1 On average 1 in 20 members of the population of this country has a particular DNA feature. Members of the population are selected at random until one is found who has this feature.

(i) Find the probability that the first person to have this feature is

(a) the sixth person selected, [3]

(b) not among the first 10 people selected. [3]

(ii) Find the expected number of people selected. [2]

(Q5, Jan 2005)

i a) $p = 0.05 \quad q = 0.95$

$P(X=6) = 0.95^5 \times 0.05$
$= 0.038689$
≈ 0.039

b) $P(X > 10) = 0.95^{10}$
$= 0.599$

ii) $\mu = \frac{1}{p} = \frac{1}{0.05} = 20$

2 It is known that, on average, one match box in 10 contains fewer than 42 matches. Eight boxes are selected, and the number of boxes that contain fewer than 42 matches is denoted by Y.

(i) State two conditions needed to model Y by a binomial distribution. [2]

Assume now that a binomial model is valid.

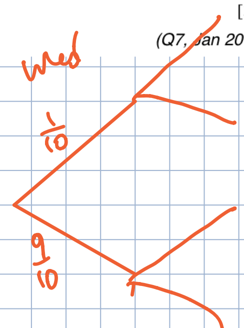
(ii) Find

(a) $P(Y = 0)$, [2]

(b) $P(Y \geqslant 2)$. [2]

(iii) On Wednesday 8 boxes are selected, and on Thursday another 8 boxes are selected. Find the probability that on one of these days the number of boxes containing fewer than 42 matches is 0, and that on the other day the number is 2 or more. [3]

(Q7, Jan 2005)



i)

ii) a) $X \sim B(8, \frac{1}{10})$

${}^8C_0 \times \left(\frac{1}{10}\right)^0 \times \left(\frac{9}{10}\right)^8$

$= 0.4305$

b) $P(Y \geqslant 2)$

$= P(Y > 1)$

$1 - P(Y \leq 1)$

$1 - \left[\left({}^8C_1 \times \left(\frac{1}{10}\right)^1 \times \left(\frac{9}{10}\right)^7\right) + 0.4305\right] = 0.187$

ii) $2 \times 0.4305 \times 0.187 = 0.161$

?

3 The probability that a certain sample of radioactive material emits an alpha-particle in one unit of time is 0.14. In one unit of time no more than one alpha-particle can be emitted. The number of units of time up to and including the first in which an alpha-particle is emitted is denoted by T.

(i) Find the value of

(a) $P(T = 5)$, [3]

(b) $P(T < 8)$. [3]

(ii) State the value of $E(T)$. [2]

(Q2, June 2005)

3(i)(a) $0.86^4 \times 0.14^1 = 0.0766$

(b) $1 - 0.86^7 = 0.652$

ii) $\frac{1}{0.14} = 7.14$

4 In a supermarket the proportion of shoppers who buy washing powder is denoted by p. 16 shoppers are selected at random.

(i) Given that $p = 0.35$, use tables to find the probability that the number of shoppers who buy washing powder is

(a) at least 8, [3]

(b) between 4 and 9 inclusive. [2]

(ii) Given instead that $p = 0.38$, find the probability that the number of shoppers who buy washing powder is exactly 6. [3]

(Q3, June 2005)

$^{16}C_6 \times 0.38^6 \times 0.62^{10}$

$= 0.202$

5 **(i)** The random variable X has the distribution $\mathrm{B}(25, 0.2)$. Using the tables of cumulative binomial probabilities, or otherwise, find $\mathrm{P}(X \geqslant 5)$. [2]

(ii) The random variable Y has the distribution $\mathrm{B}(10, 0.27)$. Find $\mathrm{P}(Y = 3)$. [2]

(iii) The random variable Z has the distribution $\mathrm{B}(n, 0.27)$. Find the smallest value of n such that $\mathrm{P}(Z \geqslant 1) > 0.95$. [3]

(Q4, June 2006)

i)

iii) $P(X \geq 0)$

$1 - p(x = 0)$

$1 - 0.73^n > 0.95$

$0.73^n < 0.05$

$n \ln 0.73 < \ln 0.05$

$n > \dfrac{\ln 0.05}{\ln 0.73}$

$n > 9.5\ldots$

$n = 10$

6 Henry makes repeated attempts to light his gas fire. He makes the modelling assumption that the probability that the fire will light on any attempt is $\frac{1}{3}$.

Let X be the number of attempts at lighting the fire, up to and including the successful attempt.

(i) Name the distribution of X, stating a further modelling assumption needed. [2]

In the rest of this question, you should use the distribution named in part **(i)**.

(ii) Calculate

(a) $\mathrm{P}(X = 4)$, [3]

(b) $\mathrm{P}(X < 4)$. [3]

(iii) State the value of $\mathrm{E}(X)$. [1]

(iv) Henry has to light the fire once a day, starting on March 1st. Calculate the probability that the first day on which fewer than 4 attempts are needed to light the fire is March 3rd. [3]

(Q8, June 2006)

6i) Geometric

ii) a) $\left(\frac{2}{3}\right)^3 \times \left(\frac{1}{3}\right) = \frac{8}{81} = 0.0988$

b) $1 - \left(\frac{2}{3}\right)^3 = 0.704$

iii) $\dfrac{1}{\frac{1}{3}} = 3$

iv)

2 An ordinary fair die is thrown until a 6 is obtained. $\frac{1}{6}$

(a) Find the probability that obtaining a 6 takes more than 8 throws. [2]

$p(x>8)$

$\left(\frac{5}{6}\right)^8 = 0.335$

Two ordinary fair dice are thrown together until a pair of 6s is obtained. The number of throws taken is denoted by the random variable X.

(b) Find the expected value of X. [1]

$\frac{1}{\left(\frac{1}{6}\right)^2}$

(c) Find the probability that obtaining a pair of 6s takes either 10 or 11 throws. [2]

$\frac{1}{36}$

$\frac{35}{36}$

$\left(\frac{35}{36}\right)^9\left(\frac{1}{36}\right) + \left(\frac{35}{36}\right)^{10}\left(\frac{1}{36}\right)$

0.0425

7 A coin is biased so that the probability that it will show heads on any throw is $\frac{2}{3}$. The coin is thrown repeatedly.

The number of throws up to and including the first head is denoted by X. Find

(i) $P(X = 4)$, [3]

(ii) $P(X < 4)$, [3]

(iii) $E(X)$. [2]

(Q6, Jan 2007)

i) $\left(\frac{1}{3}\right)^3 \frac{2}{3} = 0.0247$

ii) $\frac{2}{3} + \left(\frac{1}{3}\right)\left(\frac{2}{3}\right) + \left(\frac{1}{3}\right)^2 \times \frac{2}{3} = 0.963$

$= 0.321$

iii) $\frac{1}{\left(\frac{2}{3}\right)} = \frac{3}{2}$ ✓

8 A variable X has the distribution $B(11, p)$.

(i) Given that $p = \frac{3}{4}$, find $P(X = 5)$. [2]

(ii) Given that $P(X = 0) = 0.05$, find p. [4]

(iii) Given that $Var(X) = 1.76$, find the two possible values of p. [5]

(Q9, Jan 2007)

i) ${}^{11}C_5 \times 0.75^5 \times 0.25^6 = 0.026766$

≈ 0.0268

ii) ${}^{11}\cancel{C}_0 \times p^0 \times (1-p)^{11} = 0.05$

$1 - p = \sqrt[11]{0.05}$

$p = 1 - \sqrt[11]{0.05}$

$= 0.238$

iii)

9 On average, 25% of the packets of a certain kind of soup contain a voucher. Kim buys one packet of soup each week for 12 weeks. The number of vouchers she obtains is denoted by X.

(i) State two conditions needed for X to be modelled by the distribution $B(12, 0.25)$. [2]

In the rest of this question you should assume that these conditions are satisfied.

(ii) Find $P(X \leqslant 6)$. [2]

In order to claim a free gift, 7 vouchers are needed.

(iii) Find the probability that Kim will be able to claim a free gift at some time during the 12 weeks. [1]

(iv) Find the probability that Kim will be able to claim a free gift in the 12th week but not before. [4]

(Q7, June 2007)

i). • The probability that a packet contains a voucher is same of every packet
• The trials are independent.

ii) x x

iii)