Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2016**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **3** printed pages and **1** blank page.

1.5 hours, go: 3:54

1 Use logarithms to solve the equation $4^{3x-1} = 3(5^x)$, giving your answer correct to 3 decimal places. [4]

2 Expand $\dfrac{1}{\sqrt{(1-2x)}}$ in ascending powers of x, up to and including the term in x^3, simplifying the coefficients. [4]

3 Find the exact value of $\displaystyle\int_0^{\frac{1}{2}\pi} x^2 \sin 2x \,\mathrm{d}x$. [5]

4 The curve with equation $y = \dfrac{(\ln x)^2}{x}$ has two stationary points. Find the exact values of the coordinates of these points. [6]

5 **(i)** Prove the identity $\cos 4\theta - 4\cos 2\theta \equiv 8\sin^4\theta - 3$. [4]

(ii) Hence solve the equation

$$\cos 4\theta = 4\cos 2\theta + 3,$$

for $0° \leqslant \theta \leqslant 360°$. [4]

6 The variables x and θ satisfy the differential equation

$$(3 + \cos 2\theta)\frac{\mathrm{d}x}{\mathrm{d}\theta} = x \sin 2\theta,$$

and it is given that $x = 3$ when $\theta = \frac{1}{4}\pi$.

(i) Solve the differential equation and obtain an expression for x in terms of θ. [7]

(ii) State the least value taken by x. [1]

7 Let $\mathrm{f}(x) = \dfrac{4x^2 + 7x + 4}{(2x+1)(x+2)}$.

(i) Express $\mathrm{f}(x)$ in partial fractions. [5]

(ii) Show that $\displaystyle\int_0^4 \mathrm{f}(x)\,\mathrm{d}x = 8 - \ln 3$. [5]

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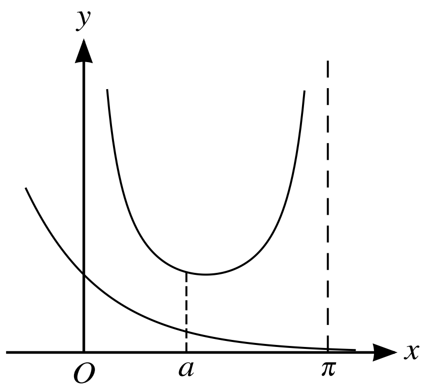

The diagram shows the curve $y = \operatorname{cosec} x$ for $0 < x < \pi$ and part of the curve $y = e^{-x}$. When $x = a$, the tangents to the curves are parallel.

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(ii) By equating the gradients of the curves at $x = a$, show that

$$a = \tan^{-1}\left(\frac{e^a}{\sin a}\right).$$ [2]

(iii) Verify by calculation that a lies between 1 and 1.5. [2]

(iv) Use an iterative formula based on the equation in part **(ii)** to determine a correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

9 The points A, B and C have position vectors, relative to the origin O, given by $\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$, $\overrightarrow{OB} = 4\mathbf{j} + \mathbf{k}$ and $\overrightarrow{OC} = 2\mathbf{i} + 5\mathbf{j} - \mathbf{k}$. A fourth point D is such that the quadrilateral $ABCD$ is a parallelogram.

(i) Find the position vector of D and verify that the parallelogram is a rhombus. [5]

(ii) The plane p is parallel to OA and the line BC lies in p. Find the equation of p, giving your answer in the form $ax + by + cz = d$. [5]

10 **(a)** Showing all necessary working, solve the equation $iz^2 + 2z - 3i = 0$, giving your answers in the form $x + iy$, where x and y are real and exact. [5]

(b) **(i)** On a sketch of an Argand diagram, show the locus representing complex numbers satisfying the equation $|z| = |z - 4 - 3i|$. [2]

(ii) Find the complex number represented by the point on the locus where $|z|$ is least. Find the modulus and argument of this complex number, giving the argument correct to 2 decimal places. [3]

1 Use logarithms to solve the equation $4^{3x-1} = 3(5^x)$, giving your answer correct to 3 decimal places. [4]

1)

$$\ln 4^{3x-1} = \ln 3 \times 5^x$$

$$(3x-1)\ln 4 = \ln 3 + x\ln 5$$

$$3x\ln 4 - \ln 4 - x\ln 5 = \ln 3 + \ln 4$$

$$x\ln 64 - x\ln 5 = \ln 12$$

$$x\ln \frac{64}{5} = \ln 12$$

$$x = \frac{\ln 12}{\ln\left(\frac{64}{5}\right)}$$

$$x = 0.974685$$

$$x = 0.975 \ (3\text{dp})$$

2 Expand $\dfrac{1}{\sqrt{(1-2x)}}$ in ascending powers of x, up to and including the term in x^3, simplifying the coefficients. [4]

$$\frac{1}{(1-2x)^{\frac{1}{2}}} = (1-2x)^{-\frac{1}{2}} = \left(1 + \left(-\tfrac{1}{2}\right)(-2x) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)(-2x)^2}{2!} + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)(-2x)^3}{3!}\right)$$

$$= 1 + x + \frac{3}{2}x^2 + \frac{5}{2}x^3$$

3 Find the exact value of $\displaystyle\int_0^{\frac{1}{2}\pi} x^2 \sin 2x \, dx$. [5]

u v'

$u = x^2 \qquad u' = 2x$

$v = -\frac{1}{2}\cos 2x \qquad v' = \sin 2x$

$-\frac{1}{2}\int \cos 2x$

$-\frac{1}{2}\left(\frac{1}{2}\right)\sin 2x$

$-\frac{1}{4}\sin 2x$

$$-\frac{x^2\cos 2x}{2} - \int\left(-\frac{1}{2}\cos 2x\right)2x \, dx$$

v' u

$u' = 2$

$v = -\frac{1}{4}\sin 2x$

$$-\frac{x^2\cos 2x}{2} - \left[2x\left(-\frac{1}{4}\sin 2x\right) - \int\left(-\frac{1}{4}\sin 2x\right)(2)\, dx\right]$$

$$-\frac{x^2}{2}\cos 2x - \left[-\frac{x}{2}\sin 2x + \int \frac{1}{2}\sin 2x \, dx\right]$$

$$-\frac{x^2}{2}\cos 2x - \left[-\frac{x}{2}\sin 2x + \frac{1}{2}\left(-\frac{1}{2}\cos 2x\right)\right]$$

$$-\frac{x^2}{2}\cos 2x - \left[-\frac{x}{2}\sin 2x - \frac{1}{4}\cos 2x\right]$$

$$\left[-\frac{x^2}{2}\cos 2x + \frac{x}{2}\sin 2x + \frac{1}{4}\cos 2x\right]$$

$$-\frac{\left(\frac{1}{2}\pi\right)^2}{2}\cos\pi + \frac{1}{2}\pi\sin\pi + \frac{1}{4}\cos\pi \Rightarrow -\frac{\pi^2}{8}(-1) - \frac{1}{4}$$

-1.733

$$-\quad \frac{1}{4}\cos 0 = \frac{1}{4}$$

$$\frac{\pi^2}{8} - \frac{1}{4} - \frac{1}{4} \Rightarrow \frac{\pi^2}{8} - \frac{1}{2} \Rightarrow \frac{1}{8}(\pi^2 - 4)$$

4 The curve with equation $y = \frac{(\ln x)^2}{x}$ has two stationary points. Find the exact values of the coordinates of these points. [6]

u v

$u' = \frac{2\ln x}{x} = \frac{2\ln x}{x}$ $v' = 1$

$y = \frac{(\ln x)^2}{x}$ $\frac{dy}{dx} \Rightarrow \frac{\cancel{x}\left(\frac{2\ln x}{\cancel{x}}\right) - (\ln x)^2}{x^2}$

$\frac{2\ln x - (\ln x)^2}{x^2} = 0$

$2\ln(x) - (\ln x)^2 = 0$

$\ln x(2 - \ln x) = 0$

$\ln x = 0$

$x = 1$, $y = \frac{(\ln(1))^2}{1} = 0$

$(1, 0)$

$\ln x = 2$

$x = e^2$

$y = \frac{(\ln e^2)^2}{e^2} = \frac{(\ln e^2)(\ln e^2)}{e^2}$

$= \frac{(2\ln e)(2\ln e)}{e^2}$

$= \frac{4(1)}{e^2} \Rightarrow \frac{4}{e^2}$

$\left(e^2, \frac{4}{e^2}\right)$

5 **(i)** Prove the identity $\cos 4\theta - 4\cos 2\theta \equiv 8\sin^4\theta - 3$. [4]

(ii) Hence solve the equation

$$\cos 4\theta = 4\cos 2\theta + 3,$$

for $0° \leqslant \theta \leqslant 360°$. [4]

i) $(2\cos^2 2\theta - 1) - 4(1 - 2\sin^2\theta)$

$(2(1-2\sin^2\theta)^2 - 1) - 4 + 8\sin^2\theta$

$(2(1-2\sin^2\theta)(1-2\sin^2\theta) - 1) - 4 + 8\sin^2\theta$

$(2(1 - 4\sin^2\theta + 4\sin^4\theta) - 1)$

$2 - \cancel{8\sin^2\theta} + 8\sin^4\theta - 1 - 4 + \cancel{8\sin^2\theta}$

$8\sin^4\theta - 3$

ii) $\cos 4\theta - 4\cos 2\theta = 3$

$8\sin^4\theta - 3 = 3$

$8\sin^4\theta = 6$

$\sin^4\theta = \frac{3}{4}$

$\sin\theta = +\sqrt[4]{\frac{3}{4}}$, $\sin\theta = -\sqrt[4]{\frac{3}{4}}$

$\theta = 68.5°$, $\theta = 248.5$

or or

$\theta = 111.47° = 111.5°$, $\theta = 291.5$

6 The variables x and θ satisfy the differential equation

$$(3 + \cos 2\theta)\frac{dx}{d\theta} = x \sin 2\theta,$$

and it is given that $x = 3$ when $\theta = \frac{1}{4}\pi$.

(i) Solve the differential equation and obtain an expression for x in terms of θ. [7]

(ii) State the least value taken by x. [1]

i) $\int \frac{1}{x}\,dx = \int \frac{\sin 2\theta}{3+\cos 2\theta}\,d\theta$

$\ln x = -\frac{1}{2}\int \frac{-2\sin 2\theta}{3+\cos 2\theta}\,d\theta$

$\ln x = -\frac{1}{2}\ln(3+\cos 2\theta) + C$

$\ln 3 = -\frac{1}{2}\ln(3+0) + C$

$\ln 3 + \frac{1}{2}\ln 3$

$\frac{3}{2}\ln 3$

$\ln 3^{\frac{3}{2}} \Rightarrow \underline{\ln 3\sqrt{3}} = C$

$\ln x = \ln 3 + \frac{1}{2}\ln 3 - \frac{1}{2}\ln(3+\cos 2\theta)$

$\ln x = \ln 3 + \frac{1}{2}\ln\left(\frac{3}{3+\cos 2\theta}\right)$

$\ln x = \frac{1}{2}\ln 9 + \frac{1}{2}\ln\left(\frac{3}{3+\cos 2\theta}\right)$

$\ln x = \frac{1}{2}\ln\left(\frac{27}{3+\cos 2\theta}\right)$

$\ln x^2 = \ln \frac{27}{3+\cos 2\theta}$

$x = \sqrt{\frac{27}{3+\cos 2\theta}}$

ii) $x = \sqrt{\frac{27}{3+1}}$

$x = \frac{3\sqrt{3}}{2} = \underline{\underline{2.60}}$

7 Let $f(x) = \dfrac{4x^2 + 7x + 4}{(2x+1)(x+2)}$. $= A + \dfrac{B}{2x+1} + \dfrac{C}{x+2}$

(i) Express $f(x)$ in partial fractions. [5]

(ii) Show that $\int_0^4 f(x)\,dx = 8 - \ln 3$. [5]

i) $4x^2 + 7x + 4 = A(2x+1)(x+2) + B(x+2) + C(2x+1)$

$= A(2x^2 + 5x + 2) + Bx + 2B + 2Cx + C$

$= 2Ax^2 + 5Ax + 2A + Bx + 2B + 2Cx + C$

$4\cancel{x^2} = 2A\cancel{x^2}$

$A = 2$

$7\cancel{x} = 5(2)\cancel{x} + B\cancel{x} + 2C\cancel{x}$

$-3 = B + 2C$

$4 = 2(2) + 2B + C$

$0 = 2B + C$

$C = -2B$

$-3 = B + 2(-2B)$

$-3 = B - 4B$

$\cancel{-}3 = \cancel{-}3B$

$B = 1$

$\therefore C = -2$

$2 + \dfrac{1}{2x+1} - \dfrac{2}{x+2}$

ii) $\int_0^4 2 + \dfrac{1}{2x+1} - \dfrac{2}{x+2}\,dx$

$2x + \frac{1}{2}\ln(2x+1) - 2\ln(x+2)$

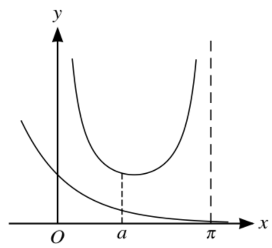
sub,4: $8 + \frac{1}{2}\ln(9) - 2\ln(6) \Rightarrow 8 + \ln 3 - \ln 36$

sub,0: $0 + \frac{1}{2}\ln 1 - 2\ln 2 \Rightarrow -2\ln 2 \Rightarrow -\ln 4$

$8 + \ln 3 - \ln 36 + \ln 4$

$8 + \ln 12 - \ln 36 \Rightarrow 8 + \ln\frac{1}{3} = 8 - \ln 3$

8



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$$a = \tan^{-1}\left(\frac{e^a}{\sin a}\right).$$ [2]

(iii) Verify by calculation that a lies between 1 and 1.5. [2]

(iv) Use an iterative formula based on the equation in part **(ii)** to determine a correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

i) $\frac{d}{dx}\left(\frac{1}{\sin x}\right) = \frac{\sin x(0) - \cos x}{\sin^2 x} = -\frac{\cos x}{\sin^2 x} = -\frac{\cos x}{\sin x} \times \frac{1}{\sin x} =$

$= -\operatorname{cosec} x \cot x$

$\frac{1}{\sin x} = \operatorname{cosec} x \quad \therefore$ if $y = \operatorname{cosec} x \therefore \frac{dy}{dx}$

ii) $y = e^{-x} \quad \frac{dy}{dx} = -e^{-x}$

$\cancel{-} e^{-x} = \cancel{-} \frac{\operatorname{cosec} x}{\tan x} \Rightarrow$ $x = a$ $\quad \frac{1}{e^a} = \frac{\operatorname{cosec} a}{\tan a}$

$\tan a = \frac{e^a}{\sin a}$

$a = \tan^{-1}\left(\frac{e^a}{\sin a}\right)$

iii)

	LHS		RHS
$x = 1$	1	$<$	1.271
$x = 1.5$	1.5	$>$	1.352

change of sign $\therefore$ root lies b/w 1 and 1.5

iv) let $x_1 = 1$

$$x_2 = \tan^{-1}\left(\frac{e^1}{\sin 1}\right) = 1.27059$$

$x_3 = 1.30884$

$x_4 = 1.31557$

$x_5 =$

$x_6 = 1.31678$

$x_7 = 1.31699$

$x_8 = 1.31703$

$x_9 = 1.31704$

$\therefore x = 1.317$

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(i) Find the position vector of D and verify that the parallelogram is a rhombus. [5]

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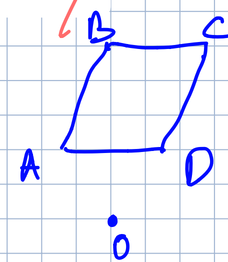

9i) $AB = OB - OA = \begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix}$

$BC = OC - OB = \begin{pmatrix} 2 \\ 5 \\ -1 \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$

$AD = OD - OA = \begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} x-1 \\ y-2 \\ z-3 \end{pmatrix}$

$CD = D - C = \begin{pmatrix} x-2 \\ y-5 \\ z+1 \end{pmatrix}$

$AB + BC + CD = AD$

$\begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix} + \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix} + \begin{pmatrix} x-2 \\ y-5 \\ \cancel{z}+1 \end{pmatrix} = \begin{pmatrix} x-1 \\ y-2 \\ \cancel{z}-3 \end{pmatrix}$

$2 + x - 2 = x - 1$

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(b) **(i)** On a sketch of an Argand diagram, show the locus representing complex numbers satisfying the equation $|z| = |z - 4 - 3i|$. [2] 1

$z-(4+3i)$

(ii) Find the complex number represented by the point on the locus where $|z|$ is least. Find the modulus and argument of this complex number, giving the argument correct to 2 decimal places. [3]

10a)

$$\frac{-2 \pm \sqrt{4 - 4(i)(-3i)}}{2i}$$

$i^2 = -1$

$$\frac{-2 \pm \sqrt{4-12}}{2i}$$

$$\frac{-2 \pm \sqrt{-8}}{2i}$$

$$= \frac{-\cancel{2} \pm \cancel{2}\sqrt{2}\, i}{\cancel{2} i}$$

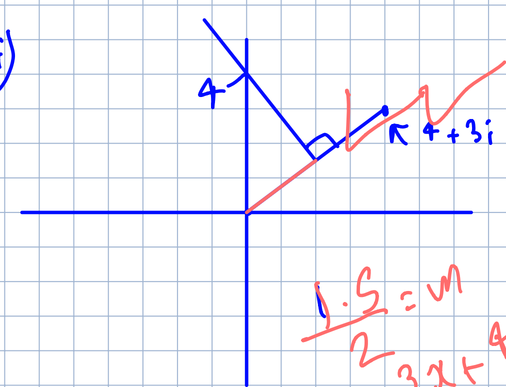
$= i \pm \sqrt{2}$

$\sqrt{2} \pm \cancel{i}$ $\pm\sqrt{2} + i$

$x = \sqrt{2}$

$y = 1$

bi)



$\frac{1.5}{2} = m$

$y = \frac{3}{4}x + 4$

ii) $2 + 1.5i$

$\sqrt{2^2 + 1.5^2} = 2.5$

mod $= 2.5$

$\theta = \tan^{-1}\left(\frac{1.5}{2}\right) = 0.\cancel{6}4$

$4y = 3x + 12$

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