Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

20/75

CANDIDATE NAME Fuzail

CENTRE NUMBER

CANDIDATE NUMBER

* 9 0 3 5 1 3 2 6 9 3 *

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2017**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **19** printed pages and **1** blank page.

CAMBRIDGE International Examinations

[Turn over

1 Solve the equation $\ln(x^2 + 1) = 1 + 2\ln x$, giving your answer correct to 3 significant figures. [3]

$\ln(x^2+1) - \ln x^2 = 1$

$\ln \frac{x^2+1}{x^2} = 1$

$\frac{x^2+1}{x^2} = e$

$x^2+1 = ex^2$

$ex^2 - x^2 = 1$

$x^2(e-1) = 1$

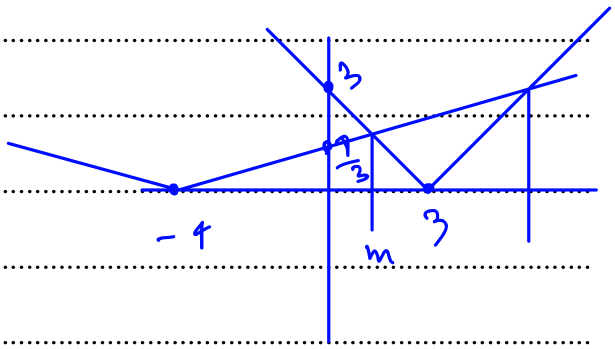
$x = \sqrt{\frac{1}{e-1}}$

$x = 0.763$

2 Solve the inequality $|x - 3| < 3x - 4$. [4]

$3x - 4 = 0$

$x = \frac{4}{3}$



$\frac{\left(\frac{4}{3}\right)}{4}x + \frac{4}{3} = -x + 3$

$\frac{1}{3}x + \frac{4}{3} = -x + 3$

$\frac{4}{3}x = \frac{5}{3}$

$x > \frac{5}{4}$

$\frac{1}{3}x + \frac{4}{3} = x - 3$

$\frac{13}{3} = \frac{4}{3}x$

$x < \frac{13}{4}$

$\frac{5}{4} < x < \frac{13}{4}$

3 **(i)** Express the equation $\cot\theta - 2\tan\theta = \sin 2\theta$ in the form $a\cos^4\theta + b\cos^2\theta + c = 0$, where a, b and c are constants to be determined. [3]

$$\frac{\cos\theta}{\sin\theta} - 2\frac{\sin\theta}{\cos\theta} = 2\sin\theta\cos\theta$$

$$\frac{\cos^2\theta - 2\sin^2\theta}{\sin\theta\cos\theta} = 2\sin\theta\cos\theta$$

$$\cos^2\theta - 2\sin^2\theta = 2\sin^2\theta\cos^2\theta$$

$$2(1-\cos^2\theta)\cos^2\theta = \cos^2\theta - 2(1-\cos^2\theta)$$

$$2\cancel{\cos^2\theta} - 2\cos^4\theta = \cos^2\theta - 2 + \cancel{2\cos^2\theta}$$

$$2\cos^4\theta + \cos^2\theta - 2$$

(ii) Hence solve the equation $\cot\theta - 2\tan\theta = \sin 2\theta$ for $90° < \theta < 180°$. [2]

$2\cos^4\theta + \cos^2\theta - 2$

$x_{1,2} = \dfrac{-1 \pm \sqrt{1 - 4(2)(-2)}}{4}$

$\cos^2\theta = \dfrac{-1 \pm \sqrt{17}}{4}$

$\cos\theta = \pm\sqrt{\dfrac{-1-\sqrt{17}}{4}}$ or $\pm\sqrt{\dfrac{-1+\sqrt{17}}{4}}$

$\times$

$\theta = 27.918$ or 152.082

$152.1°$

4 The parametric equations of a curve are

$$x = t^2 + 1, \quad y = 4t + \ln(2t - 1).$$

(i) Express $\frac{dy}{dx}$ in terms of t. [3]

correct diff

$\frac{dy}{d\theta} = 4 + \frac{1}{2t-1}$ (2)

$= 4 + \frac{2}{2t-1} = \frac{4(2t-1)+2}{2t-1} = \frac{8t-2}{2t-1}$

$\frac{dx}{d\theta} = 2t \quad \therefore \quad \frac{d\theta}{dx} = \frac{1}{2t}$

$\frac{8t-2}{2t-1} \times \frac{1}{2t}$

$\frac{8t-2}{4t^2-2t} = \frac{\cancel{2}(4t-1)}{\cancel{2}(2t^2-t)} = \frac{4t-1}{2t^2-t}$

(ii) Find the equation of the normal to the curve at the point where $t = 1$. Give your answer in the form $ax + by + c = 0$. [3]

$\frac{4(1)-1}{2(1)-1} = \frac{3}{1} = 3$

$\therefore$ normal: $-\frac{1}{3}$

finding x and y

$x = (1)+1 = 2$

$y = 4 + \ln 1 = 4$

$y = -\frac{1}{3}x + c$

$4 = -\frac{1}{3}(2) + c$

$4 = \frac{-2}{3} + c$

$4 + \frac{2}{3} = c$

$c = \frac{14}{3}$

$y = -\frac{1}{3}x + \frac{14}{3}$

$3y + x - 14 = 0$

5 In a certain chemical process a substance A reacts with and reduces a substance B. The masses of A and B at time t after the start of the process are x and y respectively. It is given that $\frac{dy}{dt} = -0.2xy$ and $x = \frac{10}{(1+t)^2}$. At the beginning of the process $y = 100$.

(i) Form a differential equation in y and t, and solve this differential equation. [6]

$A + B = k$

$$\int \frac{1}{y}\,dy = -0.2\int\left(\frac{10}{(1+t)^2}\right) dt$$

$$\ln y = -2\int (1+t)^{-2}\,dt$$

$$\ln y = \cancel{+}2\left(\frac{(1+t)^{-1}}{\cancel{-}1\,(1)}\right)$$

$$\ln y = \frac{2}{(1+t)} + C$$

$$\ln 100 = \frac{2}{1} + c$$

$$\ln(100) - 2 = c$$

$$\ln y = \frac{2}{1+t} + \ln 100 - 2$$

(ii) Find the exact value approached by the mass of B as t becomes large. State what happens to the mass of A as t becomes large. [2]

mass of B = $y = e^{\left(\frac{2}{1+\infty} + \ln 100 - 2\right)}$

$y = e^{0 + \ln 100 - 2}$

$= \frac{e^{\ln 100}}{e^2} = \frac{100}{e^2}$

As t becomes larger, mass of A tends to 0

6 Throughout this question the use of a calculator is not permitted.

The complex number $2 - \mathrm{i}$ is denoted by u.

(i) It is given that u is a root of the equation $x^3 + ax^2 - 3x + b = 0$, where the constants a and b are real. Find the values of a and b. [4]

$(2-i)^2 = 4 - 4i + i^2 = 3 - 4i$

$\therefore u^3 = (3-4i)(2-i) = 6 - 3i - 8i + 4i^2$

$= 6 - 11i - 4$

$2 - 11i$

$2 - 11i + a(3-4i) - 3(2-i) + b = 0$

$2 - 11i + 3a - 4ai - 6 + 3i + b = 0$

$-4 - 8i + 3a - 4ai + b = 0$

$-8\cancel{i} - 4a\cancel{i} = 0$

$-4a = 8$

$a = -2$

$-4 + 3(-2) + b = 0$

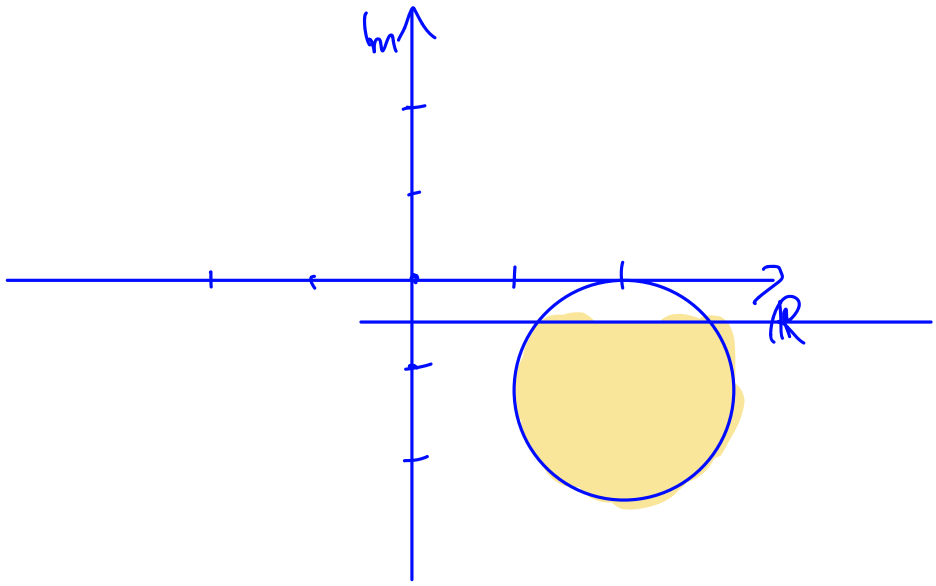
$-4 - 6 + b = 0$

$b = 10$

(ii) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying both the inequalities $|z - u| < 1$ and $|z| < |z + i|$. [4]

$z-(2-i)$ $z-(0-i)$

3



7 (i) Prove that if $y = \dfrac{1}{\cos\theta}$ then $\dfrac{dy}{d\theta} = \sec\theta\tan\theta$. [2]

$\dfrac{dy}{dx} = \dfrac{\cos\theta(0) - 1(-\sin\theta)}{\cos^2\theta}$

$= \dfrac{\sin\theta}{\cos^2\theta} = \dfrac{1}{\cos\theta}\tan\theta = \sec\theta\tan\theta$

(ii) Prove the identity $\dfrac{1+\sin\theta}{1-\sin\theta} \equiv 2\sec^2\theta + 2\sec\theta\tan\theta - 1$. [3]

$\dfrac{(1+\sin\theta)(1+\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}$

$\dfrac{1+2\sin\theta+\sin^2\theta}{1-\sin^2\theta}$

$\dfrac{1+2\sin\theta+1-\cos^2\theta}{1-(1-\cos^2\theta)}$

$\dfrac{2+2\sin\theta-\cos^2\theta}{\cos^2\theta}$

$2\left(\dfrac{1}{\cos^2\theta}\right) + \dfrac{2\sin\theta}{\cos\theta}\left(\dfrac{1}{\cos\theta}\right) - \dfrac{\cos^2}{\cos^2\theta}$

$2\sec^2\theta + 2\tan\theta\sec\theta - 1$

(iii) Hence find the exact value of $\displaystyle\int_0^{\frac{1}{4}\pi} \frac{1+\sin\theta}{1-\sin\theta}\,d\theta$. [4]

$$\int_0^{\frac{1}{4}\pi} 2\sec^2\theta + 2\tan\theta\sec\theta - 1\,d\theta$$

$$2\tan\theta + 2\sec\theta - \theta$$

$$2\tan\tfrac{1}{4}\pi + 2\sec\tfrac{1}{4}\pi - \tfrac{1}{4}\pi \qquad 0 + 2 - 0$$

$$2 + 2\sqrt{2} - \tfrac{1}{4}\pi$$

$$\cancel{2} + 2\sqrt{2} - \tfrac{1}{4}\pi - \cancel{2}$$

$$2\sqrt{2} - \tfrac{1}{4}\pi$$

8 Let $f(x) = \dfrac{5x^2 - 7x + 4}{(3x+2)(x^2+5)} = \dfrac{A}{3x+2} + \dfrac{Bx+C}{x^2+5}$

(i) Express f(x) in partial fractions. [5]

$5x^2 - 7x + 4 = A(x^2+5) + (Bx+C)(3x+2)$

$Ax^2 + 5A + 3Bx^2 + 2Bx + 3Cx + 2C$ $(B+C)(5)$

$5 = A + 3B \rightarrow$

$-7 = 2B + 3C \rightarrow \dfrac{-7-3C}{2} = B$

$4 = 5A + 2C \rightarrow \dfrac{4-5A}{2} = C$

$5 = A + 3\left(\dfrac{-7 - 3\left(\frac{4-5A}{2}\right)}{2}\right)$

$5 = A + 3\left(\dfrac{-7 - \frac{12+15A}{2}}{2}\right)$

$5 = A + 3\left(\dfrac{\frac{-14-12+15A}{2}}{2}\right)$

$5 = A + 3\left(\dfrac{-26+15A}{4}\right)$

$5 = A - \dfrac{78+45A}{4}$

$20 = 4A - 78 + 45A$

$20 = 49A - 78$

$98 = 49A$

$A = 2$

$\therefore \quad B = \dfrac{5-2}{3} = 1$

$C = \dfrac{4-10}{2} = -3$

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

$2(3x+2)^{-1} \quad + \quad (x-3)(x^2+5)^{-1}$

$1\left(1+\frac{3x}{2}\right)^{-1}$

$1+(-1)\left(\frac{3}{2}x\right)+\frac{(-1)(-2)\left(\frac{3}{2}x\right)^2}{2!}$

$1-\frac{3}{2}x+\frac{9}{4}x^2$

$(x-3)\times 5^{-1}\left(1+\frac{x^2}{5}\right)^{-1}$

$(x-3)\times\frac{1}{5}\left(1-\frac{x^2}{5}\right) \Rightarrow (x-3)\left(\frac{1}{5}-\frac{x^2}{25}\right)$

$=\frac{x}{5}-\frac{x^3}{25}-\frac{3}{5}+\frac{3x^2}{25}$

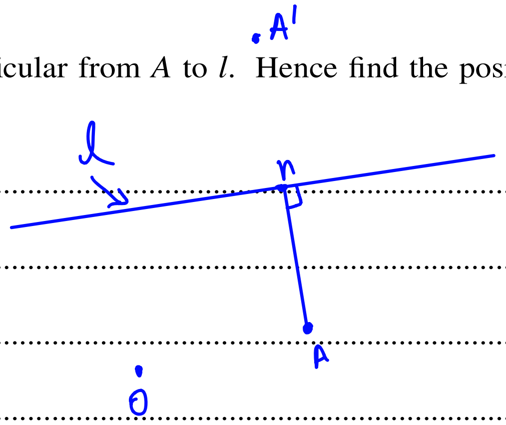
$1-\frac{3}{2}x+\frac{9}{4}x^2+\frac{x}{5}-\frac{x^3}{25}-\frac{3}{5}+\frac{3x^2}{25}$

$\frac{2}{5}-\frac{13x}{10}+\frac{237}{100}x^2$

9 Relative to the origin O, the point A has position vector given by $\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$. The line l has equation $\mathbf{r} = 9\mathbf{i} - \mathbf{j} + 8\mathbf{k} + \mu(3\mathbf{i} - \mathbf{j} + 2\mathbf{k})$.

(i) Find the position vector of the foot of the perpendicular from A to l. Hence find the position vector of the reflection of A in l. [5]

4



$AN = ON - OA$

$= \begin{pmatrix} 9+3\mu \\ -1-\mu \\ 8+2\mu \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$

$= \begin{pmatrix} 8+3\mu \\ -3-\mu \\ 4+2\mu \end{pmatrix}$

$\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 8+3\mu \\ -3-\mu \\ 4+2\mu \end{pmatrix} = 0$

$3(8+3\mu) + (-1)(-3-\mu) + 2(4+2\mu) = 0$

$24 + 9\mu + 3 + \mu + 8 + 4\mu = 0$

$14\mu + 35 = 0$

$\mu = -\frac{5}{2}$

$\overrightarrow{ON} = \begin{pmatrix} 9+3(-2.5) \\ -1-1(-2.5) \\ 8+2(-2.5) \end{pmatrix} = \begin{pmatrix} 1.5 \\ 1.5 \\ 3 \end{pmatrix}$ ✓

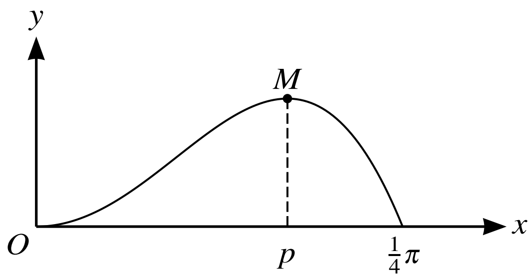
$2AN$ = reflection of A: $2\begin{pmatrix} 8+3(-2.5) \\ -3-1(-2.5) \\ 4+2(-2.5) \end{pmatrix} = 2\begin{pmatrix} 0.5 \\ -0.5 \\ -1 \end{pmatrix}$

$= \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$

(ii) Find the equation of the plane through the origin which contains l. Give your answer in the form $ax + by + cz = d$. [3]

(iii) Find the exact value of the perpendicular distance of A from this plane. [3]

10



The diagram shows the curve $y = x^2 \cos 2x$ for $0 \leqslant x \leqslant \frac{1}{4}\pi$. The curve has a maximum point at M where $x = p$.

(i) Show that p satisfies the equation $p = \frac{1}{2}\tan^{-1}\left(\frac{1}{p}\right)$. [3]

$\frac{dy}{dx} = 2x\cos 2x + (-2\sin 2x)(x^2) = 0$

$2p\cos 2p - 2p^2\sin 2p$

$2p\cos 2p = 2p^2\sin 2p$

$1 = p\tan 2p$

$\tan 2p = \frac{1}{p}$

$2p = \tan^{-1}\left(\frac{1}{p}\right)$

$\therefore\ p = \frac{1}{2}\tan^{-1}\left(\frac{1}{p}\right)$

(ii) Use the iterative formula $p_{n+1} = \frac{1}{2}\tan^{-1}\left(\frac{1}{p_n}\right)$ to determine the value of p correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Let $p_1 = \frac{1}{5}\pi$

$\therefore P_2 = \frac{1}{2}\tan\left(\frac{1}{\frac{1}{5}\pi}\right) = 0.5049$

$P_3 = 0.5516$

$P_4 = 0.5334$

$P_5 = 0.5404$

$P_6 = 0.5376$

$P_7 = 0.5387$

$\therefore P = 0.54$

(iii) Find, showing all necessary working, the exact area of the region bounded by the curve and the x-axis. [5]

$$\int_0^{\frac{1}{4}\pi} \underbrace{x^2}_{u} \underbrace{\cos 2x}_{v'} \, dx$$

$u = x^2 \qquad u' = 2x$

$v = \frac{1}{2}\sin 2x \qquad v' = \cos 2x$

$(x^2)\left(\frac{1}{2}\sin 2x\right) - \int \cancel{2}x \times \frac{1}{\cancel{2}}\sin 2x \, dx$

$\frac{x^2 \sin 2x}{2} - \int x \sin 2x \, dx$

$u = x \qquad u' = 1$

$v = -\frac{1}{2}\cos 2x \qquad v' = \sin 2x$

$- \left[(x)\left(-\frac{1}{2}\cos 2x\right) - \int -\frac{1}{2}\cos 2x \, dx\right]$

$- \left[\frac{-x\cos 2x}{2} + \frac{1}{2}\left(\frac{1}{2}\sin 2x\right)\right]$

$\frac{1}{2}x^2 \sin 2x + \frac{1}{2}x\cos 2x - \frac{1}{4}\sin 2x$

$\frac{1}{2}\left(\frac{1}{4}\pi\right)^2(1) \quad \frac{1}{2}\left(\frac{1}{4}\pi\right)\cos\frac{1}{2}\pi - \frac{1}{4}$

$\frac{\pi^2}{32} - \frac{1}{4}$

$= \frac{1}{32}(\pi^2 - 8)$

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