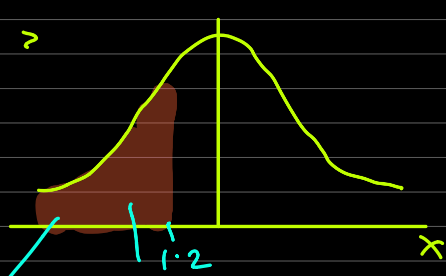
Normal Distribution

- $X \sim N(\mu, \sigma^2)$
- $Z = \dfrac{x-\mu}{\sigma}$
-



$P(X<1.2)$ Use data booklet or calc $(P(1.2))$.

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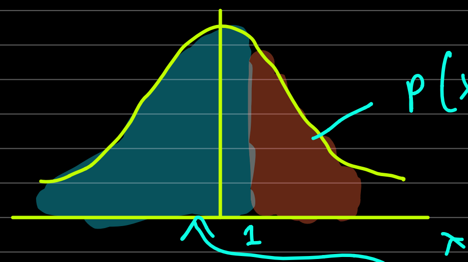


$p(x>1) = 1 - p(x<1)$

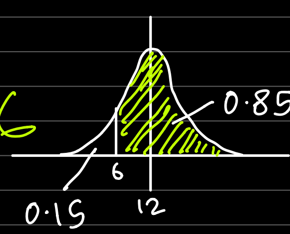
on calc $R(1)$

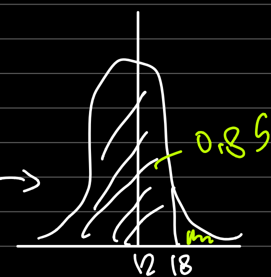
Q1) if $X \sim N(12, \sigma^2)$. Given $P(X>6) = 0.85$, find σ

0.15

$P(X<6) = 0.15$

$p(0.85) = 1.036$





0.85

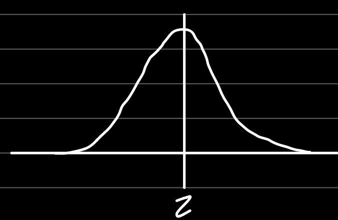
$\Phi z =$

$$\frac{18-12}{\sigma} = 1.036$$

$$\frac{18-12}{\sigma} = 1.36$$

$\sigma = 5.8$

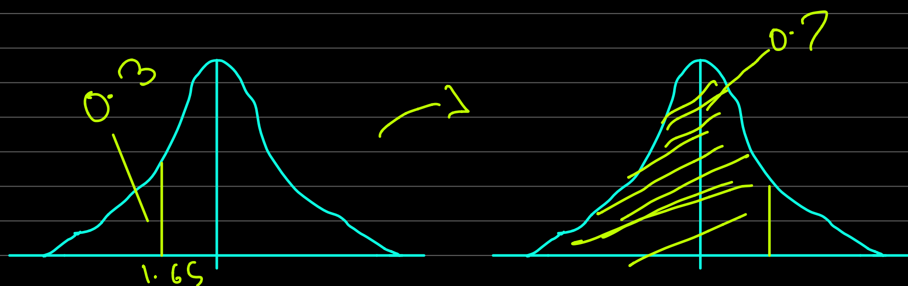
$\sigma = 4.41$

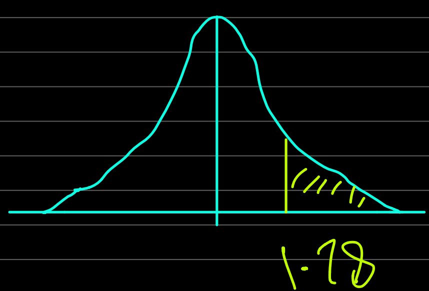


A high jumper knows from experience that she can clear a height of at least 1.65m on 7 out of 10 attempts and clear a height of at least 1.78m once in 5 attempts.
Assuming that the height she jumps in cm. is given by the r.v. X and is normally distributed.
Find to 1 decimal place the mean and standard deviation of the heights the jumper can reach.

$P(X \geq 1.65) = 0.7$

$P(X \geq 1.78) = 0.2$





① $\dfrac{1.65-\mu}{\sigma} = -0.524$

$P(0.8)$

② $\dfrac{1.78-\mu}{\sigma} = 0.842$

① $1.65 - \mu = -0.524\sigma$

$\mu = 1.65 - 0.524\sigma$

② $1.78 - \mu = 0.842\sigma$

$\mu = 1.78 - 0.842\sigma$

$1.65 + 0.524\sigma = 1.78 - 0.842\sigma$

$1.366\sigma = 0.13$

$\sigma = 0.095 \approx 9.5$

$\mu = 1.65 + 0.524(0.0951)$

$= 1.6998$

$\approx 1.7 = 170$ cm

2. ON/14/62/no.5

(a) The time, X hours, for which people sleep in one night has a normal distribution with mean 7.15 hours and standard deviation 0.88 hours.

(i) Find the probability that a randomly chosen person sleeps for less than 8 hours in a night. [2]

(ii) Find the value of q such that $P(X < q) = 0.75$. [3]

(b) The random variable Y has the distribution $N(\mu, \sigma^2)$, where $2\sigma = 3\mu$ and $\mu \neq 0$. Find $P(Y > 4\mu)$. [3]

2a) $X \sim N(7.15, \overset{\sigma}{0.88})$

i) $\dfrac{8 - 7.15}{0.88} = 0.9659 \approx 0.966$

$\phi(0.966) = 0.8315 + 0.0015$

$= 0.833$

ii) $\phi(z) = 0.75$

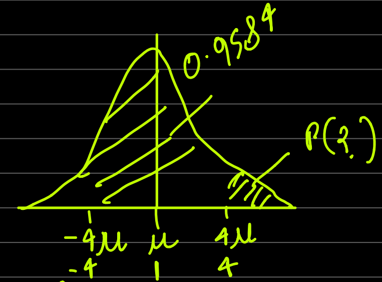
$z = 0.674$

$\dfrac{x - 7.15}{0.88} = 0.674$

$x = 7.74312$

$= 7.74$

iii)



$\sigma = \dfrac{3\mu}{2} = 1.5$

$\dfrac{4 - 1}{1.5} =$

$\dfrac{3}{1.5} = 2$

$\phi(2) = 0.9772$

$= 0.9772$

$1 - 0.9772 = \underline{\underline{0.0228}}$

3. ON/14/62/no.7

In Marumbo, three quarters of the adults own a cell phone.

(i) A random sample of 8 adults from Marumbo is taken. Find the probability that the number of adults who own a cell phone is between 4 and 6 inclusive. [3]

(ii) A random sample of 160 adults from Marumbo is taken. Use an approximation to find the probability that more than 114 of them own a cell phone. [5]

(iii) Justify use of your approximation in part (ii). [1]

i) $P(4 \le x \le 6) = 0.606$

$- {}^8C_4 \times 0.75^4 \times 0.25^4 = 0.086517$

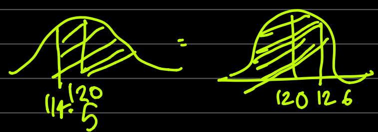
$- \quad : n=5 = 0.20764$

$- \quad : n=6 = 0.31146$

$\therefore \; 0.6056$

ii) $X \sim B(160, 0.75)$

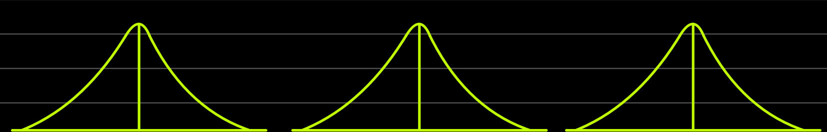
$X \sim N(120, 30)$



$P(X > 114.5) \quad \frac{5.5}{\sqrt{30}} = z = 1.0042$

0.8413
$+.0009$
8422

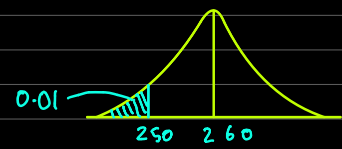
$P = 0.842$



4. ON/14/63/no.1

Packets of tea are labelled as containing $250g$. The actual weight of tea in a packet has a normal distribution with mean $260g$ and standard deviation σ g. Any packet with a weight less than $250g$ is classed as 'underweight'. Given that 1% of packets of tea are underweight, find the value of σ. [3]

$X \sim N(260, \sigma^2)$



$\phi(z) = 0.99$

$z = 2.326$

$\frac{270-260}{\sigma} = 2.326$

$10 = 2.326\sigma$

$\sigma = 4.2992$

$\sigma = 4.30$

5. ON/14/63/no.5

Gem stones from a certain mine have weights, X grams, which are normally distributed with mean $1.9g$ and standard deviation $0.55g$. These gem stones are sorted into three categories for sale depending on their weights, as follows.

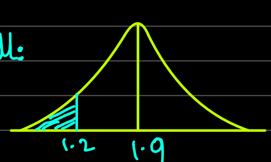
Small: under $1.2g$ Medium: between $1.2g$ and $2.5g$ Large: over $2.5g$

i) Find the proportion of gem stones in each of these three categories. [5]

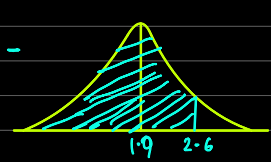
ii) Find the value of k such that $P(k < X < 2.5) = 0.8$. [4]

$X \sim N(1.9, 0.55)$

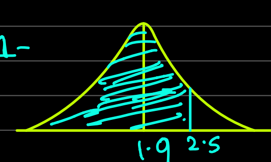
i) small:



$= 1 -$



large: $1 -$



$\frac{2.6-1.9}{0.55} = 1.272727 \approx 1.273$

$\Phi(1.273) = 0.8980 + 0.0006$
$= 0.8986$

$1 - 0.8986 = 0.1014 = 0.101$

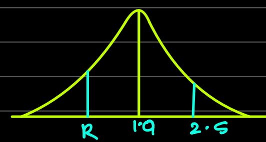
$\frac{2.5-1.9}{0.55} = 1.0909$
≈ 1.091

$\Phi(1.091) = 0.8623$

$1 - 0.8623 = 0.1377$
$= 0.138$

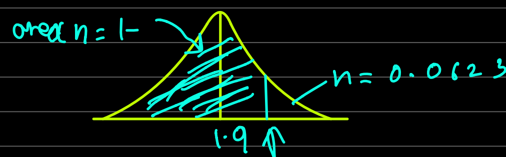
$\therefore$ medium: $1-(0.1014+0.1377) = 0.7609 = 0.761$

ii) $P(k < x < 2.5) = 0.8$



$P(x < 2.5) = 0.8623$

$0.8623 - 0.8 = 0.0623$



area n $= 1 - 0.0623$
$= 0.9377$

$(1.9-k)+1.9$

$$\frac{(1.9-k)+\cancel{1.9}-\cancel{1.9}}{0.55} = z$$

$$\Phi\left(\frac{1.9-k}{0.55}\right) = 0.9377$$

$$\frac{1.9-k}{0.55} = 1.536$$

$1.9 - k = 0.8448$

$k = 1.0552$

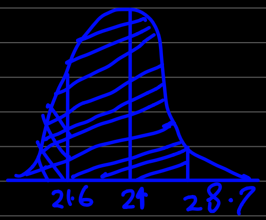
$= 1.06$

6. MJ/14/61/no.1

The petrol consumption of a certain type of car has a normal distribution with mean 24 kilometres per litre and standard deviation 4.7 kilometres per litre. Find the probability that the petrol consumption of a randomly chosen car of this type is between 21.6 kilometres per litre and 28.7 kilometres per litre. [4]

6) let X be the r.v consumption of the Km/L

$X \sim N(24, 4.7^2)$



$P(21.6 < x < 28.7) =$

$P(x < 28.7) - P(x < 21.6)$

$P(x < 28.7) = \phi\left(\frac{28.7-24}{4.7}\right)$

$= \phi(1)$

$= 0.8413$

$P(x < 21.6) = 1 - P(x < 26.4)$

$= 1 - \phi\left(\frac{26.4-24}{4.7}\right)$

$= 1 - \phi(0.51063\,8)$

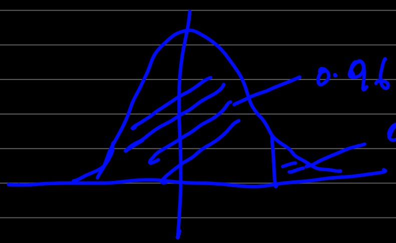
$= 1 - (0.6950 + 0.0003)$

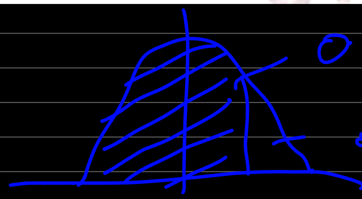
$= 0.3047$

$0.8413 - 0.3047 = 0.5366 \approx 0.537$

7. MJ/14/61/no.2

Lengths of a certain type of white radish are normally distributed with mean μ cm and standard deviation σ cm. 4% of these radishes are longer than 12cm and 32% are longer than 9cm. Find μ and σ. [5]





① $\frac{12-\mu}{\sigma} = 1.751$

$12 - \mu = 1.751\sigma$

② $\frac{9-\mu}{\sigma} = 0.468$

$9 - \mu = 0.468\sigma$

$$\begin{array}{rcl} 12-\mu & = & 1.751\sigma \\ -\quad 9-\mu & = & 0.468\sigma \\ \hline 3 & = & 1.283\sigma \end{array}$$

$\sigma = 2.33826$

$= 2.34$

$\mu = 9 - 0.468(2.33826)$

$\mu = 7.90569$

$\mu = 7.91$

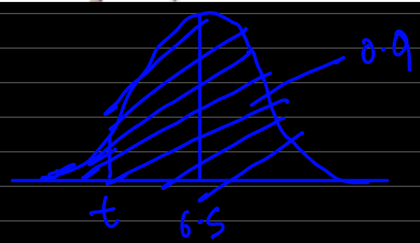
10. MJ/14/63/no.5

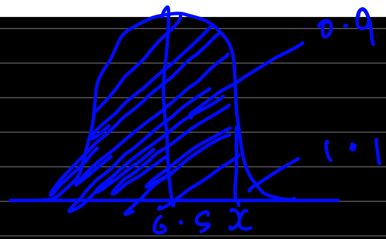
When ~~Moses~~ Ali makes a phone call, the amount of time that the call takes has a normal distribution with mean 6.5 minutes and standard deviation 1.76 minutes.

(i) 90% of Moses's phone calls take longer than t minutes. Find the value of t. **[3]**

(ii) Find the probability that, in a random sample of 9 phone calls made by Moses, more than 7 take a time which is within 1 standard deviation of the mean. **[5]**

i)





$$\frac{x-6.5}{1.76} = 1.281$$

$$x = 8.90456$$

$$6.5 - t = x - 6.5$$

$$13 - x = t$$

$$t = 13 - 8.90456$$

$$= 4.09544 \approx \underline{4.10}$$

ii) $X \sim B(9, 0.683)$

$$P(X>7) = P(X=8) + P(X=9)$$

$$^9C_8 \times 0.683^8 \times 0.317^1 = 0.139$$

$$= 0.03234$$

$$= 0.16384$$

$$= 0.164$$

12. ON/15/61/no.4

(a) Amy measured her pulse rate while resting, x beats per minute, at the same time each day on 30 days. The results are summarised below.

$$\Sigma(x - 80) = -147 \qquad \Sigma(x - 80)^2 = 952$$

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S1 (Cambridge)

Find the mean and standard deviation of Amy's pulse rate. [4]

(b) Amy's friend Marok measured her pulse rate every day after running for half an hour. Marok's pulse rate, in beats per minute, was found to have a mean of 148.6 and a standard deviation of 18.5. Assuming that pulse rates have a normal distribution, find what proportion of Marok's pulse rates, after running half an hour, were above 160 beats per minute. [3]

a) $\dfrac{-147 + 80 \times 30}{30}$

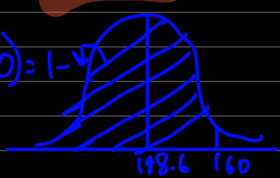
mean $= 75.1$ ✓

$\sqrt{\dfrac{\Sigma(x-80)^2}{30} - (75.1-80)^2}$

$= 2.779$

SD: ≈ 2.78 ✓

b) $P(X > 160) = 1 - \Phi$



$\dfrac{160 - 148.6}{18.5} = 0.616216$

$\Phi(0.616) = 0.7291 + 0.0019$

$= 0.731$

$1 - 0.731 = \underline{0.269}$ ✓

8. MJ/14/62/no.7

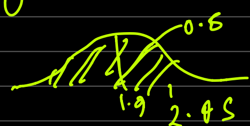
The time Rafa spends on his homework each day in term-time has a normal distribution with mean 1.9 hours and standard deviation σ hours. On 80% f these days he spends more than 1.35 hours on his homework.

(i) Find the value of σ. **[3]**

(ii) Find the probability that, on a randomly chosen day in term-time, Rafa spends less than 2 hours on his homework. **[2]**

(iii) A random sample of 200 days in term-time is taken. Use an approximation to find the probability that the number of days on which Rafa spends more than 1.35 hours on his homework is between 163 and 173 inclusive. **[6]**

i)



$\frac{0.55}{\sigma} = z$

$\phi(0.8)$, $\therefore z = 0.842$

$\sigma = \frac{0.55}{0.842} = \underline{\underline{0.653}}$

ii)

$\frac{2-1.9}{0.653} = 0.1531$

$\phi(0.1531) = 0.5596$

$.0012$

0.561

iii)

$np = 200 \times .8$
$= 160$

$npq = 32$

$X \sim N(160, 32)$

$162.5 \leq X \leq 173.5$

$\frac{2.5}{\sqrt{32}} = z_1 = 0.4419$

$\phi(z_1) = 0.6707$

$\frac{13.5}{\sqrt{32}} = z_2$
$= 2.38648$

$.9913$
$+ \quad 2$

0.9915

$0.9915 - 0.6707$

$\underline{\underline{0.321}}$

9. MJ/14/63/no.2

There is a probability of $\frac{1}{7}$ that Wenjie goes out with her friends on any particular day. 252 days are chosen at random.

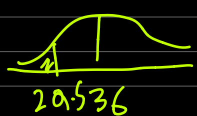
(i) Use a normal approximation to find the probability that the number of days on which Wenjie goes out with her friends is less than 30 or more than 44. **[5]**

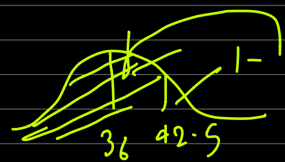
(ii) Give a reason why the use of normal approximation is justified. **[1]**

$X \sim B(252, \frac{1}{7})$

$\frac{1}{7} \times 252 = 36$, $252 \times \frac{1}{7} \times (1 - \frac{1}{7}) = 30.857$

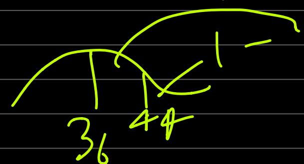
$X \sim N(36, 30.857)$





$\frac{6.5}{\sqrt{30.857}} = z = 1.1701$

$P(X < 30) = 1 - 0.8790 = 0.121$



$\sqrt{30.857} = 1.5302 = z$

$\Phi(z) = 0.9370$

$1 - 0.9370 = 0.063$

$0.121 + 0.063 = \underline{0.184}$

(ii) np and nq are both > 5.

30. MJ/16/61/no.5

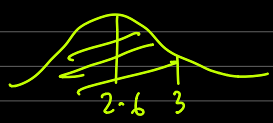
Plastic drinking straws are manufactured to fit into drinks cartons which have a hole in the top. A straw fits into the hole if the diameter of the straw is less than $3mm$. The diameters of the straws have a normal distribution with mean $2.6mm$ and standard deviation $0.25mm$.

(i) A straw is chosen at random. Find the probability that it fits into the hole in a drinks carton. [3]

(ii) 500 straws are chosen at random. Use a suitable approximation to find the probability that at least 480 straws fit into the holes in drinks cartons. [5]

(iii) Justify the use of your approximation. [1]

i)



$\frac{0.4}{0.25} = 1.6$

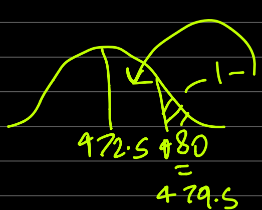
$\Phi(1.60) = 0.9452$

$p = 0.945$

ii) $np = 500 \times 0.9452 = 472.6$

$npq = 500 \times 0.945 \times (1 - 0.945) = 25.89875$

$X \sim N(472.6,\ 25.898)$



$\frac{479.5 - 472.6}{\sqrt{25.898}} = 1.\cancel{373}\,3558$

$1 - \Phi(1.3558)$

$1 - 0.9125$

$\cancel{0.0848}$

0.0875

iii) $np > 5$ and $npq > 5$ $\therefore$ normal distribution can be used

56. MJ/18/62/no.7

In a certain country, 60% of mobile phones sold are made by Company A, 35% are made by Company B and 5% are made by other companies.

(i) Find the probability that, out of a random sample of 13 people who buy a mobile phone, fewer than 11 choose a mobile phone made by Company A. [3]

(ii) Use a suitable approximation to find the probability that, out of a random sample of 130 people who buy a mobile phone, at least 50 choose a mobile phone made by Company B. [5]

(iii) A random sample of n mobile phones sold is chosen. The probability that at least one of these phones is made by Company B is more than 0.98. Find the least possible value of n. [3]

i) $p(x<11) = 1 - p(x \geq 11)$

$p(x=11) = {}^{13}C_{11} \times 0.6^{11} \times 0.4^{2} = 0.045277$

$p(x=12) = {}^{13}C_{12} \times 0.6^{12} \times 0.4^{1} = 0.011319$

$p(x=13) = {}^{13}C_{13} \times 0.6^{13} = 0.001306$

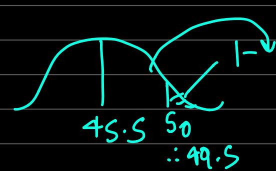
0.057902

$1 - 0.057902 = 0.942$

ii) $x \sim B(\overset{n}{130}, \overset{p}{0.35})$

$x \sim N(np, np(1-p))$

$x \sim N(45.5, 29.575)$



$\dfrac{49.5 - 45.5}{\sqrt{29.575}} = 0.7355$

$\Phi(0.7355) = 0.7682$

$1 - 0.7682 = 0.232$

iii) $p(B \geq 1) = 1 - p(x=0)$

$p(B=0) = {}^{n}C_{0} \times 1 \times 0.65^{n}$

$1 - 0.65^{n} = 0.98$

$0.65^{n} = 0.02$

$n = \dfrac{\ln 0.02}{\ln 0.65} = 9.08 \quad \therefore 10$