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Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE NAME	Fezail	
CENTRE NUMBER		CANDIDATE NUMBER

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MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2017**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **19** printed pages and **1** blank page.

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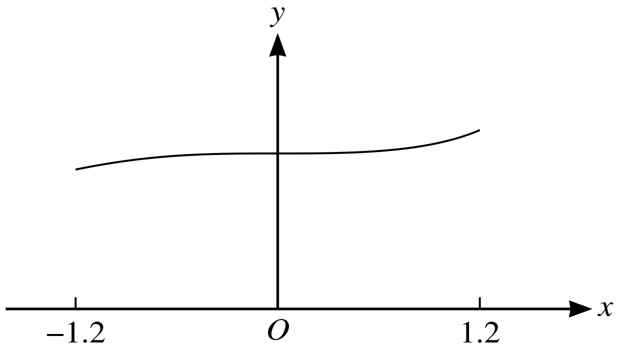

The diagram shows a sketch of the curve $y = \frac{3}{\sqrt{(9 - x^3)}}$ for values of x from -1.2 to 1.2.

(i) Use the trapezium rule, with two intervals, to estimate the value of

$$\int_{-1.2}^{1.2} \frac{3}{\sqrt{(9 - x^3)}}\,dx,$$

giving your answer correct to 2 decimal places. [3]

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(ii) Explain, with reference to the diagram, why the trapezium rule may be expected to give a good approximation to the true value of the integral in this case. [1]

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2 Showing all necessary working, solve the equation $2\log_2 x = 3 + \log_2(x+1)$, giving your answer correct to 3 significant figures. [5]

A

$\log_2 x^2 - \log_2(x+1) = 3$

$\frac{x^2}{x+1} = 2^3$

$x^2 = 8x + 8$

$x^2 - 8x - 8 = 0$

$x = 8.90$ or

~~$x = -0.899$~~

only 8.91, log cant have -ve

so alway check main eqn

3 By expressing the equation $\tan(\theta + 60°) + \tan(\theta - 60°) = \cot\theta$ in terms of $\tan\theta$ only, solve the equation for $0° < \theta < 90°$. [5]

$$\frac{\tan\theta + \tan 60}{1 - \tan 60 \tan\theta} + \frac{\tan\theta - \tan 60}{1 + \tan 60 \tan\theta} = \frac{1}{\tan\theta}$$

$$\frac{\tan\theta + \sqrt{3}}{1 - \sqrt{3}\tan\theta} + \frac{\tan\theta - \sqrt{3}}{1 + \sqrt{3}\tan\theta} = \frac{1}{\tan\theta}$$

$$\frac{(\tan\theta + \sqrt{3})(1 + \sqrt{3}\tan\theta) + (1 - \sqrt{3}\tan\theta)(\tan\theta - \sqrt{3})}{(1 - \sqrt{3}\tan\theta)(1 + \sqrt{3}\tan\theta)} = \frac{1}{\tan\theta}$$

$$\frac{(\tan\theta + \cancel{\sqrt{3}\tan^2\theta} + \cancel{\sqrt{3}} + 3\tan\theta) + (\tan\theta - \cancel{\sqrt{3}} - \cancel{\sqrt{3}\tan^2\theta} + 3\tan\theta)}{(1 - \sqrt{3}\tan\theta)(1 + \sqrt{3}\tan\theta)} = \frac{1}{\tan\theta}$$

$$(8\tan\theta)\tan\theta = 1 - 3\tan^2\theta$$

$$11\tan^2\theta = 1$$

$$\tan^2\theta = \frac{1}{11}$$

$\tan\theta = \sqrt{\frac{1}{11}}$

$\theta = 16.77865°$

$\theta = 16.8°$

$\tan\theta = -\sqrt{\frac{1}{11}}$

X outside range

4 The curve with equation $y = \dfrac{2 - \sin x}{\cos x}$ has one stationary point in the interval $-\frac{1}{2}\pi < x < \frac{1}{2}\pi$.

4

(i) Find the exact coordinates of this point. [5]

$$\frac{dy}{dx} = \frac{\cos x(-\cos x) - (2-\sin)(-\sin x)}{\cos^2 x} = 0$$

$$-\cos^2 x - (-2\sin x + \sin^2 x) = 0$$

$$-\cos^2 x + 2\sin x - \sin^2 x = 0$$

$$-(1-\sin^2 x) + 2\sin x - \sin^2 x = 0$$

$$-1 + \cancel{\sin^2 x} + 2\sin x - \cancel{\sin^2 x} = 0$$

$$2\sin x = 1$$

$$\frac{dy}{dx} = \frac{-1 + 2\sin x}{\cos^2 x}$$

$$\sin x = \frac{1}{2}$$

$$x = \sin^{-1}\left(\frac{1}{2}\right) = \frac{1}{6}\pi$$

$y = 2$?

(ii) Determine whether this point is a maximum or a minimum point. [2]

$$\frac{dy}{dx} = \frac{-1+2\sin x}{\cos^2 x}$$

$$\frac{d^2y}{dx^2} = \frac{(\cos^2 x)(2\cos x) - (-1+2\sin x)(-2\cos x \sin x)}{\cos^2 x}$$

$$= \frac{2\cos^3 x - 2\cos x \sin x + 4\sin^2 x \cos x}{\cos^2 x}$$

$\sin 2x$

sub $x = \frac{1}{6}\pi$

$$= \frac{2\cos\left(\frac{1}{6}\pi\right)^3 - \sin\left(\frac{1}{3}\pi\right) + 4\sin\left(\frac{1}{6}\pi\right)^2 \cos\left(\frac{1}{6}\pi\right)}{\cos\left(\frac{1}{6}\pi\right)^2}$$

$$= \sqrt{3}$$

as $\frac{d^2y}{dx^2} > 0$ $\therefore$ minimum

x	0	$\frac{1}{6}\pi$	$\frac{2}{6}\pi$
m	\	—	/

5 The variables x and y satisfy the differential equation

$$(x+1)\frac{dy}{dx} = y(x+2),$$

and it is given that $y = 2$ when $x = 1$. Solve the differential equation and obtain an expression for y in terms of x. [7]

$$\int \frac{1}{y}\,dy = \int \frac{x+2}{x+1}\,dx$$

$$\frac{x+2}{x+1} = A + \frac{B}{x+1}$$

$$x+2 = Ax + A + B$$

$$Ax = x$$
$$A = 1$$

$$2 = 1 + B$$

$$B = 1$$

$$\ln y = \int 1 + \frac{1}{x+1}\,dx$$

$$\ln y = x + \ln(x+1) + C$$

$$\cancel{\ln 2} = 1 + \cancel{\ln 2} + C$$

$$C = -1$$

$$\ln y = x + \ln(x+1) - 1$$

$$y = e^{x + \ln(x+1) - 1}$$

$$y = \frac{e^x \times e^{\ln x+1}}{e}$$

$$y = \frac{e^x \times (x+1)}{e}$$

$$y = (x+1)e^{x-1}$$

6 The equation of a curve is $x^3y - 3xy^3 = 2a^4$, where a is a non-zero constant.

(i) Show that $\dfrac{dy}{dx} = \dfrac{3x^2y - 3y^3}{9xy^2 - x^3}$. [4]

$$3x^2y + \frac{dy}{dx}(x^3) - \left[3y^3 + (3x)\left(3y^2\frac{dy}{dx}\right)\right] = 0$$

$$3x^2y + x^3\frac{dy}{dx} - 3y^3 - 9xy^2\frac{dy}{dx}$$

$$3x^2y - y^3) = (9xy^2 - x^3)\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{3x^2y - y^2}{9xy^2 - x^3}$$

(ii) Hence show that there are only two points on the curve at which the tangent is parallel to the x-axis and find the coordinates of these points. [4]

$$\frac{3x^2y - y^2}{9xy^2 - x^3} = 0$$

$$3x^2y - y^2 = 0$$

$$3x^2\cancel{y} = y^2$$

$$x = \sqrt{\frac{y}{3}}$$

$3x^2 = y$

2nd

$y(3x^2 - y) = 0$

$y = 0$

$x^3y - 3xy^2 = 2a^4$

$0 - 0 = 2a^4$

$(2a^4, 0)$

$$x^3y - 3xy^2 = 2a^4$$

$$x^3(3x^2) - 3x(3x^2)^2 = 2a^4$$

$$3x^5 - 3x(9x^4) = 2a^4$$

$$3x^5 - 27x^5 = 2a^4$$

$$-24x^5 = 2a^4$$

$$x^5 = \frac{-a^4}{12}$$

$$x = \sqrt[5]{\frac{-a^4}{12}}$$

$$\left(\sqrt[5]{\frac{-a^4}{12}}, ?\right)$$

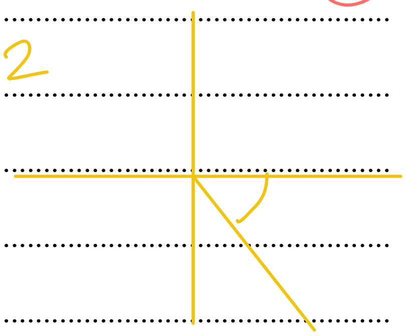
7 Throughout this question the use of a calculator is not permitted.

The complex number $1 - (\sqrt{3})\mathrm{i}$ is denoted by u.

(i) Find the modulus and argument of u. [2]

$\sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{3+1} = 2$

modulus = 2



$\tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = \frac{1}{3}\pi$

$\therefore \arg(u) = -\frac{1}{3}\pi$

(ii) Show that $u^3 + 8 = 0$. [2]

$u^2 = (1-\sqrt{3}i)(1-\sqrt{3}i)$

$= 1 - 2\sqrt{3}i + 3i^2$

$u^2 = (-2 - 2\sqrt{3}i)$

$u^3 = (-2 - 2\sqrt{3}i)(1-\sqrt{3}i)$

$(-2 + \cancel{2\sqrt{3}i - 2\sqrt{3}i} + 6i^2)$

$(-2 + 6(-1))$

$u^3 = -8$

$\therefore u^3 + 8 = 0$

(iii) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying both the inequalities $|z - u| \leqslant 2$ and $\mathrm{Re}\, z \geqslant 2$, where $\mathrm{Re}\, z$ denotes the real part of z. [4]

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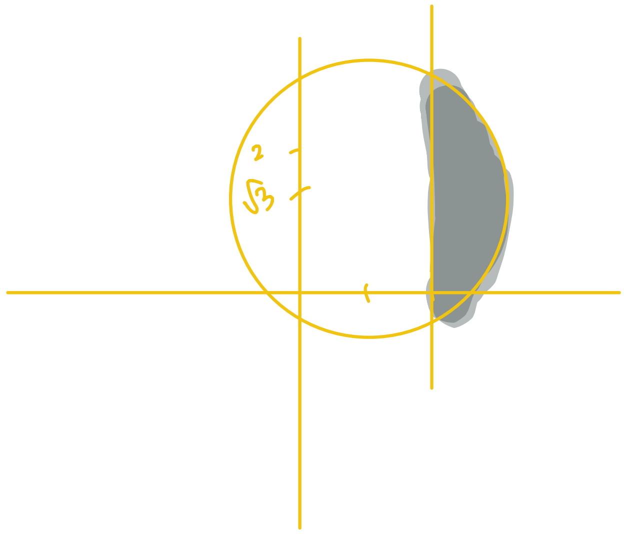

8 Let $f(x) = \dfrac{8x^2 + 9x + 8}{(1-x)(2x+3)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

$\frac{A}{1-x} + \frac{B}{(2x+3)} + \frac{C}{(2x+3)^2}$

$A(2x+3)(2x+3) + B(3+2x)(1-x) + C(1-x)$

$A(4x^2+12x+9) + B(3-x-2x^2) + C - Cx$

$4Ax^2 + 12Ax + 9A + 3B - Bx - 2Bx^2 + C - Cx$

① $8 = 4A - 2B$	$9 = 12A - B - C$	$8 = 9A + 3B + C$
$2B = 4A - 8$	$B = 12A - C - 9$	$C = 8 - 9A - 3B$
$4A = 2B + 8$	$2B = 24(\frac{1}{2}B + 2) - 2C - 18$	$C = 8 - 9A - 3(12A - C - 9)$
$A = \frac{1}{2}B + 2$		$C = 8 - 9A - 36A + 3C + 27$
		$-2C = -45A + 35$
		$-2C = -45(\frac{1}{2}B + 2) + 35$
	$2B = 24(\frac{1}{2}B+2) + (\frac{-45}{2}B - 55) - 18$	$-2C = \frac{-45}{2}B - 90 + 35$
	$2B = 12B + 48 \; \frac{-45}{2}B - 55 - 18$	$-2C = -\frac{45}{2}B - 55$

$\frac{25}{2}B = -25$

$B = -2$

$C = \dfrac{\left(\frac{-45}{2}(-2) - 55\right)}{-2}$

$= 5$

$\therefore A = 1$

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

$$\frac{1}{1-x} - \frac{2}{2x+3} + \frac{5}{(2x+3)^2}$$

① ② ③

$$(1-x)^{-1} - 2(2x+3)^{-1} + 5(2x+3)^{2}$$

① $1+(-1)(-x)+\frac{(-1)(-2)(-x)^2}{2!} = 1 + x + x^2$

② $2\times3^{-1}\left(1+\frac{2}{3}x\right)^{-1} = \frac{2}{3}\left(1-\frac{2}{3}x+\frac{(-1)(-2)(\frac{2}{3}x)^2}{2!}\right)$

$= \frac{2}{3}\left(1-\frac{2}{3}x+\frac{4}{9}x^2\right)$

$= \frac{2}{3} - \frac{4}{9}x + \frac{8}{27}x^2$

③ $2\times3^{-2}\left(1+\frac{2}{3}x\right)^{-2} = \frac{2}{9}\left(1+(-2)\left(\frac{2}{3}x\right)+\frac{(-2)(-3)(\frac{2}{3}x)^2}{2!}\right)$

$= \frac{2}{9}\left(1-\frac{4}{3}x+\frac{4}{3}x^2\right)$

$= \frac{2}{9} - \frac{8}{27}x + \frac{8}{27}x^2$

$1 + x + x^2 \quad -\frac{2}{3} + \frac{4}{9}x - \frac{8}{27}x^2 + \frac{2}{9} - \frac{8}{27}x + \frac{8}{27}x^2$

$\frac{5}{9} + \frac{31}{27}x + x^2$

[Turn over

9 It is given that $\int_1^a x^{\frac{1}{2}} \ln x \, dx = 2$, where $a > 1$.

(i) Show that $a^{\frac{3}{2}} = \dfrac{7 + 2a^{\frac{3}{2}}}{3 \ln a}$. [5]

$\int_1^a x^{\frac{1}{2}} \ln x \, dx$

v' u

$u = \ln x$ $u' = \frac{1}{x}$

$v = \frac{x^{\frac{3}{2}}}{\frac{3}{2}}$ $v' = x^{\frac{1}{2}}$

$= \frac{2}{3} x^{\frac{3}{2}}$

$\frac{2x^{\frac{3}{2}}(\ln x)}{3} - \int \frac{2x^{\frac{3}{2}}}{3x} \, dx$

$\frac{2x^{\frac{3}{2}} \ln x}{3} - \frac{2}{3} \int x^{\frac{1}{2}} \, dx$

$\frac{2x^{\frac{3}{2}} \ln x}{3} - \frac{2}{3}\left(\frac{2x^{\frac{3}{2}}}{3}\right)$

$\frac{2x^{\frac{3}{2}} \ln x}{3} - \frac{4}{9} x^{\frac{3}{2}}$

$\left[x^{\frac{3}{2}}\left(\frac{2}{3} \ln x - \frac{4}{9}\right) \right]_0^a$

sub a: $a^{\frac{3}{2}}\left(\frac{6 \ln a - 4}{9}\right) = \frac{6a^{\frac{3}{2}} \ln a - 4a^{\frac{3}{2}}}{9}$

sub 0: $1\left(\frac{6 \ln 0 - 4}{9}\right) = \frac{-4}{9}$

$\frac{6a^{\frac{3}{2}} \ln a - 4a^{\frac{3}{2}}}{9} + \frac{4}{9} = 2$

$6a^{\frac{3}{2}} \ln a - 4a^{\frac{3}{2}} + 4 = 18$

$6a^{\frac{3}{2}} \ln a - 4a^{\frac{3}{2}} = 14$

$\cancel{6}^{3} a^{\frac{3}{2}} \ln a = \cancel{14}^{7} + \cancel{4}^{2} a^{\frac{3}{2}}$

$a^{\frac{3}{2}} = \frac{7 + 2a^{\frac{3}{2}}}{3 \ln a}$

(ii) Show by calculation that a lies between 2 and 4. [2]

let $f(x) = \frac{7+2a^{\frac{3}{2}}}{3\ln a} - a^{3/2}$

$f(2) = 3.258$

$f(4) = -2.469$

change of sign, ∴ root

(iii) Use the iterative formula

$$a_{n+1} = \left(\frac{7 + 2a_n^{\frac{3}{2}}}{3\ln a_n}\right)^{\frac{2}{3}}$$

to determine a correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

let $a_1 = 3$

$a_2 = \left(\frac{7+2(3)^{\frac{3}{2}}}{3\ln(3)}\right)^{\frac{2}{3}} = 3.03106$

$a_3 = 3.03091$

$a_4 = 3.03091$

$a_5 = 3.03091$

$a_6 = 3.03091$

$\therefore a = 3.031$

10 Two planes p and q have equations $x + y + 3z = 8$ and $2x - 2y + z = 3$ respectively.

(i) Calculate the acute angle between the planes p and q. [4]

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(ii) The point A on the line of intersection of p and q has y-coordinate equal to 2. Find the equation of the plane which contains the point A and is perpendicular to both the planes p and q. Give your answer in the form $ax + by + cz = d$. [7]

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