خورشیدنوش ۹۸۲۳۱۰۹۸

سؤال یک

الف)

$$\cos\theta = \frac{1}{2}\left(e^{j\theta}+e^{-j\theta}\right), \quad \sin\theta = \frac{1}{2}\left(e^{j\theta}-e^{-j\theta}\right)$$

$$x_1(t) = 1 + \frac{1}{2}\left(e^{j\omega_0 t} - e^{-j\omega_0 t}\right) + \frac{3}{2}\left(e^{j(2\omega_0 t+\pi/4)} + e^{-j(2\omega_0 t+\pi/4)}\right).$$

$$= 1 + \frac{1}{2j}e^{j\omega_0 t} - \frac{1}{2j}e^{-j\omega_0 t} + \frac{3}{2}e^{\frac{j\pi}{4}}e^{2\omega_0 t j} + \frac{3}{2}e^{-\frac{j\pi}{4}}e^{-2\omega_0 t j}$$

$$\longrightarrow a_0 = 1,\ a_1 = \frac{1}{2j},\ a_{-1} = \frac{-1}{2j},\ a_{-2} = \frac{3}{2}e^{-\frac{j\pi}{4}},\ a_2 = \frac{3}{2}e^{j\frac{\pi}{4}}$$

$$\{a_K = 0 \mid K \notin \{-2,-1,0,1,2\}\}$$

ب)

$$a_K = \frac{1}{T}\int_T x(t)e^{-jK\omega_0 t}\,dt$$

سیگنال متناوب است ــــ یک دوره تناوب: $\omega_0 = \pi,\ T = 2$

$$a_K = \frac{1}{2}\int_T x(t)e^{-jK\omega t}\,dt \longrightarrow \frac{1}{2}\int_0^{2} x(t)e^{-jK\omega_0 t}\,dt \longrightarrow$$

$$\frac{1}{2}\left[2\int_0^1 e^{-jK\omega_0 t}\,dt + \int_1^2 e^{-jK\omega_0 t}\,dt\right] = \frac{1}{2}\left[2\,\frac{e^{-jK\pi t}}{-jK\pi}\Bigg|_0^1 + \frac{e^{-jK\pi t}}{jK\pi}\Bigg|_1^2\right]$$

$$= \frac{-e^{-jK\pi}}{jK\pi} + \frac{1}{jK\pi} - \frac{e^{-2jK\pi}}{2jK\pi} + \frac{e^{-jK\pi}}{2jK\pi}$$

$$\longrightarrow a_K = \frac{-e^{-jK\pi}}{2jK\pi} + \frac{1}{jK\pi} - \frac{e^{-2jK\pi}}{2jK\pi} = \frac{-1}{2jK\pi}\left[e^{-jK\pi} + e^{-2jK\pi} - 2\right]$$

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۲) با توجه به سوال ۱ $\omega = \frac{\pi}{5}$, $T=10$ بدست می‌آید. در یک دوره تناوب خواهیم داشت ←

$$a_k = \frac{1}{10}\int_T x(t)\, e^{\frac{-jk\pi t}{5}}\, dt = \frac{1}{10}\left[\int_0^1 t e^{\frac{-jk\pi t}{5}} dt + \int_1^2 e^{\frac{-jk\pi t}{5}} + \int_2^3 (3-t) e^{\frac{-jk\pi t}{5}} \right.$$

$$\to a_k = \frac{1}{10}\left[\frac{-25}{k^2\pi^2} + \frac{(5\pi jk+25)e^{\frac{-\pi jk}{5}}}{k^2\pi^2} - \frac{5e^{\frac{-2\pi jk}{5}}}{\pi jk} + \frac{5e^{\frac{-\pi jk}{5}}}{\pi jk} - \frac{25e^{-\frac{3\pi jk}{5}}}{k^2\pi^2} - \frac{(5\pi jk-25)e^{\frac{-2\pi jk}{5}}}{\pi^2 k^2}\right]$$

answer $\to$
$$a_k = \frac{-5}{2k^2\pi^2} + \frac{(\pi jk+5)e^{\frac{-\pi jk}{5}}}{2k^2\pi^2} + \frac{e^{\frac{-\pi jk}{5}}}{2\pi jk} - \frac{e^{\frac{-2\pi jk}{5}}}{\pi jk} - \frac{5e^{\frac{-3\pi jk}{5}}}{2k^2\pi^2} - \frac{(\pi jk-5)e^{\frac{-2\pi jk}{5}}}{2k^2\pi^2}$$

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الف)

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t} = \sum_{p=-\infty}^{\infty} e^{j(2p)\omega_0 t} + \sum_{p=-\infty}^{\infty} 2e^{j(2p+1)\omega_0 t}$$

$$= \sum_{p=-\infty}^{\infty} e^{2pj\omega_0 t}\left(1+2e^{j\omega_0 t}\right)$$

ب)

$$x(t) = \sum_{k=-\infty}^{+\infty} x(t) e^{jk\omega_0 t} = \sum_{k=-3}^{3} jk e^{jk\omega_0 t} = -3je^{-3j\omega_0 t} - 2je^{-2j\omega_0 t} - je^{-j\omega_0 t} + je^{j\omega_0 t} + 2je^{2j\omega_0 t} + 3je^{3\omega_0 t j}$$

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الف)

$$x(t-t_0)+x(t+t_0) \xleftrightarrow[\text{با استفاده از خاصیت شیفت}]{FS} a_k\left(e^{-jk\omega_0 t}+e^{jk\omega_0 t}\right)$$

ب)

$$Even[x(t)] = \frac{x(t)+x(-t)}{2} \longleftrightarrow$$

time reversal $\rightarrow x(-t) = a_{-k}$, linearity $\rightarrow \frac{1}{2}(x(t)+x(-t)) = \frac{1}{2}(a_k+a_{-k})$

ج)

$$Re[x(t)] = \frac{1}{2}(x(t)+x^*(t)) \xleftrightarrow{F} \frac{1}{2}(a_k+a^*_{-k})$$

$$\xrightarrow{\text{درنتیجه}} x^*(t) \longleftrightarrow a^*_{-k}$$

د)

Differentiation : $x(t) \rightarrow \frac{dx}{dt} : jk\omega_0 a_k$

Differentiation : $\frac{dx}{dt} \rightarrow \frac{d^2x}{dt^2} = -k^2\omega_0^2 a_k$

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ه)

$$a_K = \frac{1}{T} \int_{-\infty}^{\infty} x(t)\, e^{-jKt\omega_0}\, dt$$

$$a'_K = \frac{1}{T/3} \int_{-\infty}^{\infty} x(3t-1)\, e^{-jKt\frac{2\pi}{T/3}}\, dt$$

$$a'_K = \frac{3}{T} \int_{-\infty}^{\infty} x(t')\, e^{-j\frac{2\pi}{T}K(t'+1)}\, \frac{dt'}{3}$$

$$a'_K = \frac{1}{T} \int_{-\infty}^{\infty} x(t')\, e^{-j\frac{2\pi}{T}K(t'+1)}\, dt'$$

$$t' = 3t-1 \longrightarrow t = \frac{t'+1}{3} \quad , \quad a'_K = e^{-j\omega_0 K} a_K$$

$$\longrightarrow x(3t-1) = e^{-j\omega_0 K} a_K$$

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$$x(t)=\frac{3\pi}{2}\cos\left(\frac{3\pi}{2}t+\pi\right)\sin\left(\frac{3\pi}{4}t+\frac{\pi}{2}\right)=\frac{3\pi}{2}\left(-\cos\left(\frac{3\pi}{2}\right)\right)\left(\cos\left(\frac{3\pi}{4}t\right)\right)$$ -۴

$$\rightarrow\begin{cases}\frac{3}{2}\pi T=2K\pi\\ \frac{3\pi}{4}T=2K'\pi\end{cases}\longrightarrow K=2K' \;,\; K'=1\rightarrow K=2\rightarrow T=8/3 \;,\; \omega_0=3/4\,\pi$$

$$\begin{cases}\sin(S)=\frac{1}{2}(e^{Sj}-e^{-Sj})\\ \cos(S)=\frac{1}{2}(e^{Sj}+e^{-Sj})\end{cases}\xrightarrow{\text{جایگذاری}}$$

$$x(t)=-\frac{3\pi}{8}\left(e^{\frac{9}{4}\pi jt}+e^{\frac{3}{4}\pi jt}+e^{-\frac{3\pi}{4}\pi jt}+e^{-\frac{9}{4}\pi jt}\right)$$

$$\longrightarrow a_1=a_{-1}=a_{-3}=a_3=-\frac{3\pi}{8}$$

$$\{a_L=0\;|\;L\neq\{1,-1,-3,3\}\}$$

$$\frac{1}{T}\int_T|x(t)|^2dt=\sum_{K=-\infty}^{\infty}|a_K|^2=4\,\frac{9\pi^2}{64}=\frac{9\pi^2}{16}$$

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$$T=4,\;\omega_0=\frac{\pi}{2}\;,\;x(t)=\sum_{K=-\infty}^{\infty}j^K e^{\frac{jK\pi}{2}t}$$

$$\begin{cases}\sin(S)=\frac{1}{2}(e^{Sj}-e^{-Sj})\\ \cos(S)=\frac{1}{2}(e^{Sj}+e^{-Sj})\end{cases}\longrightarrow y(t)=\sum_{K=\infty}^{\infty}\left[\frac{j^K}{2}e^{j\left(\frac{\pi K}{2}+\frac{\pi}{4}\right)t}+\frac{j^K}{2}e^{jt\left(\frac{K\pi}{2}-\frac{\pi}{4}\right)}+j^K e^{j2k\pi t}\right]$$

دوره تناوب

$$\begin{aligned}&x(t)\rightarrow 4\\&x(4t)\rightarrow 1\\&\cos\left(\tfrac{\pi}{4}t\right)\rightarrow 8\rightarrow\left(\frac{2\pi}{\frac{\pi}{4}}\right)\end{aligned}\longrightarrow y_{\text{دوره تناوب}}=8\longrightarrow\omega_0'=\frac{\pi}{4}$$

$$y(t)=\sum_{K=-\infty}^{\infty}\frac{j^K+j^{K+1}}{2}e^{j\left(\frac{2K+1}{4}\right)t}+j^K e^{2K\pi jt}$$

$$\longrightarrow\begin{aligned}&a_0=1\\&a_1=\frac{1-j}{2}\\&a_2=0\\&a_3=\frac{j-1}{2}\\&a_4=0\\&a_5=\frac{-1-j}{2}\end{aligned}$$

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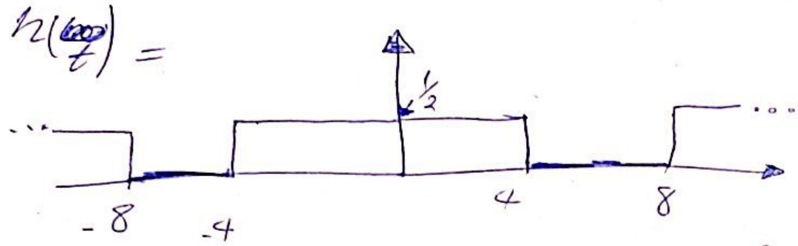
$$H(j\omega) = \int_{-\infty}^{\infty} h(t) e^{-j\omega t}\, dt = \frac{\sin(4\omega)}{\omega}$$

$x(t) =$



$$x(j\omega) = e^{-2j\omega} \frac{2\sin(2\omega)}{\omega} - e^{2j\omega} \frac{2\sin(2\omega)}{\omega}$$

$$a_k = \frac{1}{T} x(j\omega) = \frac{\sin(2\omega)}{4\omega}\left(e^{-2j\omega} - e^{2j\omega}\right) \quad \omega = k\omega_0 = \frac{k\pi}{4} \leftrightarrow \omega_0 = \frac{\pi}{4}$$

$$a_k = \frac{\sin(2\omega)(-2j\sin(2\omega))}{4\omega} \longrightarrow a_k = \frac{-2j\sin^2\left(\frac{k\pi}{2}\right)}{k\pi}$$

$$a_k = \begin{cases} 0 & k \text{ is even} \\ \frac{-2j}{k\pi} & k \text{ is odd} \end{cases}$$

$$y(t) = \sum_{k=-\infty}^{\infty} \frac{2\sin^2\left(\frac{k\pi}{2}\right)}{jk\pi} \cdot \frac{\sin(k\pi)}{\frac{k\pi}{4}} e^{j\frac{k\pi}{4}t}$$

$$= \sum_{k=-\infty}^{\infty} \frac{8\sin^2\left(\frac{k\pi}{2}\right)\sin(k\pi)}{jk^2\pi^2} e^{j\frac{k\pi}{4}t}$$

$$a_k = \frac{2\sin^2\left(\frac{k\pi}{2}\right)}{jk\pi}$$

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