Cambridge Assessment International Education

Cambridge International AS & A Level

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CENTRE NUMBER

CANDIDATE NUMBER

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MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **October/November 2020**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Blank pages are indicated.

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1 Solve the inequality $2 - 5x > 2|x - 3|$. [4]

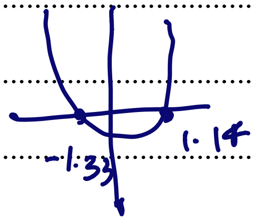
$(2-5x)^2 > (2(x-3))^2$

$25x^2 - 20x + 4 > 4(x^2 - 6x + 9)$

$25x^2 - 20x + 4 > 4x - 24x + 36$

$21x^2 + 4x - 32 > 0$

$x = -1.333 \;,\; 1.1428$



$x < -\frac{4}{3}$ and $x > 1.4228$

Trying $x > 1.4228$

let $x = 2$

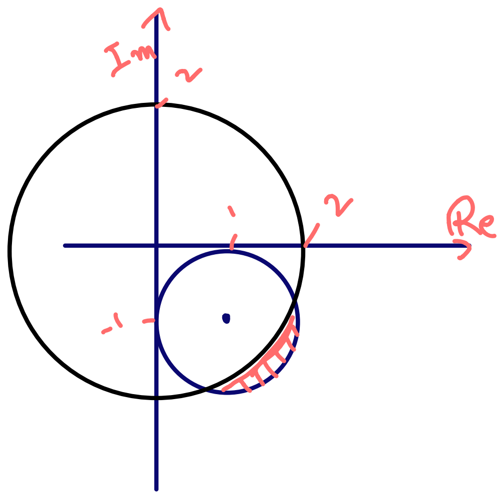
$2 - 5(2) > 2|2-3|$

$-8 \not> 2$

$\therefore \quad x < \frac{-4}{3}$

2 On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z| \geqslant 2$ and $|z - 1 + \mathrm{i}| \leqslant 1$. [4]

$1 - i$



3 The parametric equations of a curve are

$$x = 3 - \cos 2\theta, \quad y = 2\theta + \sin 2\theta,$$

for $0 < \theta < \frac{1}{2}\pi$.

Show that $\dfrac{dy}{dx} = \cot\theta$. [5]

$\dfrac{dx}{d\theta} = 2\sin 2\theta$ $\qquad$ $\dfrac{dy}{d\theta} = 2 + 2\cos 2\theta$

$\dfrac{2+2\cos\theta}{2\sin 2\theta} = \dfrac{\cancel{2}(1+\cos 2\theta)}{\cancel{2}\sin 2\theta} = \dfrac{1+\cos 2\theta}{2\sin\theta\cos\theta}$

$= \dfrac{\cancel{1}+2\cos^2\theta-\cancel{1}}{2\sin\theta\cos\theta}$

2.

$= \dfrac{\cos^{\cancel{2}}\theta}{\sin\theta\cancel{\cos\theta}}$

$= \cot\theta$

4 Solve the equation

$$\log_{10}(2x + 1) = 2\log_{10}(x + 1) - 1.$$

Give your answers correct to 3 decimal places. [6]

$\log_{10}(2x+1) = \log_{10}(x+1)^2 - 1$

$\log_{10}\left(\frac{2x+1}{(x+1)^2}\right) = -1$

$\frac{2x+1}{(x+1)^2} = \frac{1}{10}$

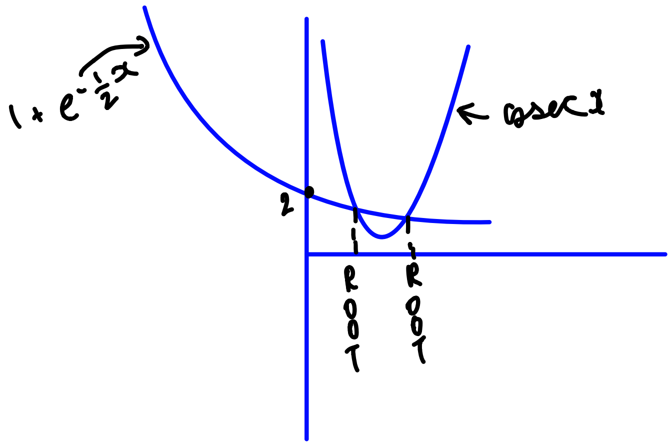
$10(2x+1) = (x+1)^2$

$20x + 10 = x^2 + 2x + 1$

$x^2 - 18x - 9 = 0$

$x = -0.487, 18.487$ ✓

5 **(a)** By sketching a suitable pair of graphs, show that the equation $\operatorname{cosec} x = 1 + e^{-\frac{1}{2}x}$ has exactly two roots in the interval $0 < x < \pi$. [2]



(b) The sequence of values given by the iterative formula

$$x_{n+1} = \pi - \sin^{-1}\left(\frac{1}{e^{-\frac{1}{2}x_n} + 1}\right),$$

with initial value $x_1 = 2$, converges to one of these roots.

Use the formula to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

$x_1 = \pi - \sin^{-1}\left(\frac{1}{e^{-x} + 1}\right) = 2.3217$

$x_2 = 2.2760$

$x_3 = 2.2824$

$x_4 = 2.2815$

$x_5 = 2.2816$

$x_6 = 2.2816$

$x_7 = 2.2816$

$\therefore x = 2.28$

6 **(a)** Express $\sqrt{6}\cos\theta + 3\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. State the exact value of R and give α correct to 2 decimal places. [3]

$R = \sqrt{(\sqrt{6})^2 + 3^2} = \sqrt{15}$

$\alpha = \tan^{-1}\left(\frac{3}{\sqrt{6}}\right) = 50.77$

$\sqrt{15}\cos(\theta - 50.77)$ ✓

(b) Hence solve the equation $\sqrt{6}\cos\frac{1}{3}x + 3\sin\frac{1}{3}x = 2.5$, for $0° < x < 360°$. [4]

$\sqrt{15}\cos\left(\frac{1}{3}x - 50.77\right) = 2.5$

$\frac{1}{3}x - 50.77 = \cos^{-1}\left(\frac{2.5}{\sqrt{15}}\right)$

$\frac{1}{3}x = 49.797 + 50.77$

$x = 301.7$

or

$\frac{1}{3}x = 310.202 + 50.77$

$\frac{1}{3}x = 360.9729$

$\frac{1}{3}x = 0.9729$

$x = 2.92$

7 **(a)** Verify that $-1 + \sqrt{5}\mathrm{i}$ is a root of the equation $2x^3 + x^2 + 6x - 18 = 0$. [3]

$-1 - \sqrt{5}i$

$-(1 + 5i$

2

(b) Find the other roots of this equation. [4]

$-1-\sqrt{5}i$

$1+\sqrt{5}i \times (\quad) = 2x^3+x^2+6x-18$

$(\quad) = \frac{(2x^3+x^2+6x-18)(-1-\sqrt{5}i)}{(-1+\sqrt{5}i)(-1-\sqrt{5}i)} = 0$

$(2x^3+x^2+6x-18)(-1-\sqrt{5}i) = 0$

$-2x^3 - x^2 - 6x + 18 - 2\sqrt{5}x^3 i - x^2\sqrt{5}i - 6\sqrt{5}xi - 18\sqrt{5}i = 0$

$[x-(1-\sqrt{5}i)] \times [x-(1+\sqrt{5}i)] \times (ax+b) = 2x^3 + x^2 + 6x - 18$

$(x-1+\sqrt{5}i)(x-1-\sqrt{5}i)(ax+b) =$

$a = 2$

$b = -1$

$x^2 - x - \cancel{\sqrt{5}xi} - x + 1 + \cancel{\sqrt{5}i} + \cancel{\sqrt{5}xi} - \cancel{\sqrt{5}i} - (\sqrt{5}i)^2$

$(x^2 - 2x + 1 - 5(-1))(ax+b) = 2x^3 + x^2 + 6x - 18$

$(x^2 - 2x + 6)(ax+b) = 2x^3 + x^2 + 6x - 18$

$2x^3 = x^2 \times ax$ $\quad bx^2 - 2x(ax) = x^2$

$a = 2$ $\quad bx^2 - 4x^2 = x^2$

$b = 5$

$2x + 5 = 0$

$2x = -5$

$x = -\frac{5}{2}$

$\therefore (\quad (-1+\sqrt{5}i)(-1-\sqrt{5}i)$

8 The coordinates (x, y) of a general point of a curve satisfy the differential equation

$$x\frac{dy}{dx} = (1 - 2x^2)y,$$

for $x > 0$. It is given that $y = 1$ when $x = 1$.

Solve the differential equation, obtaining an expression for y in terms of x. [6]

$\int \frac{1}{y}\,dy = \int \frac{1}{x} - \frac{2x^2}{x}\,dx$

$\ln y = \int \frac{1}{x}dx - 2\int x\,dx$

$\ln y = \ln x - \frac{\cancel{2}x^2}{\cancel{2}} + c$

$\ln y = \ln x - x^2 + c$

sub $(1,1)$

$\ln 1 = \ln 1 - 1 + c$

$0 = 0 - 1 + c$

$c = 1$

$\ln y = \ln x - x^2 + 1$

$y = e^{\ln x - x^2 + 1}$

$y = \frac{e^{\ln x}}{e^{x^2}} \times e$

$y = \frac{xe}{e^{x^2}}$

$y = xe^{1-x^2}$

9 Let $f(x) = \dfrac{8 + 5x + 12x^2}{(1 - x)(2 + 3x)^2}$.

(a) Express $f(x)$ in partial fractions. [5]

$$\frac{A}{1-x} + \frac{B}{2+3x} + \frac{C}{(2+3x)^2}$$

$$8 + 5x + 12x^2 = A(2+3x)^2 + B(1-x)(2+3x) + C(1-x)$$

$$8+5x+12x^2 = A(4+12x+9x^2) + B(2+x-3x^2) + C - Cx$$

$$8+5x+12x^2 = 4A + 12Ax + 9Ax^2 + 2B + Bx - 3Bx^2 + C - Cx$$

$$12\cancel{x^2} = 9A\cancel{x^2} - 3B\cancel{x^2}$$

$$5\cancel{x} = 12A\cancel{x} + B\cancel{x} - C\cancel{x} \quad B = 5 - 12A + C$$

$$8 = 4A + 2B + C$$

$A = 1$

$B = -1$

$C = 6$

$$\frac{1}{1-x} - \frac{1}{2+3x} + \frac{6}{(2+3x)^2}$$

(b) Hence obtain the expansion of f(x) in ascending powers of x, up to and including the term in x^2. [5]

$$\frac{1}{1-x} - \frac{1}{2+3x} + \frac{6}{(2+3x)^2}$$

$$(1-x)^{-1} - (2+3x)^{-1} + 6(2+3x)^{-2}$$

$$\left[1+(-1)(-x)+\frac{-1(-2)(-x)^2}{2!}\right] - 2^{-1}\left(1+\frac{3x}{2}\right)^{-1} + 6\times2^{-2}\left(1+\frac{3x}{2}\right)^{-2}$$

$$1+x+x^2 - \left[\frac{1}{2}\left(1+(-1)\left(\frac{3x}{2}\right)+\frac{(-1)(-2)\left(\frac{3x}{2}\right)^2}{2!}\right]+\left[\frac{3}{2}\left(1+(-2)\left(\frac{3}{2}x\right)+\frac{(-2)(-3)\left(\frac{3}{2}x\right)^2}{2!}\right]$$

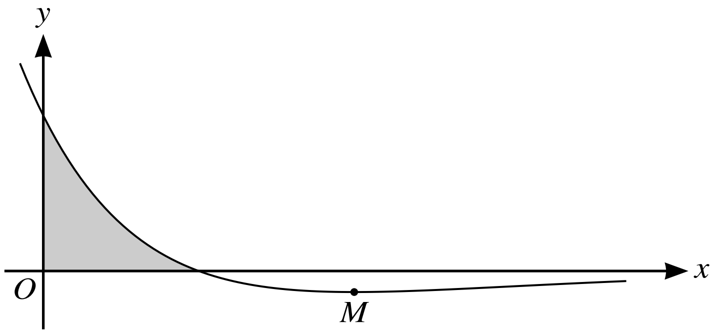
$$1+x+x^2-\left[\frac{1}{2}\left(1-\frac{3x}{2}+\frac{9x^2}{4}\right]+\left[\frac{3}{2}\left(1-3x+\frac{27x^2}{4}\right]$$

$$1+x+x^2-\left(\frac{1}{2}-\frac{3x}{4}+\frac{9x^2}{8}\right)+\left(\frac{3}{2}-\frac{9x}{2}+\frac{81x^2}{8}\right)$$

$$1+x+x^2-\frac{1}{2}+\frac{3x}{4}-\frac{9x^2}{8}+\frac{3}{2}-\frac{9x}{2}+\frac{81x^2}{8}$$

$$2-\frac{11x}{4}+10x^2$$

10



The diagram shows the curve $y = (2 - x)e^{-\frac{1}{2}x}$, and its minimum point M.

(a) Find the exact coordinates of M. [5]

$\frac{dy}{dx} = (2-x)(-\frac{1}{2})(e^{-\frac{1}{2}x}) + (-1)(e^{-\frac{1}{2}x})$

$= (2-x)(-\frac{1e^{-\frac{1}{2}x}}{2}) - e^{-\frac{1}{2}x}$

$= -e^{-\frac{1}{2}x} + \frac{xe^{-\frac{1}{2}x}}{2} - e^{-\frac{1}{2}x}$

$= \frac{xe^{-\frac{1}{2}x}}{2} - 2e^{-\frac{1}{2}x}$

$\frac{1}{2}e^{-\frac{1}{2}x}(x - 4) = 0$

$x - 4 = 0$

$x = 4$

$y = (2-4)e^{-\frac{1}{2}(4)}$

$= -2e^{-2}$

$M(4, -2e^{-2})$

(b) Find the area of the shaded region bounded by the curve and the axes. Give your answer in terms of e. [5]

$y = (2-x)e^{-\frac{1}{2}x} = 0$

$(2-x) = 0$

$x = 2$

$\cancel{e^{-\frac{1}{2}x} = 0}$

Mistake where

$\int_0^2 (2-x)e^{-\frac{1}{2}x}\,dx$

$2\int_0^2 e^{-\frac{1}{2}x}\,dx - \int_0^2 xe^{-\frac{1}{2}x}$ $\quad u = x \quad u' = 1$

$\quad v = -2e^{-\frac{1}{2}x} \quad v' = e^{-\frac{1}{2}x}$

3

$2\left(\frac{1}{-\frac{1}{2}}\right)e^{-\frac{1}{2}x} - \left[\frac{-xe^{-\frac{1}{2}x}}{2} - \int -\frac{1}{2}e^{\frac{1}{2}x}\,dx\right]$

$-4e^{-\frac{1}{2}x} - \left[\frac{-xe^{-\frac{1}{2}x}}{2} + \frac{1}{2}\left(-2e^{-\frac{1}{2}x}\right)\right]$

$-4e^{-\frac{1}{2}x} + \frac{xe^{-\frac{1}{2}x}}{2} + e^{-\frac{1}{2}x}$

$\left[\frac{xe^{-\frac{1}{2}x}}{2} - 3e^{-\frac{1}{2}x}\right]_0^2$

$\frac{\cancel{2}e^{-1}}{\cancel{2}} - 3e^{-1}$

$-2e^{-1}$

2.

11 Two lines have equations $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(a\mathbf{i} + 2\mathbf{j} - \mathbf{k})$ and $\mathbf{r} = 2\mathbf{i} + \mathbf{j} - \mathbf{k} + \mu(2\mathbf{i} - \mathbf{j} + \mathbf{k})$, where a is a constant.

(a) Given that the two lines intersect, find the value of a and the position vector of the point of intersection. [5]

$1 + \lambda a = 2 + 2\mu$ ①

$2 + 2\lambda = 1 - \mu$ ②

$1 - \lambda = -1 + \mu$ ③

① $\lambda a - 2\mu = 1$

② $2\lambda + \mu = -1$

③ $-\lambda - \mu = -2$

② + 3

$$\begin{array}{rl} & 2\lambda + \mu = -1 \\ + & -\lambda - \mu = -2 \\ \hline & \lambda = -3 \end{array}$$

$\therefore \mu = 3 + 2 = 5$

$-3a - 10 = 1$

$3a = -11$

$a = \frac{-11}{3}$

$$\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + (-3)\begin{pmatrix} -\frac{11}{3} \\ 2 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 + 11 \\ 2 - 6 \\ 1 + 3 \end{pmatrix} = \begin{pmatrix} 12 \\ -4 \\ 4 \end{pmatrix}$$

(b) Given instead that the acute angle between the directions of the two lines is $\cos^{-1}(\frac{1}{6})$, find the two possible values of a. [6]

$$\begin{pmatrix} a \\ 2 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

$$\frac{2a-2-1}{\sqrt{a^2+2^2+1^2} \times \sqrt{2^2+1^2+1^2}} = \frac{1}{6}$$

$$6(2a-3) = \sqrt{a^2+5} \times \sqrt{6}$$

$$(12a-18)^2 = (a^2+5)(6)$$

$$144a^2-432a+324 = 6a^2+30$$

$$138a^2-432a+294 = 0$$

$$a = \frac{49}{23} \quad \text{or} \quad a = 1$$

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge Assessment International Education

Cambridge International A Level

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **October/November 2020**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2020 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M	Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
B	Mark for a correct result or statement independent of method marks.
DM or **DB**	When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
FT	Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Make a recognisable sketch graph of $y = 2\|x-3\|$ and the line $y = 2 - 5x$	**B1**	Need to see correct V at $x = 3$, roughly symmetrical, $x = 3$ stated, domain at least $(-2, 5)$.
	Find x-coordinate of intersection with $y = 2 - 5x$	**M1**	Find point of intersection with $y = 2\|x-3\|$ or solve $2 - 5x$ with $2(x-3)$ or $-2(x-3)$
	Obtain $x = -\frac{4}{3}$	**A1**	
	State final answer $x < -\frac{4}{3}$	**A1**	Do not accept $x < -1.33$ [Do not condone $\leqslant$ for $<$ in the final answer.]
	Alternative method for question 1		
	State or imply non-modular inequality/equality $(2-5x)^2 >, \geqslant, =, 2^2(x-3)^2$, or corresponding quadratic equation, or pair of linear equations $(2-5x) >, \geqslant, =, \pm 2(x-3)$	**B1**	Two correct linear equations only
	Make reasonable attempt at solving a 3-term quadratic, or solve one linear equation, or linear inequality for x	**M1**	$21x^2 + 4x - 32 = (3x+4)(7x-8) = 0$ $2 - 5x$ or $-(2-5x)$ with $2(x-3)$ or $-2(x-3)$
	Obtain critical value $x = -\frac{4}{3}$	**A1**	
	State final answer $x < -\frac{4}{3}$	**A1**	Do not accept $x < -1.33$ [Do not condone $\leqslant$ for $<$ in the final answer.]
		4	

Question	Answer	Marks	Guidance
2	Show a circle with centre the origin and radius 2	**B1**	
	Show the point representing 1 – i	**B1**	
	Show a circle with centre 1 – i and radius 1	**B1 FT**	The FT is on the position of 1 – i.
	Shade the appropriate region	**B1 FT**	The FT is on the position of 1 – i. Shaded region outside circle with centre the origin and radius 2 and inside circle with centre $\pm 1 \pm$ i and radius 1
		4	

Question	Answer	Marks	Guidance
3	State or imply $\frac{dx}{d\theta} = 2\sin 2\theta$ or $\frac{dy}{d\theta} = 2+2\cos 2\theta$	**B1**	
	Use $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$	**M1**	
	Obtain correct answer $\frac{dy}{dx} = \frac{2+2\cos 2\theta}{2\sin 2\theta}$	**A1**	OE
	Use correct double angle formulae	**M1**	
	Obtain the given answer correctly $\frac{dy}{dx} = \cot\theta$	**A1**	AG. Must have simplified numerator in terms of $\cos\theta$.
	Alternative method for question 3		
	Start by using both correct double angle formulae e.g. $x = 3 - (2\cos^2\theta - 1)$, $y = 2\theta + 2\sin\theta\cos\theta$	**M1**	
	$\frac{dx}{d\theta}$ or $\frac{dy}{d\theta}$	**B1**	
	$\frac{dy}{dx} = \frac{\left(2+2\left(\cos^2\theta - \sin^2\theta\right)\right)}{4\cos\theta\sin\theta}$	**M1 A1**	
	Simplify to given answer correctly $\frac{dy}{dx} = \cot\theta$	**A1**	AG

Question	Answer	Marks	Guidance
3	**Alternative method for question 3**		
	Set $= 2\theta$. State $\frac{dx}{dt} = \sin t$ or $\frac{dy}{dt} = 1 + \cos t$	**B1**	
	Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	**M1**	
	Obtain correct answer $\frac{dy}{dx} = \frac{1+\cos t}{\sin t}$	**A1**	OE
	Use correct double angle formulae	**M1**	
	Obtain the given answer correctly $\frac{dy}{dx} = \cot\theta$	**A1**	
		5	
4	State or imply $\log_{10} 10 = 1$	**B1**	$\log_{10} 10^{-1} = -1$
	Use law of the logarithm of a power, product or quotient	**M1**	
	Obtain a correct equation in any form, free of logs	**A1**	e.g. $(2x + 1)/(x + 1)^2 = 10^{-1}$ or $10(2x + 1)/(x + 1)^2 = 10^0$ or 1 or $x^2 + 2x + 1 = 20x + 10$
	Reduce to $x^2 - 18x - 9 = 0$, or equivalent	**A1**	
	Solve a 3-term quadratic	**M1**	
	Obtain final answers $x = 18.487$ and $x = -0.487$	**A1**	Must be 3 d.p. Do not allow rejection.
		6	

Question	Answer	Marks	Guidance
5(a)	Sketch a relevant graph, e.g. $y = \operatorname{cosec} x$	**B1**	cosec x, U shaped, roughly symmetrical about $x = \frac{\pi}{2}$, $y\left(\frac{\pi}{2}\right) = 1$ and domain at least $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$.
	Sketch a second relevant graph, e.g. $y = 1 + e^{-\frac{1}{2}x}$, and justify the given statement	**B1**	Exponential graph needs $y(0) = 2$, negative gradient, always increasing, and $y(\pi) > 1$ Needs to mark intersections with dots, crosses, or say roots at points of intersection, or equivalent
		2	
5(b)	Use the iterative formula correctly at least twice	**M1**	2, 2.3217, 2.2760, 2.2824… Need to see 2 iterations and following value inserted correctly
	Obtain final answer 2.28	**A1**	Must be supported by iterations
	Show sufficient iterations to at least 4 d.p. to justify 2.28 to 2 d.p., or show there is a sign change in the interval (2.275, 2.285)	**A1**	
		3	

Question	Answer	Marks	Guidance
6(a)	State $R = \sqrt{15}$	B1	
	Use trig formulae to find α	M1	$\frac{\sin\alpha}{\cos\alpha} = \frac{3}{\sqrt{6}}$ with no error seen or $\tan\alpha = \frac{3}{\sqrt{6}}$ quoted then allow M1
	Obtain $\alpha = 50.77$	A1	Must be 2 d.p. If radians 0.89 A0 MR
		3	
6(b)	Evaluate $\beta = \cos^{-1}\frac{2.5}{\sqrt{15}}$ (49.797° to 4 d.p.)	B1 FT	The FT is on incorrect R. $\frac{x}{3} = \beta - \alpha$ [–2.9° and –301.7°]
	Use correct method to find a value of $\frac{x}{3}$ in the interval	M1	Needs to use $\frac{x}{3}$
	Obtain answer rounding to $x = 301.6°$ to $301.8°$	A1	
	Obtain second answer rounding to $x = 2.9(0)°$ to $2.9(2)°$ and no others in the interval	A1	
		4	

Question	Answer	Marks	Guidance
7(a)	Substitute $-1+\sqrt{5}\,\mathrm{i}$ in the equation and attempt expansions of x^2 and x^3	**M1**	All working must be seen. Allow M1 if small errors in $1-2\sqrt{5}\mathrm{i}-5$ or $1-\sqrt{5}\mathrm{i}-\sqrt{5}\mathrm{i}-5$ and $4-2\sqrt{5}\mathrm{i}+10$ or $4-4\sqrt{5}\mathrm{i}+2\sqrt{5}\mathrm{i}+10$
	Use $\mathrm{i}^2 = -1$ correctly at least once	**M1**	$1-5$ or $4+10$ seen
	Complete the verification correctly	**A1**	$2\left(14-2\sqrt{5}\mathrm{i}\right)+\left(-4-2\sqrt{5}\mathrm{i}\right)+6\left(-1+\sqrt{5}\mathrm{i}\right)-18=0$
		3	
7(b)	State second root $-1-\sqrt{5}\mathrm{i}$	**B1**	
	Carry out a complete method for finding a quadratic factor with zeros $-1+\sqrt{5}\mathrm{i}$ and $-1-\sqrt{5}\mathrm{i}$	**M1**	
	Obtain $x^2 + 2x + 6$	**A1**	
	Obtain root $x=\dfrac{3}{2}$	**A1**	OE
	Alternative method for question 7(b)		
	State second root $-1-\sqrt{5}\mathrm{i}$	**B1**	
	$(x + 1 - \sqrt{5}\mathrm{i})\,(x + 1 + \sqrt{5}\mathrm{i})\,(2x + a) = 2x^3 + x^2 + 6x - 18$	**M1**	
	$(1 - \sqrt{5}\,\mathrm{i})\,(1 + \sqrt{5}\,\mathrm{i})\,a = -18$	**A1**	
	$6a = -18$ $a = -3$ leading to $x=\dfrac{3}{2}$	**A1**	OE

Question	Answer	Marks	Guidance
7(b)	**Alternative method for question 7(b)**		
	State second root $-1-\sqrt{5}\mathrm{i}$	**B1**	
	POR = 6 SOR = – 2	**M1**	
	Obtain $x^2 + 2x + 6$	**A1**	
	Obtain root $x = \frac{3}{2}$	**A1**	OE
	Alternative method for question 7(b)		
	State second root $-1-\sqrt{5}\mathrm{i}$	**B1**	
	POR $\left(-1-\sqrt{5}\mathrm{i}\right)\left(-1+\sqrt{5}\mathrm{i}\right)a = 9$	**M1 A1**	
	Obtain root $x = \frac{3}{2}$	**A1**	OE
	Alternative method for question 7(b)		
	State second root $-1-\sqrt{5}\mathrm{i}$	**B1**	
	SOR $\left(-1-\sqrt{5}\mathrm{i}\right)+\left(-1+\sqrt{5}\mathrm{i}\right)+a = -\frac{1}{2}$	**M1 A1**	
	Obtain root $x = \frac{3}{2}$	**A1**	OE
		4	

Question	Answer	Marks	Guidance
8	Separate variables correctly and attempt integration of at least one side	**B1**	$\frac{1}{y}\mathrm{d}y = \frac{1-2x^2}{x}\mathrm{d}x$
	Obtain term $\ln y$	**B1**	
	Obtain terms $\ln x - x^2$	**B1**	
	Use $x = 1$, $y = 1$ to evaluate a constant, or as limits, in a solution containing at least 2 terms of the form $a\ln y$, $b\ln x$ and cx^2	**M1**	The 2 terms of required form must be from correct working e.g. $\ln y = \ln x - x^2 + 1$
	Obtain correct solution in any form	**A1**	
	Rearrange and obtain $y = xe^{1-x^2}$	**A1**	OE
		6	

Question	Answer	Marks	Guidance
9(a)	State or imply the form $\dfrac{A}{1-x}+\dfrac{B}{2+3x}+\dfrac{C}{(2+3x)^2}$	**B1**	
	Use a correct method for finding a coefficient	**M1**	
	Obtain one of $A=1$, $B=-1$, $C=6$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	In the form $\dfrac{A}{1-x}+\dfrac{Dx+E}{(2+3x)^2}$, where $A=1$, $D=-3$ and $E=4$ can score B1 M1 A1 A1 A1 as above.
		5	

Question	Answer	Marks	Guidance
9(b)	Use a correct method to find the first two terms of the expansion of $(1-x)^{-1}$, $(2+3x)^{-1}$, $\left(1+\frac{3}{2}x\right)^{-1}$, $(2+3x)^{-2}$ or $\left(1+\frac{3}{2}x\right)^{-2}$	**M1**	Symbolic coefficients are not sufficient for the M1 $A\left[\frac{1+(-1)(-x)+(-1)(-2)(-x)^2}{2\ldots}\right]$ $A = 1$ $\frac{B}{2}\left[\frac{1+(-1)\left(\frac{3x}{2}\right)+(-1)(-2)\left(\frac{3x}{2}\right)^2}{2\ldots}\right]$ $B = 1$ $\frac{C}{4}\left[\frac{1+(-2)\left(\frac{3x}{2}\right)+(-2)(-3)\left(\frac{3x}{2}\right)^2}{2\ldots}\right]$ $C = 6$
	Obtain correct un-simplified expansions up to the term in of each partial fraction	**A1 FT** +	$\left(1+x+x^2\right)+\left(-\frac{1}{2}+\left(\frac{3}{4}\right)x-\left(\frac{9}{8}\right)x^2\right)$
		A1 FT +	$+\left(\frac{6}{4}-\left(\frac{18}{4}\right)x+\left(\frac{81}{8}\right)x^2\right)$ [The FT is on A, B, C]
		A1 FT	$\left(1-\frac{1}{2}+\frac{6}{4}\right)+\left(1+\frac{3}{4}-\frac{18}{4}\right)x+\left(1-\frac{9}{8}+\frac{81}{8}\right)x^2$
	Obtain final answer $2-\frac{11}{4}x+10x^2$, or equivalent	**A1**	Allow unsimplified fractions $\frac{(Dx+E)}{4}\left[\frac{1+(-2)\left(\frac{3x}{2}\right)+(-2)(-3)\left(\frac{3x}{2}\right)^2}{2\ldots}\right]$ $D = -3, E = 4$ The FT is on A, D, E.
		5	

Question	Answer	Marks	Guidance
10(a)	Use correct product or quotient rule	*M1	$\frac{\mathrm{d}y}{\mathrm{d}x} = \left(-\frac{1}{2}\right)(2-x)\mathrm{e}^{-\frac{1}{2}x} - \mathrm{e}^{-\frac{1}{2}x}$ M1 requires at least one of derivatives correct
	Obtain correct derivative in any form	A1	
	Equate derivative to zero and solve for x	DM1	
	Obtain $x = 4$	A1	ISW
	Obtain $y = -2\mathrm{e}^{-2}$, or exact equivalent	A1	
		5	

Question	Answer	Marks	Guidance
10(b)	Commence integration and reach $a(2-x)e^{-\frac{1}{2}x}+b\int e^{-\frac{1}{2}x}dx$	*M1	Condone omission of dx $-2(2-x)e^{-\frac{1}{2}x}+4e^{-\frac{1}{2}x}$ or $2xe^{-\frac{1}{2}x}$
	Obtain $-2(2-x)e^{-\frac{1}{2}x}-2\int e^{-\frac{1}{2}x}dx$	A1	OE
	Complete integration and obtain $2xe^{-\frac{1}{2}x}$	A1	OE
	Use correct limits, $x = 0$ and $x = 2$, correctly, having integrated twice	DM1	Ignore omission of zeros and allow max of 1 error
	Obtain answer $4e^{-1}$, or exact equivalent	A1	ISW
	Alternative method for question 10(b)		
	$\frac{d\left(2xe^{-\frac{1}{2}x}\right)}{dx}=2e^{-\frac{1}{2}x}-xe^{-\frac{1}{2}x}$	*M1 A1	
	$\therefore 2xe^{-\frac{1}{2}x}$	A1	
	Use correct limits, $x = 0$ and $x = 2$, correctly, having integrated twice	DM1	Ignore omission of zeros and allow max of 1 error
	Obtain answer $4e^{-1}$, or exact equivalent	A1	ISW
		5	

Question	Answer	Marks	Guidance
11(a)	Express general point of at least one line correctly in component form, i.e. $(1 + a\lambda, 2 + 2\lambda, 1 - \lambda)$ or $(2 + 2\mu, 1 - \mu, -1 + \mu)$	**B1**	
	Equate at least two pairs of corresponding components and solve for λ or for μ	**M1**	May be implied $1 + a\lambda = 2 + 2\mu \quad 2 + 2\lambda = 1 - \mu \quad 1 - \lambda = -1 + \mu$
	Obtain $\lambda = -3$ or $\mu = 5$	**A1**	
	Obtain $a = -\dfrac{11}{3}$	**A1**	Allow $a = -3.667$
	State that the point of intersection has position vector $12\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}$	**A1**	Allow coordinate form (12, – 4, 4)
		5	

Question	Answer	Marks	Guidance
11(b)	Use correct process for finding the scalar product of direction vectors for the two lines	M1	$(a, 2, -1) \cdot (2, -1, 1) = 2a - 2 - 1$ or $2a - 3$
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and equate the result to $\pm\frac{1}{6}$	*M1	
	State a correct equation in a in any form, e.g. $\frac{2a-2-1}{\sqrt{6}\sqrt{(a^2+5)}} = \pm\frac{1}{6}$	A1	
	Solve for a	DM1	Solve 3-term quadratic for a having expanded $(2a - 3)^2$ to produce 3 terms e.g. $36(2a-3)^2 = 6(a^2+5)$ $138a^2 - 432a + 294 = 0$ $23a^2 - 72a + 49 = 0$ $(23a - 49)(a - 1) = 0$
	Obtain $a = 1$	A1	
	Obtain $a = \frac{49}{23}$	A1	Allow $a = 2.13$

Question	Answer	Marks	Guidance
11(b)	**Alternative method for question 11(b)**		
	$\cos(\theta) = [\lvert a^2 + 2^2 + (-1)^2\rvert^2 + \lvert 2^2 + (-1)^2 + 1^2\rvert^2$ $- \lvert(a-2)^2 + 3^2 + (-2)^2\rvert^2] / [2\lvert a^2 + 2^2 + (-1)^2\rvert.\lvert 2^2 + (-1)^2 + 1^2\rvert]$	**M1**	Use of cosine rule. Must be correct vectors.
	Equate the result to $\pm\frac{1}{6}$	***M1** **A1**	Allow M1* here for any two vectors
	Solve for a	**DM1**	Solve 3-term quadratic for a having expanded $(2a-3)^2$ to produce 3 terms e.g. $36(2a-3)^2 = 6(a^2+5)$ $138a^2 - 432a + 294 = 0$ $23a^2 - 72a + 49 = 0$ $(23a-49)(a-1) = 0$
	Obtain $a = 1$	**A1**	
	Obtain $a = \frac{49}{23}$	**A1**	Allow $a = 2.13$
		6	