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۱-

a) $x(t) = e^{-2|t-1|} \longrightarrow X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t}\, dt = \int_{-\infty}^{1} e^{2t-2-j\omega t}\, dt$

$+ \int_{1}^{\infty} e^{-2t+2-j\omega t}\, dt = \frac{e^{-2}}{2-j\omega} e^{2t-j\omega t} \Big|_{-\infty}^{1} + \frac{e^{2}}{-2-j\omega} e^{-2t-j\omega t} \Big|_{1}^{\infty}$

$$= \boxed{\frac{e^{-j\omega}}{2-j\omega} + \frac{e^{-j\omega}}{2+j\omega}}$$

b) $x(t) = 1 + 3 \underbrace{\cos(4\pi t + \pi/3)}_{\cos(4\pi t)\cos(\pi/3) - \sin(4\pi t)\sin(\pi/3)}$

$\longrightarrow x(t) = 1 + \frac{3}{2}\cos(4\pi t) - \frac{3\sqrt{3}}{2}\sin(4\pi t) = 1 + \frac{3}{2}\left(\frac{e^{j4\pi t} + e^{-4j\pi t}}{2}\right)$

$- \frac{3\sqrt{3}}{2}\left(\frac{e^{4j\pi t} - e^{-4j\pi t}}{2}\right) = 1 + \left(\frac{3}{4} + \frac{3\sqrt{3}}{4}\right) e^{4j\pi t}$

$+ \left(\frac{3}{4} - \frac{3\sqrt{3}}{4}\right) e^{-4j\pi t}$

$a_0 = 1, \; a_1 = \frac{3}{4} + \frac{3\sqrt{3}}{4}, \; a_{-1} = \frac{3}{4} - \frac{3\sqrt{3}}{4}, \; a_k = 0 \quad |k| > 1$

c) $x(t) = e^{5j\pi t} - \frac{\sin(2\pi t)}{\pi t} \longleftrightarrow X(j\omega) = 2\pi\,\delta(\omega - 5\pi) - \begin{cases} 1, & |\omega| < 2\pi \\ 0, & |\omega| > 2\pi \end{cases} \longrightarrow$

$$\begin{cases} X(j\omega) = 2\pi\,\delta(\omega - 5\pi) - 1 & |\omega| < 2\pi \\ X(j\omega) = 2\pi\,\delta(\omega - 5\pi) & |\omega| > 2\pi \end{cases}$$

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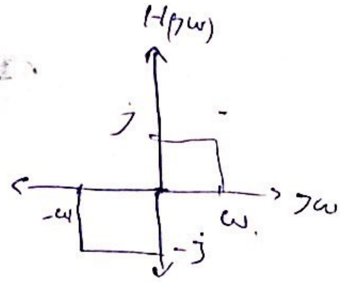
d) $x(t) = \frac{1}{t}\left(\frac{t\sin(t)}{\pi t}\right)^2 = \frac{1}{t}\left(\frac{\sin^2(t)}{\pi^2}\right) = \frac{1-\cos(2t)}{2\pi^2 t} = \frac{1}{2\pi^2 t} - \frac{\cos(2t)}{2\pi^2 t}$

$\longrightarrow x(t) = \frac{1}{2\pi^2 t} - \frac{1}{2\pi}\left(\frac{\cos(2t)}{\pi t}\right)$



$h(t) = \frac{1}{2\pi}\int_{-\omega_0}^{\omega_0} H(j\omega)\, e^{j\omega t} d\omega = \frac{1}{2\pi}\left[-j\int_{-\omega_0}^{0} e^{j\omega t} d\omega + j\int_{0}^{\omega_0} e^{j\omega t} d\omega\right] = \frac{2\cos(\omega_0 t) - 2}{2\pi t} = \frac{\cos(\omega_0 t) - 1}{\pi t}$

$x(t) = \frac{-1}{2\pi} h(t) = \frac{-1}{2\pi}\left(\frac{\cos(2t)-1}{\pi t}\right)$ $\quad X(j\omega) = \frac{1}{2\pi} X(j\omega)$

e) $x(t) = \frac{4t}{(1+t^2)^2}$, $y(t) = h(t) * x(t) \rightarrow Y(j\omega) = H(j\omega) X(j\omega)$

$y(t) = \int_{-\infty}^{t} \frac{4\tau}{(1+\tau^2)^2} d\tau = \frac{2}{1+t^2}$ $\quad h(t) = u(t)$

$Y(j\omega) = H(j\omega) * X(j\omega) \longrightarrow 2\pi e^{-|\omega|} = \frac{1}{j\omega} X(j\omega) + \pi\delta(\omega)X(0) \leftrightarrow X(j\omega) = \frac{2\pi e^{-|\omega|}}{\frac{1}{j\omega} + \pi\delta(\omega)}$

f) $x(t) = \begin{cases} e^{-t} & 0 \leq t \leq 1 \\ 0 & \text{else} \end{cases} \longrightarrow X(j\omega) = \int_0^1 e^{-t(1+j\omega)} dt = \frac{1}{-1-j\omega} e^{-t(1+j\omega)}\Big|_0^1$

$\longrightarrow X(j\omega) = \frac{1 - e^{-(1+j\omega)}}{1+j\omega}$

۲-

a) $X(j\omega) = 2(\delta(\omega+6)) \longrightarrow x(t) = \frac{\int_{-\infty}^{\infty} 2\delta(\omega+6)\, e^{j\omega t} d\omega}{2\pi} = \frac{1}{\pi} e^{-j6t}$

b) $X(j\omega) = 2\frac{\sin[3(\omega - 2\pi)]}{\omega - 2\pi}$

نکته: $y(t) = \begin{cases} e^{j\alpha t} & |t| < T_0 \\ 0 & |t| > T_0 \end{cases}$

$Y(j\omega) = \frac{2\sin(T_0(\omega - \alpha))}{\omega - \alpha}$

$x(t) = \begin{cases} e^{j2\pi t} & -3 < t < 3 \\ 0 & \text{else} \end{cases}$

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c) $X(j\omega) = 2[\delta(\omega-1) - \delta(\omega+1)] + 3[\delta(\omega-2\pi) + \delta(\omega+2\pi)]$

$\hookrightarrow x(t) = \frac{1}{\pi} e^{jt} - \frac{1}{\pi} e^{-jt} + \frac{3}{2\pi} e^{j2\pi t} + \frac{3}{2\pi} e^{-j2\pi t}$

d) $X(j\omega) = \frac{7j\omega + 46}{-\omega^2 + 13j\omega + 42} = \frac{4}{j\omega + 6} + \frac{3}{j\omega + 7} \longrightarrow x(t) = 3e^{-7t}u(t) + 4e^{-6t}u(t)$

$$\frac{7x + 46}{x^2 + 13x + 42} = \frac{4}{(x+6)} + \frac{3}{(x+7)}$$

-3

الف) سیگنال فرد و موهومی ← نادرست ⟷

داریم → $x(t) = -x(-t)$ و $x^*(t) = -x(t)$

$x(t) \xleftrightarrow{FS} X(j\omega)$ و ضرب در -1 → $-x(t) \longleftrightarrow -X(j\omega)$

$x(-t) \longleftrightarrow X(-j\omega) \longrightarrow x^*(t) \longleftrightarrow -X^*(-j\omega)$

$X^*(-j\omega) = X(-j\omega) \longrightarrow X(j\omega)$ حقیقی است

$X(-j\omega) = -X(j\omega) \longrightarrow X(j\omega)$ فرد است

ب) کانولوشن دو تابع حقیقی فرد و زوج ← درست ⟷

$Y(j\omega) = \underbrace{H(j\omega)}_{\text{فرد}} * \underbrace{X(j\omega)}_{\text{زوج}} \longrightarrow Y(-j\omega) = H(-j\omega) * X(-j\omega) =$

$H(j\omega) = -H(-j\omega)$

$X(j\omega) = X(-j\omega)$

$-H(j\omega) * X(j\omega) = -Y(j\omega)$

-4

$y(t) = x(t) * h(t)$

$g(t) = x(3t) * h(3t)$

$Y(j\omega) = X(j\omega)H(j\omega)$

$G(j\omega) = \frac{1}{3} X(j\frac{\omega}{3}) \times \frac{1}{3} H(j\frac{\omega}{3}) \rightarrow G(j\omega) = \frac{1}{9} X(j\frac{\omega}{3}) H(j\frac{\omega}{3})$

$Y(j\frac{\omega}{3}) = 9G(j\omega)$

$y(3t) \xleftrightarrow{FS} \frac{1}{3} Y(j\frac{\omega}{3})$ | $g(t) = \frac{1}{3} y(3t)$ $B = 3,$ $A = \frac{1}{3}$

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$$X(j\omega) = \delta(\omega) + \delta(\omega-\pi) + \delta(\omega-5)$$ ۵-

$$h(t) = u(t) - u(t-2)$$

الف) $$x(t) = \frac{1}{2\pi} + \frac{1}{2\pi} e^{j\pi t} + \frac{1}{2\pi} e^{j5t} \quad , x(t+T) = x(t)$$

$$\left.\begin{aligned} 5T &= 2K_1\pi \\ \pi T &= 2K_2\pi \end{aligned}\right\} \rightarrow \frac{\pi}{5} = \frac{K_2}{K_1}$$

$$\rightarrow \pi K_1 = 5K_2$$

چون K_1, K_2 اعداد صحیح نیستند پس $x(t)$ متناوب نیست.

ب) $x(t) * h(t)$

$$H(j\omega) = \left(\frac{1}{j\omega} + \pi\delta(\omega)\right)(1-e^{-2j\omega}), \quad Y(j\omega) = H(j\omega)X(j\omega)$$

$$\rightarrow y(t) = \frac{1}{2\pi}\underbrace{\int_{-\infty}^{\infty} Y(j\omega)e^{j\omega t}\,d\omega}_{e^{j5t}\left(\frac{1}{5j}\right)(1-e^{-j10}) + e^{j\pi t}\left(\frac{1}{\pi j}\cdot 0\right)}$$

$$\rightarrow y(t) = \frac{1}{10\pi j} e^{j5t}(1-e^{-j10}) \qquad 5T = 2\pi \rightarrow T = \frac{2\pi}{5}$$

ج) با علامت سوال بالا.

۶-

$$y(t) = e^{-3t}u(t) - e^{-4t}u(t) \rightarrow Y(j\omega) = \frac{1}{3+j\omega} - \frac{1}{4+j\omega}$$

$$\rightarrow X(j\omega) = Y(j\omega)\,H(j\omega) = 1 - \frac{3+j\omega}{4+j\omega} = \frac{1}{4+j\omega} \rightarrow x(t) = e^{-4t}u(t)$$

۷-

$$\int_{-\infty}^{\infty} |x(t)|^2 d(t) = \int_{-\infty}^{\infty} \left|t\left(\frac{\sin(t)}{\pi t}\right)^2\right|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$$

$$x(t) = t\,\frac{\sin^2(t)}{\pi^2 t^2} = \frac{\sin^2(t)}{\pi^2 t}, \quad y(t) = \frac{\sin^2(t)}{\pi^2 t^2}$$

$$Y(j\omega) = \frac{(u(\omega+1) - u(\omega-1)) * (u(\omega+1) - u(\omega-1))}{2\pi} = \frac{(\omega+2)u(\omega+2) - 2\omega u(\omega) + (\omega-2)u(\omega-2)}{2\pi}$$

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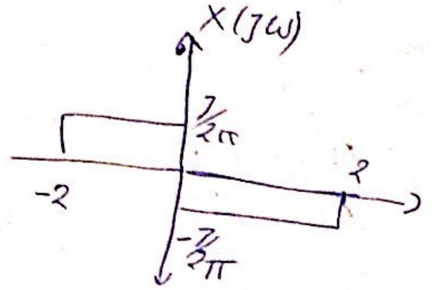
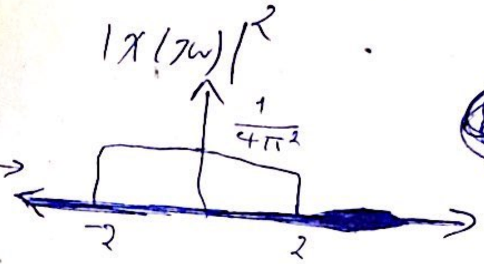
$$X(j\omega) = j\frac{d}{d\omega}Y(j\omega) = \frac{j}{2\pi}\Big[u(\omega+2)+(\omega+2)\,\delta(\omega+2)-2u(\omega)-2\omega\,\delta(\omega) + u(\omega-2)+(\omega-2)\,\delta(\omega-2)\Big]$$

$$= \frac{j}{2\pi}\Big[u(\omega+2)-2u(\omega)+u(\omega-2)\Big]$$

$$\longrightarrow \frac{1}{2\pi}\int_{-\infty}^{\infty}|X(j\omega)|^2\,d\omega = \frac{1}{2\pi}\cdot 4\cdot\frac{1}{4\pi^2} = \frac{1}{2\pi^3}$$





۸- داریم ⟵

$$(j\omega)^2 Y(j\omega) + 6(j\omega)Y(j\omega) + 8Y(j\omega) = 2X(j\omega)$$

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{2}{(j\omega)^2+(j\omega)6+8} = \frac{1}{j\omega+2}-\frac{1}{j\omega+4}$$

الف)

$$h(t) = e^{-2t}u(t) - e^{-4t}u(t) \longleftrightarrow H(j\omega) = \frac{1}{j\omega+2}-\frac{1}{j\omega+4}$$

ب)

$$X(j\omega) = j\frac{d\left(\frac{1}{j\omega+2}\right)}{d\omega} = \frac{1}{(j\omega+2)^2}$$

$$\longrightarrow Y(j\omega) = \frac{2}{(j\omega+4)(j\omega+2)}\cdot\frac{1}{(j\omega+2)^2} = \frac{2}{(j\omega+2)^3(j\omega+4)} \xrightarrow{\text{باید تجزیه کرد بشود}}$$

تجزیه کسر

$$\frac{A}{s+4}+\frac{B}{s+2}+\frac{A's+B'}{(s+2)^2}+\frac{A''s^2+B''s+C}{(s+2)^3} = \frac{2}{(s+2)^3(s+4)}$$

$$\longrightarrow A''=B''=A'=0,\ C=1,\ B'=-\tfrac{1}{2},\ A=-\tfrac{1}{4}=-B$$

خواهیم داشت

$$\longrightarrow y(t) = -\tfrac{1}{4}e^{-4t}u(t) + \tfrac{1}{4}e^{-2t}u(t) - \tfrac{1}{2}te^{-2t}u(t) + \tfrac{1}{2}t^2e^{-2t}u(t)$$

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9-

الف)

$$h(t) = \delta(t) - 3e^{-t}u(t) \longrightarrow H(j\omega) = 1 - \frac{3}{1+j\omega}$$

$$H(j\omega) = \frac{-2-j\omega}{1+j\omega} \longrightarrow H^{-1}(j\omega) = \underbrace{\frac{1+j\omega}{2+j\omega}}_{1-\frac{1}{2+j\omega}} \longrightarrow \boxed{H^{-1}(t) = \delta(t) - e^{-2t}u(t)}$$

ب)

$$x(t) = e^{-3t}u(t) - e^{-4t}u(t) \longrightarrow X(j\omega) = \frac{1}{3+j\omega} + \frac{1}{4+j\omega}$$

$$\longrightarrow Y(j\omega) = X(j\omega)H(j\omega) = \frac{1}{3+j\omega} + \frac{1}{4+j\omega} - \frac{3}{1+j\omega}\left(\frac{1}{3+j\omega} + \frac{1}{4+j\omega}\right)$$

$$= \frac{1}{3+j\omega} + \frac{1}{4+j\omega} - \left(\frac{1}{2+j\omega} - \frac{1}{3+j\omega} + \left[\frac{\frac{-1}{4+j\omega} + \frac{1}{2+j\omega}}{2}\right]\right) = \frac{2}{3+j\omega} + \frac{\frac{3}{2}}{4+j\omega} - \frac{\frac{3}{2}}{2+j\omega}$$

$$\rightarrow Y(j\omega) = \frac{2}{3+j\omega} + \frac{3}{2}\left(\frac{1}{4+j\omega} - \frac{1}{2+j\omega}\right)$$

$$\longrightarrow \boxed{y(t) = 2e^{-3t}u(t) + \frac{3}{2}\left(e^{-4t}u(t) - e^{-2t}u(t)\right)}$$

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