Cambridge International AS & A Level

Cambridge International Examinations
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS **9709/61**

Paper 6 Probability & Statistics 1 **(S1)** **October/November 2016**

1 hour 15 minutes

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

An answer booklet and a graph paper booklet are provided inside this question paper. You should follow the instructions on the front cover of both booklets. If you need additional answer paper or graph paper ask the invigilator for a continuation booklet or graph paper booklet.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.

This document consists of **3** printed pages, **1** blank page and **2** inserts.

CAMBRIDGE International Examinations

1 The random variable X is such that $X \sim \mathrm{N}(20, 49)$. Given that $\mathrm{P}(X > k) = 0.25$, find the value of k. [3]

2 Two fair six-sided dice with faces numbered 1, 2, 3, 4, 5, 6 are thrown and the two scores are noted. The difference between the two scores is defined as follows.

- If the scores are equal the difference is zero.
- If the scores are not equal the difference is the larger score minus the smaller score.

Find the expectation of the difference between the two scores. [5]

3 Visitors to a Wildlife Park in Africa have independent probabilities of 0.9 of seeing giraffes, 0.95 of seeing elephants, 0.85 of seeing zebras and 0.1 of seeing lions.

(i) Find the probability that a visitor to the Wildlife Park sees all these animals. [1]

(ii) Find the probability that, out of 12 randomly chosen visitors, fewer than 3 see lions. [3]

(iii) 50 people independently visit the Wildlife Park. Find the mean and variance of the number of these people who see zebras. [2]

4 Packets of rice are filled by a machine and have weights which are normally distributed with mean 1.04 kg and standard deviation 0.017 kg.

(i) Find the probability that a randomly chosen packet weighs less than 1 kg. [3]

(ii) How many packets of rice, on average, would the machine fill from 1000 kg of rice? [1]

The factory manager wants to produce more packets of rice. He changes the settings on the machine so that the standard deviation is the same but the mean is reduced to μ kg. With this mean the probability that a packet weighs less than 1 kg is 0.0388.

(iii) Find the value of μ. [3]

(iv) How many packets of rice, on average, would the machine now fill from 1000 kg of rice? [1]

5 **(a)** Find the number of different ways of arranging all nine letters of the word PINEAPPLE if no vowel (A, E, I) is next to another vowel. [4]

(b) A certain country has a cricket squad of 16 people, consisting of 7 batsmen, 5 bowlers, 2 all-rounders and 2 wicket-keepers. The manager chooses a team of 11 players consisting of 5 batsmen, 4 bowlers, 1 all-rounder and 1 wicket-keeper.

(i) Find the number of different teams the manager can choose. [2]

(ii) Find the number of different teams the manager can choose if one particular batsman refuses to be in the team when one particular bowler is in the team. [3]

6 Deeti has 3 red pens and 1 blue pen in her left pocket and 3 red pens and 1 blue pen in her right pocket. 'Operation T' consists of Deeti taking one pen at random from her left pocket and placing it in her right pocket, then taking one pen at random from her right pocket and placing it in her left pocket.

(i) Find the probability that, when Deeti carries out operation T, she takes a blue pen from her left pocket and then a blue pen from her right pocket. [2]

The random variable X is the number of blue pens in Deeti's left pocket after carrying out operation T.

(ii) Find $P(X = 1)$. [3]

(iii) Given that the pen taken from Deeti's right pocket is blue, find the probability that the pen taken from Deeti's left pocket is blue. [4]

7 The masses, in grams, of components made in factory A and components made in factory B are shown below.

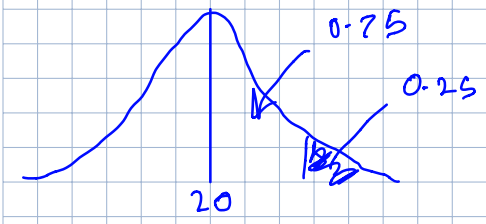
Factory A	0.049	0.050	0.053	0.054	0.057	0.058	0.058
	0.059	0.061	0.061	0.061	0.063	0.065	
Factory B	0.031	0.056	0.049	0.044	0.038	0.048	0.051
	0.064	0.035	0.042	0.047	0.054	0.058	

(i) Draw a back-to-back stem-and-leaf diagram to represent the masses of components made in the two factories. [5]

(ii) Find the median and the interquartile range for the masses of components made in factory B. [3]

(iii) Make two comparisons between the masses of components made in factory A and the masses of those made in factory B. [2]

1 The random variable X is such that $X \sim N(20, 49)$. Given that $P(X > k) = 0.25$, find the value of k. [3]



$\Phi\left(\frac{x-20}{7}\right) = 0.75$

$\frac{x-20}{7} = 0.674$

$x = 24.718$

$= 24.7$ (3 s.f.)

2 Two fair six-sided dice with faces numbered 1, 2, 3, 4, 5, 6 are thrown and the two scores are noted. The difference between the two scores is defined as follows.

- If the scores are equal the difference is zero.
- If the scores are not equal the difference is the larger score minus the smaller score.

Find the expectation of the difference between the two scores. [5]

	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

x	0	1	2	3	4	5
$P(X=x)$	$\frac{6}{36}$	$\frac{10}{36}$	$\frac{8}{36}$	$\frac{6}{36}$	$\frac{4}{36}$	$\frac{2}{36}$

Exp $\frac{10}{36} + \frac{16}{36} + \frac{18}{36} + \frac{12}{36} + \frac{10}{36} = \frac{35}{18} = 1.94$

3 Visitors to a Wildlife Park in Africa have independent probabilities of 0.9 of seeing giraffes, 0.95 of seeing elephants, 0.85 of seeing zebras and 0.1 of seeing lions.

(i) Find the probability that a visitor to the Wildlife Park sees all these animals. [1]

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(iii) 50 people independently visit the Wildlife Park. Find the mean and variance of the number of these people who see zebras. [2]

i) $0.9 \times 0.95 \times 0.85 \times 0.1 = 0.072$

ii) $X \sim B(12, 0.1)$, $P(X < 3) = P(0, 1, 2)$

0: ${}^{12}C_0 \times 0.1^0 \times 0.9^{12} = 0.28243$

1: $= 0.3765727$

2: $= 0.23012777$

$= 0.889$

iii) $X \sim B(50, 0.85)$

$np = \mu = 50 \times 0.85 = 42.5$

$\sigma = 6.38$ (3sf)

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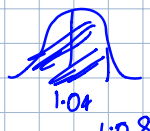
(iii) Find the value of μ. [3]

(iv) How many packets of rice, on average, would the machine now fill from 1000 kg of rice? [1]

i)

$= 1 -$



$= 1 - \Phi\left(\frac{0.04}{0.017}\right)$

$= 1 - 0.9907$

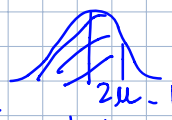
$= 0.0093$

ii) $\frac{1000}{1.04} = 961.538$

$\therefore$ 962

iii)

$= 1 -$



$1 - \Phi\left(\frac{\mu - 1}{0.017}\right) = 0.0388$

$\Phi\left(\frac{\mu - 1}{0.017}\right) = 0.9612$

$\frac{\mu - 1}{0.017} = 1.765$

$\mu = 1.029988$

$= 1.03$

iv) $\frac{1000}{1.029988} = 970$

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a) Vowels = A E E I
Consonant = P P N L

• _ • _ • _ • _ • _ •

$\frac{5!}{3!} \times {}^{6}P_{4} = 3600$ ways

b) i

$_ \; _ \; _ \; _ \; _ \; _ \; _ \; _ \; _ \; _ \; _$

${}^{7}C_{5} \times {}^{5}C_{4} \times {}^{2}C_{1} \times {}^{2}C_{1} = 420$

ii)

iii) bowler only : ${}^{6}C_{5} \times {}^{5}C_{4} \times {}^{2}C_{1} \times {}^{2}C_{1}$ | 120
Batter only : ${}^{7}C_{5} \times {}^{4}C_{4} \times {}^{2}C_{1} \times {}^{2}C_{1}$ | 84
none : ${}^{6}C_{5} \times {}^{4}C_{4} \times {}^{2}C_{1} \times {}^{2}C_{1}$ | 24

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i) Left 3R 1B Right 3R 1B 3R2B

$\frac{1}{4} \times \frac{2}{5} = \frac{1}{10} = 0.1$

ii) $\frac{3}{4} \times \frac{4}{5} = \frac{3}{5}$

$\frac{1}{4} \times \frac{2}{5} = \frac{1}{10}$

$\frac{7}{10} = 0.7$

iii) $P(B_L / B_R) = \frac{P(B_L \cap B_R)}{P(B_R)}$

$= \frac{0.1}{\frac{1}{4}} = \frac{2}{5} = 0.4$

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(iii) Make two comparisons between the masses of components made in factory *A* and the masses of those made in factory *B*. [2]

i)

B		A
8 5 1	3	
9 8 7 4 2	4	9
8 6 4 1	5	0 3 4 1 8 8 9
4	6	1 1 1 6 3

ii)

Median = 0.0048

LQ = 0.0038 + 0.0042

= 0.004

UQ = 0.0055

IQR = 0.0015

iii) Mass B are smaller and more spread

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