A sample of 36 data values, x, gave $\Sigma(x-45) = -148$ and $\Sigma(x-45)^2 = 3089$.

(i) Find the mean and standard deviation of the 36 values. [3]

(ii) One extra data value of 29 was added to the sample. Find the standard deviation of all 37 values. [4]

(i) You should know that when you increase or decrease observations x by a given constant say 'a' then the mean will go up or down by 'a'.

$$\text{ie} \quad E(X \pm a) = E(X) \pm a$$

$$\therefore \text{ the coded mean} = \frac{\Sigma(x-45)}{36}$$

$$= -\frac{148}{36}$$

but the new mean will be 45 more as we reduced every observation by 45.

$$\therefore \text{ original mean} = -\frac{148}{36} + 45$$

$$= \frac{368}{9} = 40.\dot{8}$$

$$\text{Standard dev.} = \sqrt{\frac{\Sigma x^2}{n} - \text{mean}^2}$$

2

Now adding or subtracting a value from all observations does not change the scattering of points about the mean. So the standard deviation would be the same.

$$\text{So stand. dev.} = \sqrt{\frac{\Sigma(x-45)^2}{n} - (\text{transformed mean}^2)}$$

So standard deviation $= \sqrt{\frac{\sum(x-45)^2}{n} - \left[\frac{\sum(x-45)}{n}\right]^2}$

$$= \sqrt{\frac{3089}{36} - \left(-\frac{148}{36}\right)^2}$$

$$= 8.3008\ldots$$

(ii) When 29 is added the new mean $= \frac{\sum x + 29}{37}$

Remember $\frac{\sum x}{36} = 40.8$

$$= \frac{(40.8)(36) + 29}{37}$$

$$= 40.4\dot{8}1\dot{0}$$

$\therefore$ stand deviation $= \sqrt{\frac{\sum x^2 + 29^2}{37} - 40.4\dot{8}1\dot{0}^2}$

but from (i) $\sqrt{\frac{\sum x^2}{36} - (40.8)^2} = 8.3008$

$\therefore \sum x^2 = 36(8.3008^2 + 40.8^2)$

$= 62407.5581$

$\therefore$ stand dev $= \sqrt{\frac{62407.5581 + 29^2}{37} - 40.4\dot{8}1\dot{0}^2}$

$= 8.408\ldots$

$= 8.41$ (3sf)

4 **(a)** Amy measured her pulse rate while resting, x beats per minute, at the same time each day on 30 days. The results are summarised below.

$$\Sigma(x - 80) = -147 \qquad \Sigma(x - 80)^2 = 952$$

Find the mean and standard deviation of Amy's pulse rate. [4]

$x - 80$

a)

60

50

80

61

63

x

$\Sigma(x-80) = -147$

$\Sigma(x-80)^2 = 952$

$\Sigma x - 80(30) = -147$

$\Sigma x = 2253$

$\bar{x} = \frac{2253}{30} = 75.1$

$75.1 - 80 = -4.9$

$\sqrt{\frac{\Sigma(x-80)^2}{30} - (-4.9)^2}$

$\sqrt{\frac{952}{30} - (24.01)}$

$\sigma = 2.78$

1 The time taken, t hours, to deliver letters on a particular route each day is measured on 250 working days. The mean time taken is 2.8 hours. Given that $\Sigma(t - 2.5)^2 = 96.1$, find the standard deviation of the times taken. [3]

1) $\sqrt{\dfrac{96.1}{250} - (0.3)^2} = 0.543$

a)

$\Sigma x = 745$

$\mu = \frac{745}{18} = 41.4$

$\sigma = \sqrt{\frac{33951}{18} - 41.4^2}$

$= 13.158$

b) i)

$\frac{745 - x}{17} = 41$

$745 - x = 697$

$x = 48$ ✓

ii)

$\sigma = \sqrt{\frac{31647}{17} - 41^2}$

$\sigma = 13.44$

$48^2 = 2304$

$33951 - 2304$

31647

$$\text{mean} = \frac{\Sigma x + \Sigma y}{n_1 + n_2}$$

$$\sigma = \sqrt{\frac{\Sigma x^2 + \Sigma y^2}{n_1 + n_2} - (\text{mean})^2}$$

$$\Sigma(x - a)$$
$$\Sigma(x - a)^2$$

$$\mu = \frac{\Sigma(x - a)}{n} + a$$

$$\sigma = \sqrt{\frac{\Sigma(x-a)^2}{n} - (\text{mean})^2}$$

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i)

$$\frac{-73.2}{24} + a = 8.95$$

$$a = 12$$

ii)

$$\sigma = \sqrt{\frac{2115}{24} - \left(\frac{-73.2}{24}\right)^2}$$

$$= 8.89$$

$$\mu = \frac{\Sigma(x-60)}{n} + 60$$

$$\mu = \frac{245}{70} + 60$$

$$= 63.5$$

$$\frac{\Sigma(x-50)}{70} + 50 = 63.5$$

$$\Sigma(x-5) = \underline{\underline{945}}$$

$$\frac{\Sigma(x-50)^2}{70} - (63.5-50)^2 = 112.36$$

$$\Sigma(x-50)^2 = 20622.7$$

$$\frac{\Sigma(x-80)}{n} + 80 = \text{mean}$$

$$\frac{-147}{30} + 80 = \text{mean}$$

$\mu = \text{mean}$

$\mu = 75.1$

$$\sqrt{\frac{952}{30} - (75.1 - 80)^2} = 2.78$$