Agency-Based Firefighting

Ethan Kelly

March 18, 2021

Contents

1	Res	sults 2
	1.1	Complete Graph
		1.1.1 Deterministic Protection
		1.1.2 Mixed Protection
		1.1.3 Random Protection
	1.2	4-Regular Graph
		1.2.1 Deterministic Protection
		1.2.2 Mixed Protection
		1.2.3 Random Protection
	1.3	Erdős–Rényi Graph
		1.3.1 Deterministic Protection
		1.3.2 Mixed Protection
		1.3.3 Random Protection
	1.4	Tree Graph
		1.4.1 Deterministic Protection
		1.4.2 Mixed Protection
		1.4.3 Random Protection

1 Results

1.1 Complete Graph

For the Complete graph, the Graph Generator used the following parameters:

• Type of graph: Complete

• Number of vertices: 20

• Number of edges: 190

• Random generator seed: 1615819871401

and the model took the following parameters:

• Total defence quota each turn: 1.0

• Probability with which the infection propagates: 1.0

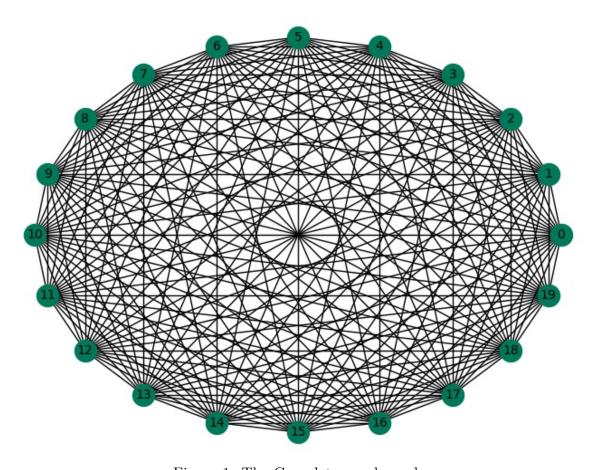


Figure 1: The Complete graph used.

1.1.1 Deterministic Protection

The winning strategies by source node are listed in the table below. This really isn't too surprising - by *deterministic*, we mean that the initial protection is equal to the "peril" rating of the given agent, which - in a complete graph - must be 1, since every vertex is adjacent to every other vertex and so adjacent to the source node. We can see this in the following initial agent states for source node 0:

Vertex Location	Peril	Protection	State
0	1.00	1.00	Infected
1	1.00	1.00	Susceptible
2	1.00	1.00	Susceptible
3	1.00	1.00	Susceptible
4	1.00	1.00	Susceptible
5	1.00	1.00	Susceptible
6	1.00	1.00	Susceptible
7	1.00	1.00	Susceptible
8	1.00	1.00	Susceptible
9	1.00	1.00	Susceptible
10	1.00	1.00	Susceptible
11	1.00	1.00	Susceptible
12	1.00	1.00	Susceptible
13	1.00	1.00	Susceptible
14	1.00	1.00	Susceptible
15	1.00	1.00	Susceptible
16	1.00	1.00	Susceptible
17	1.00	1.00	Susceptible
18	1.00	1.00	Susceptible
19	1.00	1.00	Susceptible

Thus, all vertices become protected in the second turn as their protection rating is assigned to 1 and the model ends. In future, this would become a more interesting scenario when we introduce the notion of protection decay - the protection rating of a given agent decays over time, moving the agent back into the susceptible state.

Source node	Winning Strategy	Infections	Protections	End-Turn
0	All	1	19	1
1	All	1	19	1
2	All	1	19	1
3	All	1	19	1
4	All	1	19	1
5	All	1	19	1
6	All	1	19	1
7	All	1	19	1
8	All	1	19	1
9	All	1	19	1
10	All	1	19	1
11	All	1	19	1
12	All	1	19	1
13	All	1	19	1
14	All	1	19	1
15	All	1	19	1
16	All	1	19	1
17	All	1	19	1
18	All	1	19	1
19	All	1	19	1

Defence Strategy Performance Number of infected agents for each defence strategy by source node (smaller the better). Infected Agents Source vertex ■ PROXIMITY ■ DEGREE ■ PROTECTION Defence Strategy Performance Number of protected agents for each defence strategy by source node (larger the better). 16 14 Protected Agents 10 13 15 Source vertex ■ PROXIMITY ■ DEGREE ■ PROTECTION **Defence Strategy Performance** End turn count for each defence strategy by source node (smaller the better). End turn count 10 13

Figure 2: Model results on a Complete graph by source node for each defence strategy with deterministic initial protection allocation.

Source vertex

PROXIMITY DEGREE PROTECTION

1.1.2 Mixed Protection

Here, we allocate protection by first generating a baseline pseudo-random number and then increasing based on proximity to closest infection.

Source node	Winning Strategy	Infections	Protections	End-Turn
0	Protection	15	5	2
1	Protection	16	4	2
2	Protection	15	5	2
3	Protection	16	4	2
4	Protection	16	4	2
5	Protection	13	7	2
6	Protection	15	5	2
7	Protection	15	5	2
8	Protection	15	5	2
9	Protection	17	3	2
10	Protection	15	5	2
11	Protection	13	7	2
12	Protection	16	4	2
13	Protection	15	5	2
14	Protection	14	6	2
15	Protection	13	7	2
16	Protection	16	4	2
17	Protection	13	7	2
18	Protection	13	7	2
19	Protection	13	7	2

Defence Strategy Performance Number of protected agents for each defence strategy by source node (larger the better).

■ PROXIMITY ■ DEGREE ■ PROTECTION

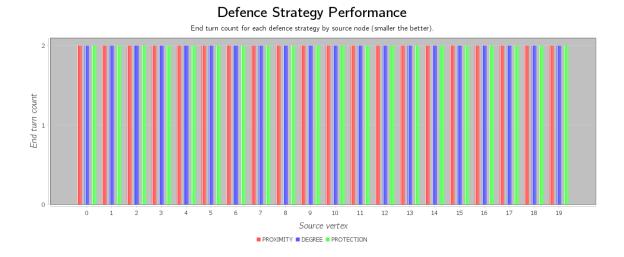
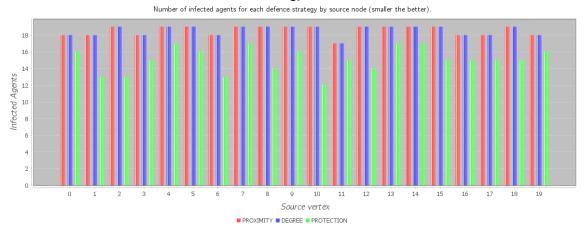


Figure 3: Model results on a Complete graph by source node for each defence strategy with mixed initial protection allocation.

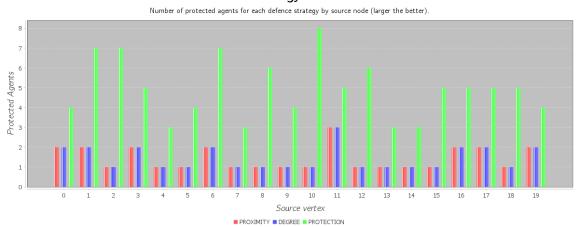
1.1.3 Random Protection

Here, we generate a pseudo-random number and assigning this as the protection rating of the given vertex

Source node	Winning Strategy	Infections	Protections	End-Turn
0	Protection	14	6	2
1	Protection	16	4	2
2	Protection	13	7	2
3	Protection	13	7	2
4	Protection	14	6	2
5	Protection	15	5	2
6	Protection	16	4	2
7	Protection	13	7	2
8	Protection	13	7	2
9	Protection	14	6	2
10	Protection	15	5	2
11	Protection	14	6	2
12	Protection	16	4	2
13	Protection	14	6	2
14	Protection	15	5	2
15	Protection	16	4	2
16	Protection	14	6	2
17	Protection	16	4	2
18	Protection	13	7	2
19	Protection	13	7	2



Defence Strategy Performance



Defence Strategy Performance

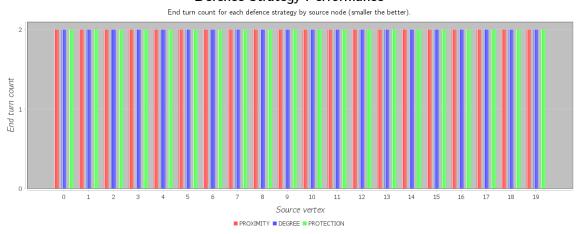


Figure 4: Model results on a Complete graph by source node for each defence strategy with mixed initial protection allocation.

1.2 4-Regular Graph

For the 4-Regular (Quartic) graph, the Graph Generator used the following parameters:

• Type of graph: 4-Regular

• Number of vertices: 20

• Number of edges: 40

 \bullet Random generator seed: 1615826375197

and the model took the following parameters:

 $\bullet\,$ Total defence quota each turn: $1.0\,$

• Probability with which the infection propagates: 1.0

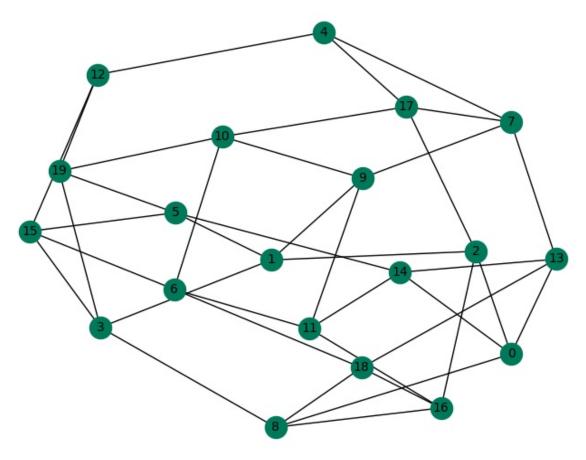
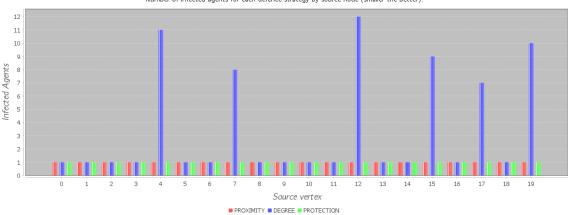


Figure 5: The 4-Regular (Quartic) graph used.

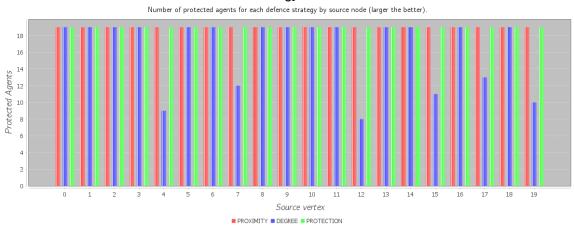
1.2.1 Deterministic Protection

Source node	Winning Strategy	Infections	Protections	End-Turn
0	Proximity	1	19	17
0	Degree	1	19	17
1	Proximity	1	19	17
1	Degree	1	19	17
2	Proximity	1	19	17
2	Degree	1	19	17
3	Proximity	1	19	17
3	Degree	1	19	17
4	Proximity	1	19	19
5	Proximity	1	19	17
5	Degree	1	19	17
6	Proximity	1	19	17
6	Degree	1	19	17
7	Proximity	1	19	17
8	Proximity	1	19	17
8	Degree	1	19	17
9	Proximity	1	19	17
9	Degree	1	19	17
10	Proximity	1	19	17
10	Degree	1	19	17
11	Proximity	1	19	17
11	Degree	1	19	17
12	Proximity	1	19	19
13	Proximity	1	19	17
13	Degree	1	19	17
14	Proximity	1	19	17
14	Degree	1	19	17
15	Proximity	1	19	17
16	Proximity	1	19	17
16	Degree	1	19	17
17	Proximity	1	19	17
18	Proximity	1	19	17
18	Degree	1	19	17
19	Proximity	1	19	17

Defence Strategy Performance Number of infected agents for each defence strategy by source node (smaller the better).



Defence Strategy Performance



Defence Strategy Performance

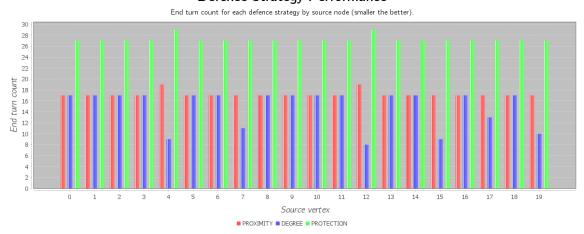
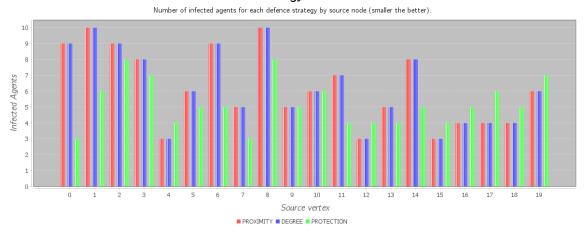


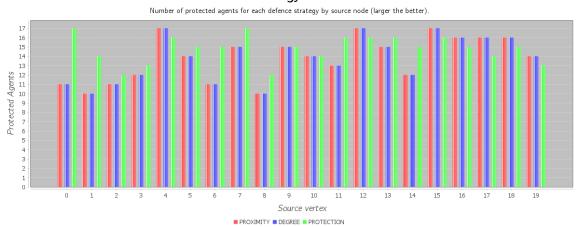
Figure 6: Model results on a 4-Regular graph by source node for each defence strategy with deterministic initial protection allocation.

1.2.2 Mixed Protection

Source node	Winning Strategy	Infections	Protections	End-Turn
0	Protection	3	17	9
1	Protection	6	14	7
2	Protection	8	12	4
3	Protection	7	13	4
4	Proximity	3	17	7
4	Degree	3	17	7
5	Protection	5	15	4
6	Protection	5	15	5
7	Protection	3	17	3
8	Protection	8	12	6
9	Protection	5	15	3
10	Protection	6	14	5
11	Protection	4	16	5
12	Proximity	3	17	7
12	Degree	3	17	7
13	Protection	4	16	7
14	Protection	5	15	7
15	Proximity	3	17	7
15	Degree	3	17	7
16	Proximity	4	16	5
16	Degree	4	16	5
17	Proximity	4	16	9
17	Degree	4	16	9
18	Proximity	4	16	5
18	Degree	4	16	5
19	Proximity	6	14	7
19	Degree	6	14	7



Defence Strategy Performance



Defence Strategy Performance

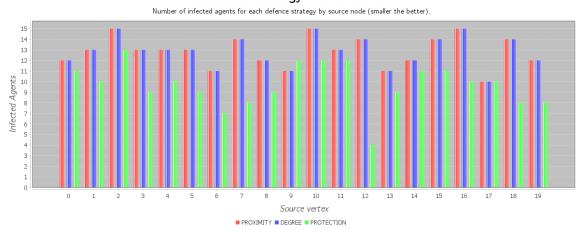
End turn count for each defence strategy by source node (smaller the better).

Possible of the property of the

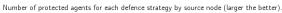
Figure 7: Model results on a 4-Regular graph by source node for each defence strategy with mixed initial protection allocation.

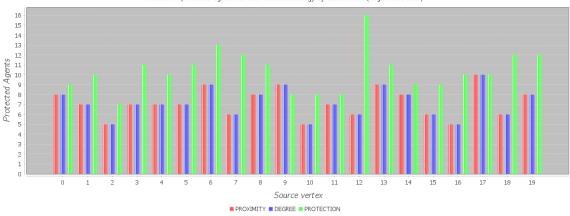
1.2.3 Random Protection

Source node	Winning Strategy	Infections	Protections	End-Turn
0	Protection	11	9	8
1	Protection	10	10	7
2	Protection	13	7	6
3	Protection	9	11	9
4	Protection	10	10	9
5	Protection	9	11	7
6	Protection	7	13	7
7	Protection	8	12	11
8	Protection	9	11	9
9	Proximity	11	9	10
9	Degree	11	9	10
10	Protection	12	8	6
11	Protection	12	8	7
12	Protection	4	16	17
13	Protection	9	11	7
14	Protection	11	9	8
15	Protection	11	9	6
16	Protection	10	10	9
17	Protection	10	10	6
18	Protection	8	12	9
19	Protection	8	12	5



Defence Strategy Performance





Defence Strategy Performance

End turn count for each defence strategy by source node (smaller the better).

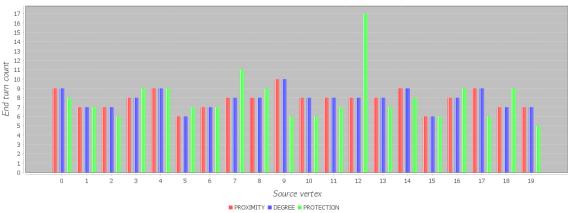


Figure 8: Model results on a 4-Regular graph by source node for each defence strategy with random initial protection allocation.

1.3 Erdős–Rényi Graph

For the Erdős–Rényi graph, the Graph Generator used the following parameters:

• Type of graph: Erdős–Rényi

• Number of vertices: 20

• Number of edges: 82

 \bullet Probability: 82 / (20 * (20 - 1) / 2) = 0.43

• Random generator seed: 1615826141222

and the model took the following parameters:

• Total defence quota each turn: 1.0

• Probability with which the infection propagates: 1.0

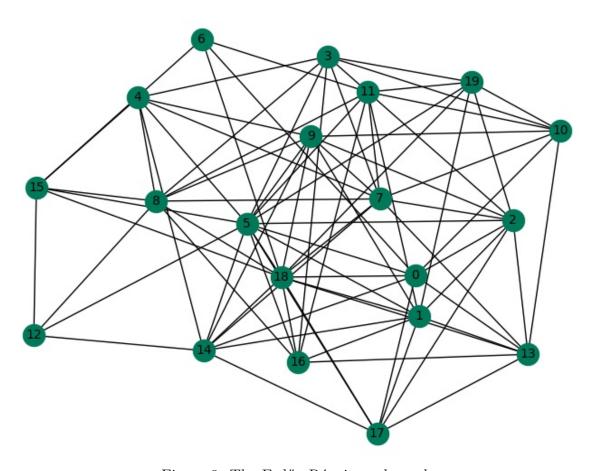
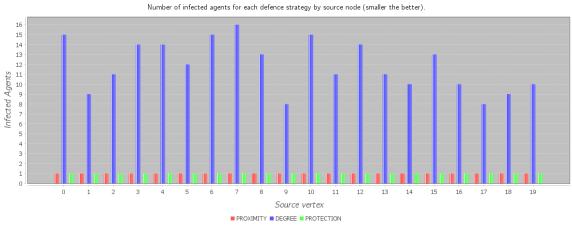


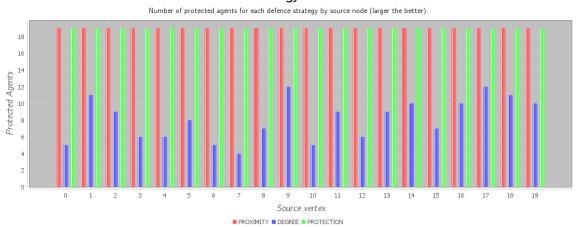
Figure 9: The Erdős–Rényi graph used.

1.3.1 Deterministic Protection

Source node	Winning Strategy	Infections	Protections	End-Turn
0	Proximity	1	19	11
1	Proximity	1	19	9
2	Proximity	1	19	11
3	Proximity	1	19	11
4	Proximity	1	19	13
5	Proximity	1	19	5
6	Proximity	1	19	15
7	Proximity	1	19	9
8	Proximity	1	19	11
9	Proximity	1	19	11
10	Proximity	1	19	13
11	Proximity	1	19	11
12	Proximity	1	19	15
13	Proximity	1	19	11
14	Proximity	1	19	11
15	Proximity	1	19	13
16	Proximity	1	19	13
17	Proximity	1	19	13
18	Proximity	1	19	7
19	Proximity	1	19	13



Defence Strategy Performance



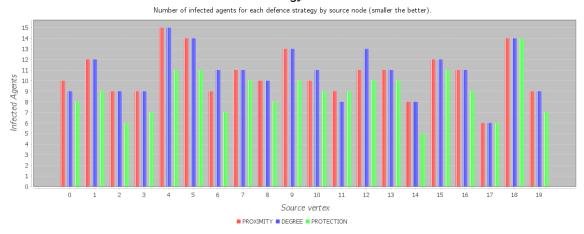
Defence Strategy Performance



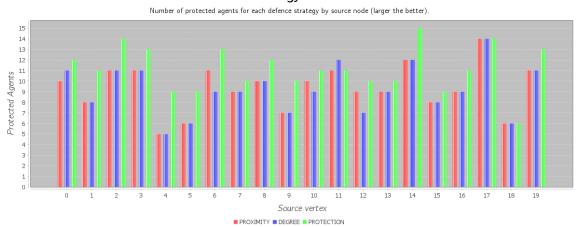
Figure 10: Model results on an Erdős–Rényi graph by source node for each defence strategy with deterministic initial protection allocation.

1.3.2 Mixed Protection

Source node	Winning Strategy	Infections	Protections	End-Turn
0	Protection	8	12	4
1	Protection	9	11	4
2	Protection	6	14	4
3	Protection	7	13	4
4	Protection	11	9	4
5	Protection	11	9	3
6	Protection	7	13	4
7	Protection	10	10	4
8	Protection	8	12	3
9	Protection	10	10	4
10	Protection	9	11	4
11	Degree	8	12	3
12	Protection	10	10	4
13	Protection	10	10	4
14	Protection	5	15	3
15	Protection	11	9	4
16	Protection	9	11	4
17	Protection	6	14	4
18	Proximity	14	6	4
18	Degree	14	6	4
18	Protection	14	6	4
19	Protection	7	13	3



Defence Strategy Performance



Defence Strategy Performance

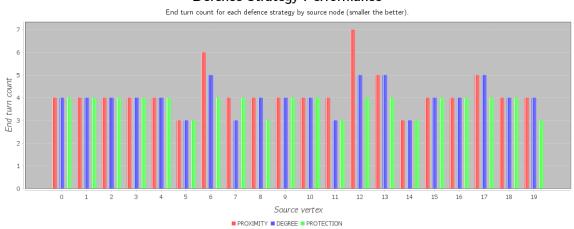


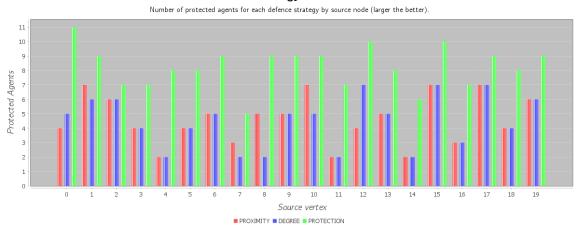
Figure 11: Model results on an Erdős–Rényi graph by source node for each defence strategy with mixed initial protection allocation.

1.3.3 Random Protection

Source node	Winning Strategy	Infections	Protections	End-Turn
0	Protection	9	11	5
1	Protection	11	9	4
2	Protection	13	7	4
3	Protection	13	7	4
4	Protection	12	8	4
5	Protection	12	8	4
6	Protection	11	9	5
7	Protection	15	5	4
8	Protection	11	9	4
9	Protection	11	9	4
10	Protection	11	9	4
11	Protection	13	7	4
12	Protection	10	10	6
13	Protection	12	8	5
14	Protection	14	6	4
15	Protection	10	10	5
16	Protection	13	7	4
17	Protection	11	9	5
18	Protection	12	8	4
19	Protection	11	9	4

Defence Strategy Performance Number of infected agents for each defence strategy by source node (smaller the better).

Defence Strategy Performance



Defence Strategy Performance

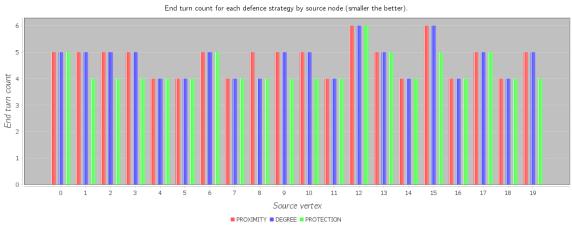


Figure 12: Model results on an Erdős–Rényi graph by source node for each defence strategy with random initial protection allocation.

1.4 Tree Graph

For the Tree graph, the Graph Generator used the following parameters:

• Type of graph: Tree

• Number of vertices: 20

• Number of edges: 19

• Random generator seed: 1615826088771

and the model took the following parameters:

• Total defence quota each turn: 1.0

• Probability with which the infection propagates: 1.0

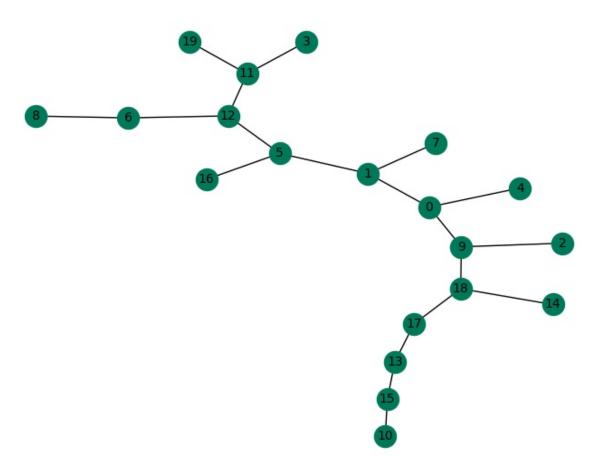


Figure 13: The Tree graph used.

1.4.1 Deterministic Protection

Source node	Winning Strategy	Infections	Protections	End-Turn
0	Proximity	1	19	21
1	Proximity	1	19	21
2	Proximity	1	19	27
2	Degree	1	19	27
3	Proximity	1	19	29
3	Degree	1	19	29
3	Protection	1	19	29
4	Proximity	1	19	27
4	Degree	1	19	27
5	Proximity	1	19	23
6	Proximity	1	19	25
7	Proximity	1	19	27
7	Degree	1	19	27
8	Proximity	1	19	29
8	Protection	1	19	29
9	Proximity	1	19	23
10	Proximity	1	19	29
10	Protection	1	19	29
11	Proximity	1	19	25
11	Protection	1	19	25
12	Proximity	1	19	23
13	Proximity	1	19	27
14	Proximity	1	19	27
14	Degree	1	19	27
15	Proximity	1	19	27
16	Proximity	1	19	27
16	Degree	1	19	27
17	Proximity	1	19	25
18	Proximity	1	19	23
19	Proximity	1	19	29
19	Degree	1	19	29
19	Protection	1	19	29

Defence Strategy Performance Number of infected agents for each defence strategy by source node (smaller the better).

■ PROXIMITY ■ DEGREE ■ PROTECTION

Defence Strategy Performance Number of protected agents for each defence strategy by source node (larger the better).

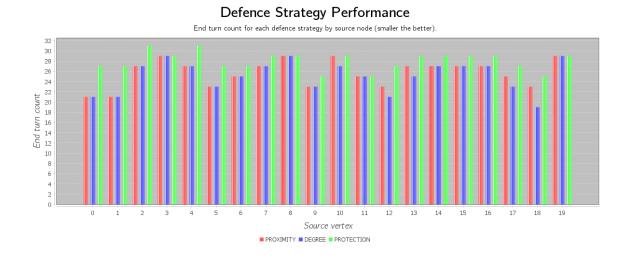
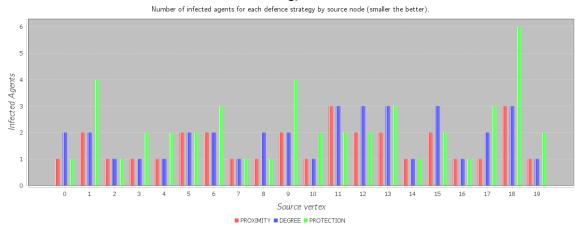


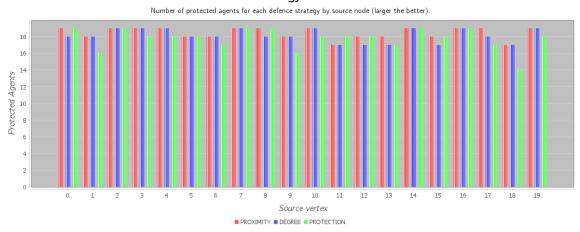
Figure 14: Model results on a Tree graph by source node for each defence strategy with deterministic initial protection allocation.

1.4.2 Mixed Protection

Source node	Winning Strategy	Infections	Protections	End-Turn
0	Proximity	1	19	3
0	Protection	1	19	3
1	Proximity	2	18	7
1	Degree	2	18	7
2	Proximity	1	19	7
2	Degree	1	19	7
2	Protection	1	19	7
3	Proximity	1	19	5
3	Degree	1	19	5
4	Proximity	1	19	5
4	Degree	1	19	5
5	Degree	2	18	5
5	Protection	2	18	5
6	Proximity	2	18	7
6	Degree	2	18	7
7	Proximity	1	19	5
7	Degree	1	19	5
7	Protection	1	19	5
8	Proximity	1	19	7
8	Protection	1	19	7
9	Degree	2	18	5
10	Proximity	1	19	3
10	Degree	1	19	3
11	Protection	2	18	5
12	Protection	2	18	5
13	Proximity	2	18	5
14	Proximity	1	19	7
14	Degree	1	19	7
14	Protection	1	19	7
15	Protection	2	18	3
16	Proximity	1	19	5
16	Degree	1	19	5
16	Protection	1	19	5
17	Proximity	1	19	7
18	Proximity	3	17	11
18	Degree	3	17	11
19	Proximity	1	19	3
19	Degree	1	19	3



Defence Strategy Performance



Defence Strategy Performance

End turn count for each defence strategy by source node (smaller the better).

End turn count for each defence strategy by source node (smaller the better).

End turn count for each defence strategy by source node (smaller the better).

Source vertex

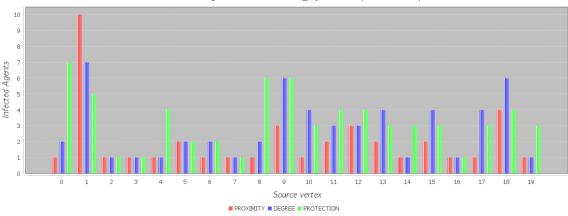
PROXIMITY DEGREE PROTECTION

Figure 15: Model results on a Tree graph by source node for each defence strategy with mixed initial protection allocation.

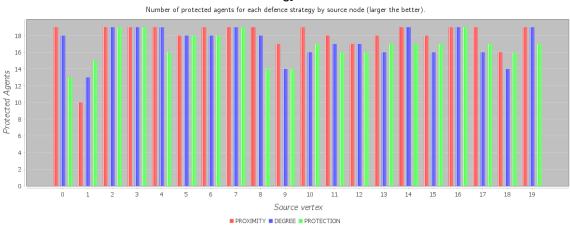
1.4.3 Random Protection

Source node	Winning Strategy	Infections	Protections	End-Turn
0	Proximity	1	19	17
1	Protection	5	15	13
2	Proximity	1	19	17
2	Degree	1	19	17
2	Protection	1	19	17
3	Proximity	1	19	19
3	Degree	1	19	19
3	Protection	1	19	19
4	Proximity	1	19	19
4	Degree	1	19	19
5	Degree	2	18	11
5	Protection	2	18	11
6	Proximity	1	19	15
7	Proximity	1	19	17
7	Degree	1	19	17
7	Protection	1	19	17
8	Proximity	1	19	21
9	Proximity	3	17	19
10	Proximity	1	19	21
11	Proximity	2	18	21
12	Proximity	3	17	17
12	Degree	3	17	17
13	Proximity	2	18	19
14	Proximity	1	19	23
14	Degree	1	19	23
15	Proximity	2	18	21
16	Proximity	1	19	17
16	Degree	1	19	17
16	Protection	1	19	17
17	Proximity	1	19	17
18	Protection	4	16	13
19	Proximity	1	19	17
19	Degree	1	19	17

Defence Strategy Performance Number of infected agents for each defence strategy by source node (smaller the better).



Defence Strategy Performance



Defence Strategy Performance

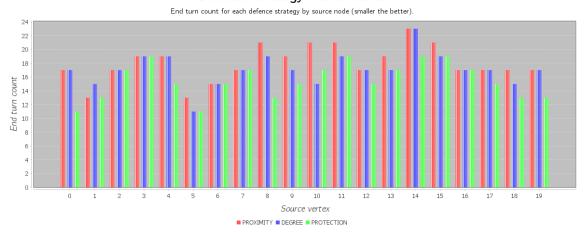


Figure 16: Model results on a Tree graph by source node for each defence strategy with random initial protection allocation.