Cambridge Assessment
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Cambridge International AS & A Level

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MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **May/June 2020**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

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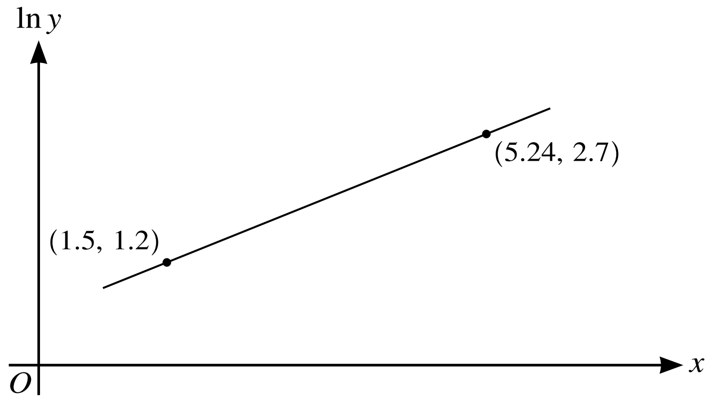
1 Find the quotient and remainder when $6x^4 + x^3 - x^2 + 5x - 6$ is divided by $2x^2 - x + 1$. [3]

$$
\begin{array}{r|l}
 & 3x^2 + 2x - 1 \\
2x^2 - x + 1 & 6x^4 + x^3 - x^2 + 5x - 6 \\
- & 6x^4 - 3x^3 + 3x^2 \\
 & 4x^3 - 4x^2 + 5x \\
 & 4x^3 - 2x^2 + 2x \\
 & -2x^2 + 3x - 6 \\
 & -2x^2 + x - 1 \\
 & 0 \quad 2x - 5
\end{array}
$$

quotient = $3x^2 + 2x - 1$

remainder = $2x - 5$

2



The variables x and y satisfy the equation $y^2 = Ae^{kx}$, where A and k are constants. The graph of $\ln y$ against x is a straight line passing through the points (1.5, 1.2) and (5.24, 2.7) as shown in the diagram.

Find the values of A and k correct to 2 decimal places. [5]

$2\ln y = \ln A + kx$

$\ln y = \frac{\ln A}{2} + \frac{kx}{2}$

y intercept $= \frac{\ln A}{2}$, $\frac{k}{2} =$ gradient

$y = mx + c$

$m = \frac{2.7 - 1.2}{5.24 - 1.5} = 0.4012$

$c = y - mx$
$= 1.2 - 0.4012(1.5)$
$= 0.5982$

$\frac{k}{2} = 0.4012$

$k = 0.80$

$\therefore \ln A = 1.1964$
$A = e^{1.1964}$
$A = 3.31$

3 Find the exact value of

$$\int_1^4 x^{\frac{3}{2}} \ln x \, dx.$$ LIATE [5]

$u = \ln x \qquad u' = \frac{1}{x}$

$v = \frac{2x^{2.5}}{5} \qquad v' = x^{1.5}$

$\ln x\left(\frac{2x^{2.5}}{5}\right) - \frac{2}{5}\int x^{2.5}\left(\frac{1}{x}\right) dx$

$\frac{2}{5}x^{2.5}\ln x - \frac{2}{5}\int x^{1.5}\, dx$

$\frac{2}{5}x^{2.5}\ln x - \frac{2}{5}\left(\frac{2}{5}x^{2.5}\right)$

$\frac{2}{5}x^{2.5}\ln x - \frac{4x^{2.5}}{25}$

$\frac{2x^{2.5}}{5}\left(\ln x - \frac{2}{5}\right)$

$\left[\frac{2(4)^{2.5}}{5}\left(\ln 4 - \frac{2}{5}\right)\right] - \left[\frac{2(1)^{2.5}}{5}\left(-\frac{2}{5}\right)\right]$

$\frac{64}{5}\ln 4 - \frac{128}{25} + \frac{4}{25}$

$\frac{64\ln 4}{5} - \frac{124}{25}$

4 A curve has equation $y = \cos x \sin 2x$.

2

Find the x-coordinate of the stationary point in the interval $0 < x < \frac{1}{2}\pi$, giving your answer correct to 3 significant figures. [6]

$y = \cos x \sin 2x$

$\frac{dy}{dx} = -\sin x \sin 2x + 2\cos 2x \cos x$

at stationary point $\frac{dy}{dx} = 0$

$\therefore 2\cos 2x \cos x = \sin x \sin 2x$

$2(2\cos^2 x - 1)\cos x = \sin x \; 2\sin x \cos x$

$\frac{4\cos^{2} x - 2\cancel{\cos x}}{\cancel{\cos x}} = 2\sin x^2$

$4\cos^2 x - 2 = 2 - 2\cos^2 x$

$6\cos^2 x - 4 = 0$

$\cos x = \pm\sqrt{\frac{4}{6}}$

$x = 0.615$

5 **(a)** Express $\sqrt{2}\cos x - \sqrt{5}\sin x$ in the form $R\cos(x + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give the exact value of R and the value of α correct to 3 decimal places. [3]

$R = \sqrt{(\sqrt{2})^2 + (\sqrt{5})^2}$ $\alpha = \tan^{-1}\left(\frac{\sqrt{5}}{\sqrt{2}}\right)$

$= \sqrt{2+5}$

$= \sqrt{7}$ $= 57.688$

$\therefore$ $\sqrt{7}\cos(x + 57.688)$

(b) Hence solve the equation $\sqrt{2}\cos 2\theta - \sqrt{5}\sin 2\theta = 1$, for $0° < \theta < 180°$. [4]

$\sqrt{7}\cos(2x + 57.688) = 1$

$2x + 57.688 = \cos^{-1}\left(\frac{1}{\sqrt{7}}\right)$

$2x + 57.688 = 67.792 \quad , 292.208$

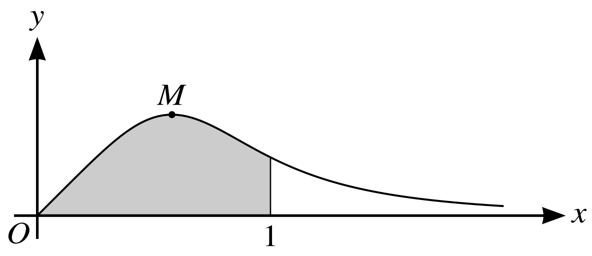
$2x + 57.688 = 67.792$

$\underline{\underline{x = 5.1}}$

$2x + 57.688 = 292.208$

$\underline{\underline{x = 117.3}}$

6



The diagram shows the curve $y = \dfrac{x}{1 + 3x^4}$, for $x \geqslant 0$, and its maximum point M.

(a) Find the x-coordinate of M, giving your answer correct to 3 decimal places. [4]

$= \dfrac{(1+3x^4)(1) - (x)(12x^3)}{(1+3x^4)^2}$

$= \dfrac{1+3x^4 - 12x^4}{(1+3x^4)^2}$

$\dfrac{1-9x^4}{(1+3x^4)^2} = 0$

$1 - 9x^4 = 0$

$x^4 = \dfrac{1}{9}$

$x = 0.577$

(b) Using the substitution $u = \sqrt{3}x^2$, find by integration the exact area of the shaded region bounded by the curve, the x-axis and the line $x = 1$. [5]

$y = \frac{x}{1+3x^4}$

$u = \sqrt{3}x^2$

$du = 2\sqrt{3}x \, dx$

$\int_0^1 \frac{\cancel{x}}{1+u^2} \times \frac{du}{2\sqrt{3}\cancel{x}}$

when $x = 0$ $u = 0$

$x = 1$ $u = \sqrt{3}$

$\frac{1}{2\sqrt{3}} \times \int_0^1 \frac{1}{u^2+1} \, du$

$\frac{1}{2\sqrt{3}} \times \tan^{-1}\left(\frac{u}{1}\right)$

$\left[\frac{\tan^{-1}\left(\frac{u}{1}\right)}{2\sqrt{3}}\right]_0^{\sqrt{3}}$

$\left(\frac{1}{2\sqrt{3}} \times \frac{1}{3}\pi\right) - \left(0\right)$

$\frac{\pi}{6\sqrt{3}}$

7 The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = \frac{y-1}{(x+1)(x+3)}.$$

It is given that $y = 2$ when $x = 0$.

Solve the differential equation, obtaining an expression for y in terms of x. [9]

$$\int \frac{1}{y-1}\,dy = \int \frac{1}{(x+1)(x+3)}\,dx$$

$$\ln(y-1) = \frac{1}{2}\left[\int \frac{1}{(x+1)}\,dx - \int \frac{1}{x+3}\,dx\right]$$

$$\frac{A}{x+1} + \frac{B}{x+3}$$

$$1 = Ax + 3A + Bx + B$$

$$3A + B = 1$$

$$A + B = 0$$

$$\therefore A = -B$$

$$3(-B) + B = 1$$

$$-2B = 1$$

$$B = -\frac{1}{2} \quad \therefore A = \frac{1}{2}$$

$$\ln(y-1) = \frac{1}{2}\left[\ln(x+1) - \ln(x+3)\right] + C$$

$$\ln(y-1) = \frac{1}{2}\ln\left(\frac{x+1}{x+3}\right) + C$$

$$C = \ln(1) - \frac{1}{2}\ln\left(\frac{1}{3}\right)$$

$$C = -\frac{1}{2}\ln\frac{1}{3}$$

$$\therefore y - 1 = e^{\frac{1}{2}\ln\left(\frac{x+1}{x+3}\right) - \frac{1}{2}\ln\left(\frac{1}{3}\right)} + 1$$

$$y = e^{\frac{1}{2}\ln\left(\frac{3x+3}{x+3}\right)} + 1$$

$$y = \sqrt{\frac{3x+3}{x+3}} + 1$$

8 **(a)** Solve the equation $(1 + 2i)w + iw^* = 3 + 5i$. Give your answer in the form $x + iy$, where x and y are real. [4]

let $w = x + iy$

$(1+2i)(x+iy) + i(x-iy) = 3+5i$

$x + iy + 2xi + 2y(-1) + xi - (-1)y = 3 + 5i$

$x + iy + 3xi - 2y + y = 3 + 5i$

$3 = x - y$

$\therefore x = 3 + y$ — ①

$5 = y + 3x$ ②

sub ① in ②

$5 = y + 3(3+y)$

$5 = y + 9 + 3y$

$4y = -4$

$y = -1$

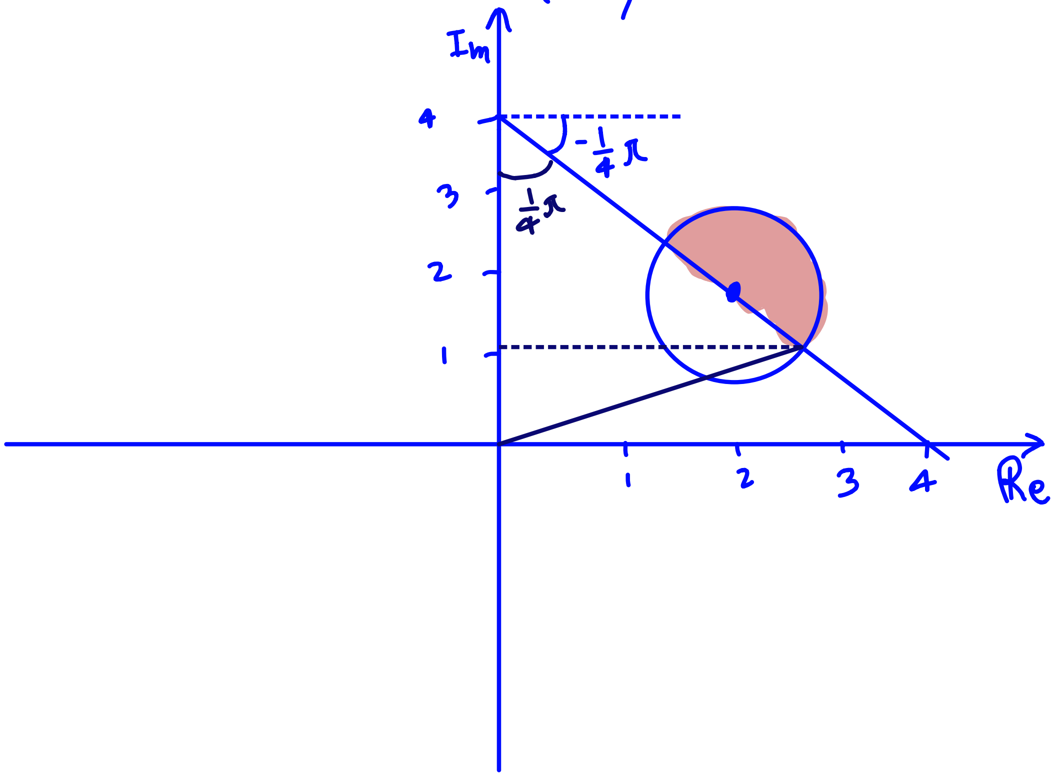
$\therefore x = 3 + (-1)$

$x = 2$

$z = 2 - i$

(b) **(i)** On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - 2 - 2i| \leqslant 1$ and $\arg(z - 4i) \geqslant -\frac{1}{4}\pi$. [4]

$z - (2 + 2i)$



(ii) Find the least value of Im z for points in this region, giving your answer in an exact form. [2]

$(x-2)^2 + (y-2)^2 = 1^2$ $\quad y = -x + 4$

$x^2 - 4x + 4 + (-x + 2)^2 = 1$

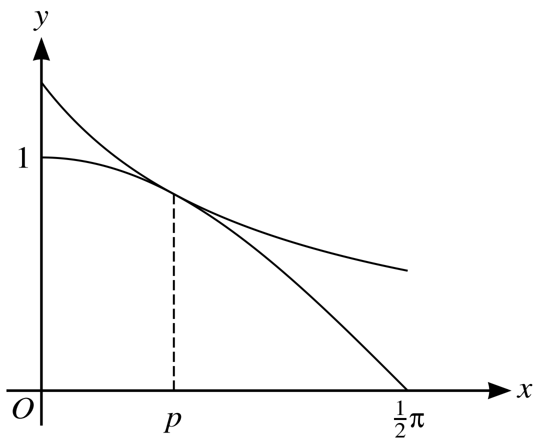
$x^2 - 4x + 4 + x^2 - 4x + 4 = 1$

$2x^2 - 8x + 7 =$

$x = 2 + \frac{1}{\sqrt{2}}$

$\therefore y = 2 - \frac{1}{\sqrt{2}}$ ✓

9



The diagram shows the curves $y = \cos x$ and $y = \dfrac{k}{1+x}$, where k is a constant, for $0 \leqslant x \leqslant \frac{1}{2}\pi$. The curves touch at the point where $x = p$.

(a) Show that p satisfies the equation $\tan p = \dfrac{1}{1+p}$. [5]

$\cos p = \dfrac{k}{1+p}$

$-\sin p = \dfrac{(1+p)(0) - k(1)}{(1+p)^2}$

$k = (\cos p)(1+p)$ ②

$-\sin p = \dfrac{-k}{(1+p)^2}$

$k = (\sin p)(1+p)^2$ — ①

$\dfrac{①}{②}$: $\dfrac{k}{k} = \dfrac{(\sin p)(1+p)^{\cancel{2}}}{(\cos p)\cancel{(1+p)}}$

$1 = \tan p\,(1+p)$

$\boxed{\tan p = \dfrac{1}{1+p}}$

[proved

(b) Use the iterative formula $p_{n+1} = \tan^{-1}\left(\frac{1}{1+p_n}\right)$ to determine the value of p correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

$P_{n+1} = \tan^{-1}\left(\frac{1}{1+P_n}\right)$

$P_1 = \frac{1}{4}\pi$

$P_2 = 0.51056$

$P_3 = 0.58978$

$P_4 = 0.56291$

$P_5 = 0.56920$

$P_6 = 0.56737$

$P_7 = 0.56790$

$P_8 = 0.56774$

$P_9 = 0.56779$

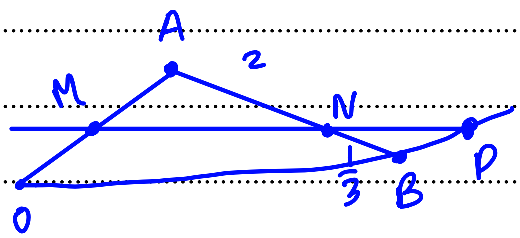
$\therefore p = 0.568$

(c) Hence find the value of k correct to 2 decimal places. [2]

$k = \cos 0.568 \times (1 + 0.568)$
$= 1.32$

10 With respect to the origin O, the points A and B have position vectors given by $\overrightarrow{OA} = 6\mathbf{i} + 2\mathbf{j}$ and $\overrightarrow{OB} = 2\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$. The midpoint of OA is M. The point N lying on AB, between A and B, is such that $AN = 2NB$.

(a) Find a vector equation for the line through M and N. [5]



$\overrightarrow{OM} = \frac{1}{2}\overrightarrow{OA} = \frac{1}{2}\begin{pmatrix}6\\2\\0\end{pmatrix} = \begin{pmatrix}3\\1\\0\end{pmatrix}$

$\overrightarrow{MN} = \overrightarrow{MA} + \overrightarrow{AN}$

$= \begin{pmatrix}3\\1\\0\end{pmatrix} + \frac{2}{3}AB$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= \begin{pmatrix}2\\2\\3\end{pmatrix} - \begin{pmatrix}6\\2\\0\end{pmatrix} = \begin{pmatrix}-4\\0\\3\end{pmatrix}$

$= \begin{pmatrix}3\\1\\0\end{pmatrix} + \frac{2}{3}\begin{pmatrix}-4\\0\\3\end{pmatrix}$

$\overrightarrow{MN} = \begin{pmatrix}\frac{1}{3}\\1\\2\end{pmatrix}$

$r = \begin{pmatrix}3\\1\\0\end{pmatrix} + \lambda\begin{pmatrix}1/3\\1\\2\end{pmatrix}$

$r = 3i + j + \lambda\left(\frac{1}{3}i + j + 2k\right)$

The line through M and N intersects the line through O and B at the point P.

(b) Find the position vector of P. [3]

MN: $\begin{pmatrix}3\\1\\0\end{pmatrix} + \lambda\begin{pmatrix}1/3\\1\\2\end{pmatrix}$

OB: $0 + \mu\begin{pmatrix}2\\2\\3\end{pmatrix}$

① $3 + \frac{1}{3}\lambda = 2\mu$

② $1 + \lambda = 2\mu$

$\to \therefore \lambda = 2\mu - 1$ ③

sub ③ into ①

$3 + \frac{1}{3}(2\mu - 1) = 2\mu$

$3 + \frac{2}{3}\mu - \frac{1}{3} = 2\mu$

$\frac{8}{3} = \frac{4}{3}\mu$

$\mu = 2$

$\therefore P = 2\begin{pmatrix}2\\2\\3\end{pmatrix} = \begin{pmatrix}4\\4\\6\end{pmatrix}$

$P = 4i + 4j + 6k$

(c) Calculate angle OPM, giving your answer in degrees. [3]

$PO = \begin{pmatrix}-4\\-4\\-6\end{pmatrix}$

PM: $3\begin{pmatrix}-\frac{1}{3}\\-1\\-2\end{pmatrix} = \begin{pmatrix}-1\\-3\\-6\end{pmatrix}$

$\lambda = 4 - 1$
$= 3$

$\therefore \cos\theta = \dfrac{4 + 12 + 36}{\sqrt{4^2+4^2+6^2}\cdot\sqrt{1^2+3^2+6^2}}$

$\cos\theta = \dfrac{52}{2\sqrt{782}}$

$\cos\theta = 0.92976$

$\theta = 21.6$

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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