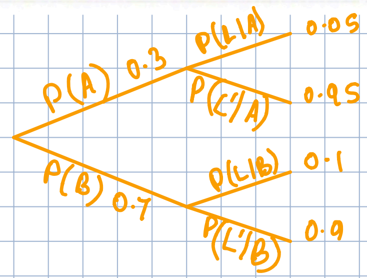
Susan goes to work by one of two routes A or B. The probability of going by route A is 30%. If she goes by route A the probability of being late for school is 5% and if she goes by route B, the probability of being late is 10%.

Draw a probability tree diagram, and then

(i) Find the probability that Susan is late for school
(ii) Given that Susan is late for school, find the probability that she went via route A.

i



$P(L) = (0.3 \times 0.05) + (0.7 \times 0.1)$

$= 0.085$

ii $P(A|L) = \dfrac{P(A) \times P(L)}{P(L)} = \dfrac{0.3 \times 0.05}{0.085} \approx 0.176$

In a factory, three machines, J, K and L, are used to make biscuits.

Machine J makes 25% of the biscuits.

Machine K makes 45% of the biscuits.

The rest of the biscuits are made by machine L. 30%.

It is known that 2% of the biscuits made by machine J are broken, 3% of the biscuits made by machine K are broken and 5% of the biscuits made by machine L are broken.

(a) Draw a tree diagram to illustrate all the possible outcomes and associated probabilities. **(2)**

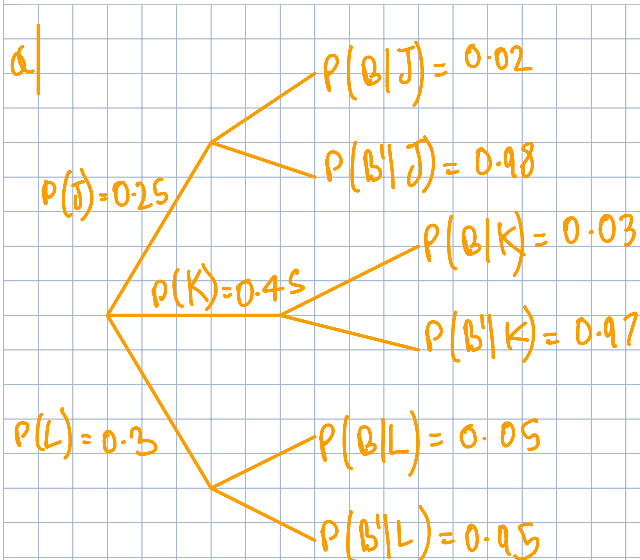
A biscuit is selected at random.

(b) Calculate the probability that the biscuit is made by machine J and is not broken. **(2)**

(c) Calculate the probability that the biscuit is broken. **(2)**

(d) Given that the biscuit is broken, find the probability that it was not made by machine K. **(3)**

a



b $P(J \cap B') = 0.25 \times 0.98 = 0.245$

c $P(B) = (0.25 \times 0.02) + (0.45 \times 0.03) + (0.3 \times 0.05)$

$= 0.0335$

d $1 - P(K|B) = 1 - \dfrac{P(K \cap B)}{P(B)}$

$= 1 - \dfrac{0.45 \times 0.03}{0.0335} = 1 - \dfrac{27}{67} = \dfrac{40}{67} \approx 0.597$

A manufacturer carried out a survey of the defects in their soft toys. It is found that the probability of a toy having poor stitching is 0.03 and that a toy with poor stitching has a probability of 0.7 of splitting open. A toy without poor stitching has a probability of 0.02 of splitting open.

(a) Draw a tree diagram to represent this information.

(3)

(b) Find the probability that a randomly chosen soft toy has exactly one of the two defects, poor stitching or splitting open.

(3)

The manufacturer also finds that soft toys can become faded with probability 0.05 and that this defect is independent of poor stitching or splitting open. A soft toy is chosen at random.

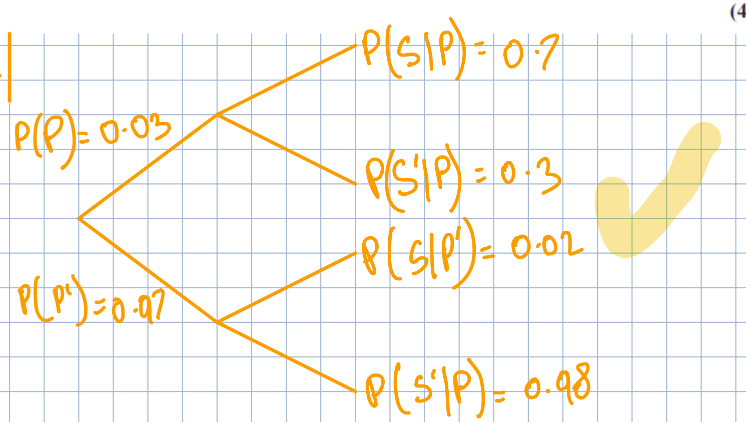
(c) Find the probability that the soft toy has none of these 3 defects.

(2)

(d) Find the probability that the soft toy has exactly one of these 3 defects.

(4)

a

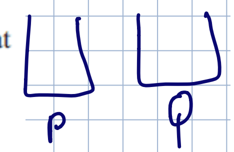


b $(0.3 \times 0.03) + (0.97 \times 0.02) = 0.0284$

c $(0.97 \times 0.98) \times (0.95) = 0.90307$

d $(0.03 \times 0.3 \times 0.95) + (0.97 \times 0.02 \times 0.95) + (0.97 \times 0.98 \times 0.05)$
$= 0.07451$

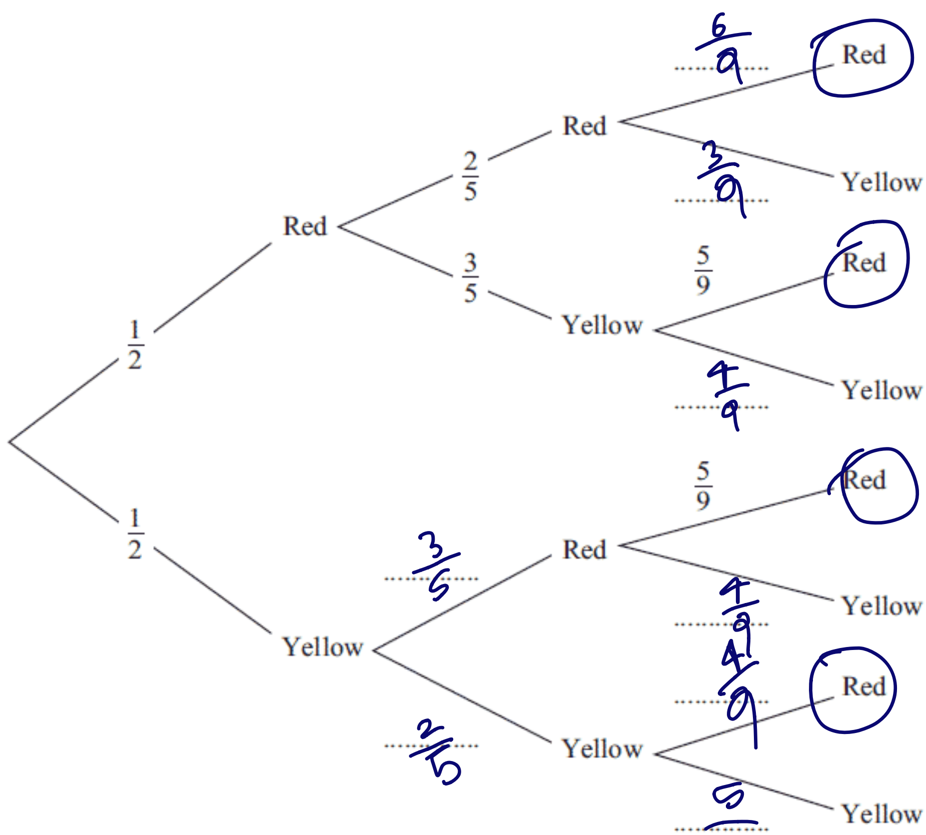
The bag P contains 6 balls of which 3 are red and 3 are yellow.
The bag Q contains 7 balls of which 4 are red and 3 are yellow.
A ball is drawn at random from bag P and placed in bag Q. A second ball is drawn at random from bag P and placed in bag Q.
A third ball is then drawn at random from the 9 balls in bag Q.



The event A occurs when the 2 balls drawn from bag P are of the same colour.
The event B occurs when the ball drawn from bag Q is red.

(a) Complete the tree diagram shown below.

(4)



(b) Find P(A) $\left(\frac{1}{2}\times\frac{2}{5}\right)+\left(\frac{1}{2}\times\frac{2}{5}\right)=\frac{2}{5}$

(3)

(c) Show that P(B) $=\frac{5}{9}$ $\left(\frac{1}{2}\right)\left(\frac{2}{5}\right)\left(\frac{2}{3}\right)+\frac{1}{2}\left(\frac{3}{5}\right)\left(\frac{5}{9}\right)+\left(\frac{1}{2}\right)\left(\frac{3}{5}\right)\left(\frac{5}{9}\right)+\left(\frac{1}{2}\right)\left(\frac{2}{5}\right)\left(\frac{4}{9}\right)=\frac{5}{9}$

(3)

(d) Show that P($A\cap B$) $=\frac{2}{9}$ $P(R,R,R)+P(Y,Y,R)=\frac{1}{2}\left(\frac{2}{5}\right)\left(\frac{2}{3}\right)+\left(\frac{1}{2}\right)\left(\frac{2}{5}\right)\left(\frac{4}{9}\right)=\frac{2}{9}$

(2)

(e) Hence find P($A\cup B$) $P(A)+P(B)-P(A\cap B)$

$\left(\frac{2}{5}\right)+\left(\frac{5}{9}\right)-\left(\frac{2}{9}\right)=\frac{11}{15}$

(2)

(f) Given that all three balls drawn are the same colour, find the probability that they are all red.

$P(\text{all }R \mid \text{all 3 same})=\frac{P(\text{all Red})}{P(\text{3 same})}=\frac{P(R,R,R)}{P(RRR)+P(YYY)}=\frac{6}{11}$

(3)

A disease is known to be present in 2% of a population. A test is developed to help determine whether or not someone has the disease.

Given that a person has the disease, the test is positive with probability 0.95

Given that a person does not have the disease, the test is positive with probability 0.03

(a) Draw a tree diagram to represent this information.

(3)

A person is selected at random from the population and tested for this disease.

(b) Find the probability that the test is positive.

(3)

A doctor randomly selects a person from the population and tests him for the disease. Given that the test is positive,

(c) find the probability that he does not have the disease.

(2)

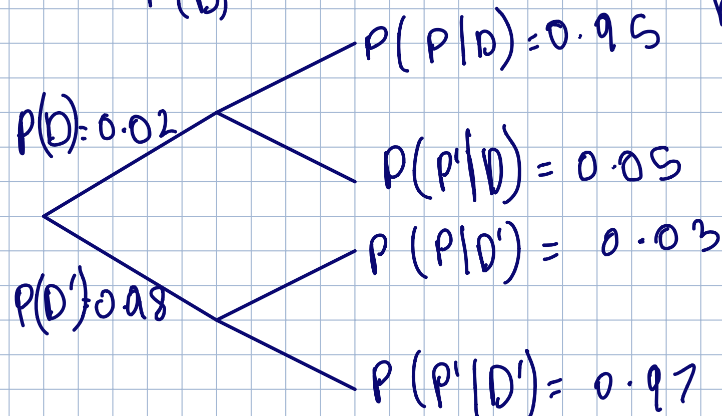
(d) Comment on the usefulness of this test.

(1)

a) Has ~~Diseases~~ $= D$

is positive $= P$

$P(P|D) = \dfrac{P(P \cap D)}{P(D)}$ $\qquad$ $P(P|D') = \dfrac{P(P \cap D')}{P(D')}$



b) $P(P) = [P(D) \times P(P|D)] + [P(D') \times P(P|D')]$

$= (0.02 \times 0.95) + (0.98 \times 0.03)$

$= 0.0484$

c) $P(D'|P) = \dfrac{P(D' \cap P)}{P(P)} = \dfrac{0.98 \times 0.03}{0.0484} = 0.607$

d) $P(+)$ is very small when we would expect a high probability.

$P(\bar{D}|+)$ is high when we would ~~expect~~ expect a small probability.

May/June 2002

1 Events A and B are such that $P(A) = 0.3$, $P(B) = 0.8$ and $P(A \text{ and } B) = 0.4$. State, giving a reason in each case, whether events A and B are

$P(A \cup B) =$

(i) independent, [2]

(ii) mutually exclusive. [2]

i) if A and B are independent, then $P(A \overset{\text{and}}{\cap} B) = P(A) \cdot P(B)$

$0.4 \neq 0.8 \times 0.3$

so not independent

ii) if A and B are mutually exclusive, then $P(A \cap B) = 0$

so not mutually exclusive

May/June 2003

6 The people living in 3 houses are classified as children (C), parents (P) or grandparents (G). The numbers living in each house are shown in the table below.

House number 1	House number 2	House number 3
$4C$, $1P$, $2G$	$2C$, $2P$, $3G$	$1C$, $1G$
7	7	2

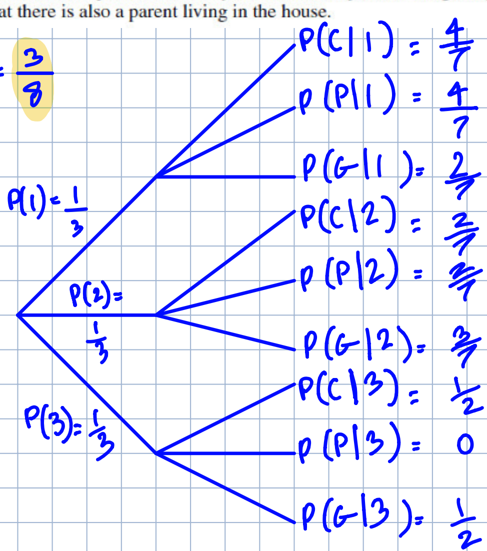
(i) All the people in all 3 houses meet for a party. One person at the party is chosen at random. Calculate the probability of choosing a grandparent. [2]

(ii) A house is chosen at random. Then a person in that house is chosen at random. Using a tree diagram, or otherwise, calculate the probability that the person chosen is a grandparent. [3]

(iii) Given that the person chosen by the method in part **(ii)** is a grandparent, calculate the probability that there is also a parent living in the house. [4]

i) $\frac{6}{16} = \frac{3}{8}$

ii)



iii) $P(P|G) = \frac{P(P \cap G)}{P(G)}$

$=$

$P(G) = \left(\frac{1}{3} \times \frac{2}{7}\right) + \left(\frac{1}{3} \times \frac{3}{7}\right) + \left(\frac{1}{3} \times \frac{1}{2}\right)$

$= \frac{17}{42}$

May/June 2004

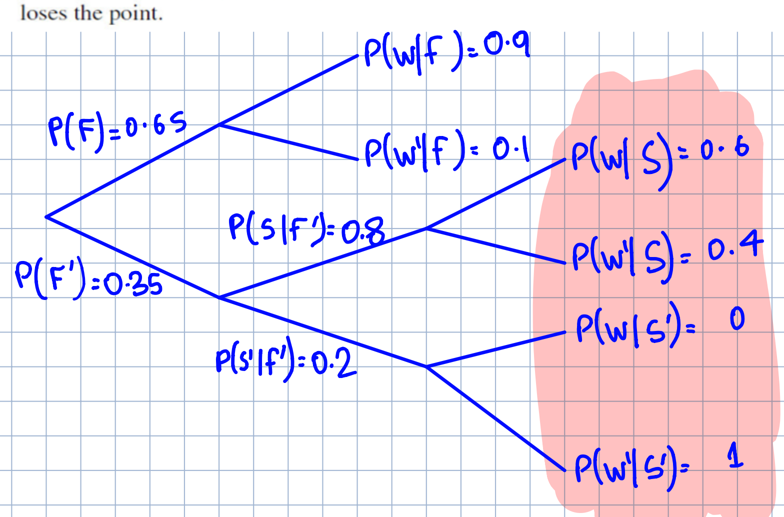
6 When Don plays tennis, 65% of his first serves go into the correct area of the court. If the first serve goes into the correct area, his chance of winning the point is 90%. If his first serve does not go into the correct area, Don is allowed a second serve, and of these, 80% go into the correct area. If the second serve goes into the correct area, his chance of winning the point is 60%. If neither serve goes into the correct area, Don loses the point.

(i) Draw a tree diagram to represent this information. [4]

(ii) Using your tree diagram, find the probability that Don loses the point. [3]

(iii) Find the conditional probability that Don's first serve went into the correct area, given that he loses the point. [2]

i)



ii) $P(W') = (0.65 \times 0.1) + (0.35 \times 0.8 \times 0.4) + (0.35 \times 0.2 \times 1)$

$= 0.247$

iii) $P(F|W') = \dfrac{P(F \cap W')}{P(W')} \quad \dfrac{0.65 \times 0.1}{0.247} = 0.263$

May/June 2005

5 Data about employment for males and females in a small rural area are shown in the table.

	Unemployed	Employed
Male	206	412
Female	358	305

A person from this area is chosen at random. Let M be the event that the person is male and let E be the event that the person is employed.

(i) Find $P(M)$. [2]

(ii) Find $P(M \text{ and } E)$. [1]

(iii) Are M and E independent events? Justify your answer. [3]

(iv) Given that the person chosen is unemployed, find the probability that the person is female. [2]

i) $\frac{206}{1281}$

ii) $\frac{412}{1281} = 0.322$

iii) $P(M \text{ and } E) = P(M) \times P(E)$

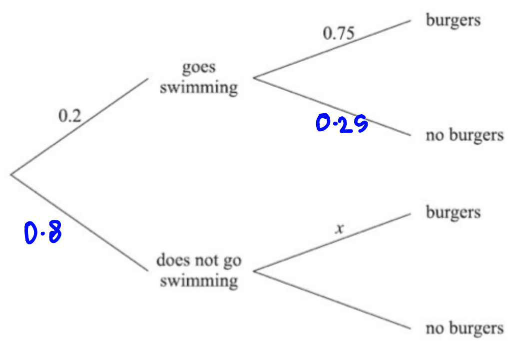
$0.322 \neq \frac{206}{1281} \times \frac{412+305}{1281}$

Not indopentent

iv) $P(F|U) = \frac{P(F \cap U)}{P(U)} = \frac{\left(\frac{358}{1281}\right)}{\left(\frac{564}{1281}\right)} = 0.635$

2 The probability that Henk goes swimming on any day is 0.2. On a day when he goes swimming, the probability that Henk has burgers for supper is 0.75. On a day when he does not go swimming the probability that he has burgers for supper is x. This information is shown on the following tree diagram.

May/June 2006



The probability that Henk has burgers for supper on any day is 0.5.

(i) Find x. [4]

(ii) Given that Henk has burgers for supper, find the probability that he went swimming that day. [2]

i) $(0\cdot 2 \times 0\cdot 75) + 0\cdot 8x = 0\cdot 5$

$0\cdot 8x = 0\cdot 35$

$x = 0\cdot 4375 \approx 0\cdot 438$

ii) $P(S|B) = \dfrac{P(S \cap B)}{P(B)} = \dfrac{0\cdot 2 \times 0\cdot 75}{0\cdot 5} = 0\cdot 3$

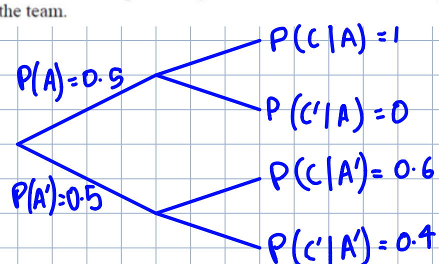
May/June 2007

2 Jamie is equally likely to attend or not to attend a training session before a football match. If he attends, he is certain to be chosen for the team which plays in the match. If he does not attend, there is a probability of 0.6 that he is chosen for the team.

(i) Find the probability that Jamie is chosen for the team. [3]

(ii) Find the conditional probability that Jamie attended the training session, given that he was chosen for the team. [3]

i)



$P(C) = (0\cdot 5 \times 1) + (0\cdot 5 \times 0\cdot 6) = 0\cdot 8$

ii) $P(A|C) = \dfrac{P(A \cap C)}{P(C)} = \dfrac{0\cdot 5 \times 1}{0\cdot 8} = 0\cdot 625$

May/June 2008

2 In country A 30% of people who drink tea have sugar in it. In country B 65% of people who drink tea have sugar in it. There are 3 million people in country A who drink tea and 12 million people in country B who drink tea. A person is chosen at random from these 15 million people.

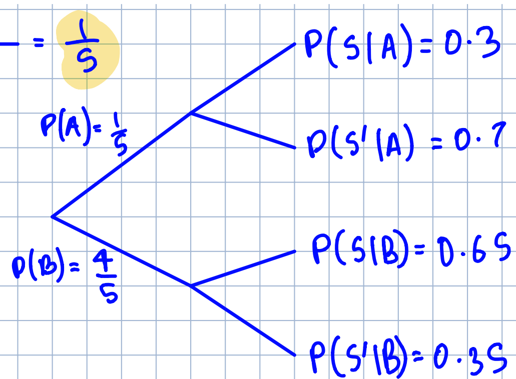
(i) Find the probability that the person chosen is from country A. [1]

(ii) Find the probability that the person chosen does not have sugar in their tea. [2]

(iii) Given that the person chosen does not have sugar in their tea, find the probability that the person is from country B. [2]

i) $\frac{3}{15} = \frac{1}{5}$

ii)



$P(S') = \left(\frac{1}{5} \times 0.7\right) + \left(\frac{4}{5} \times 0.35\right) = 0.42$

iii) $P(B \mid S') = \frac{P(B \cap S')}{P(S')} = \frac{\frac{4}{5} \times 0.35}{0.42} = \frac{2}{3}$

May/June 2009

5 At a zoo, rides are offered on elephants, camels and jungle tractors. Ravi has money for only one ride. To decide which ride to choose, he tosses a fair coin twice. If he gets 2 heads he will go on the elephant ride, if he gets 2 tails he will go on the camel ride and if he gets 1 of each he will go on the jungle tractor ride.

(i) Find the probabilities that he goes on each of the three rides. [2]

The probabilities that Ravi is frightened on each of the rides are as follows:

elephant ride $\frac{6}{10}$, camel ride $\frac{7}{10}$, jungle tractor ride $\frac{8}{10}$.

(ii) Draw a fully labelled tree diagram showing the rides that Ravi could take and whether or not he is frightened. [2]

Ravi goes on a ride.

(iii) Find the probability that he is frightened. [2]

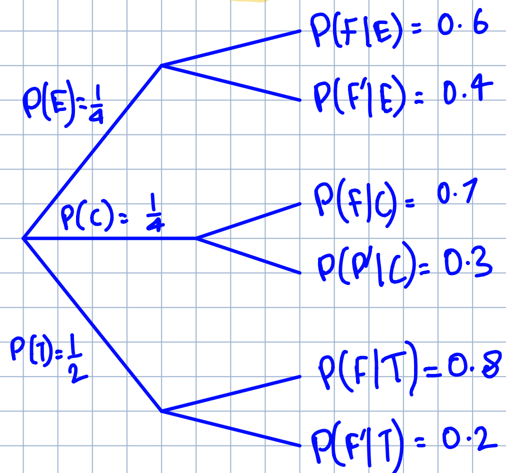
(iv) Given that Ravi is **not** frightened, find the probability that he went on the camel ride. [3]

i) Elephants : $0.5 \times 0.5 = 0.25$

Camel : $0.5 \times 0.5 = 0.25$

Tractor : $(0.5 \times 0.5) \times 2 = 0.50$

ii)



iii) $P(F) = (0.25 \times 0.6) + (0.25 \times 0.7) + (0.5 \times 0.8)$

$= 0.725$

iv) $P(C|F') = \frac{P(C \cap F')}{P(F)} \quad \frac{0.25 \times 0.3}{(1 - 0.725)} = 0.273$

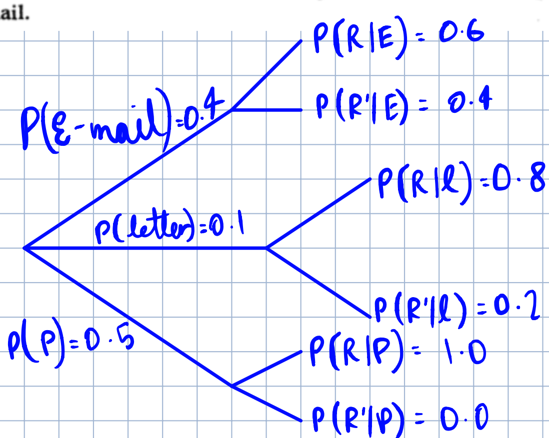
Oct/Nov 2001

3 A lecturer wishes to give a message to a student. The probabilities that she uses e-mail, letter or personal contact are 0.4, 0.1 and 0.5 respectively. She uses only one method. The probabilities of the student receiving the message if the lecturer uses e-mail, letter or personal contact are 0.6, 0.8 and 1 respectively.

(i) Find the probability that the student receives the message. [3]

(ii) Given that the student receives the message, find the conditional probability that he received it via e-mail. [3]

3i)



$$P(R) = (0.4 \times 0.6) + (0.1 \times 0.8) + (0.5 \times 1)$$
$$= 0.82$$

ii) $$P(E|R) = \frac{P(E \cap R)}{P(R)} = \frac{0.4 \times 0.6}{0.82} = \frac{12}{41} \approx 0.293$$

Question

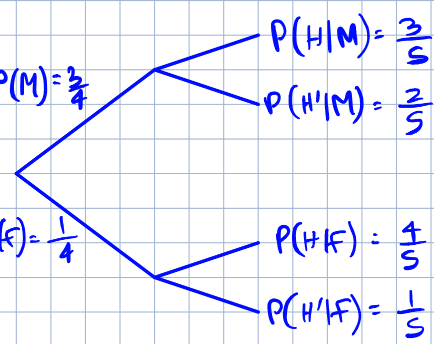
Q2. In a group of students, 3/4 are male. The proportion of male students who like their curry hot is 3/5 and the proportion of female students who like their curry hot is 4/5. One student is chosen at random

(i) Find the probability that the student chosen is either female, **or** likes their curry hot, **or** is both female and likes their curry hot

(ii) Showing your working, determine whether the events 'the student chosen is male' and 'the student chosen likes their curry hot' are independent

@lemniscates_math

i)



~~$= 0.25 + \left[(0.25\times 0.8)+(0.75\times 0.6)\right]$~~

~~$+(0.25\times 0.8)$~~

~~$= 1.1$~~ X

$1-(0.75\times 0.4)= 0.7$

ii) if $P(M)$ and $P(H)$ are independent, then $P(M\cap H)=P(M)\times P(H)$

$P(M\cap H)=0.75\times 0.6=0.45$

$P(M)=0.75$

$P(H)=(0.75\times 0.6)+(0.25\times 0.8)=0.65$

$0.45\neq 0.75\times 0.65$

$\therefore$ they are dependent

6 Three coins A, B and C are each thrown once.

- Coins A and B are each biased so that the probability of obtaining a head is $\frac{2}{3}$.
- Coin C is biased so that the probability of obtaining a head is $\frac{4}{5}$.

(a) Show that the probability of obtaining exactly 2 heads and 1 tail is $\frac{4}{9}$. [3]

A B C

H T H $= \frac{2}{3} \times \frac{1}{3} \times \frac{4}{5} = \frac{8}{45}$

H H T $= \frac{2}{3} \times \frac{2}{3} \times \frac{1}{5} = \frac{4}{45}$

T H H $= \frac{1}{3} \times \frac{2}{3} \times \frac{4}{5} = \frac{8}{45}$

$\frac{20}{45} = \frac{4}{9}$

The random variable X is the number of heads obtained when the three coins are thrown.

(b) Draw up the probability distribution table for X. [3]

x	0	1	2	3
$P(X=x)$	$\frac{1}{45}$	$\frac{8}{45}$	$\frac{20}{45}$	$\frac{16}{45}$

$0: \frac{1}{3} \times \frac{1}{3} \times \frac{1}{5}$

$3: \frac{2}{3} \times \frac{2}{3} \times \frac{4}{5}$

(c) Given that $E(X) = \frac{32}{15}$, find $Var(X)$. [2]

$$\left(0^2 \times \frac{1}{45}\right) + \left(1^2 \times \frac{8}{45}\right) + \left(2^2 \times \frac{20}{45}\right) + \left(3^2 \times \frac{16}{45}\right)$$

$E(x)^2 = 5.1555$

$\therefore Var(x) = 5.1555 - \left(\frac{32}{15}\right)^2$

$= 0.60448$

$= 0.604$