

MATHEMATICS

9709/61

Paper 6 Probability & Statistics 1 (S1)

October/November 2015

1 hour 15 minutes

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

An answer booklet and a graph paper booklet are provided inside this question paper. You should follow the instructions on the front cover of both booklets. If you need additional answer paper or graph paper ask the invigilator for a continuation booklet or graph paper booklet.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

DO NOT WRITE IN ANY BARCODES.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 50.

This document consists of **3** printed pages, **1** blank page and **2** inserts.

- 1 In a certain town, 76% of cars are fitted with satellite navigation equipment. A random sample of 11 cars from this town is chosen. Find the probability that fewer than 10 of these cars are fitted with this equipment. [4]
- 2 The random variable X has the distribution $N(\mu, \sigma^2)$. It is given that $P(X < 54.1) = 0.5$ and $P(X > 50.9) = 0.8665$. Find the values of μ and σ . [4]
- 3 Robert has a part-time job delivering newspapers. On a number of days he noted the time, correct to the nearest minute, that it took him to do his job. Robert used his results to draw up the following table; two of the values in the table are denoted by a and b .

Time (t minutes)	60 – 62	63 – 64	65 – 67	68 – 71
Frequency (number of days)	3	9	6	b
Frequency density	1	a	2	1.5

- (i) Find the values of a and b . [3]
- (ii) On graph paper, draw a histogram to represent Robert's times. [3]
- 4 (a) Amy measured her pulse rate while resting, x beats per minute, at the same time each day on 30 days. The results are summarised below.

$$\Sigma(x - 80) = -147 \qquad \Sigma(x - 80)^2 = 952$$

Find the mean and standard deviation of Amy's pulse rate. [4]

- (b) Amy's friend Marok measured her pulse rate every day after running for half an hour. Marok's pulse rate, in beats per minute, was found to have a mean of 148.6 and a standard deviation of 18.5. Assuming that pulse rates have a normal distribution, find what proportion of Marok's pulse rates, after running for half an hour, were above 160 beats per minute. [3]
- 5 (a) Find the number of ways in which all nine letters of the word TENNESSEE can be arranged
- (i) if all the letters E are together, [3]
- (ii) if the T is at one end and there is an S at the other end. [3]
- (b) Four letters are selected from the nine letters of the word VENEZUELA. Find the number of possible selections which contain exactly one E. [3]

- 6** Nadia is very forgetful. Every time she logs in to her online bank she only has a 40% chance of remembering her password correctly. She is allowed 3 unsuccessful attempts on any one day and then the bank will not let her try again until the next day.

(i) Draw a fully labelled tree diagram to illustrate this situation. [3]

(ii) Let X be the number of unsuccessful attempts Nadia makes on any day that she tries to log in to her bank. Copy and complete the following table to show the probability distribution of X . [4]

x	0	1	2	3
$P(X = x)$		0.24		

(iii) Calculate the expected number of unsuccessful attempts made by Nadia on any day that she tries to log in. [2]

- 7** The faces of a biased die are numbered 1, 2, 3, 4, 5 and 6. The probabilities of throwing odd numbers are all the same. The probabilities of throwing even numbers are all the same. The probability of throwing an odd number is twice the probability of throwing an even number.

(i) Find the probability of throwing a 3. [3]

(ii) The die is thrown three times. Find the probability of throwing two 5s and one 4. [3]

(iii) The die is thrown 100 times. Use an approximation to find the probability that an even number is thrown at most 37 times. [5]

- 1 In a certain town, 76% of cars are fitted with satellite navigation equipment. A random sample of 11 cars from this town is chosen. Find the probability that fewer than 10 of these cars are fitted with this equipment. [4]

let x be the r.v. "cars fitted with this equipment"

$$x \sim B(11, 0.76)$$

$$P(X < 10) = 1 - [P(X=11) + P(X=10)]$$

$$P(X=11) = {}^{11}C_{11} \times 0.76^{11} = 0.048859$$

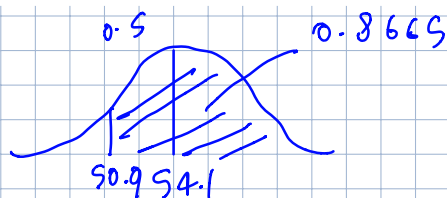
$$P(X=10) = 0.16922$$

$$1 - (0.16922 + 0.048859)$$

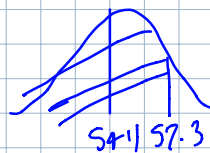
$$= 0.781921$$

$$= 0.782$$

- 2 The random variable X has the distribution $N(\mu, \sigma^2)$. It is given that $P(X < 54.1) = 0.5$ and $P(X > 50.9) = 0.8665$. Find the values of μ and σ . [4]



$$\mu = 54.1$$



$$P\left(\frac{X - \mu}{\sigma} > \frac{50.9 - 54.1}{\sigma}\right) = 0.8665$$

$$\frac{3.2}{\sigma} = 1.1$$

$$\sigma = 2.88$$

- 3 Robert has a part-time job delivering newspapers. On a number of days he noted the time, correct to the nearest minute, that it took him to do his job. Robert used his results to draw up the following table; two of the values in the table are denoted by a and b .

Time (t minutes)	60 – 62	63 – 64	65 – 67	68 – 71
Frequency (number of days)	3	9	6	b
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- (i) Find the values of a and b .

[3]

- (ii) On graph paper, draw a histogram to represent Robert's times.

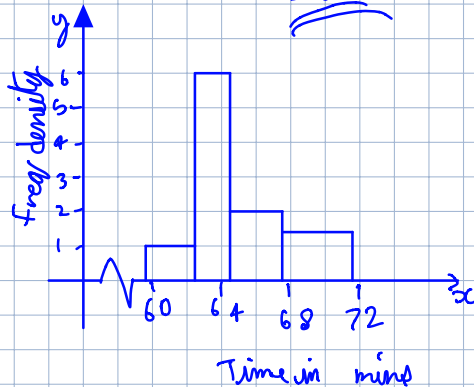
[3]

i) $a = \frac{9}{2} = 4.5$

$b = \frac{6}{1} = 6$

$\frac{6}{4} = 1.5$
 $b = 6$

ii)



- 4 (a) Amy measured her pulse rate while resting, x beats per minute, at the same time each day on 30 days. The results are summarised below.

$$\Sigma(x - 80) = -147$$

$$\Sigma(x - 80)^2 = 952$$

Find the mean and standard deviation of Amy's pulse rate.

[4]

- (b) Amy's friend Marok measured her pulse rate every day after running for half an hour. Marok's pulse rate, in beats per minute, was found to have a mean of 148.6 and a standard deviation of 18.5. Assuming that pulse rates have a normal distribution, find what proportion of Marok's pulse rates, after running for half an hour, were above 160 beats per minute.

[3]

a)

$$\Sigma x - 30 \times 80 = -147$$

$$\Sigma x = 2253 \therefore \text{mean} = \frac{2253}{30} = 75.1$$

$$s = \sqrt{\frac{952}{30} - (75.1)^2}$$

$$= 2.78$$

b) $x \sim N(148.6, 18.5^2)$

$$1 - \Phi\left(\frac{160 - 148.6}{18.5}\right)$$

$$1 - \Phi(0.6162)$$

$$1 - 0.731 = 0.269$$



- 5 (a) Find the number of ways in which all nine letters of the word TENNESSEE can be arranged
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- (ii) if the T is at one end and there is an S at the other end. [3]
- (b) Four letters are selected from the nine letters of the word VENEZUELA. Find the number of possible selections which contain exactly one E. [3]

a) i) $\frac{6!}{2! \times 2!} = \underline{\underline{180}}$ E T N N S S

ii) ~~T~~ E N N E ~~S~~ S E E

$$\frac{7!}{2! \times 4!} + \frac{7!}{2! \times 4!}$$

210 ways

b) E

1 x ${}^6C_3 = \underline{\underline{20}}$

- 6 Nadia is very forgetful. Every time she logs in to her online bank she only has a 40% chance of remembering her password correctly. She is allowed 3 unsuccessful attempts on any one day and then the bank will not let her try again until the next day.

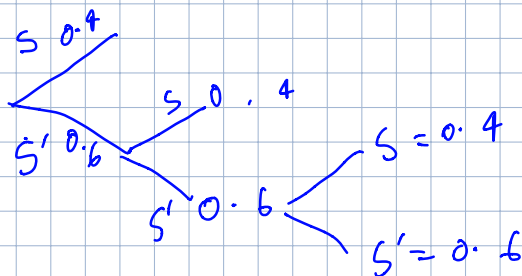
(i) Draw a fully labelled tree diagram to illustrate this situation. [3]

- (ii) Let X be the number of unsuccessful attempts Nadia makes on any day that she tries to log in to her bank. Copy and complete the following table to show the probability distribution of X . [4]

x	0	1	2	3
$P(X = x)$	0.4	0.24	0.144	0.216

- (iii) Calculate the expected number of unsuccessful attempts made by Nadia on any day that she tries to log in. [2]

i)



ii)

$$0.24 + 0.288 + 0.648 = \underline{\underline{1.18}}$$

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(i) Find the probability of throwing a 3. [3]

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i) $p(\text{odd}) = 2 \times p(\text{even})$

$$p(O) + p(E) = 1$$

$$2p(E) + p(E) = 1$$

$$3p(E) = 1$$

$$p(E) = \frac{1}{3}$$

$$p(O) = \frac{2}{3}$$

$$p(1, 2, 3) = \frac{2}{3}$$

$$p(3) = \frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$$

ii) $SS4 = \left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right) \times 3 = 0.0165$

4SS =

SSS =

iii) $X \sim B(100, \frac{1}{3})$

$X \sim N(33.333, 22.222)$



$$\frac{37.5 - 33.3}{\sqrt{22.2}} = 2$$

$$2 = 0.82388$$

$$\Phi(0.82388) = \frac{0.8106}{0.0011}$$

$$0.8117$$

$$0.812$$

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