Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME Fuzail

CENTRE NUMBER

CANDIDATE NUMBER

* 1 2 1 4 6 4 7 7 5 3 *

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **May/June 2020**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Blank pages are indicated.

1 Find the set of values of x for which $2(3^{1-2x}) < 5^x$. Give your answer in a simplified exact form. [4]

$\log 2 + \log 3^{1-2x} < \log 5^x$

$\log 2 + (1-2x)\log 3 < x\log 5$

$(1-2x)\log 3 - x\log 5 < -\log 2$

$\log 3 - 2x\log 3 - x\log 5 < -\log 2$

$\log 3 + \log 2 < 2x\log 3 + x\log 5$

$\log 3 + \log 2 < x(2\log 3 + \log 5)$

$x > \dfrac{\log 6}{\log 45}$

2 **(a)** Expand $(2 - 3x)^{-2}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [4]

$2^{-2}\left(1 - \frac{3x}{2}\right)^{-2}$

$\frac{1}{4}\left(1 - \frac{3x}{2}\right)^{-2}$

$\frac{1}{4}\left[1 + (-2)(-\frac{3}{2}x) + \frac{(-2)(-3)(-\frac{3}{2}x)^2}{2!}\right]$

$\frac{1}{4}\left(1 + 3x + \frac{27}{4}x^2\right]$

$\frac{1}{4} + \frac{3x}{4} + \frac{27x^2}{16}$

(b) State the set of values of x for which the expansion is valid. [1]

$|-\frac{3}{2}x| < 1$

$\frac{3}{2}|x| < 1$

$|x| < \frac{2}{3}$

3 Express the equation $\tan(\theta + 60°) = 2 + \tan(60° - \theta)$ as a quadratic equation in $\tan\theta$, and hence solve the equation for $0° \leqslant \theta \leqslant 180°$. [6]

$$\frac{\tan\theta + \sqrt{3}}{1 - \sqrt{3}\tan\theta} = 2 + \frac{\sqrt{3} - \tan\theta}{1 + \sqrt{3}\tan\theta}$$

$$\frac{(\tan\theta + \sqrt{3})}{(1 - \sqrt{3}\tan\theta)} - \frac{(\sqrt{3} - \tan\theta)}{(1 + \sqrt{3}\tan\theta)} = 2$$

$$\frac{(\tan\theta + \sqrt{3})(1 + \sqrt{3}\tan\theta) - (\sqrt{3} - \tan\theta)(1 - \sqrt{3}\tan\theta)}{1 - 3\tan^2\theta} = 2$$

$$\frac{\tan\theta + \sqrt{3}\tan^2\theta + \sqrt{3} + 3\tan\theta - (\sqrt{3} - 3\tan\theta - \tan\theta + \sqrt{3}\tan^2\theta)}{1 - 3\tan^2\theta} = 2$$

$$4\tan\theta \cancel{+ \sqrt{3}\tan^2\theta} \cancel{+\sqrt{3}} \cancel{-\sqrt{3}} + 4\tan\theta \cancel{- \sqrt{3}\tan^2\theta} = 2 - 6\tan^2\theta$$

$$6\tan^2\theta + 8\tan\theta - 2 = 0$$

4 The curve with equation $y = e^{2x}(\sin x + 3\cos x)$ has a stationary point in the interval $0 \leqslant x \leqslant \pi$.

(a) Find the x-coordinate of this point, giving your answer correct to 2 decimal places. [4]

$\frac{dy}{dx} = e^{2x}(\cos x - 3\sin x) + 2e^{2x}(\sin x + 3\cos x)$

$e^{2x}\cos x - 3e^{2x}\sin x + 2e^{2x}\sin x + 6e^{2x}\cos x = 0$

$e^{2x}(7\cos x - \sin x) = 0$

$7\cos x - \sin x = 0$

$7 - \tan x = 0$

$\tan x = 7$

$x = 1.43$

(b) Determine whether the stationary point is a maximum or a minimum. [2]

$\frac{d^2y}{dx^2} = e^{2x}(-7\sin x - \cos x) + 2e^{2x}(-7\sin x - \cos x)$

sub $x = 1.43$

$-ve \therefore$ maximum.

5 **(a)** Find the quotient and remainder when $2x^3 - x^2 + 6x + 3$ is divided by $x^2 + 3$. [3] 2

$$
\begin{array}{r|l}
 & 0x^2 + 2x - 1 \\
\hline
x^2 + 3 & 0x^4 + 2x^3 - x^2 + 6x + 3 \\
 & \underline{0 + 2x^3 + 6x} \\
 & 0 - x^2 \cancel{- 6x + 6x} + 3 \\
 & - \quad \underline{-x^2 \quad - 3} \\
 & 0 \quad + \quad 6
\end{array}
$$

quotient = $2x - 1$

remainder = 6

(b) Using your answer to part **(a)**, find the exact value of $\int_1^3 \frac{2x^3 - x^2 + 6x + 3}{x^2 + 3}\,dx$. [5]

$\int_1^3 2x - 1 + \frac{6}{x^2 + 3}\, dx$

$\int_1^3 2x - 1 + 6\int_1^3 \frac{1}{3+x^2}\, dx$

$x^2 - x\Big|_1^3 + 6\left(\frac{1}{\sqrt{3}} \times \tan^{-1}\left(\frac{x}{\sqrt{3}}\right)\right)\Big|_1^3$

$(9-3) - 0 + 6\left(\frac{\tan^{-1}(\sqrt{3})}{\sqrt{3}} - \frac{\tan^{-1}(\sqrt{3}/3)}{\sqrt{3}}\right)$

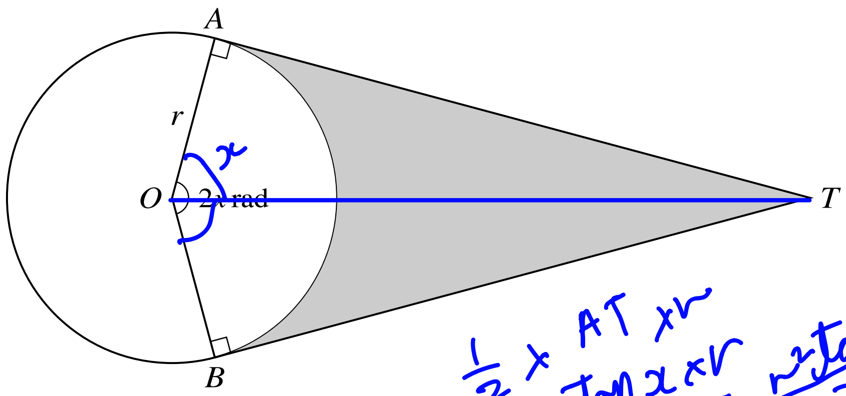
6

$6 + 6\left(\frac{\pi}{3\sqrt{3}} - \frac{\pi}{6\sqrt{3}}\right)$

$6 + \frac{2\pi}{\sqrt{3}} - \frac{\pi}{\sqrt{3}}$

$6 + \frac{\pi}{\sqrt{3}}$

6



$\frac{1}{2} \times AT \times r$

$\frac{1}{2} \times r\tan x \times r = \frac{r^2\tan x}{2} + \frac{r^2\tan x}{2}$

$= r^2\tan x - 2xr$

The diagram shows a circle with centre O and radius r. The tangents to the circle at the points A and B meet at T, and angle AOB is $2x$ radians. The shaded region is bounded by the tangents AT and BT, and by the minor arc AB. The area of the shaded region is equal to the area of the circle.

(a) Show that x satisfies the equation $\tan x = \pi + x$. [3]

shaded region $= (AT \times r) - \frac{1}{2}r^2(2x)$ $\quad \tan x = \frac{AT}{r}$

$AT = r\tan x \rightarrow r^2\tan x - \frac{r^2 \cancel{2} x}{\cancel{2}}$

Area of circle $= \pi r^2$

$\cancel{r^2}(\tan x - x) = \pi \cancel{r^2}$

$\underline{\underline{\tan x = \pi + x}}$

(b) This equation has one root in the interval $0 < x < \frac{1}{2}\pi$. Verify by calculation that this root lies between 1 and 1.4. [2]

$x = 1$ $\qquad$ $x = 1.4$

LHS RHS $\qquad$ LHS RHS

$1.557 < 4.142$ $\qquad$ $5.798 > 4.542$

$\therefore$ since we have sign change, x lie between $x = 1$, $x = 1.4$

(c) Use the iterative formula

$$x_{n+1} = \tan^{-1}(\pi + x_n)$$

to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

$x_{n+1} = \tan^{-1}(\pi + x_n)$

$x_0 = 1.2$

$x_1 = 1.3441$

$x_2 = 1.3515$

$x_3 = 1.3518$

$x_4 = 1.3518$

$\therefore x = 1.35$

7 Let $f(x) = \dfrac{\cos x}{1 + \sin x}$.

(a) Show that $f'(x) < 0$ for all x in the interval $-\frac{1}{2}\pi < x < \frac{3}{2}\pi$. [4]

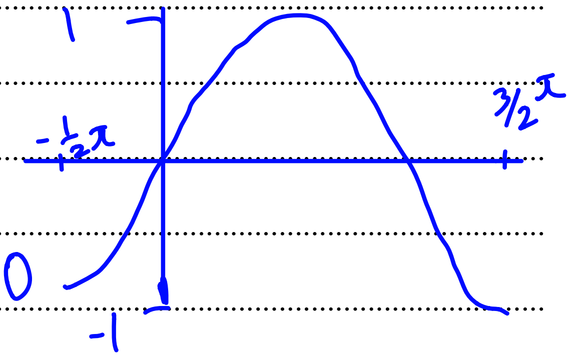
$f(x) = \dfrac{\cos x}{1+\sin x}$ v v

$f'(x) = \dfrac{(1+\sin x)(-\sin x) - (\cos x)(\cos x)}{(1+\sin x)(1+\sin x)}$

$= \dfrac{-\sin x - \sin^2 x - \cos^2 x}{1 + 2\sin x + \sin^2 x}$

$= \dfrac{-1(\cancel{\sin x + 1})}{(1+\sin x)^{\cancel{2}}}$

$f'(x) = \dfrac{-1}{1+\sin x}$



$\therefore f'(x) < 0$ since $(1+\sin x) > 0$

(b) Find $\int_{\frac{1}{6}\pi}^{\frac{1}{2}\pi} f(x)\,dx$. Give your answer in a simplified exact form. [4]

$f(x) = \dfrac{\cos x}{1+\sin x}$

$\displaystyle\int_{\frac{1}{6}\pi}^{\frac{1}{2}\pi} \frac{\cos x}{1+\sin x}\,dx$

$u = 1+\sin x$

$\dfrac{du}{dx} = \cos x$

$dx = \dfrac{du}{\cos x}$

$= \displaystyle\int_{1.5}^{2} \frac{\cancel{\cos x}}{u} \times \frac{du}{\cancel{\cos x}}$

when $x = \frac{1}{6}\pi$ $u = \frac{3}{2}$

$x = \frac{1}{2}\pi$ $u = 2$

$\ln|u|\Big|_{1.5}^{2}$

$\ln \dfrac{2}{1.5} = \ln \dfrac{4}{3}$

8 A certain curve is such that its gradient at a point (x, y) is proportional to $\frac{y}{x\sqrt{x}}$. The curve passes through the points with coordinates $(1, 1)$ and $(4, e)$. (e^1)

(a) By setting up and solving a differential equation, find the equation of the curve, expressing y in terms of x. [8]

$$\frac{dy}{dx} \propto \frac{y}{x\sqrt{x}}$$

$$\frac{dy}{dx} = \frac{k\,y}{x\sqrt{x}}$$

$$\int \frac{1}{y}\,dy = k\int \frac{1}{x\sqrt{x}}\,dx$$

$x \times x^{0.5}$

$x^{1.5}$

$$\ln y = k \int x^{-\frac{3}{2}}\,dx$$

$$\ln y = k\,\frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + C$$

$$\ln y = -2k\,x^{-\frac{1}{2}} + C$$

whe (1, 1)

$\ln 1 = -2k + C$

$c = 2k$

when (4, e)

$1 = -2k\left(\frac{1}{2}\right) + 2k \longrightarrow y = e^{\left(-\frac{2}{\sqrt{x}}+2\right)}$

$1 = -k + 2k$

$k = 1$

$\therefore C = 2$

(b) Describe what happens to y as x tends to infinity. [1]

$y = e^{-\frac{2}{\sqrt{3x}} + 2}$

$y = e^2$

$\therefore$ Value of y will approach e^2

9 With respect to the origin O, the vertices of a triangle ABC have position vectors

$$\overrightarrow{OA} = 2\mathbf{i} + 5\mathbf{k}, \quad \overrightarrow{OB} = 3\mathbf{i} + 2\mathbf{j} + 3\mathbf{k} \quad \text{and} \quad \overrightarrow{OC} = \mathbf{i} + \mathbf{j} + \mathbf{k}.$$

(a) Using a scalar product, show that angle ABC is a right angle. [3]

$AB = OB - OA$

$= \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 \\ 0 \\ 5 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$

$BA = \begin{pmatrix} -1 \\ -2 \\ 2 \end{pmatrix}$

$BC = \begin{pmatrix} -2 \\ -1 \\ -2 \end{pmatrix}$

$BC = OC - OB$

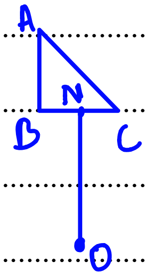
$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -2 \\ -1 \\ -2 \end{pmatrix}$

$\dfrac{BA \cdot BC}{|BA| \cdot |BC|} = \dfrac{2 + 2 - 4}{|BA| \cdot |BC|} = 0$

$BA \cdot BC = 0$

$\therefore$ right angle

(b) Show that triangle ABC is isosceles. [2]



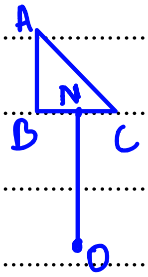
$|BA| = |BC|$

$\sqrt{1^2 + 2^2 + 2^2} = \sqrt{1^2 + 2^2 + 2^2}$

$3 = 3$

$\therefore AB = BC$

(c) Find the exact length of the perpendicular from O to the line through B and C. [4]



$ON = OB + BN$

$= \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ -1 \\ -2 \end{pmatrix}$

$= \begin{pmatrix} 3-2\lambda \\ 2-\lambda \\ 3-2\lambda \end{pmatrix} \times \begin{pmatrix} -2 \\ -1 \\ -2 \end{pmatrix} = 0$

$-6 + 4\lambda \quad -2 + \lambda \quad -6 + 4\lambda = 0$

$9\lambda = 14$

$\lambda = \frac{14}{9}$

$ON = \begin{pmatrix} -1/9 \\ 4/9 \\ -1/9 \end{pmatrix}$ $\sqrt{\left(\frac{1}{9}\right)^2 + \left(\frac{4}{9}\right)^2 + \left(\frac{1}{9}\right)^2} = \frac{\sqrt{2}}{3}$

10 **(a)** The complex number u is defined by $u = \frac{3\mathrm{i}}{a + 2\mathrm{i}}$, where a is real.

(i) Express u in the Cartesian form $x + \mathrm{i}y$, where x and y are in terms of a. [3]

$$\frac{3i}{a+2i} \times \frac{a-2i}{a-2i}$$

$$\frac{3ai - 6(-1)}{a^2 - 4(-1)}$$

$$= \frac{3ai + 6}{a^2 + 4}$$

$$= \frac{6}{a^2+4} + \frac{3ai}{a^2+4}$$

(ii) Find the exact value of a for which $\arg u^* = \frac{1}{3}\pi$. [3]

$$\arg = \tan^{-1}\left(\frac{b}{a}\right)$$

$$\frac{b}{a} = \frac{-3a}{a^2+4} \div \frac{6}{a^2+4}$$

$$= \frac{-3a \times \cancel{(a^2+4)}}{\cancel{(a^2+4)} \times 6}$$

$$= -\frac{1a}{2}$$

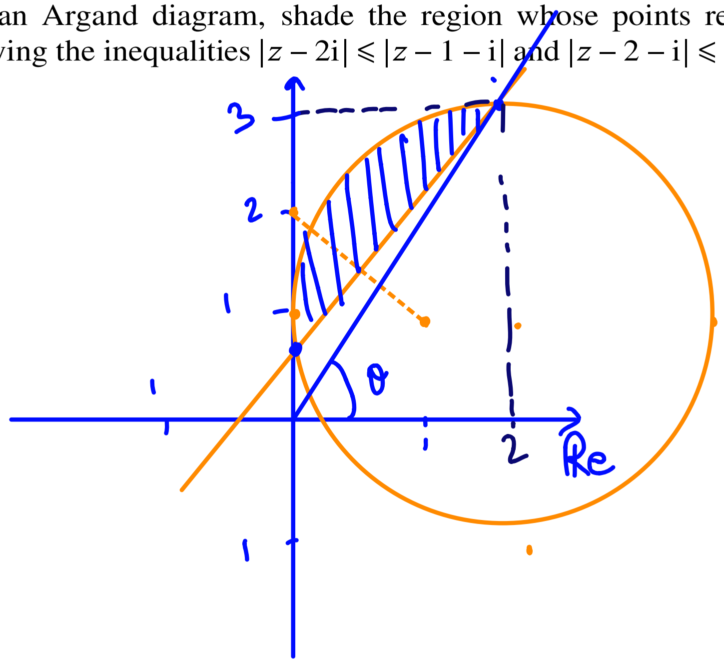
$$\tan^{-1}\left(-\frac{1a}{2}\right) = \frac{1}{3}\pi$$

$$-\frac{1}{2}a = \tan\frac{1}{3}\pi$$

$$a = -2 \times \tan\frac{1}{3}\pi$$

$$= -2\sqrt{3}$$

(b) **(i)** On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - 2i| \leqslant |z - 1 - i|$ and $|z - 2 - i| \leqslant 2$. [4]



(ii) Calculate the least value of $\arg z$ for points in this region. [2]

$2+3i$

$\arg z = \tan^{-1}\left(\frac{3}{2}\right) = 0.983$ radians

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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