6S

Cambridge International A Level

Cambridge Assessment International Education
Cambridge International Advanced Level

CANDIDATE NAME Feng

CENTRE NUMBER

CANDIDATE NUMBER

4678410875

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2019**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **19** printed pages and **1** blank page.

Cambridge Assessment International Education

BLANK PAGE

1 Solve the equation $5\ln(4-3^x)=6$. Show all necessary working and give the answer correct to 3 decimal places. [3]

$\ln(4-3^x) = \dfrac{6}{5}$

$4-3^x = e^{\frac{6}{5}}$

$3^x = 4-e^{\frac{6}{5}}$

$x\ln 3 = \ln 4-e^{\frac{6}{5}}$

$x = \dfrac{\ln(4-e^{\frac{6}{5}})}{\ln 3}$

$x = -0.3512016$

$x = -0.351$

2 The curve with equation $y = \dfrac{e^{-2x}}{1 - x^2}$ has a stationary point in the interval $-1 < x < 1$. Find $\dfrac{dy}{dx}$ and hence find the x-coordinate of this stationary point, giving the answer correct to 3 decimal places. [5]

$\frac{d}{dx} = -2e^{-2x}$

$= -2x$

$$\frac{dy}{dx} = \frac{(1-x^2)(-2e^{-2x}) - (e^{-2x})(-2x)}{(1-x^2)^2}$$

$$= \frac{-2e^{-2x} + 2x^2e^{-2x} + 2xe^{-2x}}{(1-x^2)^2} = 0$$

$$2e^{-2x}(-1 + x^2 + x) = 0$$

$2e^{-2x} = 0$

$e^{-2x} = 0$

$\ln e^{-2x} \cancel{=} \ln 0$

$x^2 + x - 1 = 0$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-1 \pm \sqrt{1 - 4(1)(-1)}}{2}$$

$$\frac{-1 \pm \sqrt{5}}{2} \;:\; x = 0.61803$$

or

$\cancel{x = -1.618}$

$x = 0.618$

3 The polynomial $x^4 + 3x^3 + ax + b$, where a and b are constants, is denoted by $\mathrm{p}(x)$. When $\mathrm{p}(x)$ is divided by $x^2 + x - 1$ the remainder is $2x + 3$. Find the values of a and b. [5]

$p(x) = x^4 + 3x^3 + ax + b$

$x^2 + 2x - 1$

$x^2 + x - 1 \,\big)\, x^4 + 3x^3 + 0x^2 + ax + b$

$-\ x^4 + x^3 - x^2$

$0 + 2x^3 + x^2 + ax$

$-\ 2x^3 + 2x^2 - 2x$

$0 - x^2 + ax + 2x + b$

$-\ -x^2 - x + 1$

$0 \quad ax + 3x + b - 1$

$3x + ax + b - 1 = 2x + 3$

$2x = 3x + ax$ $\quad$ $b - 1 = 3$

$a = -1$ $\quad$ $b = 4$

4 **(i)** Express $(\sqrt{6})\sin x + \cos x$ in the form $R\sin(x+\alpha)$, where $R > 0$ and $0° < \alpha < 90°$. State the exact value of R and give α correct to 3 decimal places. [3]

$R = \sqrt{1^2 + (\sqrt{6})^2}$

$= \sqrt{1+6}$

$= \sqrt{7}$

$\alpha = \tan^{-1}\left(\frac{1}{\sqrt{6}}\right) = 22.208$

$\sqrt{7}\,\sin(x + 22.208)$

(ii) Hence solve the equation $(\sqrt{6})\sin 2\theta + \cos 2\theta = 2$, for $0° < \theta < 180°$. [4]

$\sqrt{7}\sin(2\theta + 22.208) = 2$

$\sin(2\theta + 22.208) = \frac{2}{\sqrt{7}}$

① $2\theta + 22.208 = 49.1066$

$\theta = 13.4493$

② $2\theta + 22.208 = 130.89339$

$\theta = 54.342695$

$\theta = 13.45$ or $\theta = 54.34$

5 The equation of a curve is $2x^2y - xy^2 = a^3$, where a is a positive constant. Show that there is only one point on the curve at which the tangent is parallel to the x-axis and find the y-coordinate of this point. [7]

3

$2x^2y - xy^2 = a^3$

$(4xy) + \left(2x^2\frac{dy}{dx}\right) - (1y^2) + \left(2y\frac{dy}{dx}\right)(x) = 0$

$4xy + 2x^2\frac{dy}{dx} - y^2 + 2xy\frac{dy}{dx} = 0$

$(2x^2 + 2xy)\frac{dy}{dx} = y^2 - 4xy$

$\frac{dy}{dx} = \frac{y^2 - 4xy}{2x^2 + 2xy} = \frac{y(y-4x)}{2x(x+y)}$

$\frac{dy}{dx} = 0 = \frac{y(y-4x)}{2x(x+y)} = 0$

$= y(y-4x) = 0$

$y = 0$, or $y = +4x$

$2x^2(-4x) - x(-4x)^2 = a^3$

$-4x^3 - 16x^3 = a^3$

$4x^3(-1-4) = a^3$

as a has to be +ve,

$\therefore y = 0$

6 The variables x and θ satisfy the differential equation

$$\sin \tfrac{1}{2}\theta \frac{dx}{d\theta} = (x+2)\cos \tfrac{1}{2}\theta$$

for $0 < \theta < \pi$. It is given that $x = 1$ when $\theta = \frac{1}{3}\pi$. Solve the differential equation and obtain an expression for x in terms of $\cos\theta$. [8]

7

$$\int \frac{1}{x+2} = \int \frac{\cos\frac{1}{2}\theta}{\sin\frac{1}{2}\theta}\, d\theta$$

$$\frac{d}{dx}\sin\tfrac{1}{2}\theta = \tfrac{1}{2}\cos\tfrac{1}{2}\theta$$

$$\ln|x+2| = 2\int \frac{\frac{1}{2}\cos\frac{1}{2}\theta}{\sin\frac{1}{2}\theta}\, d\theta$$

$$\ln|x+2| = 2\ln\left|\sin\tfrac{1}{2}\theta\right| + C$$

sub $x = 1$, $\theta = \frac{1}{3}\pi$

$$\ln(3) = 2\ln\left(\tfrac{1}{2}\right) + C$$

$$\sin\theta = 2\sin\tfrac{1}{2}\theta\cos\tfrac{1}{2}\theta$$

$$\ln(3) = \ln\left(\tfrac{1}{4}\right) + C$$

$$\sin\tfrac{1}{2}\theta = \frac{\sin\theta}{2\cos\frac{1}{2}\theta}$$

$$\ln(3) - \ln\left(\tfrac{1}{4}\right) = C$$

$$\cos\theta = \cos^2\tfrac{1}{2}\theta - \sin^2\tfrac{1}{2}\theta$$

$$\ln(\cancel{6}) \ln(12) = C$$

$$\sin\left(\tfrac{1}{2}\theta\right)^2 = \cos^2\tfrac{1}{2}\theta - \cos\theta$$

$$\ln|x+2| = 2\ln\left|\sin\tfrac{1}{2}\theta\right| + \ln 6$$

$$\ln|x+2| = \ln\left(\sin^2\tfrac{1}{2}\theta\right| + \ln 6$$

$$\ln\sin\theta - \ln 2\cos\tfrac{1}{2}\theta$$

$\ln|x+2| = \ln|\cos^2\frac{1}{2}\theta - \cos\theta| + \ln 6$

$\cancel{x+2 = \cos^2\frac{1}{2}\theta - \cos\theta + 6}$

$\cancel{x = \cos^2\frac{1}{2}\theta - \cos\theta + 4}$

$\frac{x+2}{6} = \cos^2\frac{1}{2}\theta - \cos\theta$

$x = 6\cos^2\frac{1}{2}\theta - 6\cos\theta - 2$

7 (a) Find the complex number z satisfying the equation

$$z + \frac{\mathrm{i}z}{z^*} - 2 = 0,$$

where z^* denotes the complex conjugate of z. Give your answer in the form $x + \mathrm{i}y$, where x and y are real. [5]

let $z = x+iy$

$$x+iy + \frac{i(x+iy)}{x-iy} = 2$$

$$x+iy + \frac{xi-y}{x-iy} = 2$$

$$\frac{(x+iy)(x-iy)+xi-y}{(x-iy)} = 2$$

$$\frac{x^2-y^2(-1) \quad +xi-y}{x-iy} = 2$$

$$\frac{x^2 \quad +y^2+xi-y}{x-iy} = 2$$

$$x^2+y^2 \quad +xi-y = 2x-2iy$$

$x^2+y^2-y = 2x$

$xi = -2iy$

$x = -2y$

$(-2y)^2 + y^2 - y = 2(-2y)$

$4y^2+y^2-y = -4y$

$5y^2+3y = 0$

$y = \frac{-3}{5}$ or $\cancel{y} \neq 0$

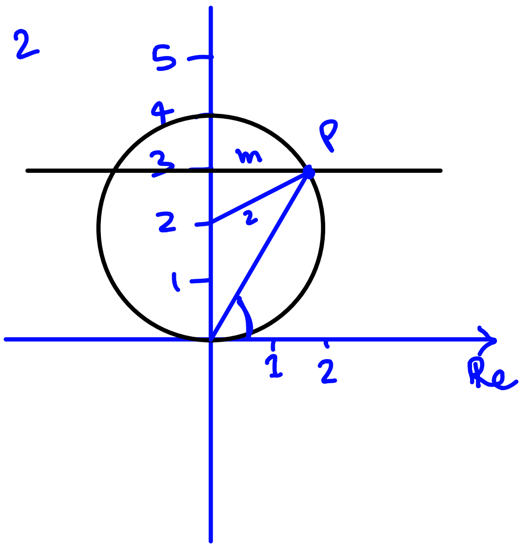
$x = -2\left(\frac{-3}{5}\right)$

$x = \frac{6}{5}$

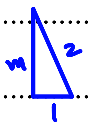
$\frac{6}{5} - \frac{3}{5}i$

(b) **(i)** On a single Argand diagram sketch the loci given by the equations $|z - 2i| = 2$ and $\text{Im}\, z = 3$, where $\text{Im}\, z$ denotes the imaginary part of z. [2]

$|z - (0 + 2i)| = 2$



(ii) In the first quadrant the two loci intersect at the point P. Find the exact argument of the complex number represented by P. [2]



$m = \sqrt{2^2 - 1^2} = \sqrt{3}$

$\therefore \tan^{-1}\left(\frac{3}{\sqrt{3}}\right) = \frac{1}{3}\pi$

8 Let $f(x) = \dfrac{2x^2 + x + 8}{(2x-1)(x^2+2)}$.

(i) Express $f(x)$ in partial fractions. [5]

$$\frac{2x^2+x+8}{(2x-1)(x^2+2)} = \frac{A}{2x-1} + \frac{Bx+C}{x^2+2}$$

$$2x^2+x+8 = A(x^2+2) + (Bx+C)(2x-1)$$
$$= Ax^2 + 2A + 2Bx^2 - Bx + 2Cx - C$$

can solve on calc2

① $2\cancel{x^2} = A\cancel{x^2} + 2B\cancel{x^2}$

$A = 2 - 2B$

② $1\cancel{x} = -B\cancel{x} + 2C\cancel{x}$

$B = 2C - 1$

③ $8 = 2A - C$

$C = 2A - 8$

$\therefore$ $A = 2 - 2(2C-1)$

$A = 2 - 2(2(2A-8)-1)$

$A = 2 - 2(4A-17)$

$A = 2 - 8A + 34$

$9A = 36$

$A = 4$

$2B = 2 - A$

$B = \frac{2-(4)}{2}$

$C = 2(4) - 8$

$C = 0$

$B = -1$

$\therefore$ $\dfrac{4}{2x-1} - \dfrac{x}{x^2+2}$

(ii) Hence, showing full working, find $\int_1^5 f(x)\,dx$, giving the answer in the form $\ln c$, where c is an integer. [5]

4

$$\int \frac{4}{2x-1}\,dx - \int \frac{x}{x^2+2}\,dx$$

$$2\int \frac{2}{2x-1}\,dx - \frac{1}{2}\int \frac{2x}{x^2+2}\,dx$$

Always straight?!

$$\left[2\ln|2x-1| - \frac{1}{2}\ln|x^2+2|\right]^5$$

sub $x=5$: $2\ln 9 - \frac{1}{2}\ln 27$

$x=1$: $2\ln 1 - \frac{1}{2}\ln 3$

$= -\frac{1}{2}\ln 3$

$\therefore$ $2\ln 9 - \frac{1}{2}\ln 27 - \left(-\frac{1}{2}\ln 3\right)$

$2\ln 9 - \frac{1}{2}\ln 27 + \frac{1}{2}\ln 3$

$2\ln 9 - \frac{1}{2}\ln 81$

$2\ln 9 - \ln 81^{\frac{1}{2}} = 2\ln 9 - \ln 9$

$= \ln 9$

9 It is given that $\int_0^a x \cos \frac{1}{3}x \, dx = 3$, where the constant a is such that $0 < a < \frac{3}{2}\pi$.

(i) Show that a satisfies the equation

$$a = \frac{4 - 3\cos \frac{1}{3}a}{\sin \frac{1}{3}a}.$$

[5]

$u = x \quad u' = 1$

$v' = 3\sin\frac{1}{3}x \quad v' = \cos\frac{1}{3}x$

$3x \sin\frac{1}{3}x - 3\int \sin\frac{1}{3}x \; dx$

$3x \sin\frac{1}{3}x - 3(3)(-\cos\frac{1}{3}x)$

$\left[3x \sin\frac{1}{3}x + 9\cos\frac{1}{3}x\right]_0^a$

$\left(3a \sin\frac{1}{3}a + 9\cos\frac{1}{3}a\right) - \left(0 + 9(1)\right) = 3$

$3a \sin\frac{1}{3}a + 9\cos\frac{1}{3}a - 9 = 3$

$3a\sin\frac{1}{3}a = 3 + 9 - 9\cos\frac{1}{3}a$

$3a\sin\frac{1}{3}a = 12 - 9\cos\frac{1}{3}a$

$\cancel{3}a\sin\frac{1}{3}a = \cancel{3}(4 - 3\cos\frac{1}{3}a)$

$a = \frac{4 - 3\cos\frac{1}{3}a}{\sin\frac{1}{3}a}$

(ii) Verify by calculation that a lies between 2.5 and 3. [2]

$$y = \frac{4 - 3\cos\frac{1}{3}a}{\sin\frac{1}{3}a} - a$$

sub $a = 2.5$, $y = 0.1781$

$a = 3$, $y = -0.1727$

change of sign, so root is between 2.5 and 3

(iii) Use an iterative formula based on the equation in part **(i)** to calculate a correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

$$a = \frac{4 - 3\cos\frac{1}{3}a}{\sin\frac{1}{3}a}$$

let $a_1 = 2.5$

$$a_2 = \frac{4 - 3\cos(\frac{1}{3}(2.5))}{\sin(\frac{1}{3}(2.5))} = 2.67877$$

$a_3 = 2.71994$

$a_4 = 2.73142$

$a_5 = 2.73474$

$a_6 = 2.73572$

$a_7 = 2.73600$

$a_8 = 2.73609$ $\therefore a = 2.736$

$a_9 = 2.73611$

10 The line l has equation $\mathbf{r} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$. The plane p has equation $2x + y - 3z = 5$.

(i) Find the position vector of the point of intersection of l and p. [3]

(ii) Calculate the acute angle between l and p. [3]

(iii) A second plane q is perpendicular to the plane p and contains the line l. Find the equation of q, giving your answer in the form $ax + by + cz = d$. [5]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.