Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2015**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **3** printed pages and **1** blank page.

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1 Sketch the graph of $y = e^{ax} - 1$ where a is a positive constant. [2]

2 Given that $\sqrt[3]{(1 + 9x)} \approx 1 + 3x + ax^2 + bx^3$ for small values of x, find the values of the coefficients a and b. [3]

3 A curve has equation

$$y = \frac{2 - \tan x}{1 + \tan x}.$$

Find the equation of the tangent to the curve at the point for which $x = \frac{1}{4}\pi$, giving the answer in the form $y = mx + c$ where c is correct to 3 significant figures. [6]

4 A curve has parametric equations

$$x = t^2 + 3t + 1, \qquad y = t^4 + 1.$$

The point P on the curve has parameter p. It is given that the gradient of the curve at P is 4.

(i) Show that $p = \sqrt[3]{(2p + 3)}$. [3]

(ii) Verify by calculation that the value of p lies between 1.8 and 2.0. [2]

(iii) Use an iterative formula based on the equation in part **(i)** to find the value of p correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

5 Use the substitution $u = 4 - 3\cos x$ to find the exact value of $\displaystyle\int_0^{\frac{1}{2}\pi} \frac{9\sin 2x}{\sqrt{(4 - 3\cos x)}}\,\mathrm{d}x$. [8]

6 The angles A and B are such that

$$\sin(A + 45°) = (2\sqrt{2})\cos A \quad \text{and} \quad 4\sec^2 B + 5 = 12\tan B.$$

Without using a calculator, find the exact value of $\tan(A - B)$. [8]

7 **(i)** Show that $(x + 1)$ is a factor of $4x^3 - x^2 - 11x - 6$. [2]

(ii) Find $\displaystyle\int \frac{4x^2 + 9x - 1}{4x^3 - x^2 - 11x - 6}\,\mathrm{d}x$. [8]

8 A plane has equation $4x - y + 5z = 39$. A straight line is parallel to the vector $\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$ and passes through the point $A\,(0, 2, -8)$. The line meets the plane at the point B.

(i) Find the coordinates of B. [3]

(ii) Find the acute angle between the line and the plane. [4]

(iii) The point C lies on the line and is such that the distance between C and B is twice the distance between A and B. Find the coordinates of each of the possible positions of the point C. [3]

9 **(a)** It is given that $(1 + 3i)w = 2 + 4i$. Showing all necessary working, prove that the exact value of $|w^2|$ is 2 and find $\arg(w^2)$ correct to 3 significant figures. [6]

(b) On a single Argand diagram sketch the loci $|z| = 5$ and $|z - 5| = |z|$. Hence determine the complex numbers represented by points common to both loci, giving each answer in the form $re^{i\theta}$. [4]

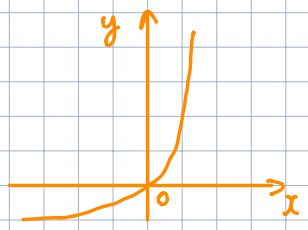
10 Naturalists are managing a wildlife reserve to increase the number of plants of a rare species. The number of plants at time t years is denoted by N, where N is treated as a continuous variable.

(i) It is given that the rate of increase of N with respect to t is proportional to $(N - 150)$. Write down a differential equation relating N, t and a constant of proportionality. [1]

(ii) Initially, when $t = 0$, the number of plants was 650. It was noted that, at a time when there were 900 plants, the number of plants was increasing at a rate of 60 per year. Express N in terms of t. [7]

(iii) The naturalists had a target of increasing the number of plants from 650 to 2000 within 15 years. Will this target be met? [2]

1 Sketch the graph of $y = e^{ax} - 1$ where a is a positive constant. [2]



2 Given that $\sqrt[3]{(1+9x)} \approx 1 + 3x + ax^2 + bx^3$ for small values of x, find the values of the coefficients a and b. [3]

$(1+9x)^{\frac{1}{3}}$

$1 + 3x + \frac{(\frac{1}{3})(-\frac{2}{3})(9x)^2}{2!} + \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})(9x)^3}{3!}$

$1 + 3x - 9x^2 + 45x^3$

3 A curve has equation

$$y = \frac{2 - \tan x}{1 + \tan x}.$$

Find the equation of the tangent to the curve at the point for which $x = \frac{1}{4}\pi$, giving the answer in the form $y = mx + c$ where c is correct to 3 significant figures. [6]

$\frac{dy}{dx} = \frac{(1+\tan x)(-\sec^2 x) - (\sec^2 x)(2 - \tan x)}{(1+\tan x)^2}$

$= \frac{(2)(-2) - (2)(1)}{(1+1)^2}$

$= \frac{-4-2}{4} = \frac{-6}{4} = -\frac{3}{2}$

$y = -\frac{3}{2}x + c$

$\frac{2-1}{1+1} = \frac{1}{2}$

$\frac{1}{2} = -\frac{3}{8}\pi + c$

$c = 1.68$

$y = -\frac{3}{2}x + 1.68$

4 A curve has parametric equations

$$x = t^2 + 3t + 1, \qquad y = t^4 + 1.$$

The point P on the curve has parameter p. It is given that the gradient of the curve at P is 4.

(i) Show that $p = \sqrt[3]{(2p + 3)}$. [3]

(ii) Verify by calculation that the value of p lies between 1.8 and 2.0. [2]

(iii) Use an iterative formula based on the equation in part **(i)** to find the value of p correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

4i) $\frac{dx}{dt} = 2t + 3$ $\qquad \frac{dy}{dt} = 4t^3$

$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$

$= \frac{dy}{dt} \times \frac{dt}{dx} = \frac{4t^3}{2t+3}$

$\frac{4p^3}{2p+3} = 4$

$4p^3 = 8p + 12$

$2p^3 = 2p + 3$

$p = \sqrt[3]{2p + 3}$

ii)

	LHS		RHS
when $x=1.8$	1.8	$<$	1.88
$x=2.0$	2.0	$>$	1.91

iii)

$p_1 = 1.9$

$p_2 = \sqrt[3]{2(1.9) + 3} = 1.8945$

$p_3 = 1.8935$

$p_4 = 1.8933$

$p_5 = 1.89333$

$p_6 = 1.89332$

$\therefore \; p = 1.89$

5 Use the substitution $u = 4 - 3\cos x$ to find the exact value of $\int_0^{\frac{1}{2}\pi} \frac{9\sin 2x}{\sqrt{(4-3\cos x)}}\,\mathrm{d}x$. [8]

$u = 4 - 3\cos x$

$\frac{du}{dx} = 3\sin x$

$dx = \frac{du}{3\sin x}$

$\frac{4-u}{3} = \cos x$

$9(2\sin x \cos x)$

$18\sin x \cos x$

$\int_1^6 \frac{18\sin x\cos x}{\sqrt{u}} \cdot \frac{du}{3\sin x}$

$\int_1^4 \frac{6\left(\frac{4-u}{3}\right)}{u^{\frac{1}{2}}}\,du$

$\int_1^4 \frac{24-6u}{3} \times \frac{1}{u^{\frac{1}{2}}}\,du$

$\int_1^4 \frac{24-6u}{3u^{\frac{1}{2}}}\,du$

$\int_1^4 \frac{8-2u}{u^{\frac{1}{2}}}\,du$

$\int_1^4 8u^{-\frac{1}{2}} - 2u^{\frac{1}{2}}\,du$

$16u^{\frac{1}{2}} - \frac{2u^{\frac{3}{2}}}{\frac{3}{2}}$ 4

$\left[16u^{\frac{1}{2}} - \frac{4}{3}u^{\frac{3}{2}}\right]_1$

$16(2) - \frac{4}{3}(8) = \frac{64}{3}$

$16 - \frac{4}{3} = \frac{44}{3}$

$\frac{20}{3}$

6 The angles A and B are such that

$$\sin(A + 45°) = (2\sqrt{2})\cos A \quad \text{and} \quad 4\sec^2 B + 5 = 12\tan B.$$

Without using a calculator, find the exact value of $\tan(A - B)$. [8]

$\frac{\sqrt{2}}{2}\sin A + \frac{\sqrt{2}}{2}\cos A = 2\sqrt{2}\cos A$

$\frac{\sqrt{2}}{2}\tan A = \frac{3\sqrt{2}}{2}$

$\sqrt{2}\tan A = \frac{6\sqrt{2}}{2}$

$\tan A = \frac{6\sqrt{2}}{2\sqrt{2}}$

$= 3$

$4(\tan^2 B + 1) + 5 = 12\tan B$

$4\tan^2 B + 9 = 12\tan B$

$12\tan B - 4\tan^2 B - 9 = 0$

$\tan B = \frac{3}{2}$

$\frac{\tan A - \tan B}{1 + \tan A \tan B}$

$\frac{3 - \frac{3}{2}}{1 + 3\,\frac{3}{2}} = \frac{3}{11}$

7 **(i)** Show that $(x + 1)$ is a factor of $4x^3 - x^2 - 11x - 6$. [2]

(ii) Find $\int \frac{4x^2 + 9x - 1}{4x^3 - x^2 - 11x - 6} \, dx$. [8]

i) $x = -1$

$-4 - 1 + 11 - 6 = 0$

ii) $(x+1)(cx^2 + dx + e)$

$cx^3 + dx^2 + ex + cx^2 + dx + e$

$c = 4 \quad d + c = -1 \quad e = -6$

$d = -1 - c = -5$

$4x^2 - 5x - 6$

$(x+1)(x-2)(4x+3)$

$= \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{4x+3}$

$4x^2 + 9x - 1 = A(x-2)(4x+3) + B(x+1)(4x+3) + C(x+1)(x-2)$

$4 = 4A + 4B + C$

$16 + 18 - 1 = B(3)(11)$

$33 = 33B$

$B = 1$

$4 - 9 - 1 = 4A + 5A - 6A$

$-6 = 3A$

$A = -2$

$-1 = -6A + 3 - 2C$

$-1 = 12 + 3 - 2C$

$C = 8$

$\int \frac{-2}{x+1} + \frac{1}{x-2} + \frac{8}{4x+3}$

$-2\ln(x+1) + \ln(x-2) + 2\ln(4x+3)$

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(b) On a single Argand diagram sketch the loci $|z| = 5$ and $|z - 5| = |z|$. Hence determine the complex numbers represented by points common to both loci, giving each answer in the form $re^{i\theta}$. [4]

a) $w = \frac{2+4i}{1+3i}\frac{(1-3i)}{(1-3i)}$

$= \frac{2-2i-12i^2}{1+9} = \frac{14-2i}{10}$

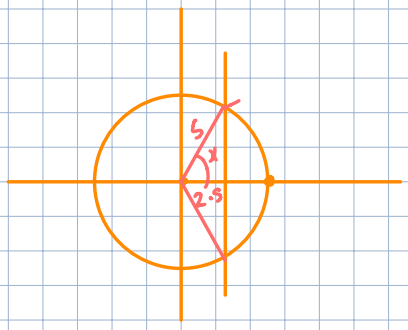
$\left(\frac{7}{5} - \frac{1}{5}i\right)\left(\frac{7}{5} - \frac{1}{5}i\right)$

$\frac{49}{25} - \frac{14}{25}i + \frac{1}{25}(-1)$

$\frac{48}{25} - \frac{14}{25}i$

$\sqrt{\left(\frac{48}{25}\right)^2 + \left(\frac{14}{25}\right)^2} = 2$

b)



4.33

$5e^{\frac{\pi}{3}i}$ and $5e^{-\frac{\pi}{3}i}$

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(iii) The naturalists had a target of increasing the number of plants from 650 to 2000 within 15 years. Will this target be met? [2]

i) $\frac{dN}{dt} = k(N-150)$

ii) $60 = 750k$

$k = 0.08$

$\int \frac{1}{N-150} = 0.08t + C$

$\ln N-150 = 0.08t + C$

$\ln 500 = C$

$\ln N-150 = 0.08t + \ln 500$

$N-150 = e^{0.08t} \times 500$

$N = 500e^{0.08t} + 150$

iii) $N = 500\, e^{0.08(15)} + 150$

$= 1810...$

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