Cambridge International AS & A Level

Cambridge International Examinations
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS **9709/61**

Paper 6 Probability & Statistics 1 **(S1)** **May/June 2016**

1 hour 15 minutes

Additional Materials: List of Formulae (MF9)

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READ THESE INSTRUCTIONS FIRST

An answer booklet and a graph paper booklet are provided inside this question paper. You should follow the instructions on the front cover of both booklets. If you need additional answer paper or graph paper ask the invigilator for a continuation booklet or graph paper booklet.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.

This document consists of **3** printed pages, **1** blank page and **2** inserts.

CAMBRIDGE International Examinations

1 The height of maize plants in Mpapwa is normally distributed with mean 1.62 m and standard deviation σ m. The probability that a randomly chosen plant has a height greater than 1.8 m is 0.15. Find the value of σ. [3]

2 The faces of a biased die are numbered 1, 2, 3, 4, 5 and 6. The random variable X is the score when the die is thrown. The following is the probability distribution table for X.

x	1	2	3	4	5	6
$P(X = x)$	p	p	p	p	0.2	0.2

The die is thrown 3 times. Find the probability that the score is 4 on not more than 1 of the 3 throws. [5]

3 The probability that the school bus is on time on any particular day is 0.6. If the bus is on time the probability that Sam the driver gets a cup of coffee is 0.9. If the bus is not on time the probability that Sam gets a cup of coffee is 0.3.

(i) Find the probability that Sam gets a cup of coffee. [2]

(ii) Given that Sam does not get a cup of coffee, find the probability that the bus is not on time. [3]

4 A box contains 2 green sweets and 5 blue sweets. Two sweets are taken at random from the box, without replacement. The random variable X is the number of green sweets taken. Find $E(X)$ and $Var(X)$. [6]

5 Plastic drinking straws are manufactured to fit into drinks cartons which have a hole in the top. A straw fits into the hole if the diameter of the straw is less than 3 mm. The diameters of the straws have a normal distribution with mean 2.6 mm and standard deviation 0.25 mm.

(i) A straw is chosen at random. Find the probability that it fits into the hole in a drinks carton. [3]

(ii) 500 straws are chosen at random. Use a suitable approximation to find the probability that at least 480 straws fit into the holes in drinks cartons. [5]

(iii) Justify the use of your approximation. [1]

6 **(a)** **(i)** Find how many numbers there are between 100 and 999 in which all three digits are different. [3]

(ii) Find how many of the numbers in part **(i)** are odd numbers greater than 700. [4]

(b) A bunch of flowers consists of a mixture of roses, tulips and daffodils. Tom orders a bunch of 7 flowers from a shop to give to a friend. There must be at least 2 of each type of flower. The shop has 6 roses, 5 tulips and 4 daffodils, all different from each other. Find the number of different bunches of flowers that are possible. [4]

7 The amounts spent by 160 shoppers at a supermarket are summarised in the following table.

Amount spent ($\$x$)	$0 < x \leqslant 30$	$30 < x \leqslant 50$	$50 < x \leqslant 70$	$70 < x \leqslant 90$	$90 < x \leqslant 140$
Number of shoppers	16	40	48	26	30

(i) Draw a cumulative frequency graph of this distribution. [4]

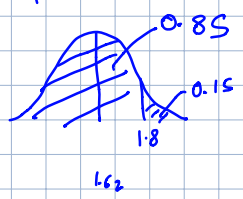
(ii) Estimate the median and the interquartile range of the amount spent. [3]

(iii) Estimate the number of shoppers who spent more than \$115. [2]

(iv) Calculate an estimate of the mean amount spent. [2]

1 The height of maize plants in Mpapwa is normally distributed with mean 1.62 m and standard deviation σ m. The probability that a randomly chosen plant has a height greater than 1.8 m is 0.15. Find the value of σ. [3]

$X \sim N(1.62, \sigma^2)$



$\Phi\left(\frac{1.8 - 1.62}{\sigma}\right) = 0.85$

$\frac{0.18}{\sigma} = 1.0365$

$\sigma = 0.17366$

≈ 0.174

2 The faces of a biased die are numbered 1, 2, 3, 4, 5 and 6. The random variable X is the score when the die is thrown. The following is the probability distribution table for X.

x	1	2	3	4	5	6
$P(X = x)$	p	p	p	p	0.2	0.2

The die is thrown 3 times. Find the probability that the score is 4 on not more than 1 of the 3 throws. [5]

$4p + 0.4 = 1$

$p = 0.15$

$P(x \leq 1) = P(x = 0, 1)$

$X \sim B(3, 0.15)$

$x = 0: \ {}^3C_0 \times 0.15^0 \times 0.85^3 = 0.614125$

$x = 1: \quad = 0.325125$

0.939250

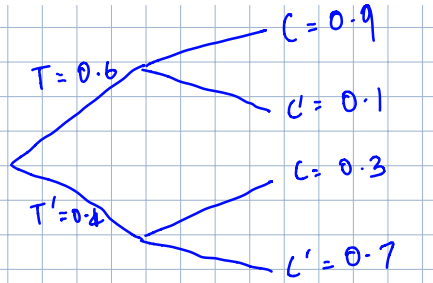
0.939

3 The probability that the school bus is on time on any particular day is 0.6. If the bus is on time the probability that Sam the driver gets a cup of coffee is 0.9. If the bus is not on time the probability that Sam gets a cup of coffee is 0.3.

(i) Find the probability that Sam gets a cup of coffee. [2]

(ii) Given that Sam does not get a cup of coffee, find the probability that the bus is not on time. [3]

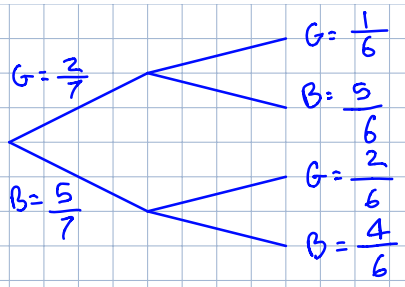
i)



$(0.6 \times 0.9) + (0.4 \times 0.3) = 0.66$

ii) $P(T' \mid C') = \dfrac{P(T' \cap C')}{P(C')} = \dfrac{0.4 \times 0.7}{1 - 0.66} = 0.824$

4 A box contains 2 green sweets and 5 blue sweets. Two sweets are taken at random from the box, without replacement. The random variable X is the number of green sweets taken. Find E(X) and Var(X). [6]



x	0	1	2
$P(X=x)$	$\frac{10}{21}$	$\frac{10}{21}$	$\frac{1}{21}$

0: $\frac{10}{21}$

1: $\frac{10}{21}$

$E(x) = \frac{10}{21} + \frac{2}{21} = \frac{12}{21}$

$Var(x) = E(x^2) - E(x)^2$

$= \frac{14}{21} - \left(\frac{12}{21}\right)^2 = \frac{50}{147} = 0.340$

5 Plastic drinking straws are manufactured to fit into drinks cartons which have a hole in the top. A straw fits into the hole if the diameter of the straw is less than 3 mm. The diameters of the straws have a normal distribution with mean 2.6 mm and standard deviation 0.25 mm.

(i) A straw is chosen at random. Find the probability that it fits into the hole in a drinks carton. [3]

(ii) 500 straws are chosen at random. Use a suitable approximation to find the probability that at least 480 straws fit into the holes in drinks cartons. [5]

(iii) Justify the use of your approximation. [1]

i) 

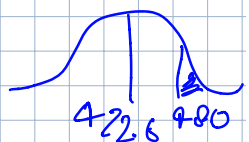
$\Phi\left(\frac{3-2.6}{0.25}\right)$ (2.6)

$\Phi(1.6) = 0.9452$

$= 0.945$

ii) $X \sim B(500, 0.945)$

$X \sim N(472.6,\ 25.89848)$



$\Phi\left(\frac{479.5 - 472.6}{\sqrt{25.89848}}\right)$

$\Phi(1.35589) = 0.9125$, $1 - = 0.0875$

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(ii) Find how many of the numbers in part **(i)** are odd numbers greater than 700. [4]

(b) A bunch of flowers consists of a mixture of roses, tulips and daffodils. Tom orders a bunch of 7 flowers from a shop to give to a friend. There must be at least 2 of each type of flower. The shop has 6 roses, 5 tulips and 4 daffodils, all different from each other. Find the number of different bunches of flowers that are possible. [4]

ai) 0 1 2 3 4 5 6 7 8 9

$9 \times 9 \times 8 = 648$

1 2 3
1 3 2
1 3 4
1

$^6C_2 \times {}^5C_2 \times {}^4C_2$

2 3 2 : $^6C_2\ {}^5C_3\ {}^4C_2 = 900$
2 2 3 : $= 600$
3 2 2 : $= 1200$
2700

ii)

7 _ _ _ _ _ _ _

1 3 5 9

$4 \times 8 = 32$

8 _ _ _ _ _ _ _

5×8

1 3 5 7 9

9 _ _ _ _ _ _ _

1 3 5 7 9

$4 \times 8 = 32$

$\therefore$ 104 ways

iii)

7 The amounts spent by 160 shoppers at a supermarket are summarised in the following table.

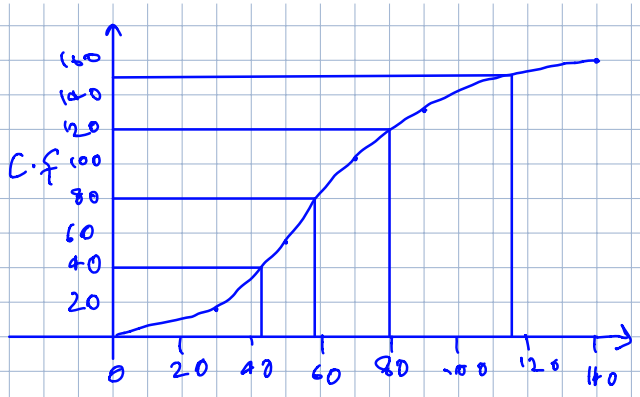
Amount spent ($\$x$)	$0 < x \leqslant 30$	$30 < x \leqslant 50$	$50 < x \leqslant 70$	$70 < x \leqslant 90$	$90 < x \leqslant 140$
Number of shoppers	16	40	48	26	30
	16	56	104	130	160

(i) Draw a cumulative frequency graph of this distribution. [4]

(ii) Estimate the median and the interquartile range of the amount spent. [3]

(iii) Estimate the number of shoppers who spent more than \$115. [2]

(iv) Calculate an estimate of the mean amount spent. [2]



(i) median = 57.5
LQ = 43
UQ = 80
IQ = 37

(ii) 10

(iv)

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