◦ Fuzzy phenomenon

1. incomplete (資訊不完整)
2. ambiguity (曖昧)
   → 譬如同個概念有不同圖片表示法
3. imprecision (不精確)
4. randomness (隨機)
5. fuzziness (模糊)

1965. Zadeh, "Fuzzy Sets"
information and Control

Δ. Fuzzy Logics:

1. Fuzzy logic is much closer to human decision making and natural language than traditional logic system.

2. Fuzzy logic provides a new parading such that system can be designed for complex, ill-conditioned process without knowing quantitative data regarding then I/O relations which are generally required by conventional method.

3. Applications :

Artificial intelligence, control engineering
decision theory, pattern recognition,
robot, or research, education.....

Rule 1:

function $f(x_1, x_2) = y$ x_2

If project-funding is adequate
or project-staffing is small then
risk is low. x_2

y

Rule 2:

If project-funding is imaginal
and project-staffing is large
then risk is normal.

理論出來,都要驗證規則, try and error

4. Fuzzy sets

Let U be a collection of objects

$U = \{u\}$, U can be discrete or continuous

A Fuzzy set A in U is

① If U is a continuous universe of discourse.

$$A = \int_{u \in U} \mu_A(u)/u$$

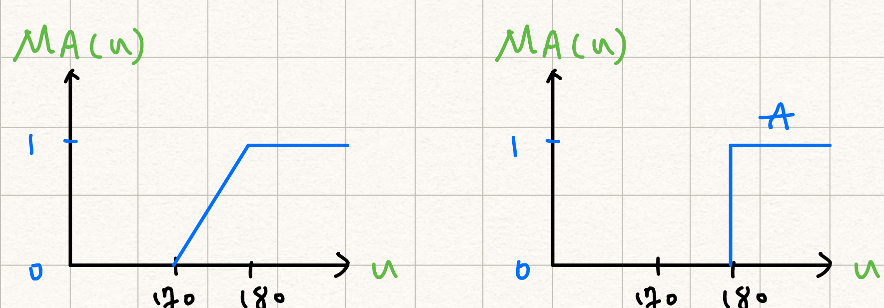
② If .... discrete ....

$$A = \sum_{i=1}^{n} \mu_A(u_i)/u_i$$

where $\mu_A : U \to [0, 1]$ is a membership function or fuzzy grade.

Note : $\mu_A \begin{cases} U \to \{0, 1\} & \text{crisp set} \\ U \to [0, 1] & \text{fuzzy set} \end{cases}$

Ex: Tall (A)

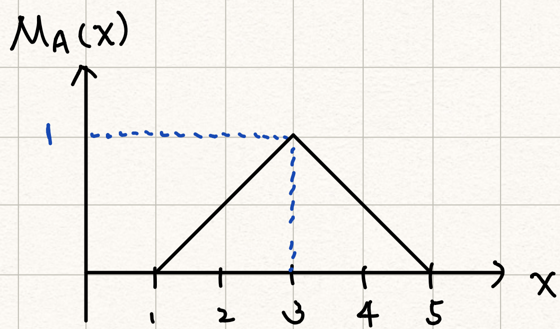


Ex: discrete universe (離散)

A ≡ number close to 3

$U = \{1, 2, 3, 4, 5\}$

U: 論域



$$A = \sum_i \frac{\mu_A(x_i)}{x_i}$$

$$= \frac{0}{1} + \frac{0.5}{2} + \frac{1}{3} + \frac{0.5}{4} + \frac{0}{5}$$

$$= \{(1, 0), (2, 0.5), (3, 1), (4, 0.5), (5, 0)\}$$

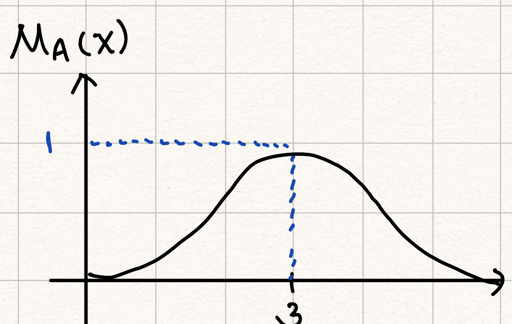
$$= \{(x, \mu_A(x))\}$$

Ex: continuous universe (連續)

A: x is real number and x is close to 3.

$U = \{(x, \mu_A(x)) \mid x \in U\}$

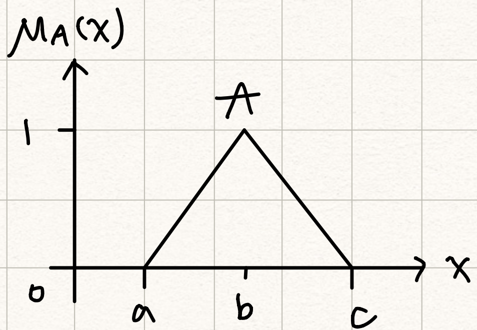
U: 論域



$$\mu_A(x) = \frac{1}{1 + (x-3)^2}$$

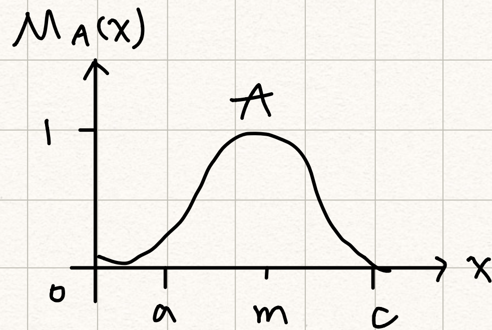
Δ. Membership functions

1. Triangular m.f.



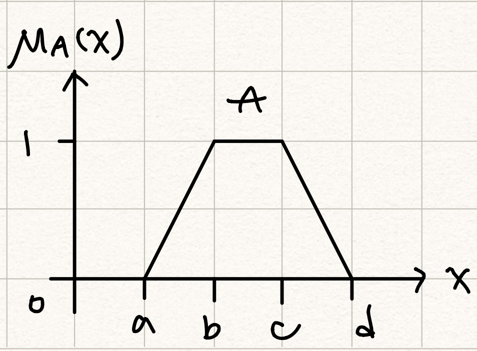
$$\mu_A(x) = \begin{cases} 0 & x \le a \\ \dfrac{x-a}{b-a} & a \le x \le b \\ \dfrac{c-x}{c-b} & b \le x \le c \\ 0 & c \le x \end{cases}$$

2. Gaussian m.f.

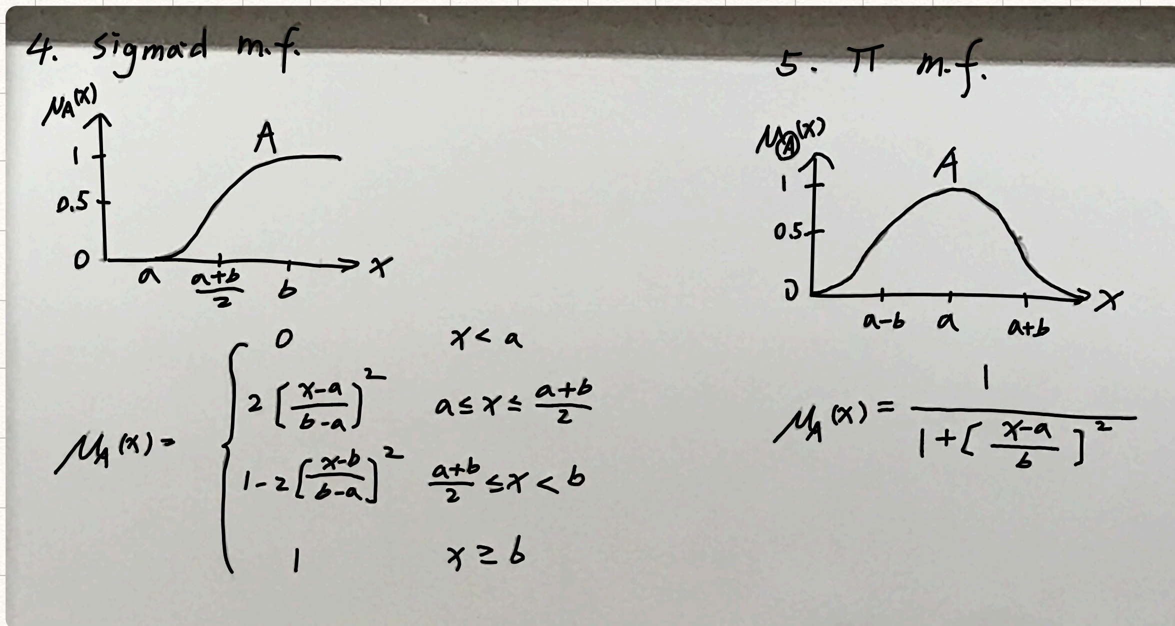


$$\mu_A(x) = \exp\left[\frac{-(x-m)^2}{2\sigma^2}\right]$$

3. Trapejoid m.f.



$$\mu_A(x) = \begin{cases} 0, & x \le a \\ \dfrac{x-a}{b-a} & a \le x \le b \\ 1 & b \le x \le c \\ \dfrac{d-x}{d-c} & c \le x \le d \\ 0 & d \le x \end{cases}$$



△. Fuzzy sets operator

1. complement : $\overline{A}$

$$\mu_{\bar{A}}(x) \triangleq 1 - \mu_A(x), \ \forall x \in U$$

2. Intersection : $A \cap B$

$$\mu_{A \cap B}(x) \triangleq \min\left[\mu_A(x), \mu_B(x)\right]$$
$$= \mu_A(x) \wedge \mu_B(x), \ \forall x \in U$$

論域

3. Union : $A \cup B$

$$\mu_{A \cup B}(x) \triangleq \max\left[\mu_A(x), \mu_B(x)\right]$$
$$= \mu_A(x) \vee \mu_B(x)$$

4. Equility (相似程度)

$$\mu_A(x) = \mu_B(x) \quad \forall x \in U$$

$$E(A,B) \equiv \text{degree}\,(A = B) \triangleq \frac{|A \cap B|}{|A \cup B|}$$

相似評量

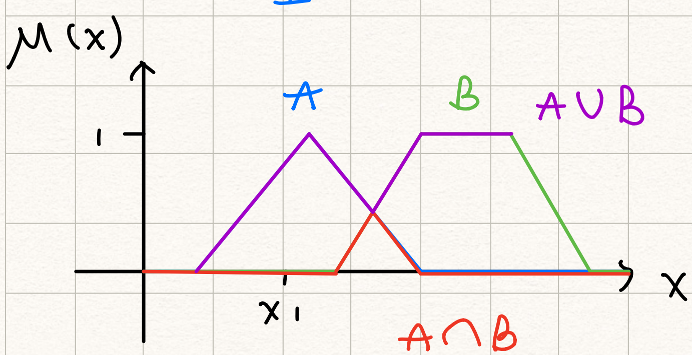
$$|A| = \sum_{x \in U} \mu_A(x)$$

5. Subet (subsethood measure) $A \subseteq B$

if only if $\mu_A(x) \le \mu_B(x) \quad \forall x \subseteq U$

$$S(A,B) = \text{degree}\,(A \subseteq B) \triangleq \frac{|A \cap B|}{|A|}$$

子集評量



$$\left.\begin{array}{l}\mu_A(x_1) = 0.2 \\ \mu_B(x_1) = 0\end{array}\right\rangle \rightarrow \min = 0$$

記得 A．B 兩sets為零的部分也是元素一種

Δ. Properties of fuzzy set operators

1. Idempotence

$A \cup A = A \quad A \cap A = A$

2. Double-negation law

$\bar{\bar{A}} = A$

3. Law of identity

$A \cup \phi = A \;,\; A \cap \underset{= 宇集}{U} = A$

4. Distributivity

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

5. Communtativity

$A \cap B = B \cap A \;,\; A \cup B = B \cup A$

6. Associativity

$$(A \cap B) \cap C = A \cap (B \cap C)$$
$$(A \cup B) \cup C = A \cup (B \cup C)$$

7. Absorption

$$A \cup (A \cap B) = A, \quad A \cap (A \cup B) = A$$

8. De Morgan's laws

$$\overline{A \cup B} = \bar{A} \cap \bar{B}, \quad \overline{A \cap B} = \bar{A} \cup \bar{B}$$

9. Law of zero

$$A \cup U = U, \quad A \cap \phi = \phi$$

Δ. Fuzzy complement

c function

$\bar{A}$: complement of A

$$c : [0, 1] \longrightarrow [0, 1], \quad \mu_{\bar{A}}(x) = c(\mu_A\, x)$$

$\mu_A \to \mu_{\bar{A}}$?

complement function satisfies following
$C(\cdot)$
conditions:

1. Boundary condition $c(0) = 1, \quad c(1) = 0$

2. Monotonic property for x_1 and $x_2 \in U$

if $\mu_A(x_1) < \mu_A(x_2)$ then $c(\mu_A(x_1)) > c(\mu_A(x_2))$

3. continuity $c(\cdot)$ is continuous function

4. Involution $c(c(\mu_A(x))) = \mu_A(x), \ \forall x \in U$

Typical nonparametric fuzzy complement
Negation complement.

$$\mu_{\bar{A}}(x) = c(\mu_A(x)) \triangleq 1 - \mu_A(x), \ \forall x \in U$$

$\lambda = -1, \ \mu_A = U$ $\qquad$ $\lambda = 0, \ \mu_A = 1 - \mu_A(x)$

Typical parametric fuzzy complement

① λ complement (Sugeno's complement)

$$\mu_{A^\lambda}(x) = c(\mu_A(x)) \triangleq \frac{1-\mu_A(x)}{1+\mu_A(x)} \quad -1 < \lambda < \infty$$

② w complement (yager's complement)

$$\mu_{A^w}(x) = c(\mu_A x) \triangleq (1-\mu_A^w(x))^{\frac{1}{w}},$$

$0 < w < \infty$

△. Fuzzy intersection (trangular norms, t-norms)

$t: [0,1] \times [0,1] \longrightarrow [0,1]$

交集

$$\mu_{A\cap B}(x) = t[\mu_A(x), \mu_B(x)]$$

$t(\cdot,\cdot)$ satisfies the following condition

① Boundary condition

$t(\mu_A(x), 0) = 0, \; t(\mu_A(x), 1) = t(1, \mu_A(x))$

$\mu_A(x) = \mu_A(x)$

② Monotonicity

if $\mu_A(x) < \mu_C(x)$ and $\mu_B(x) < \mu_D(x)$

then $t(\mu_A(x), \mu_B(x)) \leq t(\mu_C(x), \mu_D(x))$

③ Commutativety

$t(\mu_A(x), \mu_B(x)) = t(\mu_B(x), \mu_A(x))$

④ Associativity

$t(\mu_A(x), t(\mu_B(x), \mu_C(x)))$ $\quad A \cap (B \cap C)$

$= t(t(\mu_A(x), \mu_B(x)), \mu_C(x))$ $\quad (A \cap B) \cap C$

Typical nonparametric t-norms

$a = \mu_A(x)$, $b = \mu_B(x)$

1. Minimum

$t_{min}(a, b) = a \wedge b = \min(a, b)$

2. Algebra preduct

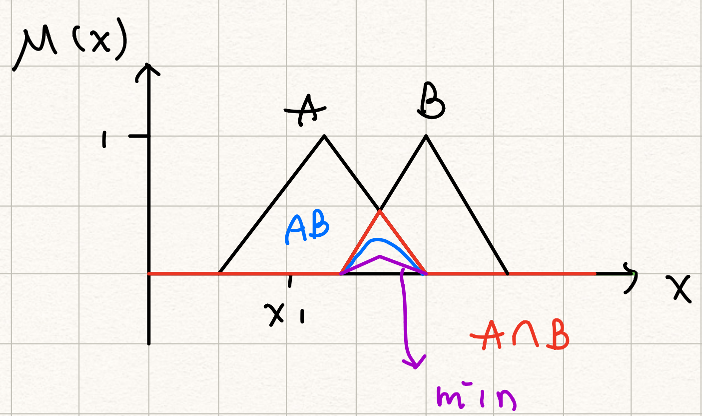
$$t_{ap}(a,b) = a \cdot b = ab$$

3. Bounded product

$$t_{bp}(a,b) = a \odot b = \max(0, a+b-1)$$

4. Drastic product

$$t_{ap}(a,b) = a \hat{\cdot} b = \begin{cases} a, & b=1 \\ b, & a=1 \\ 0, & ab<1 \end{cases}$$



Typical parametric t-norms:

1. Yager:

$$t_w(a,b) = 1 - \min\{1, ((1-a)^w + (1-b)^w)^{\frac{1}{w}}\},$$

$w \in (0, 8)$

$w \to 0 \to$ Algebraic product

$w \to \infty \to$ min

$w \to 1 \to$ Bounded product

2. Sugene

$t_\lambda(a,b) = \max[0, (\lambda+1)(a+b-1) - \lambda ab]$,

$\lambda \in [-1, \infty)$

$w \to 1 \to$ Algebraic product

$w \to 0 \to$ Bounded product

實務,在不同區間,用不同的參數造成不同的結果

Fuzzy Union (t-conorms) (s-norms)

$[0, 1] \times [0, 1] \to [0, 1]$

$\mu_{A \cup B}(x) = S[\mu_A(x), \mu_B(x)]$

$S(\cdot, \cdot)$ satisfies the following conditions

1. Boundary condition:

$$S(1,1)=1, \quad S(\mu_A(x), 0) = S(0, \mu_A(x)) = \mu_A(x)$$

2. Monotonicity

if $\mu_A(x) < \mu_C(x)$ and $\mu_B(x) < \mu_D(x)$

then $S(\mu_A(x), \mu_B(x)) \leq S(\mu_C(x), \mu_D(x))$

3. Commutativity

$$S(\mu_A(x), \mu_B(x)) = S(\mu_B(x), \mu_A(x))$$

4. Associativity

$$S(\mu_A(x), S(\mu_B(x), \mu_C(x))) = S(S(\mu_A(x), \mu_B(x)), \mu_C(x))$$

Typical nonparametric s-norms are:

1. Maximum

$$S_{max}(a,b) = a \vee b = \max(a,b)$$

2. Algebra sum

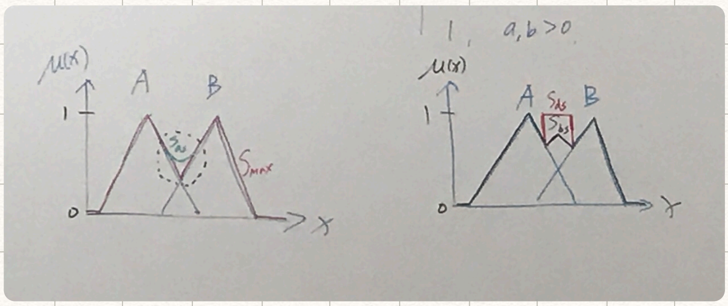
$$S_{as}(a,b) = a \hat{+} b = a+b-ab$$

3. Boundary sum

$$S_{bs}(a,b) = a \oplus b = \min(1, a+b)$$

4. Drastic sum

$$S_{ds}(a,b) = a \dot{\vee} b = \begin{cases} a, & b=0 \\ b, & a=0 \\ 1, & a,b>0 \end{cases}$$



Typical parametric s-norms are:

1. Yager

$$S_w(a,b) = \min\{1, (a^w + b^w)^{\frac{1}{w}}\}, \; w \in (0, \infty)$$

2. Sugeno

$$S_\lambda(a,b) = \min[1, a+b-\lambda ab], \quad \lambda \in (-1, \infty)$$

crisp relation :

Assume X and Y are two crisp sets, where $x \in X$, $y \in Y$.

There are two conditions for x and Y

1. There is a relation R between x and Y
2. .... is no ....

<Ex> Assume two crisp sets

$x = \{1, 2, 3\}$ $Y = \{a, b, c\}$

$R(x, Y) = \{(1,a), (2,b), (3,c)\} \subseteq X \times Y$

mapping description matrix description

X Y Y

$$1 \overset{1}{\text{——}} a$$
$$2 \overset{1}{\text{——}} b$$
$$3 \overset{1}{\text{——}} c$$

$$x \quad \begin{array}{c|ccc} & a & b & c \\ \hline 1 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ 3 & 0 & 0 & 1 \end{array}$$

Fuzzy relation :

$$R(x,y) = \{((x,y), \mu_R(x,y)) \mid (x,y) \in X \times Y\}$$

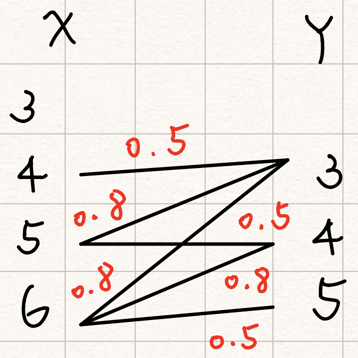
<Ex> Let universe of discourse of X and Y is real axis. R is define $x >> y$
$(x,y) \in X \times Y$

$$\mu_R(x,y) = \begin{cases} 0 & x \le y \\ \dfrac{1}{1+(y-x)^{-2}} & x > y \end{cases}$$

Assume

$X = \{3, 4, 5, 6\}$
$Y = \{3, 4, 5\}$

越近1,表越符合這句話的描述
0 → 完全不符合



$$\begin{array}{c} \\ \\ \end{array} \begin{array}{cccc} & & Y & \\ & 3 & 4 & 5 \end{array}$$
$$\begin{array}{c} 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{bmatrix} 0 & 0 & 0 \\ 0.5 & 0 & 0 \\ 0.8 & 0.5 & 0 \\ 0.9 & 0.8 & 0.5 \end{bmatrix}$$

$R(X, Y) = \{((4,3), 0.5), ((5,3), 0.8), ((5,4), 0.5), ((6,3), 0.9), ((6,4), 0.8), ((6,5), 0.5)\}$

Operation of fuzzy relation

Fuzzy relation can be considered as fuzzy set. Then fuzzy operations can be used in fuzzy relation.

Let $R(x, y) = \{((x,y), \mu_R(x,y)) \mid (x,y) \in X \times Y\}$

$S(x, y) = \{ \text{.....} \ \mu_S \ \text{.......} \ \text{------} \ \text{...} \}$

1. union : $\mu_{R \cup S}(x,y) = \max(\mu_R(x,y), \mu_S(x,y))$
2. intersection : $\mu_{R \cap S}(x, y) = \min(\mu_R(x,y), \mu_S(x,y))$
3. complement : $\mu_{\bar{R}}(x, y) = 1 - \mu_R(x, y)$
4. subset : $R \subseteq S \Leftrightarrow \mu_R(x, y) \leq \mu_S(x, y), \ \forall (x, y)$

Composition Operator:

1. relation to relation
2. set to relation

P is defined as the relation between X and Y, $x \in X$, $y \in Y$

Q .... Y and Z, $y \in Y$, $z \in Z$

P and Q can be composed as:

$$R(X, Z) = P(X, Y) \circ Q(Y, Z)$$

$$= \{(x, z) \mid \exists y, \exists (x, y) \in P \text{ and } (y, z) \in Q\}$$

<Ex> composition of crisp relations

$X \xrightarrow{P} Y$, $Y \xrightarrow{Q} Z$ find $X \xrightarrow{R} Z$

$$P = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cccc} y_1 & y_2 & y_3 & y_4 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \end{array} \qquad Q = \begin{array}{c} \\ y_1 \\ y_2 \\ y_3 \\ y_4 \end{array} \begin{array}{c} \begin{array}{cc} z_1 & z_2 \end{array} \\ \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \end{array}$$

$$R = P \circ Q = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cc} z_1 & z_2 \end{array} \\ \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} \end{array}, \quad \max\{1 \times 0,\ 0 \times 1,\ 1 \times 0,\ 0 \times 0\} = 0$$

Composition of fuzzy relations, where

$$P(X,Y) = \{((x,y), \mu_P(x,y)) \mid (x,y) \in X \times Y\}$$

$$Q(Y,Z) = \{((y,z), \mu_Q(y,z)) \mid (y,z) \in Y \times Z\}$$

R is the composition of P and Q

$$R = P \circ Q$$

$$= \{(x,y), \max t[\mu_P(x,y), \mu_Q(y,z)] \mid x \in X, \; y \in Y, \; z \in Z\}$$

for max-min operator:

(沒講都是用這個
最常見之 operator)

$$R = P \circ Q$$

$$= \{[(x,y), \max \min(\mu_{RP}(x,y), \mu_{RB}(y,z)] \mid x \in X, \; y \in Y, \; z \in Z\}$$

$$\mu_R(x,y) = \bigvee_{y=Y} (\mu_P(x,y) \wedge \mu_Q(y,z))$$

$$= \max_{y=Y} [\min(\mu_P(x,y), \mu_Q(y,z)]$$

<EX> composition of fuzzy relations

$X \xrightarrow{P} Y$, $Y \xrightarrow{Q} Z$, find $X \xrightarrow{R} Z$

$$P = \begin{array}{c|cc} & y_1 & y_2 \\ \hline x_1 & 0.5 & 0.3 \\ x_2 & 0.4 & 0.8 \end{array} \qquad Q = \begin{array}{c|ccc} & y_1 & y_2 & y_3 \\ \hline y_1 & 0.8 & 0.5 & 0.1 \\ y_2 & 0.3 & 0.7 & 0.5 \end{array}$$

$$R = P \circ Q$$

$$= \begin{bmatrix} \overset{\text{max}}{\vee}\{(0.5 \overset{\text{min}}{\wedge} 0.8), (0.3 \wedge 0.3)\} & \vee\{(0.5 \wedge 0.5), (0.3 \wedge 0.7)\} \\ \vee\{(0.4 \wedge 0.8), (0.8 \wedge 0.3)\} & \cdots \quad \cdot \quad \cdots \end{bmatrix}$$

$$= \begin{bmatrix} \vee(0.5, 0.3) & \vee(0.5, 0.3) & \vee(0.1, 0.3) \\ \vee(0.4, 0.3) & \vee(0.4, 0.7) & \vee(0.1, 0.5) \end{bmatrix}$$

$$= \begin{array}{c|ccc} & z_1 & z_2 & z_3 \\ \hline x_1 & 0.5 & 0.5 & 0.3 \\ x_2 & 0.4 & 0.7 & 0.5 \end{array}$$

composition of fuzzy relation and fuzzy set

fuzzy set A and fuzzy relation R compose to fuzzy set B $\mu_B(y) = \max\{\min(\mu_A(x), \mu_R(x,y))\}$

$\forall y \in Y,\ B = A \circ R$

Fuzzy rule

If x is A then y is B:

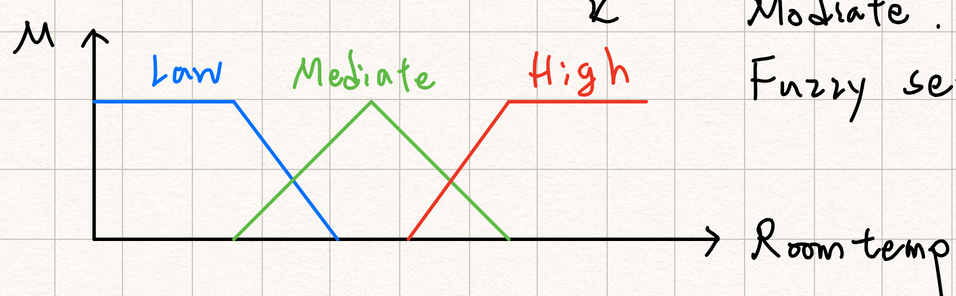
If RoomTemp is Low Then A.C_Speed is High.

linguistic variable

linguistic value

a.語意式的

有Low的 Fuzzy set 跟 Modiate, High 的 Fuzzy set.



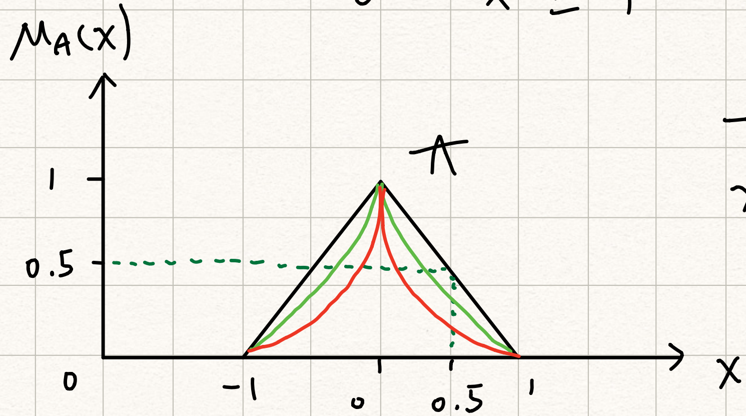
Hedge: (修飾詞, 運算詞)

1. highly $A = (\mu_A(x))^3$
2. very $A = (\mu_A(x))^2$
3. more or less $= (\mu_A(x))^{\frac{1}{2}}$

4. roughly $= (\mu_A(x))^{\frac{1}{4}}$

<Ex>

$$\mu_A(x) = \begin{cases} 0 & x = -1 \\ 1+x & -1 \le x \le 0 \\ 1-x & 0 \le x \le 1 \\ 0 & x \ge 1 \end{cases}$$



A: fuzzy set of x close to zero.

very A Fuzzy set

Highly A Fuzzy set

$\mu(0.5) = 0.25$

very A

$\mu_A(0.5) = 0.5$

有EX例題的部分要詳看

Fuzzy Implication: (一種relation, 可計算)

If x is A Then y is B else nothing

If A then B

$A \to B$

Globally

locally

Globally:

1. Diness - Rescher Implication

$$\mu_{R_{DR}}(x, y) = \max\{1-\mu_A(x), \mu_B(y)\}$$

2. Lukasiewicz Implication

$$\mu_{R_L}(x, y) = \min\{1, 1-\mu_A(x)+\mu_B(y)\}$$

3. Zadeh Implication ← 用最多,取大取小

$$\mu_{R_Z}(x, y) = \max[\min(\mu_A(x), \mu_B(y)), 1-\mu_A(x)]$$

4. Global Implication

$$\mu_{R_G}(x, y) = \begin{cases} 1 & \text{if } \mu_A(x) \le \mu_B(y) \\ \mu_B(y) & \text{otherwise} \end{cases}$$

Locally:

1. Mamdan Implication

$$\mu_{R_m}(x, y) = \min\{\mu_A(x), \mu_B(y)\}$$

$$\mu_{A\to B}(x,y) = \underline{\mu_A(x) \wedge \mu_B(y)}$$

A.B 交集

2. Product Implication

$$\mu_{R_p}(x,y) = \mu_A(x) \cdot \mu_B(y)$$

Fuzzy Reasoning (Approximate Reasoning)

① Binary Logic : (modus poneas)

能力 大 體積 小

premise 1: x is A

premise 2: If x is A Then y is B.

---

consequence: y is B

②

premise 1: x is A′ 非常大

premise 2: If x is A Then y is B.

---

consequence: y is B′ 非常小

A′跟A很像，但不是A

$$Y \text{ is } [A' \circ (A\to B)],\ (=B')$$

$\uparrow$

compose

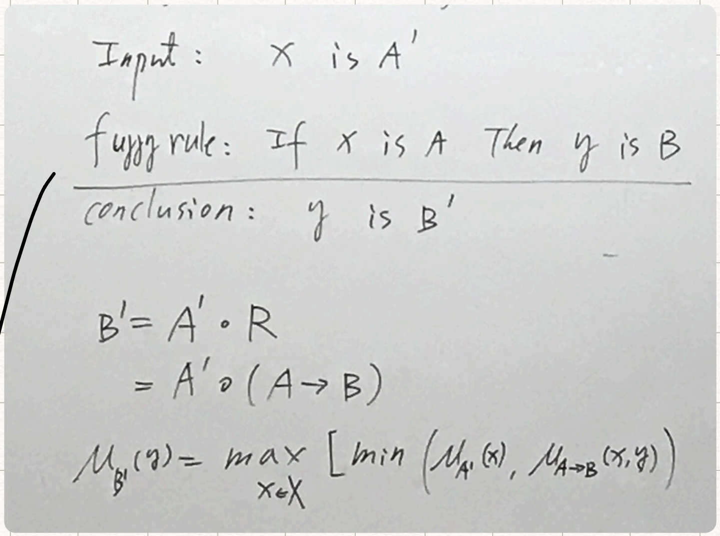
① for max-min composition

$$\mu_B(y) = \max_x [\min(\mu_{A'}(x), \mu_{A\to B}(x,y)]$$

$$= \bigvee_x [\wedge(\mu_{A'}(x), \mu_{A\to B}(x,y)]$$

1. single variable, single rule (1 input, 1 output, 1 rule)

題目


Input: x is A'

fuzzy rule: If x is A Then y is B

conclusion: y is B'

$$B' = A' \circ R$$
$$= A' \circ (A \to B)$$
$$\mu_{B'}(y) = \max_{x\in X} \left[\min\left(\mu_{A'}(x), \mu_{A\to B}(x,y)\right)\right)$$

if we use ② Mamdani implication

$\mu_{A\to B}(x,y) = \mu_A(x) \wedge \mu_B(y)$. Then,

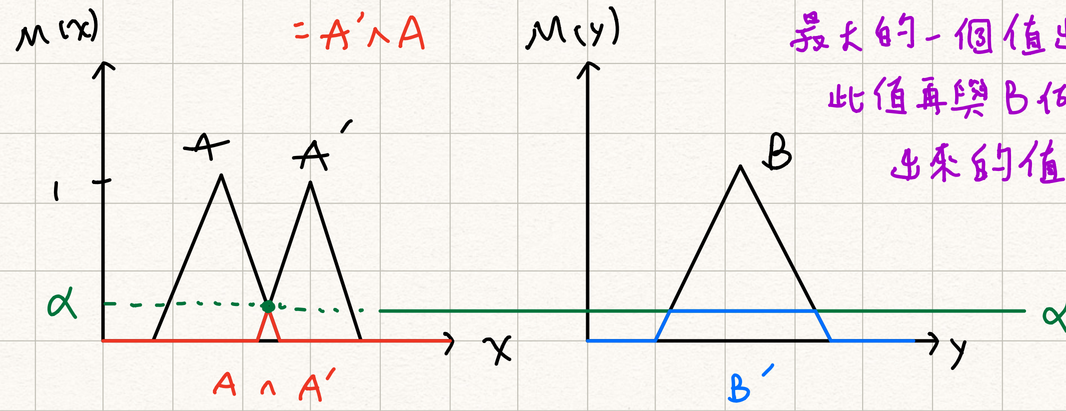
$$\mu_{B'}(y) = \bigvee_{x=X} [\wedge(\mu_{A'}(x), \mu_A(x), \mu_B(y)]$$

$$= \left[\bigvee_{x=X}\left(\mu_{A'}(x) \wedge \mu_A(x)\right)\right] \wedge \mu_B(y)$$

$= \underline{\alpha} \wedge \mu_B(y)$

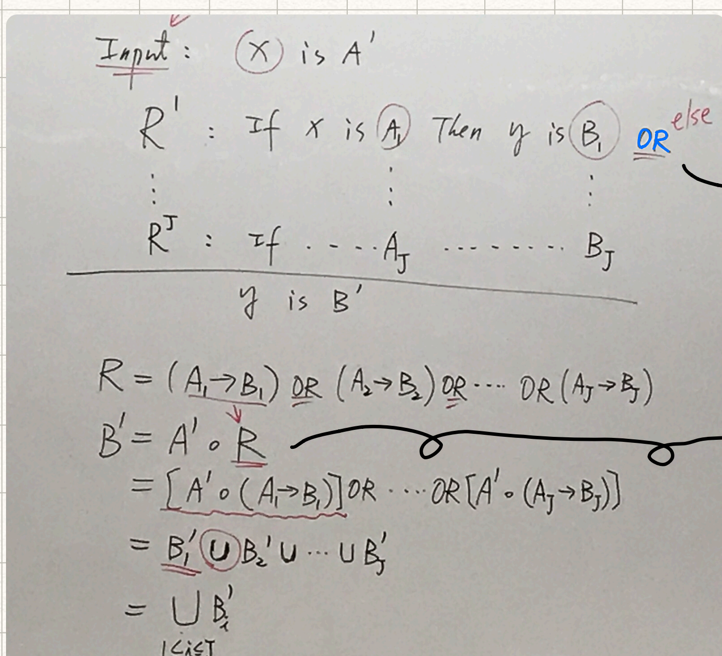
α cut
$= A' \wedge A$

先取 $A' \wedge A$，再從 $A' \wedge A$ 取最大的一個值出來，最後此值再與 B 做 min 出來的值便為 B'



求 $B' = ?$ 以上述公式解

2. single variable, multiple rules (1 Input
1 Output
多 rules)



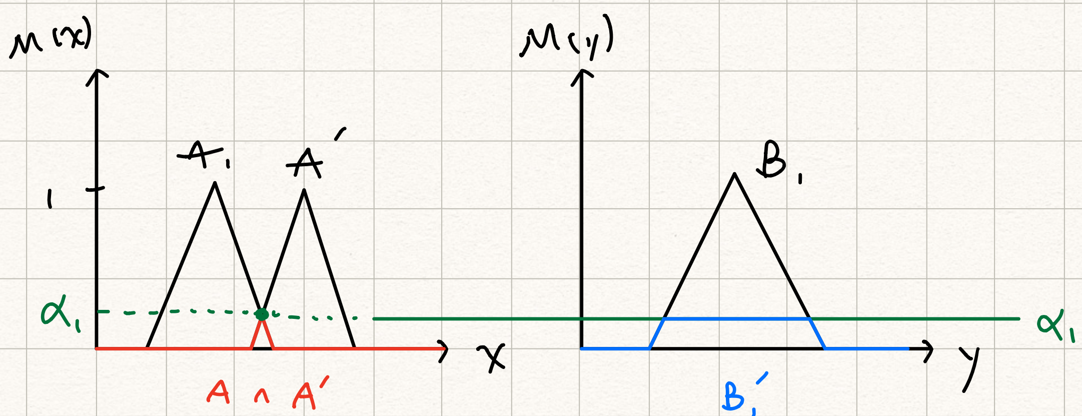
or 表聯集

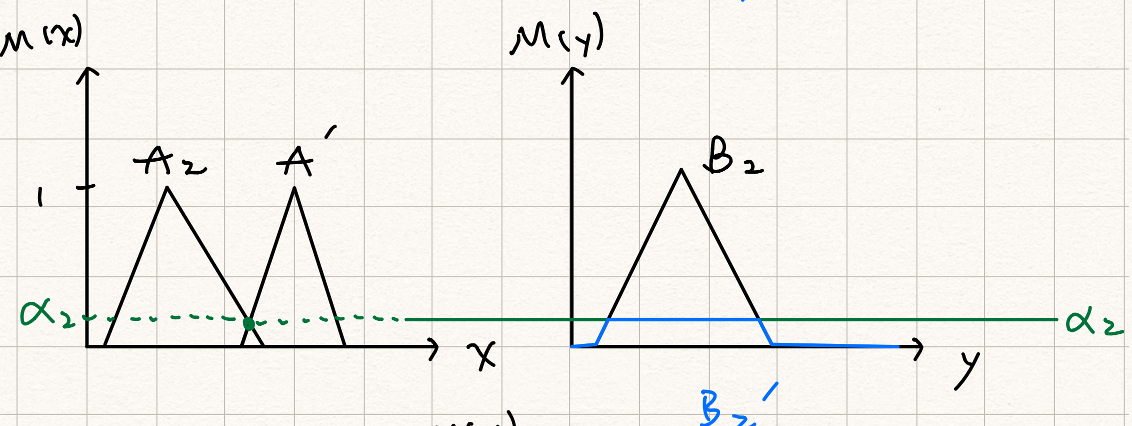
若 single rule,
則 $R = A \rightarrow B$

$$\mu_{B'_i}(y) = \bigvee_{x \in X} \left\{ \left[\mu_{A'}(x) \wedge \mu_{A_i}(x) \right] \wedge \mu_{B_i}(y) \right\}$$

$$= \alpha_i \wedge \mu_{B_i}(y)$$

$$\mu_{B'}(y) = \bigvee_{1 \le i \le J} \underbrace{\left[\alpha_i \wedge \mu_{B_i}(y) \right]}_{R_1 \cdot R_2 \cdot R_3 \cdots\cdots}$$





$B_1' \cup B_2' \Rightarrow$

3. multiple variables, single rule (多 Input, 1 Output, 1 rule)

Input: x is A' $\boxed{\text{and } y \text{ is } B'}$

R: If x is A and y is B Then z is C

---

z is C'

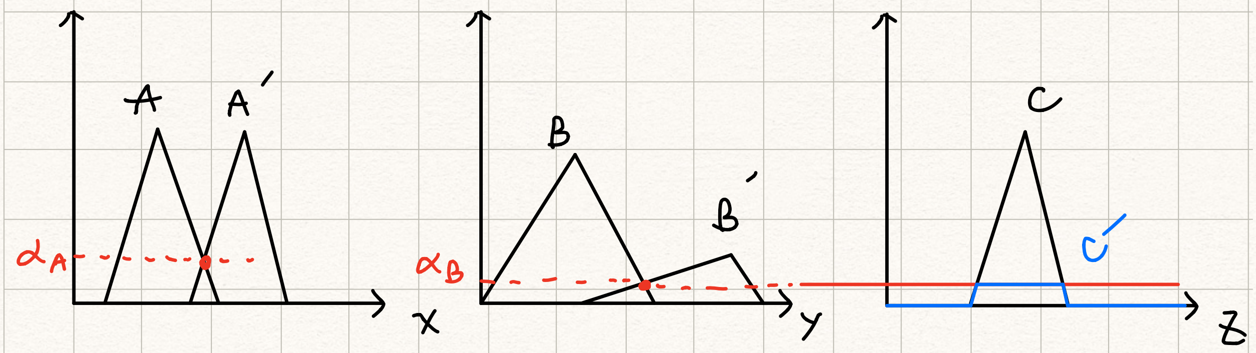
$$\mu_R(x,y,z) = \mu_{(A \text{ and } B)\to C}(x,y,z)$$
$$= \mu_{A \text{ and } B}(x,y) \wedge \mu_C(z)$$
$$= \mu_A(x) \boxed{\wedge\, \mu_B(y)} \wedge \mu_C(z)$$

唯有此處與前幾式相異

$$C' = (A' \text{ and } B') \circ R$$
$$= (A' \text{ and } B') \circ \{(A \text{ and } B) \to C\}$$

$$\mu_{C'}(z) = \bigvee_{x\in X} \bigvee_{y\in Y} \left[\mu_{A' \text{ and } B'}(x,y) \wedge \underbrace{\mu_{(A \text{ and } B)\to C}(x,y,z)}_{R}\right]$$
$$= \bigvee_{x\in X} \bigvee_{y\in Y} \left[(\mu_{A'}(x) \wedge \mu_{B'}(y)) \wedge (\mu_A(x) \wedge \mu_B(y) \wedge \mu_C(z))\right]$$
$$= \left[\bigvee_{x\in X} \bigvee_{y\in Y} (\mu_{A'}(x) \wedge \mu_A(x) \wedge \mu_{B'}(y) \wedge \mu_B(y))\right] \wedge \mu_C(z)$$
$$= \bigvee_{x\in X} \left\{\mu_{A'}(x) \wedge \mu_A(x) \wedge \left[\bigvee_{y\in Y} (\mu_{B'}(y) \wedge \mu_B(y))\right]\right\} \wedge \mu_C(z)$$
$$= \left[\bigvee_{x\in X} (\mu_{A'}(x) \wedge \mu_A(x))\right] \wedge \left[\bigvee_{y\in Y} (\mu_{B'}(y) \wedge \mu_B(y))\right] \wedge \mu_C(z)$$
$$= (\alpha_A \wedge \alpha_B) \wedge \mu_C(z)$$
$$= \alpha \wedge \mu_C(z)$$

若多其他變數C，則照 $(\alpha_A \wedge \alpha_n \wedge \alpha_n)$ …… 做



$\alpha_A \wedge \alpha_B = \alpha_B$ (取小)

4. multiple variables, multiple rules (多 Input
1 Output
多 rule)

Input: x is A' and y is B'

R^1: If x is A_1 and y is B_1 Then z is C_1 OR
R^2: - - - - A_2 . . . . B_2 - - - - C_2 OR
⋮
R^J: - - - - - A_J - - - - B_J . . . . . . C_J

z is C'

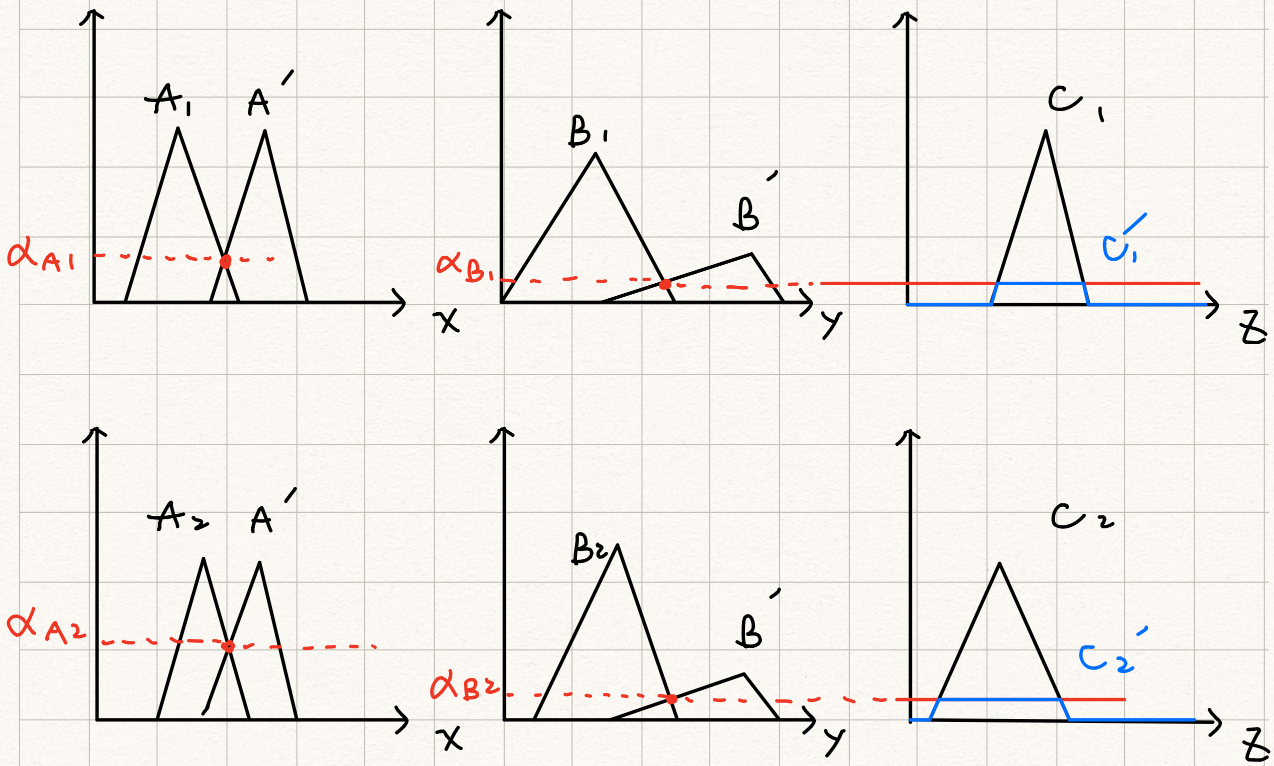
$$R = \bigcup_{i=1}^{J} R_i = \bigcup_{i=1}^{J} [(A_i \text{ and } B_i) \to C_i]$$

$$
\begin{aligned}
C' &= (A' \text{ and } B') \circ R \\
&= (A' \text{ and } B') \circ \left[\bigcup_i (A_i \text{ and } B_i) \to C_i\right] \\
&= \bigcup_i \left[(A' \text{ and } B') \circ ((A_i \text{ and } B_i) \to C_i)\right] \leftarrow \\
&= \bigcup_i \left\{[A' \circ (A_i \to C_i)] \wedge [B' \circ (B_i \to C_i)]\right\} \leftarrow
\end{aligned}
$$

$$
\begin{aligned}
\mu_{C'}(z) &= \bigvee_i \left[\bigvee_{x \in X} \bigvee_{y \in Y} \left(\mu_{A' \text{ and } B'}(x,y) \wedge \mu_{(A_i \text{ and } B_i) \to C_i}(x,y,z)\right)\right] \\
&= \bigvee_i \left\{\left[\bigvee_{x \in X} (\mu_{A'}(x) \wedge \mu_{A_i}(x))\right] \wedge \left[\bigvee_{y \in Y} (\mu_{B'}(y) \wedge \mu_{B_i}(y))\right] \wedge \mu_{C_i}(z)\right\} \\
&= \bigvee_i \left[(\underline{\alpha_{A_i} \wedge \alpha_{B_i}}) \wedge \mu_{C_i}(z)\right] \\
&= \bigvee_i \left[\alpha_i \wedge \mu_{C_i}(z)\right]
\end{aligned}
$$

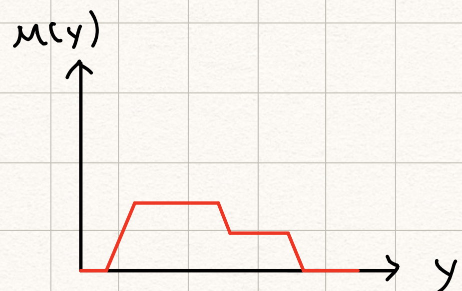
中括號裡的東西便是③ single rule 所描述之事

A_1', B_1' 是另外給的輸入，造成 C_1' 的輸出



Input 1   Input 2

$C_1' \cup C_2' \xrightarrow[MAX]{} $



Input : x is x_0 and y is y_0

A' 輸入是一個值的話 B'

Output : If x is A and y is B

Then z is C。

---

z is C

fuzzy singletion

多變數的輸出如何做?

把輸出之多變數拆成一個一個的,並分別把各個變數與 α cut 做取小運算後所得之圖形做取大運算即可

△. Defuzzifier (解模糊)

1. for consequence is fuzzy set
2. for consequence is crisp value

C: fuzzy set after fuzzy reasoning
y: output variable
y^*: crisp value after deffuzification

(1-1) center of gravity or center of Area (COG)

continuous:

$$y^* = \frac{\int \mu_C(y) \cdot y \, dy}{\int \mu_C(y) \, dy}$$

較常用

discrete:

最大值對應
下來之X軸值為 y_i

$$y^* = \frac{\sum_{i=1}^{L} \mu_c(y_i)\cdot y_i}{\sum_{i=1}^{L} \mu_c(y_i)}$$

L: # of quantization levels of output variable.

y_i: i^{th} ....

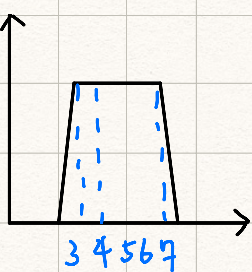
(1-2) mean of maxima (MOM)

$$y^* = \frac{1}{N}\sum_{i=1}^{N} y_i \quad 3+4+5+6+7 \quad \mu_c(y_i) = \max_{y\in Y} \mu_c(y) = \text{height}(c).$$

$i = 1, 2, \cdots, N.$

(1-3) modified MOM

$$y^* = \frac{\max_i(y_i) + \min_i(y_i)}{2}$$

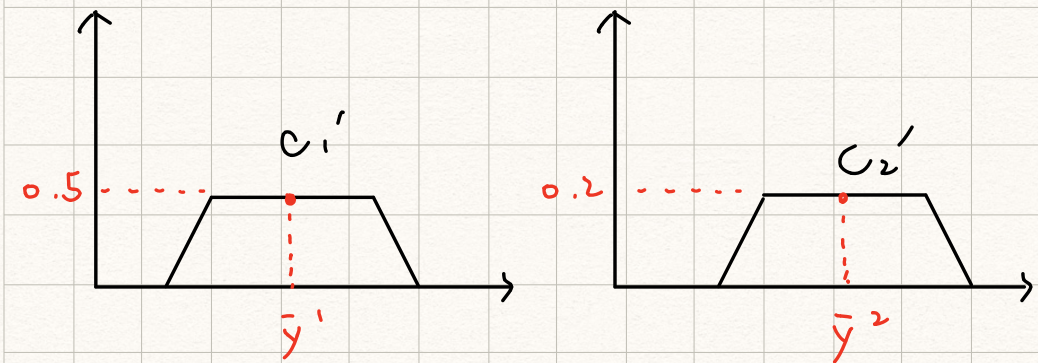


(1-4) center average

$$y^* = \frac{\sum_{j=1}^{J} \bar{y}^j \mu_{c^j}(\bar{y}^j)}{\sum_{j=1}^{J} \mu_{c^j}(\bar{y}^j)}$$

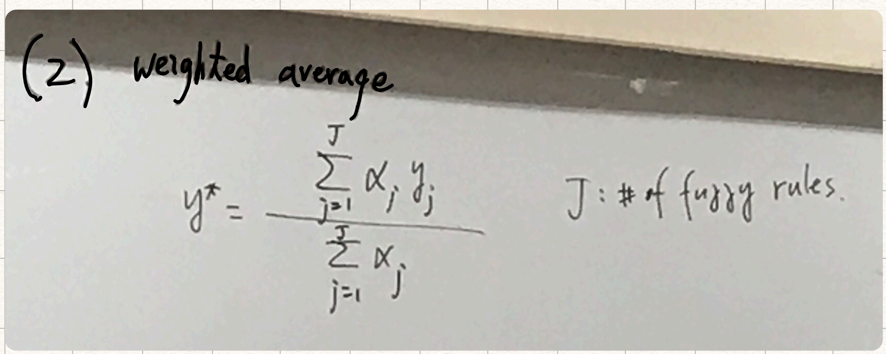
$\bar{y}^j$: center of y of the j^{th} fuzzy rule,
where y is the value that makes $\mu_{c^j}(y)$ maxima.

J: # of fuzzy rules



先獨立處理，不要並在一起計算

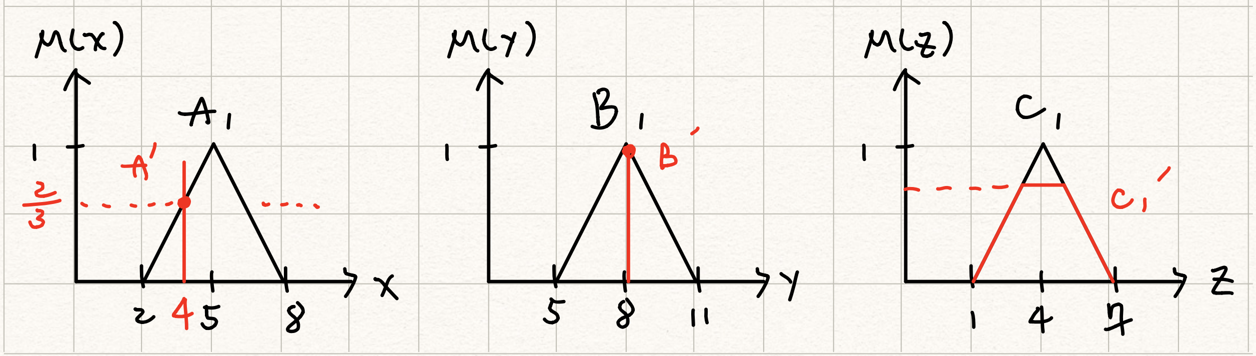
→ 必定只有一高峰 ⌒，故沒有兩個 y 的問題

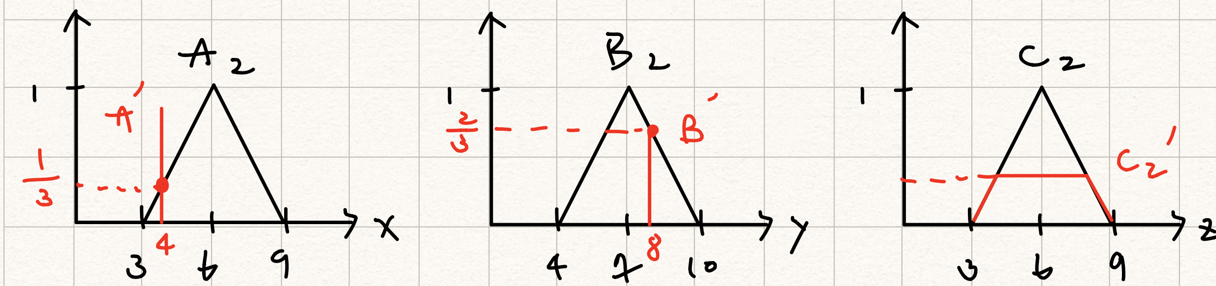


<EX> I: $x=4\,(A')$, $y=8\,(B')$, 求 z^* ?

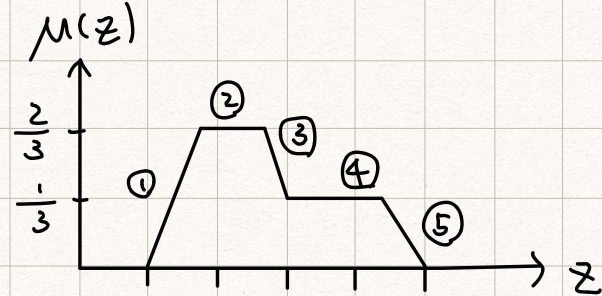
R^1: If x is A_1 and y is B_1 Then y is C_1

R^2: — — - - A_2 - - - - - - - B_2 - - - - - - C_1





$C_1' \cup C_2' \underset{\text{MAX}}{\Rightarrow}$



$$
\mu_C(z)\begin{cases} \frac{z-1}{3} & 1 \le z \le 3 \;\cdots\cdots\; ① \\ \frac{2}{3} & 3 \le z \le 5 \;\cdots\cdots\; ② \\ \frac{7-z}{3} & 5 \le z \le 6 \;\cdots\cdots\; ③ \\ \frac{1}{3} & 6 \le z \le 8 \;\cdots\cdots\; ④ \\ \frac{9-z}{3} & 8 \le z \le 9 \;\cdots\cdots\; ⑤ \end{cases} \qquad \text{otherwise, } 0
$$

(1-1)

$$
z^* = \frac{\int_1^3 \left(\frac{z-1}{3}\right) z\,dz + \int_3^5 \left(\frac{2}{3}\right) z\,dz + \cdots\cdots + \int_8^9 \left(\frac{9-z}{3}\right) dz}{\int_1^3 \left(\frac{z-1}{3}\right) dz + \int_3^5 ②\,dz + \cdots + \int_8^9 ⑤\,dz}
$$

$= 4.7$ ※

(1-1)

$\bar{y} = 7$　　$\mu(7) = \frac{1}{3}$

$$z^* = \frac{1\times 0 + 2\times\frac{1}{3} + 3\times\frac{2}{3} + 4\times\frac{2}{3} + 5\times\frac{2}{3} + 6\times\frac{1}{2} + 7\times\frac{1}{3} + 8\times\frac{1}{3} + 9\times 0}{0 + \frac{1}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{1}{2} + \frac{1}{3} + \frac{1}{3} + 0}$$

$L = 9$

$= 4.7$ ※

(1-2)

$$z^* = \frac{3+4+5}{3} = 4$$ ※

$\frac{2}{3}$封下來，有 3 4 5 三個值

(1-3)

$$z^* = \frac{3+5}{2} = 4$$ ※

(1-4)

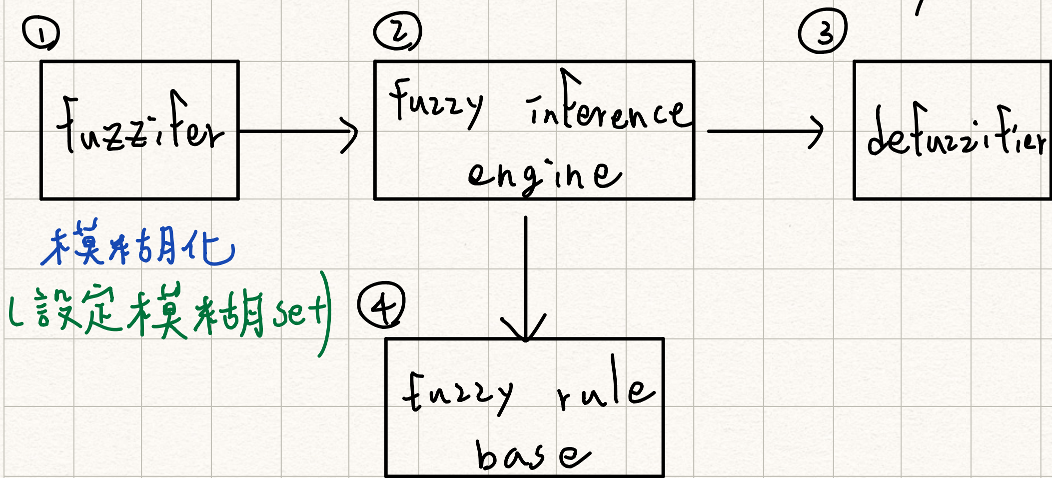
$$z^* = \frac{(4\times\frac{2}{3}) + (6\times\frac{1}{3})}{\frac{2}{3} + \frac{1}{3}} = 4.7$$ ※

第4法，$4\times\frac{2}{3}$ (Rule 1)，$6\times\frac{1}{3}$ (Rule 2)

Δ. Fuzzy systems

Advantage:

1. accurate mathematic modeling isn't required
2. human Knowledge can be implimented in the system design.



① transfer inputs to linguistic fuzzy information
② solve problem through fuzzy reasoning
③ transfer fuzzy consequence to real inputs
④ store fuzzy rules

① Fuzzifier

1. Fuzzy singleton $\mu_A(x) \begin{cases} 1, & x = x_0 \\ 0, & x \neq x_0 \end{cases}$

2. Gaussian membership function

$$\mu_A(x) = \exp\left[-\frac{(x-x_0)^2}{\sigma^2}\right]$$

3. triangular m.f.

4. trap. m.f.

② fuzzy interference engine

premise 1: x is A'

premise 2: if x is A, Then y is B

---

consequence : y is B'

$$B' = A' \circ R = A' \circ (A \to B)$$

1. operations of composition

$$\mu_{B'}(y) = \max_x \left[\min(\mu_{A'}(x), \mu_{A\to B}(x,y))\right]$$

2. max product operation

$$\mu_{B'}(y) = \max_x \{ \mu_{A'}(x), \mu_{A\to B}(x, y)$$

3. max bounded product operation

$$\mu_{B'}(y) = \max_x \{ \mu_{A'}(x) \odot \mu_{A\to B}(x, y) \}$$

4. max drastic product operation

$$\mu_B(y) = \max_x \{ \mu_{A'}(x) \hat{\cdot} \mu_{A\to B}(x, y) \}$$

③. fuzzy rule base

1. If - then type

2. MEMO $\rightarrow$ MISO

3. MISO rule types: linguistic fuzzy rule
   Functional fuzzy rule
   Tsukamoto f.r.

linguistic fuzzy rule :

R^{δ}: If x_1 is A_1^{δ} and ... and x_p is

A_p^j, Then y is B^j.

A_i^j, B^j are linguistic value

$\mu_{A_i^j}$, μ_{B^j} are membership function

x_i is input variable of rule j

y is output - - - - - - - - - - ...

(x and y are linguistic variables)

EX:

<EX>

premise: x is $\overset{A'}{x_0}$ and y is $\overset{B'}{y_0}$ oR

R^1: If x is A_1 $\underset{\wedge}{\text{and}}$ y is B_1 Then z is C_1

R^2: - - - - - A_2 - - - - - B_2 - - - - - C_2

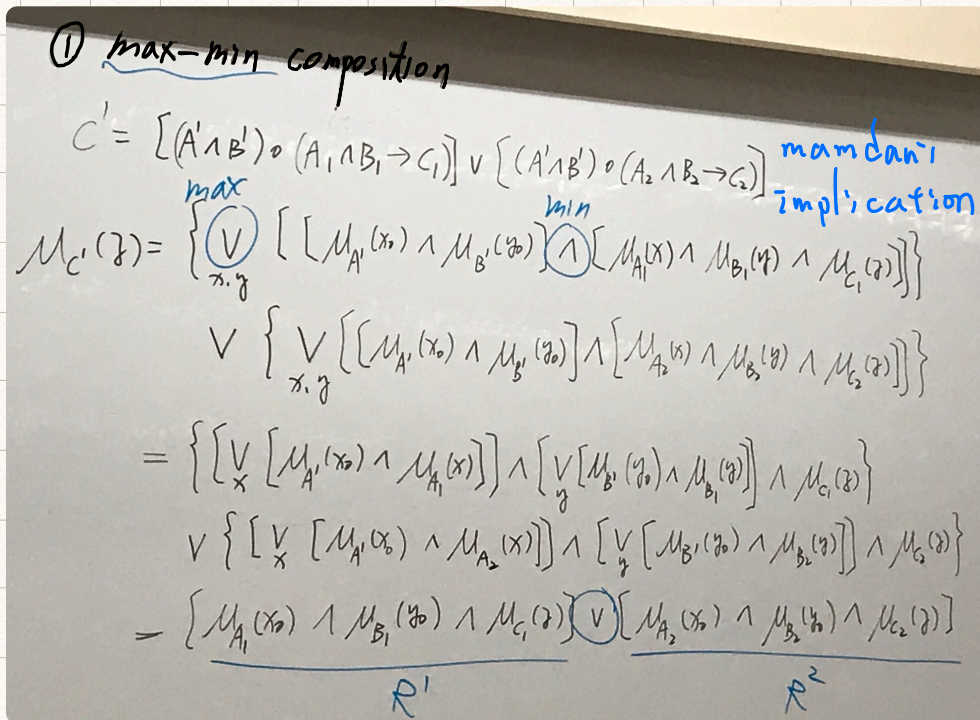
$$\Rightarrow C' = (A' \wedge B') \circ [\overset{R^1}{\underline{(A_1 \wedge B_1 \to C_1)}} \vee \overset{R^2}{\underline{(A_2 \wedge B_2 \to C_2)}}]$$

$$= [(A' \wedge B') \circ (A_1 \wedge B_1 \to C_1)] \vee [(A' \wedge B') \circ (A_2 \wedge B_2 \to C_2)]$$

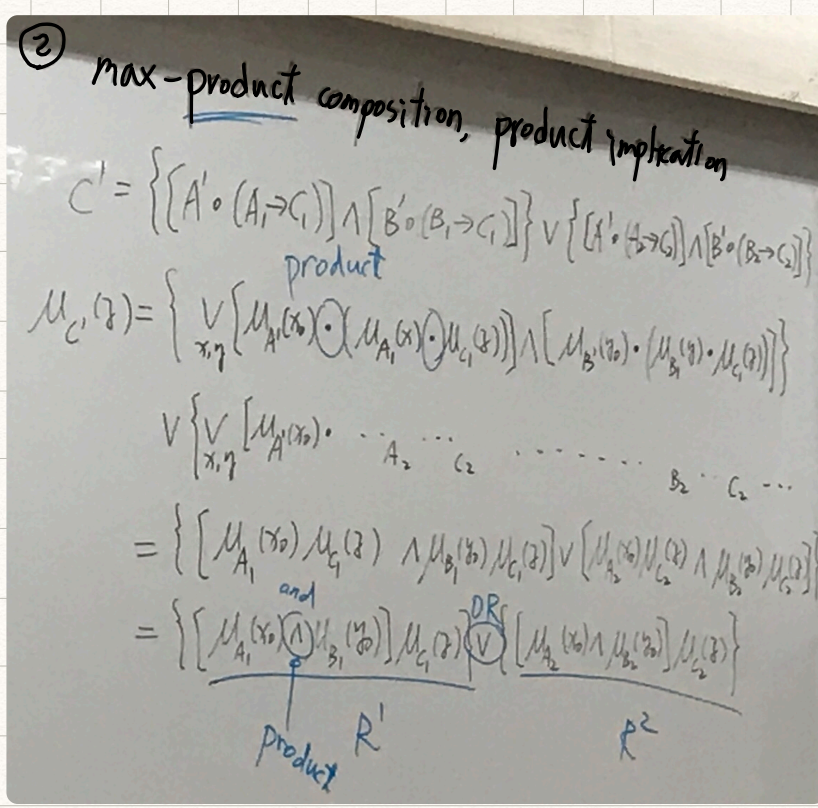
$$= \{[A' \circ (A_1 \to C_1)] \wedge [B' \circ (B_1 \to C_1)]\} \vee \{[A' \circ (A_2 \to C_2)] \wedge [B' \circ (B_2 \to C_2)]\}$$

c′ 的解法：

① max-min composition



② Max-product composition, product implication

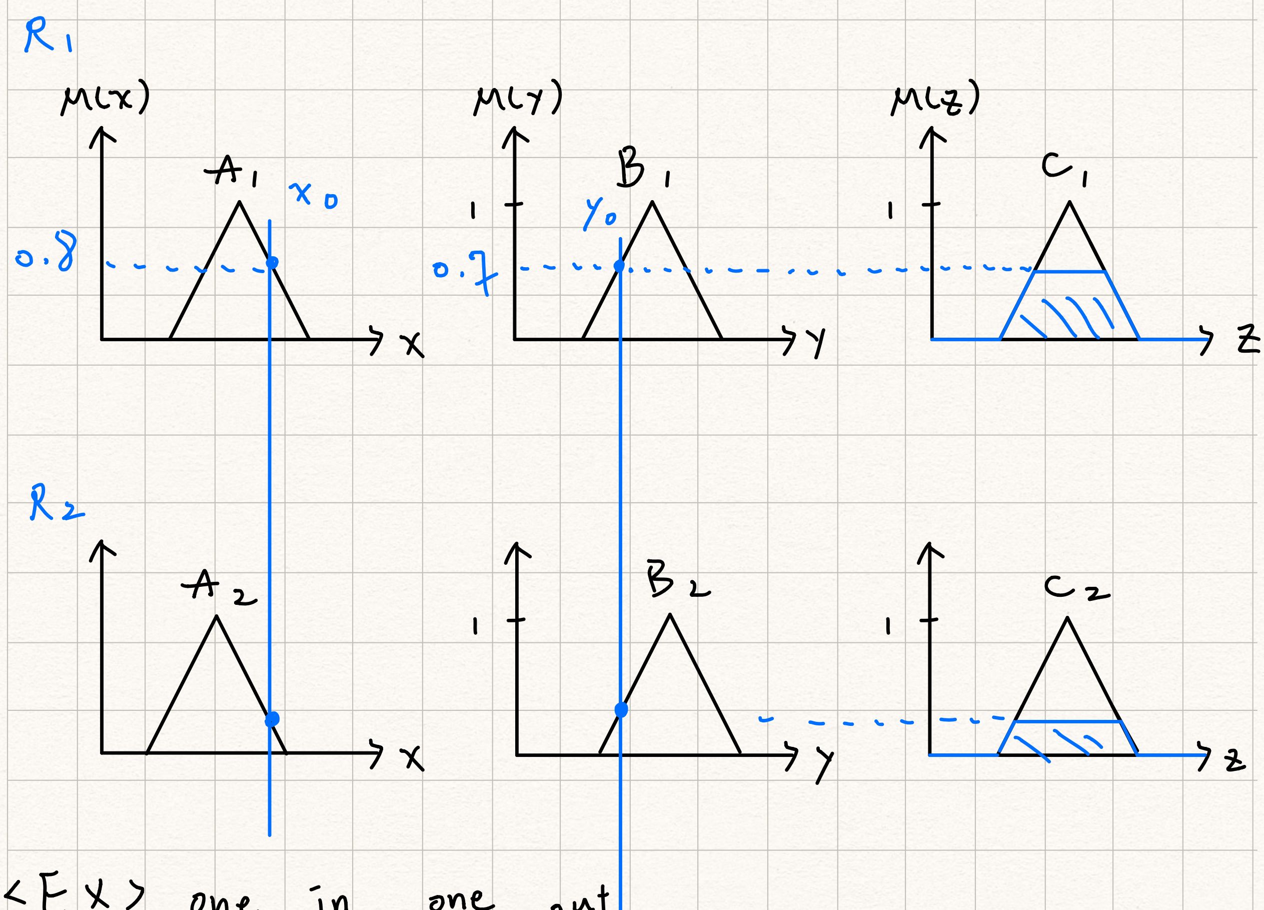


以取大取小法算之(法①)，α cut = 0.7

α cut 跟 C_1 做取小運算

product (法②)，α cut = 0.8 × 0.7 = 0.56

α cut 跟 C_1 做相乘運算

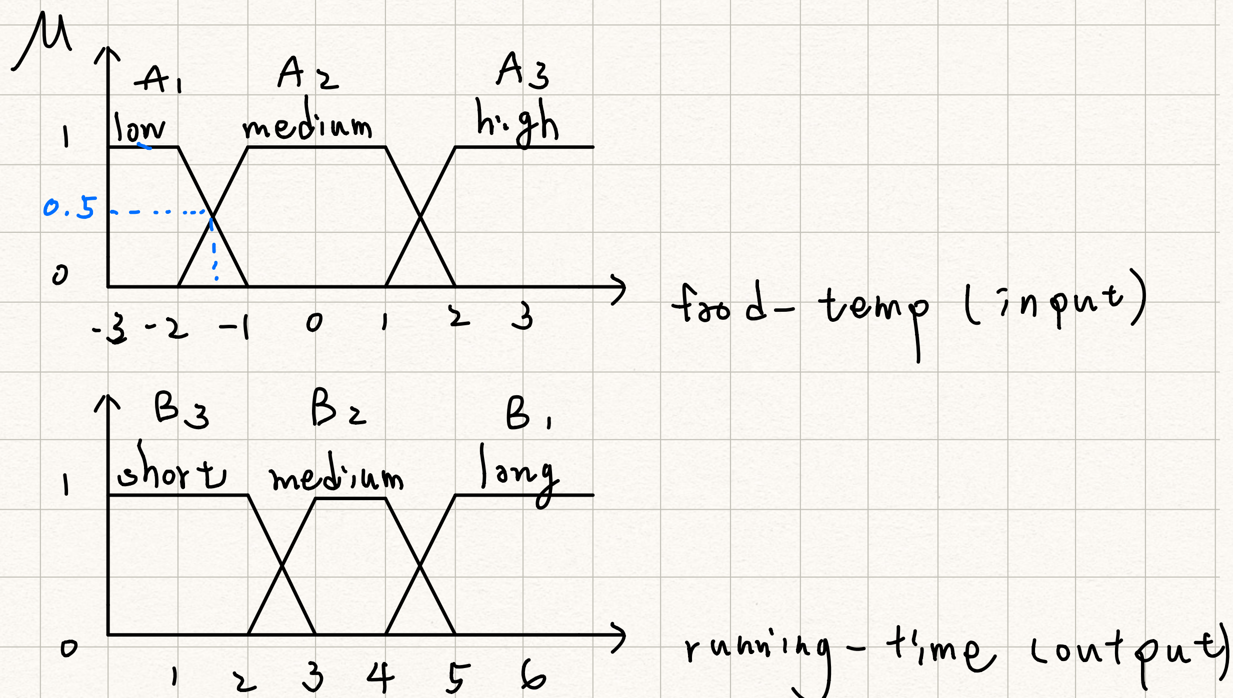


<Ex> one in one out

R' : If food-temp is low then runing-time is long

R^2 : ... medium ... is medium

R^3 : - - - - - - - - - - high ...... is short

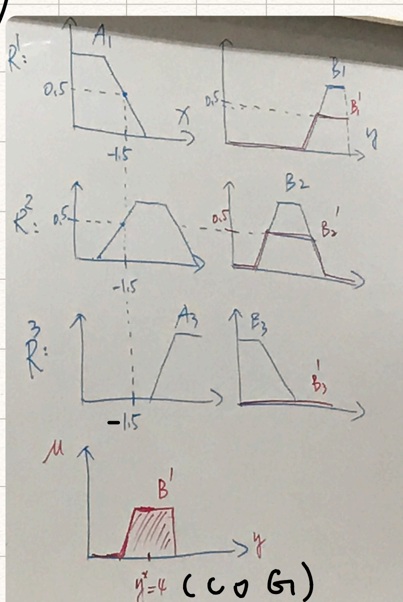


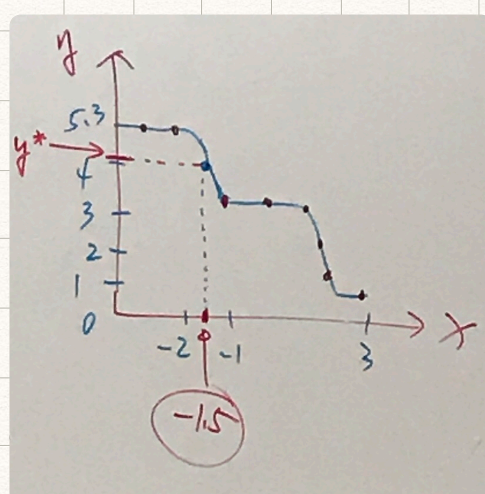
temp = -1.5

$x_0 = -1.5$

時，R_1 R_2 R_3 三條規則的 B_n' 為 ⇒

輸入
是 -1.5
而不是0.5



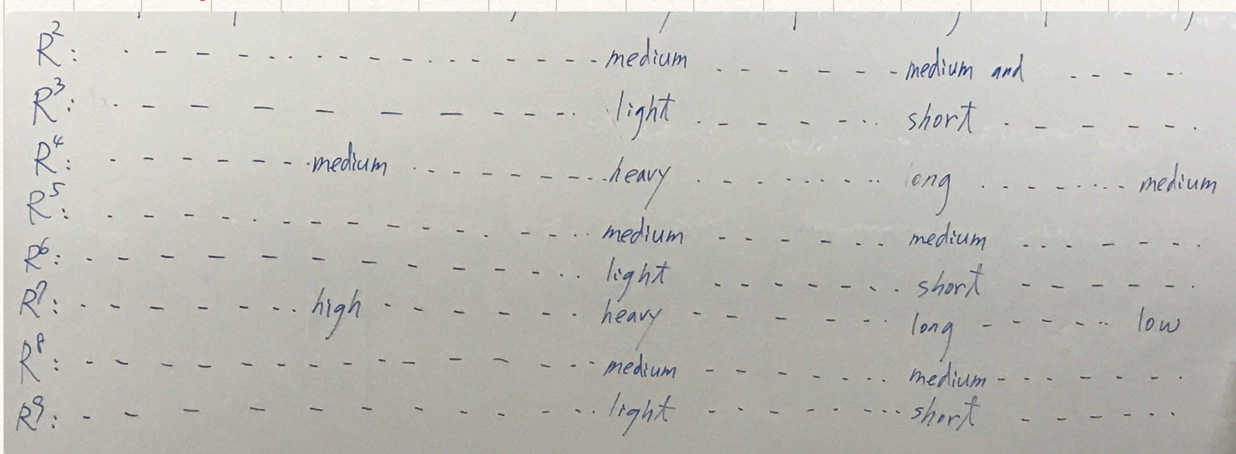


Complete solution

Simulation Homework

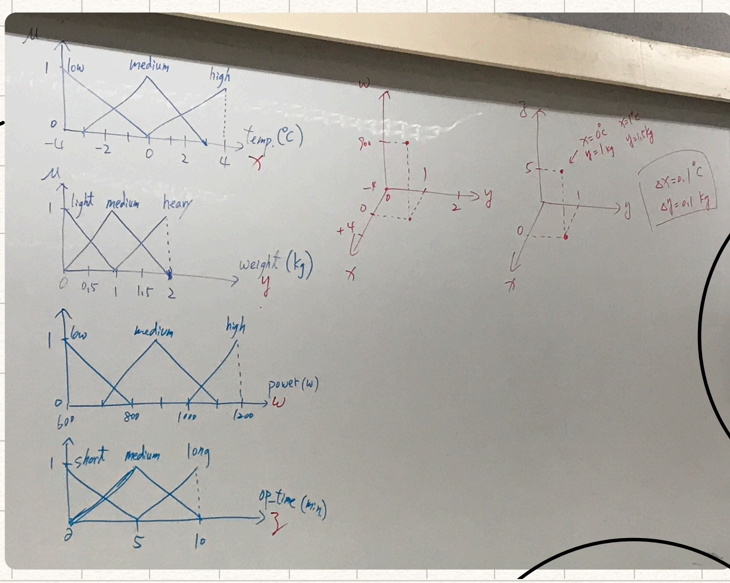
simulation #1 : microwave oven (2 in, 2 out)

R^1: If temp is low and weight is heavy
Then op-time is long and power is high.



-4:0.1:4

0:0.1:2



程式碼
電子檔
email給
老師

間隔都用 0.1，(for loop)

$\Delta x = 0.1°C$
$\Delta y = 0.1kg$

三個禮拜後交

紙本報告，簡報(上台3~5 min)

3個禮拜，結論要結

用<u>離散重心法</u>做，用圖

COG　　多做加分

---

functional f.r. (Sugeno f.r. or TSK f.r.)

R^j: If x_1 is A_1^j and .... and x_p is A_p^j Then y is $f_j(x_1, x_2 \cdots x_p)$

① <u>linear function</u> f.r.

$$y = c_0^j + c_1^j \underline{\underline{x_1}} + c_2^j \underline{\underline{x_2}} + \cdots + c_p^j \underline{\underline{x_p}}$$

② <u>singleton</u> f.r. $y = c_0^j$

c_i^j are real numbers

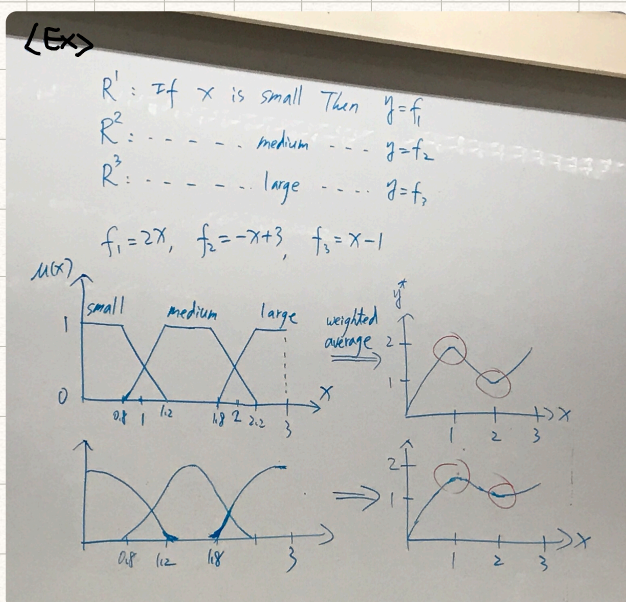
<Ex>

R_1 : If x is small Then $y = f_1$

R_2 : - - - - - medium - - - - - $y = f_2$

R_3 : - - - - - - large - - - - $y = f_3$

$f_1 = 2x$, $f_2 = -x + 3$, $f_3 = x - 1$



函數式模糊規則

Input : x_0, y_0

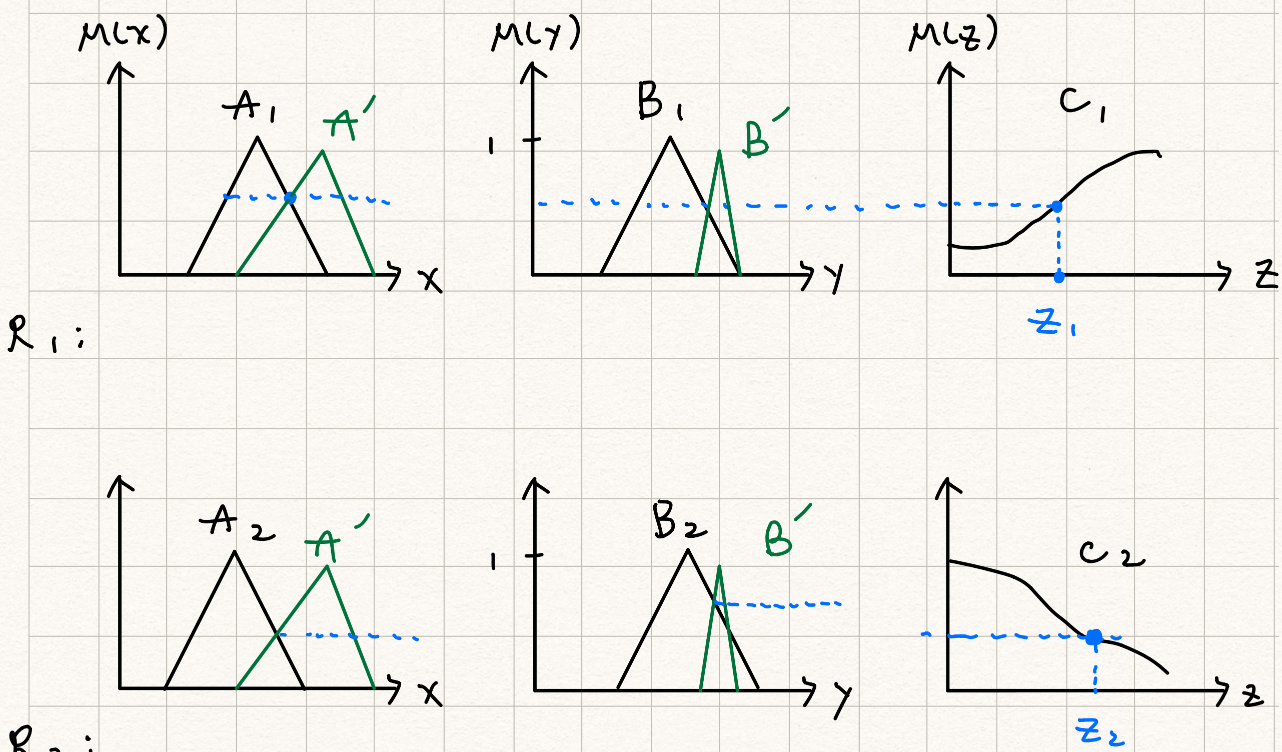
output : z^*

weight average method

$$z^* = \frac{\alpha_1 f_1(x_0, y_0) + \alpha_2 f_2(x_0, y_0)}{\alpha_1 + \alpha_2}$$

3. Tsukamoto f.r.

Same as linguistic f.r., except the m.f. of consequence is monotonical, which implies that a crisp value will be obtained after fuzzy reasoning.



Inputs: x is A' and y is B'

Output: $z^* = \dfrac{\alpha_1 z_1 + \alpha_2 z_2}{\alpha_1 + \alpha_2}$ (weighted average)

R_1: If x is A_1 and y is B_1, Then z is $f_1(x, y)$.

交集(取小運算)

R_2: - ---- -------...... $f_2(x, y)$

② $f_1(x,y)=2x+y^2$ 值直接帶入得值,

③ c 跟 x, y 沒有直接關係

① 語意式, 輸出為 c_1, c_2

② 輸出為 function ..

③ 輸出為單調遞↑↓ function

Determine fuzzy rule:

1. From expert's experience
2. From training algorithm through measured data.

自己生出來

以 data 回授

Fuzzy set 如何擺？

已有三個Δ，便在論域range內，將這三
個Δ平均分配

斜率 X 中心點的移動 !

將Fuzzy set看成高斯函數