

The discrete random variable X has the probability function

$$P(X = x) = \begin{cases} kx & x = 2, 4, 6 \\ k(x - 2) & x = 8 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant.

- (a) Show that $k = \frac{1}{18}$
- (b) Find the exact value of $F(5)$.
- (c) Find the exact value of $E(X)$.
- (d) Find the exact value of $E(X^2)$.
- (e) Calculate $\text{Var}(3 - 4X)$ giving your answer to 3 significant figures.

$$\text{(a)} \quad 2 \quad 4 \quad 6 \quad 8$$

$$2k + 4k + 6k + 6k = 1$$

$$18k = 1$$

$$k = \frac{1}{18}$$

$$\text{(1)} \quad \text{b) } F(5) = P(X \leq 5) = P(X=2) + P(X=4) = 6k = \frac{1}{3}$$

(2)	$\begin{array}{ c c c c c } \hline x & 2 & 4 & 6 & 8 \\ \hline P(X=x) & \frac{2}{18} & \frac{4}{18} & \frac{6}{18} & \frac{6}{18} \\ \hline \end{array}$
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$$\text{(3)} \quad E(X) = \left(2 \times \frac{2}{18}\right) + \left(4 \times \frac{4}{18}\right) + \left(6 \times \frac{6}{18}\right) + \left(8 \times \frac{6}{18}\right)$$

$$= \frac{52}{9}$$

$$\text{d) } E(X^2) = \left(2^2 \times \frac{2}{18}\right) + \left(4^2 \times \frac{4}{18}\right) + \left(6^2 \times \frac{6}{18}\right) + \left(8^2 \times \frac{6}{18}\right) = \frac{112}{3}$$

$$\text{e) } \text{Var}(X) = E(X^2) - E(X)^2$$

$$= \frac{112}{3} - \left(\frac{52}{9}\right)^2$$

$$= \frac{320}{81}$$

$$\therefore \text{Var}(3 - 4X) = 4^2 \text{Var}(X)$$

$$= 16 \times \left(\frac{320}{81}\right)$$

$$= \frac{5120}{81}$$

A biased die with six faces is rolled. The discrete random variable X represents the score on the uppermost face. The probability distribution of X is shown in the table below.

x	1	2	3	4	5	6
$P(X=x)$	a	a	a	b	b	0.3

(a) Given that $E(X) = 4.2$ find the value of a and the value of b .

(b) Show that $E(X^2) = 20.4$

(c) Find $\text{Var}(5 - 3X)$

A biased die with five faces is rolled. The discrete random variable Y represents the score which is uppermost. The cumulative distribution function of Y is shown in the table below.

y	1	2	3	4	5
$F(y)$	$\frac{1}{10}$	$\frac{2}{10}$	$3k$	$4k$	$5k$

(d) Find the value of k .

(e) Find the probability distribution of Y .

Each die is rolled once. The scores on the two dice are independent.

(f) Find the probability that the sum of the two scores equals 2

$$d) 5k = 1$$

$$k = \frac{1}{5}$$

$$e) \begin{array}{c|cccccc} y & 1 & 2 & 3 & 4 & 5 \\ \hline P(Y=y) & \frac{1}{10} & \frac{2}{10} & \frac{4}{10} & \frac{2}{10} & \frac{2}{10} \end{array}$$

$$f) \left(\frac{1}{10} \times \frac{1}{10}\right) = \frac{1}{100} = 0.01$$

$$a) a + 2a + 3a + 4b + 5b + 6(0.3) = 4.2$$

$$6a + 9b = 2.4 \quad \textcircled{1}$$

$$3a + 2b = 1.03 \quad \textcircled{2}$$

$$a = 0.1, b = 0.2$$

$$b) 1^2(0.1) + 2^2(0.1) + 3^2(0.1) + 4^2(0.2) + 5^2(0.2) + 6^2(0.3)$$

$$= 20.4$$

$$c) \text{Var}(X) = 20.4 - 4.2^2 \\ = 2.76$$

$$3^2 \text{Var}(X) = 9 \times 2.76 = 24.84$$

The discrete random variable X can take only the values 1, 2 and 3. For these values the cumulative distribution function is defined by

$$F(x) = \frac{x^3 + k}{40} \quad x = 1, 2, 3$$

- (a) Show that $k = 13$

(2)

- (b) Find the probability distribution of X .

(4)

Given that $\text{Var}(X) = \frac{259}{320}$

- (c) find the exact value of $\text{Var}(4X - 5)$.

(2)

a) $f(x) = P(X \leq x)$

$$\therefore f(3) = 1$$

$$\frac{3^3 + k}{40} = 1$$

$$k = 13$$

b)

x	1	2	3
$P(X=x)$	$\frac{14}{40}$	$\frac{7}{40}$	$\frac{9}{40}$

$$\Rightarrow \frac{14 + 13}{40}$$

$$\Rightarrow \frac{27 + 13}{40} - \frac{14}{40} = \frac{7}{40}$$

c) $\sigma^2 = \frac{259}{320} = 12.95$

A fair blue die has faces numbered 1, 1, 3, 3, 5 and 5. The random variable B represents the score when the blue die is rolled.

(a) Write down the probability distribution for B .

(b) State the name of this probability distribution.

(c) Write down the value of $E(B)$.

A second die is red and the random variable R represents the score when the red die is rolled.

The probability distribution of R is

r	2	4	6
$P(R=r)$	$\frac{2}{3}$	$\frac{1}{6}$	$\frac{1}{6}$

(d) Find $E(R)$.

(e) Find $\text{Var}(R)$.

Tom invites Avisha to play a game with these dice.

Tom spins a fair coin with one side labelled 2 and the other side labelled 5. When Avisha sees the number showing on the coin she then chooses one of the dice and rolls it. If the number showing on the die is greater than the number showing on the coin, Avisha wins, otherwise Tom wins.

Avisha chooses the die which gives her the best chance of winning each time Tom spins the coin.

(f) Find the probability that Avisha wins the game, stating clearly which die she should use in each case.

(2)	a)	x	1	3	5
(1)		$P(x=x)$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

b) Discrete uniform distribution

$$\text{(c)} \quad E(x) = \frac{1}{3} + 1 + \frac{5}{3}$$

$$= 3$$

$$\text{(d)} \quad E(x) = \frac{9}{3} + \frac{4}{6} + \frac{6}{6}$$

$$= 3$$

$$\text{(e)} \quad \text{Var}(R) = E(R^2) - E(R)^2$$

$$= \left[4\left(\frac{2}{3}\right) + 16\left(\frac{1}{6}\right) + 36\left(\frac{1}{6}\right) \right] - 3^2$$

$$= \frac{34}{3} - 3^2 = \frac{7}{3}$$

$$\text{(f)} \quad P(\text{win}) = P(2)P(B>2) + P(5)P(R=6)$$

$$= \frac{1}{2}\left(\frac{4}{6}\right) + \frac{1}{2}\left(\frac{1}{6}\right)$$

$$= \frac{5}{12}$$

A discrete random variable X has the probability function

$$P(X = x) = \begin{cases} k(1-x)^2 & x = -1, 0, 1 \text{ and } 2 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Show that $k = \frac{1}{6}$

$$\begin{array}{cccccc} & -1 & 0 & 1 & 2 \\ | & 4k & k & 0 & k \end{array}$$

(3)

- (b) Find $E(X)$

(2)

- (c) Show that $E(X^2) = \frac{4}{3}$

(2)

- (d) Find $\text{Var}(1-3X)$

(3)

$$a) E(X) = k(-1)^2 + k(-0)^2 + k(0)^2 + k(1-2)^2$$

$$4k + k + k = 1$$

$$\begin{aligned} 6k &= 1 \\ k &= \frac{1}{6} \end{aligned}$$

$$b) E(X) = \sum x P(X=x)$$

$$= -4k + 2k$$

$$= -\frac{4}{6} + \frac{2}{6}$$

$$= -\frac{1}{3}$$

$$c) E(X^2) = \sum x^2 P(X=x)$$

$$4k + 4k$$

$$8\left(\frac{1}{6}\right) = \frac{4}{3}$$

$$d) \text{Var}(X) = E(X^2) - E(X)^2$$

$$= \frac{4}{3} - \left(-\frac{1}{3}\right)^2$$

$$= \frac{11}{9}$$

$$\text{Var}(1-3X) = 3^2 \times \frac{11}{9} = 11$$

The discrete random variable Y has probability distribution

y	1	2	3	4
$P(Y=y)$	a	b	0.3	c

$$0.1 \quad 0.4 \quad 0.2$$

where a , b and c are constants.

The cumulative distribution function $F(y)$ of Y is given in the following table

y	1	2	3	4
$F(y)$	0.1	0.5	d	1.0

where d is a constant.

- (a) Find the value of a , the value of b , the value of c and the value of d .

(5)

- (b) Find $P(3Y+2 \geq 8)$.

(2)

a) $0.1 + 0.4 + 0.3 = 0.8 \quad a = 0.1$

$b = 0.4$

$c = 0.2$

b) $3y + 2 \geq 8$

$$3y \geq 6$$

$$y \geq 2$$

$$P(3Y+2 \geq 8) = P(Y \geq 2)$$

$$= 0.4 + 0.3 + 0.2$$

$$= 0.9$$

The discrete random variable X has the probability distribution

x	1	2	3	4
$P(X=x)$	k	$2k$	$3k$	$4k$

- (a) Show that $k = 0.1$

$$\begin{aligned} 10k &= 1 \\ k &= 0.1 \end{aligned}$$

Find

(b) $E(X) = 0.1 + 2(0.2) + 3(0.3) + 4(0.4) = 3$

(c) $E(X^2) = 0.1 + 4(0.2) + 9(0.3) + 16(0.4) = 10$

(d) $\text{Var}(2-5X) = 5^2(10-3^2) = 25$

Two independent observations X_1 and X_2 are made of X .

- (e) Show that $P(X_1 + X_2 = 4) = 0.1$

(1)

(2)

(2)

(3)

(2)

- (f) Complete the probability distribution table for $X_1 + X_2$

(2)

y	2	3	4	5	6	7	8
$P(X_1 + X_2 = y)$	0.01	0.04	0.10	0.2	0.25	0.24	0.16

- (g) Find $P(1.5 < X_1 + X_2 \leq 3.5)$

(2)

$$\begin{aligned} P(X_1 + X_2 = 2) &= P(1,1) + P(2,1) \\ &= (0.1 \times 0.1) + (0.2 \times 0.1) + (0.2)^2 \\ &= 0.1 \end{aligned}$$

$$P(8) = P(4,4) = 0.4^2$$

$$P(8) = 0.01 + 0.04 = 0.05$$

The discrete random variable X has probability distribution given by

x	-1	0	1	2	3
$P(X = x)$	$\frac{1}{5}$	$\cancel{\frac{a}{5}}$	$\frac{1}{10}$	$\cancel{\frac{4}{5}}$	$\frac{1}{5}$

where a is a constant.

(a) Find the value of a .

(2)

(b) Write down $E(X)$.

(1)

(c) Find $\text{Var}(X)$.

(3)

The random variable $Y = 6 - 2X$

(d) Find $\text{Var}(Y)$.

(2)

(e) Calculate $P(X \geq Y)$.

(3)

$$\text{a) } 2a + 0.5 = 1$$

$$a = \frac{1}{4}$$

$$\text{b) } E(X) = 1$$

$$\text{c) } \frac{21}{10}$$

$$\text{d) } E(2X) = 2E(X) = 2 \cdot 1 = 2$$

$$\text{e) } P(X \geq 6 - 2X)$$

$$P(3X \geq 6)$$

$$P(X \geq 2)$$

$$0.25 + 0.2 = \frac{9}{20}$$

The random variable X has probability distribution

x	1	3	5	7	9
$P(X=x)$	0.2	p	0.2	q	0.15

- (a) Given that $E(X) = 4.5$, write down two equations involving p and q .

(3)

Find

- (b) the value of p and the value of q ,

(3)

- (c) $P(4 < X \leq 7)$.

(2)

Given that $E(X^2) = 27.4$, find

- (d) $\text{Var}(X)$,

(2)

- (e) $E(19 - 4X)$,

(1)

- (f) $\text{Var}(19 - 4X)$.

(2)

$$\begin{array}{l|l} \text{a)} \quad 0.55 + p + q = 1 & 0.2 + 3p + 1 + 7q + 1.35 = 4.5 \\ & 3p + 7q = 1.95 \\ p + q = 0.45 & 3p + 7q = 1.95 \end{array}$$

$$\text{b)} \quad p = 0.45 - q$$

$$3(0.45 - q) + 7q = 1.95$$

$$1.35 - 3q + 7q = 1.95$$

$$4q = 0.6$$

$$q = 0.15$$

$$\therefore p = 0.3$$

$$\text{c)} \quad 0.2 + 0.15 = 0.35$$

$$\text{d)} \quad 27.4 - 4 \cdot 3.5^2 = 7.15$$

$$\text{Var}(X) = 7.15$$

$$\text{e)} \quad E(19 - 4X) = E(19) - E(4X) = 19 - 4(4 - 3.5) = 11$$

$$\text{f)} \quad 4^2(7.15) = 114.4$$

In a large restaurant an average of 3 out of every 5 customers ask for water with their meal.

A random sample of 10 customers is selected.

(a) Find the probability that

- (i) exactly 6 ask for water with their meal,
- (ii) less than 9 ask for water with their meal.

(5)

A second random sample of 50 customers is selected.

(b) Find the smallest value of n such that

$$P(X < n) \geq 0.9$$

where the random variable X represents the number of these customers who ask for water.

(3)

i) $X \sim B(10, \frac{3}{5})$

$${}^{10}C_6 \times \left(\frac{3}{5}\right)^6 \times \left(\frac{2}{5}\right)^4 = 0.251$$

ii) $1 - \left[\left({}^{10}C_{10} \times \left(\frac{3}{5}\right)^{10} \times 1 \right) + \left({}^{10}C_9 \times \left(\frac{3}{5}\right)^9 \times \left(\frac{2}{5}\right)^1 \right) \right]$

$$1 - (0.0060466 + 0.04031)$$

0.954 (3sf)

b) $X \sim B(50, \frac{3}{5})$

Let $Y \sim B(50, \frac{2}{5})$

if $P(X < n) \geq 0.9$

$$\therefore P(Y > 50-n) \geq 0.1$$

$$1 - P(Y \leq 50-n) \geq 0.1$$

$$P(Y \leq 50-n) \leq 0.1$$

Check graph!

$$50-n \leq 15$$

$n \geq 35$

$\therefore n = 35$

The probability of a telesales representative making a sale on a customer call is 0.15

Find the probability that

- (a) no sales are made in 10 calls,

(2)

- (b) more than 3 sales are made in 20 calls.

(2)

Representatives are required to achieve a mean of at least 5 sales each day.

- (c) Find the least number of calls each day a representative should make to achieve this requirement.

(2)

- (d) Calculate the least number of calls that need to be made by a representative for the probability of at least 1 sale to exceed 0.95

(3)

a) $X \sim B(10, 0.15)$

$$P(X=0) = {}^{10}C_0 (0.15)^0 (0.85)^{10}$$
$$= 0.197$$

b) $X \sim B(20, 0.15)$

$$P(X > 3) = 1 - P(X \leq 3)$$

$$= 1 - [0.197 + {}^1C_1 (0.15)^1 (0.85)^{19} + {}^2C_2 (0.15)^2 (0.85)^{18} + {}^3C_3 (0.15)^3 (0.85)^{17}]$$
$$\approx 1 - 0.6977$$

$$= 0.3022$$

May/June 2002

- 3 A fair cubical die with faces numbered 1, 1, 1, 2, 3, 4 is thrown and the score noted. The area A of a square of side equal to the score is calculated, so, for example, when the score on the die is 3, the value of A is 9.

(i) Draw up a table to show the probability distribution of A . [3]

(ii) Find $E(A)$ and $\text{Var}(A)$. [4]

- 7 (i) A garden shop sells polyanthus plants in boxes, each box containing the same number of plants. The number of plants per box which produce yellow flowers has a binomial distribution with mean 11 and variance 4.95.

(a) Find the number of plants per box. [4]

(b) Find the probability that a box contains exactly 12 plants which produce yellow flowers. [2]

x	1	4	9	16
$P(x=x)$	$\frac{3}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$$\text{i)} E(A) = \frac{3}{6} + \frac{4}{6} + \frac{9}{6} + \frac{16}{6}$$

$$= \frac{16}{6}$$

$$\text{Var}(A) = E(A^2) - E(A)^2$$

$$E(A^2) = \frac{3}{6} + \frac{16}{6} + \frac{81}{6} + \frac{256}{6} = \frac{178}{3}$$

$$\therefore \text{Var}(A) = \frac{178}{3} - \left(\frac{16}{3}\right)^2$$

$$= \frac{278}{9}$$

$$7) \text{e } np = 11 \rightarrow P = \frac{11}{n}$$

$$np(1-p) = 4.95$$

$$\frac{11}{n} \left(1 - \frac{11}{n}\right) = 4.95$$

$$11 - \frac{121}{n} = 4.95$$

$$\frac{121}{n} = 6.05$$

$$n = 20$$

$$\therefore P = 0.55$$

$$b) x \sim B(20, 0.5)$$

$$\begin{aligned} & {}^{20}C_8 (0.5)^{12} (0.45)^8 \\ & = 0.16 \end{aligned}$$

May/June 2003

- 2 A box contains 10 pens of which 3 are new. A random sample of two pens is taken.
- (i) Show that the probability of getting exactly one new pen in the sample is $\frac{7}{15}$. [2]
 - (ii) Construct a probability distribution table for the number of new pens in the sample. [3]
 - (iii) Calculate the expected number of new pens in the sample. [1]
- 4 Kamal has 30 hens. The probability that any hen lays an egg on any day is 0.7. Hens do not lay more than one egg per day, and the days on which a hen lays an egg are independent.
- (i) Calculate the probability that, on any particular day, Kamal's hens lay exactly 24 eggs. [2]

$$2) \quad \frac{2}{10} \times \frac{1}{9} \times 2 = \frac{2}{15}$$

$$\begin{array}{ccc} 0 & 1 & 2 \\ \frac{2}{15} & \frac{1}{15} & \frac{1}{15} \end{array}$$

$$3) \quad \frac{2}{15} + \frac{1}{15} = \frac{3}{5}$$

$$4) \quad 30C_{24} \times 0.7^{24} \times 0.3^6$$

$$= 0.0829$$

May/June 2004

- 3 Two fair dice are thrown. Let the random variable X be the smaller of the two scores if the scores are different, or the score on one of the dice if the scores are the same.

(i) Copy and complete the following table to show the probability distribution of X .

[3]

x	1	2	3	4	5	6
$P(X = x)$	$\frac{11}{36}$	$\frac{9}{36}$	$\frac{7}{36}$	$\frac{5}{36}$	$\frac{3}{36}$	$\frac{1}{36}$

(ii) Find $E(X)$.

[2]

x	1	2	3	4	5	6
1	1	1	1	1	1	1
2	1	2	2	2	2	2
3	1	2	3	3	3	3
4	1	2	3	4	4	4
5	1	2	3	4	5	5
6	1	2	3	4	5	6

$$\text{i) } \begin{array}{|c|c|} \hline x & 1 \ 2 \ 3 \ 4 \ 5 \ 6 \\ \hline 1 & 1 \ 1 \ 1 \ 1 \ 1 \ 1 \\ 2 & 1 \ 2 \ 2 \ 2 \ 2 \ 2 \\ 3 & 1 \ 2 \ 3 \ 3 \ 3 \ 3 \\ 4 & 1 \ 2 \ 3 \ 4 \ 4 \ 4 \\ 5 & 1 \ 2 \ 3 \ 4 \ 5 \ 5 \\ 6 & 1 \ 2 \ 3 \ 4 \ 5 \ 6 \\ \hline \end{array} \quad \text{ii) } E(x) = \frac{11}{36} + \frac{18}{36} + \frac{21}{36} + \frac{20}{36} + \frac{15}{36} + \frac{6}{36} \\ = \frac{91}{36}$$

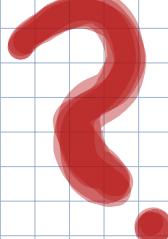
- 7 A shop sells old video tapes, of which 1 in 5 on average are known to be damaged.

(i) A random sample of 15 tapes is taken. Find the probability that at most 2 are damaged. [3]

(ii) Find the smallest value of n if there is a probability of at least 0.85 that a random sample of n tapes contains at least one damaged tape. 9 [3]

$$\text{i) } [{}^{15}C_0 \times 0.2^0 \times 0.8^{15}] + [{}^{15}C_1 \times 0.2^1 \times 0.8^{14}] + [{}^{15}C_2 \times 0.2^2 \times 0.8^{13}] \\ 0.398$$

$$\text{ii) } 0.85 = {}^{15}C_n \times 0.2^n \times 0.8^{15}$$



May/June 2005

P

O

G

- 3 A fair dice has four faces. One face is coloured pink, one is coloured orange, one is coloured green and one is coloured blue. Five such dice are thrown and the number that fall on a green face are counted. The random variable X is the number of dice that fall on a green face.

(i) Show that the probability of 4 dice landing on a green face is 0.0146, correct to 4 decimal places. [2]

(ii) Draw up a table for the probability distribution of X , giving your answers correct to 4 decimal places. [5]

i) $5C_4 \times 0.25^4 \times 0.75^1 = 0.0146$ ✓

ii)

x	0	1	2	3	4	5
$P(X=x)$	243/1024	405/1024	270/1024	90/1024	15/1024	1/1024
						✓

May/June 2006

- 6 32 teams enter for a knockout competition, in which each match results in one team winning and the other team losing. After each match the winning team goes on to the next round, and the losing team takes no further part in the competition. Thus 16 teams play in the second round, 8 teams play in the third round, and so on, until 2 teams play in the final round.

(i) How many teams play in only 1 match? [1]

(ii) How many teams play in exactly 2 matches? [1]

(iii) Draw up a frequency table for the numbers of matches which the teams play. [3]

(iv) Calculate the mean and variance of the numbers of matches which the teams play. [4]

- 7 A survey of adults in a certain large town found that 76% of people wore a watch on their left wrist, 15% wore a watch on their right wrist and 9% did not wear a watch.

(i) A random sample of 14 adults was taken. Find the probability that more than 2 adults did not wear a watch. [4]

6i) 16

ii) 8 22

iii)

No. of Wrist	1	2	3	4	5
Sample	16	8	4	2	2

iv) $\frac{16+16+12+8+10}{32} = 1.9375 = \bar{x}$

$16+32+36+32+50 = 166$

$$\begin{aligned}\sigma^2 &= \frac{166}{32} - 1.9375^2 \\ &= 1.43\end{aligned}$$

7i) $P(X > 2) = 1 - P(X \leq 2)$

$$= 1 - \left[\left({}^{14}C_0 \times 0.09^0 \times 0.91^{14} \right) + \left({}^{14}C_1 \times 0.09^1 \times 0.91^{13} \right) + \left({}^{14}C_2 \times 0.09^2 \times 0.91^{12} \right) \right]$$

$$= 1 - 0.874489 = 0.126$$

O

May/June 2007

- 7 A vegetable basket contains 12 peppers, of which 3 are red, 4 are green and 5 are yellow. Three peppers are taken, at random and without replacement, from the basket.

(i) Find the probability that the three peppers are all different colours. [3]

(ii) Show that the probability that exactly 2 of the peppers taken are green is $\frac{12}{55}$. [2]

(iii) The number of green peppers taken is denoted by the discrete random variable X . Draw up a probability distribution table for X . [5]

i)

X different chomp

x	0	1	2	3	4
$P(X=x)$	0.296	0.414	0.222	0.032	

$$P(X=x) = \left(\frac{1}{3}\right)^x \times \left(\frac{2}{3}\right)^{3-x}$$

May/June 2008

- 6 Every day Eduardo tries to phone his friend. Every time he phones there is a 50% chance that his friend will answer. If his friend answers, Eduardo does not phone again on that day. If his friend does not answer, Eduardo tries again in a few minutes' time. If his friend has not answered after 4 attempts, Eduardo does not try again on that day.

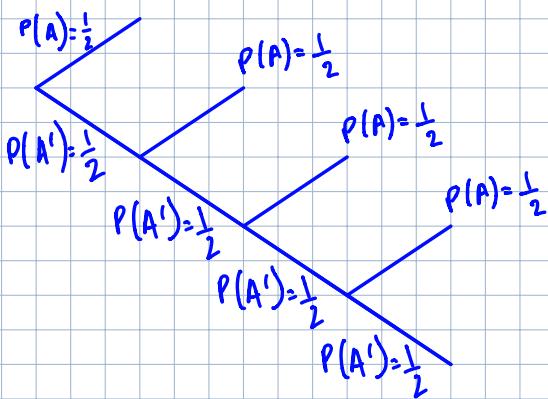
(i) Draw a tree diagram to illustrate this situation. [3]

(ii) Let X be the number of unanswered phone calls made by Eduardo on a day. Copy and complete the table showing the probability distribution of X . [4]

x	0	1	2	3	4
$P(X=x)$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{16}$

(iii) Calculate the expected number of unanswered phone calls on a day. [2]

i)



ii)

$$\text{(iii)} \quad \frac{1}{4} + \frac{2}{8} + \frac{3}{16} + \frac{4}{16} = \frac{15}{16}$$

May/June 2009

- 1 The volume of milk in millilitres in cartons is normally distributed with mean μ and standard deviation 8. Measurements were taken of the volume in 900 of these cartons and it was found that 225 of them contained more than 1002 millilitres.

(i) Calculate the value of μ . [3]

(ii) Three of these 900 cartons are chosen at random. Calculate the probability that exactly 2 of them contain more than 1002 millilitres. [2]

- 2 Gohan throws a fair tetrahedral die with faces numbered 1, 2, 3, 4. If she throws an even number then her score is the number thrown. If she throws an odd number then she throws again and her score is the sum of both numbers thrown. Let the random variable X denote Gohan's score.

(i) Show that $P(X = 2) = \frac{5}{16}$. [2]

(ii) The table below shows the probability distribution of X .

x	2	3	4	5	6	7
$P(X = x)$	$\frac{5}{16}$	$\frac{1}{16}$	$\frac{3}{8}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{16}$

Calculate $E(X)$ and $\text{Var}(X)$. [4]

Ez

	1	2	3	4
1	2	2		
2	3	8		
3	1	2		
4	5	2		

R 2.5.4

6 The results of a survey by a large supermarket show that 35% of its customers shop online.

(i) Six customers are chosen at random. Find the probability that more than three of them shop online. [3]

(ii) For a random sample of n customers, the probability that at least one of them shops online is greater than 0.95. Find the least possible value of n . [3]

i) $X \sim B(6, 0.35)$

$$\begin{aligned} P(X > 3) &= P(X=4) + P(X=5) + P(X=6) \\ &= ({}^6C_4 \times 0.35^4 \times 0.65^2) + ({}^6C_5 \times 0.35^5 \times 0.65) + ({}^6C_6 \times 0.35^6 \times 0.65^0) \\ &= 0.0951 + 0.02048 \approx 0.001838 \\ &= 0.11791 \end{aligned}$$

ii) $X \sim (n, 0.35)$

$$\begin{aligned} P(X \geq 1) &> 0.95 \\ \therefore P(X \geq 0) &> 0.95 \\ 1 - P(X=0) &\geq 0.95 \\ 1 - {}^nC_0 \times 0.35^0 \times 0.65^n &\geq 0.95 \end{aligned}$$

$$1 - 0.65^n \geq 0.95$$

$$1 - 0.95 \geq 0.65^n$$

$$0.05 \geq 0.65^n$$

$$\ln 0.65^n \leq \ln 0.05$$

$$n \leq \frac{\ln 0.05}{\ln 0.65}$$

$$n \leq 6.95$$

$$n = 7$$