Introducción a la Inteligencia Artificial
Clase 7

.UBAfiuba
FACULTAD DE INGENIERÍA

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Clase 7

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Aprendizaje no supervisado

Machine Learning Supervisado	Machine Learning no Supervisado
Proceso aleatorio $\bar{X}, y$	Proceso aleatorio $\bar{X}$
¿$f_{y/\bar{x}}(y\|\bar{x})$? $\longrightarrow$ Bayes y M.V.	¿$f_{\bar{x}}(\bar{x})$? $\longrightarrow$ Bayes y M.V.
Inferencias, predicciones	Clusterización, Reducción Dimensionalidad

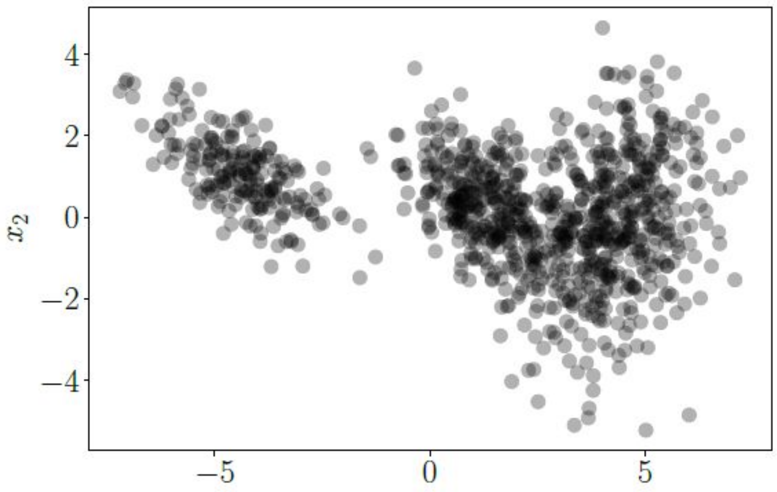
Aprendizaje no supervisado

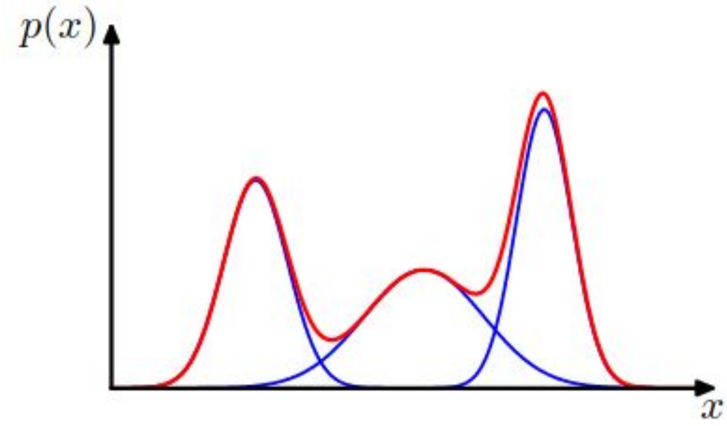
Aplicaciones Generales

- Data Mining
- Pattern Recognition
- Statistical Analysis

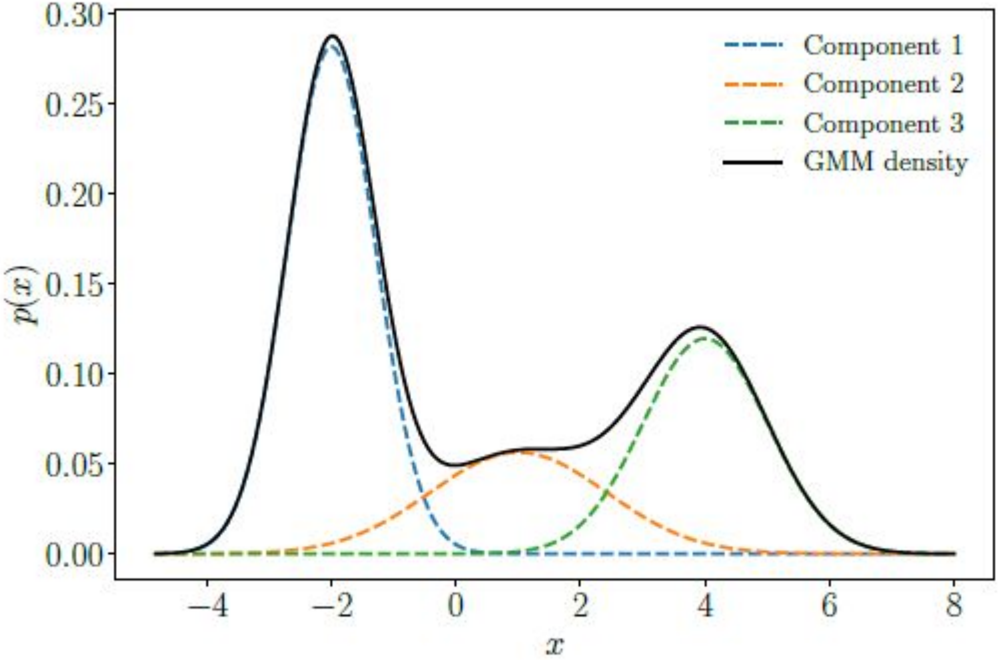
Aplicaciones Específicas

- Density Estimation
- Clustering
- Anomaly Detection
- Object Tracking
- Speech Feature Extraction





Formulación

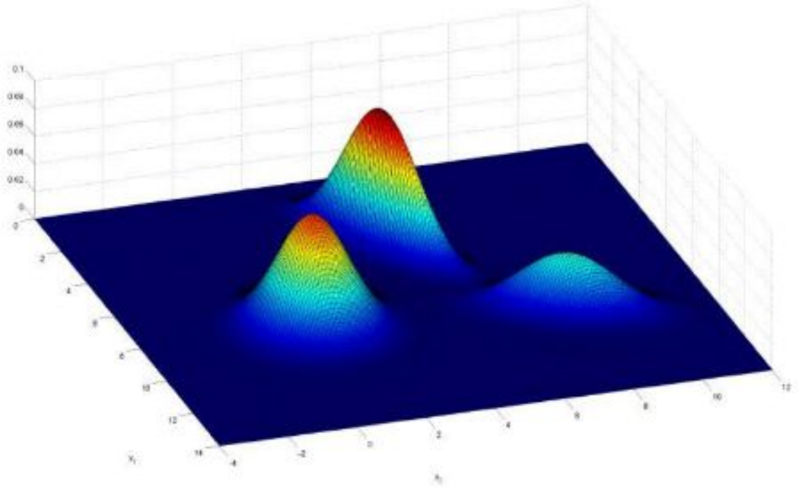


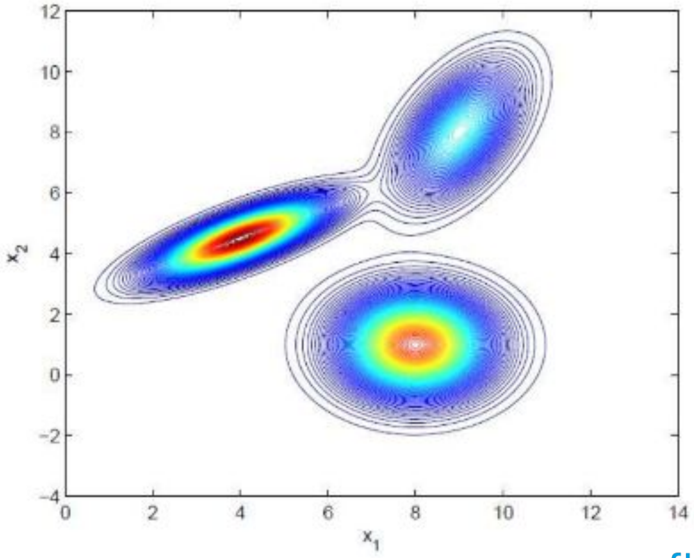
$$p(x \mid \boldsymbol{\theta}) = 0.5\mathcal{N}\left(x \mid -2, \tfrac{1}{2}\right) + 0.2\mathcal{N}\left(x \mid 1, 2\right) + 0.3\mathcal{N}\left(x \mid 4, 1\right)$$

Gaussian Mixture Models

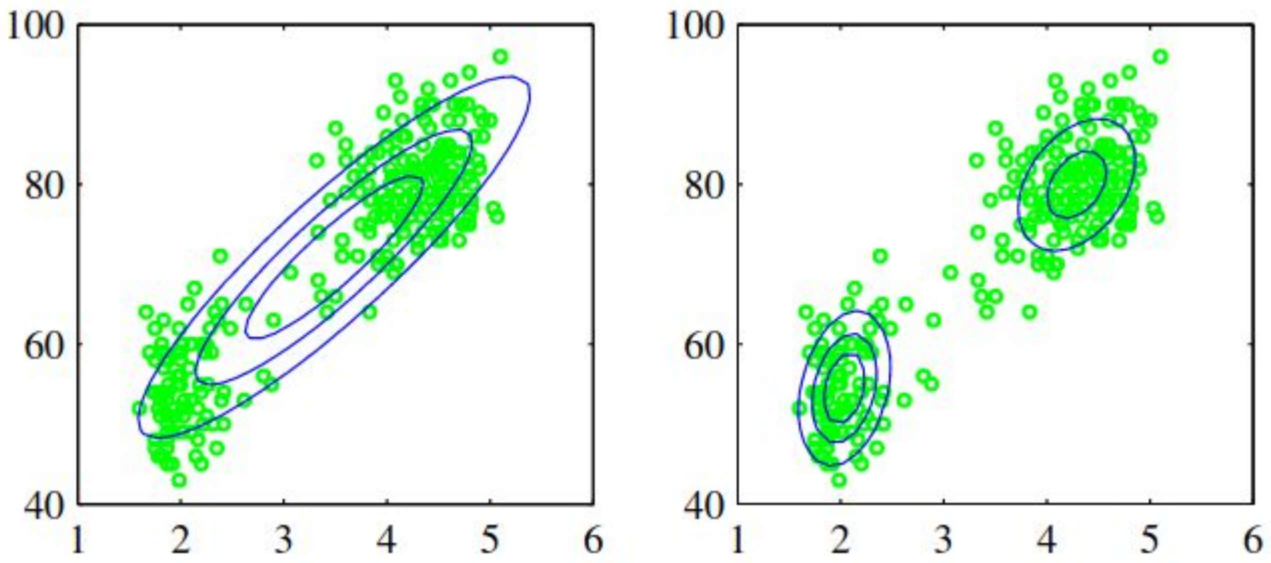
Formulación

$$p(x) = \underbrace{0.3}_{\pi_1} \mathcal{N}\left(x \,\middle|\, \underbrace{\begin{pmatrix} 4 \\ 4.5 \end{pmatrix}}_{\mu_1}, \underbrace{\begin{pmatrix} 1.2 & 0.6 \\ 0.6 & 0.5 \end{pmatrix}}_{\Sigma_1}\right) + \underbrace{0.5}_{\pi_2} \mathcal{N}\left(x \,\middle|\, \underbrace{\begin{pmatrix} 8 \\ 1 \end{pmatrix}}_{\mu_2}, \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}_{\Sigma_2}\right) + \underbrace{0.2}_{\pi_3} \mathcal{N}\left(x \,\middle|\, \underbrace{\begin{pmatrix} 9 \\ 8 \end{pmatrix}}_{\mu_3}, \underbrace{\begin{pmatrix} 0.6 & 0.5 \\ 0.5 & 1.5 \end{pmatrix}}_{\Sigma_3}\right)$$





Gaussian Mixture Models: Estudio de fenómenos naturales



“Old Faithful” dataset. 272 mediciones de erupciones del “Old Faithful” geyser en el Parque Nacional Yellowstone. El eje horizontal representa la duración de una erupción (medida en minutos) y el vertical el tiempo hasta la próxima erupción.

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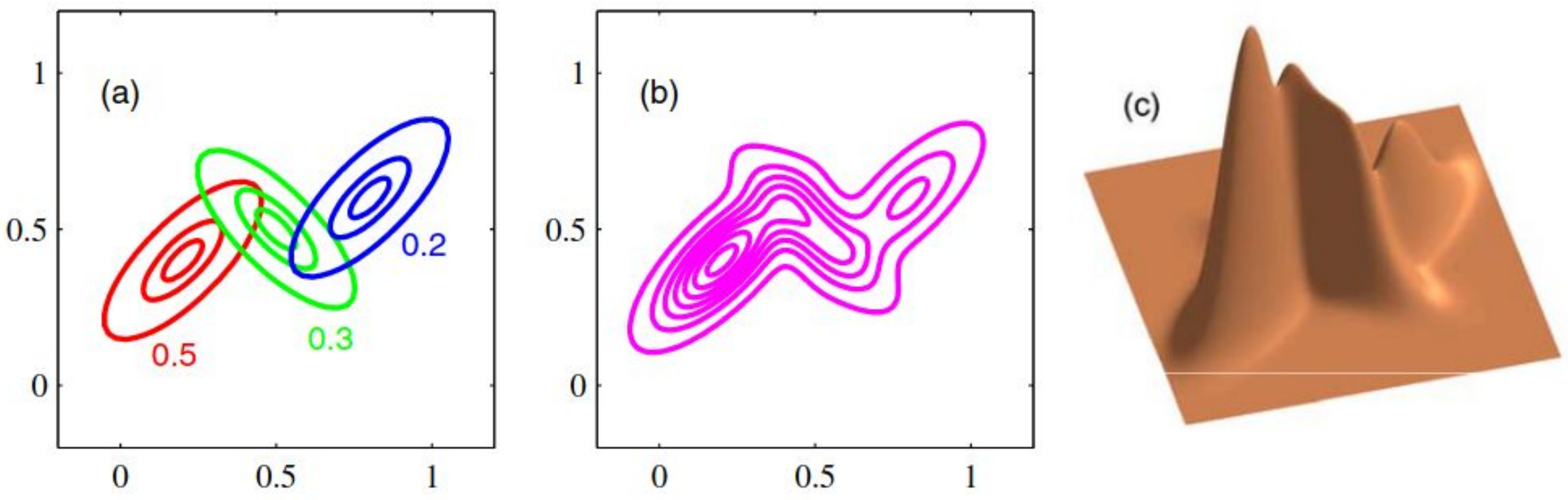
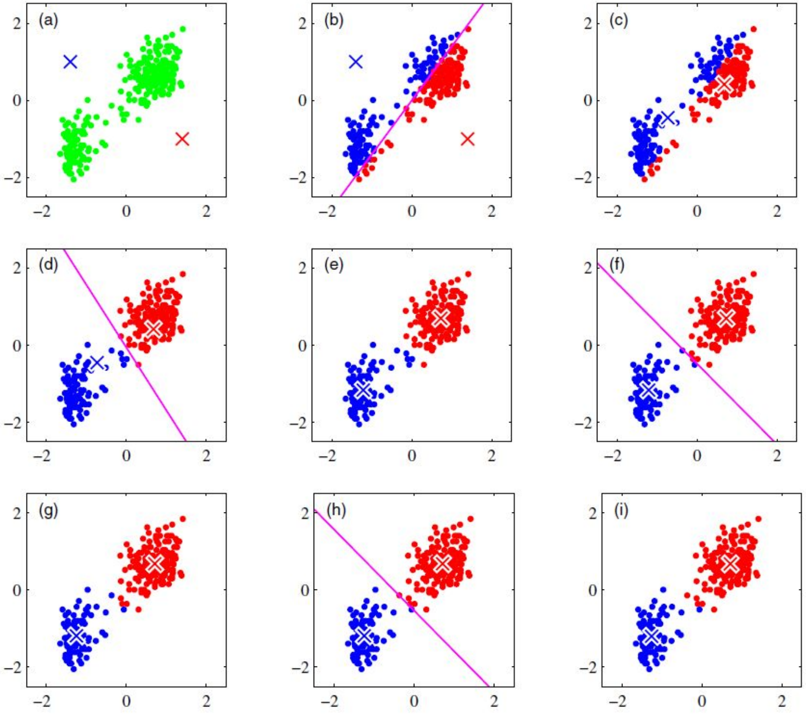
Gaussian Mixture Models: Clustering

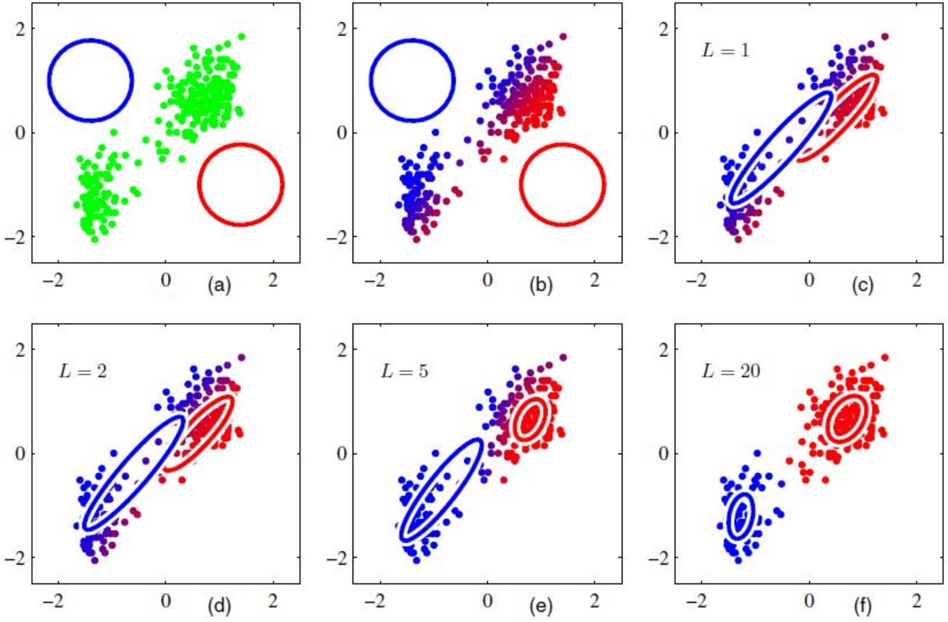


Ejemplo de Gaussian Mixture. En la imagen (a) se muestran las tres distribuciones subyacentes indicando con colores sus variables latentes. En la imagen (b) las curvas de nivel de la distribución conjunta y en la (c) la densidad.

Gaussian Mixture Models: Clustering

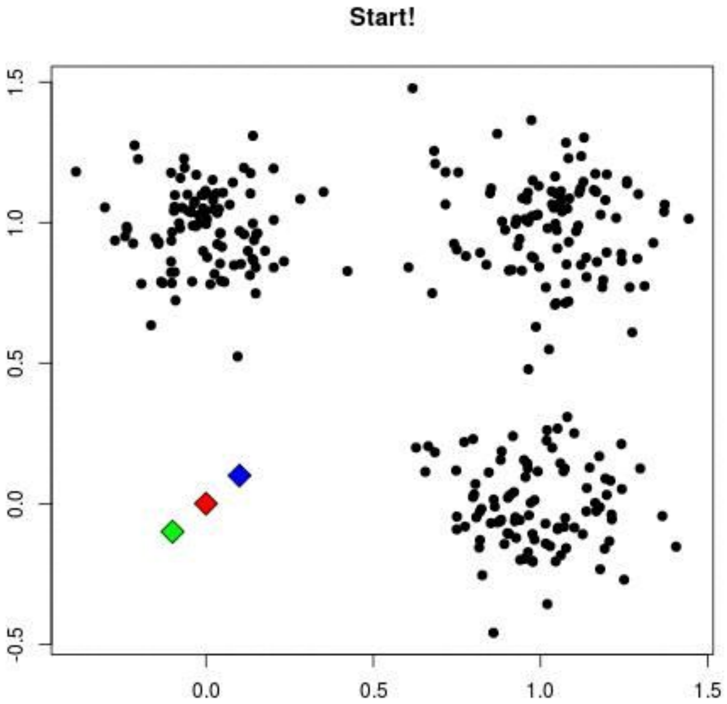


KMeans

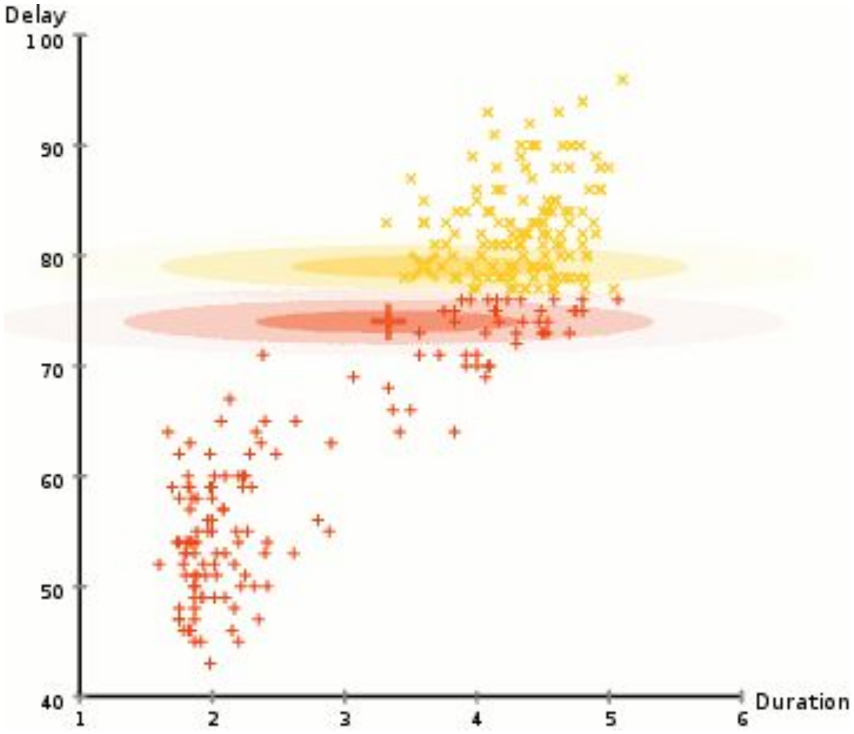


GMM

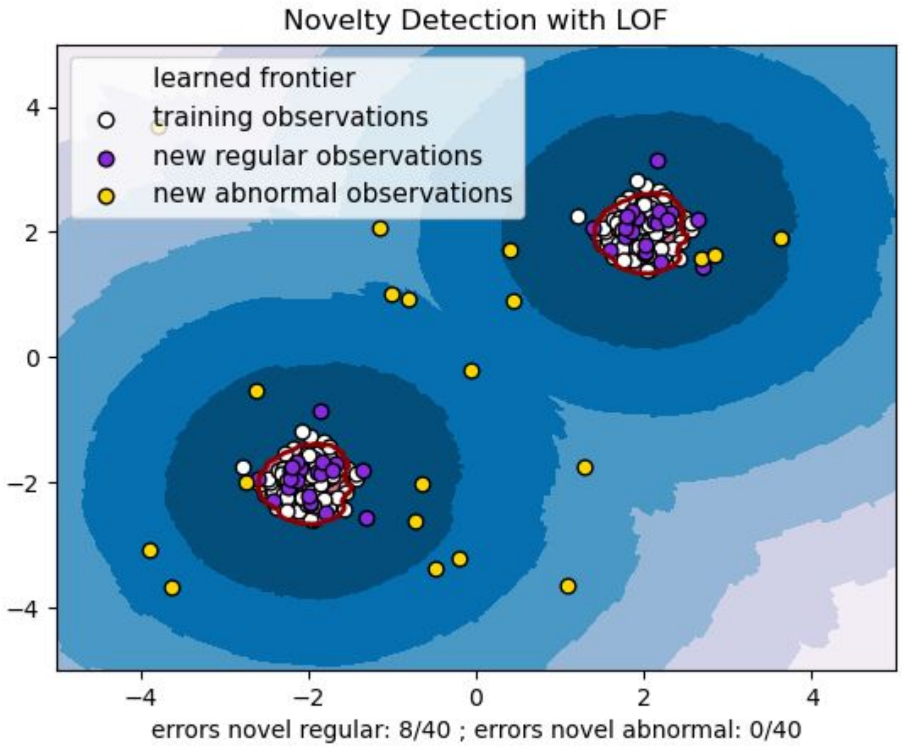
Gaussian Mixture Models - kMeans



Gaussian Mixture Models: Clustering

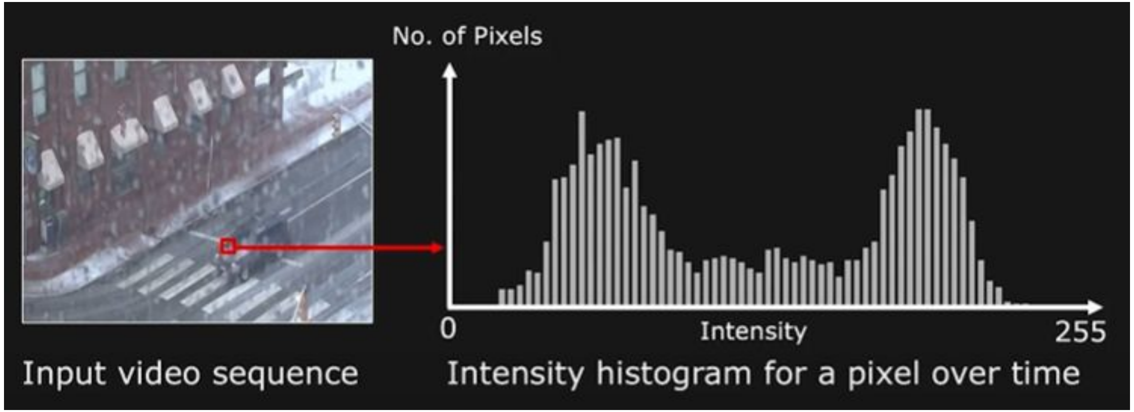


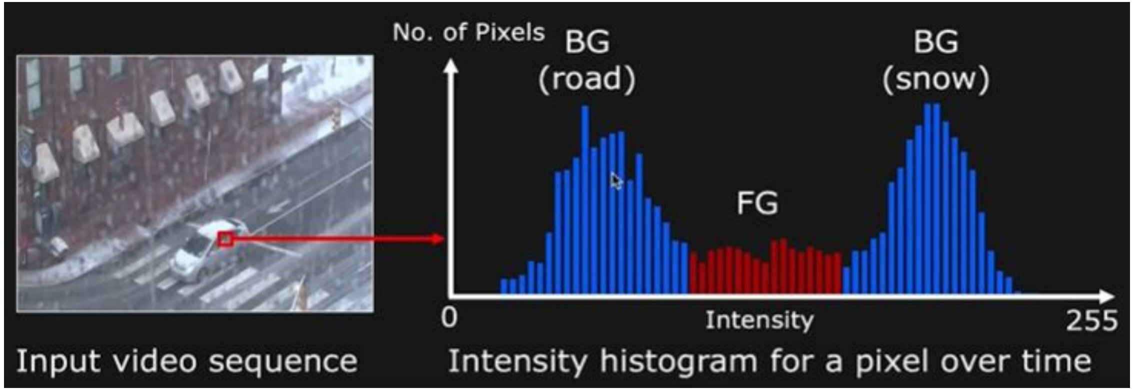
Gaussian Mixture Models: Detección de anomalías



LOF: Local outlier factor

Gaussian Mixture Models - Object Tracking





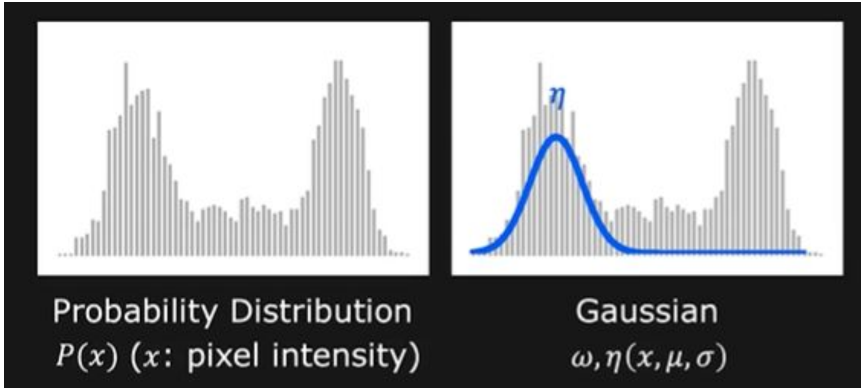
Gaussian Mixture Models - Object Tracking

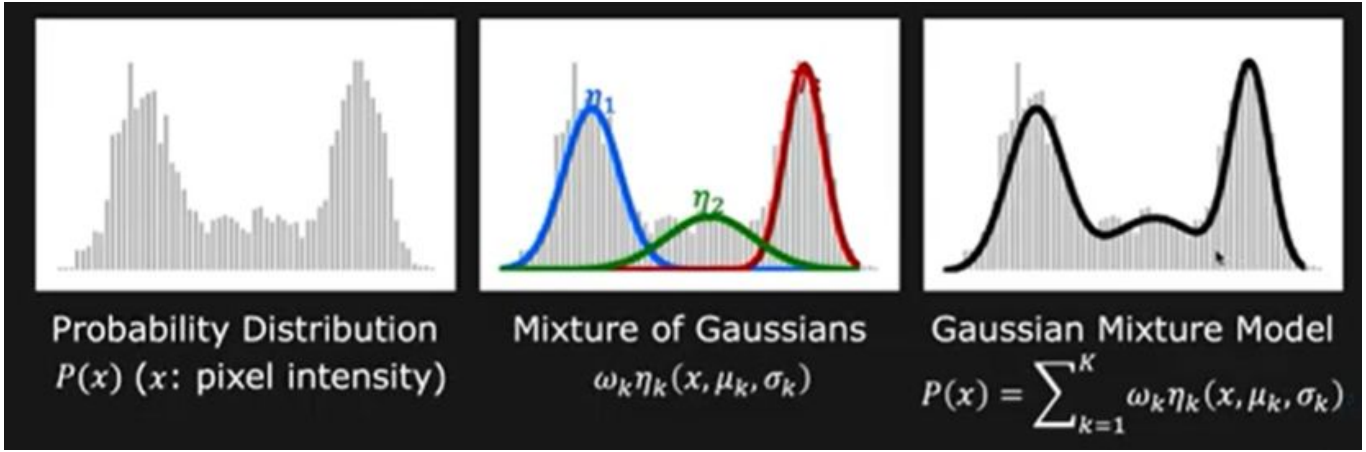
$$\boxed{P(\mathbf{X}) \cong \sum_{k=1}^{K} \omega_k \eta_k(\mathbf{X}, \boldsymbol{\mu}_k, \Sigma_k)} \quad \text{such that} \quad \sum_{k=1}^{K} \omega_k = 1$$

where: $$\boxed{\eta(\mathbf{X}, \boldsymbol{\mu}, \Sigma) = \frac{1}{(2\pi)^{D/2}|\Sigma|^{1/2}} e^{-\frac{1}{2}(\mathbf{X}-\boldsymbol{\mu})^T(\Sigma)^{-1}(\mathbf{X}-\boldsymbol{\mu})}}$$

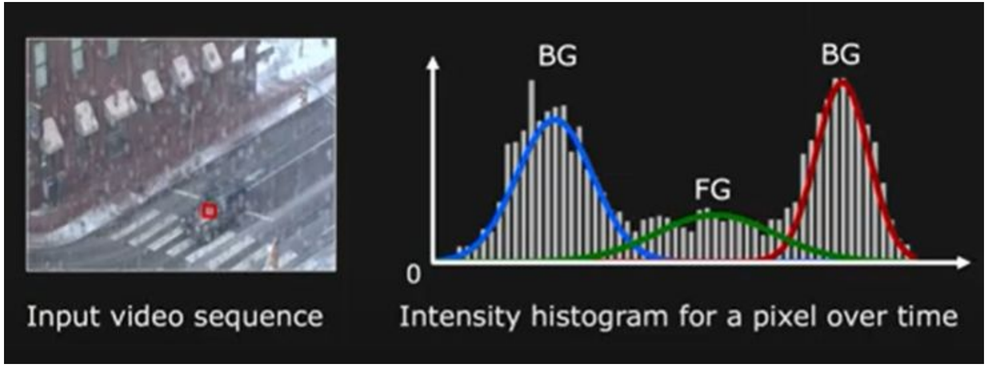
Mean $\boldsymbol{\mu} = \begin{bmatrix} \mu_r \\ \mu_g \\ \mu_b \end{bmatrix}$ Covariance matrix $\Sigma = \begin{bmatrix} \sigma^2 & 0 & 0 \\ 0 & \sigma^2 & 0 \\ 0 & 0 & \sigma^2 \end{bmatrix}$ (can be a full matrix)

Gaussian Mixture Models - Object Tracking





Gaussian Mixture Models - Object Tracking



$\uparrow \dfrac{\omega}{\sigma}$ **Background**

$\downarrow \dfrac{\omega}{\sigma}$ **Foreground**

Formulación

$$p(\boldsymbol{x}) = \sum_{k=1}^{K} \pi_k p_k(\boldsymbol{x})$$

$$0 \leqslant \pi_k \leqslant 1\,, \quad \sum_{k=1}^{K} \pi_k = 1\,,$$

Mixture Models - General

$$p(\boldsymbol{x} \mid \boldsymbol{\theta}) = \sum_{k=1}^{K} \pi_k \mathcal{N}(\boldsymbol{x} \mid \boldsymbol{\mu}_k,\, \boldsymbol{\Sigma}_k)$$

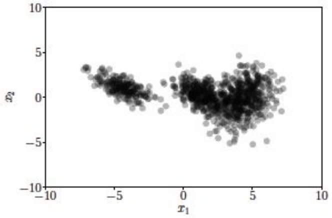
$$0 \leqslant \pi_k \leqslant 1\,, \quad \sum_{k=1}^{K} \pi_k = 1\,,$$

$$\boldsymbol{\theta} := \{\boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k, \pi_k : \; k = 1, \ldots, K\}$$

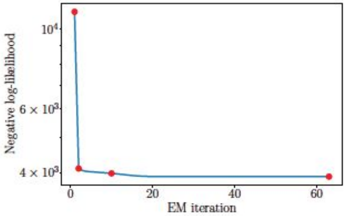
Gaussian Mixture Models

GMM y EM - JAMBOARD

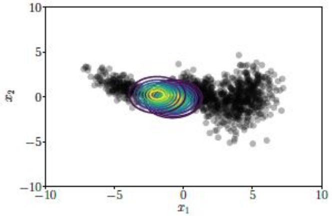
Gaussian Mixture Models - Teoría



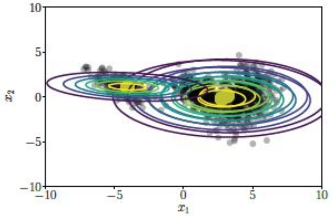
(a) Dataset.



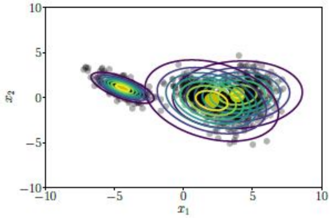
(b) Negative log-likelihood.



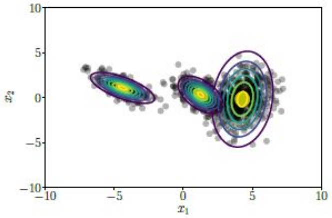
(c) EM initialization.



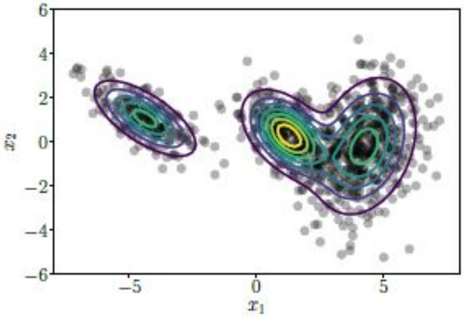
(d) EM after one iteration.



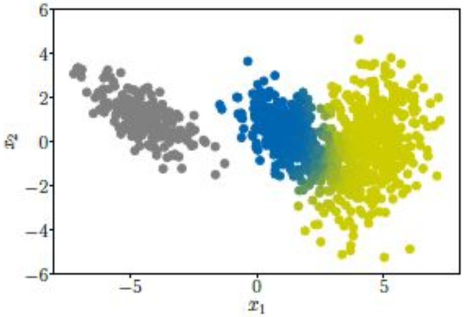
(e) EM after 10 iterations.



(f) EM after 62 iterations.



(a) GMM fit after 62 iterations.



(b) Dataset colored according to the responsibilities of the mixture components.

Notebooks

Bibliografía

- Mathematics for Machine Learning | Deisenroth, Faisal, Ong
- Pattern Recognition and Machine Learning | Bishop
- Gaussian Mixture Model | John McGonagle, Geoff Pilling, Andrei Dobre
- Expectation-Maximization Algorithms | Stanford CS229: Machine Learning
- First Principles of Computer Vision| Computer Science Department, School of Engineering and Applied Sciences, Columbia University