Jupyter notebook for proving the Kalman filter online regression

First it creates synthesized raw gas resistance, temperature, relative humidity time series with selectable parameters and linear dependencies:

- gas_resistance_compensated = f1(g0, dg, f_g0); f1=g0+dg*np.power(np.sin(df['index']f_g02*math.pi),3)
- temperature = f2(T0, dT, f_T0); f2=T0+dT*np.power(np.cos(df['index']f_T02*math.pi),5)
- relative_humidity = f3(rH0, dH, f_rH0); f3=rH0+dH*np.power(np.sin(df['index']f_rH02*math.pi),7)

The synthesized gas_resistance_raw is then calculated by the following linear equation:

- gas_resistance_raw = gas_resistance_compensated + alpha * temperature + beta * relative_humidity

In Python Pandas syntax this translates into

df["gas_resistance_raw"] = df["gas_resistance_compensated"] + alpha*df["temperature"] + beta*df["relative_humidity"]

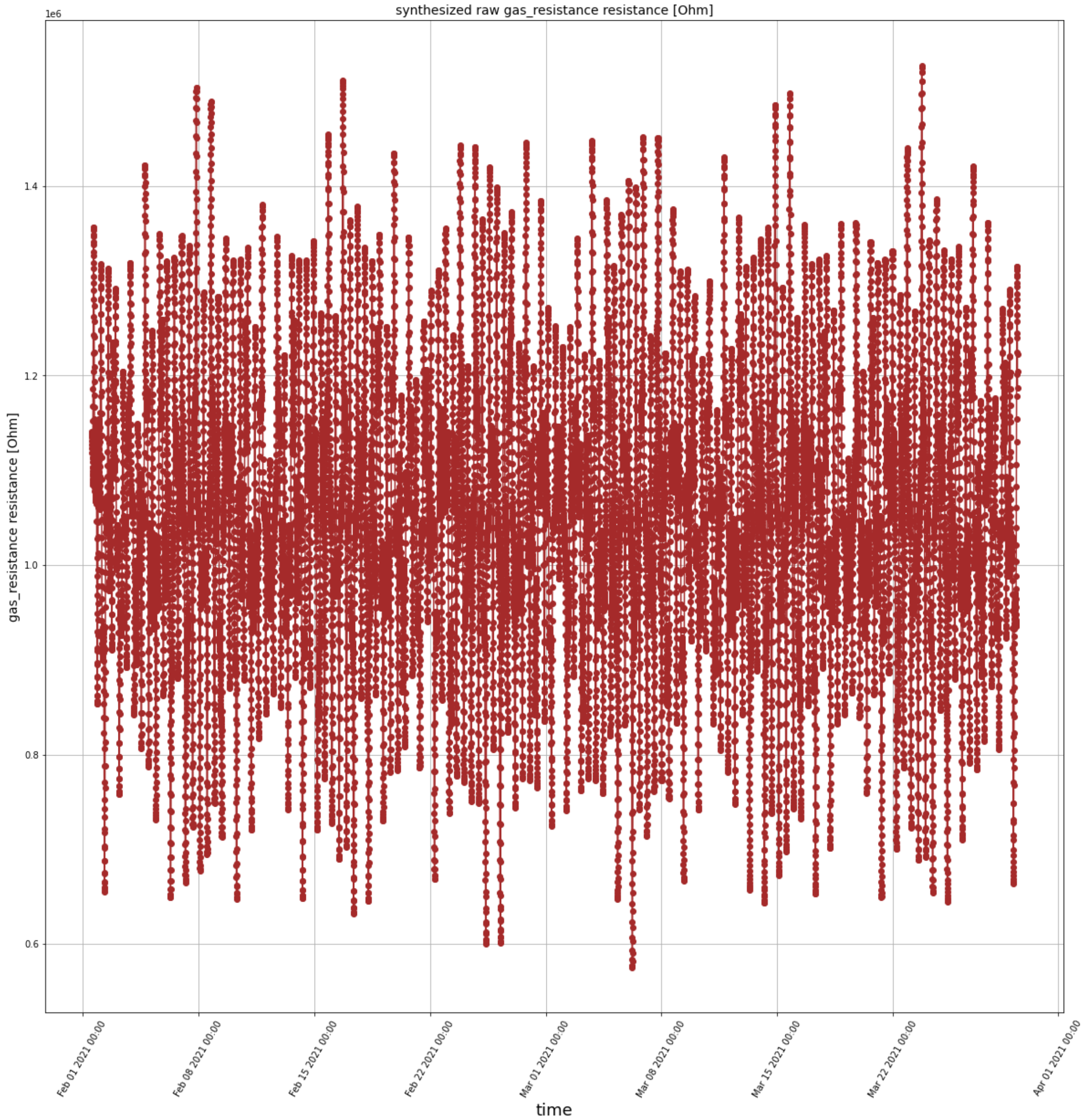
The synthesized triple ['gas_resistance_raw','temperature','relative_humidity'] is then fed into the Kalman filter The Kalman filter is set up in such a way that is executing an online linear regression for estimation of the coefficients alpha and beta.

Please check at the 'Results' section at the bottom of the notebook the estimated alpha and beta coefficients.

There may be a small deviating between defined and estimated regression coefficients if the number of ventilation cycles is quite small, e.g. few days only.

In [1]:

```
import numpy as np
import matplotlib.pyplot as plt
from datetime import datetime
import pandas as pd
import math
import matplotlib.dates as mdates
from matplotlib.dates import DateFormatter
from matplotlib.ticker import (MultipleLocator, FormatStrFormatter,
                               AutoMinorLocator)
from pandas.plotting import register_matplotlib_converters

register_matplotlib_converters()

number_of_days   = 56 # duration of the synthesized data set used for the online regi
time_step        = 4  # in minutes; sampling time of sensor

number_of_points = int(number_of_days * 24 * 60 / time_step) + 1  # cxalculated; do n

g0               = 250000          # base value of gas_resistance_raw
dg               = g0/2            # modulation of gas_resistance_raw
f_g0             = 1               # frequency of gas_resistance_raw in days

T0               = 23              # base value of temperature
dT               = 4.567           # modulation of temperature
f_T0             = 2.37            # frequency of temperature in days

rH0              = 48.5            # base value of relative_humidity
dH               = 40.5            # modulation of relative_humidity
f_rH0            = 2.2557          # frequency of relative_humidity in days

                                   # range of relative humidity should be 0..100


# change the cooefficients 'alpha' and 'beta' here and check at the end of the notebo
alpha            = 20345            # linear dependency coefficient of temperature
beta             = 6800.56          # linear dependency coefficient of relative_humi

fig, ax = plt.subplots(figsize=(20, 20))
plt.xticks(rotation=60)
ax.xaxis.set_major_formatter(DateFormatter('%b %d %Y %H:%M'))

# you may choose another start date in 'pd.date_range('2021-02-01 12:59:59.50', peric

start_date  = '2021-02-01 12:59:59.50'

df=pd.DataFrame({"time"                       : pd.date_range(start_date, periods=numb
                 "index"                      : np.linspace(0,number_of_days, num=numb

df["index"] = df["index"].astype(np.float64)

df=pd.DataFrame({"time"                       : pd.date_range(start_date, periods=numb
                 "index"                      : np.linspace(0,number_of_days, num=numb
                 "gas_resistance_compensated" : g0+dg*np.power(np.sin(df['index']*f_g0
                 "temperature"                : T0+dT*np.power(np.cos(df['index']*f_T0
                 "relative_humidity"          : rH0+dH*np.power(np.sin(df['index']*f_r

df["gas_resistance_raw"] =  df["gas_resistance_compensated"] + alpha*df["temperature"

ax.plot_date(df['time'], df['gas_resistance_raw'], linestyle='solid', color='brown')
plt.title('synthesized raw gas_resistance resistance [Ohm]', fontsize=14)
plt.xlabel('time', fontsize=18)
plt.ylabel('gas_resistance resistance [Ohm]', fontsize=14)
plt.grid(True)

plt.show()
```



Multilinear Regression (MLR) for comparison

In [2]:

```python
from sklearn import linear_model
import statsmodels.api as sm

X = df[['temperature','relative_humidity']] # here we have 2 variables for multiple
Y = df['gas_resistance_raw']

# with sklearn
regr = linear_model.LinearRegression()
regr.fit(X, Y)

print('Intercept: \n', regr.intercept_)
print('Coefficients: \n', regr.coef_)

X = sm.add_constant(X)
model = sm.OLS(Y, X).fit()
predictions = model.predict(X)

print_model = model.summary()
print(print_model)
print(model.rsquared)

print("\n\n\nResults of multilinear regression (MLR):\n")
print("\n\nset temperature coefficent 'alpha' of synthesis              = %11.1lf" %
print("\ntemperature coefficent 'alpha' of MLR prediction             = %11.1lf" %
print("\nprediction error of MLR temperature coefficent 'alpha'       = %11.2lf %%"
 
print("\n\nset relative humidity coefficent 'beta' of synthesis         = %11.1lf" %
print("\nrelative humidity coefficent 'beta' of MLR prediction        = %11.1lf" %
print("\nprediction error of MLR relative humditiy coefficent 'beta'  = %11.2lf %%"
print("\n")
```

```
Intercept: 
 251662.3805782711
Coefficients: 
 [20257.05460028  6807.97867921]
                            OLS Regression Results                            
==============================================================================
Dep. Variable:     gas_resistance_raw   R-squared:                       0.784
Model:                            OLS   Adj. R-squared:                  0.783
Method:                 Least Squares   F-statistic:                 3.648e+04
Date:                Sat, 03 Jun 2023   Prob (F-statistic):               0.00
Time:                        15:07:10   Log-Likelihood:            -2.5349e+05
No. Observations:               20161   AIC:                         5.070e+05
Df Residuals:                   20158   BIC:                         5.070e+05
Df Model:                           2                                         
Covariance Type:            nonrobust                                         
=====================================================================================
                        coef    std err          t      P>|t|      [0.025      0.975]
-------------------------------------------------------------------------------------
const              2.517e+05   5225.061     48.164      0.000    2.41e+05    2.62e+05
temperature        2.026e+04    217.315     93.215      0.000    1.98e+04    2.07e+04
relative_humidity  6807.9787     26.549    256.432      0.000    6755.941    6860.017
==============================================================================
Omnibus:                     1013.506   Durbin-Watson:                   0.001
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              399.825
Skew:                           0.000   Prob(JB):                     1.51e-87
Kurtosis:                       2.310   Cond. No.                         598.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specif
ied.
0.7835191814614665



Results of multilinear regression (MLR):



set temperature coefficent 'alpha' of synthesis              =     20345.0

temperature coefficent 'alpha' of MLR prediction             =     20257.1
```

```
prediction error of MLR temperature coefficent 'alpha'        =        0.43 %

set relative humidity coefficent 'beta' of synthesis          =      6800.6

relative humidity coefficent 'beta' of MLR prediction         =      6808.0

prediction error of MLR relative humditiy coefficent 'beta'   =       -0.11 %
```

Please check whether the R-squared (uncentered) of the multiple linear regression above is sufficiently good (should be > 0.7): R-squared (also called coefficient of determination) is the portion of variance in the dependent variables that can be explained by the independent variables. Hence, as a rule of thumb for interpreting the strength of a relationship based on its R-squared value is:

```
if R-squared value < 0.3 this value is generally considered as None or very weak
effect size
if R-squared value 0.3 < r < 0.5 this value is generally considered as weak or lo
w effect size
if R-squared value 0.5 < r < 0.7 this value is generally considered as moderate e
ffect size
if R-squared value 0.7 < r < 1.0 this value is generally considered as strong eff
ect size
```

If R-squared value is < 0.3, the collected history may be too short. Please try to collect datapoints for a longer timeframe!

In [3]:
```
print("\n\nR-squared (uncentered) of the multiple linear regression       = %11.2lf\r
```

```
R-squared (uncentered) of the multiple linear regression       =        0.78
```

Kalman Filter

In [4]:

```
class KalmanFilter(object):
    def __init__(self, F = None, B = None, H = None, Q = None, R = None, P = None, x0

        if(F is None or H is None):
            raise ValueError("Set proper system dynamics.")

        self.n = F.shape[1]
        self.m = H.shape[1]

        self.F = F
        self.H = H
        self.B = 0 if B is None else B
        self.Q = np.eye(self.n) if Q is None else Q
        self.R = np.eye(self.n) if R is None else R
        self.P = np.eye(self.n) if P is None else P
        self.x = np.zeros((self.n, 1)) if x0 is None else x0

    def predict(self, u = 0):
        self.x = np.dot(self.F, self.x) + np.dot(self.B, u)             # Predicted (
        self.P = np.dot(np.dot(self.F, self.P), self.F.T) + self.Q      # Predicted (
        return self.x

    def update(self, z):
        y = z - np.dot(self.H, self.x)                                  # Innovation
        S = self.R + np.dot(self.H, np.dot(self.P, self.H.T))           # Innovation
        #print("\nUpdate: self.H = ", self.H)
        #print("\nUpdate: self.P = ", self.P)
        #print("\nUpdate: self.R = ", self.R)
        K = np.dot(np.dot(self.P, self.H.T), np.linalg.inv(S))          # Optimal Kal
        #print("\nUpdate: Kalman gain matrix K  = ", K)
        self.x = self.x + np.dot(K, y)
        I = np.eye(self.n)

        self.P = np.dot(np.dot(I - np.dot(K, self.H), self.P), (I - np.dot(K, self.H)

```

In [5]:

```
my_observations = df[['gas_resistance_raw','temperature','relative_humidity']]
my_observations.head()
```

Out[5]:

	gas_resistance_raw	temperature	relative_humidity
0	1.140678e+06	27.567000	48.500000
1	1.140282e+06	27.547501	48.500000
2	1.139105e+06	27.489435	48.500001
3	1.137178e+06	27.394087	48.500013
4	1.134547e+06	27.263548	48.500094

In [6]:

```
list_of_rows = [list(row) for row in my_observations.values]
print(list_of_rows[:4])
```

```
[[1140677.775, 27.567, 48.5], [1140281.727597354, 27.547500766092345, 48.5000000059262
6], [1139105.0396710136, 27.489435301132655, 48.500000754456146], [1137177.8588636708,
27.39408666132805, 48.50001277448958]]
```

In [7]:

```
np.array(list_of_rows)
measurements = np.array(list_of_rows)
print("number of measurement datapoints = ", len(measurements))
```

```
number of measurement datapoints =  20161
```

In [8]:

```
F = np.eye(3)
H = np.array([ [1, 1, 1] ]).reshape(1, 3)
# key ist to set Q to a zero matrix, in this case the Kalman filter works as an ordi
q0 = 0.0
Q = np.array([ [q0, q0, q0], [q0, q0, q0], [q0, q0, q0] ]).reshape(3, 3)
# set covariance of gas_resistancet resistance measurements also to a very small valu
R = np.array([ [0.0001] ]).reshape(1, 1)

print("\nF = ",F)  # the state-transition model;
print("\nInitial H = ",H)  # the observation model;
print("\nQ = ",Q)  # covariance of the process noise
print("\nR = ",R)  # covariance of the observation noise
```

```
F =  [[1. 0. 0.]
 [0. 1. 0.]
 [0. 0. 1.]]

Initial H =  [[1 1 1]]

Q =  [[0. 0. 0.]
 [0. 0. 0.]
 [0. 0. 0.]]

R =  [[0.0001]]
```

In [9]:

```
kf = KalmanFilter(F = F, H = H, Q = Q, R = R)
predictions = []
raw_gas=[]
compensated_gas_resistance_resistance=[]
states=[]

#print("raw gas_resistance resistance measurements =", measurements[:,0])

print("dim measurements : ", measurements.shape)

last_index = len(measurements)

print ("last index of measurement array = ", last_index)
```

```
dim measurements :  (20161, 3)
last index of measurement array =  20161
```

In [10]:

```
it = 0  # iteration index
#print("\nState vector kf.x= ", kf.x)
for z in measurements:
    zg = z[0] # raw_gas_resistance_resistance
    raw_gas.append(zg)
    # make observation model matrix state dependant
    H = np.array([[1, z[1], z[2]]]).reshape(1, 3)
    # z[1]: measured temperature T
    # z[2]: measured relatuive humidity rH
    # estimated state vector x:
    # x[0]: estimated VOC resistance
    # x[1]: estimated regression coefficient for T temperatureerature dependency
    # x[2]: estimated regression coefficient for relative_humidity relative humidity
    kf.H = H
    it = it + 1
    #print("\nState vector kf.x= ", kf.x)
    #print results for the last sample of the measurement sequence
    if ((it == 1) or (it == last_index)):  # print results of first and last measuren
        print ("\nIteration index = ", it)
        print ("\n")
        print("\nState vector kf.x= ", kf.x)
        print("\nObservation vector z = ", z)
        print("\nObservation transition matrix kf.H = ", kf.H)
        print("\nKalman filter prediction = ", kf.predict())
        print("\nKalman filter update = ",np.dot(H,  kf.predict()))
        print("\nraw gas = ",zg)
        print ("\n\n")
    predictions.append(np.dot(H,  kf.predict()))

    compensated_gas_resistance_resistance.append(zg-kf.predict()[1,0]*z[1]-kf.predict
    #compensatedensated_gas_resistance_resistance.append(-kf.predict()[1,0]*z[1]-kf.p
    #compensatedensated_gas_resistance_resistance.append(-kf.predict()[1,0]*z[1])
    #rint("\nraw gas_resistance resistance                = ",zg)
    #print("\ntemperatureerature coefficent prediction = ",kf.predict()[1,0])
    #print("\ntemperatureerature                           = ",z[1])
    #print("\ntemperatureerature compensatedensation          = ",-kf.predict()[1,0]*
    #print("\nhumidity coefficent prediction    = ",kf.predict()[2,0])
    #print("\nrelative humidity                 = ",z[2])
    #print("\nhumidity compensatedensation             = ",-kf.predict()[2,0]*z[2])
    #print("\nKalman state prediction          = ",kf.predict())
    #print("\ntemperature coefficent prediction = ",kf.predict()[1,0])
    #print("\ncompensatedensated gas_resistance resistance       = ",zg-kf.predict()
    states.append(kf.x)
    kf.update(zg)  #only zg raw_gas_resistance_resistance is an observation variable.
```

```
Iteration index =  1



State vector kf.x=  [[0.]
 [0.]
 [0.]]

Observation vector z =  [1.14067777e+06 2.75670000e+01 4.85000000e+01]

Observation transition matrix kf.H =  [[ 1.    27.567 48.5  ]]

Kalman filter prediction =  [[0.]
 [0.]
 [0.]]

Kalman filter update =  [[0.]]

raw gas =  1140677.775




Iteration index =  20161



State vector kf.x=  [[251662.22341898]
 [ 20257.06034695]
```

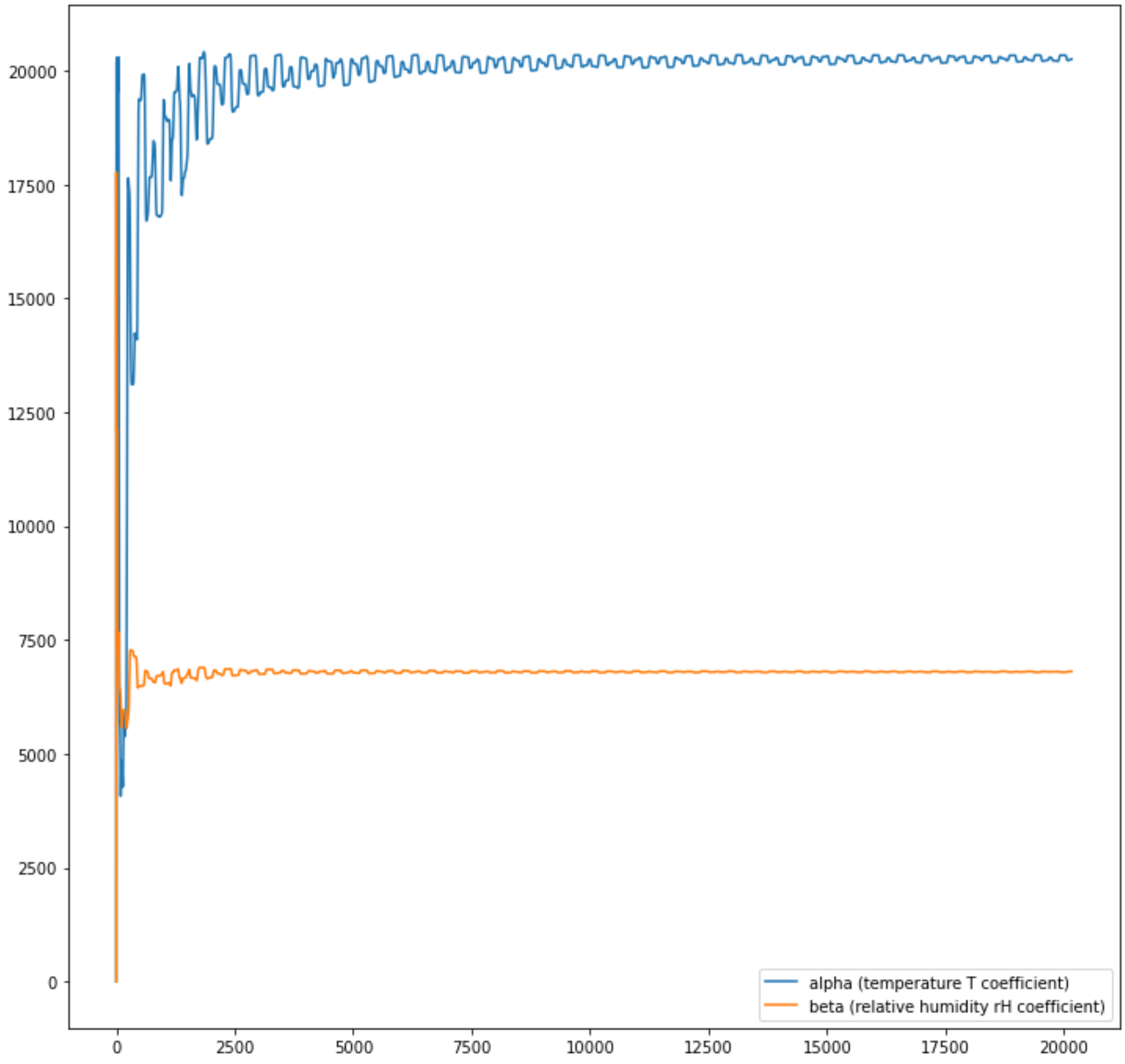
```
 [  6807.97932307]]

Observation vector z =  [1.18676880e+06 2.29989450e+01 6.89436258e+01]

Observation transition matrix kf.H =  [[ 1.         22.99894498 68.94362583]]

Kalman filter prediction =  [[251662.22341898]
 [ 20257.06034695]
 [  6807.97932307]]

Kalman filter update =  [[1186920.01880196]]

raw gas =  1186768.7995909627
```

Results

```
## Plot alpha (temperature T coefficient) and beta (relative humidity rH
coefficient) regression coefficients
```

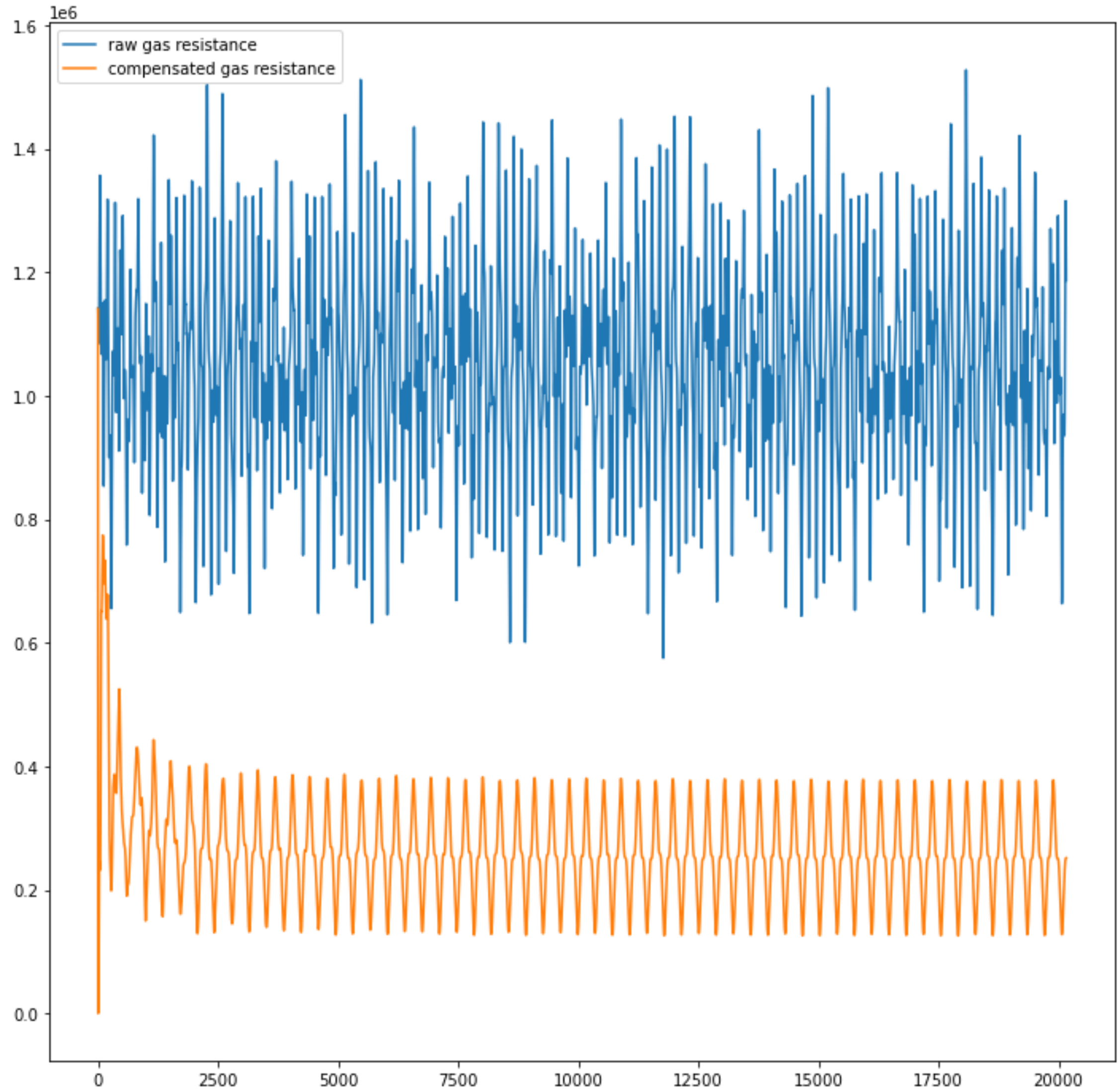
In [11]:

```
import matplotlib.pyplot as plt
fig, ax = plt.subplots(figsize=(12, 12))
ax.plot(range(len(predictions)), np.array(states)[:,1], label = 'alpha (temperature T
ax.plot(range(len(predictions)), np.array(states)[:,2], label = 'beta (relative humid
ax.legend()
plt.show()
```



Plot compenasted gas resistance

In [12]:
```
import matplotlib.pyplot as plt
fig, ax = plt.subplots(figsize=(12, 12))
ax.plot(range(len(raw_gas)), np.array(raw_gas), label = 'raw gas resistance')
ax.plot(range(len(compensated_gas_resistance_resistance)), np.array(compensated_gas_r
ax.legend()
plt.show()
```



Summary of results

In [13]:
```
print("number of days of synthesized data = ", number_of_days)
print("number of measurement datapoints   = ", len(measurements))
```

```
number of days of synthesized data =  56
number of measurement datapoints   =  20161
```

Online regression with Kalman filter

In [14]:

```
print("\n\n\nResults of an online regression using a Kalman filter:\n")
print("\n\nset temperature coefficent 'alpha'                         = %11.1lf" % alp
print("\ntemperature coefficent 'alpha' prediction                  = %11.1lf" % kf.p
print("\nprediction error of temperature coefficent 'alpha'         = %11.2lf %%" % ((
```

```
Results of an online regression using a Kalman filter:



set temperature coefficent 'alpha'                         =     20345.0

temperature coefficent 'alpha' prediction                  =     20257.1

prediction error of temperature coefficent 'alpha'         =        0.43 %
```

In [15]:

```
print("\n\nset relative_humidity coefficent 'beta'                    = %11.1lf" % bet
print("\nrelative_humidity coefficent 'beta' prediction             = %11.1lf" % kf.p
print("\nprediction error of relative_humidity coefficent 'beta'    = %11.2lf %%" % ((
```

```
set relative_humidity coefficent 'beta'                    =      6800.6

relative_humidity coefficent 'beta' prediction             =      6808.0

prediction error of relative_humidity coefficent 'beta'    =       -0.11 %
```

Classical multilinear regression (see above)

In [16]:

```
print("\n\n\nResults of multilinear regression (MLR):\n")
print("\n\nset temperature coefficent 'alpha' of synthesis                = %11.1lf" %
print("\ntemperature coefficent 'alpha' of MLR prediction              = %11.1lf" %
print("\nprediction error of MLR temperature coefficent 'alpha'        = %11.2lf %%"

print("\n\nset relative humidity coefficent 'beta' of synthesis           = %11.1lf" %
print("\nrelative humidity coefficent 'beta' of MLR prediction         = %11.1lf" %
print("\nprediction error of MLR relative humditiy coefficent 'beta'    = %11.2lf %%"
print("\n")
```

```
Results of multilinear regression (MLR):



set temperature coefficent 'alpha' of synthesis                =     20345.0

temperature coefficent 'alpha' of MLR prediction               =     20257.1

prediction error of MLR temperature coefficent 'alpha'         =        0.43 %


set relative humidity coefficent 'beta' of synthesis           =      6800.6

relative humidity coefficent 'beta' of MLR prediction          =      6808.0

prediction error of MLR relative humditiy coefficent 'beta'    =       -0.11 %
```

Outcomes:

1. The online regression with Kalman filter and the offline multilinear regression (MLR) are resulting in identical regression coefficients for the same synthesized data set
2. Both methods are quite accurately predicting the synthesized parameters if data squence is long enough, i.e. several (>>14) days
3. The Kalman filter can easily be realized in a micro controller since it requires to store just one state in the RAM

memory

Please play with the parameter 'number_of_days' above

Increase set value of 'number_of_days above' and check the influence on the accuracy of the estimation.

- We can see that we need at least 14 days in order to get a reasonably low error of the estimation.
- After 4 days the estimation error is still significantly high!
- The estimation accuracy will improve when the Kalman filter will run for a longer time, e.g. for >> 14 days
- Since we have an online regression, the Kalman filter will be able to adapt to slowly changing paramaters, e.g. in the different seasons or aging of tthe sensor

Done