for (int j = i+1; j < n; j++) {</pre>
 for (int k = i+1: k < n: k++) {</pre>
     if (j == k) continue;
     double xn = (y[j]-y[i])*(z[k]-z[i]) - (y[k]-y[i])*(z[j])
     double yn = (x[k]-x[i])*(z[j]-z[i]) - (x[j]-x[i])*(z[k
     double zn = (x[j]-x[i])*(y[k]-y[i]) - (x[k]-x[i])*(y[j])
          l-v[i]):
     bool flag = zn < 0:
     for (int m = 0; flag && m < n; m++)</pre>
   flag = flag && ((x[m]-x[i])*xn +
    (y[m]-y[i])*yn +
    (z[m]-z[i])*zn <= 0);
     if (flag) ret.push_back(triple(i, j, k));
 }
return ret;
int main()
   T xs[]={0, 0, 1, 0.9};
   T ys[]={0, 1, 0, 0.9};
   vector<T> x(&xs[0], &xs[4]), y(&ys[0], &ys[4]);
   vector<triple> tri = delaunayTriangulation(x, y);
   //expected: 0 1 3
   for(i = 0: i < tri.size(): i++)</pre>
       printf("%d %d %d\n", tri[i].i, tri[i].j, tri[i].k);
   return 0;
```

4.8 Latitude and Longitude

```
Converts from rectangular coordinates to latitude/longitude
versa. Uses degrees (not radians).
using namespace std;
struct 11
 double r, lat, lon;
struct rect
 double x, y, z;
11 convert(rect& P)
 11 Q;
 0.r = sart(P.x*P.x+P.v*P.v+P.z*P.z):
 Q.lat = 180/M_PI*asin(P.z/Q.r);
 Q.lon = 180/M_PI*acos(P.x/sqrt(P.x*P.x+P.y*P.y));
 return Q;
rect convert(11& 0)
 rect P;
 P.x = Q.r*cos(Q.lon*M_PI/180)*cos(Q.lat*M_PI/180);
 P.v = Q.r*sin(Q.lon*M_PI/180)*cos(Q.lat*M_PI/180);
 P.z = Q.r*sin(Q.lat*M_PI/180);
 return P;
int main()
 rect A;
 11 B;
 A.x = -1.0; A.y = 2.0; A.z = -3.0;
 B = convert(A):
 cout << B.r << " " << B.lat << " " << B.lon << endl;</pre>
 A = convert(B);
 cout << A.x << " " << A.y << " " << A.z << endl;
```

Line Intersection

```
// Ax + By = C
A = v2 - v1
B = x1 - x2
C = A*x1 + B*v1
double det = A1*B2 - A2*B1
double x = (B2*C1 - B1*C2)/det
double y = (A1*C2 - A2*C1)/det
typedef pair<double, double> pointd;
#define X first
#define Y second
bool eqf(double a, double b) {
   return fabs(b - a) < 1e-6;</pre>
int crossVecs(pointd a, pointd b) {
   return a.X * b.Y - a.Y*b.X;
int cross(pointd o, pointd a, pointd b){
   return crossVecs(make_pair(a.X - o.X, a.Y - o.Y),
        make pair(b.X - o.X. b.Y - o.Y);
int dotVecs(pointd a, pointd b) {
   return a.X * b.X + a.Y * b.Y;
}
int dot(pointd o, pointd a, pointd b) {
   return dotVecs(make_pair(a.X - o.X, a.Y - o.Y), make_pair | 4.10 Point in Polygon
        (b.X - o.X, b.Y - o.Y)):
bool on The Line (const point d& a, const point d& p, const
    pointd& b) {
   return eqf(cross(p, a, b), 0) && dot(p, a, b) < 0 ;</pre>
class LineSegment {
   public:
   double A, B, C;
   pointd from, to;
   LineSegment(const pointd& a, const pointd& b) {
       A = b.Y - a.Y:
       B = a.X - b.X;
       C = A*a.X + B*a.Y;
       from = a:
       to = b:
   bool between(double 1, double a, double r) const {
```

```
if(1 > r) {
           swap(1, r);
       return 1 <= a && a <= r:
   }
   bool pointOnSegment(const pointd& p) const {
       return eqf(A*p.X + B*p.Y, C) && between(from.X, p.X,
            to.X) && between(from.Y, p.Y, to.Y);
   pair<br/>bool, pointd> segmentsIntersect(const LineSegment& l
        ) const {
       double det = A * 1.B - B * 1.A;
       pair<bool, pointd> ret;
       ret.first = false;
       if(det != 0) {
           pointd inter((1.B*C - B*1.C)/det, (A*1.C - 1.A*C)
           if(1.pointOnSegment(inter) && pointOnSegment(
               inter)) {
              ret.first = true:
              ret.second = inter;
       return ret;
};
```

```
// determine if point is in a possibly non-convex polygon (
    by William
// Randolph Franklin): returns 1 for strictly interior
    points, 0 for
// strictly exterior points, and 0 or 1 for the remaining
// Note that it is possible to convert this into an *exact*
    test using
// integer arithmetic by taking care of the division
    appropriately
// (making sure to deal with signs properly) and then by
    writing exact
// tests for checking point on polygon boundary
bool PointInPolygon(const vector<PT> &p. PT a) {
 bool c = 0:
 for (int i = 0; i < p.size(); i++){</pre>
   int j = (i+1)%p.size();
  if ((p[i].y <= q.y && q.y < p[j].y ||</pre>
```

```
p[j].y \le q.y && q.y \le p[i].y) &&
    q.x < p[i].x + (p[j].x - p[i].x) * (q.y - p[i].y) / (p[
        j].y - p[i].v))
    c = !c:
}
return c:
```

4.11 Polygon Centroid

```
// This code computes the area or centroid of a (possibly
    nonconvex)
// polygon, assuming that the coordinates are listed in a
    clockwise or
// counterclockwise fashion. Note that the centroid is often
// the "center of gravity" or "center of mass".
double ComputeSignedArea(const vector<PT> &p) {
 double area = 0:
 for(int i = 0; i < p.size(); i++) {</pre>
  int j = (i+1) % p.size();
  area += p[i].x*p[j].y - p[j].x*p[i].y;
 return area / 2.0;
double ComputeArea(const vector<PT> &p) {
 return fabs(ComputeSignedArea(p)):
PT ComputeCentroid(const vector<PT> &p) {
 PT c(0,0);
 double scale = 6.0 * ComputeSignedArea(p);
 for (int i = 0: i < p.size(): i++){</pre>
   int j = (i+1) % p.size();
   c = c + (p[i]+p[j])*(p[i].x*p[j].y - p[j].x*p[i].y);
 return c / scale;
```

Rotation Around Origin by t

```
x = x.Cos(t) - y.Sin(t)
y = x.Sin(t) + y.Cos(t)
```

5 Graph

5.1 2-SAT

```
//From "You Know Izad?" team cheat sheet
//fill the v array
//e.g. to push (p v !q) use the following code:
// v[VAR(p)].push_back( NOT( VAR(q) ) )
// v[NOT( VAR(q) )].push_back( VAR(p) )
//the result will be in color array
#define VAR(X) (X << 1)</pre>
#define NOT(X) (X ^ 1)
#define CVAR(X,Y) (VAR(X) | (Y))z
#define COL(X) (X & 1)
#define NVAR 400
int n;
vector<int> v[2 * NVAR];
int color[2 * NVAR];
int bc[2 * NVAR]:
bool dfs( int a, int col ) {
   color[a] = col:
   int num = CVAR( a, col );
   for( int i = 0; i < v[num].size(); i++ ) {</pre>
       int adi = v[num][i] >> 1:
       int ncol = NOT( COL( v[num][i] ) );
       if( ( color[adi] == -1 && !dfs( adi, ncol ) ) ||
           ( color[adj] != -1 && color[adj] != ncol ) ) {
           color[a] = -1;
           return false:
   }
   return true;
bool twosat() {
   memset( color, -1, sizeof color ):
   for( int i = 0; i < n; i++ ){</pre>
       if( color[i] == -1 ){
           memcpy(bc, color, sizeof color);
           if( !dfs( i, 0 )){
              memcpy(color, bc, sizeof color);
              if(!dfs( i, 1 ))
                  return false:
          }
       }
   return true;
```

5.2 Bidirectional Min Cost

```
define MAX V 1+2*100
#define MAX_E 2*10001
typedef long long edge_type;
struct edge
int start, to:
11 cap. cost:
edge(int _s, int _d, ll _c, ll _co)
 start=_s, to=_d, cost=_co, cap=_c;
edge(){}
const edge type INF = 111<<60:</pre>
int V,E,prevee[MAX_V],last[MAX_V];
edge_type flowVal, flowCost, pot[MAX_V], dist[MAX_V];
vector<int> adj[MAX_V];
vector<edge> val;
void add(edge b)
yal.push_back(b):
adj[b.start].push_back(yal.size()-1);
swap(b.start, b.to):
b.cost*=-1, b.cap=0;
val.push back(b):
adj[b.start].push_back(yal.size()-1);
bool Bellman Ford (int s)
bool f:
Rep(i, V+1) pot[i]=INF:
pot[s]=0;
for(int i=1:i<V:i++)</pre>
 for(int i=0:i<val.size():i++)</pre>
  int k1=yal[j].start, k2=yal[j].to,w=yal[j].cost;
  if(pot[k2]>w+pot[k1])
   pot[k2]=w+pot[k1];
   f=1;
 }
 if(f==0)
  break;
for(int i=0;i<val.size();i++)</pre>
```

```
int k1=yal[i].start,k2=yal[i].to,w=yal[i].cost;
 if(pot[k2]>w+pot[k1])
  return 0:
return 1:
void mcmf(int s. int t){
flowVal = flowCost = 0:
memset(pot,0,sizeof(pot));
Bellman Ford(s):
while(true){
 for(int i = 0;i<V;++i) dist[i] = INF, prevee[i]=-1;</pre>
 priority_queue<pair<11, 11> > q;
 q.push(MP(0, s));
 dist[s] = prevee[s] = 0;
 while(!q.empty()){
  int aux = q.top().second: q.pop():
  for(int i = 0;i<adj[aux].size(); i++){</pre>
   int e=adi[aux][i]:
   if(val[e].cap<=0) continue;</pre>
   edge_type new_dist = dist[aux]+yal[e].cost+pot[aux]-pot[
        val[e].to];
   if(new dist<dist[val[e].to]){</pre>
    dist[val[e].to] = new_dist;
    prevee[val[e].to] = e:
    q.push(MP(-1*new_dist, yal[e].to));
  }
 }
 if (prevee[t]==-1) break:
 edge_type f = INF;
 for(int i = t;i!=s;i = yal[prevee[i]^1].to)
  f = min(f.val[prevee[i]].cap);
 for(int i = t:i!=s:i = val[prevee[i]^1].to){
  val[prevee[i]].cap -= f;
  yal[prevee[i]^1].cap += f;
 flowVal += f:
 flowCost += f*(dist[t]-pot[s]+pot[t]);
```

```
for(int i = 0:i<V:++i) if (prevee[i]!=-1) pot[i] += dist[i</pre>
}
}
int main(){
 int N,M,u[5000],v[5000];
 long long cst[5000],D,K;
 freopen("a.in", "r", stdin);
 while(scanf("%d %d",&N,&M)==2){
 val.clear():
 V = 2*N+1:
 for(int i=0; i<=V; i++) adj[i].clear();</pre>
 for(int i = 0;i<M;++i){</pre>
  scanf("%d %d %lld",&u[i],&v[i],&cst[i]);
  --u[i]; --v[i];
 scanf("%lld %lld",&D,&K);
 add(edge(0, 1, D, 0));
 for(int i = 0; i < N; ++i) add(edge(1+2*i,1+2*i+1,INF,0));
 for(int i = 0;i<M;++i){</pre>
  add(edge(1+2*u[i]+1,1+2*v[i],K,cst[i]));
  add(edge(1+2*v[i]+1,1+2*u[i],K,cst[i]));
 mcmf(0.2*N-1):
 if(flowVal!=D) printf("Impossible.\n");
 else printf("%lld\n",flowCost);
 return 0;
```

5.3 Bipartite Matching and Vertex Cover

```
//Bipartite Matching is O(M * N)
#define M 128
#define N 128
bool graph[M][N];
bool seen[N];
int matchL[M], matchR[N];
int n, m;
bool bpm( int u )
{
    for( int v = 0; v < n; v++ ) if( graph[u][v] )
    {</pre>
```

```
if( seen[v] ) continue;
       seen[v] = true:
       if( matchR[v] < 0 || bpm( matchR[v] ) )</pre>
           matchL[u] = v:
           matchR[v] = u;
           return true;
   return false:
vector<int> vertex cover()
// Comment : Vertices on the left side (n side) are labeled
      like this : m+i where i is the index
set<int> s. t. um: // um = UnMarked
vector<int> vc:
for(int i = 0; i < m; i++)</pre>
 if (matchL[i]==-1)
  s.insert(i), um.insert(i);
while( um.size() )
 int v = *(um.begin());
 for(int i = 0; i < n; i++)</pre>
  if( graph[v][i] && matchL[v]!=i)
   t.insert(i):
   if( s.find(matchR[i]) == s.end())
    s.insert(matchR[i]), um.insert(matchR[i]);
  }
 um.erase(v);
for(int i = 0; i < m; i++)</pre>
 if( s.find(i) == s.end() )
  vc.push back(i):
for(set<int>::iterator i = t.begin(); i != t.end(); i++)
 vc.push_back((*i) + m);
return vc;
int main()
   // Read input and populate graph[][]
   // Set m. n
   memset( matchL, -1, sizeof( matchL ) );
   memset( matchR, -1, sizeof( matchR ) );
   int cnt = 0;
   for( int i = 0: i < m: i++ )</pre>
       memset( seen, 0, sizeof( seen ) );
```

```
if( bpm( i ) ) cnt++;
}
vector<int> vc = vertex_cover();
// cnt contains the number of happy pigeons
// matchL[i] contains the hole of pigeon i or -1 if
    pigeon i is unhappy
// matchR[j] contains the pigeon in hole j or -1 if hole
    j is empty
// vc contains the Vertex Cover
return 0;
```

5.4 Bridge and Articulate Point Finding

```
typedef struct {
 int deg;
 int adj[MAX_N];
} Node:
Node alist[MAX_N];
bool art[MAX N]:
int df_num[MAX_N], low[MAX_N], father[MAX_N], count;
int bridge[MAX_N*MAX_N][2], bridges;
void add_bridge(int v1, int v2) {
 bridge[bridges][0] = v1;
 bridge[bridges][1] = v2;
 ++bridges;
void search(int v. bool root) {
 int w. child = 0:
 low[v] = df num[v] = count++:
 for (int i = 0; i < alist[v].deg; ++i) {</pre>
   w = alist[v].adi[i]:
   if (df num[w] == -1) {
     father[w] = v:
     ++child:
     search(w, false);
     if (low[w] > df_num[v]) add_bridge(v, w);
     if (low[w] >= df_num[v] && !root) art[v] = true;
     low[v] = min(low[v], low[w]);
   else if (w != father[v]) {
     low[v] = min(low[v], df num[w]):
```

```
8
```

```
if (root && child > 1) art[v] = true;
}

void articulate(int n) {
  int child = 0;

for (int i = 0; i < n; ++i) {
    art[i] = false;
    df_num[i] = -1;
    father[i] = -1;
}

count = bridges = 0;

search(0, true);
}</pre>
```

5.5 Center of Tree

```
struct node
   char ch:
   int col, big, sz;
   vector<int> adj;
}nd[MAXN];
int n, col;
vector<int> vec;
void DFS(int pos, int col)
   nd[pos].sz=1:
   nd[pos].col=col;
   int k:
   nd[pos].big=0;
   Rep(i, nd[pos].adj.size())
       k=nd[pos].adj[i];
       if(nd[k].col==col || nd[k].col==-1) continue:
       DFS(k. col):
       nd[pos].sz+=nd[k].sz;
       nd[pos].big=max(nd[pos].big, nd[k].sz);
   vec.push_back(pos);
void div(int r, char ch,int col)
   vec.clear():
   DFS(r, col);
```

```
r=vec[0];
int sz=vec.size();
Rep(i, vec.size())
{
    nd[vec[i]].big=max(nd[vec[i]].big, sz-nd[vec[i]].sz);
    if(nd[vec[i]].big<nd[r].big) r=vec[i];
}
nd[r].col=-1;
nd[r].ch=ch;
Rep(i, nd[r].adj.size()) if(nd[nd[r].adj[i]].col==col)
    div(nd[r].adj[i], ch+1, col+1);</pre>
```

5.6 Count Triangles

```
vector <int> adj[maxn], Adj[maxn];
int ord[maxn], f[maxn], fi[maxn], se[maxn], ans[maxn];
bool get(int v,int u) {
int idx = lower_bound(adj[v].begin(), adj[v].end(), u) -
     adj[v].begin();
if (idx != adj[v].size() && adj[v][idx] == u)
 return true:
return false;
bool cmp(int v,int u) {
if (adj[v].size() < adj[u].size())</pre>
 return true;
if (adj[v].size() > adj[u].size())
 return false:
return (v < u);</pre>
int main() {
int n, m, q;
cin >> n >> m >> q;
for (int i = 0: i < m: i++) {</pre>
 cin >> fi[i] >> se[i]:
 fi[i]--, se[i]--;
 adj[fi[i]].push_back(se[i]);
 adj[se[i]].push_back(fi[i]);
 Adj[fi[i]].push_back(se[i]);
 Adj[se[i]].push_back(fi[i]);
for (int i = 0; i < n; i++)</pre>
 sort(adj[i].begin(), adj[i].end()),
 sort(Adj[i].begin(), Adj[i].end(), cmp);
```

```
for (int i = 0: i < n: i++)
ord[i] = i:
sort (ord, ord + n, cmp);
for (int i = 0: i < n: i++)</pre>
f[ord[i]] = i;
for (int v = 0: v < n: v++) {
int idx = -1;
 for (int j = 0; j < adj[v].size(); j++) {</pre>
 int u = Adj[v][j];
 if (f[u] > f[v])
  break:
  idx = i:
 for (int i = 0; i <= idx; i++)</pre>
  for (int j = 0; j < i; j++) {
  int u = Adj[v][i];
  int w = Adj[v][j];
  if (get(u,w))
   ans[v]++, ans[u]++, ans[w]++:
for (int i = 0; i < q; i++) {</pre>
int v:
 cin >> v:
cout << ans[v] << '\n';
return 0:
```

5.7 DFS on Complement Graph

```
#include <bits/stdc++.h>
using namespace std;

const int maxn = 5e5 + 10;
int nxt[maxn], cmp, n;
vector <int> adj[maxn], ver[maxn];

bool con(int v,int u) {
   int idx = lower_bound(adj[v].begin(), adj[v].end(), u) -
        adj[v].begin();
   return (idx != adj[v].size() && adj[v][idx] == u);
}

int get(int v) {
   if (nxt[v] == v)
        return v;
```

```
g
```

```
return (nxt[v] = get(nxt[v]));
}
void dfs(int v) {
    nxt[v] = get(v + 1);
    ver[cmp].push back(v):
    for (int u = get(0); u < n; u = get(u + 1)) {</pre>
       if (!con(u, v))
           dfs(u):
}
int main() {
    int m:
    scanf("%d%d", &n, &m);
    for (int i = 0; i < m; i++) {</pre>
       int v. u:
       scanf("%d%d", &v, &u);
       v--. u--:
       adj[v].push_back(u);
       adj[u].push_back(v);
    for (int i = 0; i <= n; i++)</pre>
        sort (adj[i].begin(), adj[i].end());
    for (int i = 0; i < maxn; i++)</pre>
       nxt[i] = i;
    for (int i = 0: i < n: i++)
       if (get(i) == i)
           dfs(i), cmp++;
    printf("%d\n", cmp);
    for (int i = 0; i < cmp; i++) {</pre>
       printf("%d ", (int)ver[i].size());
       for (int j = 0; j < ver[i].size(); j++)</pre>
           printf("%d ", ver[i][j] + 1);
       printf("\n"):
    return 0;
```

5.8 DSU on Tree

```
// How many vertices in subtree of vertice v has some
    property in O(n lg n) time (for all of the queries).
// Approach 1
//sz[i] = size of subtree of node i
int cnt[maxn];
```

```
bool big[maxn]:
void add(int v, int p, int x){
   cnt[ col[v] ] += x;
   for(auto u: g[v])
      if(u != p && !big[u])
          add(u, v, x)
void dfs(int v, int p, bool keep){
   int mx = -1, bigChild = -1;
   for(auto u : g[v])
      if(u != p && sz[u] > mx)
          mx = sz[u], bigChild = u:
   for(auto u : g[v])
      if(u != p && u != bigChild)
          dfs(u, v, 0); // run a dfs on small childs and
               clear them from cnt
   if(bigChild != -1)
       dfs(bigChild, v, 1), big[bigChild] = 1; // bigChild
           marked as big and not cleared from cnt
   add(v, p, 1);
   //now cnt[c] is the number of vertices in subtree of
        vertice v that has color c. You can answer the
        queries easily.
   if(bigChild != -1)
       big[bigChild] = 0;
   if(keep == 0)
       add(v, p, -1);
```

5.9 Dinic Max Flow

```
while( true )
   // find an augmenting path
   memset( prev, -1, sizeof( prev ) );
   int af = 0, ab = 0:
   prev[q[qb++] = s] = -2;
   while( qb > qf && prev[t] == -1 )
       for( int u = q[qf++], i = 0, v; i < deg[u]; i++ )</pre>
           if( prev[v = adj[u][i]] == -1 && cap[u][v] )
              prev[a[ab++] = v] = u:
    // see if we're done
    if( prev[t] == -1 ) break;
   // try finding more paths
   for (int z = 0; z < n; z++) if (cap[z][t] && prev[z]
         ! = -1 )
       int bot = cap[z][t];
       for( int v = z, u = prev[v]; u >= 0; v = u, u =
            prev[v])
           bot <?= cap[u][v];
       if( !bot ) continue:
       cap[z][t] -= bot;
       cap[t][z] += bot;
       for( int v = z, u = prev[v]; u >= 0; v = u, u =
            prev[v])
           cap[u][v] -= bot;
           cap[v][u] += bot;
       flow += bot;
}
return flow;
```

5.10 Floyd Cycle Finding

```
11 a, b, c;
vector<11> vec;
11 f(11 x)
{
   return (a*x+(x%b))%c;
}
pii floydCycleFinding(11 x0)
```

```
// mu : start of cycle , lambda : lenght of cycle, cnt for
     seting limits
// 1st part: finding k*mu, hares speed is 2x tortoises
ll tortoise = f(x0), hare = f(f(x0)); // f(x0) is the
     element/node next to x0
int cnt=0:
while (tortoise != hare && cnt<=20000000)</pre>
{ tortoise = f(tortoise); hare = f(f(hare)); cnt++;}
if(cnt>20000000) return MP(-1, -1);
// 2nd part: finding mu, hare and tortoise move at the same
11 \text{ mu} = 0; \text{ hare} = x0;
while (tortoise != hare) { tortoise = f(tortoise); hare = f
     (hare): mu++: }
// 3rd part: finding lambda, hare moves, tortoise stays
11 lambda = 1; hare = f(tortoise);
while (tortoise != hare)
{ hare = f(hare): lambda++: }
return pii(mu, lambda);
int main(){
cin>>a>>b>>c:
pii ans=floydCycleFinding(1);
if(ans.first!=-1) cout<<ans.first+ans.second<<endl;</pre>
else cout<<-1<<endl:</pre>
return 0:
```

5.11 Hopcroft Karp Max Flow

```
u=adi[v][i]:
  if(dis2[u]==0)
   dis2[u]=dis1[v]+1:
   if(g2[u]==-1) f=1;
   else
    dis1[g2[u]]=dis2[u]+1;
     q.push(g2[u]);
return f:
bool DFS(int v)
for(int i=0: i<adi[v].size(): i++)</pre>
 u=adj[v][i];
 if(dis2[u] == dis1[v]+1)
  dis2[u]=0:
  if(g2[u] ==-1 || DFS(g2[u]))
   g1[v]=u;
   g2[u]=v:
   return 1;
 }
return 0:
int Hopcroft_Karp()
for(int i=0; i<n1; i++) g1[i]=-1;</pre>
for(int i=0; i<n2; i++) g2[i]=-1;</pre>
int matching=0;
while (BFS()) for(int i=0; i<n1; i++)</pre>
 if(g1[i]==-1 && DFS(i)) matching++;
return matching;
int col[MAXN];
int ind[MAXN];
void paint_geraph()
memset(col, -1, sizeof col);
int c, k;
```

```
ind[0]=0:
col[n1++]=1:
q.push(0);
while (!q.empty())
 c=q.front(), q.pop();
 for(int i=0; i<adj[c].size(); i++)</pre>
  k=adj[c][i];
  if(col[k]!=-1) continue;
  col[k]=!col[c]:
  if(col[k]) ind[k]=n1++;
  else ind[k]=n2++;
  q.push(k);
int main(){
int m. u. v:
vector<pii> ed;
cin>>n>>m;
for(int i=0; i<m; i++)</pre>
 cin>>u>>v:
 ed.push_back(make_pair(u-1, v-1));
 adj[u-1].push_back(v-1);
 adj[v-1].push_back(u-1);
n1=n2=0;
paint_geraph();
for(int i=0; i<n; i++) adj[i].clear();</pre>
for(int i=0; i<m; i++)</pre>
 u=ind[ed[i].first], v=ind[ed[i].second];
 if(col[ed[i].first]) adj[u].push_back(v);
 else adi[v].push back(u);
int ans=Hopcroft_Karp();
cout<<ans<<endl;
bool f:
for(int i=0: i<m: i++)</pre>
 u=ind[ed[i].first], v=ind[ed[i].second];
 if(col[ed[i].first])
 if(g1[u]==v)
   f=1:
 else if(g2[u]==v) f=1;
```

```
if(f) cout<<ed[i].first+1<<' ''<<ed[i].second+1<<endl;
}
return 0;
}</pre>
```

5.12 Hungarian Algorithm

```
#define N 55 //max number of vertices in one part
#define INF 100000000 //just infinity
int cost[N][N]; //cost matrix
int n, max_match; //n workers and n jobs
int lx[N], ly[N]; //labels of X and Y parts
int xy[N]; //xy[x] - vertex that is matched with x,
int yx[N]; //yx[y] - vertex that is matched with y
bool S[N], T[N]; //sets S and T in algorithm
int slack[N]; //as in the algorithm description
int slackx[N]; //slackx[y] such a vertex, that
// 1(\operatorname{slackx}[v]) + 1(v) - w(\operatorname{slackx}[v], v) = \operatorname{slack}[v]
int prv[N]; //array for memorizing alternating paths
void init labels()
    memset(lx, 0, sizeof(lx));
    memset(ly, 0, sizeof(ly));
    for (int x = 0; x < n; x++)
       for (int y = 0; y < n; y++)
           lx[x] = max(lx[x], cost[x][y]);
}
void update labels()
    int x, y, delta = INF; //init delta as infinity
    for (y = 0; y < n; y++) //calculate delta using slack
       if (!T[v])
           delta = min(delta, slack[v]):
    for (x = 0; x < n; x++) //update X labels
       if (S[x]) lx[x] -= delta;
    for (y = 0; y < n; y++) //update Y labels
       if (T[v]) lv[v] += delta;
    for (y = 0; y < n; y++) //update slack array
       if (!T[v])
           slack[v] -= delta;
}
void add_to_tree(int x, int prevx)
//x - current vertex,prevx - vertex from X before x in the
     alternating path,
```

```
//so we add edges (prevx. xv[x]), (xv[x], x)
   S[x] = true; //add x to S
   prv[x] = prevx; //we need this when augmenting
   for (int y = 0; y < n; y++) //update slacks, because we
        add new vertex to S
       if (lx[x] + ly[y] - cost[x][y] < slack[y])
           slack[y] = lx[x] + ly[y] - cost[x][y];
           slackx[v] = x;
       }
void augment() //main function of the algorithm
   if (max_match == n) return; //check wether matching is
        already perfect
   int x, y, root; //just counters and root vertex
   int q[N], wr = 0, rd = 0; //q - queue for bfs, wr,rd -
        write and read
   //pos in queue
   memset(S, false, sizeof(S)); //init set S
   memset(T, false, sizeof(T)); //init set T
   memset(prv, -1, sizeof(prv)); //init set prev - for the
        alternating tree
   for (x = 0; x < n; x++) //finding root of the tree
       if (xy[x] == -1)
           q[wr++] = root = x;
          prv[x] = -2;
           S[x] = true;
           break:
   for (v = 0: v < n: v++) //initializing slack array</pre>
       slack[y] = lx[root] + ly[y] - cost[root][y];
       slackx[v] = root;
   //second part of augment() function
   while (true) //main cycle
       while (rd < wr) //building tree with bfs cycle</pre>
           x = q[rd++]; //current vertex from X part
           for (y = 0; y < n; y++) //iterate through all
               edges in equality graph
              if (cost[x][y] == lx[x] + ly[y] && !T[y])
```

```
if (yx[y] == -1) break; //an exposed
               vertex in Y found, so
           //augmenting path exists!
           T[y] = true; //else just add y to T,
           q[wr++] = vx[y]; //add vertex vx[y], which
                 is matched
           //with v, to the queue
           add_to_tree(yx[y], x); //add edges (x,y)
               and (y,yx[y]) to the tree
   if (y < n) break; //augmenting path found!</pre>
if (y < n) break; //augmenting path found!</pre>
update_labels(); //augmenting path not found, so
    improve labeling
wr = rd = 0:
for (y = 0; y < n; y++)
   //in this cycle we add edges that were added to
        the equality graph as a
   //result of improving the labeling, we add edge (
        slackx[y], y) to the tree if
   //and only if !T[y] && slack[y] == 0, also with
        this edge we add another one
   //(y, yx[y]) or augment the matching, if y was
        exposed
   if (!T[y] && slack[y] == 0)
       if (yx[y] == -1) //exposed vertex in Y found -
             augmenting path exists!
           x = slackx[y];
           break:
       else
           T[y] = true; //else just add y to T,
           if (!S[yx[y]])
              q[wr++] = yx[y]; //add vertex yx[y],
                   which is matched with
              //v, to the queue
              add_to_tree(yx[y], slackx[y]); //and
                   add edges (x,y) and (y,
              //yx[y]) to the tree
if (y < n) break; //augmenting path found!</pre>
```

```
if (y < n) //we found augmenting path!</pre>
       max_match++; //increment matching
       //in this cycle we inverse edges along augmenting
       for (int cx = x, cy = y, ty; cx != -2; cx = prv[cx],
            cy = ty)
           ty = xy[cx];
           vx[cv] = cx:
          xv[cx] = cv:
       augment(); //recall function, go to step 1 of the
            algorithm
}//end of augment() function
int hungarian()
   int ret = 0; //weight of the optimal matching
   max_match = 0; //number of vertices in current matching
   memset(xy, -1, sizeof(xy));
   memset(yx, -1, sizeof(yx));
   init_labels(); //step 0
   augment(); //steps 1-3
   for (int x = 0; x < n; x++) //forming answer there
       ret += cost[x][xv[x]];
   return ret;
```

5.13 List Dinic

```
//From "You Know Izad?" team cheat sheet
const int MAXN = 300;
struct Edge
{
    int a, b, cap, flow;
};
int n, s, t, d[MAXN], ptr[MAXN];
vector<Edge> e;
vi adj[MAXN];
void init(){
    e.clear();
    fore(i, 0, MAXN)
        adj[i].clear();
}
void add_edge (int a, int b, int cap) {
    Edge e1 = { a, b, cap, 0 };
```

```
Edge e2 = \{ b, a, 0, 0 \}:
    adj[a].push_back ((int) e.size());
    e.push_back (e1);
    adj[b].push_back ((int) e.size());
    e.push_back (e2);
bool bfs() {
    queue <int> q;
    q.push(s);
    memset(d, -1, sizeof d);
    d[s] = 0:
    while (!q.emptv() && d[t] == -1){
       int v = q.front();
       q.pop();
       for (int i = 0; i < L(adj[v]); ++i)</pre>
           int id = adi[v][i].
               to = e[id].b:
           if (d[to] == -1 && e[id].flow < e[id].cap)</pre>
               q.push(to);
               d[to] = d[v] + 1;
    return d[t] != -1;
int dfs (int v, int flow){
    if (!flow) return 0;
    if (v == t) return flow:
    for (; ptr[v] < L(adj[v]); ++ptr[v]){</pre>
       int id = adj[v][ptr[v]],
           to = e[id].b:
       if (d[to] != d[v] + 1)
           continue:
       int pushed = dfs (to, min (flow, e[id].cap - e[id].
            flow)):
       if (pushed) {
           e[id].flow += pushed;
           e[id ^ 1].flow -= pushed;
           return pushed:
       }
    return 0;
int dinic(){
    int flow = 0;
    while(true){
       if (!bfs())
           break:
```

```
memset(ptr, 0, sizeof ptr);
    // overflow?
    while (int pushed = dfs (s, INF32))
        flow += pushed;
}
    return flow;
}
int main()
{
    init();
    // set n, s, t
    // add edges using add_edge (directed edge)
    int result = dinic();
}
```

5.14 Matrix Dinic

```
//From "You Know Izad?" team cheat sheet
#define MAXN 400
struct Edge
   int a, b;
   ll cap, flow;
int n, c[MAXN] [MAXN], f[MAXN] [MAXN], s, t, d[MAXN], ptr[MAXN
    ];
bool bfs()
   queue <int> q;
   q.push(s);
   memset (d, -1, sizeof d);
   d[s] = 0:
   while(!q.empty()){
       int v = q.front();
       q.pop();
       for (int to=0; to<n; ++to){</pre>
           if (d[to] == -1 && f[v][to] < c[v][to])
              q.push(to);
              d[to] = d[v] + 1;
       }
   return d[t] != -1;
int dfs (int v. int flow)
   if (!flow) return 0;
   if (v == t) return flow:
   for (int & to=ptr[v]; to<n; ++to){</pre>
```

```
if (d[to] != d[v] + 1) continue:
       int pushed = dfs (to, min (flow, c[v][to] - f[v][to])
            );
       if (pushed){
          f[v][to] += pushed;
          f[to][v] -= pushed;
          return pushed;
      }
   return 0;
}
int dinic() {
   int flow = 0;
   // flow between any two vertices is initially zero
   memset(f, 0, sizeof f);
   while(true){
       if (!bfs()) break;
       memset(ptr, 0, sizeof ptr);
       // overflow?
       while (int pushed = dfs (s, INF32))
          flow += pushed;
   return flow;
int main()
   // set s (source) , t (sink) , n (nodes)
   memset(c, 0, sizeof c):
   // add edges in capacity (c) matrix
   // call dinic function to get Maxflow
```

5.15 Max Flow

```
int cap[MAXN][MAXN],n;
vector<int> adj[MAXN];
int BFS(int s, int e)
{
  bool v[MAXN]={0};
  int p[MAXN], i, t, k, c;
  for(i=1;i<=n;i++)
   p[i] = -1;
  queue <int> q;
  q.push(s);
  v[s]=1;
  while(!q.empty())
  {
  t=q.front();
  q.pop();
```

```
for(i=0:i<adi[t].size():i++)</pre>
  k=adj[t][i];
  if(v[k]==0 \&\& cap[t][k]>0)
   q.push(k);
   v[k]=1;
   p[k]=t;
   if(k==e)
    break:
 }
 if(i<adj[t].size())</pre>
  break;
k=e,c=1<<28;
while(p[k] > -1)
 c=min(c,cap[p[k]][k]);
k=e:
while(p[k] > -1)
 cap[p[k]][k]-=c;
 cap[k][p[k]]+=c;
 k=p[k];
if(c==1<<28)
 return 0:
return c;
int max flow(int s. int e)
int ans=0, c;
while(1)
 c=BFS(s, e);
 if(c==0)
 break:
 ans+=c:
return ans:
void add_edge(int u, int v, int c)
adj[u].push_back(v), adj[v].push_back(u);
if(c!=MOD) cap[u][v]+=c;
else cap[u][v]=MOD;
```

5.16 Min Cost Bipartite Matching

```
//From "You Know Izad?" team cheat sheet
vi u (n+1), v (m+1), p (m+1), way (m+1);
for (int i=1; i<=n; ++i) {</pre>
   i = [0]q
   int j0 = 0;
   vi minv (m+1, INF):
   vector<char> used (m+1, false);
       used[j0] = true;
       int i0 = p[j0], delta = INF, j1;
       for (int j=1; j<=m; ++j)</pre>
           if (!used[j]) {
              int cur = a[i0][j]-u[i0]-v[j];
               if (cur < minv[j])</pre>
                   minv[j] = cur, way[j] = j0;
              if (minv[j] < delta)</pre>
                  delta = minv[i]. i1 = i;
       for (int j=0; j<=m; ++j)</pre>
           if (used[i])
               u[p[j]] += delta, v[j] -= delta;
               minv[j] -= delta;
       j0 = i1:
   } while (p[j0] != 0);
   do {
       int j1 = way[j0];
       p[j0] = p[j1];
       i0 = i1;
   } while (j0);
int cost = -v[0]; // minimum cost
// ans -> printable matching result
vi ans (n+1);
for (int j=1; j<=m; ++j)</pre>
   ans[p[j]] = j;
```

5.17 Min Cost Max Flow

```
//From "You Know Izad?" team cheat sheet
struct rib {
   int b, u, c, f;
   size_t back;
};
void add_rib (vector < vector<rib> > & g, int a, int b, int
   u, int c) {
   // u = capacity
```

```
// c = cost per flow (maybe double)
   // add edge between a and b
   rib r1 = { b, u, c, 0, g[b].size() };
   rib r2 = \{a, 0, -c, 0, g[a].size()\};
   g[a].push_back (r1);
   g[b].push_back (r2);
int main() {
   // k = the exact amount of flow (cost is calculated
        according to this)
    // set k to infinity --> gives maxFlow (in flow variable)
   vector < vector<rib> > g (n);
   int s, t;
   //reading the graph
   int flow = 0, cost = 0;
   while (flow < k) {</pre>
       vector<int> id (n, 0);
       vector<int> d (n. INF):
       vector<int> q (n);
       vector<int> p (n);
       vector<size_t> p_rib (n);
       int qh=0, qt=0;
       q[qt++] = s;
       d[s] = 0;
       while (qh != qt) {
          int v = q[qh++];
          id[v] = 2:
           if (qh == n) qh = 0;
           for (size_t i=0; i<g[v].size(); ++i) {</pre>
              rib & r = g[v][i];
              if (r.f < r.u \&\& d[v] + r.c < d[r.b]) {
                  d[r.b] = d[v] + r.c:
                  if (id[r.b] == 0) {
                      q[qt++] = r.b;
                      if (at == n) at = 0:
                  else if (id[r.b] == 2) {
                      if (--qh == -1) qh = n-1;
                     q[qh] = r.b;
                  }
                  id[r.b] = 1;
                  p[r.b] = v:
                  p_rib[r.b] = i;
       if (d[t] == INF) break:
       int addflow = k - flow:
       for (int v=t; v!=s; v=p[v]) {
```

5.18 Tarjan SCC

```
int n, low_link[MAXN], index[MAXN], ind, group, gr[MAXN];
stack<int> st;
vector<int> adj[MAXN];
bool instack[MAXN];
void tarjan(int c)
index[c]=low_link[c]=ind++;
instack[c]=1:
st.push(c);
int k:
Rep(i, (int)adj[c].size())
 k=adi[c][i]:
 if(index[k]==-1)
 ł
  tarian(k):
  low_link[c]=min(low_link[c], low_link[k]);
 else if(instack[k]) low link[c]=min(low link[c]. index[k])
if(low link[c]==index[c])
 group++;
  k=st.top(), st.pop();
  gr[k]=group;
  instack[k]=0;
 }while (k!=c);
```

```
}
int main(){
   Set(index, -1), Set(instack, 0);
   ind=group=0;
   Rep(i, n) if(index[i]==-1) tarjan(i);
   cout<<group<<endl;
   return 0;
}</pre>
```

5.19 Weighted Min Cut

```
// Maximum number of vertices in the graph
#define NN 256
// Maximum edge weight (MAXW * NN * NN must fit into an int)
#define MAXW 1000
// Adjacency matrix and some internal arrays
int g[NN][NN], v[NN], w[NN], na[NN];
bool a[NN];
int minCut( int n )
   // init the remaining vertex set
   for( int i = 0; i < n; i++ ) v[i] = i;
   // run Stoer-Wagner
   int best = MAXW * n * n;
   while (n > 1)
       // initialize the set A and vertex weights
       a[v[0]] = true:
       for( int i = 1; i < n; i++ )</pre>
          a[v[i]] = false;
          na[i - 1] = i;
          w[i] = g[v[0]][v[i]];
       // add the other vertices
       int prev = v[0]:
       for( int i = 1; i < n; i++ )</pre>
          // find the most tightly connected non-A vertex
          int zi = -1:
          for( int j = 1; j < n; j++ )</pre>
              if( !a[v[j]] && ( zj < 0 || w[j] > w[zj] ) )
```

```
// add it to A
           a[v[zi]] = true;
           // last vertex?
           if(i == n - 1)
              // remember the cut weight
              best <?= w[zi];
              // merge prev and v[zi]
              for( int j = 0; j < n; j++ )</pre>
                  g[v[j]][prev] = g[prev][v[j]] += g[v[zj]][
                       v[i]];
              v[zj] = v[--n];
              break:
           prev = v[zj];
           // update the weights of its neighbours
           for( int j = 1; j < n; j++ ) if( !a[v[j]] )</pre>
              w[i] += g[v[zi]][v[i]];
    return best;
int main()
   // read the graph's adjacency matrix into g[][]
   // and set n to equal the number of vertices
   int n, answer = minCut( n );
   return 0;
```

6 Math

6.1 Discrete Logarithm Solver

```
// discrete-logarithm, finding y for equation k = x^y % mod
int discrete_logarithm(int x, int mod, int k) {
  if (mod == 1) return 0;
  int s = 1, g;
  for (int i = 0; i < 64; ++i) {
    if (s == k) return i;
    s = (111 * s * x) % mod;
  }
  while ((g = gcd(x, mod)) != 1) {
    if (k % g) return -1;</pre>
```

```
mod /= g;
}
static unordered_map<int, int> M; M.clear();
int q = int(sqrt(double(euler(mod)))) + 1; // mod-1 is
    also okay
for (int i = 0, b = 1; i < q; ++i) {
    if (M.find(b) == M.end()) M[b] = i;
    b = (111 * b * x) % mod;
}
int p = fpow(x, q, mod);
for (int i = 0, b = 1; i <= q; ++i) {
    int v = (111 * k * inverse(b, mod)) % mod;
    if (M.find(v) != M.end()) {
        int y = i * q + M[v];
        if (y >= 64) return y;
    }
    b = (111 * b * p) % mod;
}
return -1;
}
```

6.2 Euler Totient Function

```
/* Returns the number of positive integers that are
 * relatively prime to n. As efficient as factor().
 * REQUIRES: factor()
 * REQUIRES: sqrt() must work on Int.
 * REQUIRES: the constructor Int::Int( double ).
 **/
int phi( int n ) {
 vector< int > p;
 factor( n, p );
 for( int i = 0; i < ( int )p.size(); i++ ) {
  if( i && p[i] == p[i - 1] ) continue;
  n /= p[i];
  n *= p[i] - 1;
 }
 return n;
}</pre>
```

6.3 Extended GCD

```
template< class Int >
struct Triple
{
  Int d, x, y;
  Triple( Int q, Int w, Int e ) : d( q ), x( w ), y( e ) {}
```

```
};

/* Given nonnegative a and b, computes d = gcd( a, b )
 * along with integers x and y, such that d = ax + by
 * and returns the triple (d, x, y).
 * WARNING: needs a small modification to work on
 * negative integers (operator% fails).
 **/

template< class Int >
Triple< Int > egcd( Int a, Int b )
{
   if( !b ) return Triple< Int >( a, Int( 1 ), Int( 0 ) );
   Triple< Int > q = egcd( b, a % b );
   return Triple< Int >( q.d, q.y, q.x - a / b * q.y );
}
```

6.4 Gaussian Elimination

```
//Amin Anvari's solution to Shortest XOR Path problem
const int maxl = 31:
int s, w[maxn], pat[maxn], b[maxn];
int get(int x) {
   for (int i = maxl - 1: i >= 0: i--)
       if (x & (1 << i))</pre>
           return i:
   return -1;
void add(int x) {
   for (int i = 0; i < s; i++)</pre>
       if (get(b[i]) != -1 && (x & (1 << get(b[i]))))</pre>
           x ^= b[i]:
   if (x == 0)
       return:
   for (int i = 0; i < s; i++)
       if (b[i] < x)
           swap(x, b[i]);
   b[s++] = x;
int GET(int x) {
   for (int i = 0; i < s; i++)</pre>
       if (get(b[i]) != -1 && (x & (1 << get(b[i]))))</pre>
          x ^= b[i]:
   return x;
```

6.5 Linear Diophantine Equation Solver

```
/* Solves integer equations of the form ax + by = c
* for integers x and y. Returns a triple containing
* the answer (in .x and .y) and a flag (in .d).
* If the returned flag is zero, then there are no
* solutions. Otherwise, there is an infinite number
* of solutions of the form
* x = t.x + k * b / t.d.
* v = t.v - k * a / t.d;
* where t is the returned triple, and k is any
* REQUIRES: struct Triple, egcd
template< class Int >
Triple< Int > ldioph( Int a, Int b, Int c ) {
Triple< Int > t = egcd( a, b );
if( c % t.d ) return Triple< Int >( 0, 0, 0 );
t.x *= c / t.d: t.v *= c / t.d:
return t:
```

6.6 Modular Linear Equation Solver

```
/* Given a, b and n, solves the equation ax = b (mod n)
* for x. Returns the vector of solutions, all smaller
* than n and sorted in increasing order. The vector is
* empty if there are no solutions.
* REQUIRES: struct Triple, egcd
template< class Int >
vector< Int > msolve( Int a, Int b, Int n ) {
if(n < 0) n = -n;
Triple< Int > t = egcd( a, n );
vector< Int > r:
if( b % t.d ) return r;
Int x = (b / t.d * t.x) % n:
if( x < Int( 0 ) ) x += n;</pre>
for( Int i = 0: i < t.d: i++ )</pre>
r.push_back( ( x + i * n / t.d ) % n );
return r;
```

6.7 Number of Divisors

```
/* Returns the number of positive divisors of n.
 * Complexity: about O(sqrt(n)).
```

```
* REQUIRES: factor()
* REQUIRES: sqrt() must work on Int.
* REQUIRES: the constructor Int::Int( double ).
template< class Int >
Int divisors( Int n ) {
vector< Int > f;
factor( n, f );
int k = f.size():
vector< Int > table( k + 1, Int( 0 ) );
table[k] = Int(1):
for( int i = k - 1; i \ge 0; i-- ) {
 table[i] = table[i + 1];
 for( int j = i + 1; ; j++ )
 if( j == k || f[j] != f[i] )
 { table[i] += table[j]; break; }
return table[0]:
```

6.8 Prime Factors in n Factorial

```
using namespace std;
typedef long long 11;
typedef pair<ll ,int> pii;
vector <pii> v;
//////// bozorgtarin i b shekli k N!%k^i==0
void fact(ll n) {
11 x = 2:
while (x * x \le n)
 11 num = 0:
 while (n \% x == 0) {
  num++;
 if (num) v.push_back(MP(x, num));
 if (n == 111) break:
if(n > 1) v.push_back(MP(n, 1));
11 getfact(ll n) {
ll ret = n:
Rep(i, v.size()) {
 ll k = v[i].first;
```

```
11 cnt = 0:
 11 t = n:
 while (k <= n) {</pre>
 cnt += n / k:
 n /= k;
 ret = min(ret, cnt / v[i].second);
return ret;
int main() {
int to:
11 n. k:
cin >> tc;
while (tc--) {
 v.clear();
 cin >> n >> k:
 fact(k):
 cout << getfact(n) << endl;</pre>
return 0;
```

6.9 Reduced Row Echelon Form (Gaussian Elimination)

16

```
// Reduced row echelon form via Gauss-Jordan elimination
// with partial pivoting. This can be used for computing
// the rank of a matrix.
//
// Running time: O(n^3)
//
// INPUT: a[][] = an nxm matrix
//
// OUTPUT: rref[][] = an nxm matrix (stored in a[][])
// returns rank of a[][]

#include <iostream>
#include <vector>
#include <cmath>

using namespace std;

const double EPSILON = 1e-10;

typedef double T;
```

```
17
```

```
typedef vector<T> VT;
typedef vector<VT> VVT;
int rref(VVT &a) {
 int n = a.size();
 int m = a[0].size():
 int r = 0;
 for (int c = 0; c < m && r < n; c++) {
   int i = r:
   for (int i = r + 1; i < n; i++)
     if (fabs(a[i][c]) > fabs(a[j][c])) j = i;
   if (fabs(a[i][c]) < EPSILON) continue;</pre>
   swap(a[j], a[r]);
   T s = 1.0 / a[r][c]:
   for (int j = 0; j < m; j++) a[r][j] *= s;</pre>
   for (int i = 0; i < n; i++) if (i != r) {
    T t = a[i][c];
    for (int j = 0; j < m; j++) a[i][j] -= t * a[r][j];</pre>
   r++;
 return r;
int main() {
 const int n = 5, m = 4;
 double A[n][m] = {
   {16, 2, 3, 13},
   { 5, 11, 10, 8},
   { 9, 7, 6, 12},
   { 4, 14, 15, 1},
   {13, 21, 21, 13}};
 VVT a(n);
 for (int i = 0; i < n; i++)</pre>
   a[i] = VT(A[i], A[i] + m):
 int rank = rref(a);
 // expected: 3
 cout << "Rank: " << rank << endl:
 // expected: 1 0 0 1
 11
             0 1 0 3
 11
             0 0 1 -3
             0 0 0 3.10862e-15
             0 0 0 2.22045e-15
 cout << "rref: " << endl:</pre>
 for (int i = 0; i < 5; i++) {</pre>
   for (int j = 0; j < 4; j++)
```

```
cout << a[i][j] << ' ';
cout << endl;
}
</pre>
```

6.10 Solving Recursive Functions

```
//From "You Know Izad?" team cheat sheet
a[i] = b[i] (for i \le k)
a[i] = c[1]*a[i-1] + c[2]a[i-2] + ... + c[k]a[i-k] (for i > i)
Given:
b[1], b[2], ..., b[k]
c[1], c[2], ..., c[k]
a[N]=?
typedef vector<vector<ll> > matrix;
matrix mul(matrix A, matrix B){
   matrix C(K+1, vector<ll>(K+1)):
   REP(i, K) REP(j, K) REP(k, K)
       C[i][j] = (C[i][j] + A[i][k] * B[k][j]) % INF32;
   return C:
matrix pow(matrix A, 11 p){
   if (p == 1) return A;
   if (p % 2) return mul(A, pow(A, p-1));
   matrix X = pow(A, p/2);
   return mul(X, X);
ll solve() {
   // base (initial) values
   vector<ll> F1(K+1):
   REP (i. K)
       cin >> F1[i];
   matrix T(K+1, vector<ll>(K+1));
   REP(i, K) {
       REP(j, K) {
           if(j == i + 1) T[i][j] = 1;
           else if(i == K) cin >> T[i][K - j + 1]; //
               multipliers
           else T[i][j] = 0;
   }
   11 N;
   cin >> N;
   if (N == 1) return 1;
   T = pow(T, N-1);
```

```
ll res = 0;
REP(i, K)
    res = (res + T[1][i] * F1[i]) % INF32; // Mod Value
    return res;
}
int main() {
    cin >> K;
    cout << solve() << endl;
}</pre>
```

7 Sequences

7.1 FFT and Multiply

```
#define base complex<double>
void fft (vector<base> & a, bool invert){
   if (L(a) == 1) return:
   int n = L(a);
   vector \langle base \rangle a0(n / 2), a1(n / 2);
   for (int i = 0, j = 0; i < n; i += 2, ++j){
       a0[i] = a[i];
       a1[j] = a[i + 1];
   fft (a0, invert);
   fft (a1. invert):
   double ang = 2 * PI / n * (invert ? -1 : 1);
   base w(1), wn(cos(ang), sin(ang));
   fore(i, 0, n / 2) {
       a[i] = a0[i] + w * a1[i];
       a[i + n / 2] = a0[i] - w * a1[i]:
       if (invert)
          a[i] /= 2, a[i + n / 2] /= 2;
   }
void multiply (const vector<int> &a, const vector<int> & b,
    vector<int> &res){
   vector <base> fa(all(a)), fb(all(b));
   size t n = 1:
   while (n < max(L(a), (L(b)))) n <<= 1;
   n <<= 1;
   fa.resize(n), fb.resize(n);
   fft(fa, false), fft(fb, false);
   fore(i, 0, n)
   fa[i] *= fb[i];
   fft (fa. true):
   res.resize (n);
```

```
18
```

```
fore(i, 0, n)
  res[i] = int (fa[i].real() + 0.5);
```

7.2 LIS

```
void reconstruct print(int end, int a[], int p[]) {
 int x = end;
 stack<int> s;
 for (; p[x] >= 0; x = p[x]) s.push(a[x]);
 printf("[%d", a[x]);
 for (; !s.empty(); s.pop()) printf(", %d", s.top());
 printf("]\n");
}
int main() {
 int n = 11, A[] = \{-7, 10, 9, 2, 3, 8, 8, 1, 2, 3, 4\};
 int L[MAX N], L id[MAX N], P[MAX N];
 int lis = 0, lis end = 0:
 for (int i = 0: i < n: ++i) {</pre>
   int pos = lower_bound(L, L + lis, A[i]) - L;
   L[pos] = A[i]:
   L_id[pos] = i;
   P[i] = pos ? L_id[pos - 1] : -1;
   if (pos + 1 > lis) {
     lis = pos + 1;
     lis end = i:
   printf("LIS ending at A[%d] is of length %d: ", i, pos +
   reconstruct_print(i, A, P);
   printf("\n");
 printf("Final LIS is of length %d: ", lis);
 reconstruct_print(lis_end, A, P);
 return 0:
```

7.3 Manacher Longest Palindrome

```
string preProcess(string s) {
  int n = s.length();
  if (n == 0) return "^$";
  string ret = "^";
  for (int i = 0; i < n; i++)</pre>
```

```
ret += "#" + s.substr(i, 1):
 ret += "#$":
 return ret:
string longestPalindrome(string s) {
 string T = preProcess(s);
 int n = T.length();
 int *P = new int[n];
 int C = 0, R = 0:
 for (int i = 1: i < n-1: i++) {</pre>
   int i_mirror = 2*C-i; // equals to i' = C - (i-C)
   P[i] = (R > i) ? min(R-i, P[i_mirror]) : 0;
   // Attempt to expand palindrome centered at i
   while (T[i + 1 + P[i]] == T[i - 1 - P[i]])
    P[i]++:
   // If palindrome centered at i expand past R,
   // adjust center based on expanded palindrome.
   if (i + P[i] > R) {
     C = i;
     R = i + P[i]:
 // Find the maximum element in P.
 int maxLen = 0;
 int centerIndex = 0:
 for (int i = 1: i < n-1: i++) {
   if (P[i] > maxLen) {
    maxLen = P[i]:
     centerIndex = i;
 delete[] P;
 return s.substr((centerIndex - 1 - maxLen)/2, maxLen);
```

8 Strings

8.1 Aho Corasick

```
#include <bits/stdc++.h>
#define FOR(i, n) for (int i = 0; i < (n); ++i)
#define REP(i, n) for (int i = 1; i <= (n); ++i)
using namespace std;

struct AC_trie {</pre>
```

```
int N. P:
vector<map<char, int>> next; // trie
vector<int> link, out_link;
vector<vector<int>> out:
AC_trie(): N(0), P(0) { node(); }
int node() {
 next.emplace_back(); // trie
 link.emplace_back(0);
 out_link.emplace_back(0);
 out.emplace_back(0);
 return N++:
int add_pattern(const string T) {
 int u = 0:
 for (auto c : T) {
   if (!next[u][c]) next[u][c] = node();
   u = next[u][c]:
 out[u].push back(P):
 return P++:
void compute() {
 queue<int> q;
 for (q.push(0); !q.empty(); ) {
   int u = q.front(); q.pop();
   // trie:
   for (auto e : next[u]) {
     int v = e.second:
     link[v] = u ? advance(link[u], e.first) : 0;
     out_link[v] = out[link[v]].empty() ? out_link[link[v]
         ]] : link[v];
     q.push(e.second);
 }
int advance(int u. char c) {
 // trie:
 while (u && next[u].find(c) == next[u].end())
   u = link[u];
 if (next[u].find(c) != next[u].end())
   u = next[u][c]:
 return u;
void match(const string S) {
 int u = 0:
 for (auto c : S) {
   u = advance(u, c);
   for (int v = u; v; v = out_link[v])
    for (auto p : out[v])
       cout << "match " << p << endl;</pre>
```

```
}:
struct AC_automaton {
 int N, P;
 vector<vector<int>> next: // automaton
 vector<int> link, out_link;
 vector<vector<int>> out;
 AC_automaton(): N(0), P(0) { node(); }
 int node() {
   next.emplace_back(26, 0); // automaton
   link.emplace back():
   out_link.emplace_back();
   out.emplace_back();
   return N++:
 int add_pattern(const string T) {
   int u = 0:
   for (auto c : T) {
     if (!next[u][c - 'a']) next[u][c - 'a'] = node():
    u = next[u][c - 'a'];
   out[u].push_back(P);
   return P++;
 void compute() {
   queue<int> q;
   for (q.push(0); !q.empty(); ) {
     int u = q.front(); q.pop();
    // automaton:
     for (int c = 0; c < 26; ++c) {
      int v = next[u][c];
      if (!v) next[u][c] = next[link[u]][c];
      else {
        link[v] = u ? next[link[u]][c] : 0;
        out link[v] = out[link[v]].emptv() ? out link[link[
             vll : link[v]:
        q.push(v);
 int advance(int u. char c) {
   // automaton:
   while (u && !next[u][c - 'a']) u = link[u];
   u = next[u][c - 'a']:
   return u;
 void match(const string S) {
   int u = 0:
```

```
for (auto c : S) {
    u = advance(u, c):
    for (int v = u; v; v = out_link[v])
      for (auto p : out[v])
        cout << "match " << p << endl;</pre>
 }
int main() {
 int P:
 string T:
 cin >> P:
 AC trie match1:
 AC automaton match2:
 REP (i, P) {
   cin >> T:
   match1.add_pattern(T); match2.add_pattern(T);
 match1.compute();
 match2.compute();
 cin >> T:
 match1.match(T);
 match2.match(T);
 return 0;
```

8.2 KMP

```
//From "You Know Izad?" team cheat sheet
int fail[100005]:
void build(const string &kev){
   fail[0] = 0;
   fail[1] = 0:
   fore(i, 2, L(key)) {
      int j = fail[i - 1];
       while (true) {
          if (kev[i] == kev[i - 1]) {
             fail[i] = j + 1;
              break;
          else if (j == 0) break;
          j = fail[j];
   }
int KMP(const string &text, const string &key) {
   build(kev):
   int i = 0, j = 0;
```

```
while (true) {
    if (j == L(text)) return -1;
    if (text[j] == key[i]) {
        i++;
        j++;
        if (i == L(key)) return j - i;
    }
    else if (i > 0) i = fail[i];
    else j++;
}
```

8.3 Suffix Array and LCP

```
//From "You Know Izad?" team cheat sheet
const int MAXLG = 15:
const int MAXN = 3000;
int P[MAXLG][MAXN], stp;
string s;
struct entry {
   int nr[2], p;
} L[MAXN]:
int cmp(entry a, entry b) {
   return a.nr[0] == b.nr[0] ? (a.nr[1] < b.nr[1] ? 1 : 0) :
         (a.nr[0] < b.nr[0] ? 1 : 0);
void init() {
   memset(P, 0, sizeof P);
   stp = 1:
   fore(i, 0, MAXN) {
      L[i].nr[0] = 0:
      L[i].nr[1] = 0:
      L[i].p = 0;
int rangeComp(int idx, const string &t, int len){
   fore(i, 0, len) {
       if(i \ge L(t) && i + idx \ge L(s)) return 0;
       else if(i + idx >= L(s)) return 1:
       else if(i >= L(t)) return -1:
       if(s[i + idx] == t[i]) continue:
      if(s[i + idx] > t[i]) return 1;
      return -1;
   return 0:
void construct() {
   init():
   for (int i = 0; i < L(s); i++)</pre>
```

```
P[0][i] = s[i] - 'a':
    for (int cnt = 1; (cnt >> 1) < L(s); stp++, cnt <<= 1){</pre>
       for (int i = 0; i < L(s); i++){</pre>
           L[i].nr[0] = P[stp - 1][i];
           L[i].nr[1] = i + cnt < L(s) ? P[stp-1][i+cnt] :
                -1:
           L[i].p = i;
       }
       sort(L, L + L(s), cmp);
       for (int i = 0; i < L(s); i ++)</pre>
           P[stp][L[i].p] = i > 0 \&\& L[i].nr[0] == L[i - 1].
                nr[0] \&\& L[i].nr[1] == L[i - 1].nr[1] ? P[stp
                ][L[i - 1].p] : i;
}
ii stringMatching(const string &t){
    int low = 0, high = L(s) - 1, mid = low:
    while (low < high){</pre>
       mid = (low + high) / 2:
       int res = rangeComp(L[mid].p , t, L(t));
       if (res >= 0) high = mid;
       else low = mid + 1:
    if (rangeComp(L[low].p , t, L(t)) != 0) return ii(-1, -1)
    ii ans:
    ans.first = low:
    low = 0: high = L(s) - 1: mid = low:
    while (low < high) {</pre>
       mid = (low + high) / 2:
       int res = rangeComp(L[mid].p, t, L(t));
       if (res > 0) high = mid;
       else low = mid + 1:
    if (rangeComp(L[high].p, t, L(t)) != 0) high--;
    ans.second = high:
    return ans:
int lcp(int x, int y) {
    int ret = 0:
    if (x == v) return L(s) - x:
    for (int k = stp - 1; k \ge 0 && x < L(s) && y < L(s); k
        --) {
       if (P[k][x] == P[k][y])
           x += (1 << k), y += (1 << k), ret += (1 << k);
   }
    return ret;
}
int main() {
    cin >> s:
```

```
construct();
  string t;
// rangeComp and stringMatching are optional
  while(cin >> t) {
     ii ans = stringMatching(t);
     cout << ans.first << " " << ans.second << endl;
  }
}</pre>
```

8.4 Z Algorithm

```
// Z[i] => max len perfixi az s k az khuneye i e S shoru
mishe

int L = 0, R = 0;
n=s.size();
for (int i = 1; i < n; i++)
{
    if (i > R) {
        L = R = i;
        while (R < n && s[R-L] == s[R]) R++;
        z[i] = R-L; R--;
    }
    else
    {
        int k = i-L;
        if (z[k] < R-i+1) z[i] = z[k];
        else
        {
        L = i;
        while (R < n && s[R-L] == s[R]) R++;
        z[i] = R-L; R--;
    }
}
}
```

9 Tips, Tricks and Theorems

9.1 C++ Tricks

9.2 Contest Tips

```
READ THE STATEMENT AGAIN. TELL YOUR TEAMMATE IF
 NECESSARY
Double check spell of literals
Graph: Multiple components, Multiple edges, Loops
Geometry: Be careful about +pi.-pi
Initialization: Use memset/clear(). Dont expect global
variables to be zero. Care about multiple tests
Precision and Range: Use long long if necessary. Use
BigInteger/BigDecimal
Derive recursive formulas that use sum instead of
multiplication to avoid overflow.
Small cases (n=0,1,negative)
0-based <=> 1-based
Division by zero. Integer division a/(double)b
Stack overflow (DFS on 1e5)
Infinite loop?
array bound check. maxn or x*maxn
Dont use .size()-1 !
   (int)-3 < (unsigned int) 2 is false!
Check copy-pasted codes!
Be careful about -0.0
Remove debug info!
Output format: Spaces at the end of line. Blank lines.
View the output in VIM if necessary
Add eps to double before getting floor or round
Convex Hull: Check if points are collinear
Geometry: Distance may not overflow, but its square may
 does
```

9.3 Gallai

```
\begin{split} &a(G) \; := \; max\{|C| \; | \; C \; \text{is a stable set}\}, \\ &b(G) \; := \; min\{|W| \; | \; W \; \text{is a vertex cover}\}, \\ &c(G) \; := \; max\{|M| \; | \; M \; \text{is a matching}\}, \\ &d(G) \; := \; min\{|F| \; | \; F \; \text{is an edge cover}\}. \\ &Gallais \; \; \text{theorem: If } G = (V, \; E) \; \text{is a graph without isolated vertices, then} \\ &a(G) \; + \; b(G) \; = \; |V| \; = \; c(G) \; + \; d(G) \; . \end{split}
```

9.4 Konig

Knig theorem can be proven in a way that provides additional useful information beyond just its truth: the proof provides a way of constructing a minimum vertex cover from a maximum matching. Let $\{G=(V,E)\}$ be a bipartite graph, and let the vertex set $\{V\}$ be

partitioned into left set { L} and right set { R}. Suppose that { M} is a maximum matching for { G}. No vertex in a vertex cover can cover more than one edge of { M} (because the edge half-overlap would prevent { M} from being a matching in the first place), so if a vertex cover with { |M|} vertices can be constructed, it must be a minimum cover.[10]

To construct such a cover, let { U} be the set of unmatched vertices in { L} (possibly empty), and let { Z} be the set of vertices that are either in { U} or are connected to { U} by alternating paths (paths that alternate between edges that are in the matching and edges that are not in the matching). Let { K=(L - Z) Union (R Intersect Z).}

Every edge { e} in { E} either belongs to an alternating path (and has a right endpoint in { K}), or it has a left endpoint in { K}. For, if { e} is matched but not in an alternating path, then its left endpoint cannot

be in an alternating path (for such a path could only

end at { e}) and thus belongs to { L- Z}. Alternatively , if { e} is unmatched but not in an alternating path, then its left endpoint cannot be in an alternating path , for such a path could be extended by adding { e} to it. Thus, { K} forms a vertex cover.[11]

Additionally, every vertex in { K} is an endpoint of a matched edge. For, every vertex in { L - Z} is matched because Z is a superset of U, the set of unmatched left vertices. And every vertex in { R intersect Z} must also be matched, for if there existed an alternating path to an unmatched vertex then changing the matching by removing the matched edges from this path and adding the unmatched edges in their place would increase the size of the matching. However, no matched edge can have both of its endpoints in { K}. Thus, { K} is a vertex cover of cardinality equal to { M}, and must be a minimum vertex cover.[11]