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Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **February/March 2016**

1 hour 45 minutes

0572757842

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **3** printed pages and **1** blank page.

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1 Solve the equation $\ln(x^2 + 4) = 2\ln x + \ln 4$, giving your answer in an exact form. [3]

2 Express the equation $\tan(\theta + 45°) - 2\tan(\theta - 45°) = 4$ as a quadratic equation in $\tan\theta$. Hence solve this equation for $0° \leqslant \theta \leqslant 180°$. [6]

3 The equation $x^5 - 3x^3 + x^2 - 4 = 0$ has one positive root.

(i) Verify by calculation that this root lies between 1 and 2. [2]

(ii) Show that the equation can be rearranged in the form

$$x = \sqrt[3]{\left(3x + \frac{4}{x^2} - 1\right)}.$$ [1]

(iii) Use an iterative formula based on this rearrangement to determine the positive root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

4 The polynomial $4x^3 + ax + 2$, where a is a constant, is denoted by $\mathrm{p}(x)$. It is given that $(2x + 1)$ is a factor of $\mathrm{p}(x)$.

(i) Find the value of a. [2]

(ii) When a has this value,

(a) factorise $\mathrm{p}(x)$, [2]

(b) solve the inequality $\mathrm{p}(x) > 0$, justifying your answer. [3]

5 Let $I = \int_0^1 \frac{9}{(3 + x^2)^2}\,\mathrm{d}x$.

(i) Using the substitution $x = (\sqrt{3})\tan\theta$, show that $I = \sqrt{3}\int_0^{\frac{1}{6}\pi} \cos^2\theta\,\mathrm{d}\theta$. [3]

(ii) Hence find the exact value of I. [4]

6 A curve has equation

$$\sin y \ln x = x - 2\sin y,$$

for $-\frac{1}{2}\pi \leqslant y \leqslant \frac{1}{2}\pi$.

(i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y. [5]

(ii) Hence find the exact x-coordinate of the point on the curve at which the tangent is parallel to the x-axis. [3]

7 The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = xe^{x+y},$$

and it is given that $y = 0$ when $x = 0$.

(i) Solve the differential equation and obtain an expression for y in terms of x. [7]

(ii) Explain briefly why x can only take values less than 1. [1]

8 The line l has equation $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$. The plane p has equation $\mathbf{r}.\begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} = 6$.

(i) Show that l is parallel to p. [3]

(ii) A line m lies in the plane p and is perpendicular to l. The line m passes through the point with coordinates $(5, 3, 1)$. Find a vector equation for m. [6]

9 Let $f(x) = \dfrac{3x^3 + 6x - 8}{x(x^2 + 2)}$.

(i) Express $f(x)$ in the form $A + \dfrac{B}{x} + \dfrac{Cx + D}{x^2 + 2}$. [5]

(ii) Show that $\int_1^2 f(x)\,dx = 3 - \ln 4$. [5]

10 **(a)** Find the complex number z satisfying the equation $z^* + 1 = 2iz$, where z^* denotes the complex conjugate of z. Give your answer in the form $x + iy$, where x and y are real. [5]

(b) **(i)** On a sketch of an Argand diagram, shade the region whose points represent complex numbers satisfying the inequalities $|z + 1 - 3i| \leqslant 1$ and $\operatorname{Im} z \geqslant 3$, where $\operatorname{Im} z$ denotes the imaginary part of z. [4]

(ii) Determine the difference between the greatest and least values of $\arg z$ for points lying in this region. [2]

1 Solve the equation $\ln(x^2 + 4) = 2\ln x + \ln 4$, giving your answer in an exact form. [3]

$$\ln\left(\frac{x^2+4}{x^2}\right) = \ln 4$$

$$\ln\left(\frac{\left(\frac{x^2+4}{x^2}\right)}{4}\right) = 0$$

$$\ln\left(\frac{x^2+4}{x^2} \times \frac{1}{4}\right) = 0$$

$$\frac{x^2+4}{4x^2} = e^0$$

$$x^2+4 = 4x^2$$

$$3x^2 = 4$$

$$x^2 = \frac{4}{3}$$

$$x = \pm\sqrt{\frac{4}{3}}$$

$-$ve rejected as ln cannot be of a $-$ve number.

2 Express the equation $\tan(\theta + 45°) - 2\tan(\theta - 45°) = 4$ as a quadratic equation in $\tan\theta$. Hence solve this equation for $0° \leqslant \theta \leqslant 180°$. [6]

$$\frac{\tan\theta + \tan 45}{1 - \tan\theta \tan 45} - 2\left[\frac{\tan\theta - \tan 45}{1 + \tan\theta \tan 45}\right] = 4$$

$$\frac{\tan\theta + 1}{1 - \tan\theta} - 2\left[\frac{\tan\theta - 1}{1 + \tan\theta}\right] = 4$$

$$\frac{(\tan\theta + 1)(1 + \tan\theta) - (2\tan\theta - 2)(1 - \tan\theta)}{(1 - \tan\theta)(1 + \tan\theta)} = 4$$

$$\frac{2\tan\theta + \tan^2\theta + 1 - (4\tan\theta - 2\tan^2\theta - 2)}{1 - \tan^2\theta} = 4$$

$$2\tan\theta + \tan^2\theta + 1 - 4\tan\theta + 2\tan^2\theta + 2 = 4 - 4\tan^2\theta$$

$$3\tan^2 - 2\tan\theta + 3 = 4 - 4\tan^2\theta$$

$$7\tan^2\theta - 2\tan\theta - 1 = 0$$

$\tan\theta = 0.5469$, $\tan\theta = -0.2612 \rightarrow \theta = 360 - \tan^{-1}(0.2612) - 180$

$\theta = 28.674$ $\qquad$ $\theta = 165.361$

$= 28.7$ $\qquad$ $\theta = 165.4$

3 The equation $x^5 - 3x^3 + x^2 - 4 = 0$ has one positive root.

(i) Verify by calculation that this root lies between 1 and 2. [2]

(ii) Show that the equation can be rearranged in the form

$$x = \sqrt[3]{\left(3x + \frac{4}{x^2} - 1\right)}.$$ [1]

(iii) Use an iterative formula based on this rearrangement to determine the positive root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

i) let $f(x) = x^5 - 3x^3 + x^2 - 4$

$f(1) = 1 - 3 + 1 - 4 = -5$

$f(2) = 32 - 24 + 4 - 4 = 8$

change of sign $\therefore$ root lies between 1 and 2

ii) $x^5 - 3x^3 + x^2 - 4 = 0$

$x^3 - 3x^2 + 1 - \frac{4}{x^2} = 0$

$x^3 = 3x^2 + \frac{4}{x^2} - 1$

$x = \sqrt[3]{3x^2 + \frac{4}{x^2} - 1}$

iii)

4 The polynomial $4x^3 + ax + 2$, where a is a constant, is denoted by $p(x)$. It is given that $(2x + 1)$ is a factor of $p(x)$.

(i) Find the value of a. [2]

(ii) When a has this value,

(a) factorise $p(x)$, [2]

(b) solve the inequality $p(x) > 0$, justifying your answer. [3]

i) $(2x+1)(2x^2+?x+2)$

$a=3$

$$
\begin{array}{r|l}
 & 2x^2 - x + 2 \\
\hline
2x+1 & 4x^3 + 0x^2 + ax + 2 \\
 & -\ 4x^3 + 2x^2 \\
 & 0 \quad - 2x^2 + ax \\
 & -\ -2x^2 - x \\
 & 0 + ax + x + 2 \\
 & -\ 4x + 2
\end{array}
$$

$ax + x - 4x = 0$

$ax - 3x = 0$

$x(a-3) = 0$

$a - 3 = 0$

$a = 3$

ii) a) $4x^3 + 3x + 2$

$x(4x^2+3) + 2$

b) $x(4x^2+3) + 2 > 0$

$x(4x^2+3) > -2$

$4x^3 + 3x > -2$

$(2x+1)(2x^2-x+2)$

c) $2x+1 > 0$

$x > -\frac{1}{2}$

5 Let $I = \int_0^1 \frac{9}{(3+x^2)^2}\,dx$.

(i) Using the substitution $x = (\sqrt{3})\tan\theta$, show that $I = \sqrt{3}\int_0^{\frac{1}{6}\pi} \cos^2\theta\,d\theta$. [3]

(ii) Hence find the exact value of I. [4]

i) when $x = 0$ $\quad \theta = 0$ $\quad \theta = \tan^{-1}\left(\frac{x}{\sqrt{3}}\right)$

$x = 1$ $\quad \theta = \frac{1}{6}\pi$

$\frac{S^2}{C^2} + \frac{C^2}{C^2} = \frac{1}{C^2}$

$T^2 + 1 = \sec^2$

$x = \sqrt{3}\tan\theta \longrightarrow x^2 = 3\tan^2\theta$

$\frac{dx}{d\theta} = (\sqrt{3})(\sec^2\theta)$ $\qquad \tan^2\theta = \sec^2\theta - 1$

$dx = \sqrt{3}\sec^2\theta\,d\theta$

$$\int_0^{\frac{1}{6}\pi} \frac{9}{(3+3\tan^2\theta)^2} \times \sqrt{3}\sec^2\theta\,d\theta$$

$$\int_0^{\frac{1}{6}\pi} \frac{9\sqrt{3}\sec^2\theta}{(3+3(\sec^2\theta-1))^2}\,d\theta$$

$$\sqrt{3}\int_0^{\frac{1}{6}\pi} \frac{9\sec^2\theta}{(\cancel{3}+3\sec^2\theta-\cancel{3})^2}\,d\theta$$

$$\sqrt{3}\int_0^{\frac{1}{6}\pi} \frac{\cancel{9}\,\cancel{\sec^2\theta}}{\cancel{9}\,\sec^{\cancel{4}}\theta}\,d\theta$$

$$\sqrt{3}\int_0^{\frac{1}{6}\pi} \frac{1}{\sec^2\theta}\,d\theta \implies \sqrt{3}\int_0^{\frac{1}{6}\pi}\cos^2\theta\,d\theta$$

$\cos 2\theta = 2\cos^2\theta - 1$

$\cos^2\theta = \frac{\cos 2\theta}{2} + \frac{1}{2}$

ii) $\sqrt{3}\int_0^{\frac{1}{6}\pi} \frac{1}{2}(\cos 2\theta + 1)\,d\theta$

$\frac{\sqrt{3}}{2}\int_0^{\frac{1}{6}\pi} \cos 2\theta + 1\,d\theta \implies \left[\frac{\sqrt{3}}{2}\left(\frac{1}{2}\sin 2\theta + \theta\right)\right]_0^{\frac{1}{6}\pi}$ $\quad$ sub 0: $\frac{\sqrt{3}}{2}(0+0) = 0$

sub $\frac{1}{6}\pi$: $\left(\frac{\sqrt{3}}{2}\left(\frac{1}{2}\sin\left(\frac{1}{3}\pi\right) + \frac{1}{6}\pi\right)\right) = \frac{\sqrt{3}}{2}\left(\frac{\sqrt{3}}{4} + \frac{1}{6}\pi\right) = \frac{3}{8} + \frac{\sqrt{3}\pi}{12}$

6 A curve has equation

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for $-\frac{1}{2}\pi \leqslant y \leqslant \frac{1}{2}\pi$.

(i) Find $\frac{dy}{dx}$ in terms of x and y. cos cant be 2?? or sin **[5]**

(ii) Hence find the exact x-coordinate of the point on the curve at which the tangent is parallel to the x-axis. **[3]**

6i) $\left(\cos y \frac{dy}{dx}\right)(\ln x) + \frac{\sin y}{x} = 1 - 2\cos y \frac{dy}{dx}$

$(\cos y \ln x + 2\cos y)\frac{dy}{dx} = \frac{x - \sin y}{x}$

$\frac{dy}{dx} = \frac{x - \sin y}{x} \times \frac{1}{\cos y \ln x + 2\cos y}$

$\frac{dy}{dx} = \frac{x - \sin y}{x\cos y \ln x + 2x\cos y}$

ii) $x - \sin y = 0$

$x = \sin y$

$\sin y \ln \sin y = \sin - 2\sin y$

$\sin y \ln \sin y = -\sin y$

$\ln \sin y = -1$

$\sin y = \frac{1}{e}$

$y = \sin^{-1}\left(\frac{1}{e}\right)$ $\therefore x = \sin\left(\sin^{-1}\left(\frac{1}{e}\right)\right)$

$x = \frac{1}{e}$

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and it is given that $y = 0$ when $x = 0$.

(i) Solve the differential equation and obtain an expression for y in terms of x. [7]

(ii) Explain briefly why x can only take values less than 1. [1]

3

$dy = xe^{x} + x$

$\frac{dy}{dx} = x \times (e^{x} \times e^{y})$

$\frac{dy}{dx} = \cancel{xe^{x} + x}e^{y}$

$\frac{dy}{xe^{y}} = xe^{x}\,dx$

move

$\int \frac{1}{e^{y}}\,dy = \int x^2 e^{x}\,dx$

$\int e^{-y} = x^2e^{x} - xe^{x} + 2e^{x} + C$

$-e = x^2e^{x} - xe^{x} + 2e^{x} + C$

$-e^{0} = 0 - 0 + 2 + c$

$-1 - 2 = C$

$C = -3$

$e^{-y} = xe^{x} - x^2e^{x} - 2e^{x} + 3$

$y = -\ln(xe^{x} - x^2e^{x} - 2x^{x} + 3)$

$u = x^2$ $\quad u' = 2x$

$v = e^{x}$ $\quad v' = e^{x}$

$x^2e^{x} - 2\int xe^{x}\,dx$ $\quad u = x$ $\quad u' = 1$ $\quad v = e^{x}$ $\quad v' = e^{x}$

$x^2e^{x} - \left[xe^{x} - 2\int e^{x}\,dx\right]$

$x^2e^{x} - (xe^{x} - 2e^{x})$

$x^2e^{x} - xe^{x} + 2e^{x} + C$

ii) because otherwise we would get a -ve value inside ln, and ln of a -ve value does not exist.

8 The line l has equation $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$. The plane p has equation $\mathbf{r}.\begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} = 6$.

(i) Show that l is parallel to p. [3]

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(ii) Show that $\int_1^2 f(x)\,dx = 3 - \ln 4$. [5]

i) $3x^3 + 6x - 8 = A(x)(x^2+2) + B(x^2+2) + (Cx+D)(x)$

$3x^3 + 6x - 8 \quad Ax^3 + 2Ax + Bx^2 + 2B + Cx^3 + Dx$

$3 = A + C$

$A = 3 - C$

$6 = 2A + Dx$

$-8 = 2B$

$-8 = 2B$

$B = -4$

$0 = Bx^2$

$$\begin{array}{r} 3 \\ x^3+2x \overline{\big) 3x^3 + 6x - 8} \\ \underline{3x^3 + 6x} \\ 0 \quad -8 \end{array}$$

$\therefore A = 3$

$\dfrac{-8}{x^3+2x} = \dfrac{B}{x} + \dfrac{Cx + D}{x^2+2}$

$-8 = Bx^2 + 2B + Cx^2 + Dx$

$2B = -8$

$B = -4$

$0 = B + C$

$-4 + C = 0$

$C = 4$

$D = 0$

10 **(a)** Find the complex number z satisfying the equation $z^* + 1 = 2\mathrm{i}z$, where z^* denotes the complex conjugate of z. Give your answer in the form $x + \mathrm{i}y$, where x and y are real. [5]

(b) **(i)** On a sketch of an Argand diagram, shade the region whose points represent complex numbers satisfying the inequalities $|z + 1 - 3\mathrm{i}| \leqslant 1$ and $\operatorname{Im} z \geqslant 3$, where $\operatorname{Im} z$ denotes the imaginary part of z. [4]

$z-(-1+3i)$

(ii) Determine the difference between the greatest and least values of $\arg z$ for points lying in this region. [2]

$(x-iy)+1 = 2i(x+iy)$

$x-iy+1 = 2ix-2y$

$x+1=-2y$

$x+2y=-1$

$2x=-y$

$y=-2x$

$z = \frac{1}{3} - \frac{2}{3}i$

$x-2(-2x)=-1$

$z^* = \frac{1}{3}+\frac{2}{3}i$

$x-4x=-1$

$-3x=-1$

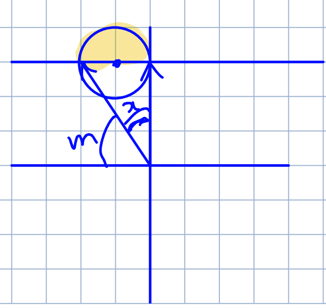
$x=\frac{1}{3}$

$z = \frac{1}{3} - \frac{2}{2}i$

$y = \frac{-2}{3}$

ii) $\angle m = \tan^{-1}\left(\frac{3}{2}\right) = 0.9827$

bi)



$\angle x = \frac{\pi}{2} - 0.9827 = \underline{0.588}$

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