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Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER | | | | |

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MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2018**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **20** printed pages.

CAMBRIDGE International Examinations

12:21am

1 Find the set of values of x satisfying the inequality $2|2x - a| < |x + 3a|$, where a is a positive constant. [4] 1

$(2|2x-a|)^2 < (x+3a)^2$

$4(2x-a)(2x-a) < (x+3a)(x+3a)$

$4(4x^2-4ax+a^2) < x^2+6a+9a^2$

$16x^2-16ax+4a^2 < x^2+6a+9a^2$

$5a^2+16ax+6a-15x^2 > 0$

$\underbrace{-15x^2}_{a} + \underbrace{16ax}_{b} + \underbrace{6a+5a^2}_{c} > 0$

#

2 Showing all necessary working, solve the equation $\frac{2e^x + e^{-x}}{e^x - e^{-x}} = 4$, giving your answer correct to 2 decimal places. [4]

$$2e^x + \frac{1}{e^x} = 4e^x - \frac{4}{e^x}$$

$$\frac{2e^{2x} + 1}{\cancel{e^x}} = \frac{4e^{2x} - 4}{\cancel{e^x}}$$

$$2e^{2x} = 5$$

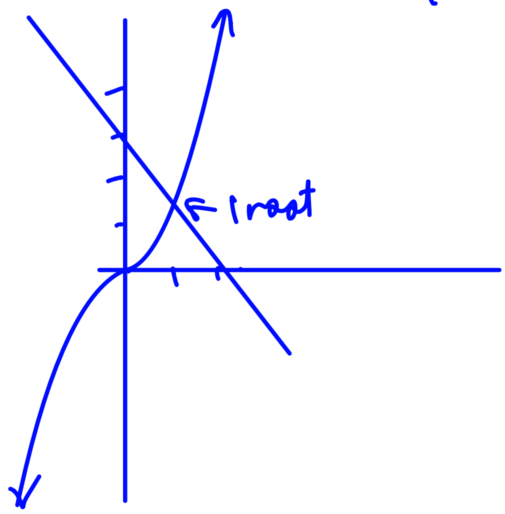
$$e^{2x} = \frac{5}{2}$$

$$2x = \ln\frac{5}{2}$$

$$x = 0.46$$

3 **(i)** By sketching a suitable pair of graphs, show that the equation $x^3 = 3 - x$ has exactly one real root. [2]

	-2	-1	0	1	2	3	4
y	5	4	3	2	1	0	-1



(ii) Show that if a sequence of real values given by the iterative formula

$$x_{n+1} = \frac{2x_n^3 + 3}{3x_n^2 + 1}$$

converges, then it converges to the root of the equation in part **(i)**. [2]

$3x^3 + x = 2x^3 + 3$

$x^3 = 3 - x$

(iii) Use this iterative formula to determine the root correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

$x_1 = 1$

$x_2 = \dfrac{2(1)^3 + 3}{3(1)^2 + 1} = 1.25000$

$x_3 = 1.21428$

$x_4 = 1.21341$

$x_5 = 1.21341$

$x_6 = 1.21341$

x is converges to 1.213

4 The parametric equations of a curve are

$$x = 2\sin\theta + \sin 2\theta, \qquad y = 2\cos\theta + \cos 2\theta,$$

where $0 < \theta < \pi$.

(i) Obtain an expression for $\frac{dy}{dx}$ in terms of θ. [3]

2

$\frac{dx}{d\theta} = 2(\sin\theta)^0(\cos\theta) + 2\cos 2\theta$ $\qquad \frac{dy}{d\theta} = -2\sin\theta - 2\sin 2\theta$

$= 2\cos\theta + 2\cos 2\theta$

$\frac{-\cancel{2}(\sin\theta + \sin 2\theta)}{\cancel{2}(\cos\theta + \cos 2\theta)}$

$= \frac{dy}{dx} = -\frac{\sin\theta + \sin 2\theta}{\cos\theta + \cos 2\theta}$

$-\sin\theta - \sin 2\theta = 0$

$(\sin\theta + \sin 2\theta) = 0$

$-2\sin\theta - 2\sin 2\theta = 0$ $\qquad \sin\theta + 2\sin\theta\cos\theta = 0$

$\cancel{2}\sin\theta = -\cancel{2}\sin 2\theta$ $\qquad \sin\theta(1 + 2\cos\theta) = 0$

$\sin\theta = -\sin 2\theta$ $\qquad 1 + 2\cos\theta = 0$

$\cancel{\sin\theta} = -2\cancel{\sin\theta}\cos\theta$ $\qquad \cos\theta = -\frac{1}{2}$

$1 = -2\cos\theta$

$\cos\theta = -\frac{1}{2}$

(ii) Hence find the exact coordinates of the point on the curve at which the tangent is parallel to the y-axis. [4] 0

$-\frac{1}{\infty} = 0$

$\therefore \frac{dy}{dx} = \frac{0}{0} = \sin\theta + 2\sin\theta\cos\theta$ ✓

$= -\sin\theta(1 + 2\cos\theta) = 0$

$\sin\theta = 0$ | $\cos\theta = -\frac{1}{2}$

$\cancel{\theta = 0}$

$\theta = \frac{2}{3}\pi$

$x = 2\sin\frac{2}{3}\pi + \sin\frac{4\pi}{3}$ $\quad$ $y = 2\cos\frac{2}{3}\pi + \cos\frac{4\pi}{3}$

$\sqrt{3} \quad -\frac{\sqrt{3}}{2}$ $\quad$ $= -1 \quad -\frac{1}{2}$

$= \frac{\sqrt{3}}{2}$ $\quad$ $= -\frac{3}{2}$

$-\sin\theta - 2\sin\theta\cos\theta$

$\sin\theta(-1 - 2\cos\theta)$

$\left(\frac{\sqrt{3}}{2}, -\frac{3}{2}\right)$

5 The coordinates (x, y) of a general point on a curve satisfy the differential equation

$$x\frac{dy}{dx} = (2 - x^2)y.$$

The curve passes through the point $(1, 1)$. Find the equation of the curve, obtaining an expression for y in terms of x. [7]

$$\int \frac{1}{y}\,dy = \int \frac{2}{x} - x \;dx$$

$$\ln y = 2\ln x - \frac{x^2}{2} + C$$

sub(1,2)

$$0 = 0 - \frac{1}{2} + c$$

$$c = \frac{1}{2}$$

$$\ln y = 2\ln x - \frac{x^2}{2} + \frac{1}{2}$$

$$y = e^{2\ln x - \frac{x^2}{2} + \frac{1}{2}}$$

$$y = \frac{e^{\ln x^2} \times e^{\frac{1}{2}}}{e^{\frac{x^2}{2}}}$$

$$y = x^2 e^{\frac{1}{2} - \frac{x^2}{2}}$$

6 **(i)** Show that the equation $(\sqrt{2})\operatorname{cosec} x + \cot x = \sqrt{3}$ can be expressed in the form $R\sin(x - \alpha) = \sqrt{2}$, where $R > 0$ and $0° < \alpha < 90°$. [4]

$R = \sqrt{1^2 + \sqrt{2}^2} = \sqrt{3}$

$\alpha = \tan^{-1}\left(\frac{1}{\sqrt{2}}\right) = 35.264$

$\sqrt{3}\sin(x - 35.26) =$

$\frac{\sqrt{2}}{\sin\theta} + \frac{\cos\theta}{\sin\theta} = \sqrt{3}$

$\sqrt{2} + \cos\theta = \sqrt{3}\sin\theta$

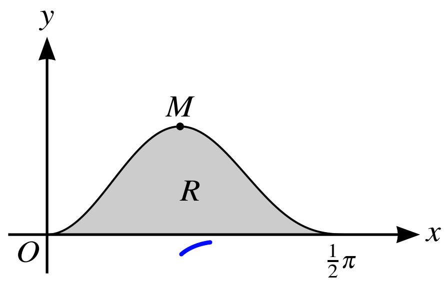
$\sqrt{3}\sin\theta - \cos\theta = \sqrt{2}$

0

(ii) Hence solve the equation $(\sqrt{2})\operatorname{cosec} x + \cot x = \sqrt{3}$, for $0° < x < 180°$. [4]

7

$10\sin x\cos x$



$-3\cos^2 x\sin x$

The diagram shows the curve $y = 5\sin^2 x\cos^3 x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$, and its maximum point M. The shaded region R is bounded by the curve and the x-axis.

(i) Find the x-coordinate of M, giving your answer correct to 3 decimal places. [5]

$\frac{dy}{dx} = 5\sin^2 x(-3\cos^2 x\sin x) + \cos^3 x(10\sin x\cos x)$

$= -15\sin^3 x\cos^2 x + 10\sin x\cos^4 x = 0$

$2\,\cancel{10\sin x}\cos^{\cancel{4}\,2} x = 3\,\cancel{15}\sin^{\cancel{3}\,2} x\,\cancel{\cos^2 x}$

$2\cos^2 x = 3\sin^2 x$

$\div\cos^2 x$

$2 = 3\tan^2 x$

$\tan^2 x = \frac{2}{3}$

$\tan x = \sqrt{\frac{2}{3}}$ $\quad$ $\cancel{\tan x = -\sqrt{\frac{2}{3}}}$

$x = 0.6847$

$x = 0.685$

(ii) Using the substitution $u = \sin x$ and showing all necessary working, find the exact area of R. [4]

$y = 5\sin^2 x \cos^3 x$

$u = \sin x$

$5\int \sin^2 x \cos^3 x \, dx$

$\frac{du}{dx} = \cos x$

$5\int u^3 \cos^{2} x \frac{du}{\cancel{\cos x}}$

$dx = \frac{du}{\cos x}$

$5\int u^3 (1-u^2)\, du$

$5\int_0^{\frac{1}{2}\pi} u^3 - u^5 \, du$

$5\left[\frac{u^4}{4} - \frac{u^6}{6}\right]_0^{\frac{1}{2}\pi}$

let $f(u) = 5\left(\frac{\sin^4 x}{4} - \frac{\sin^6 x}{6}\right)_0^{\frac{1}{2}\pi}$

$f(\frac{1}{2}\pi) = \frac{5}{12}$

$f(0) = 0$

$\therefore$ area $= \frac{5}{12}$

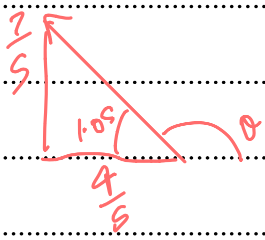
8 **(a)** Showing all necessary working, express the complex number $\dfrac{2+3\mathrm{i}}{1-2\mathrm{i}}$ in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$. Give the values of r and θ correct to 3 significant figures. [5]

4

$$\frac{(2+3i)(1+2i)}{(1-2i)(1+2i)}$$

$$\frac{2+7i+6(-1)}{1-4(-1)}$$

$$\frac{-4+7i}{5}$$



$\therefore \theta = \pi - 1.05 = 2.09$

$$r = \sqrt{\left(\frac{4}{5}\right)^2 + \left(\frac{7}{5}\right)^2}$$

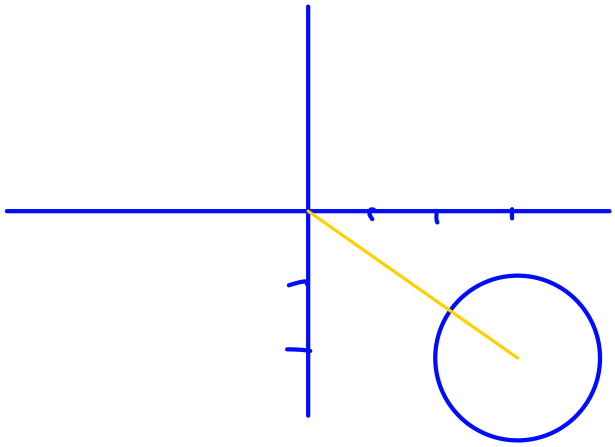
$$r = \frac{\sqrt{65}}{5} = 1.61$$

$$\theta = \tan^{-1}\left(\frac{7/5}{-4/5}\right) = -1.05$$

$$1.61\, r^{i(-1.05)}$$

(b) On an Argand diagram sketch the locus of points representing complex numbers z satisfying the equation $|z - 3 + 2i| = 1$. Find the least value of $|z|$ for points on this locus, giving your answer in an exact form. [4]

$z - (3 - 2i)$



least value $= \sqrt{3^2 + 2^2} - 1$

$= \sqrt{13} - 1$

9 Let $f(x) = \dfrac{6x^2 + 8x + 9}{(2-x)(3+2x)^2}$. $= \dfrac{A}{2-x} + \dfrac{B}{3+2x} + \dfrac{C}{(3+2x)^2}$

(i) Express $f(x)$ in partial fractions. [5]

$6x^2 + 8x + 9 = A(3+2x)^2 + B(3+2x)(2-x) + C(2-x)$

$= A(9+12x+4x^2) + B(6+x-2x^2) + 2C - Cx$

$= 9A + 12Ax + 4Ax^2 + 6B + Bx - 2Bx^2 + 2C - Cx$

$6 = 4A - 2B$

$8 = 12A + B - C$

$9 = 9A + 6B + 2C$

$A = 1$, $B = -1$, $C = 3$

(ii) Hence, showing all necessary working, show that $\int_{-1}^{0} \mathrm{f}(x)\,\mathrm{d}x = 1 + \frac{1}{2}\ln\left(\frac{3}{4}\right)$. [5]

$$\int_{-1}^{0} \frac{1}{2-x} - \frac{1}{3+2x} + \frac{3}{(3+2x)^2}\,dx$$

$$-\ln(2-x) - \frac{1}{2}\ln(3+2x) + 3\frac{(3+2x)^{-1}}{-1(2)}$$

let $f(x) = -\ln(2-x) - \frac{1}{2}\ln(3+2x) - \frac{3}{2(3+2x)}$

$f(0) = -\ln 2 - \frac{1}{2}\ln 3 - \frac{3}{6}$

$=$

$f(-1) = -\ln(3) - \frac{1}{2}\ln 1 - \frac{3}{2(1)}$

$= -\ln 3 - 0 - \frac{3}{2}$

$= -\ln 3 - \frac{3}{2}$

$-\ln 2 - \frac{1}{2}\ln 3 - \frac{3}{6} + \ln 3 + \frac{3}{2}$

$1 - \ln 4^{\frac{1}{2}} + \frac{1}{2}\ln 3$

$1 + \frac{1}{2}\ln 3 - \frac{1}{2}\ln 4$

$1 + \frac{1}{2}\ln\left(\frac{3}{4}\right)$

10 The planes m and n have equations $3x + y - 2z = 10$ and $x - 2y + 2z = 5$ respectively. The line l has equation $\mathbf{r} = 4\mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(\mathbf{i} + \mathbf{j} + 2\mathbf{k})$.

(i) Show that l is parallel to m. [3]

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(ii) Calculate the acute angle between the planes m and n. [3]

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(iii) A point P lies on the line l. The perpendicular distance of P from the plane n is equal to 2. Find the position vectors of the two possible positions of P. [4]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.