56

Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2015**

1 hour 45 minutes

8254149865

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **3** printed pages and **1** blank page.

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1 Use logarithms to solve the equation $2^{5x} = 3^{2x+1}$, giving the answer correct to 3 significant figures. [4]

2 Use the trapezium rule with three intervals to find an approximation to

$$\int_0^3 |3^x - 10| \, dx.$$

[4]

3 Show that, for small values of x^2,

$$(1 - 2x^2)^{-2} - (1 + 6x^2)^{\frac{2}{3}} \approx kx^4,$$

where the value of the constant k is to be determined. [6]

4 The equation of a curve is

$$y = 3\cos 2x + 7\sin x + 2.$$

Find the x-coordinates of the stationary points in the interval $0 \leqslant x \leqslant \pi$. Give each answer correct to 3 significant figures. [7]

5 **(a)** Find $\int (4 + \tan^2 2x) \, dx$. [3]

(b) Find the exact value of $\int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi} \frac{\sin(x + \frac{1}{6}\pi)}{\sin x} \, dx$. [5]

6 The straight line l_1 passes through the points $(0, 1, 5)$ and $(2, -2, 1)$. The straight line l_2 has equation $\mathbf{r} = 7\mathbf{i} + \mathbf{j} + \mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} + 5\mathbf{k})$.

(i) Show that the lines l_1 and l_2 are skew. [6]

(ii) Find the acute angle between the direction of the line l_2 and the direction of the x-axis. [3]

7 Given that $y = 1$ when $x = 0$, solve the differential equation

$$\frac{dy}{dx} = 4x(3y^2 + 10y + 3),$$

obtaining an expression for y in terms of x. [9]

8 The complex number w is defined by $w = \frac{22 + 4i}{(2 - i)^2}$.

(i) Without using a calculator, show that $w = 2 + 4i$. [3]

(ii) It is given that p is a real number such that $\frac{1}{4}\pi \leqslant \arg(w + p) \leqslant \frac{3}{4}\pi$. Find the set of possible values of p. [3]

(iii) The complex conjugate of w is denoted by w^*. The complex numbers w and w^* are represented in an Argand diagram by the points S and T respectively. Find, in the form $|z - a| = k$, the equation of the circle passing through S, T and the origin. [3]

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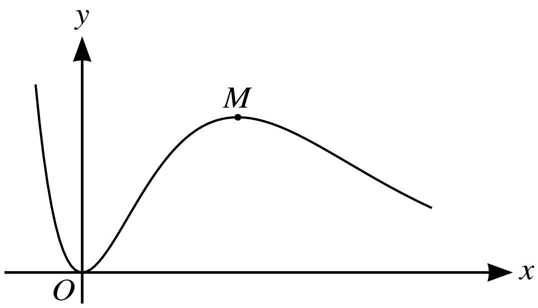

The diagram shows the curve $y = x^2 e^{2-x}$ and its maximum point M.

(i) Show that the x-coordinate of M is 2. [3]

(ii) Find the exact value of $\int_0^2 x^2 e^{2-x}\, dx$. [6]

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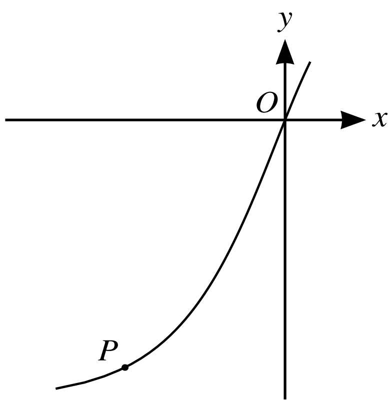

The diagram shows part of the curve with parametric equations

$$x = 2\ln(t+2), \qquad y = t^3 + 2t + 3.$$

(i) Find the gradient of the curve at the origin. [5]

(ii) At the point P on the curve, the value of the parameter is p. It is given that the gradient of the curve at P is $\frac{1}{2}$.

(a) Show that $p = \dfrac{1}{3p^2 + 2} - 2$. [1]

(b) By first using an iterative formula based on the equation in part **(a)**, determine the coordinates of the point P. Give the result of each iteration to 5 decimal places and each coordinate of P correct to 2 decimal places. [4]

1 Use logarithms to solve the equation $2^{5x} = 3^{2x+1}$, giving the answer correct to 3 significant figures. [4]

$$5x\ln 2 = (2x+1)\ln 3$$

$$5x\ln 2 = 2x\ln 3 + \ln 3$$

$$5x\ln 2 - 2x\ln 3 = \ln 3$$

$$x(\ln 2^5 - \ln 3^2) = \ln 3$$

$$x(\ln 32 - \ln 9) = \ln 3$$

$$x\ln\frac{32}{9} = \ln 3$$

$$x = \frac{\ln 3}{\ln\frac{32}{9}} \rightarrow 0.866$$

4

3 Show that, for small values of x^2,

$$(1-2x^2)^{-2} - (1+6x^2)^{\frac{2}{3}} \approx kx^4,$$

where the value of the constant k is to be determined. [6]

0

$\cancel{\text{let } f(x) = (1-2x^2)^{-2} - (1+6x^2)^{\frac{2}{3}}}$

$\cancel{f(0.5) = 2.15798 = kx^4}$

$\cancel{f(0.1) = 0.0016225 = kx^4}$

$\cancel{f(0.01) = 1.6\times10^{-7} = \text{''}}$

$\cancel{f(0.001) = 1.6\times10^{-11} = \text{''}}$

$\cancel{k = \frac{2.15798}{0.5^4} = 34.52777}$

$\cancel{k = \frac{0.001625}{0.1^4} = 16.25}$

$\cancel{= \frac{1.6\times10^{-7}}{0.01^4} = 16}$

$\cancel{= \frac{1.6\times10^{-11}}{0.001^4} = 16}$

$\cancel{k = 16}$

$(1-2x^2)^{-2}$

$1 + (-2)(-2x^2) + \frac{(-2)(-3)(-2x^2)^2}{2!}$

$1 + 4x^2 + 12x^4$

$(1+6x^2)^{\frac{2}{3}}$

$1 + \left(\frac{2}{3}\right)(6x^2) + \frac{\left(\frac{2}{3}\right)\left(-\frac{1}{3}\right)(6x^2)^2}{2!}$

$1 + 3x^2 - 4x^4$

$12x^4 - (-4x^4) = 16x^4$

$k = 16$

4 The equation of a curve is

$$y = 3\cos 2x + 7\sin x + 2.$$

Find the x-coordinates of the stationary points in the interval $0 \leqslant x \leqslant \pi$. Give each answer correct to 3 significant figures. [7]

6

$\frac{dy}{dx} = -6\sin 2x + 7\cos x = 0$

$7\cos x = 6\sin 2x$

$7\cancel{\cos x} = 12\sin x\cancel{\cos x}$

$\sin x = \frac{7}{12}$

$x = \sin^{-1}\left(\frac{7}{12}\right)$

$= 0.6228$ and $\pi - 0.6228$

$x = 0.62$ and 2.518

$x = 0.623$ and 2.52

$\cos x(12\sin x - 7) = 0$

$x = \cos^{-1}(0) = \frac{1}{2}\pi$

5 **(a)** Find $\int (4 + \tan^2 2x)\,dx$. [3]

2

(b) Find the exact value of $\int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi} \frac{\sin(x + \frac{1}{6}\pi)}{\sin x}\,dx$. [5]

a) $\int 4 + \tan^2 2x$

$4x + \int \sec^2 2x - 1 \, dx$

$4x + \frac{1}{2}\tan 2x - x$

$\cos 2x$

$-4\cos x \sin 2x$

$(\sin 2x)^2 = (2\sin x\cos x)^2$

b) $\int \frac{\sin x\cos\frac{1}{6}\pi + \cos x\sin\frac{1}{6}\pi}{\sin x}\,dx$

$\int \frac{\frac{\sqrt{3}}{2}\sin x}{\sin x} + \frac{\frac{1}{2}\cos x}{\sin x}\,dx$

$\int \frac{\sqrt{3}}{2} + \frac{1}{2}\frac{\cos x}{\sin x}\,dx$

$\left[\frac{\sqrt{3}}{2}x + \frac{1}{2}\ln\sin x\right]_{\frac{1}{4}\pi}^{\frac{1}{2}\pi}$

$\frac{\sqrt{3}\pi}{4} + 0 - \frac{\sqrt{3}\pi}{8} - \frac{1}{2}\ln\frac{1}{\sqrt{2}} = \frac{1}{8}\sqrt{3}\pi - \frac{1}{2}\ln\frac{1}{\sqrt{2}}$

6 The straight line l_1 passes through the points (0, 1, 5) and (2, −2, 1). The straight line l_2 has equation $\mathbf{r} = 7\mathbf{i} + \mathbf{j} + \mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} + 5\mathbf{k})$.

(i) Show that the lines l_1 and l_2 are skew. [6]

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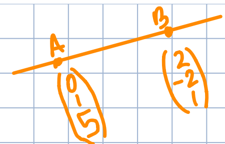


$\overrightarrow{AB} \begin{pmatrix} 2 \\ -3 \\ -4 \end{pmatrix}$

$\begin{pmatrix} 0 \\ 1 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -3 \\ -4 \end{pmatrix}$

$l_1 : \begin{pmatrix} 2\lambda \\ 1-3\lambda \\ 5-4\lambda \end{pmatrix}$ $\quad l_2 \begin{pmatrix} 7+\mu \\ 1+2\mu \\ 1+5\mu \end{pmatrix}$

$2\lambda = 7+\mu \rightarrow \mu = 2\lambda - 7$

$1-3\lambda = 1+2\mu$

$1-3\lambda = 1+4\lambda-14$

$14 = 7\lambda$

$\lambda = 2$

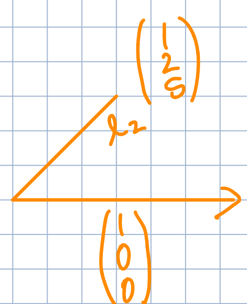
$\therefore \mu = -3$

check if satisfies last: $5-4(2) = 1+5(-3)$

$-3 \neq -14$

$\therefore$ skew

ii)



$\dfrac{1+0+0}{\sqrt{1^2+2^2+5^2} \times \sqrt{1^2}} = \cos\theta$

$\dfrac{1}{\sqrt{30} \times 1}$

$\theta = \cos^{-1}\left(\dfrac{1}{\sqrt{30}}\right) = 79.5°$

7 Given that $y = 1$ when $x = 0$, solve the differential equation

$$\frac{dy}{dx} = 4x(3y^2 + 10y + 3),$$

obtaining an expression for y in terms of x. [9]

6

i) $\int \frac{1}{3y^2+10y+3}\,dy = \int 4x\,dx$

$\int \frac{1}{(3y+1)(y+3)}\,dy = 2x^2$

$\frac{1}{8}\int \frac{3}{(3y+1)}\,dy - \frac{1}{8}\int \frac{1}{y+3}\,dy = 2x^2$

$\frac{1}{8}\ln(3y+1) - \frac{1}{8}\ln(y+3) = 2x^2$

$\ln(3y+1) - \ln(y+3) = 16x^2$

$\ln(3y+1) - \ln(y+3) = 16x^2$

$\frac{3y+1}{y+3} = e^{16x^2}$

$3y+1 = 3ye^{16x^2} + 3e^{16x^2}$

$3y(1-e^{16x^2}) = 3e^{16x^2} - 1$

$3y = \frac{3e^{16x^2}-1}{1-e^{16x^2}}$

$y = \frac{3e^{16x^2}-1}{3(1-e^{16x^2})}$

$x = 9$

$3y^2 + 9y + y + 3$

$3y(y+3) + 1(y+3)$

$(3y+1)(y+3)$

$\frac{A}{3y+1} + \frac{B}{y+3}$

$1 = A(y+3) + B(3y+1)$

$1 = Ay + 3A + 3By + B$

$3A + B = 1$

$B = -3A + 1$

$A + 3(-3A+1) = 0$

$A - 9A + 3 = 0$

$-8A = -3$

$A = \frac{3}{8}$, $B = -\frac{1}{8}$

8 The complex number w is defined by $w = \dfrac{22 + 4\mathrm{i}}{(2 - \mathrm{i})^2}$.

(i) Without using a calculator, show that $w = 2 + 4\mathrm{i}$. [3]

(ii) It is given that p is a real number such that $\frac{1}{4}\pi \leqslant \arg(w + p) \leqslant \frac{3}{4}\pi$. Find the set of possible values of p. [3]

(iii) The complex conjugate of w is denoted by w^*. The complex numbers w and w^* are represented in an Argand diagram by the points S and T respectively. Find, in the form $|z - a| = k$, the equation of the circle passing through S, T and the origin. [3]

i) $\dfrac{22+4i}{(2-i)(2-i)} = \dfrac{22+4i}{4-4i+(-1)} = \dfrac{22+4i}{3-4i}\cdot\dfrac{(3+4i)}{(3+4i)} = \dfrac{66+88i+12i-16}{9-16i^2}$

$= \dfrac{50+100i}{25}$

$= 2+4i$

$w+p=(2+p)+4i$

ii) $\frac{1}{4}\pi \leq \tan^{-1}\left(\dfrac{4}{2+p}\right)$

$\dfrac{4}{2+p} \geq \dfrac{1}{4}\pi$

$16 \geq 2\pi + p\pi$

$p\pi \leq 16-2\pi$

$p \leq \dfrac{16}{\pi} - 2$

$\dfrac{4}{2+p} \geq 1$

$p+2 \leq 4$

$p \leq 2$

$\dfrac{4}{2+p} \leq \dfrac{3}{4}\pi$

$\cancel{16 \leq 6\pi + 3p\pi}$

$3p\pi \geq 16-6\pi$

$p \geq \dfrac{16}{3\pi} - 2$

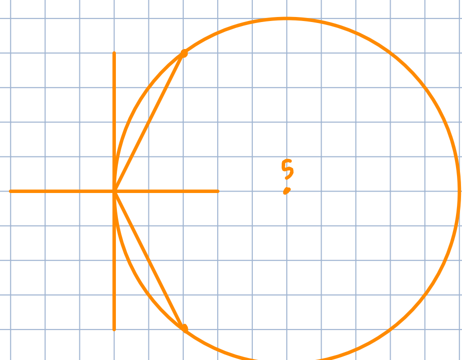
$\dfrac{4}{2+p} \leq -1$

$-2-p \geq 4$

$p \leq -6$

$\dfrac{16}{3\pi} - 2 \leq p \leq \dfrac{16}{\pi} - 2$

iii)



$z-($

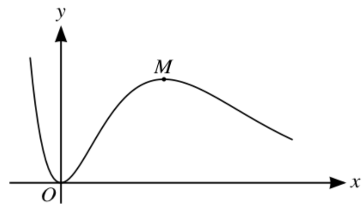
$z-5$

$z-(5+0i)$

$|z-5| = 5$

$(z-a)^2$

9



The diagram shows the curve $y = x^2e^{2-x}$ and its maximum point M.

(i) Show that the x-coordinate of M is 2. [3]

(ii) Find the exact value of $\int_0^2 x^2e^{2-x}\,dx$. [6]

i) $y = x^2 e^{2-x}$

$\frac{dy}{dx} = x^2(-1e^{2-x}) + 2x e^{2-x}$

$= -x^2e^{2-x} + 2xe^{2-x}$

$2\cancel{x}\cancel{e^{2-x}} = x^{\cancel{2}}\cancel{e^{2-x}}$

$x = 2$

ii) $\int_0^2 x^2 e^{2-x}\,dx$

$u \quad v$

$u = x^2 \qquad u' = 2x$

$v = -e^{2-x} \qquad v' = e^{2-x}$

$\int_1^2 \qquad -e^0 + e^1$

$-x^2e^{2-x} + 2\int e^{2-x}x\,dx$

$u = x \qquad u' = 1$

$v' = e^{2-x}, \quad v = -e^{2-x}$

$-x^2e^{2-x} + 2\left[-xe^{2-x} + \int e^{2-x}\,dx\right]$

$-x^2e^{2-x} + 2\left[-xe^{2-x} - e^{2-x}\right]$

$-x^2e^{2-x} - 2xe^{2-x} - 2e^{2-x}$

$\left[e^{2-x}(-x^2 - 2x - 2)\right]_0^2$

$e^0(-4 - 4 - 2) - e^2(-2)$

$-10 + 2e^2$

$2e^2 - 10$

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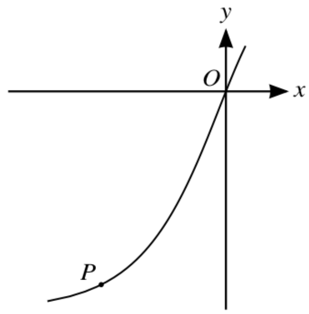

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(ii) At the point P on the curve, the value of the parameter is p. It is given that the gradient of the curve at P is $\frac{1}{2}$.

(a) Show that $p = \dfrac{1}{3p^2+2} - 2$. [1]

(b) By first using an iterative formula based on the equation in part **(a)**, determine the coordinates of the point P. Give the result of each iteration to 5 decimal places and each coordinate of P correct to 2 decimal places. [4]

2

i) $\dfrac{dy}{dt} = 3t^2+2$ $\qquad \dfrac{dx}{dt} = \dfrac{2}{t+2}$ $\qquad \therefore \dfrac{dt}{dx} = \dfrac{t+2}{2}$

$\dfrac{dy}{dx} = 3t^2+2 \times \dfrac{(t+2)}{2}$

$= \dfrac{(3t^2+2)(t+2)}{2}$

when $x=0$
$\ln t+2 = 0$
$t+2 = 1$
$t = -1$

$\dfrac{(3(-1)^2+2)(1)}{2}$

$\dfrac{3+2}{2} = \dfrac{5}{2}$

ii) $\dfrac{(3p^2+2)(p+2)}{2} = \dfrac{1}{2}$

$(3p^2+2)(p+2) = 1$

$p+2 = \dfrac{1}{3p^2+2}$

$p = \dfrac{1}{3p^2+2} - 2$

iii) let $p_1 = -0.5$

$p_2 = \dfrac{1}{3(-0.5)^2+2} - 2 = -1.63636$

$p_3 = -1.90053$

$p_4 = -1.92208$

$p_5 = -1.92357$

$p_6 = -1.92367$

$p_7 = -1.92367$

$\therefore p = -1.924$

$P(x, y)$

$x = 2\ln(1.92+2) = 2.73$

$y = 1.39$

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