Cholesky Decomposition Correlated Bivariate Normal from IID Random Draws

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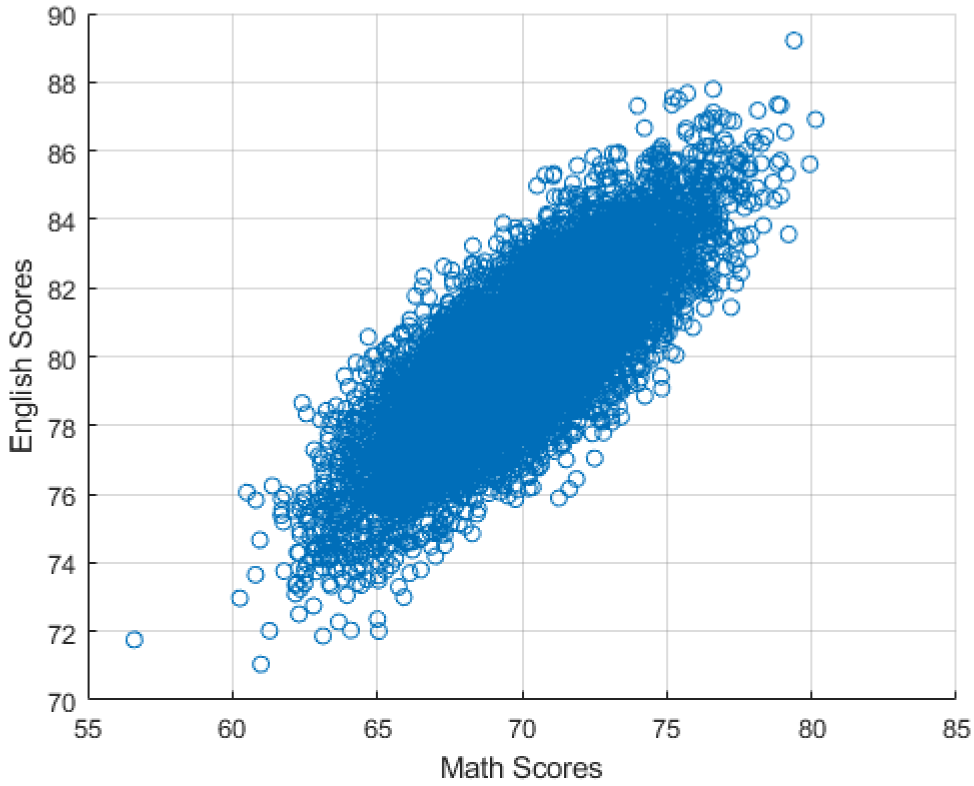
Draw two correlated normal shocks using the MVNRND function. Draw two correlated normal shocks from uniform random variables using Cholesky Decomposition.

- fs_cholesky_decomposition
- fs_cholesky_decomposition_d5
- fs_bivariate_normal

Positively Correlated Scores MVNRND

We have English and Math scores, and we draw from a bivariate normal distribution, assuming the two scores are positively correlatd. These are x_1 and x_2.

```
% mean, and varcov
ar_mu = [70,80];
mt_varcov = [8,5;5,5];
% Generate Scores
rng(123);
N = 10000;
mt_scores = mvnrnd(ar_mu, mt_varcov, N);
% graph
figure();
scatter(mt_scores(:,1), mt_scores(:,2));
ylabel('English Scores');
xlabel('Math Scores')
grid on;
```



What are the covariance and correlation statistics?

```
disp([num2str(cov(mt_scores(:,1), mt_scores(:,2)))]);
```

```
8.0557      5.0738
5.0738      5.0638
```

```
disp([num2str(corrcoef(mt_scores(:,1), mt_scores(:,2)))]);
```

```
      1     0.79441
0.79441           1
```

Bivariate Normal from Uncorrelated Draws via Cholesky Decomposition

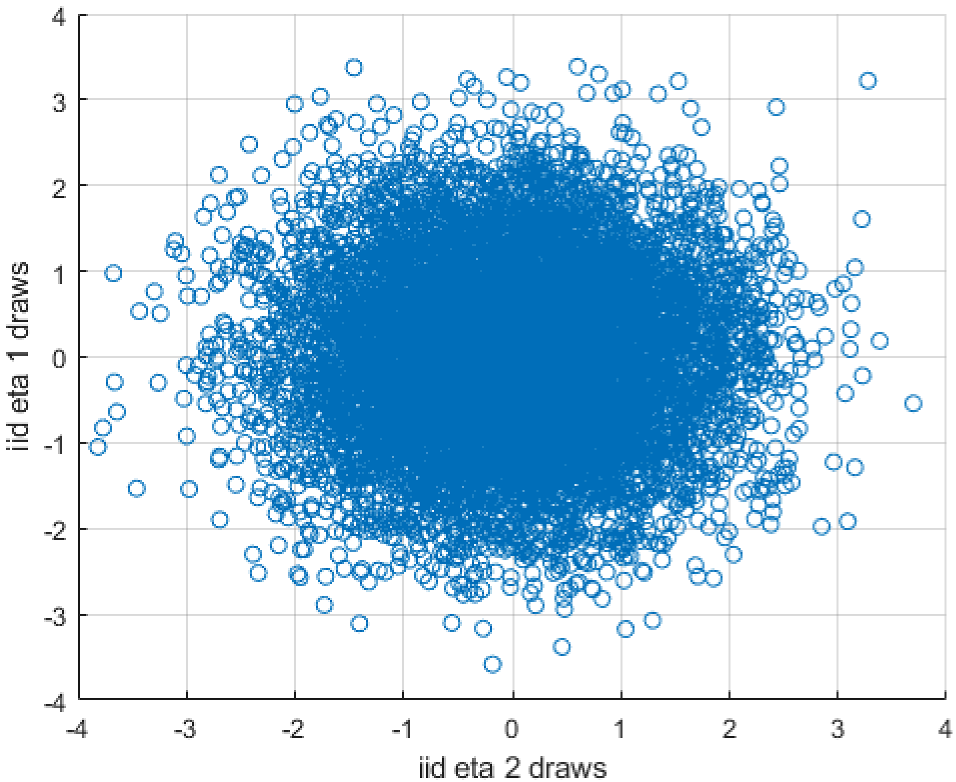
We can get the same results as above, without having to explicitly draw from a multivariate distribution by (For more details see Train (2009)):

1. Draw 2 uniform random iid vectors.
2. Convert to normal iid vectors.
3. Generate the test scores as a function of the two random variables, using Cholesky matrix.

First, draw two uncorrelated normal random variables, with mean 0, sd 1, η_1 and η_2.

```
% Draw Two Uncorrelated Normal Random Variables
% use the same N as above
rng(123);
% uniform draws, uncorrelated
```

```
ar_unif_draws = rand(1,N*2);
% normal draws, english and math are uncorreated
% ar_draws_eta_1 and ar_draws_eta_2 are uncorrelated by construction
ar_normal_draws = norminv(ar_unif_draws);
ar_draws_eta_1 = ar_normal_draws(1:N);
ar_draws_eta_2 = ar_normal_draws((N+1):N*2);
% graph
figure();
scatter(ar_draws_eta_1, ar_draws_eta_2);
ylabel('iid eta 1 draws');
xlabel('iid eta 2 draws')
grid on;
```




```
% Show Mean 1, cov = 0
disp([num2str(cov(ar_draws_eta_1, ar_draws_eta_2))]);
```

```
  0.99075   0.0056929
0.0056929     0.98517
```

```
disp([num2str(corrcoef(ar_draws_eta_1, ar_draws_eta_2))]);
```

```
        1   0.0057623
0.0057623           1
```

Second, now using the variance-covariance we already have, decompose it, we will have:

- $c_{aa}, 0$

  c_{ab}, c_{bb}

```
% Choesley decompose the variance covariance matrix
mt_varcov_chol = chol(mt_varcov, 'lower');
disp([num2str(mt_varcov_chol)]);
```

```
2.8284         0
1.7678    1.3693
```

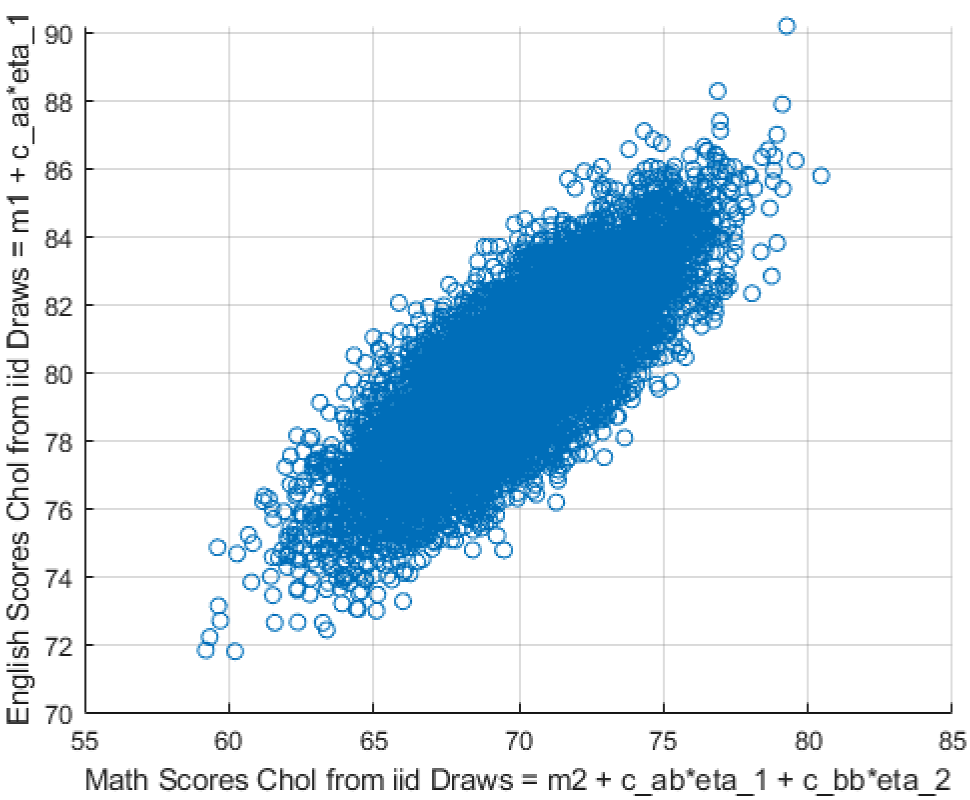
```
% The cholesky decomposed matrix factorizes the original varcov matrix
disp([num2str(mt_varcov_chol*mt_varcov_chol')]);
```

```
8         5
5         5
```

Third, We can get back to the original x_1 and x_2 variables:

- $x_1 = \mu_1 + c_{aa} * \eta_1$
- $x_2 = \mu_2 + c_{ab} * \eta_1 + c_{bb} * \eta_2$

```
% multiple the cholesky matrix by the eta draws
mt_scores_chol = ar_mu' + mt_varcov_chol*([ar_draws_eta_1; ar_draws_eta_2]);
mt_scores_chol = mt_scores_chol';
% graph
figure();
scatter(mt_scores_chol(:,1), mt_scores_chol(:,2));
ylabel('English Scores Chol from iid Draws = m1 + c\_aa*eta\_1');
xlabel('Math Scores Chol from iid Draws = m2 + c\_ab*eta\_1 + c\_bb*eta\_2')
grid on;
```



```
disp([num2str(cov(mt_scores_chol(:,1), mt_scores_chol(:,2)))]);
```

```
 7.926      4.9758
4.9758      4.9708
```

```
disp([num2str(corrcoef(mt_scores_chol(:,1), mt_scores_chol(:,2)))]);
```

```
      1     0.79272
0.79272           1
```