# 1 CFLs

**note 1**: $\Sigma = \{a, b\}$
**note 2**: $n_x(a)$ means the number of a's in the string x.

- $\{x | n_x(a) = n_x(b)\}$
  corresponding CFG :

$$S \rightarrow aSbS | bSaS | \epsilon$$

- $\{x | n_x(a) \geq n_x(b)\}$
  corresponding CFG:

$$S \rightarrow SS | aSb | bSa | a | \epsilon$$

  not surely right version :

$$\begin{aligned} S &\rightarrow SaS | T \\ T &\rightarrow aTbT | bTaT | \epsilon \end{aligned}$$

- $\{x | n_x(a) = 2n_x(b)\}$
  corresponding CFG:

$$S \rightarrow SaSaSbS | SaSbSaS | SbSaSaS | \epsilon$$

  using similar method, we can easily generalize this into k.
  important lemma needed in the proof: substring $a^x ba^y$, $(x + y = k)$ can always be found in any non-empty string s in the language.

- complement of $a^n b^n$
  corresponding CFG:

$$\begin{aligned} S &\rightarrow aSb | bY | Ya \\ Y &\rightarrow bY | aY | \epsilon \end{aligned}$$

- $\left\{a^i b^j | i \neq j \land i \neq 2j\right\}$
  corresponding CFG(not surely right):

$$\begin{aligned} S &\rightarrow A | B | N \\ E &\rightarrow aEb | \epsilon \\ A &\rightarrow Eb | aAb | Ab \\ B &\rightarrow aaabb | aBb | aaBb \\ M &\rightarrow aaMb | \epsilon \\ N &\rightarrow aM | aNb | aN \end{aligned}$$

- $\{xyz | x,\ z \in \Sigma^\star,\ y \in \Sigma^\star b \Sigma^\star\ and\ |x| = |z| \geq |y|\}$
  corresponding CFG (not surely right):

$$\begin{aligned} S &\rightarrow A | B | C \\ A &\rightarrow EEEbEE | EAE | EEAE \\ B &\rightarrow EEbEEE | EBE | EBEE \\ C &\rightarrow EbE | ECE \\ E &\rightarrow a | b \end{aligned}$$