Formulas

- $\psi = \psi_0 \sin \omega t$ or $\psi = \psi_0 \sin \omega t$

Amplitude

Equation of a wave

- $x = x_0 \sin \omega t$ or $x = x_0 \cos \omega t$

$\therefore v = x_0 \omega \cos \omega t$ or $v = -x_0 \omega \sin \omega t$

$\therefore a = -x_0 \omega^2 \sin \omega t$ or $a = -x_0 \omega^2 \cos \omega t$

$$\therefore a = -\omega^2 x$$

- $v = \omega \sqrt{x_0^2 - x^2}$

$V_{max} = V_0 = \omega x_0$

- $K_e = \frac{1}{2} m \omega^2 (x_0^2 - x^2)$

- $\frac{K_{e1}}{(x_0^2 - x_1^2)} = \frac{K_{e2}}{(x_0^2 - x_2^2)}$ as $K_e \propto (x_0^2 - x^2)$

- $P_e = U = \frac{1}{2} m\omega^2 x^2$

- $\frac{P_{e_1}}{x_1^2} = \frac{P_{e_2}}{x_2^2}$

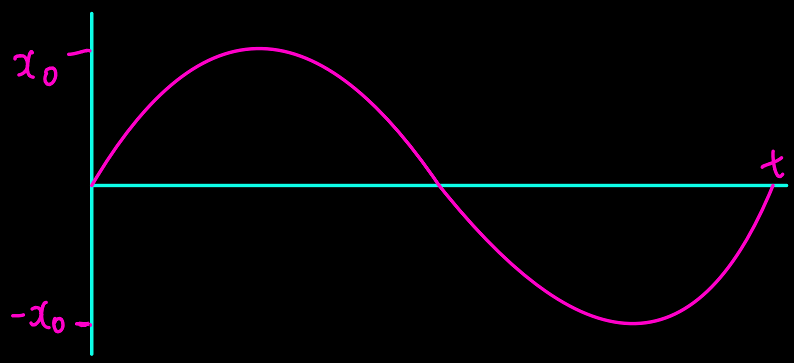
- Total ~~Energy~~ $= \frac{1}{2} m\omega^2 x_0^2$

- phase difference $= \frac{2\pi}{\lambda} \times$ (path difference)

Equation of a wave

- If the wave starts from mean position

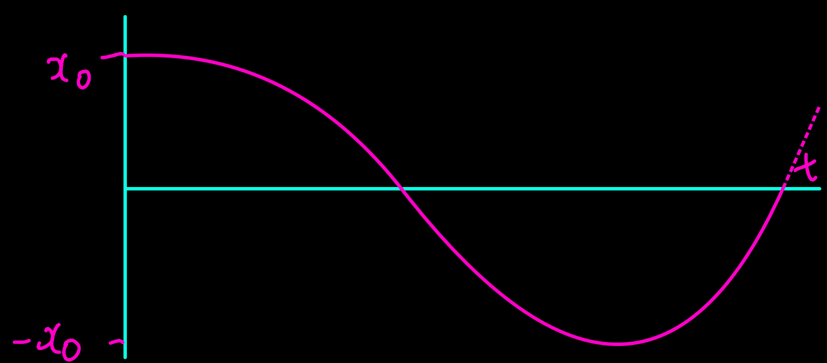
$x = x_0 \sin \omega t$



$x = x_0 \cos \omega t$

- If the wave starts from Max or Min position

$x = x_0 \cos \omega t$



- 1 oscillation is one complete movement from the starting or rest position, up, then down and then back up to the rest position

- S.H.M (Simple Harmonic Motion) is the motion of a particle about a fixed point such that its acceleration, a, is proportional to its displacement, x, from the fixed point and is directed towards the point.

$$a = -\omega^2 x$$

$$\omega = 2\pi f$$

Deriving $a = -\omega^2 x$

- $x = x_0 \sin \omega t$ (1) or $x = x_0 \cos \omega t$ (1')

$\downarrow$ Differentiate $\qquad\qquad$ $\downarrow$ Differentiate

$V = x_0 \omega \cos \omega t$ (2) or $V = -x_0 \omega \sin \omega t$

$\downarrow$ Differentiate $\qquad\qquad$ $\downarrow$ Differentiate

$a = -x_0 \omega^2 \sin \omega t$ or $a = -x_0 \omega^2 \cos \omega t$

Replacing $x_0 \sin \omega t$ and $x_0 \cos \omega t$ with x from 1 and 1' respectively.

$a = -\omega^2 x$ or $a = -\omega^2 x$

Deriving $v = w\sqrt{x_0^2 - x^2}$ and $v_0 = w^2 x_0$

- From 2 on previous page

$$v = x_0 w \cos wt$$
$$\cos wt = \frac{v}{x_0 w} \quad (3)$$

- From 1 on previous page

$$x = x_0 \sin wt$$
$$\sin wt = \frac{x}{x_0} \quad (4)$$

- using the identity $\sin^2 x + \cos^2 x = 1$

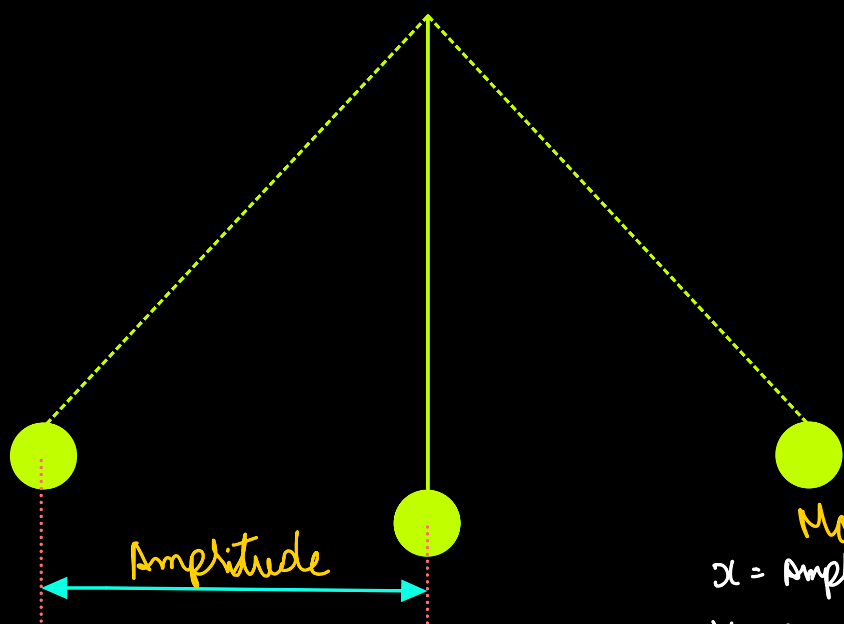
$\therefore$ $\sin^2 wt + \cos^2 wt = 1$

- $\dfrac{x^2}{x_0^2} + \dfrac{v^2}{x_0^2 w^2} = 1$
- $\dfrac{x^2 x_0^2 w^2 + v^2 x_0^2}{x_0^4 w^2} = 1$
- $x^2 x_0^2 w^2 + v^2 x_0^2 = x_0^4 w^2$
- $v^2 x_0^2 = x_0^4 w^2 - x^2 x_0^2 w^2$
- $v^2 \cancel{x_0^2} = \cancel{x_0^2} w^2 (x_0^2 - x^2)$

$\therefore$ $v = \sqrt{w^2} \times \sqrt{(x_0^2 - x^2)}$

$$v = w\sqrt{(x_0^2 - x^2)}$$

Example of a Pendulum in S.H.M



Min

$x = -\text{Amplitude} = -x_0$

$V = 0$

$a = a_{max} = a_0$

mean

$x = 0$

$V = \pm V_{max} = \pm V_0$

$a = 0$

Acceleration

Max

$x = \text{Amplitude} = x_0$

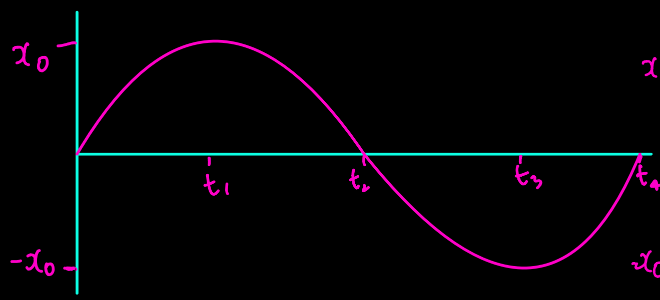
$V = 0$

$a = -a_{max} = -a_0$

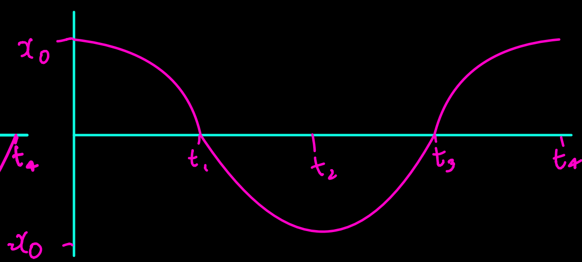
Graphs

① displacement and Acceleration vs time

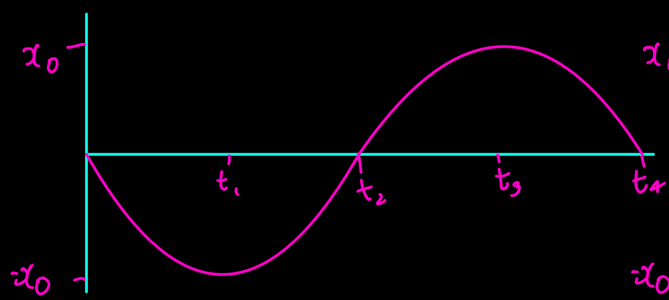
$x = x_0 \sin \omega t$



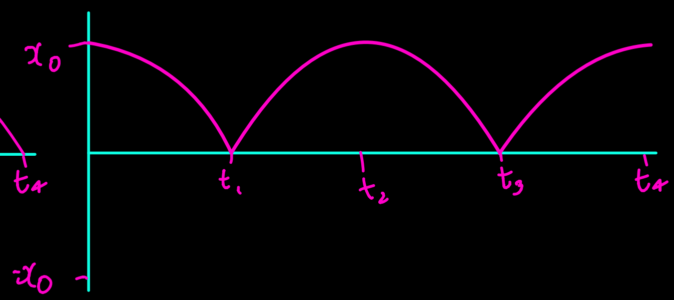
$v = x_0 \omega \cos \omega t$



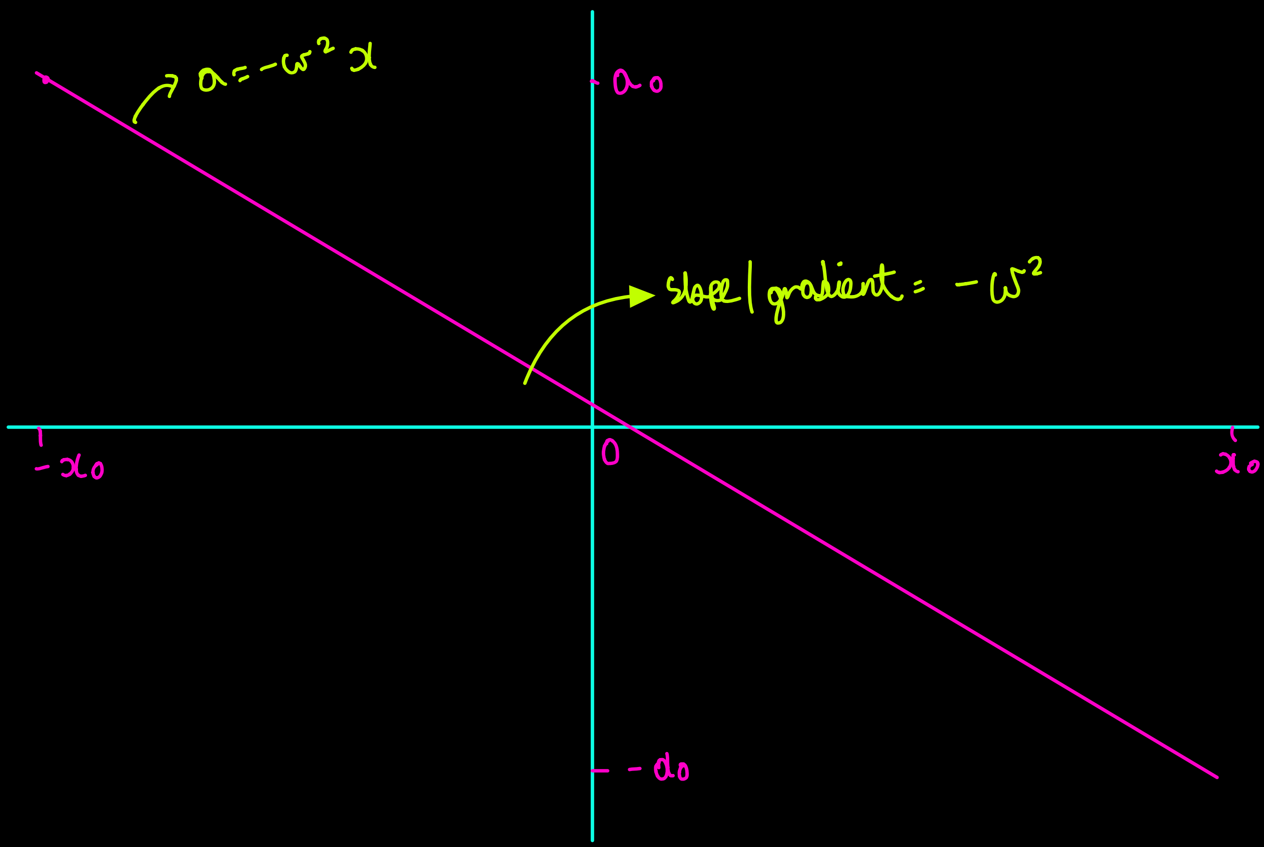
$a = - x_0 \omega^2 \sin \omega t$



$$\begin{aligned} Ke &= \frac{1}{2} m v^2 \\ &= \frac{1}{2} m (x_0 \omega \cos \omega t)^2 \\ &= \frac{1}{2} m x_0^2 \omega^2 \cos^2 \omega t \end{aligned}$$

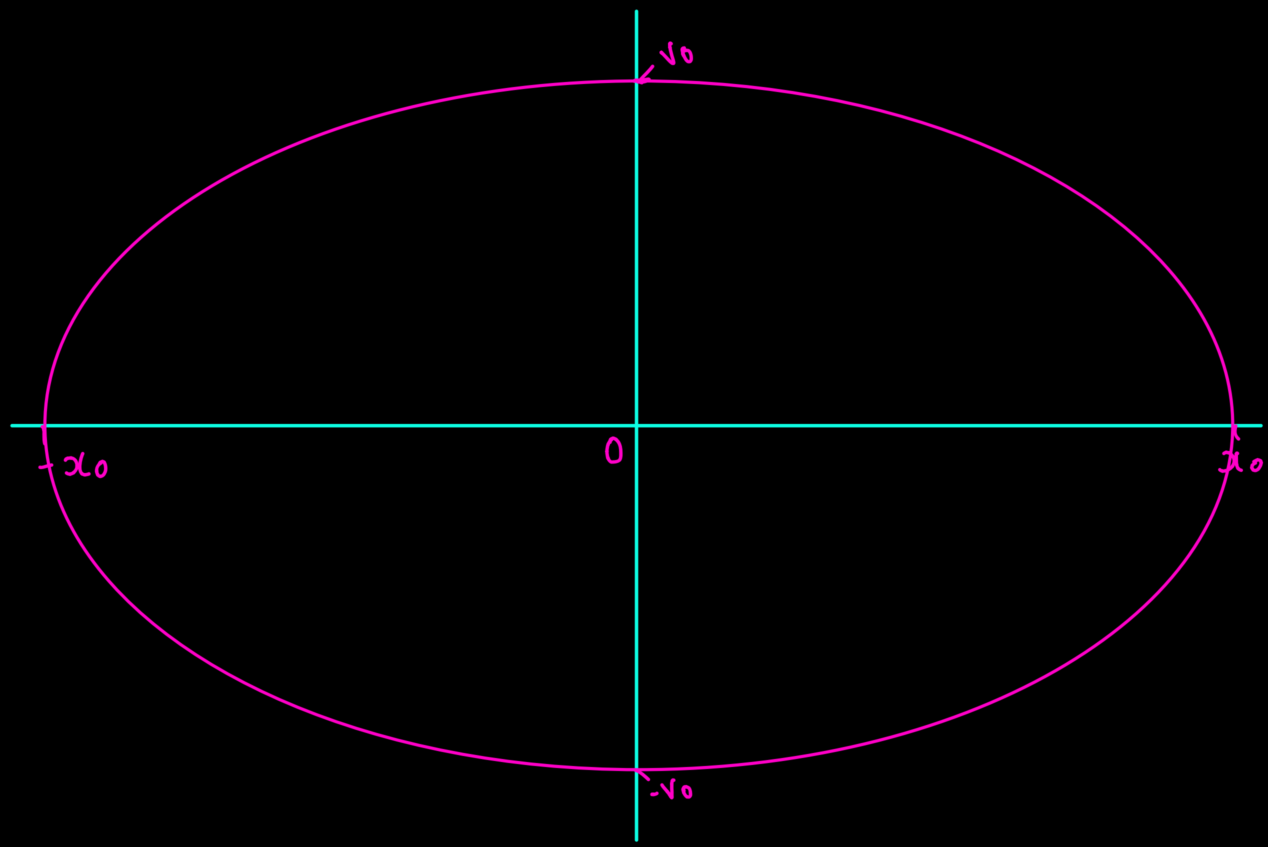


② Acceleration vs displacement ($a = -\omega^2 x$)



③ Velocity vs displacement ($v = \omega\sqrt{x_0^2 - x}$)

$v_0 = \omega^2 x_0$



Energy Graphs

① K.E vs displacement

$$KE = \frac{1}{2} m v^2$$

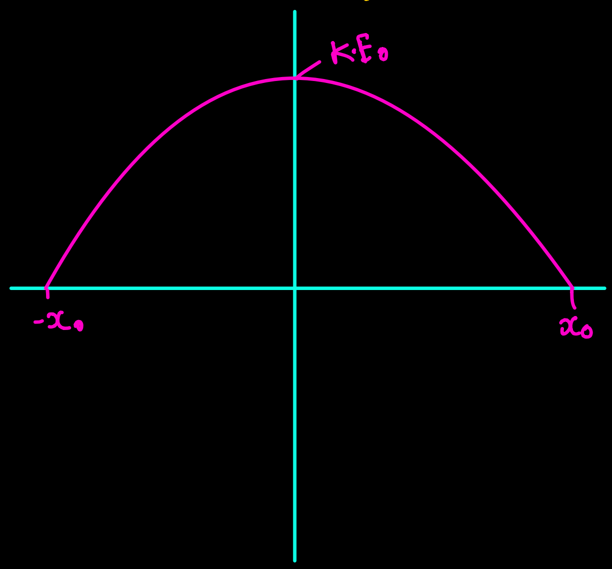
$$= \frac{1}{2} m \left(\omega \sqrt{x_0^2 - x^2}\right)^2$$

$$K.E = \frac{1}{2} m \omega^2 (x_0^2 - x^2)$$

- $K.E \propto (x_0^2 - x^2)$

$$\therefore \frac{Ke_1}{(x_0^2 - x_1^2)} = \frac{Ke_2}{(x_0^2 - x_2^2)}$$

when at Max/Min positions Ke=0 and when at Mean Position Ke= Maximum



② P.E vs displacement

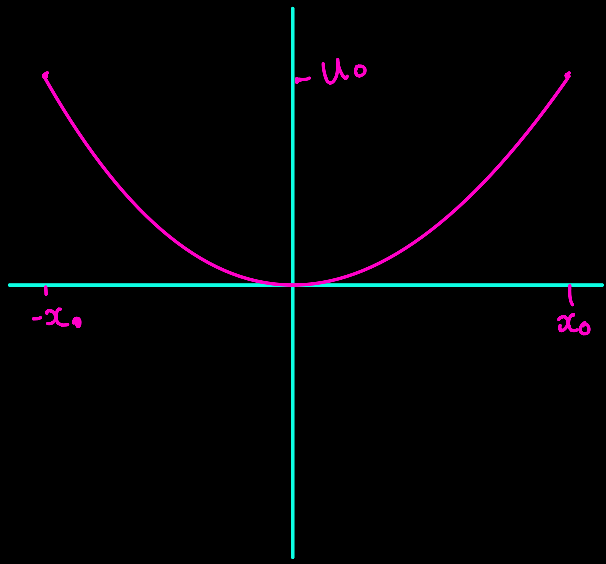
$$P.E = U = \frac{1}{2} m \omega^2 x^2$$

just the symbol of Potential Energy

- $U \propto x^2$

$$\therefore \frac{U_1}{x_1^2} = \frac{U_2}{x_2^2}$$

when at Max/Min positions Ke=0 and when at Mean Position Ke = Maximum

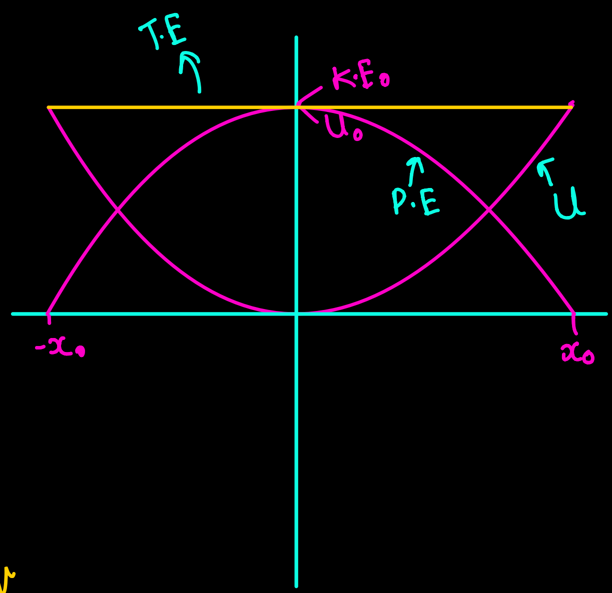


③ Total Energy

$$T.E = K.E + U$$

$$T.E = \frac{1}{2} m \omega^2 (x_0^2 - x^2) + \frac{1}{2} m \omega^2 x^2$$

$$T.E = \frac{1}{2} m \omega^2 x_0^2$$



- $T.E = K.E_0 = U_0$

* from the graph we know

- $K.E_0 = \frac{1}{2} m \omega^2 (x_0^2 - x^2)$, we know $x = 0$ when $K.E_0$

$\therefore K.E_0 = \frac{1}{2} m \omega^2 x_0^2$

- $U_0 = \frac{1}{2} m \omega^2 x^2$, we know $x = x_0$ when U_0

$\therefore U_0 = \frac{1}{2} m \omega^2 x_0^2$

Damping

- It is the loss of energy from an oscillating system to the environment due to dissipative forces (friction, viscous forces, eddy currents)

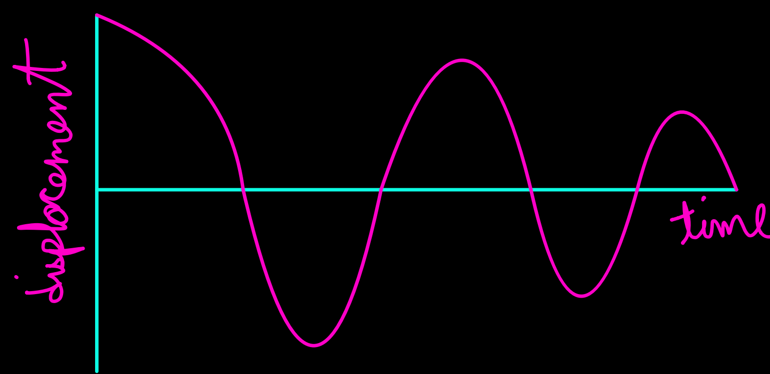
- Light damping
  - The system oscillates about the equilibrium position with decreasing amplitude over a period of time

- Heavy/Over damping
  - The damping is so strong that the displaced objects never oscillates but returns to its equilibrium position very very slowly.

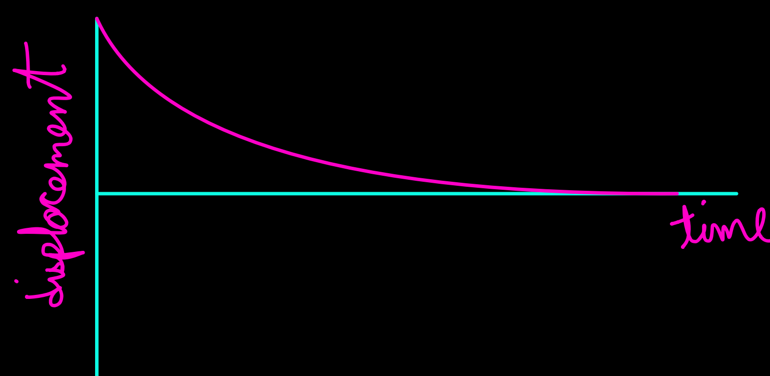
- Critical damping
  - The system does not oscillate and damping is just adequate such that the system returns to its equilibrium really fast

Graphs of damping

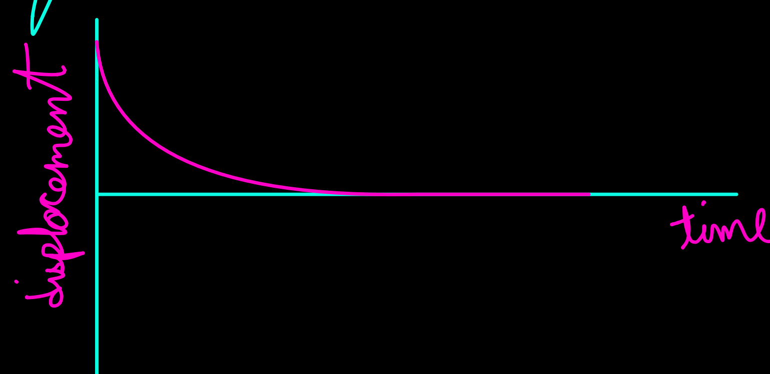
① light damping



② heavy/over damping



③ critical damping



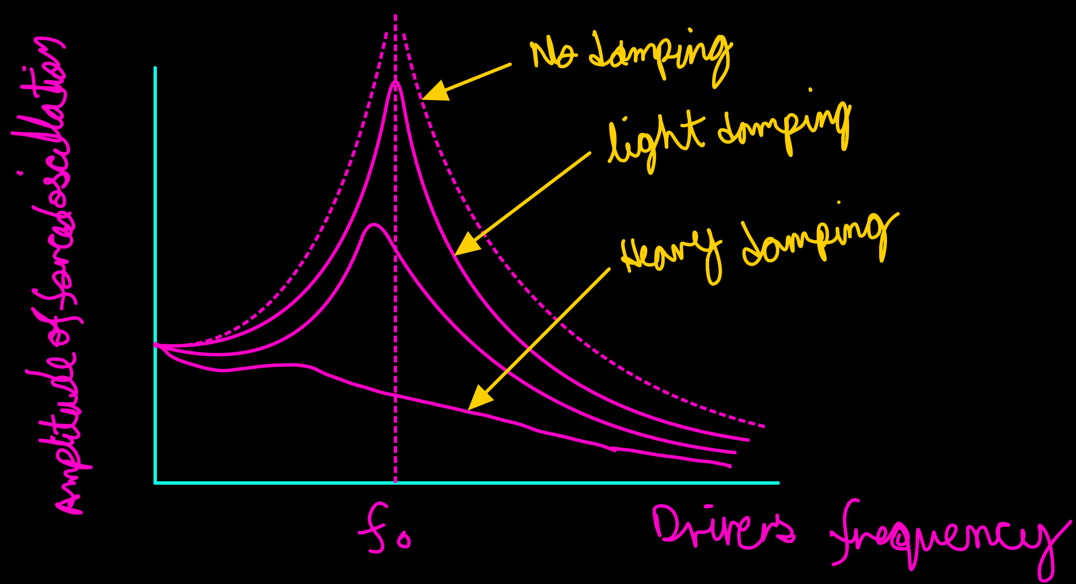
Free and Forced oscillations

- Free oscillation is when an oscillating object's motion is subjected to no external driving force (oscillating at its natural freq.

- Forced oscillation is when a driving force acts on an oscillating object, thus making it oscillate at the freq. of the driving force.

- Resonance is a phenomenon where the amplitude of a system undergoing forced oscillations increases to maximum, that happens when the frequency of the periodic driving force is equal to the natural frequency of the system.

* Effects of Damping on frequency of a system
  - Resonant freq. decreases
  - Sharpness of resonant peak decreases
  - Amplitude of forced oscillation decreases.



How Resonance can be useful in Real life

1) oscillation of a child swing

2) Tuning of radio receiver, Natural frequency of the radio is adjusted so it resonds resonantly to a specific brodcast frequency

How Resonance can of destructive nature in Real life

1) High pitched sound waves can shatter fragile objects

2) Building that vibrate at high frequencies close to the frequency of seismic waves face the possibility of collaps during earthquakes.

Examples of Useful Purposes of Resonance

(a) Oscillation of a child's swing.

(b) Tuning of musical instruments.

(c) Tuning of radio receiver - Natural frequency of the radio is adjusted so that it responds resonantly to a specific broadcast frequency.

(d) Using microwave to cook food - Microwave ovens produce microwaves of a frequency which is equal to the natural frequency of water molecules, thus causing the water molecules in the food to vibrate more violently. This generates heat to cook the food but the glass and paper containers do not heat up as much.

(e) Magnetic Resonance Imaging (MRI) is used in hospitals to create images of the human organs.

(f) Seismography - the science of detecting small movements in the Earth's crust in order to locate centres of earthquakes.

Examples of Destructive Nature of Resonance

(a) An example of a disaster that was caused by resonance occurred in the United States in 1940. The Tarcoma Narrows Bridge in Washington was suspended by huge cables across a valley. Shortly after its completion, it was observed to be unstable. On a windy day four months after its official opening, the bridge began vibrating at its resonant frequency. The vibrations were so great that the bridge collapsed.

(b) High-pitched sound waves can shatter fragile objects, an example being the shattering of a wine glass when a soprano hits a high note.

(c) Buildings that vibrate at natural frequencies close to the frequency of seismic waves face the possibility of collapse during earthquakes.