Cambridge International A Level

Cambridge Assessment International Education
Cambridge International Advanced Level

CANDIDATE NAME			
CENTRE NUMBER		CANDIDATE NUMBER	

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MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2019**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **20** printed pages.

JC19 06_9709_33/RP

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1 Use logarithms to solve the equation $5^{3-2x} = 4(7^x)$, giving your answer correct to 3 decimal places. [4]

$(3-2x)\ln 5 = \ln 4 + x\ln 7$ $\ln 5$

$3\ln 5 - 2x\ln 5 = \ln 4 + x\ln 7$

$x\ln 7 + 2x\ln 5 = 3\ln 5 - \ln 4$

$$x = \frac{3\ln 5 - \ln 4}{\ln 7 + 2\ln 5}$$

$= 0.666$

2 Show that $\int_0^{\frac{1}{4}\pi} x^2 \cos 2x \, dx = \frac{1}{32}(\pi^2 - 8)$. [5]

u v

$u = x^2$ $u' = 2x$

$v = 2(\cos 2x)^0 (-\sin 2x$ $v' = \cos 2x$

3 Let $f(\theta) = \dfrac{1 - \cos 2\theta + \sin 2\theta}{1 + \cos 2\theta + \sin 2\theta}$.

(i) Show that $f(\theta) = \tan \theta$. [3]

(ii) Hence show that $\int_{\frac{1}{6}\pi}^{\frac{1}{4}\pi} \mathrm{f}(\theta)\,\mathrm{d}\theta = \frac{1}{2}\ln\frac{3}{2}$. [4]

4 The equation of a curve is $y = \dfrac{1 + e^{-x}}{1 - e^{-x}}$, for $x > 0$.

u
v

(i) Show that $\dfrac{dy}{dx}$ is always negative. [3]

$u = 1 + e^{-x}$, $u' = -e^{-x}$

$v = 1 - e^{-x}$, $v' = e^{-x}$

$v^2 = (1 - e^{-x})(1 - e^{-x})$ $(-e)\ (-e)$

$= 1 - 2e^{-x} + (e^{-x})^2$

$= 1 - 2e^{-x} + e^{-2x}$

$$\frac{dy}{dx} = \frac{(1-e^{-x})(-e^{-x}) - (1+e^{-x})(e^{-x})}{(1-e^{-x})^2}$$

$$= \frac{-e^{-x} + \cancel{e^{2x}} - e^{-x} - \cancel{e^{-2x}}}{(1-e^{-x})^2}$$

$$= \frac{-2e^{-x}}{(1-e^{-x})^2}$$

$$= -2\underbrace{\left(\frac{e^{-x}}{(1-e^{-x})^2}\right)}_{\text{always +ve}}$$

$\therefore$ -2 mulipled by a positive value is alway going to a +ve value

#

(ii) The gradient of the curve is equal to -1 when $x = a$. Show that a satisfies the equation $e^{2a} - 4e^{a} + 1 = 0$. Hence find the exact value of a. [4]

$\frac{dy}{dx}$ = gradient $= -1 = \left(\frac{-2e^{-a}}{(1-e^{-a})^2}\right)$

$\therefore \quad \cancel{-}2e^{-a} = \cancel{-}(1-e^{-a})^2$

$2e^{-a} = 1 - 2e^{-a} + e^{-2a}$

$e^{-2a} + 4e^{-a} - 1 = 0$

5 The variables x and y satisfy the differential equation

$$(x+1)y\frac{dy}{dx} = y^2 + 5.$$

It is given that $y = 2$ when $x = 0$. Solve the differential equation obtaining an expression for y^2 in terms of x. [7]

$$\int \frac{y}{y^2+5}\,dy = \int \frac{1}{x+1}\,dx$$

$$\frac{1}{2}\int \frac{2y}{y^2+5}\,dy = \ln(x+1) + c$$

$$\frac{1}{2}\ln|y^2+5| = \ln(x+1) + c$$

when $x=0,\ y=2$

$$\frac{1}{2}\ln 9 = \ln 1 + c$$

$$\ln 9^{\frac{1}{2}} = \ln 1 + c$$

$$\ln 3 - \ln 1 = c$$

$$\ln 3 = c$$

$$\ln(y^2+5) = 2\ln(x+1) + 2\ln 3$$

$$\ln y^2+5 = 2\ln(3(x+1))$$

$$\ln y^2+5 = 2\ln(3x+3)$$

$\ln(y^2+5) = \ln(3x+3)^2$

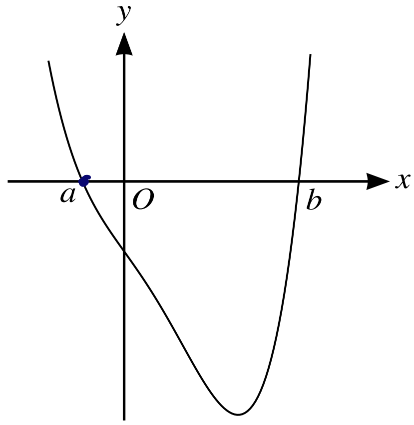
$y^2+5 = e^{\ln(3x+3)^2}$

$y^2+5 = (3x+3)^2$

$y^2+5 = 9x^2+18x+9$

$y^2 = 9x+18x+4$

6



The diagram shows the curve $y = x^4 - 2x^3 - 7x - 6$. The curve intersects the x-axis at the points $(a, 0)$ and $(b, 0)$, where $a < b$. It is given that b is an integer.

(i) Find the value of b. [1]

when $x=1$, $y \neq 0$

$x=3$, $y=0$

$x-3=0$

(ii) Hence show that a satisfies the equation $a = -\frac{1}{3}(2 + a^2 + a^3)$. [4]

$$
\begin{array}{r|l}
 & x^3 + x^2 + 3x + 2 \\
\hline
(x-3) & x^4 - 2x^3 + 0x^2 - 7x - 6 \\
 & -\ x^4 - 3x^3 \\
\hline
 & 0 + x^3 + 0x^2 \\
 & -\ x^3 - 3x^2 \\
\hline
 & 0 + 3x^2 - 7x \\
 & -\ 3x^2 - 9x \\
\hline
 & 0 + 2x - 6 \\
 & 2x - 6 \\
\hline
 & 0
\end{array}
$$

$x^3 + x^2 + 3x + 2 = 0$

(iii) Use an iterative formula based on the equation in part **(ii)** to determine *a* correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

$a = -\frac{1}{3}(2 + a^2 + a^3)$

$a_1 = 1$

$a_2 = -1.33333$

$a_3 = -0.46916$

$a_4 = -0.70561$

$a_5 = -0.71552$

$a_6 = -0.71521$

$a_7 = -0.71523$

$a = -0.715$

7 The curve $y = \sin\left(x + \frac{1}{3}\pi\right)\cos x$ has two stationary points in the interval $0 \leqslant x \leqslant \pi$.

(i) Find $\frac{dy}{dx}$. [2]

$y = \sin(x + \frac{1}{3}\pi) \cos x$

$\frac{dy}{dx} = \cos x\left(\cos\left(x + \frac{1}{3}\pi\right)\right) + \sin\left(x + \frac{1}{3}\pi\right) - \sin$

$= \cos x \cos\left(x + \frac{1}{3}\pi\right) - \sin x \sin\left(x + \frac{1}{3}\pi\right)$

$\cos\left(x + x + \frac{1}{3}\pi\right)$

$= \cos\left(2x + \frac{1}{3}\pi\right)$

(ii) By considering the formula for $\cos(A + B)$, show that, at the stationary points on the curve, $\cos\left(2x + \frac{1}{3}\pi\right) = 0$. [2]

$\cos A+B = \cos A \cos B - \sin A \sin B$

$\cos x \cos\left(x + \frac{1}{3}\pi\right) - \sin x \sin\left(x + \frac{1}{3}\pi\right)$

$\cos\left(x + \left(x + \frac{1}{3}\pi\right)\right) = \cos\left(2x + \frac{1}{3}\pi\right) = 0$

As gradient at stationary point $= 0$

(iii) Hence find the exact x-coordinates of the stationary points. [3]

$\cos^{-1}(0) = \frac{1}{2}\pi$

$2x + \frac{1}{3}\pi = \frac{1}{2}\pi$

$2x = \frac{1}{6}\pi$

$x = \frac{1}{12}\pi$

other??

8 Throughout this question the use of a calculator is not permitted.

The complex number u is defined by

$$u = \frac{4\mathrm{i}}{1 - (\sqrt{3})\mathrm{i}}.$$

(i) Express u in the form $x + \mathrm{i}y$, where x and y are real and exact. [3]

$$\frac{4i\,(1+\sqrt{3}i)}{(1-\sqrt{3}i)(1+\sqrt{3}i)}$$

$$\frac{4i + 4\sqrt{3}(-1)}{1 - 3(-1)}$$

$$\frac{4i - 4\sqrt{3}}{4}$$

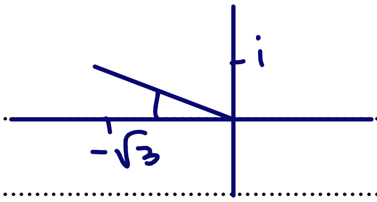
$$i - \sqrt{3}$$

$x = -\sqrt{3}$

$$-\sqrt{3} + i$$

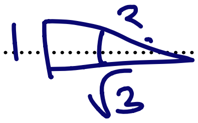
$y = 1$

(ii) Find the exact modulus and argument of u. [2]



$|u| = \sqrt{(-\sqrt{3})^2 + (1)^2}$

$= 2$



$\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

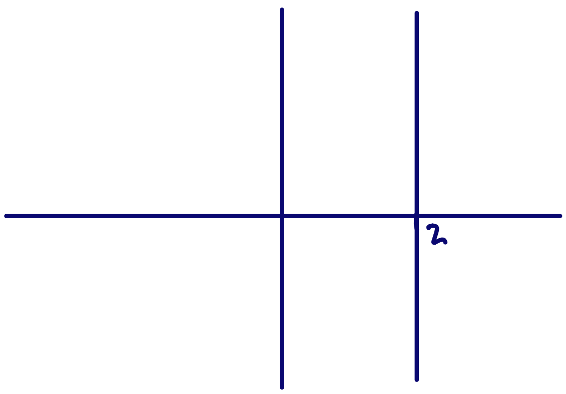
$\frac{1}{6}\pi$

$\therefore \arg(u) = \frac{5}{6}\pi$

(iii) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z| < 2$ and $|z - u| < |z|$. [4]

$z - (\sqrt{3} + i)$

z



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9 Let $\mathrm{f}(x) = \dfrac{2x(5-x)}{(3+x)(1-x)^2}$.

(i) Express $\mathrm{f}(x)$ in partial fractions. [5]

$2x(5-x) = 10x - 2x^2$

$$\frac{A}{3+x} + \frac{B}{1-x} + \frac{C}{(1-x)^2}$$

$$10x - 2x^2 = A(1-x)^2 + B(3+x)(1-x) + C(3+x)$$
$$= A(1-2x+x^2) + B(3-2x-x^2) + 3C + Cx$$
$$= A - 2Ax + Ax^2 + 3B - 2Bx - Bx^2 + 3C + Cx$$

① $0 = A + 3B + 3C$

② $10x = -2Ax - 2Bx + Cx$

③ $-2x^2 = Ax^2 - Bx^2$

$A = -\frac{9}{2}$ $B = -\frac{5}{2}$ $C = 4$

(ii) Hence obtain the expansion of $\mathrm{f}(x)$ in ascending powers of x up to and including the term in x^3. [5]

$4(1-x)^{-2} - \frac{5}{2}(1-x)^{-1} - \frac{9}{2}(3+x)^{-1}$

① $4\left[1 + (-2)(-x) + \frac{(-2)(-3)(-x)^2}{2!} + \frac{(-2)(-3)(-4)(-x)^3}{3!}\right]$

$4(1 + 2x + 3x^2 + 4x^3)$

$4 + 8x + 12x^2 + 16x^3$

② $\frac{5}{2}\left[1 + (-1)(-x) + \frac{(-1)(-2)(-x)^2}{2!} + \frac{(-1)(-2)(-3)(-x)^3}{3!}\right]$

$\frac{5}{2}(1 + x + x^2 + x^3)$

$\frac{5}{2} + \frac{5}{2}x + \frac{5}{2}x^2 + \frac{5}{2}x^3$

③ $\frac{9}{2}(3+x)^{-1} = \frac{9}{2}(3)\left(1+\frac{x}{3}\right)^{-1} \Rightarrow \frac{27}{2}\left(1+\frac{x}{3}\right)^{-1}$

$\frac{27}{2}\left[1 + (-1)\left(\frac{x}{3}\right) + \frac{(-1)(-2)\left(\frac{x}{3}\right)^2}{2!} + \frac{(-1)(-2)(-3)\left(\frac{x}{3}\right)^3}{3!}\right]$

$\frac{27}{2}\left(1 - \frac{x}{3} + \frac{1}{9}x^2 - \frac{1x^3}{27}\right)$

$\frac{27}{2} - \frac{9}{2}x + \frac{3}{2}x^2 - \frac{1}{2}x^3$

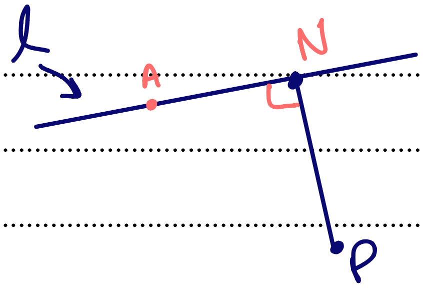
$4 + 8x + 12x^2 + 16x^3 - \frac{5}{2} - \frac{5}{2}x - \frac{5}{2}x^2 - \frac{5}{2}x^3 - \frac{27}{2} + \frac{9}{2}x - \frac{3}{2}x^2 + \frac{1}{2}x^3$

$-12 + x + 8x^2 + 14x^3$

10 The line l has equation $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \mu(2\mathbf{i} - \mathbf{j} - 2\mathbf{k})$.

$\begin{pmatrix}1\\2\\3\end{pmatrix} + \mu\begin{pmatrix}2\\-1\\-2\end{pmatrix}$

(i) The point P has position vector $4\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$. Find the length of the perpendicular from P to l. [5]



$AN \cdot NP = 0$

$\mu\begin{pmatrix}2\\-1\\-2\end{pmatrix} \cdot (\vec{OR} - \vec{ON})$

$\begin{pmatrix}2\\-1\\-2\end{pmatrix}\begin{pmatrix}3-2\mu\\ \mu\\ -6+2\mu\end{pmatrix} = 0$

NP:

$\begin{pmatrix}4\\2\\-3\end{pmatrix} - \begin{pmatrix}1+2\mu\\2-\mu\\3-2\mu\end{pmatrix} = \begin{pmatrix}3-2\mu\\ \mu\\ -6+2\mu\end{pmatrix}$

$2(3-2\mu) + (-1)(\mu) + (-2)(-6+2\mu) = 0$

$6 - 4\mu - \mu + 12 - 4\mu = 0$

$-9\mu = -18$

$\mu = 2$

$NP = \begin{pmatrix}3-2(2)\\2\\-6+4\end{pmatrix} = \begin{matrix}-1\\2\\-2\end{matrix}$

$\sqrt{1^2+2^2+2^2} = 3$

(ii) It is given that l lies in the plane with equation $ax + by + 2z = 13$, where a and b are constants. Find the values of a and b. [6]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.