Cambridge International A Level

**Cambridge International Examinations**
Cambridge International Advanced Level

**MATHEMATICS** **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2014**

**1 hour 45 minutes**

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

*1929886667*

**READ THESE INSTRUCTIONS FIRST**

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.



JC14 06_9709_32/RP


CAMBRIDGE
International Examinations

**1** Find the set of values of $x$ satisfying the inequality

$$|x + 2a| > 3|x - a|,$$

where $a$ is a positive constant. [4]

**2** Solve the equation

$$2\ln(5 - e^{-2x}) = 1,$$

giving your answer correct to 3 significant figures. [4]

**3** Solve the equation

$$\cos(x + 30°) = 2\cos x,$$

giving all solutions in the interval $-180° < x < 180°$. [5]

**4** The parametric equations of a curve are

$$x = t - \tan t, \quad y = \ln(\cos t),$$

for $-\frac{1}{2}\pi < t < \frac{1}{2}\pi$.

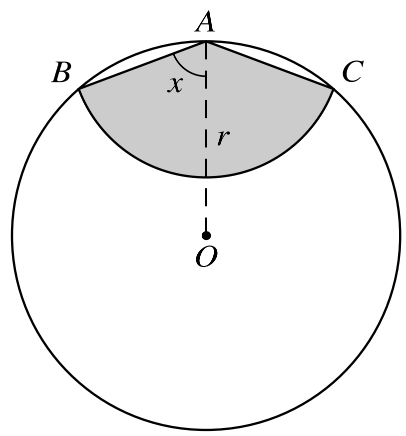
**(i)** Show that $\dfrac{dy}{dx} = \cot t$. [5]

**(ii)** Hence find the $x$-coordinate of the point on the curve at which the gradient is equal to 2. Give your answer correct to 3 significant figures. [2]

**5** **(i)** The polynomial $f(x)$ is of the form $(x - 2)^2 g(x)$, where $g(x)$ is another polynomial. Show that $(x - 2)$ is a factor of $f'(x)$. [2]

**(ii)** The polynomial $x^5 + ax^4 + 3x^3 + bx^2 + a$, where $a$ and $b$ are constants, has a factor $(x - 2)^2$. Using the factor theorem and the result of part **(i)**, or otherwise, find the values of $a$ and $b$. [5]

**6**




In the diagram, $A$ is a point on the circumference of a circle with centre $O$ and radius $r$. A circular arc with centre $A$ meets the circumference at $B$ and $C$. The angle $OAB$ is equal to $x$ radians. The shaded region is bounded by $AB$, $AC$ and the circular arc with centre $A$ joining $B$ and $C$. The perimeter of the shaded region is equal to half the circumference of the circle.

**(i)** Show that $x = \cos^{-1}\left(\dfrac{\pi}{4+4x}\right)$. [3]

**(ii)** Verify by calculation that $x$ lies between 1 and 1.5. [2]

**(iii)** Use the iterative formula

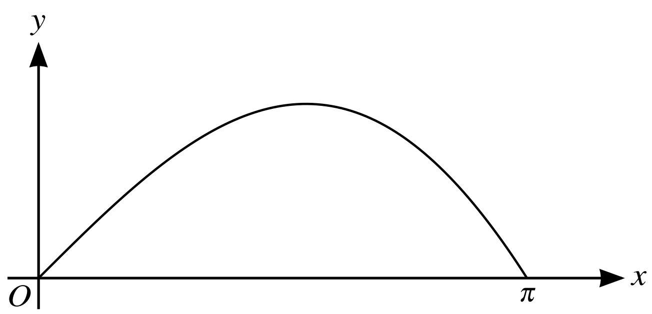
$$x_{n+1} = \cos^{-1}\left(\frac{\pi}{4+4x_n}\right)$$

to determine the value of $x$ correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

**7** **(a)** It is given that $-1 + (\sqrt{5})\mathrm{i}$ is a root of the equation $z^3 + 2z + a = 0$, where $a$ is real. Showing your working, find the value of $a$, and write down the other complex root of this equation. [4]

**(b)** The complex number $w$ has modulus 1 and argument $2\theta$ radians. Show that $\dfrac{w-1}{w+1} = \mathrm{i}\tan\theta$. [4]

**8**




The diagram shows the curve $y = x\cos\frac{1}{2}x$ for $0 \leqslant x \leqslant \pi$.

**(i)** Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and show that $4\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} + y + 4\sin\frac{1}{2}x = 0$. [5]

**(ii)** Find the exact value of the area of the region enclosed by this part of the curve and the $x$-axis. [5]

**9** The population of a country at time $t$ years is $N$ millions. At any time, $N$ is assumed to increase at a rate proportional to the product of $N$ and $(1 - 0.01N)$. When $t = 0$, $N = 20$ and $\frac{dN}{dt} = 0.32$.

**(i)** Treating $N$ and $t$ as continuous variables, show that they satisfy the differential equation

$$\frac{dN}{dt} = 0.02N(1 - 0.01N).$$ [1]

**(ii)** Solve the differential equation, obtaining an expression for $t$ in terms of $N$. [8]

**(iii)** Find the time at which the population will be double its value at $t = 0$. [1]

**10** Referred to the origin $O$, the points $A$, $B$ and $C$ have position vectors given by

$$\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}, \quad \overrightarrow{OB} = 2\mathbf{i} + 4\mathbf{j} + \mathbf{k} \quad \text{and} \quad \overrightarrow{OC} = 3\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}.$$

**(i)** Find the exact value of the cosine of angle $BAC$. [4]

**(ii)** Hence find the exact value of the area of triangle $ABC$. [3]

**(iii)** Find the equation of the plane which is parallel to the $y$-axis and contains the line through $B$ and $C$. Give your answer in the form $ax + by + cz = d$. [5]

**1** Find the set of values of $x$ satisfying the inequality

$$|x + 2a| > 3|x - a|,$$

where $a$ is a positive constant. [4]

$(x+2a)^2 > (3(x-a))^2$

$x^2+4ax+4a^2 > (9(x^2-2ax+a^2))$

$x^2+4ax+4a^2 > 9x^2-18ax+9a^2$

$8x^2-22ax+5a^2 < 0$

$\frac{22a \pm \sqrt{22^2a^2-4(5a^2)(8)}}{16}$

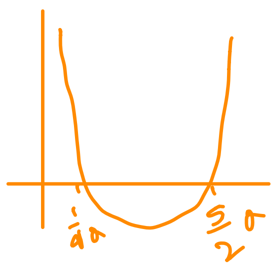
$= \frac{22a \pm \sqrt{484a^2-160a^2}}{16}$

$= \frac{22a \pm 18a}{16}$

$= \frac{11a \pm 9a}{8}$

$x_1 = \frac{20a}{8}$ $\quad x_2 = \frac{2a}{8}$

$= \frac{5a}{2}$ $\quad x_2 = \frac{1}{4}a$




$\frac{1}{4}a < x < \frac{5}{2}a$

**2** Solve the equation

$$2\ln(5 - e^{-2x}) = 1,$$

giving your answer correct to 3 significant figures. [4]

$\ln(5-e^{-2x}) = \frac{1}{2}$

$5 - e^{-2x} = e^{\frac{1}{2}}$

$e^{-2x} = 5 - e^{\frac{1}{2}}$

$-2x = \ln 5 - e^{\frac{1}{2}}$

$x = \frac{\ln(5-e^{\frac{1}{2}})}{-2}$

$x = -0.605$

**3** Solve the equation

$$\cos(x + 30°) = 2\cos x,$$

giving all solutions in the interval $-180° < x < 180°$. [5]

$\cos(x+30°) = 2\cos x$

$\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x = 2\cos x$

$\frac{\sqrt{3}}{2} + \frac{1}{2}\tan x = 2$

$\frac{1}{2}\tan x = 2 - \frac{\sqrt{3}}{2}$

$\tan x = 2.267949$

$x_1 = -66.206 \approx -66.21$ and $x_2$ $180 - 66.21 = 113.8$

$\cancel{x_2 = 246.206 \approx 246.21}$

**4** The parametric equations of a curve are

$$x = t - \tan t, \quad y = \ln(\cos t),$$

for $-\frac{1}{2}\pi < t < \frac{1}{2}\pi$.

**(i)** Show that $\frac{dy}{dx} = \cot t$. [5]

**(ii)** Hence find the $x$-coordinate of the point on the curve at which the gradient is equal to 2. Give your answer correct to 3 significant figures. [2]

i) $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$\frac{dy}{dt} = \frac{1}{\cos t}(-\sin t) = -\tan t$

$\frac{dx}{dt} = 1 - \sec^2 t = -\tan^2 t$

$\frac{dy}{dx} = -\tan t \times \frac{1}{-\tan^2 t}$

$= \frac{\tan t}{\tan^2 t} = \frac{1}{\tan t}$

ii)

**5** **(i)** The polynomial $f(x)$ is of the form $(x-2)^2 g(x)$, where $g(x)$ is another polynomial. Show that $(x-2)$ is a factor of $f'(x)$. [2]

**(ii)** The polynomial $x^5 + ax^4 + 3x^3 + bx^2 + a$, where $a$ and $b$ are constants, has a factor $(x-2)^2$. Using the factor theorem and the result of part **(i)**, or otherwise, find the values of $a$ and $b$. [5]

i) $f'(x) = 2(x-2)(1) \times g(x) + g'(x)(x-2)^2$

$= 2(x-2)g(x) + g'(x)(x-2)^2$

if $(x-2)$ is a factor of $f'(x)$, then $\dfrac{f'(x)}{(x-2)}$ is a whole number

$\dfrac{2(x-2)g(x) + g'(x)(x-2)^2}{(x-2)} = \dfrac{\cancel{(x-2)}(2g(x) + g'(x)(x-2))}{\cancel{x-2}}$

$= 2g(x) + g'(x)(x-2)$

ii) $(x-2)^2 = 0$

$x-2 = 0$

$x = 2$

$f(x) \longrightarrow 2^5 + a(2)^4 + 3(2)^3 + b(2)^2 + a = 0$

$32 + 16a + 24 + 4b + a = 0$

$17a + 4b + 56 = 0$

$f'(x) = 5x^4 + 4ax^3 + 9x^2 + 2bx$

$(x-2) = 0$

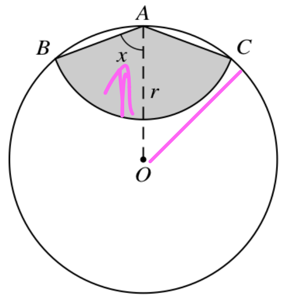
$x = 2$

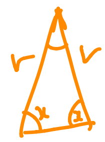
$f'(2) \quad 5(2)^4 + 4a(2)^3 + 9(2)^2 + 2b(2) = 0$

$116 + 32a + 4b = 0$

$\therefore a = -4 \qquad b = 3$

#6

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to determine the value of $x$ correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

i) $\pi r = 2x(m) + 2m$

$\pi r = 2xm + 2m$

$\pi r = 2m(x+1)$

$x + 1 = \dfrac{\pi r}{2m}$

$x = \dfrac{\pi r}{\phantom{2m}}$

Ask

ii) $x_1 = \cos^{-1}\left(\dfrac{\pi}{4+4}\right) = 66.81^\circ$

$x_2 =$

**7** **(a)** It is given that $-1 + (\sqrt{5})i$ is a root of the equation $z^3 + 2z + a = 0$, where $a$ is real. Showing your working, find the value of $a$, and write down the other complex root of this equation. [4]

**(b)** The complex number $w$ has modulus 1 and argument $2\theta$ radians. Show that $\frac{w-1}{w+1} = i\tan\theta$. [4]

$$\frac{\cos 2\theta + i\sin 2\theta - 1}{\cos 2\theta +}$$

a) $r_1 = -1+\sqrt{5}i$
$r_2 = -1-\sqrt{5}i$

$(-1+\sqrt{5}i)^3 + 2(-1+\sqrt{5}i) + a = 0$

$(-1+\sqrt{5}i)^2(-1+\sqrt{5}i) - 2 + 2\sqrt{5}i + a = 0$

$(-1-2\sqrt{5}i + 5(-1))(-1+\sqrt{5}i) - 2 + 2\sqrt{5}i + a = 0$

$(-6-2\sqrt{5}i)(-1+\sqrt{5}i) - 2 + 2\sqrt{5}i + a = 0$

$[6 - 6\sqrt{5}i + 2\sqrt{5}i - 2(5)(-1)] - 2 + 2\sqrt{5}i + a = 0$

$16 - 4\sqrt{5}i - 2 + 2\sqrt{5}i = -a$

$14 - 2\sqrt{5}i = -a$

$a = -14 + 2\sqrt{5}i$

$a^* = -14 - 2\sqrt{5}i$

b) $r = 1$
$\theta = 2\theta$

2

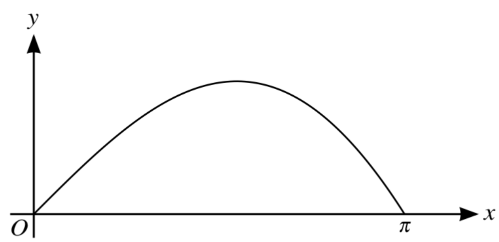
$\sqrt{a^2+b^2}$

$a = 1$
$b = 1$

$1 + i\theta$

Explain

**8**




The diagram shows the curve $y = x\cos\frac{1}{2}x$ for $0 \leqslant x \leqslant \pi$.

**(i)** Find $\frac{dy}{dx}$ and show that $4\frac{d^2y}{dx^2} + y + 4\sin\frac{1}{2}x = 0$. [5]

**(ii)** Find the exact value of the area of the region enclosed by this part of the curve and the $x$-axis. [5]

i) $y = x\cos\frac{1}{2}x$

$\frac{dy}{dx} = x\left(\frac{-1}{2}\right)\sin\frac{1}{2}x + \cos\frac{1}{2}x$

$= \frac{-x}{2}\sin\left(\frac{1}{2}x\right) + \cos\left(\frac{1}{2}x\right)$

$\frac{d^2y}{dx^2} = \frac{-x}{2}\,\frac{1}{2}\cos\frac{1}{2}x + \sin\frac{1}{2}x\left(\frac{1}{2}\right) + \frac{1}{2}\sin\frac{1}{2}x = 0$

$4\frac{d^2y}{dx^2} = -x\cos\frac{1}{2}x - 4\sin\left(\frac{1}{2}x\right)$

$4\frac{d^2y}{dx^2} + y + 4\sin\frac{1}{2}x = 0$

ii) $\int x\cos\frac{1}{2}x\,dx$

$u = x \quad u' = 1$

$v = 2\sin\frac{1}{2}x \quad v' = \cos\frac{1}{2}x$

$2x\sin\frac{1}{2}x - 2\int \sin\frac{1}{2}x\,dx$

$2x\sin\frac{1}{2}x - 2(2)\left(-\cos\frac{1}{2}x\right)$

$\left[2x\sin\frac{1}{2}x + 4\cos\frac{1}{2}x\right]_0^{\pi}$

$2\pi\sin\frac{\pi}{2} + 4\cos\frac{\pi}{2} - 4$

$2\pi - 4$

9 The population of a country at time $t$ years is $N$ millions. At any time, $N$ is assumed to increase at a rate proportional to the product of $N$ and $(1 - 0.01N)$. When $t = 0$, $N = 20$ and $\frac{dN}{dt} = 0.32$.

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**(ii)** Solve the differential equation, obtaining an expression for $t$ in terms of $N$. [8]

**(iii)** Find the time at which the population will be double its value at $t = 0$. [1]

i) $\frac{dN}{dt} \propto N(1-0.01N)$

$\frac{dN}{dt} = kN(1-0.01N)$

$0.32 = 20k(1-0.01(20))$

$0.32 = 20k(0.8)$

$20k = 0.4$
$k = 0.02$

$\frac{dN}{dt} = 0.02N(1-0.01N)$

ii) $\frac{1}{(0.02N)(1-0.01N)} dN = dt$

$\frac{A}{0.02N} + \frac{B}{1-0.01N}$

$1 = A - 0.01AN + 0.02BN$

$A = 1$, $-0.01A + 0.02B = 0$

$0.02B = 0.01$

$B = 0.5$

Explain

$\times$

$\int \frac{1}{0.02N} + \frac{1}{2(1-0.01N)} dN = t + c$

$50\ln N + \ln(2-0.02N) = t + c$

**10** Referred to the origin $O$, the points $A$, $B$ and $C$ have position vectors given by

$$\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}, \quad \overrightarrow{OB} = 2\mathbf{i} + 4\mathbf{j} + \mathbf{k} \quad \text{and} \quad \overrightarrow{OC} = 3\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}.$$

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**(iii)** Find the equation of the plane which is parallel to the $y$-axis and contains the line through $B$ and $C$. Give your answer in the form $ax + by + cz = d$. [5]

i) $\cos\theta = \dfrac{AB \cdot AC}{|AB| \cdot |AC|} = \dfrac{2+6+12}{\sqrt{1^2+2^2+3^2} \times \sqrt{2^2+3^2+6^2}} = \dfrac{10}{7}\sqrt{\dfrac{20}{7}}$

$AB = \begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$

ii) $\frac{1}{2} \times \sqrt{1^2+2^2+3^2} \times \sqrt{2^2+3^2+6^2} \times \sin\left(\cos^{-1}\left(\frac{10}{7}\sqrt{\frac{20}{7}}\right)\right)$

$= \sqrt{\dfrac{143}{2}}$

$AC = \begin{pmatrix} 3 \\ 5 \\ -3 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ -6 \end{pmatrix}$

~~iii)~~