خرشید نوش

1-

a- $N=8$, $\omega_0 = \frac{2\pi}{4}$, $x[n] = \sum_{k=\langle N\rangle} a_k e^{jk\omega_0 n}$, $a_k = \frac{1}{N} \sum_{n=\langle N\rangle} x[n] e^{-jk\omega_0 n}$

زوج و حقیقی

$$\longrightarrow a_k = \frac{\sum_{n=-4}^{3} x[n] e^{-j\frac{k\pi}{4}n}}{8} = \frac{1}{8}\left[\cos\left(\frac{3k\pi}{4}\right) + 2\cos\left(\frac{2k\pi}{4}\right) + 3\cos\left(\frac{k\pi}{4}\right) + 4\right]$$

$0 \leq k \leq 7$

b- $x[n] = \sin\left(\frac{\pi}{6}n\right) + \cos\left(\frac{\pi}{4}n\right)$, $T_1 = 12$, $T_2 = 8 \longrightarrow N = 24 = (8,12)$

$$\longrightarrow x(n) = \frac{1}{2j}\left(e^{j\frac{\pi}{6}n} - e^{-j\frac{\pi}{6}n}\right) + \frac{1}{2}\left(e^{j\frac{\pi}{4}n} + e^{-j\frac{\pi}{4}n}\right)$$

$$= \frac{1}{2} e^{j\left(\frac{3\pi}{12}\right)n} + \frac{1}{2} e^{-j\left(\frac{3\pi}{12}\right)n} + \frac{1}{2j} e^{j\left(\frac{2\pi}{12}\right)n} - \frac{1}{2j} e^{-j\left(\frac{2\pi}{12}\right)n}$$

$$\longrightarrow a_3 = a_{-3} = \frac{1}{2} , \quad a_{-2} = -a_2 = \frac{1}{2j} , \quad a_k = 0 \quad (k \notin \{2,-2,3,-3\})$$

2.

a. $x[n-n_0] + x[n_0-n] \longrightarrow e^{-j\omega n_0} X(e^{j\omega}) + e^{-j\omega n_0} X(e^{-j\omega})$

$x[n-n_0] \downarrow \underline{e^{-j\omega n_0} X(e^{j\omega})}$

$x[n+n_0] \longleftrightarrow X(e^{j\omega}) e^{j\omega n_0}$

$x[n_0-n] \longleftrightarrow \underline{X(e^{-j\omega}) e^{-j\omega n_0}}$

b- $n\,x[n] \longleftrightarrow j\frac{dX(e^{j\omega})}{d\omega} \longrightarrow X_2(e^{j\omega}) = 3j\frac{dX(e^{j\omega})}{d\omega}$

c- $n\,x[n] \longleftrightarrow j\frac{dX(e^{j\omega})}{d\omega}$

$n^2 x(n) \longleftrightarrow -1 \cdot \frac{d^2X(e^{j\omega})}{d^2\omega}$ $\longrightarrow n^2 x[n] - 2nn_1 x[n] + n_1^2 x[n] \longleftrightarrow$

$$-\frac{d^2X(e^{j\omega})}{d^2\omega} - 2n_1 j\frac{dX(e^{j\omega})}{d\omega} + n_1^2 X(e^{j\omega})$$

۹۸۲۱۰۴۸ وشیدنوش

۲-

a. $x[n] = 6 + \sin\left(\frac{n\pi}{4} + \frac{1}{2}\right) = 6 + \frac{1}{2j}\left(e^{j\left(\frac{n\pi}{4} + \frac{1}{2}\right)} - e^{-j\left(\frac{n\pi}{4}+\frac{1}{2}\right)}\right)$

$= 6 + \frac{e^{j/2}}{2j} e^{j\left(\frac{\pi}{4}\right)n} - \frac{e^{-j/2}}{2j} e^{-j\left(\frac{\pi}{4}\right)n}$

$X(e^{j\omega}) = \sum_{-\infty}^{\infty} 2\pi a_k \delta(\omega - \omega_0 k)$

$\longrightarrow a_0 = 6,\quad a_1 = \frac{e^{j/2}}{2j},\quad a_7 = -\frac{e^{-j/2}}{2j}$

$\longrightarrow X(e^{j\omega}) = 12\pi\,\delta(\omega) + \frac{\pi}{j} e^{j/2}\delta\left(\omega - \frac{\pi}{4}\right) - \frac{\pi}{j} e^{-j/2}\delta\left(\omega - \frac{7\pi}{4}\right)$ دوره تناوب 2π

$0 \leq \omega < 2\pi$

$X(e^{j\omega}) = \sum_{l=-\infty}^{\infty} 12\pi\,\delta(\omega + 2\pi l) + \frac{\pi}{j} e^{j/2}\left(\delta\left(\omega - \frac{\pi}{4} + 2\pi l\right)\right) - \frac{\pi}{j} e^{-j/2}\left(\delta\left(\omega - \frac{7\pi}{4} + 2\pi l\right)\right)$

b. $x[n] = \frac{3\sin\left(\frac{3\pi}{4}n\right)}{\pi n}$, $\omega = \frac{3\pi}{2}$

$x[n] \longleftrightarrow X(e^{j\omega}) = \begin{cases} 3 & 0 \leq |\omega| \leq \frac{3\pi}{4} \\ 0 & \frac{3\pi}{4} < |\omega| \leq \pi \end{cases}$ دوره گردش 2π

c. $x[n] = u[n] + u[n-7] \longrightarrow (1 + e^{-j7\omega})\left(\frac{1}{1-e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi\,\delta(\omega - 2\pi k)\right)$

$u[n] \longleftrightarrow \frac{1}{1-e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi\,\delta(\omega - 2\pi k)$

d. $a^n u[n] \longrightarrow \frac{1}{1-ae^{-j\omega}}$

$x[n] = n a^n u[n] + a^n u[n]$

$n a^n u[n] \longleftrightarrow j\frac{d\left(\frac{1}{1-ae^{-j\omega}}\right)}{d\omega} = \frac{ae^{-j\omega}}{(1-ae^{-j\omega})^2}$

$\longrightarrow x[n] \longleftrightarrow \frac{1}{(1-ae^{-j\omega})^2}$

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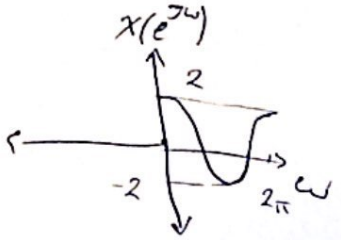
۴.

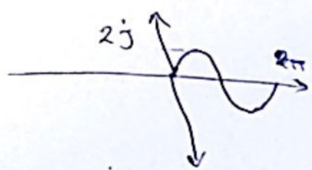
a.

$$x[n] \longleftrightarrow e^{-jw} + e^{jw} = 2\cos(w)$$



b.

$$x[n] \longleftrightarrow \underline{-e^{-2jw} + e^{2jw}} = j2\sin(2w)$$



5.

$$\omega_0 = \frac{2\pi}{5} \quad , \quad x[n] = a_0 + a_1 e^{j\frac{2\pi}{5}n} + a_2 e^{j\frac{4\pi}{5}n} + a_3 e^{j\frac{6\pi}{5}n} + a_4 e^{j\frac{8\pi}{5}n}$$

$$a_3 = a_{-2} \quad , a_{-1} = a_4 \quad , a_1 = a_{-4} \longrightarrow x[n] = 2 + e^{j\frac{\pi}{3}}\left(e^{j\frac{2\pi}{5}n} + e^{-j\frac{2\pi}{5}n}\right) + 2e^{j\frac{\pi}{6}}\left(e^{j\frac{4\pi}{5}n} + e^{-j\frac{4\pi}{5}n}\right)$$

$$x[n] = 2 + 2e^{j\frac{\pi}{3}} \cos\left(\frac{2\pi}{5}n\right) + 2e^{j\frac{\pi}{6}} \cos\left(\frac{4\pi}{5}n\right)$$

$$x[n] = 2 + 2e^{j\frac{\pi}{3}} \sin\left(\frac{2\pi}{5}n + \frac{\pi}{2}\right) + 2e^{j\frac{\pi}{6}} \sin\left(\frac{4\pi}{5} + \frac{\pi}{2}\right)$$

6. با توجه به اینکه $X(e^{jw})$ زوج و حقیقی است $\longleftarrow$ $x[n]$ زوج و حقیقی است.

$$\sum_{1}^{\infty} |x[n]|^2 = \frac{\sum_{-\infty}^{\infty} |x[n]|^2 - |x[0]|^2}{2}$$

$$|x[0]| = \frac{1}{2\pi}\int_{2\pi} X(e^{jw})\,dw = \frac{1}{2\pi}\left[\int_0^6 \cancel{X(e^{jw})\,dw}^{\,6\times\frac{1}{2}\times 2} + \int_6^{2\pi} \cancel{X(e^{jw})\,dw}^{\,4(2\pi-6)^2\frac{1}{2}}\right]$$

$$\longrightarrow |x[0]| = 4\pi - 24 + \frac{39}{\pi}$$

$$\sum_{-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi}\int |X(e^{jw})|^2 dw = \frac{1}{2\pi}\left[\int_0^6 \underbrace{\cancel{|X(e^{jw})|^2 dw}}_{6\times\frac{1}{2}\times 4} + \int_6^{2\pi} \underbrace{|X(e^{jw})|^2 dw}_{6\times(2\pi-6)^3\times\frac{1}{2}}\right]$$

$$= \frac{1}{2\pi}\left[12 + 8(2\pi-6)^3\right]$$

$$\longrightarrow ans = \frac{1}{4\pi}\left(6 + 8(2\pi-6)^3 - 2(2\pi-6)^2\right)$$

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۷-

$$x[n] = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega}) e^{j\omega n} d\omega = \frac{1}{2\pi}\int_{-\pi/4}^{\pi/4} e^{j\left(\frac{3}{2}+n\right)\omega} d\omega = \frac{\sin\left(\frac{n\pi}{4}+\frac{3\pi}{8}\right)}{\pi\left(\frac{3}{2}+n\right)}$$

$$x[n]=0 \longrightarrow \sin\left(\frac{n\pi}{4}+\frac{3\pi}{8}\right)=0 \longrightarrow \frac{n\pi}{4}+\frac{3\pi}{8}=2\pi K \longrightarrow 2K=\frac{n}{4}+\frac{3}{8} \longrightarrow K=\frac{2n+3}{16}$$

$$\xrightarrow[\text{اگر}]{K\in\mathbb{Z}} n \notin \mathbb{Z} \quad \times$$

پس تنها حالت صفر شدن این است که صورت n برود.

۸-

$$y[n]=x[n]*h[n] \longrightarrow Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega})$$

$$X(e^{j\omega}) = \pi\sum_{l=-\infty}^{\infty} \delta\left(\omega-\frac{3}{5}\pi+2\pi l\right)+\delta\left(\omega+\frac{3\pi}{5}+2\pi l\right)$$

$$h[n]=a^n u[n] \longleftrightarrow \frac{1}{1-ae^{-j\omega}} = H(e^{j\omega})$$

$$\longrightarrow y[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} \pi\,\frac{\delta\left(\omega-\frac{3\pi}{5}\right)+\delta\left(\omega+\frac{3\pi}{5}\right)}{1-ae^{-j\omega}}e^{j\omega n}d\omega = \boxed{\frac{1}{2}\left[\frac{e^{j\frac{3\pi}{5}n}}{1-ae^{-j\frac{3\pi}{5}}}+\frac{e^{-j\frac{3\pi}{5}n}}{1-ae^{j\frac{3\pi}{5}}}\right]}$$