Cambridge International A Level

Cambridge Assessment International Education
Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **February/March 2019**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **17** printed pages and **3** blank pages.

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1 **(i)** Show that the equation $\log_{10}(x-4) = 2 - \log_{10} x$ can be written as a quadratic equation in x. [3]

$\ln_{10}(x(x-4)) = 2$

$x^2 - 4x = 100$

$x^2 - 4x - 100$

(ii) Hence solve the equation $\log_{10}(x-4) = 2 - \log_{10} x$, giving your answer correct to 3 significant figures. [2]

1

$x^2 - 4x - 100 = 0$

$$\frac{4 \pm \sqrt{16 - 4(1)(-100)}}{2}$$

$$\frac{4 \pm \sqrt{416}}{2}$$

$x = 12.2$ or $x = \cancel{-8.20}$

2 The sequence of values given by the iterative formula

$$x_{n+1} = \frac{2x_n^6 + 12x_n}{3x_n^5 + 8},$$

with initial value $x_1 = 2$, converges to α.

(i) Use the formula to calculate α correct to 4 decimal places. Give the result of each iteration to 6 decimal places. [3]

$x_1 = 2$

$x_2 = \frac{2(2)^6 + 12(2)}{3(2)^5 + 8} = 1.461538$

$x_3 = 1.322263$

$x_4 = 1.319508$

$x_5 = 1.319508$

$x = 1.3195$

(ii) State an equation satisfied by α and hence find the exact value of α. [2]

$3x^6 + 8x = 2x^6 + 12x$

$x^6 - 4x = 0$

$x(x^5 - 4) =$

$x^5 - 4 = 0$

$x^5 = 4$

$x = \sqrt[5]{4}$

3 **(i)** Given that $\sin(\theta + 45°) + 2\cos(\theta + 60°) = 3\cos\theta$, find the exact value of $\tan\theta$ in a form involving surds. You need not simplify your answer. [4]

$$\frac{\sqrt{2}\sin\theta}{2} + \frac{\sqrt{2}\cos\theta}{2} + 2\left(\frac{1}{2}\cos\theta - \frac{\sqrt{3}}{2}\sin\theta\right) = 3\cos\theta$$

$$\frac{\sqrt{2}}{2}\sin\theta + \frac{\sqrt{2}}{2}\cos\theta + \cos\theta - \sqrt{3}\sin\theta = 3\cos\theta$$

$$\frac{\cos\theta}{\sin\theta} = \frac{1}{\tan\theta}$$

$$\frac{-2\sqrt{3}+\sqrt{2}}{2}\sin\theta + \frac{2+\sqrt{2}}{2}\cos\theta = 3\cos\theta$$

$$\cos\theta = \frac{\sin\theta}{\tan\theta}$$

$$3\frac{\sin\theta}{\tan\theta} = \frac{-2\sqrt{3}+\sqrt{2}}{2}\sin\theta + \frac{2+\sqrt{2}}{2}\cos\theta$$

$$\frac{3\sin\theta}{\frac{-2\sqrt{3}+\sqrt{2}}{2}\sin\theta + \frac{2+\sqrt{2}}{2}\cos\theta} = \tan\theta$$

(ii) Hence solve the equation $\sin(\theta + 45°) + 2\cos(\theta + 60°) = 3\cos\theta$ for $0° < \theta < 360°$. [2]

0

4 Show that $\int_1^4 x^{-\frac{3}{2}} \ln x \, dx = 2 - \ln 4$. [5]

$u = \ln x$ $u' = \frac{1}{x}$

$v = -2x^{-\frac{1}{2}}$ $v' = x^{-\frac{3}{2}}$ -2

$-2x^{-\frac{1}{2}} \ln x - (-2)\int x^{-\frac{1}{2}} \left(\frac{1}{x}\right) dx$ $\frac{1}{x^{\frac{1}{2}} \times x}$

$-2x^{-\frac{1}{2}} \ln x + 2 \int x^{-\frac{3}{2}} \, dx$

$-2x^{-\frac{1}{2}} \ln x + 2(-2x^{-\frac{1}{2}})$

-0.6137

let $f(x) = -2x^{-\frac{1}{2}} \ln x - 4x^{-\frac{1}{2}}$

$f(4) = -2(4)^{-\frac{1}{2}} \ln 4 - 4(4)^{-\frac{1}{2}}$

$-\ln 4 - \frac{4}{\sqrt{4}}$

$f(1) = 0 - 4$

$\therefore -\ln 4 - \frac{4}{\sqrt{4}} + 4$

$-\ln 4 + 2$

$2 - \ln 4$

5 The variables x and y satisfy the relation $\sin y = \tan x$, where $-\frac{1}{2}\pi < y < \frac{1}{2}\pi$. Show that

$$\frac{dy}{dx} = \frac{1}{\cos x\sqrt{(\cos 2x)}}.$$ [5]

$\sin y = \tan x$ $\qquad$ $y = \sin^{-1}(\tan x)$ 1

$\frac{dy}{dx} = \frac{dy}{d} \times \frac{d}{dx}$

$\frac{dy}{d} = \frac{1}{\frac{d}{dy}}$ $\therefore$ $\frac{d}{dy} = \cos y$

$\therefore \frac{dy}{d} = \frac{1}{\cos y}$

6 The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = ky^3e^{-x},$$

where k is a constant. It is given that $y = 1$ when $x = 0$, and that $y = \sqrt{e}$ when $x = 1$. Solve the differential equation, obtaining an expression for y in terms of x. [7]

$\int y^{-3}dy = k\int e^{-x}\,dx$

5

$\frac{y^{-2}}{-2} = k\cancel{(-1)}(e^{-x})$

$\frac{1}{-2y^2} = ke^{-x} + c$

sub $y=1$, $x=0$

$\frac{1}{-2(1)} = k(e^0) + c$

① $-\frac{1}{2} = k + c$

sub $y = x^{\frac{1}{2}}$, $x = 1$

$\frac{1}{-2(e)} = ke + c$

$-\frac{1}{2e} = ke + c$

$c = -\frac{1}{2} - k$

$\frac{1}{-2e} = ke - \frac{1}{2} - k$

$-\frac{1}{2e} = k(e-1) - \frac{1}{2}$

$k(e-1) = \frac{1}{2e} - \frac{1}{2}$

$k(e-1) = \frac{1-e}{2e}$

$k(e-1) = \frac{}{2e}$

$k = \frac{-1\cancel{(e-1)}}{2\cancel{(e-1)}} = \frac{-1}{2}$

$\therefore c = \frac{1}{2} + \frac{1}{2} = 1$

$$\frac{1}{2y^2} = 1 - \frac{1}{2}e^{-x}$$

$$\frac{1}{y^2} = 2 - e^{-x}$$

$$y^2 = \frac{1}{2-e^{-x}}$$

$$y = \sqrt{\frac{1}{2-e^{-x}}}$$

7 **(a)** Showing all working and without using a calculator, solve the equation

$$(1 + \mathrm{i})z^2 - (4 + 3\mathrm{i})z + 5 + \mathrm{i} = 0. \qquad x^2 +$$

Give your answers in the form $x + \mathrm{i}y$, where x and y are real. [6]

$ax^2 + bx + c$

$a = (1+i)$

$b = (-4-3i)$

$c = 5+i$

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$b^2-4ac = \left[(-4-3i)(-4-3i) - 4(1+i)(5+i)\right]$

$= \left[16 + 24i + 9(-1) - 4(1+6i-1)\right]$

$= (7+24i - 24i)$

$= 7$

$$\frac{-(-4-3i) \pm \sqrt{7}}{2(1+i)}$$

$$\frac{4+3i \pm \sqrt{7}}{2+2i}$$

$$\frac{(4+3i+\sqrt{7})(2-2i)}{(2+2i)(2-2i)}$$

$$\frac{8 - 6i + 2\sqrt{7} - 8i + 6 - 2\sqrt{7}i}{4 - 4(-1)}$$

$$\frac{14 - 14i + 2\sqrt{7} - 2\sqrt{7}i}{8}$$

$$\frac{14+2\sqrt{7}}{8} - \frac{14+2\sqrt{7}}{8}i$$

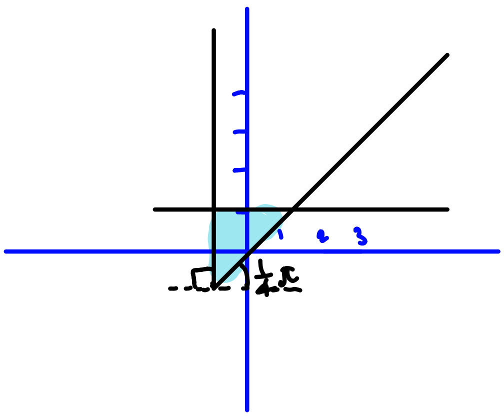
(b) The complex number u is given by

$$u = -1 - \mathrm{i}.$$

On a sketch of an Argand diagram show the point representing u. Shade the region whose points represent complex numbers satisfying the inequalities $|z| < |z - 2\mathrm{i}|$ and $\frac{1}{4}\pi < \arg(z - u) < \frac{1}{2}\pi$. [4]

$(0+2i)$

$z-(-1-i)$



8 Let $f(x) = \dfrac{12 + 12x - 4x^2}{(2 + x)(3 - 2x)}$. $= \dfrac{A}{2+x} + \dfrac{B}{3-2x} + C$

(i) Express $f(x)$ in partial fractions. [5]

4

$12 + 12x - 4x^2 \equiv A(3-2x) + B(2+x) + C(2+x)(3-2x)$

$12 + 12x - 4x^2 = 3A - 2Ax + 2B + Bx + C(6 - x - 2x^2)$

$12 + 12x - 4x^2 = 3A - 2Ax + 2B + Bx + 6C - Cx - 2Cx^2$

$-4\cancel{x^2} = -2C\cancel{x^2}$

$C = 2$

$12 = 3A + 2B + 6(2)$

$12 - 12 = 3A + 2B$

$3A + 2B = 0$

$12 = -2A + B - 2$

$-2A + B = 14$

$B = 14 + 2A$

$3A + 2(14 + 2A) = 0$

$3A + 28 + 4A = 0$

$7A = -28$

$A = -4$

$\therefore$ $3(-4) + 2B = 0$

$-12 = -2B$

$B = 6$

(ii) Hence obtain the expansion of $\mathrm{f}(x)$ in ascending powers of x, up to and including the term in x^2. [5]

$2 - \frac{4}{2+x} + \frac{6}{3-2x}$

$2 - 4(2+x)^{-1} + 6(3-2x)^{-1}$

$4(2+x)^{-1} = 4(2)^{-1}\left(1+\frac{x}{2}\right)^{-1}$

$= 2\left(1+\frac{x}{2}\right)^{-1}$

$2\left[1+(-1)\left(\frac{x}{2}\right) + \frac{(-1)(-2)\left(\frac{x}{2}\right)^2}{2}\right]$

$2\left[1 - \frac{x}{2} + \frac{1}{4}x^2\right] \Rightarrow 2 - x + \frac{1}{2}x^2$

$6(3-2x)^{-1} = 6(3)^{-1}\left(1-\frac{2x}{3}\right)^{-1}$

$= 2\left(1+(-1)\left(-\frac{2}{3}x\right) + \frac{(-1)(-2)\left(-\frac{2}{3}x\right)^2}{2!}\right)$

$= 2\left(1 + \frac{2}{3}x + \frac{4}{9}x^2\right)$

$= 2 + \frac{4}{3}x + \frac{8}{9}x^2$

$\cancel{2 - 2} + x - \frac{1}{2}x^2 + 2 + \frac{4}{3}x + \frac{8}{9}x^2$

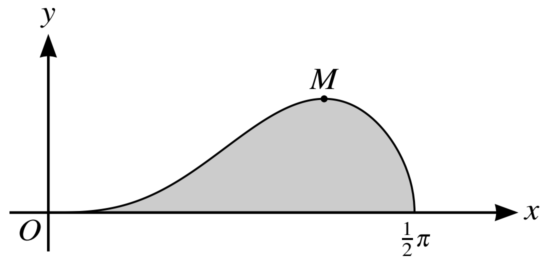
$2 + \frac{7}{3}x + \frac{7}{18}x^2$ ✓

9 Two planes have equations $2x + 3y - z = 1$ and $x - 2y + z = 3$.

(i) Find the acute angle between the planes. [4]

(ii) Find a vector equation for the line of intersection of the planes. [6]

10



The diagram shows the curve $y = \sin^3 x \sqrt{(\cos x)}$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$, and its maximum point M.

(i) Using the substitution $u = \cos x$, find by integration the exact area of the shaded region bounded by the curve and the x-axis. [6]

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$dx = \frac{du}{-\sin x}$

$\int \sin^{\cancel{3}\,2}(x)\, u^{\frac{1}{2}} \frac{du}{\cancel{-\sin x}}$

$-\int (1-u^2)\, u^{\frac{1}{2}}\, du$

$(u^2)(u^{\frac{1}{2}})$

$u^{2.5}$

$-\int u^{\frac{1}{2}} - u^{\frac{5}{2}}\, du$

$-\left[\frac{2u^{\frac{3}{2}}}{3} - \frac{2u^{\frac{7}{2}}}{7}\right]_0^{\frac{1}{2}\pi}$

let $f(x) = -\frac{2(\cos x)^{\frac{3}{2}}}{3} + \frac{2\cos^{\frac{7}{2}}(x)}{7}\Big]_0^{\frac{1}{2}\pi}$

$f\left(\frac{1}{2}\pi\right) = 0 + 0$

$f(0) = -\frac{2}{3} + \frac{2}{7}$

$= 0 - \left(-\frac{2}{3} + \frac{2}{7}\right) \quad \frac{2}{3} - \frac{2}{7} = \frac{14-6}{21} = \frac{8}{21}$

(ii) Showing all your working, find the x-coordinate of M, giving your answer correct to 3 decimal places. [6]

$y = \sin^3 x \cos^{\frac{1}{2}} x$

$\frac{dy}{dx} = \sin^3 x \left(\frac{1}{2}\cos^{-\frac{1}{2}} x\right)(-\sin x) + \cos^{\frac{1}{2}} x \,(3\sin^2 x \cos x)$

$= -\frac{1}{2}\sin^4 x \cos^{-\frac{1}{2}} x + 3\sin^2 x \cos^{\frac{3}{2}} x = 0$

$3 \cancel{\sin^2 x} \cos^{\frac{3}{2}} x = \frac{1}{2}\sin^{\cancel{4}\,2} x \cos^{-\frac{1}{2}} x$

$\frac{3\cos^{\frac{3}{2}} x}{\cos^{-\frac{1}{2}} x} = \frac{1}{2}\sin^2 x$

$6\cos^{\frac{3}{2}-(-\frac{1}{2})} x = \sin^2 x$

$6\cos^2 x = \sin^2 x$

$6\cos^2 x - \sin^2 x = 0$

$5\cos^2 x + \cos^2 x - \sin^2 x = 0$

$5\cos^2 x + \cos 2x = 0$

$5\cos^2 x + 2\cos^2 x - 1 = 0$

$7\cos^2 x = 1$

$\cos^2 x = \frac{1}{7}$

$\cos x = \pm\sqrt{\frac{1}{7}}$

$\therefore x = \cos^{-1}\left(\sqrt{\frac{1}{7}}\right)$

$= 1.183$

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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