Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

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CENTRE NUMBER

CANDIDATE NUMBER

4382153981

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2017**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **19** printed pages and **1** blank page.

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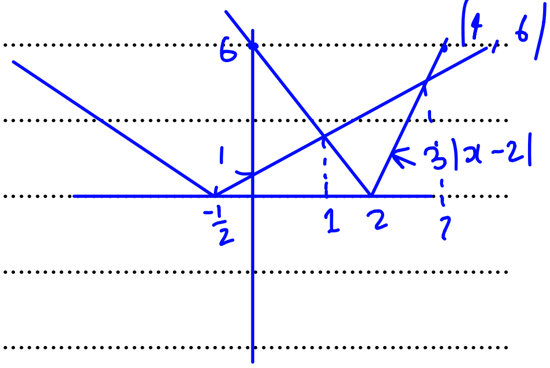
CAMBRIDGE International Examinations

1 Solve the inequality $|2x + 1| < 3|x - 2|$. [4]



$y = 2x + 1 = -3x + 6$

$5x = 5$

$x = 1$

$y = 2x + 1 = 3x - 6$

$7 = x$

x	-1, 0, 1, 2
y	

$\therefore\ x < 1$ or $x > 7$

2 Expand $\frac{1}{\sqrt[3]{(1+6x)}}$ in ascending powers of x, up to and including the term in x^3, simplifying the coefficients. [4]

Any way to check??

$(1+6x)^{-\frac{1}{3}}$

$1 + \left(-\frac{1}{3}\right)(6x) + \frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)(6x)^2}{2!} + \frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)\left(-\frac{7}{3}\right)(6x)^3}{3!}$

$1 - 2x + 8x^2 + -\frac{112}{3}x^3$

3 It is given that $x = \ln(1 - y) - \ln y$, where $0 < y < 1$.

(i) Show that $y = \dfrac{e^{-x}}{1 + e^{-x}}$. [2]

$$\ln\left(\frac{1-y}{y}\right) = x$$

$$\frac{1-y}{y} = e^{x}$$

$$1 - y = ye^{x}$$

$$ye^{x} + y = 1$$

$$y(e^{x}+1) = 1$$

$$y = \frac{1}{e^{x}+1}$$

Divide RHS's numerator and denominator by e^{x}

$$y = \frac{\left(\frac{1}{e^{x}}\right)}{\left(\frac{e^{x}+1}{e^{x}}\right)}$$

$$y = \frac{e^{-x}}{1+e^{-x}}$$

(ii) Hence show that $\int_0^1 y\,dx = \ln\left(\frac{2e}{e+1}\right)$. [4]

$y = \frac{e^{-x}}{1+e^{-x}}$

$\int_0^1 \frac{e^{-x}}{1+e^{-x}}\,dx$ $\qquad \frac{d}{dx}(1+e^{-x}) = -e^{-x}$

$-\int_0^1 \frac{-e^{-x}}{1+e^{-x}}\,dx$

let $f(x) = -\ln(1+e^{-x})$

$f(1) = -\ln(1+e^{-1})$

$f(0) = -\ln(1+e^{0}) = -\ln(2)$

$f(1) - f(0) \Rightarrow -\ln(1+e^{-1}) - (-\ln 2)$

$= \ln(2) - \ln(1+e^{-1})$

$\ln\left(\frac{2}{1+e^{-1}}\right)$

$= \ln\left(\frac{2}{1+\frac{1}{e}}\right)$

$= \ln\left(\frac{2}{\frac{e+1}{e}}\right)$

$= \ln\left(2 \times \frac{e}{e+1}\right)$

$= \ln\left(\frac{2e}{e+1}\right)$

4 The parametric equations of a curve are

$$x = \ln\cos\theta, \qquad y = 3\theta - \tan\theta,$$

where $0 \leqslant \theta < \frac{1}{2}\pi$.

(i) Express $\frac{dy}{dx}$ in terms of $\tan\theta$. [5]

$\frac{dy}{d\theta} = 3 - \sec^2\theta$

$= 3 - \frac{1}{\cos^2\theta} \Rightarrow \frac{3\cos^2\theta - 1}{\cos^2\theta}$

$\frac{dx}{d\theta} = \frac{1}{\cos\theta} \times -\sin\theta = -\frac{\sin\theta}{\cos\theta} = -\tan\theta$

$\frac{3\cos^2\theta - 1}{\cos^2\theta} \div \frac{-\sin\theta}{\cos\theta}$

$= \frac{(3\cos^2\theta - 1) \times \cancel{\cos\theta}}{-\sin\theta\cos^{\cancel{2}}\theta}$

$= \frac{3\cos^2\theta - 1}{-\sin\theta\cos\theta} = \frac{\cos^2\theta + 2\cos^2\theta - 1}{-\frac{1}{2}\sin 2\theta}$

$= \cos^2\theta + \cos 2\theta \div -\frac{1}{2}\sin 2\theta$

$= \cos^2\theta + \cos 2\theta \times \frac{-2}{\sin 2\theta}$

$= -\frac{2\cos^2\theta}{\sin 2\theta} - \frac{2\cos 2\theta}{\sin 2\theta}$

$= -\frac{\cancel{2}\cos^{\cancel{2}}\theta}{\cancel{2}\sin\theta\cancel{\cos\theta}} - 2\tan 2\theta$

$= -\frac{1}{\tan\theta} - 2\tan 2\theta$

$-\frac{1}{\tan\theta} - 2\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)$

$-\frac{1}{\tan\theta} - \frac{4\tan\theta}{1-\tan^2\theta}$

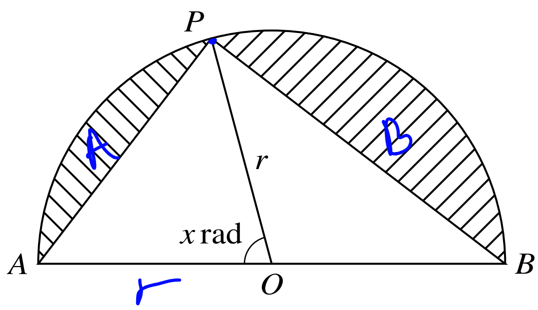
$= \frac{-1(1-\tan^2\theta) - 4\tan^2\theta}{(1-\tan^2\theta)\tan\theta}$

$= \frac{-1+\tan^2\theta - 4\tan^2\theta}{\tan\theta - \tan^3\theta}$

$= \frac{-1 - 3\tan^2\theta}{\tan\theta - \tan^3\theta}$

(ii) Find the exact y-coordinate of the point on the curve at which the gradient of the normal is equal to 1. [3]

5



The diagram shows a semicircle with centre O, radius r and diameter AB. The point P on its circumference is such that the area of the minor segment on AP is equal to half the area of the minor segment on BP. The angle AOP is x radians.

(i) Show that x satisfies the equation $x = \frac{1}{3}(\pi + \sin x)$. [3]

Area 'A' $= \frac{1}{2}r^2x - \frac{1}{2}r^2\sin x$

$= \frac{1}{2}r^2(x - \sin x)$

Area 'B' $= \frac{1}{2}r^2(\pi - x) - \frac{1}{2}r^2\sin(\pi - x)$

$= \frac{1}{2}r^2((\pi - x) - \sin \pi - x)$

$2A = B$

$2 \times \left(\frac{1}{2}r^2(x - \sin x)\right) = r^2(x - \sin x)$

$\cancel{r^2}(x - \sin x) = \frac{1}{2}\cancel{r^2}((\pi - x) - \sin(\pi - x))$

$x - \sin x = \frac{1}{2}(\pi - x) - \frac{1}{2}\sin(\pi - x)$

$\sin x = \sin \pi - x$

$x - \sin x = \frac{1}{2}\pi - \frac{1}{2}x - \frac{1}{2}\sin x$

$\frac{3}{2}x = \frac{1}{2}\pi + \frac{1}{2}\sin x$

$\frac{3}{2}x = \frac{1}{2}(\pi + \sin x)$

$x = \frac{\cancel{2}}{\cancel{2} \times 3}(\pi + \sin x)$

$x = \frac{1}{3}(\pi + \sin x)$

(ii) Verify by calculation that x lies between 1 and 1.5. [2]

LHS RHS

when $x=1$ $1 < 1.3277$

when $x=1.5$ $1.5 > 1.3197$

change of sign can be seen ∴ root lies b/w 1 and 1.5

(iii) Use an iterative formula based on the equation in part **(i)** to determine x correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

let $x_1 = 1.1$

$x_2 = \frac{1}{3}(\pi + \sin(1.1)) = 1.34427$

$x_3 = 1.37201$

$x_4 = 1.37397$

$x_5 = 1.37409$

$x_6 = 1.37410$

$x_7 = 1.37410$

$\therefore x = 1.374$

6 The plane with equation $2x + 2y - z = 5$ is denoted by m. Relative to the origin O, the points A and B have coordinates $(3, 4, 0)$ and $(-1, 0, 2)$ respectively.

$\because 5$

(i) Show that the plane m bisects AB at right angles. [5]

22 $\therefore$

$(\sqrt{2})^5 = 4\sqrt{2}$

A second plane p is parallel to m and nearer to O. The perpendicular distance between the planes is 1.

(ii) Find the equation of p, giving your answer in the form $ax + by + cz = d$. [3]

7 **Throughout this question the use of a calculator is not permitted.**

The complex numbers u and w are defined by $u = -1 + 7\mathrm{i}$ and $w = 3 + 4\mathrm{i}$.

(i) Showing all your working, find in the form $x + \mathrm{i}y$, where x and y are real, the complex numbers $u - 2w$ and $\frac{u}{w}$. [4]

$u - 2w = -1 + 7i - 2(3+4i)$

$-1 + 7i - 6 - 8i$

$\underline{-7 - i}$

$\frac{u}{w} = \frac{-1+7i}{3+4i} \times \frac{3-4i}{3-4i}$

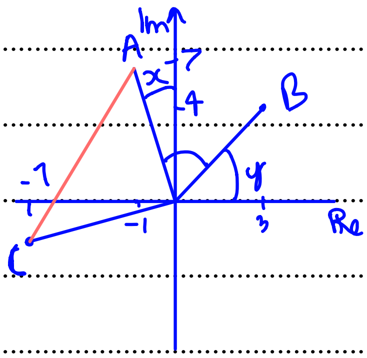
$= \frac{-3 + 4i + 21i - 28(-1)}{9 - 16(-1)}$

$= \frac{25 + 25i}{25}$

$\frac{u}{w} = \underline{1 + i}$

In an Argand diagram with origin O, the points A, B and C represent the complex numbers u, w and $u - 2w$ respectively.

(ii) Prove that angle $AOB = \frac{1}{4}\pi$. [2]



$AOB = x + \frac{\pi}{2} - y$

$x = \tan^{-1}(1/7)$

$y = \tan^{-1}(4/3)$

$\therefore \tan^{-1}\left(\frac{1}{7}\right) + \frac{\pi}{2} - \tan^{-1}\left(\frac{4}{3}\right)$

$= \frac{1}{4}\pi$

(iii) State fully the geometrical relation between the line segments OB and CA. [2]

they are parallel as their gradient is the same.

m of $CA = \frac{8}{6} = \frac{4}{3}$

m of $CA = \frac{4}{3}$

8 **(i)** By first expanding $2\sin(x - 30°)$, express $2\sin(x - 30°) - \cos x$ in the form $R\sin(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give the exact value of R and the value of α correct to 2 decimal places. [5]

4

$2\sin x\cos 30 - 2\cos x\sin 30 - \cos x$

$\sqrt{3}\sin x - \cos x - \cos x$

$\sqrt{3}\sin x - 2\cos x$

$\alpha = \tan^{-1}\left(\frac{-2}{\sqrt{3}}\right) = -49.1066 \approx -49.12$

$R = \sqrt{(\sqrt{3})^2 + (-2)^2}$

$\sqrt{7}\sin(x + 49.12)$

(ii) Hence solve the equation

$$2\sin(x - 30°) - \cos x = 1,$$

for $0° < x < 180°$. [3] 0

$\sqrt{7}\ \sin(x + 49.12) = 1$

$x + 49.12 = \sin^{-1}\left(\frac{1}{\sqrt{7}}\right)$

$\therefore\ x + 49.12 = 22.2076$ or 157.7923

$x = 22.2076 - 49.12$ X

$x = 157.7923 - 49.12$

$x = 108.67$

9 **(i)** Express $\dfrac{1}{x(2x+3)}$ in partial fractions. [2]

$$\frac{A}{x} + \frac{B}{2x+3}$$

$$1 = A(2x+3) + Bx$$
$$1 = 2Ax + 3A + Bx$$

$1 = 3A$

$A = \frac{1}{3}$ $\quad 2A + B = 0$

$$2\left(\frac{1}{3}\right) + B = 0$$

$$\frac{1}{3x} - \frac{2}{3(2x+3)}$$

$$B = -\frac{2}{3}$$

(ii) The variables x and y satisfy the differential equation

$$x(2x+3)\frac{dy}{dx} = y,$$

and it is given that $y = 1$ when $x = 1$. Solve the differential equation and calculate the value of y when $x = 9$, giving your answer correct to 3 significant figures. [7]

$$\int \frac{1}{x(2x+3)}\,dx = \int \frac{1}{y}\,dy$$

$$\int \frac{1}{3x}\,dx - \frac{1}{3}\int \frac{2}{2x+3}\,dx = \ln y$$

$$\frac{1}{3}\ln x - \frac{1}{3}\ln(2x+3) = \ln y + c$$

$$\frac{1}{3}\ln 1 - \frac{1}{3}\ln(5) = 0 + C$$

$$0 - \frac{1}{3}\ln 5 = C$$

$$\frac{1}{3}\ln\frac{9}{21} = \ln y - \frac{1}{3}\ln 5$$

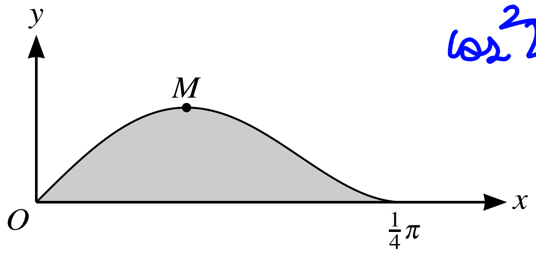
$$\frac{1}{3}\ln\frac{3}{7} + \frac{1}{3}\ln 5 = \ln y$$

$$\frac{1}{3}\ln\frac{15}{7} = \ln y$$

$$\ln y = 0.254\ 0\ 47$$

$$y = 1.29$$

10



$\cos^2 2x = (2\cos^2 x - 1)^2$

0.08954

The diagram shows the curve $y = \sin x \cos^2 2x$ for $0 \leqslant x \leqslant \frac{1}{4}\pi$ and its maximum point M.

4

(i) Using the substitution $u = \cos x$, find by integration the exact area of the shaded region bounded by the curve and the x-axis. [6]

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$(\cos 2x)^2 = (2\cos^2 x - 1)^2$

$= (2u^2 - 1)^2$

$(2u^2-1)(2u^2-1)$

$4u^4 - 4u^2 + 1$

$\cancel{-}\int_{\frac{\sqrt{2}}{2}} \cancel{\sin x}(4u^4 - 4u^2 + 1)\frac{du}{\cancel{-\sin x}}$

when $x = 0$ $u = 1$

$x = \frac{1}{4}\pi$ $u = \frac{\sqrt{2}}{2}$

$\int_{\frac{\sqrt{2}}{2}}^{1} 4u^4 - 4u^2 + 1 \, du$

$\left(\frac{4u^5}{5} - \frac{4u^3}{3} + u\right)_{\frac{\sqrt{2}}{2}}^{1}$

$\sin x \cos^2 2x$

$\boxed{2}$

(ii) Find the x-coordinate of M. Give your answer correct to 2 decimal places. [6]

$\frac{dy}{dx} = 0 = \sin x\left(2\cos(2x) \times 2 - \sin(2x)\right) + \cos^2 2x\,(\cos x)$

$= \sin x\left(-4\cos 2x \sin 2x\right) + \cos^2 2x \cos x$

$= -4\sin 2x \cos 2x \sin x + \cos^2 2x \cos x$

$= -4(2\sin x\cos x)(2\cos^2 x - 1)(\sin x) + (2\cos^2 x - 1)\cos x$

$= -4\left(4\cos^3 x \sin x - 2\sin x \cos x\right)(\sin x) + \left(2\cos^3 x - \cos x\right)$

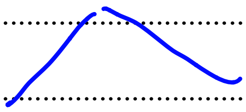
$= -16\cos^3 x \sin^2 x + 8\sin^2 x \cos x + 2\cos^3 x - \cos x = 0$

8 $= 2\cos^3 x\left(-8\sin^2 x + 1\right) - \cos x\left(-8\sin^2 x + 1\right) = 0$

9 $\left(2\cos^3 - \cos x\right)\left(-8\sin^2 x + 1\right) = 0$

$2\cos^3 x - \cos x = 0$

$\cos x\left(2\cos^2 - 1\right) = 0$



$2\cos^2 x - 1 = 0$ $\quad$ $\cos x = 0$

$\cos^2 x = \frac{1}{2}$ $\quad$ $x = \frac{1}{2}\pi$

$x = \cos^{-1}\left(\pm\sqrt{\frac{1}{2}}\right)$ $\quad$ X

$= \frac{1}{4}\pi, \frac{3}{4}\pi$

X X

$8\sin^2 x = 1$

$\sin^2 x = \frac{1}{8}$

$x = \sin^{-1}\left(\pm\sqrt{\frac{1}{8}}\right)$

$= 0.361$, -0.361 X

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