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Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE NAME Fuzail

CENTRE NUMBER

CANDIDATE NUMBER

* 9 7 8 2 7 3 3 0 5 9 *

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2018**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **19** printed pages and **1** blank page.

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1 Solve the inequality $3|2x - 1| > |x + 4|$. [4]

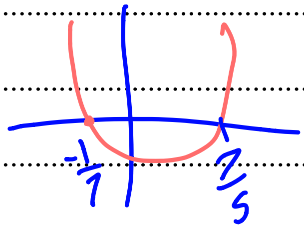
$9(2x-1)(2x-1) > (x+4)(x+4)$

$9(4x^2 - 4x + 1) > x^2 + 8x + 16$

$36x^2 - 36x + 9 > x^2 + 8x + 16$

$35x^2 - 44x - 7 > 0$

$x =$



$x < -\frac{1}{7}$

$x > \frac{7}{5}$

$3(2(2)-1) > |6|$

-1

$3|-2-1| > |-1+4$

$9 >$

$x < -\frac{1}{7}$ or $x > \frac{7}{5}$

2 Showing all necessary working, solve the equation $\sin(\theta - 30°) + \cos\theta = 2\sin\theta$, for $0° < \theta < 180°$. [4]

$\frac{\sqrt{3}}{2}\sin\theta - \frac{1}{2}\cos\theta + \cos\theta = 2\sin\theta$

$\frac{\sqrt{3}}{2}\sin\theta + \frac{1}{2}\cos\theta = 2\sin\theta$

$\frac{\sqrt{3}}{2}\tan\theta + \frac{1}{2} = 2\tan\theta$

$\left(2 - \frac{\sqrt{3}}{2}\right)\tan\theta = \frac{1}{2}$

$\theta = \tan^{-1}\left(\frac{1/2}{2 - \frac{\sqrt{3}}{2}}\right)$

$\theta = 23.79398°$

$\theta = 23.8°$

3 **(i)** Find $\displaystyle\int \frac{\ln x}{x^3}\,dx$. [3]

$\int x^{-3} \ln x$

$\uparrow v' \quad \uparrow u$

$u = \ln x \qquad u' = \frac{1}{x}$

$v = \frac{x^{-2}}{-2} \qquad v' = x^{-3}$

$= -\frac{1}{2x^2}$

$-\frac{1}{2x^2} \times \ln x + \int \frac{1}{2x^2} \times \frac{1}{x}\,dx$

$\frac{-\ln x}{2x^2} + \frac{1}{2}\int x^{-3}\,dx$

$\frac{-\ln x}{2x^2} + \frac{1}{2}\left(-\frac{1}{2x^2}\right) \Rightarrow \frac{1}{4x^2}\left(-2\ln x - 1\right)$

(ii) Hence show that $\displaystyle\int_1^2 \frac{\ln x}{x^3}\,dx = \frac{1}{16}(3 - \ln 4)$. [2]

let $f(x) = \frac{1}{4x^2}\left(-2\ln x - 1\right)$

$f(2) = \frac{1}{16}\left(-2\ln 2 - 1\right) = \frac{-1}{16} - \frac{\ln 2^2}{16}$

$f(1) = \frac{1}{4}(-1) = \frac{-1}{4}$

$\therefore \frac{3}{16} - \frac{2\ln 2}{16} = \frac{1}{16}\left(3 - \ln 4\right)$

4 Showing all necessary working, solve the equation

$$\frac{e^x + e^{-x}}{e^x + 1} = 4,$$

giving your answer correct to 3 decimal places. [5]

4

$e^x + e^{-x} = 4e^x + 4$

$(3e^x + 4) = \frac{1}{e^x}$

$e^x(3e^x + 4) = 1$

$3e^{2x} + 4e^x - 1 = 0$

let $x = e^x$

$3x^2 + 4x - 1 = 0$

$e^x = 0.215$ or $e^x = -1.548$

$x = \ln 0.215$ rejected

$x = -1.54$

End

5 The equation of a curve is $y = x\ln(8 - x)$. The gradient of the curve is equal to 1 at only one point, when $x = a$.

(i) Show that a satisfies the equation $x = 8 - \dfrac{8}{\ln(8 - x)}$. [3]

3

$\frac{dy}{dx} = x\left(\frac{1}{8-x}\right)(-1) + \ln(8-x) = 1$

$\frac{-x}{8-x} + \ln 8 - x = 1$

$\ln(8-x) = 1 + \frac{x}{8-x}$

$\frac{8-x+x}{8-x} = \ln 8 - x$

$\frac{8}{8-x} = \ln(8-x)$

$\frac{8}{\ln(8-x)} = 8 - x$

$x = 8 - \frac{8}{\ln(8-x)}$

(ii) Verify by calculation that a lies between 2.9 and 3.1. [2]

$x = 8 - \frac{8}{\ln(8-x)}$

let $f(x) = 8 - x - \frac{8}{\ln(8-x)}$

$f(2.9) = 8 - 2.9 - \frac{8}{\ln(8-2.9)} = 0.1897$

$f(3.1) = -0.1138$

change of sign $\therefore$ root lies between 2.9 and 3.1

(iii) Use an iterative formula based on the equation in part **(i)** to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

let $x_1 = 3$

$x_2 = 8 - \frac{8}{\ln(8-3)} = 3.0293$

$x_3 = 3.0111$

$x_4 = 3.0225$

$x_5 = 3.0154$

$x_6 = 3.0198$

$x_7 = 3.0170$

$x8 = 3.0188$

$\therefore x = 3.02$

6 A certain curve is such that its gradient at a general point with coordinates (x, y) is proportional to $\frac{y^2}{x}$. The curve passes through the points with coordinates $(1, 1)$ and $(e, 2)$. By setting up and solving a differential equation, find the equation of the curve, expressing y in terms of x. [8]

$\frac{dy}{dx} = \frac{ky^2}{x}$

$\int y^{-2}\,dy = k\int \frac{1}{x}\,dx$

$-\frac{1}{y} = k\ln x + c$

$-1 = 0 + c$

$c = 1$

$-\frac{1}{2} = k\ln e - 1$

$\frac{1}{2} = k(1)$

$k = \frac{1}{2}$

$-\frac{1}{y} = \frac{1}{2}\ln x - 1$

$-1 = \frac{1}{2}y\ln x - y$

$-1 = y(\frac{1}{2}\ln x - 1)$

$y = \frac{-1}{\frac{1}{2}\ln x - 1}$

7 A curve has equation $y = \dfrac{3\cos x}{2 + \sin x}$, for $-\frac{1}{2}\pi \leqslant x \leqslant \frac{1}{2}\pi$.

$-1.57 \qquad 1.57$

(i) Find the exact coordinates of the stationary point of the curve. [6]

$$0 = \frac{dy}{dx} = \frac{(2+\sin x)(3(-\sin x)) - (3\cos x)(\cos x)}{(2+\sin x)^2}$$

$$= -6\sin x - 3\sin^2 x - 3\cos^2 x = 0$$

$$-3(2\sin x + \sin^2 x + \cos^2 x) = 0$$

$$2\sin x + 1 = 0$$

$$\sin x = -\frac{1}{2}$$

$$x = \pi + \sin^{-1}\left(\frac{1}{2}\right) = 3.665$$

or

$$x = 2\pi - \sin^{-1}\left(\frac{1}{2}\right) - 2\pi = -\frac{1}{6}\pi$$

$$y = \frac{3\cos\left(-\frac{1}{6}\pi\right)}{2+\sin\left(-\frac{1}{6}\pi\right)} \qquad \frac{\frac{3\sqrt{3}}{2}}{\frac{3}{2}} = \sqrt{3}$$

$$\left(-\frac{1}{6}\pi, \sqrt{3}\right)$$

(ii) The constant a is such that $\int_0^a \frac{3\cos x}{2+\sin x}\,dx = 1$. Find the value of a, giving your answer correct to 3 significant figures. [4]

$\frac{d}{dx} = \cos x$

$3\int_0^a \frac{\cos x}{2+\sin x}\,dx = 1$

$3\ln|2+\sin x|\Big|_0^a = 1$

$3\ln|2+\sin a| - 3\ln 2 = 1$

$3\ln\left|\frac{2+\sin a}{2}\right| = 1$

$\frac{2+\sin a}{2} = e^{\frac{1}{3}}$

$\sin a = 2e^{\frac{1}{3}} - 2$

$a = \sin^{-1}(2e^{\frac{1}{3}} - 2)$

$a = 0.913$

8 Let $f(x) = \dfrac{7x^2 - 15x + 8}{(1-2x)(2-x)^2}$. $= \dfrac{A}{1-2x} + \dfrac{B}{2-x} + \dfrac{C}{(2-x)^2}$

(i) Express $f(x)$ in partial fractions. [5]

$7x^2 - 15x + 8 = A(2-x)^2 + B(1-2x)(2-x) + C(1-2x)$

$2-x$

$7x^2-15x+8 = A(4-4x+x^2) + B(2-5x+2x^2) + C-2Cx$

$7x^2-15x+8 = 4A - 4Ax + Ax^2 + 2B - 5Bx + 2Bx^2 + C - 2Cx$

$7 = A + 2B$

$-15 = -4A - 5B - 2C$

$8 = 4A + 2B + C$

$A = 1$

$B = 3$

$C = -2$

(ii) Hence obtain the expansion of $\mathrm{f}(x)$ in ascending powers of x, up to and including the term in x^2. [5]

$1(1-2x)^{-1} + 3(2-x)^{-1} - 2(2-x)^{-2}$

$(1-2x)^{-1} = 1+2x + \frac{(-1)(-2)}{2}(-2x)^2$

$= 1+2x + 4x^2$

$\frac{3}{2}\left(1-\frac{x}{2}\right)^{-1} = \frac{3}{2}\left[1 + \frac{x}{2} + \frac{(-1)(-2)}{2}\left(-\frac{1}{2}x\right)^2\right]$

$= \frac{3}{2} + \frac{3x}{4} + \frac{3}{8}x^2$

$-\frac{1}{2}\left(1-\frac{x}{2}\right)^{-2} = -\frac{1}{2}\left(1 + (-2)\left(-\frac{1}{2}x\right) + \frac{(-2)(-3)\left(-\frac{1}{2}x\right)^2}{2}\right)$

$= -\frac{1}{2} - \frac{1}{2}x - \frac{3}{8}x^2$

$1+2x+4x^2 + \frac{3}{2} + \frac{3}{4}x + \cancel{\frac{3}{8}x^2} - \frac{1}{2} - \frac{1}{2}x \cancel{- \frac{3}{8}x^2}$

$2 + \frac{9}{4}x + 4x^2$

9 **(a)** **(i)** Without using a calculator, express the complex number $\dfrac{2+6\mathrm{i}}{1-2\mathrm{i}}$ in the form $x+\mathrm{i}y$, where x and y are real. [2] 1

$\dfrac{(2+6i)(1+2i)}{(1-2i)(1+2i)} = \dfrac{2+10i+12(-1)}{1-4(-1)} = \dfrac{-10+10i}{5}$

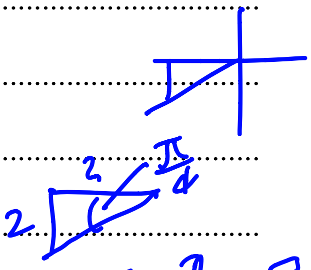
$= -2-2i$

(ii) Hence, without using a calculator, express $\dfrac{2+6\mathrm{i}}{1-2\mathrm{i}}$ in the form $r(\cos\theta + \mathrm{i}\sin\theta)$, where $r > 0$ and $-\pi < \theta \leqslant \pi$, giving the exact values of r and θ. [3] 2

$r = \sqrt{2^2+2^2} = \sqrt{4+4} = \sqrt{8}$

$\theta = -\frac{3}{4}\pi$

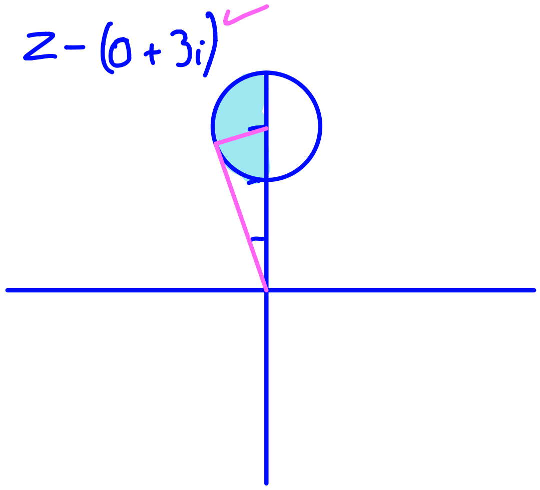
$\sqrt{8}\left(\cos -\frac{3}{4}\pi + i\sin -\frac{3}{4}\pi\right)$

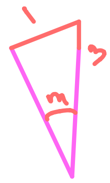


$\pi - \frac{\pi}{4} = \frac{3}{4}\pi$

(b) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying both the inequalities $|z - 3i| \leqslant 1$ and $\mathrm{Re}\, z \leqslant 0$, where $\mathrm{Re}\, z$ denotes the real part of z. Find the greatest value of $\arg z$ for points in this region, giving your answer in radians correct to 2 decimal places. [5]

5





$m = \tan^{-1}\left(\frac{1}{3}\right) = 0.32175$

Max arg $= \frac{\pi}{2} + m$

$= \frac{\pi}{2} + 0.32175 = 1.8925$

$= 1.89$ (2dp)

10 The line l has equation $\mathbf{r} = 5\mathbf{i} - 3\mathbf{j} - \mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + \mathbf{k})$. The plane p has equation

$$(\mathbf{r} - \mathbf{i} - 2\mathbf{j}) \cdot (3\mathbf{i} + \mathbf{j} + \mathbf{k}) = 0.$$

The line l intersects the plane p at the point A.

(i) Find the position vector of A. [3]

(ii) Calculate the acute angle between l and p. [4]

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[Question 10 (iii) is printed on the next page.]

(iii) Find the equation of the line which lies in p and intersects l at right angles. [4]

Additional Page

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