University of Glasgow

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Doctoral Program Research proposal

Game-theoretic and probabilistic methods applied to spatial network models of contagion

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1 Introduction

It is clear that networks are integral to human life: with COVID-19, we have experienced the alarming rate at which a pandemic can manoeuvre through networks; the prevalent discussion of 'fake news' demonstrates the harmful effects the rate of dissemination can have. While these issues are by no means novel, they show that the scope and applications of modelling contagion across given spatial domains are remarkably broad and pertinent to much of our experience as social animals.

One of the most versatile models used to understand these situations is the Firefighter Problem: each vertex of a network can catch fire, unless it is defended by a firefighter. The problem centres around game-theoretical methods and strategies for determining the least-worst outcome, given certain initial conditions and this can be easily translated to contagion across networks. Here, vertices would represent individuals who can become 'infected' or receive some idea or information under determined conditions, for instance if there is a neighbour infected or in possession of the information in the previous discrete time-step.

We aim to use game theoretic and probabilistic methods to analyse various strategies for different versions of the Firefighter Problem. For instance, we would consider the use of varying rates of infection to model the effectiveness of 'firewalls' or self-isolation in a highly contagious pandemic scenario such as COVID-19 and analyse the outcomes and most effective strategies to optimise the outcome - minimising the likelihood of the maximum worst outcome. Further, these techniques would determine how an underground revolutionary communication network could best protect itself from the implications of betrayal and subversion when no 'defenders' analogous to firefighters in the base problem are present.

2 Literature Review

In this section, we aim to outline some of the influential work that lies within the scope of the proposed work.

One very comprehensive piece of writing, that serves almost as a catalogue or textbook for the Firefighter Problem, is by Finbow and MacGillivray. [2] This paper provides a firm foundation for the formalised Firefighter Problem and mentions an array of potential applications along with previous work in such veins. They note that there is a great deal of writing surrounding contagion of diseases and ideas in certain networks and that many of these models employ probabilistic methods to some extent.

It will be helpful to begin by discussing scale-free networks, such as considered by Pastor-Satorras and Vespignani in the context of computer viruses. [8] Much work has been done on preliminary models and proof of concept, but little further than this. For instance, it has been shown that in a scale-free network, the more biased policy is towards the better connected hubs, the better the chance of eradicating a virus we have. [1] In fact, interestingly this result also shows that such policies are more cost effective and reduce the quantities of cures or immunisations required to have the epidemic threshold supersede the rate of infection. Another helpful modelling assumption is that of the 'small-world' network: here, most vertices are not neighbours of each other, but are connected to one another by a small route of connections to other vertices. [9] In such a network, we can find a simple but effective model for plant disease that gives us percolation thresholds for the control of disease. [6] By examining these examples that prove the concept is fruitful given the numerical model results that many authors use to compare their network results, we will develop more general and sophisticated networks that provide more useful and applicable results.

By extension of these modelling assumptions, we can view the internet as a scale-free network and examine the implications, for instance in the distribution of content across social networks and between platforms [7] and how this circulation of information can influence individuals in such models. [4] From a computing perspective, our infrastructure relies increasingly upon the internet. Hence, being able to study the implications of a computer virus outbreak or how media platforms can circulate information, interacting with other aspects of our lives, would give us huge insight into what may be on the way as our integration with online realities increases. We hope that, by examining spatial network models of contagion, we would be able to model these situations in increasing generality and detail than currently present in the literature on this topic.

3 Aims and Objectives

As mentioned, one of the main goals of this research would be to produce a model with enough generality as to serve a number of purposes, from disease spread to information dissemination, involving probabilities and conditions under which a factor would spread from vertex to vertex. This would be analysed using game-theoretic and probabilistic strategies to return algorithms and strategies to return the least-worst outcome of an epidemic scenario or minimising the maximum outcome in the case of false information circulation, which could apply to the current UK Government SHARE campaign to combat false information on social media. [5]

In particular, we would like to examine the measures taken to limit the spread of COVID-19, in the UK and elsewhere, and use these to test the model we developed by comparison to the real-world data. This would allow us to compare strategies used by different nations and indeed different regions and examining the types of networks where some strategies are effective and others prove less-so. For example, while social distancing appears to have been effective in densely populated areas, other strategies may be more useful in less dense populations where distances between individual places of abode are already much larger on average. To model this, we would use the notion of a geodesic and examine the results obtained by focusing resources on more populous hubs, as suggested by Dezső and Barabási. [1]

4 Timeline

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