Kalman Filter for Air Quality

see also 'Optimum Linear Estimation' at https://www.sciencedirect.com/topics/social-sciences/kalman-filter (https://www.sciencedirect.com/topics/social-sciences/kalman-filter)

- Replaces a multiple linear regression (MLR) batch run, runs recursively on an Arduino Atmega1284P as adaptive filter for the MLR regression coefficients
- Kalman Filter is derived from https://github.com/zziz/kalman-filter (https://github.com/zziz/kalman-filter)
- For theory, please read https://en.wikipedia.org/wiki/Kalman_filter (https://en.wikipedia.org/wiki/Kalman_filter)
- Key is to set the covariance of the process noise matrix to a zero matrix! See hint in 'Optimum Linear Estimation' at https://www.sciencedirect.com/topics/social-sciences/kalman-filter (https://www.sciencedirect.com/topics/social-sciences/kalman-filter).
- I send a big thank you to those who provided these great basis contributions
- Click on the button 'Re-start the kernel, and then re-run the whole notebook' above

Basic Kalman Filter class from https://github.com/zziz/kalman-filter (https://github.com/zziz/kalman-filter):

In [1]:
```
class KalmanFilter(object):
    def __init__(self, F = None, B = None, H = None, Q = None, R = None, P = None, x0 = None

        if(F is None or H is None):
            raise ValueError("Set proper system dynamics.")

        self.n = F.shape[1]
        self.m = H.shape[1]

        self.F = F
        self.H = H
        self.B = 0 if B is None else B
        self.Q = np.eye(self.n) if Q is None else Q
        self.R = np.eye(self.n) if R is None else R
        self.P = np.eye(self.n) if P is None else P
        self.x = np.zeros((self.n, 1)) if x0 is None else x0

    def predict(self, u = 0):
        self.x = np.dot(self.F, self.x) + np.dot(self.B, u)             # Predicted (a prio
        self.P = np.dot(np.dot(self.F, self.P), self.F.T) + self.Q      # Predicted (a prio
        return self.x

    def update(self, z):
        y = z - np.dot(self.H, self.x)                                  # Innovation or meas
        S = self.R + np.dot(self.H, np.dot(self.P, self.H.T))           # Innovation (or pre
        #print("\nUpdate: self.H = ", self.H)
        #print("\nUpdate: self.P = ", self.P)
        #print("\nUpdate: self.R = ", self.R)
        K = np.dot(np.dot(self.P, self.H.T), np.linalg.inv(S))          # Optimal Kalman gai
        #print("\nUpdate: Kalman gain matrix K  = ", K)
        self.x = self.x + np.dot(K, y)
        I = np.eye(self.n)

        self.P = np.dot(np.dot(I - np.dot(K, self.H), self.P), (I - np.dot(K, self.H)).T) +
```

Read historian.csv (same input file as 'Multiple linear regression for BME680 gas readings of a single sensor.ipynb' is using

In [2]:
```
import warnings
warnings.simplefilter(action='ignore', category=FutureWarning)

import pandas as pd
from datetime import datetime

import numpy as np


dateparse = lambda x: pd.datetime.strptime(x, '%d.%m.%Y %H:%M:%S,%f')

df0 = pd.read_csv("historian.csv", sep=';', thousands=".", decimal=",", skiprows = [0,1,2],c

# print first 5 lines of the pandas dataframe

df0.head(19)

```

Out[2]:

	Datum	Mode	raw_gas_resistance	relative_humidity	temperature
0	2020-12-31 16:26:56.439	2	147800	36.9	23.278
1	2020-12-31 16:30:32.862	2	147800	36.9	22.800
2	2020-12-31 16:30:32.866	2	147800	35.6	22.800
3	2020-12-31 16:30:32.873	2	157780	35.6	22.800
4	2020-12-31 16:35:04.477	2	157780	35.6	22.600
5	2020-12-31 16:35:04.486	2	157780	36.5	22.600
6	2020-12-31 16:35:04.492	2	161480	36.5	22.600
7	2020-12-31 16:39:36.091	2	161480	36.5	22.700
8	2020-12-31 16:39:36.099	2	161480	36.8	22.700
9	2020-12-31 16:39:36.106	2	162460	36.8	22.700
10	2020-12-31 16:44:07.723	2	162460	36.8	22.800
11	2020-12-31 16:44:07.734	2	162460	37.2	22.800
12	2020-12-31 16:44:07.740	2	160520	37.2	22.800
13	2020-12-31 16:48:39.333	2	160520	37.2	22.900
14	2020-12-31 16:48:39.347	2	160520	37.5	22.900
15	2020-12-31 16:48:39.353	2	158000	37.5	22.900
16	2020-12-31 16:53:10.959	2	158000	37.5	22.900
17	2020-12-31 16:53:10.968	2	158000	37.8	22.900
18	2020-12-31 16:53:10.973	2	155280	37.8	22.900

In [3]:
```
# keep every 3rd row (CCU historian is tacking every change of a datapoint separately)
df = df0[(df0.index % 3 == 0)]

df.head(7)
```

Out[3]:

	Datum	Mode	raw_gas_resistance	relative_humidity	temperature
0	2020-12-31 16:26:56.439	2	147800	36.9	23.278
3	2020-12-31 16:30:32.873	2	157780	35.6	22.800
6	2020-12-31 16:35:04.492	2	161480	36.5	22.600
9	2020-12-31 16:39:36.106	2	162460	36.8	22.700
12	2020-12-31 16:44:07.740	2	160520	37.2	22.800
15	2020-12-31 16:48:39.353	2	158000	37.5	22.900
18	2020-12-31 16:53:10.973	2	155280	37.8	22.900

Print values of Pandas dataframe. Please x-check if they meet your expectation!

In [4]:
```
1 df.values
```

Out[4]:
```
array([[Timestamp('2020-12-31 16:26:56.439000'), 2, 147800, 36.9, 23.278],
       [Timestamp('2020-12-31 16:30:32.873000'), 2, 157780, 35.6, 22.8],
       [Timestamp('2020-12-31 16:35:04.492000'), 2, 161480, 36.5, 22.6],
       ...,
       [Timestamp('2021-01-14 16:15:55.504000'), 2, 209040, 26.8, 26.4],
       [Timestamp('2021-01-14 16:20:29.140000'), 2, 205100, 27.1, 26.1],
       [Timestamp('2021-01-14 16:25:02.895000'), 2, 201900, 26.4, 26.2]],
      dtype=object)
```

In [5]:
```
1 df0.head(-1)
```

Out[5]:

	Datum	Mode	raw_gas_resistance	relative_humidity	temperature
0	2020-12-31 16:26:56.439	2	147800	36.9	23.278
1	2020-12-31 16:30:32.862	2	147800	36.9	22.800
2	2020-12-31 16:30:32.866	2	147800	35.6	22.800
3	2020-12-31 16:30:32.873	2	157780	35.6	22.800
4	2020-12-31 16:35:04.477	2	157780	35.6	22.600
...	...	...	...	...	...
14018	2021-01-14 16:20:29.135	2	209040	27.1	26.100
14019	2021-01-14 16:20:29.140	2	205100	27.1	26.100
14020	2021-01-14 16:25:02.887	2	205100	27.1	26.200
14021	2021-01-14 16:25:02.889	2	205100	26.4	26.200
14022	2021-01-14 16:25:02.895	2	201900	26.4	26.200

14023 rows × 5 columns

In [6]:
```
1 df.head(-1)
```

Out[6]:

	Datum	Mode	raw_gas_resistance	relative_humidity	temperature
0	2020-12-31 16:26:56.439	2	147800	36.9	23.278
3	2020-12-31 16:30:32.873	2	157780	35.6	22.800
6	2020-12-31 16:35:04.492	2	161480	36.5	22.600
9	2020-12-31 16:39:36.106	2	162460	36.8	22.700
12	2020-12-31 16:44:07.740	2	160520	37.2	22.800
...	...	...	...	...	...
14007	2021-01-14 16:08:06.854	2	189640	27.8	26.100
14010	2021-01-14 16:09:02.681	2	213600	28.0	25.000
14013	2021-01-14 16:11:22.754	2	218780	26.9	25.500
14016	2021-01-14 16:15:55.504	2	209040	26.8	26.400
14019	2021-01-14 16:20:29.140	2	205100	27.1	26.100

4674 rows × 5 columns

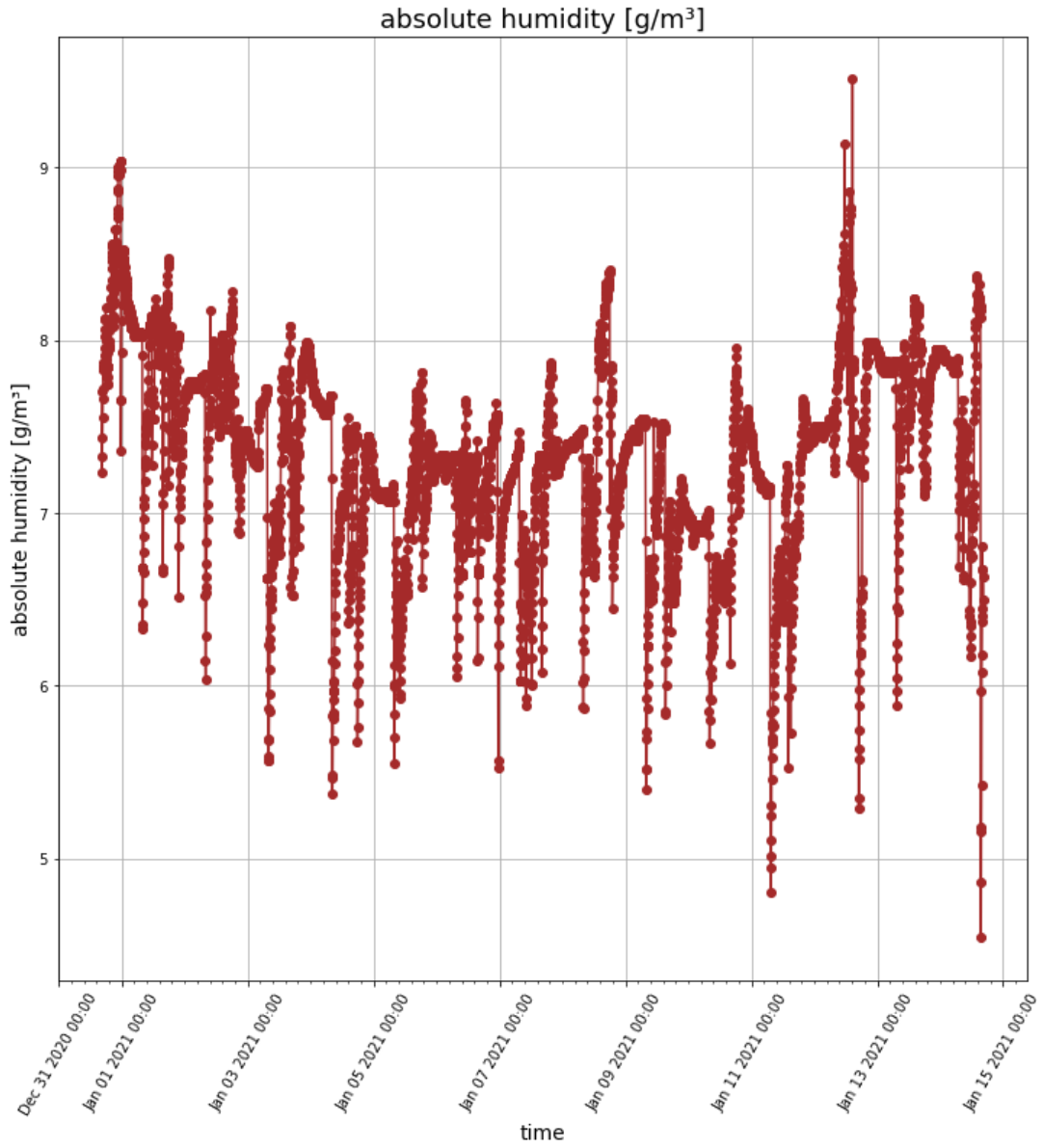
Formulas for calculating the absolute humidity

In [7]:

```
import numpy as np
# Create a function that  calculates the absolute humidity from the two arguments 'temperatu
# see for details https://www.kompf.de/weather/vent.html or  https://rechneronline.de/barome

a = 6.112
b = 17.67
c = 243.5

# Compute saturated water vapor pressure in hPa
# Param t - temperature in °C
def svp(t):
  svp = a * np.exp((b*t)/(c+t))
  return svp

# Compute actual water vapor pressure in hPa
# Param rh - relative humidity in %
# Param t - temperature in °C
def vp(rh, t):
  vp = rh/100. * svp(t)
  return vp

# Compute the absolute humidity in g/m³
# Param rh - relative humidity in %
# Param t - temperature in °C
def calculate_absolute_humidity(t, rh):
  mw = 18.016 # kg/kmol (Molekulargewicht des Wasserdampfes)
  rs = 8314.3 # J/(kmol*K) (universelle Gaskonstante)
  ah = 10**5 * mw/rs * vp(rh, t)/(t + 273.15)
  #return the absolute humidity in [g/m³]
  return ah


pd.set_option('mode.chained_assignment', None)
# now apply the above defined formulas to get the pandas dataframe column 'absolute_humidity
df['absolute_humidity'] = calculate_absolute_humidity(df['temperature'], df['relative_humidi
```

Plot the calculated absolute humidity

In [8]:
```
1  import matplotlib.pyplot as plt
2  import matplotlib.dates as mdates
3  from matplotlib.dates import DateFormatter
4  from matplotlib.ticker import (MultipleLocator, FormatStrFormatter, AutoMinorLocator)
5  fig, ax = plt.subplots(figsize=(12, 12))
6  plt.xticks(rotation=60)
7  ax.xaxis.set_major_formatter(DateFormatter('%b %d %Y %H:%M'))
8  ax.xaxis.set_minor_locator(AutoMinorLocator())
9  ax.plot_date(df['Datum'], df['absolute_humidity'], linestyle='solid', color='brown')
10 plt.title('absolute humidity [g/m³]', fontsize=18)
11 plt.xlabel('time', fontsize=14)
12 plt.ylabel('absolute humidity [g/m³]', fontsize=14)
13 plt.grid(True)
14 plt.show()
```



Print again the first 5 lines of the Pandas dataframe. Check if a column for the absolute humidity has been added.

In [9]:
```
1  df.head()
```

Out[9]:

	Datum	Mode	raw_gas_resistance	relative_humidity	temperature	absolute_humidity
0	2020-12-31 16:26:56.439	2	147800	36.9	23.278	7.704152
3	2020-12-31 16:30:32.873	2	157780	35.6	22.800	7.232304
6	2020-12-31 16:35:04.492	2	161480	36.5	22.600	7.330595
9	2020-12-31 16:39:36.106	2	162460	36.8	22.700	7.433362
12	2020-12-31 16:44:07.740	2	160520	37.2	22.800	7.557352

Create a subset of the measurement data: 'raw_gas_resistance','temperature','absolute_humidity'

In [10]:
```
my_observations = df[['raw_gas_resistance','temperature','absolute_humidity']]
my_observations.head()
```

Out[10]:

	raw_gas_resistance	temperature	absolute_humidity
0	147800	23.278	7.704152
3	157780	22.800	7.232304
6	161480	22.600	7.330595
9	162460	22.700	7.433362
12	160520	22.800	7.557352

Create a numpy array of measurements for further processing

In [11]:
```
list_of_rows = [list(row) for row in my_observations.values]
```

Print the first four elements of list of lists i.e. rows

In [12]:
```
print(list_of_rows[:4])
```

```
[[147800.0, 23.278, 7.70415152311762], [157780.0, 22.8, 7.232304207423894], [161480.0, 22.6,
7.330594890416396], [162460.0, 22.7, 7.433362386991288]]
```

Convert the selection of measurements to a numpy array

In [13]:
```
np.array(list_of_rows)
measurements = np.array(list_of_rows)
print("number of measurement datapoints = ", len(measurements))
```

```
number of measurement datapoints =  4675
```

Set the parameters of the Kalman filter

- Kalman filter with a zero covariance matrix for the process noise is well known a the recursive minimum least-square error (LMMSE) filter for a linear system with some assumptions on auto- and cross-correlations of process and measurement noise and initial state.
- observation vector y : [raw_gas_resistance]; n=1; note: 'temperature' and 'aH' are NOT part of the observation vector!
- system state vector X : [VOC_resistance, alpha_temperature, beta_ah, delta_intercept]; m=4
- state transition matrix F : identity matrix (m, n)
- observation transition matrix H : initial identidy matrix (1,m); then set to state dependant
- covariance matrix of the process noise Q : zero matrix (m,m)
- covariance matrix of the observation noise R : matrix(1,1) with very small value

In [14]:
```
F = np.eye(4)
H = np.array([ [1, 1, 1, 1] ]).reshape(1, 4)
# key ist to set Q to a zero matrix, in this case the Kalman filter works an ordinary least
Q = np.array([ [0, 0, 0, 0], [0, 0, 0, 0], [0, 0, 0, 0], [0, 0, 0, 0] ]).reshape(4, 4)
# set covariance of gast resistance measurements also to a very small value
R = np.array([ [0.0001] ]).reshape(1, 1)

print("\nF = ",F)  # the state-transition model;
print("\nInitial H = ",H)  # the observation model;
print("\nQ = ",Q)  # covariance of the process noise
print("\nR = ",R)  # covariance of the observation noise

```

```
F =  [[1. 0. 0. 0.]
 [0. 1. 0. 0.]
 [0. 0. 1. 0.]
 [0. 0. 0. 1.]]

Initial H =  [[1 1 1 1]]

Q =  [[0 0 0 0]
 [0 0 0 0]
 [0 0 0 0]
 [0 0 0 0]]

R =  [[0.0001]]
```

Initialize the Kalman filter

In [15]:
```
kf = KalmanFilter(F = F, H = H, Q = Q, R = R)
predictions = []
compensated_gas_resistance=[]
states=[]

#print("raw gas resistance measurements =", measurements[:,0])

print("dim measurements : ", measurements.shape)

last_index = len(measurements)

print ("last index of measurement array = ", last_index)


```

```
dim measurements :  (4675, 3)
last index of measurement array =  4675
```

Run the Kalman filter

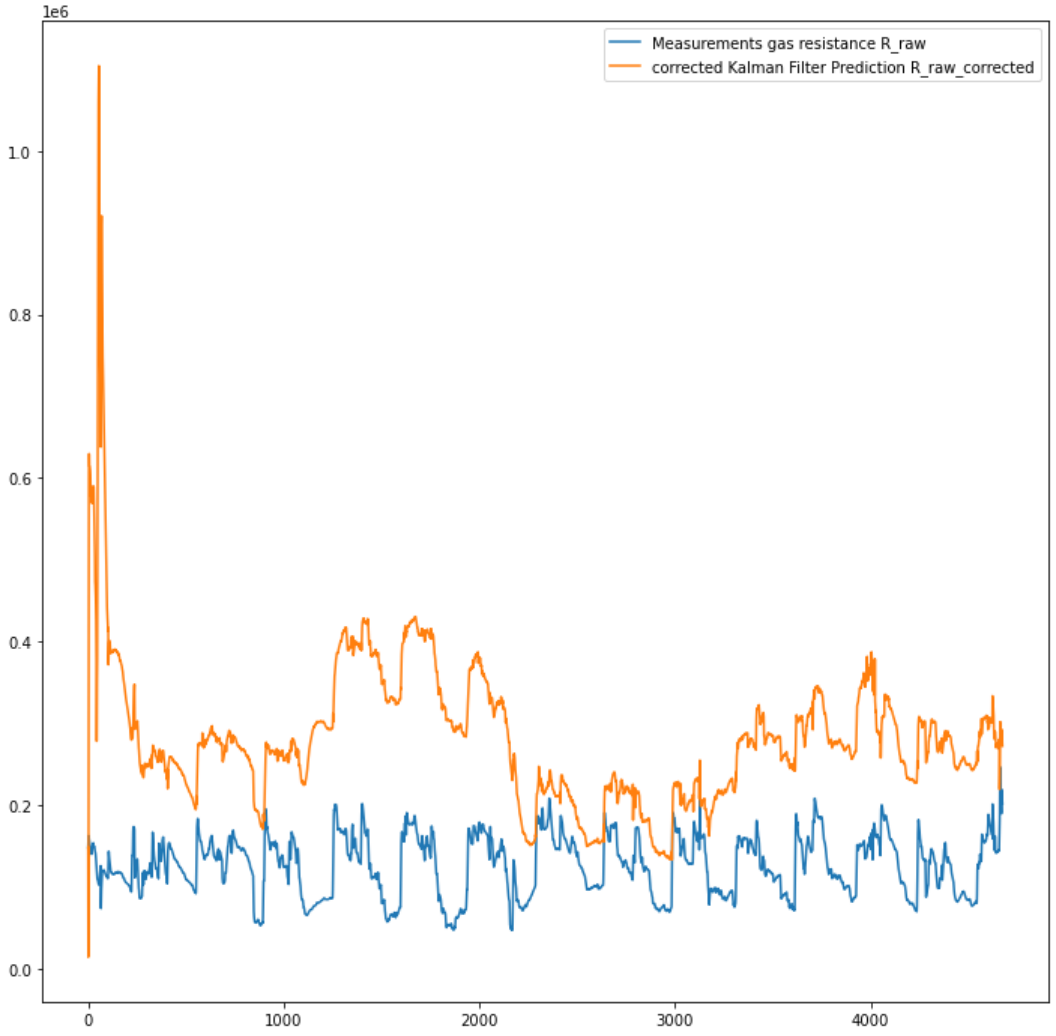
In [16]:
```
it = 0  # iteration index
#print("\nState vector kf.x= ", kf.x)
for z in measurements:
    zg = z[0] # raw_gas_resistance
    # make observation model matrix state dependant
    H = np.array([[1, z[1], z[2], 1]]).reshape(1, 4)
    # z[1]: measured temperature
    # z[2]: calculated absolute humidity ah(T, rH)
    # estimated state vector x:
    # x[0]: estimated VOC resistance
    # x[1]: estimated regression coefficient for T temperature dependency
    # x[2]: estimated regression coefficient for aH aboslute humidity dependency
    # x[3]: estimated intercept of linear regression
    kf.H = H
    it = it + 1
    #print("\nState vector kf.x= ", kf.x)
    #print results for the last sample of the measurement sequence
    if ((it == last_index)):  # print results of last measurement index
        print ("\nIteration index = ", it)
        print ("\n")
        print("\nState vector kf.x= ", kf.x)
        print("\nObservation vector z = ", z)
        print("\nObservation transition matrix kf.H = ", kf.H)
        print("\nKalman filter prediction = ", kf.predict())
        print("\nKalman filter update = ",np.dot(H,  kf.predict()))
        print ("\n\n")
    predictions.append(np.dot(H,  kf.predict()))

    compensated_gas_resistance.append(zg-kf.predict()[1,0]*z[1]-kf.predict()[2,0]*z[2])
    #compensated_gas_resistance.append(-kf.predict()[1,0]*z[1]-kf.predict()[2,0]*z[2])
    #compensated_gas_resistance.append(-kf.predict()[1,0]*z[1])
    #rint("\nraw gas resistance                = ",zg)
    #print("\ntemperature coefficent prediction = ",kf.predict()[1,0])
    #print("\ntemperature                        = ",z[1])
    #print("\ntemperature compensation           = ",-kf.predict()[1,0]*z[1])
    #print("\nhumidity coefficent prediction     = ",kf.predict()[2,0])
    #print("\nabsolute humidity                  = ",z[2])
    #print("\nhumidity compensation              = ",-kf.predict()[2,0]*z[2])
    #print("\nKalman state prediction           = ",kf.predict())
    #print("\ntmperature coefficent prediction = ",kf.predict()[1,0])
    #print("\ncompensated gas resistance       = ",zg-kf.predict()[1,0]*z[1]-kf.predict()[2,
    states.append(kf.x)
    kf.update(zg)  #only zg raw_gas_resistance is an observation variable!
```

```
Iteration index =  4675



State vector kf.x=  [[121968.07677364]
 [  6567.30461868]
 [-37295.71424912]
 [121968.07625382]]

Observation vector z =  [2.01900000e+05 2.62000000e+01 6.50024362e+00]

Observation transition matrix kf.H =  [[ 1.         26.2         6.50024362  1.        ]]

Kalman filter prediction =  [[121968.07677364]
 [  6567.30461868]
 [-37295.71424912]
 [121968.07625382]]

Kalman filter update =  [[173568.30527887]]
```

Plot the results of the Kalman filter

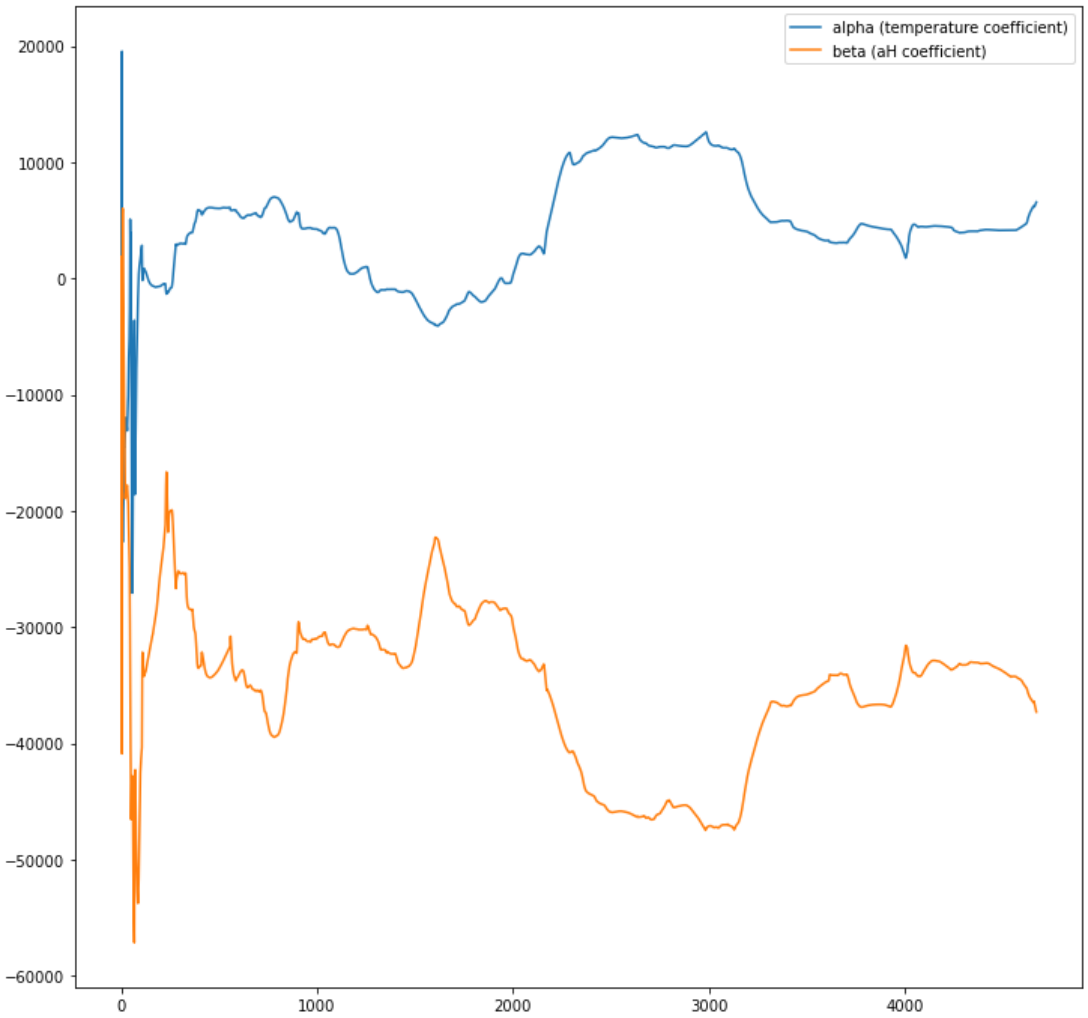
Plot measured gas resistance versus corrected gas resistance (compensation of temperature and humidity interference)

In [17]:
```
1 import matplotlib.pyplot as plt
2 fig, ax = plt.subplots(figsize=(12, 12))
3 ax.plot(range(len(measurements)), measurements[:,0], label = 'Measurements gas resistance R_
4 ax.plot(range(len(predictions)), compensated_gas_resistance[:], label = 'corrected Kalman Fi
5 ax.legend()
6 plt.show()
```



Plot alpha (temperature coefficient) and beta (aH coefficient) regression coefficients

In [18]:
```
import matplotlib.pyplot as plt
fig, ax = plt.subplots(figsize=(12, 12))
ax.plot(range(len(predictions)), np.array(states)[:,1], label = 'alpha (temperature coeffici
ax.plot(range(len(predictions)), np.array(states)[:,2], label = 'beta (aH coefficient)')
ax.legend()
plt.show()
```



Regression results of the recursive minimum least-square error (LMMSE) Kalman filter

In [19]:
```
print("\nNumber of captured data points used for online regression using Kalman filter  = %1

print("\n\nLinear regression coefficient of temperature interference alpha_LMMSE          =
print("Linear regression coefficient of absolute humidity interference beta_LMMSE     = %11.
print("\n\n")
```

```
Number of captured data points used for online regression using Kalman filter  =        4675


Linear regression coefficient of temperature interference alpha_LMMSE          =    6603.555
Linear regression coefficient of absolute humidity interference beta_LMMSE     =  -37355.402
```

You are done! Congratulations!