3DFClib: (Towards) a collection of discrete 3D Frictional Contact (3DFC) problems

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Contents

Purpose of the document

The goal of this work is to set up a collection of 3D Frictional Contact (3DFC) problems. The collection will provide a standard framework for testing available and new algorithms for solving discrete frictional contact problems.

Notation

Let us denote by the integer n_c the number of contacts. The integer n is the number of degree of freedom of the system and $m = 3n_c$ the number of unknown variables at contacts.

For each contact $\alpha \in \{1, \dots n_c\}$, the local velocity is denoted by $u^\alpha \in \mathbb{R}^3$. Its normal part is denoted by $u^\alpha_{\mathrm{N}} \in \mathbb{R}$ and its tangential part $u_{\mathrm{T}} \in \mathbb{R}^2$. One gets

$$u^\alpha = \begin{bmatrix} u^\alpha_{\mathrm{N}} \\ u^\alpha_{\mathrm{T}} \end{bmatrix} \tag{1}$$

The vector u collects all the local velocity at each contact

$$u = [[u^\alpha]^T, \alpha = 1 \dots n_c]^T \tag{2}$$

respectively for the normal part u_{N}

$$u_{\mathrm{N}} = [u^\alpha_{\mathrm{N}}, \alpha = 1 \dots n_c]^T, \tag{3}$$

and its tangential a part as

$$u_{\mathrm{T}} = [[u^\alpha_{\mathrm{T}}]^T, \alpha = 1 \dots n_c]^T. \tag{4}$$

for a contact α, the modified local velocity, denoted by $\hat{u}^\alpha$, is defined by

$$\hat{u}^\alpha = u^\alpha + \left[\begin{array}{c} \mu \|u^\alpha_{\mathrm{T}}\| \\ 0 \\ 0 \end{array} \right]^T \tag{5}$$

The vector $\hat{u}$ collects all the modified local velocity at each contact

$$\hat{u} = [[\hat{u}^\alpha]^T, \alpha = 1 \dots n_c]^T \tag{6}$$

For each contact α, the reaction vector $r^\alpha \in \mathbb{R}^3$ is also decomposed in its normal part $r^\alpha_{\mathrm{N}} \in \mathbb{R}$ and the tangential part $r_{\mathrm{T}} \in \mathbb{R}^2$ as

$$r^\alpha = \left[\begin{array}{c} r^\alpha_{\mathrm{N}} \\ r^\alpha_{\mathrm{T}} \end{array} \right] \tag{7}$$

The Coulomb friction cone for a contact α is defined by

$$C^\alpha_{\mu^\alpha} = \{ r^\alpha, \|r^\alpha_{\mathrm{T}}\| \leqslant \mu^\alpha |r^\alpha_{\mathrm{N}}| \} \tag{8}$$

and the set $C^{\alpha,\star}_{\mu^\alpha}$ is its dual.

The set C_μ is the cartesian product of Coulomb's friction cone at each contact, that

$$C_\mu = \prod_{\alpha = 1 \dots n_c} C^\alpha_{\mu^\alpha} \tag{9}$$

and $C^\star_\mu$ is dual.

1 Linear discrete problems with Coulomb's friction and unilateral contact

1.1 Reduced discrete problem. 3DFC problem

Definition 1 (Frictional contact problem (3DFC))**.** *Given*

- *a symmetric semi–positive definite matrix* $W \in \mathbb{R}^{m\times m}$
- *a vector* $q \in \mathbb{R}^m$,
- *a vector of coefficients of friction* $\mu \in \mathbb{R}^{n_c}$

the 3DFC problem is to find two vectors $u \in \mathbb{R}^m$ *and* $r \in \mathbb{R}^m$, *denoted by* $\mathrm{3DFC}(W, q, \mu)$ *such that*

$$\begin{cases} \hat{u} = Wr + q + \left[\begin{bmatrix} \mu^\alpha \|u^\alpha_T\| \\ 0 \\ 0 \end{bmatrix}^T, \alpha = 1 \ldots n_c \right]^T \\ \\ C^\star_\mu \ni \hat{u} \perp r \in C_\mu \end{cases} \tag{10}$$

□

1.2 Global/local discrete problem. G3DFC problem

Definition 2 (Global 3DFrictional contact problem (G3DFC))**.** *Given*

- *a symmetric positive definite matrix* $M \in \mathbb{R}^{n\times n}$
- *a vector* $f \in \mathbb{R}^n$,
- *a matrix* $H \in \mathbb{R}^{n\times m}$
- *a vector* $w \in \mathbb{R}^m$,
- *a vector of coefficients of friction* $\mu \in \mathbb{R}^{n_c}$

the Global 3DFC problem is to find three vectors $v \in \mathbb{R}^n$, $u \in \mathbb{R}^m$ *and* $r \in \mathbb{R}^m$, *denoted by* $\mathrm{G3DFC}(M, H, f, w, \mu)$ *such that*

$$\begin{cases} Mv = Hr + f \\ \\ \hat{u} = H^T v + w + \left[\begin{bmatrix} \mu^\alpha \|u^\alpha_T\| \\ 0 \\ 0 \end{bmatrix}^T, \alpha = 1 \ldots n_c \right]^T \\ \\ C^\star_\mu \ni \hat{u} \perp r \in C_\mu \end{cases} \tag{11}$$

□

1.3 Global Mixed Frictional contact problem (GM3DFC)

Definition 3 (Global Mixed Frictional contact problem (GM3DFC)). *Given*

- *a symmetric positive definite matrix* $M \in {\rm I\!R}^{n \times n}$
- *a vector* $f \in {\rm I\!R}^n$,
- *a matrix* $H \in {\rm I\!R}^{n \times m}$
- *a matrix* $G \in {\rm I\!R}^{n \times p}$
- *a vector* $w \in \mathbb{R}^m$,
- *a vector* $b \in \mathbb{R}^p$,
- *a vector of coefficients of friction* $\mu \in \mathbb{R}^{n_c}$

the Global Mixed 3DFC problem is to find four vectors $v \in {\rm I\!R}^n$, $u \in \mathbb{R}^m$, $r \in \mathbb{R}^m$ *and* $\lambda \in \mathbb{R}^p$ *denoted by* $\mathrm{GM3DFC}(M, H, G, w, b, \mu)$ *such that*

$$\begin{cases} Mv = Hr + G\lambda + f \\ \\ G^T v + b = 0 \\ \\ \hat{u} = H^T v + w + \left[\left[\begin{array}{c} \mu \| u^\alpha_T \| \\ 0 \\ 0 \end{array} \right]^T, \alpha = 1 \dots n_c \right]^T \\ \\ C^\star_\mu \ni \hat{u} \perp r \in C_\mu \end{cases} \tag{12}$$

□

1.4 Mixed Frictional contact problem (M3DFC)

Definition 4 (Mixed 3DFrictional contact problem (M3DFC)). *Given*

- *a matrix* $W \in {\rm I\!R}^{m \times m}$
- *a matrix* $V \in {\rm I\!R}^{m \times p}$
- *a matrix* $S \in {\rm I\!R}^{p \times m}$
- *a matrix* $R \in {\rm I\!R}^{p \times p}$
- *a vector* $q \in \mathbb{R}^m$,
- *a vector* $s \in \mathbb{R}^p$,
- *a vector of coefficients of friction* $\mu \in \mathbb{R}^{n_c}$

the Mixed 3DFC problem is to find three vectors $u \in \mathbb{R}^m$, $r \in \mathbb{R}^m$ and $\lambda \in \mathbb{R}^p$ denoted by M3DFC(S, R, V, W, q, s, μ) *such that*

$$\begin{cases} Sr + R\lambda + s = 0 \\ \\ \hat{u} = Wr + V\lambda + q + \left[\left[\begin{array}{c} \mu^\alpha \|u_T^\alpha\| \\ 0 \\ 0 \end{array} \right]^T, \alpha = 1 \ldots n_c \right]^T \\ \\ C_\mu^\star \ni \hat{u} \perp r \in C_\mu \end{cases} \tag{13}$$

□

1.5 Remarks

Note that the previous problems may be an instance of quasi-static problems: the matrix M plays the role of the stiffness matrix and the vector u is a position or a displacement.

2 Measuring errors

3 Detailed implementation

3.1 File format

The proposed file format for storing and managing data is the HDF5 data format
`http://www.hdfgroup.org/HDF5`
The data name should be defined as close as possible to the definition of this document.

3.2 Matrix storage

Three matrix storages are considered :

1. dense format
2. sparse format : row compressed format
3. sparse matrix of 3x3 dense matrices. (sparse block matrix)

The last format could be deduced from the standard sparse format. To be discussed.

3.3 C implementation

A C implementation will be proposed for reading and writing each of 3DFC problems into HDF5 files.

The storage of dense matrices will be in column major mode (FORTRAN mode).

4 Additional description of the problems

The following additional information should be added in a reference document and in the HDF5 file.

- `TITLE` : a title for the problem
- `DESCRIPTION` : The field of application. Short description on how the problem is generated.
- `MATRIX_INFO` : The sparsity and the conditioning of the matrices.
- `MATH_INFO` : Existence, uniqueness of solutions.
- ...

The following data can be optionally added in the HDF5 file

- `SOLUTION` : A reference solution
- `INITIAL_GUESS` : A initial guess
- ...

5 List of problems