Week 1:

linear algebra is defined to be

the study of vectors, vector spaces, a mapping between vector spaces

課程內容:

vectors → matrices → Google's famous Page Rank algorithm

the types of problems we might want to solve:

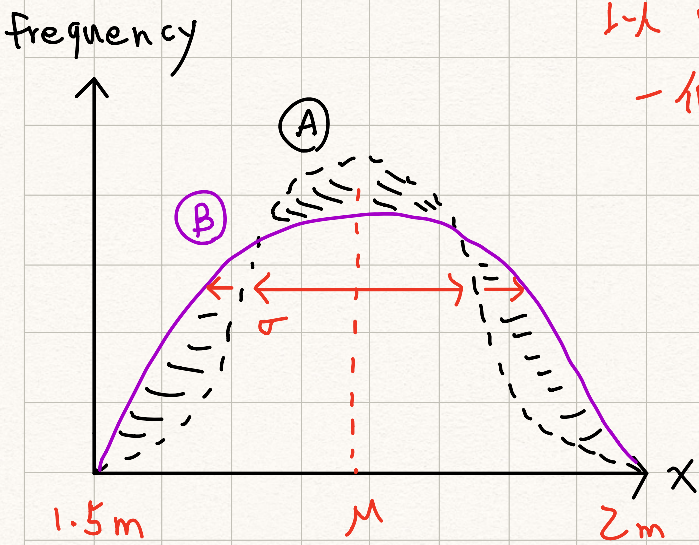
1. price discovery: 解二元一次方程式

更複雜的問題以電腦解決

vector $\begin{bmatrix} a \\ b \end{bmatrix}$ 二元一次方程式中的二元

2. the optimization problem of fitting some data with an equation with some fitting parameters

n. 最優化



以 $P = \begin{bmatrix} \mu \\ \sigma \end{bmatrix}$ 來表示
一個常態分布的 function

two paraments
→ μ (平均值)
σ (標準差)

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{\frac{-(x-\mu)^2}{2\sigma^2}\right\}$$

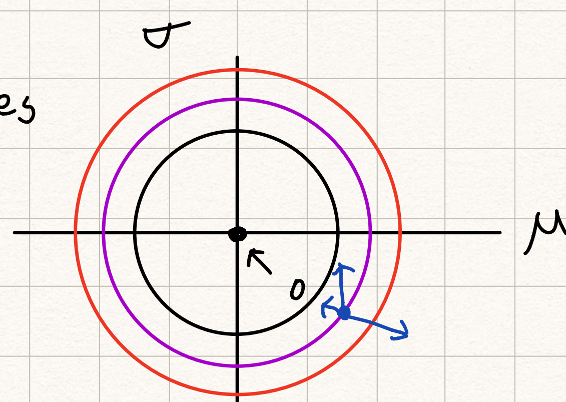
the normal Gaussian distribution
(高斯常態分布)

fitting the distribution?
(分布)

找到最好的 mu, sigma 去符合數據

B 較 A，有更大的 σ 但同樣的 μ

add up the differences
between all of our
measurements and
estiments



比喻：一台車 write down in a vector all of the things about the car

ex: $\begin{pmatrix} \$ \\ gCO_2 \\ NOx \\ V \end{pmatrix}$ time-space: $\begin{pmatrix} x \\ y \\ z \\ T \end{pmatrix}$

find the location in that space where the badness is minimized, the goodness is maximized, and the function fits the date best

What is a vector? Select all correct answers.

- [x] A list of numbers.

正確
Vectors are usually viewed by computers as an ordered list of numbers which they can perform "operations" on - some operations are very natural and, as we will see, very useful!

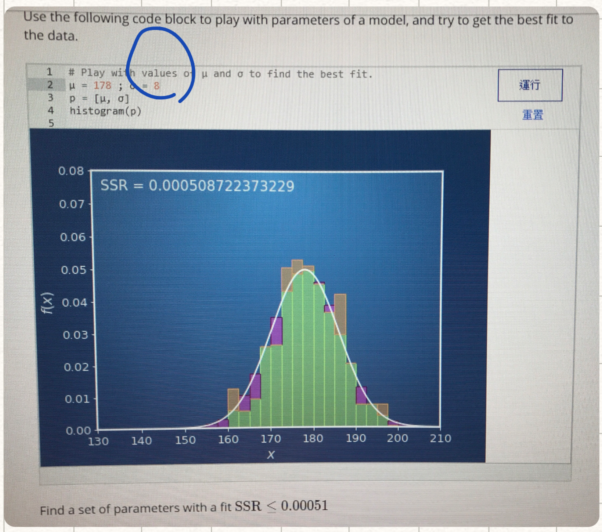
- [x] Position in three dimensions of space and in one dimension of time.

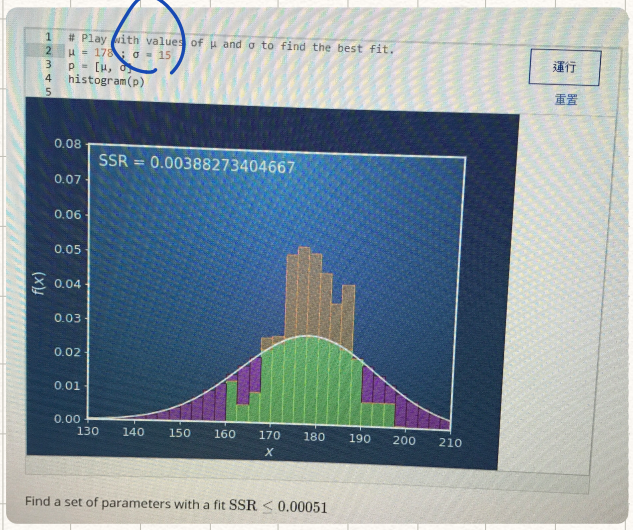
正確
A vector in space-time can be described using 3 dimensions of space and 1 dimension of time according to some co-ordinate system.

- [x] Something which moves in a space of fitting parameters.

正確
As we will see, vectors can be viewed as a list of numbers which describes some optimisation problem.

Vectors can be thought of in a variety of different ways - some geometrically, some algebraically, some numerically. In this way, there are a lot of techniques one can use to deal with vectors.





不同的 σ 值  的樣子也不同

根據 $p = \begin{bmatrix} \mu \\ \sigma \end{bmatrix}$ function 不同，符合樣本資料的程度也不同，程度以 SSR 表示

moving along contour lines has no effect on the SSR.

However moving perpendicular to them can significantly improve or reduce the quality of the fit.

Operation with Vectors =>

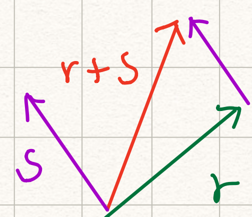
vector can be a space

高中僅以 vector 表運動狀態 or 座標

But, in Data science,
vector just a list of attributes
N. 屬性
of an object.

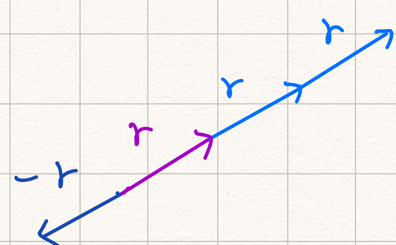
a vector is just something that are based on two rules.

① addition

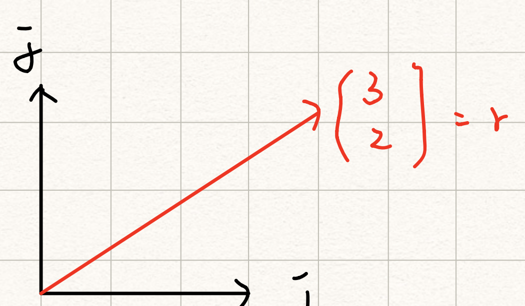


② mutiplication by a scalar number

3 r



-r



$3i + 2j$

向量具 Associative 性

ex: $(r+s)+t = r+(s+t)$

Week 2:

vector的相乘、投影、用來定義空間的向量、
基向量、線性組合……

vector: 長度、方向

$a^2+b^2=c^2$

dot (點積):

$r \cdot s = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 2 \end{bmatrix} = 3\times(-1)+2\times 2$

dot 具 commutative 性, $r \cdot s = s \cdot r$

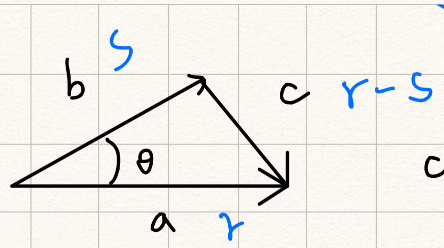
具 distributive 性, $r \cdot (s+t) = r \cdot s + r \cdot t$

具 associative over scalar multipulation, $r \cdot (a s) = a(r \cdot s)$

$r = \begin{bmatrix} r_i \\ r_j \end{bmatrix}$, $r \cdot r = r_i r_i + r_j r_j = r_i^2 + r_j^2$

$= r^2$

$r \cdot r = |r|^2$



餘弦定理證明 dot

$$c^2 = a^2 + b^2 - 2ab \cdot \cos\theta$$

$$|r-s|^2 = |r|^2 + |s|^2 - 2|r||s|\cos\theta$$

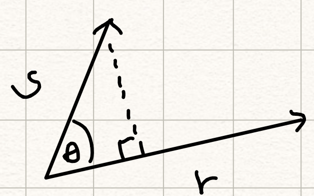
$$\rightarrow (r-s)(r-s) = r \cdot r - s \cdot r - s \cdot r - (-s)(-s)$$

$$= |r|^2 - 2s \cdot r + |s|^2 \quad \cdots\cdots ①$$

$$= |r|^2 + |s|^2 - 2|r||s|\cos\theta \quad \cdots\cdots ②$$

比較①、②式得 $s \cdot r = 2|s||r|\cos\theta$

projection:



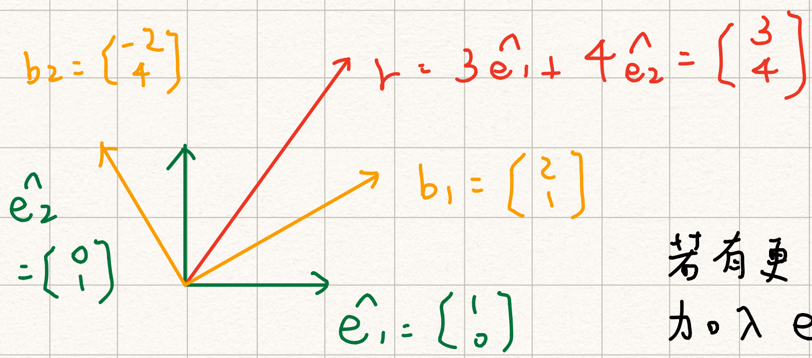
$$\cos\theta = adj / hyp$$

$$r \cdot s = |r| \underbrace{|s|\cos\theta}_{\text{adj, r x projection}}$$

$$\frac{r \cdot s}{|r|} = \underbrace{|s|\cos\theta}_{\text{scalar projection}}$$

$$r \cdot \frac{r \cdot s}{|r||r|} = \underbrace{\frac{r \cdot s}{r \cdot r} \cdot r}_{\text{vector projection}}$$

changing vector from one coordinate system to another:



若有更多維度,
加入 e_3, e_4 .... e_n

$\hat{e}_1$, $\hat{e}_2$ 互垂

稱之為 basic vectors

b_1, b_2 互垂

問題通常會問, r 在 $\hat{e}_1$, $\hat{e}_2$ 座標系統中的位置, 若改到 b_1, b_2 時的值為何

(條件: $\hat{e}_1$, $\hat{e}_2$ 及 b_1, b_2 兩兩正交)

解法:

如非正交要以矩陣計算

把 r 投影在 b_1 的 vector projection / b_1 求出
r 投影在 b_2 的 vector projection / b_1

解即為 $\begin{pmatrix} \text{vector projection} / b_1 \\ \text{vector projection} / b_1 \end{pmatrix}$

Basis is a set of n vectors that:

(i) are not linear combinations of each other (linearly-independent)

(ii) span the space

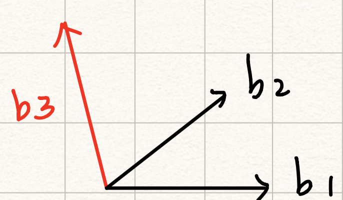
The space is then n-dimensional

線性獨立的維基百科定義：

在一組元素中，若沒有向量可用有限個其他向量的線性組合表示，則稱線性獨立

linearly dependent

$b_3 = a_1 b_1 + a_2 b_2$

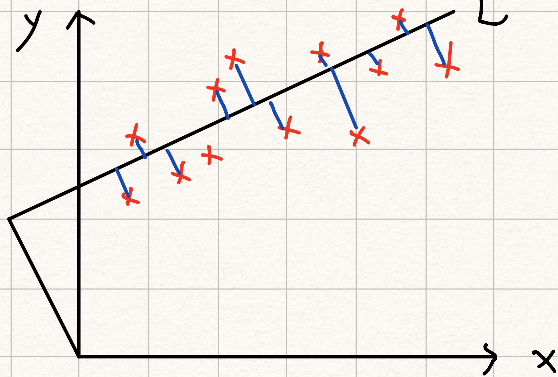


b_3對b_1、b_2來說線性相依，因b_1、b_2的線性組合可合成b_3

linearly independent

b_3跳到b_1 b_2所屬平面外

b_3對b_1、b_2來說線性獨立，因b_1、b_2的線性組合不可合成b_3



透過座標軸轉換得知直線L與數據的適配性為何

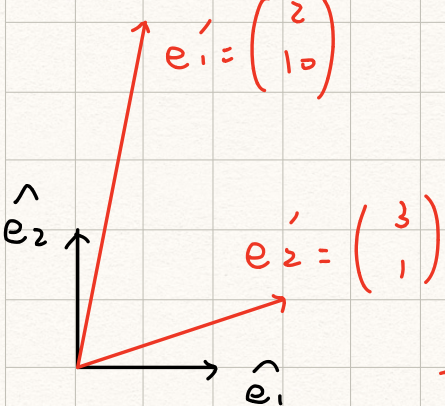
week 3:

matrix transformation:

$$2a + 3b = 8$$
$$10a + 1b = 13$$

$$\begin{pmatrix} 2 & 3 \\ 10 & 1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 8 \\ 13 \end{pmatrix} \qquad \begin{pmatrix} 2 & 3 \\ 10 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 10 \end{pmatrix}$$

$$\begin{pmatrix} 2a + 3b \\ 10a + b \end{pmatrix} = \begin{pmatrix} 8 \\ 13 \end{pmatrix} \qquad \begin{pmatrix} 2 & 3 \\ 10 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$



$$A \, r = r'$$
$$A \, (nr) = nr'$$
$$A \, (r+s) = Ar + As$$
$$A \, (n\hat{e}_1 + n\hat{e}_2) = nA\hat{e}_1 + mA\hat{e}_2$$
$$= n e_1' + n e_2'$$

$$\begin{bmatrix} 2 & 3 \\ 10 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 12 \\ 32 \end{bmatrix}$$

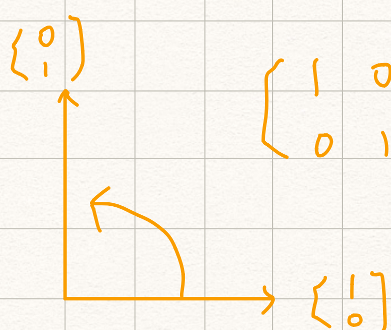
$$\begin{bmatrix} 2 & 3 \\ 10 & 1 \end{bmatrix} \left(3\begin{bmatrix} 1 \\ 0 \end{bmatrix} + 2\begin{bmatrix} 0 \\ 1 \end{bmatrix} \right)$$

$$= 3\left(\begin{bmatrix} 2 & 3 \\ 10 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right) + 2\left(\begin{bmatrix} 2 & 3 \\ 10 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right) = 3\begin{bmatrix} 2 \\ 10 \end{bmatrix} + 2\begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

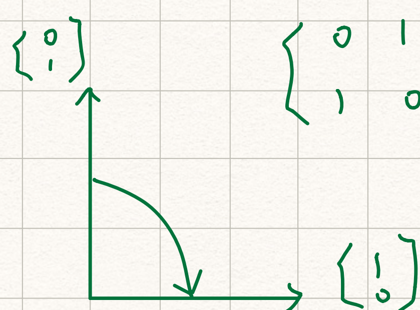
也就是說，$\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ 可拆成 $3\begin{bmatrix} 1 \\ 0 \end{bmatrix} + 2\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ 後，再和 $\begin{bmatrix} 2 & 3 \\ 10 & 1 \end{bmatrix}$ 相乘。

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

Identity Matrix I



$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

mirror

minus 90° anticlockwise: $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

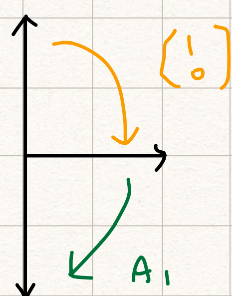
…θ若為正，為逆時針旋轉θ度….

rotation 一般式 $\begin{bmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{bmatrix}$

向量轉換：combination of rotation, shears, stretches, and inverses

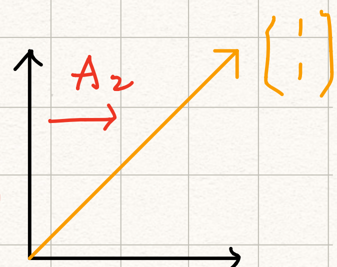
rotation：

$A_1 = \begin{bmatrix}0 & 1 \\ -1 & 0\end{bmatrix}$



shear：

$A_2 = \begin{bmatrix}1 & 1 \\ 0 & 1\end{bmatrix}$



$\begin{bmatrix}1 & 1 \\ 0 & 1\end{bmatrix}\begin{bmatrix}0 \\ -1\end{bmatrix} = \begin{bmatrix}-1 \\ -1\end{bmatrix}$

$\begin{bmatrix}1 & 1 \\ 0 & 1\end{bmatrix}\begin{bmatrix}1 \\ 0\end{bmatrix} = \begin{bmatrix}1 \\ 0\end{bmatrix}$

$\rightarrow \begin{bmatrix}-1 & 1 \\ -1 & 0\end{bmatrix} = A_2 A_1$

$A_1 A_2 \neq A_2 A_1$

一張 digit picture 是由許多有顏色的相素點構成

$\begin{bmatrix}2 & 3 \\ 10 & 1\end{bmatrix}\begin{bmatrix}a \\ b\end{bmatrix} = \begin{bmatrix}8 \\ 13\end{bmatrix}$

$A^{-1}A = I$

↑ Inverse

$$A \quad r = S$$

$$A^{-1}A \cdot r = A^{-1}S$$

$$A^{-1}A = I$$

$$\begin{array}{l} 1. \\ 2. \\ 3. \end{array}\begin{bmatrix} 1 & 1 & 3 \\ 1 & 2 & 4 \\ 1 & 1 & 2 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 15 \\ 21 \\ 13 \end{bmatrix}$$

第二、三列減掉第一列
(elimination)

$$\begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 15 \\ 6 \\ -2 \end{bmatrix}$$

第一、二列加上第三列
(back-substitution)

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 9 \\ 4 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 2 & 4 \\ 1 & 1 & 2 \end{bmatrix}\begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A \cdot B = I$$

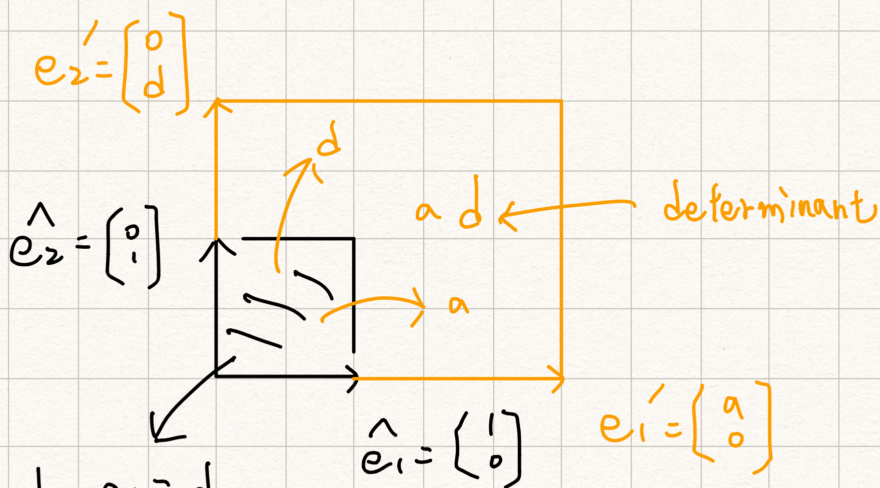
讓A矩陣經過elimination和back-substitution，並成為單位矩陣的型式，此時的I'便是A^{-1}

$$\begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} -2 & 0 & 3 \\ -2 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 0 & -1 & 2 \\ -2 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix} \leftarrow \begin{matrix} A^{-1} \\ B \end{matrix}$$

$$\begin{pmatrix} a & 0 \\ 0 & d \end{pmatrix}$$

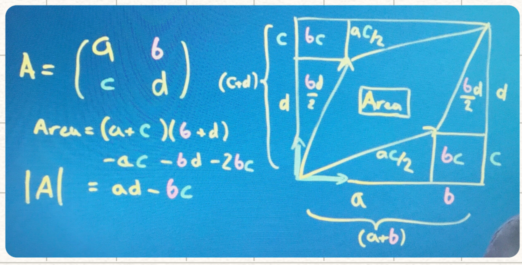


original grid.

the scale of the space is what the determinant is.

行列式的證明

$|A| = ad - bc$ ※



$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\left(\begin{array}{c|c} d & -b \\ -c & a \end{array}\right) = \begin{pmatrix} ad-bc & 0 \\ 0 & ad-bc \end{pmatrix}$$

$$\frac{1}{ad-bc}\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

向量間需為線性獨立才能求得解。

Week 4:

愛因斯坦求和定律

Einstein's convention

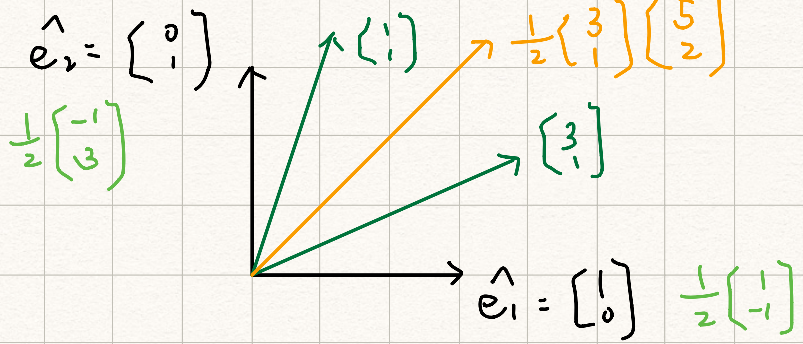
$$\begin{array}{ccc} A & B & AB \\ \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & \\ \vdots & \vdots & & \\ a_{n1} & & & \end{pmatrix} & \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ \vdots & & & \vdots \\ b_{n1} & \cdots & & b_{nn} \end{pmatrix} = & \begin{pmatrix} & 0 \\ & \end{pmatrix} \end{array}$$

$$(ab)_{23} = a_{21}b_{13} + a_{22}b_{23} + \cdots + a_{2n}b_{n3}$$

$$ab_{ik} = \sum_j a_{ij}b_{jk} = a_{ij}b_{jk}$$

$$(\quad)_{2\times 3}(\quad)_{3\times 4} = (\quad)_{2\times 4}$$

Bear's world →



$$\underbrace{\left(\begin{pmatrix} 3 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right)}_{B}$$

Bear's basis in my world.

Bear's basis vectors $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ in my frame.

$$\underbrace{\begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}}_{\text{Bear's basis in my coordinate.}} \underbrace{\begin{bmatrix} 3/2 \\ 1/2 \end{bmatrix}}_{\text{Bear's vector.}} = \underbrace{\begin{bmatrix} 5 \\ 2 \end{bmatrix}}_{\text{my vector.}}$$

將Bear視角中的向量轉為我眼中的向量

反之如何呢？

Find the inverse matrix of Bear's vector

$$\underbrace{\frac{1}{2}\begin{bmatrix} 1 & -1 \\ -1 & 3 \end{bmatrix}}_{\text{my basis in Bear's world}} \begin{bmatrix} 5 \\ 2 \end{bmatrix} = \underbrace{\frac{1}{2}\begin{bmatrix} 3 \\ 2 \end{bmatrix}}_{\text{Bear's vector.}}$$

$$\frac{1}{2}\begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix} \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \left(\begin{pmatrix} 3 \\ 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix}\right) \quad \begin{pmatrix} x \\ y \end{pmatrix}$$

B^{-1} $\quad$ R $\quad$ B

$\circlearrowleft 45°$ $\quad$ Bear's basis in my world

vector rotated in my basis $\quad \frac{1}{\sqrt{2}}\begin{pmatrix} 2 & 0 \\ 4 & 2 \end{pmatrix} = RB$

vector rotation in Bear's basis $= B^{-1}RB$

$$\frac{1}{\sqrt{2}}\begin{pmatrix} -1 & -1 \\ 5 & 3 \end{pmatrix}$$

transpose : $A^T_{ij} = A_{ji}$

$A^T = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} \quad A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

if orthonormal $\rightarrow A^T A = I$

orthonormal matrix 正交規範性

! $\quad A^T = A^{-1}$

the determinant of an orthogonal matrix must be either plus or minus one.

$$|A| = \pm 1$$

orthonormal basis vector set,

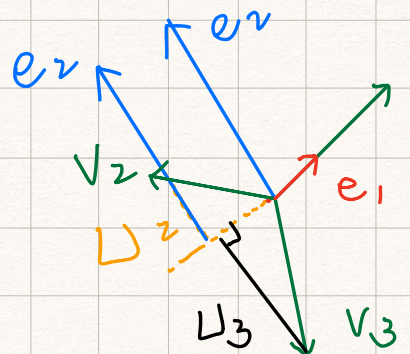
$\Rightarrow$ more easier to compute

Gram - Schmidt process

$$v = \{v_1, v_2 \ldots\ldots v_n\}$$

v_1 $\qquad$ $e_1 = \frac{v_1}{|v_1|}$ #

$$v_2 = (v_2 \cdot e_1)\frac{e_1}{|e_1|} + u_2$$

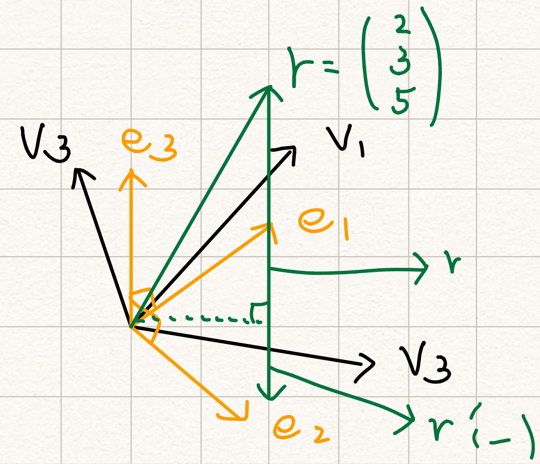


$$u_2 = v_2 - (v_2 \cdot e_1)\, e_1$$

$$\frac{v_2}{|u_2|} = e_2$$ #

$$U_3 = V_3 - (V_3 \cdot e_1)e_1 - (V_3 \cdot e_2)e_2$$

$$\frac{U_5}{|U_3|} = e_3$$



on the same plane

$$\underbrace{\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}}_{v_1} \quad \underbrace{\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}}_{v_2} \quad \underbrace{\begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}}_{v_3}$$

$$E = \begin{pmatrix} (e_1) & (e_2) & (e_3) \end{pmatrix} \quad \#$$

$$e_1 = \frac{v_1}{|v_1|} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad \#$$

$$U_2 = V_2 - (V_2 \cdot e_1)e_1 = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} - \left[\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} \cdot \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right] \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \qquad e_2 = \frac{U_2}{|U_2|} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \quad \#$$

$$U_3 = V_3 - (V_3 \cdot e_1)e_1 - (V_3 \cdot e_2)e_2$$

$$= \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} \qquad e_3 = \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} \quad \#$$

$$
\begin{tikzcd}
\boxed{r} \arrow[r, "\text{hard}"] \arrow[d, "E^{-1}"'] & \boxed{r'} \\
\boxed{r_E} \arrow[r, "T_E"'] & \boxed{r_E'} \arrow[u, "E"']
\end{tikzcd}
\qquad
T_E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}
$$

$$r' = E \cdot T_E \cdot E^{-1} \cdot r \quad , \quad E^T = E^{-1}$$

$$E T_E E^T = \frac{1}{3}\begin{pmatrix} 2 & -1 & 2 \\ -1 & 2 & 2 \\ 2 & 2 & -1 \end{pmatrix} = T$$

$$r' = Tr = T\begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 11 \\ 14 \\ 5 \end{pmatrix}$$

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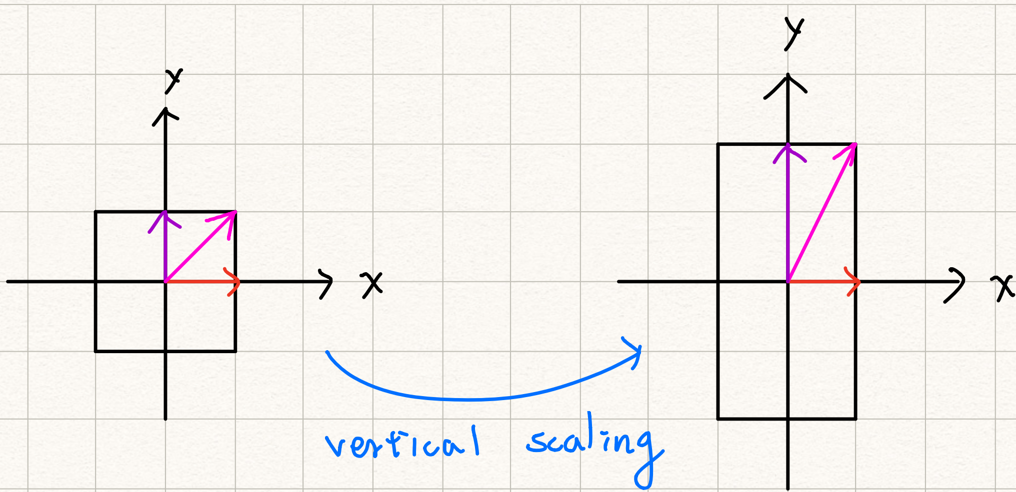
week 5

特徵問題! → App: google算法!

eigen problem

eigen → 指 characteristic



除了x軸的紅色向量大小、方向皆沒變外
其他兩條：紫的大小變為原來的2倍
粉的大小、方向都改變

horizontal and vertical vectors are special

they're characteristic of the particular transform.

eigenvectors!

horizontal vector's length is unchange, it has a corresponding eigenvalue of one.

經變換後，還在原地的便是 eigenvectors

vertical . . . . of two.

eigenvector is: a vector which, after applying the transformation, says in the same span. (1, -1 都算)

三個特殊的 eigen-cases

1. 等比放大

2. 180° 旋轉

3. a horizontal shear and a vertical scaling

If we find the eigenvector of a 3D rotation, it means we've also found the axis of rotation.

$$Ax = \lambda x$$

$$(A - \lambda I)x = 0$$

$$\det(A - \lambda I) = 0$$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\det\left(\begin{pmatrix} a & b \\ c & d \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}\right) = 0$$

$$\lambda^2 - (a+d)\lambda + ad - bc = 0$$

ex: $A = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$

$\det\begin{pmatrix} 1-\lambda & 0 \\ 0 & 2-\lambda \end{pmatrix} = (1-\lambda)(2-\lambda) = 0 \qquad \lambda = 1 \vee 2$

$\lambda = 1 \quad \begin{pmatrix} 1-1 & 0 \\ 0 & 2-1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ x_2 \end{pmatrix}$

eigenvectors

$\lambda = 2 \quad \begin{pmatrix} 1-2 & 0 \\ 0 & 2-2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -x_1 \\ 0 \end{pmatrix} \qquad = \begin{pmatrix} t \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ t \end{pmatrix}$ #

ex: $A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

$\det\begin{pmatrix} 0-\lambda & -1 \\ 1 & 0-\lambda \end{pmatrix} = \lambda^2 + 1 = 0$

characteristic polynomial

changing basis

performing efficient matrix operations

called diagonalisation

$$T^n = \begin{pmatrix} a^n & 0 & 0 \\ 0 & b^n & 0 \\ 0 & 0 & c^n \end{pmatrix}$$

$$C = \begin{pmatrix} x_1 & x_2 & x_3 \\ \vdots & \vdots & \vdots \end{pmatrix} \qquad D = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}$$

$T = CDC^{-1}$ → Diagonalization 對角化

$$T^2 = CDC^{-1}CDC^{-1} = CDDC^{-1} = CD^2C^{-1}$$

$$T^n = CD^nC^{-1}$$

$$\begin{tikzcd} \boxed{v} \arrow[r, "T^n"] \arrow[d, "C^{-1}"'] & \boxed{T^n v} \\ \boxed{[v]_E} \arrow[r, "D^n"'] & \boxed{[T^n v]_E} \arrow[u, "C"'] \end{tikzcd}$$

$$C = \left(\begin{pmatrix} \text{特徵} \\ \text{向量}_1 \end{pmatrix} \begin{pmatrix} \text{特徵} \\ \text{向量}_2 \end{pmatrix} \right)$$

$$T^n = CD^nC^{-1}$$

$$T = \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}$$

$\lambda = 1$, $x = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$\ 2$, $x = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$$C = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \qquad C^{-1} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$$

$$T^2 = CD^2C^{-1} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}^2\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$$

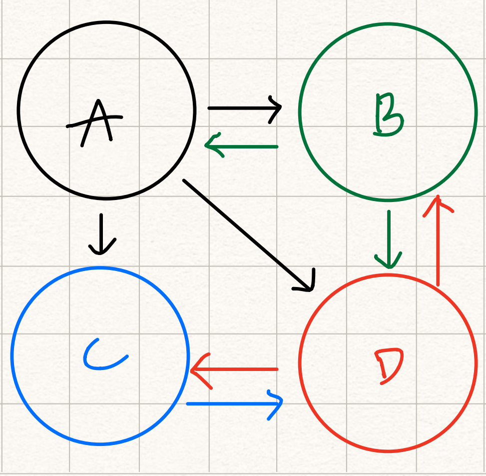
$$= \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ 0 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 0 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 3 \\ 0 & 4 \end{pmatrix}\begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \quad ※$$

PageRank ! Larry Page

website is related to its links to and from other websites



webpage

Procrastinating Pat

A. B. C. D 每個 vector 相加需 = 1

$$\begin{array}{ccccc} & A & B & C & D \\ L_A = (& 0, & 1/3, & 1/3, & 1/3) \\ L_B = (& 1/2, & 0, & 0, & 1/2) \\ L_C = (& 0, & 0, & 0, & 1) \\ L_D = (& 0, & \frac{1}{2}, & \frac{1}{2}, & 0) \end{array}$$

$$L = \begin{pmatrix} 0 & 1/2 & 0 & 0 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 1/2 & 1 & 0 \end{pmatrix}$$

$$r_A = \sum_{j=1}^{n} L_{A,j} \, r_j$$

$$r^{i+1} = L r^i$$

$$r^{i+1} = d(L r^i) + \frac{1-d}{n}$$