- Gravitational field is a region of space where a mass experiences a force.

Formulas of gravitation

1) Gravitational force $= F_g = \dfrac{GMm}{r^2}$

2) $V_g = \dfrac{\text{work done}}{\text{unit mass}} = \dfrac{\frac{GM\cancel{m}}{r^{\cancel{2}}} \times \cancel{r}}{\cancel{m}} = -\dfrac{GM}{r}$

Gravitational potential

shows masses are attractive and as r decreases, work got out.

3) $g = \dfrac{\text{Force}}{\text{mass}} = \dfrac{\frac{GM\cancel{m}}{r^2}}{\cancel{m}} = \dfrac{GM}{r^2}$

gravitational field strength
aka
acc. due to gravity

4) $U_g = V_g \times m = \frac{GM}{r} \times m = -\frac{GMm}{r}$

gravitational potential energy

5) $V = \sqrt{\frac{GM}{r}}$

Tangential velocity of a sattelite

6) $K.E = \frac{GMm}{2r}$

Kinetic energy of a sattelite

7) $\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$ or $\frac{T_1^2}{r_1^3} = \frac{T_2^2}{r_2^3}$

8) $V_e = \sqrt{\frac{2GM}{r}}$

escape velocity

- Newton's law of gravitation states that two point masses attract each other with a force that is directly proportional to their masses and inversely proportional to the square of the distance between their centers.

$$F_g = \frac{GMm}{r^2}$$

$$G = 6.67 \times 10^{-11}\ N m^2 kg^{-2}$$

- The Gravitational field strength at a point is defined as the force per unit mass acting on a small mass placed at that point.

- Gravitational field strength is equal to the acceleration due to gravity.

Deriving equations by $F_g = F_c$

- $F_g = \frac{GMm}{r^2}$
- $F_c = \frac{mv^2}{r}$ and $mr\omega^2$

- Tangential velocity (v) of a sattelite

(in circular motion)

$$\frac{GM\cancel{m}}{r^{\cancel{2}}} = \frac{\cancel{m}v^2}{\cancel{r}}$$

$$\therefore v = \sqrt{\frac{GM}{r}}$$

- Kinetic energy of a sattelite in orbit

$$\frac{1}{2} \times \frac{GMm}{r^{\cancel{2}}} = \frac{mv^2}{\cancel{r}} \times \frac{1}{2}$$

because $Ke = \frac{1}{2}mv^2$

$$\therefore \frac{GMm}{2r} = \frac{1}{2}mv^2$$

$$Ke = \frac{GMm}{2r}$$

- Time period and radius (Keplers third law)

$$\frac{GM\cancel{m}}{r^2} = \cancel{m}r\omega^2$$

$$\omega = \frac{2\pi}{T}$$

$$GM = r^3\left(\frac{2\pi}{T}\right)^2$$

$$GM = \frac{4\pi^2 r^3}{T^2}$$

$$\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$$

$\therefore$ when data of two satellites is given and you need ~~to~~ to find an unknown by using this relashianship.

$$\frac{T_1^2}{r_1^3} = \frac{T_2^2}{r_2^3}$$

- Escape velocity, is the minimum KE/velocity required to leave the gravitational influence of a Mass, M

$\therefore$ KE = PE when the escape velocity is minimum

$$\frac{1}{2}\cancel{m}V_e^2 = \frac{GM\cancel{m}}{r}$$

$$V_e = \sqrt{\frac{2GM}{r}}$$

- Gravitational potential at a point in a gravitational field is the work done per unit mass in bringing a small test mass from infinity to the point (without changing it K.E)

$$\phi = \frac{W.D}{m} = \frac{F \times d}{m} = \frac{\frac{GM\cancel{m} \times \cancel{r}}{r^{\cancel{2}}}}{\cancel{m}} = \frac{-GM}{r}$$

$$\frac{GMm}{r} \div m = \frac{GM\cancel{m}}{r} \times \frac{1}{\cancel{m}}$$

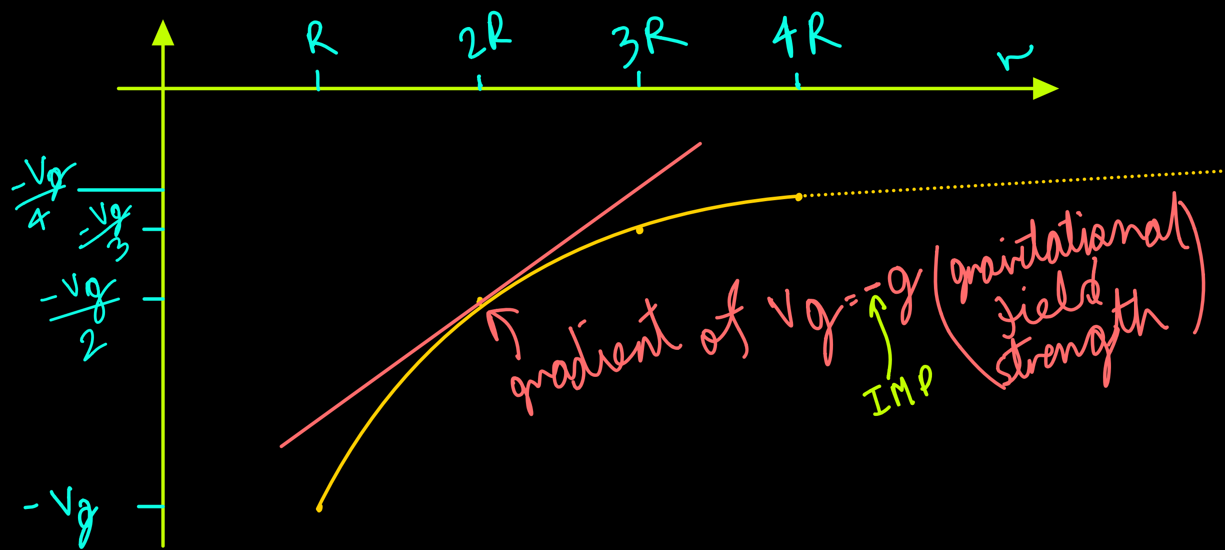
- Gravitational potential energy = mass x gravitational potential

$$G.P.E = -\frac{GMm}{r}$$

- Geostationary satellites are placed at such a height above earth so that the remain stationary above a point on earth, moving in the same direction as the rotation of earth, and thus it remains on a equatorial plane where T = 24 hours

Graphs

Gravitational potential (V_g) vs Radius

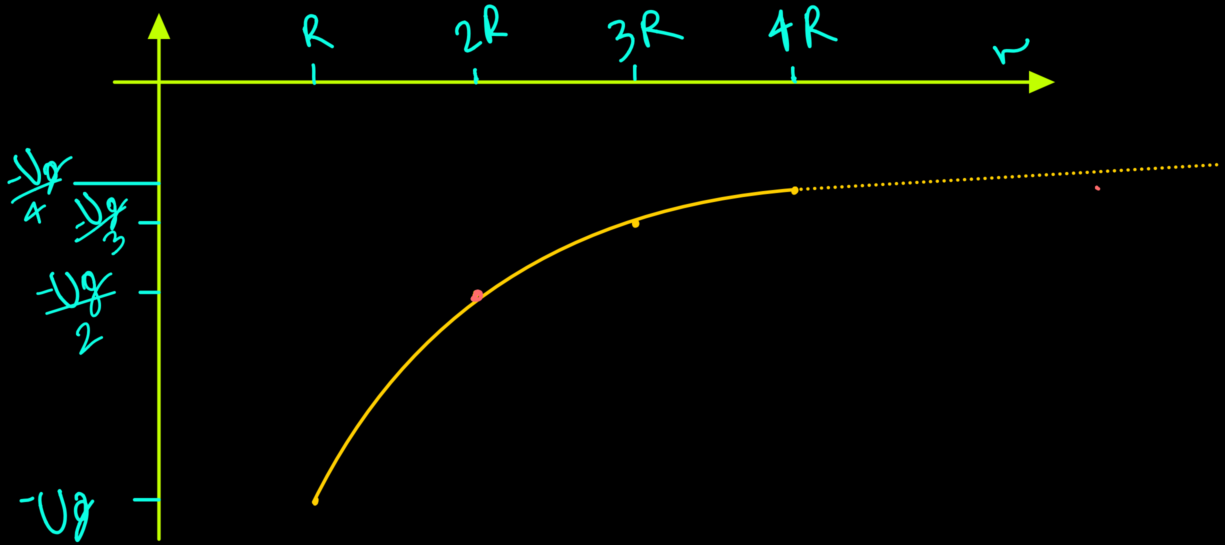


$V_g = -\frac{GM}{r}$

$\therefore V_g \propto -\frac{1}{r} \times GM$ ← constant

$g \propto$ gradient of $V_g - r$ graph $\propto F_g \propto a$

Gravitational potential energy (U_g) vs Radius

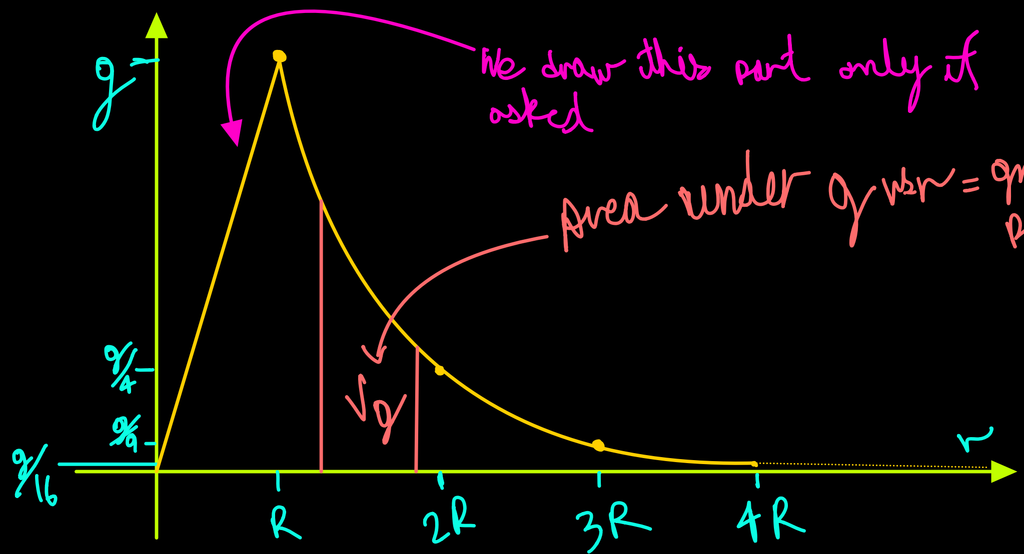


$$U_g = -\frac{GMm}{r}$$

constant

$$\therefore U_g \propto -\frac{1}{r} \times GMm$$

Gravitational field strength (g) vs Radius



$g = \frac{GM}{r^2}$ } outside sphere

$g \propto r$ } inside sphere

because

$\rho = \frac{M}{V}$

$\rho = \frac{M}{\frac{4}{3}\pi r^3}$

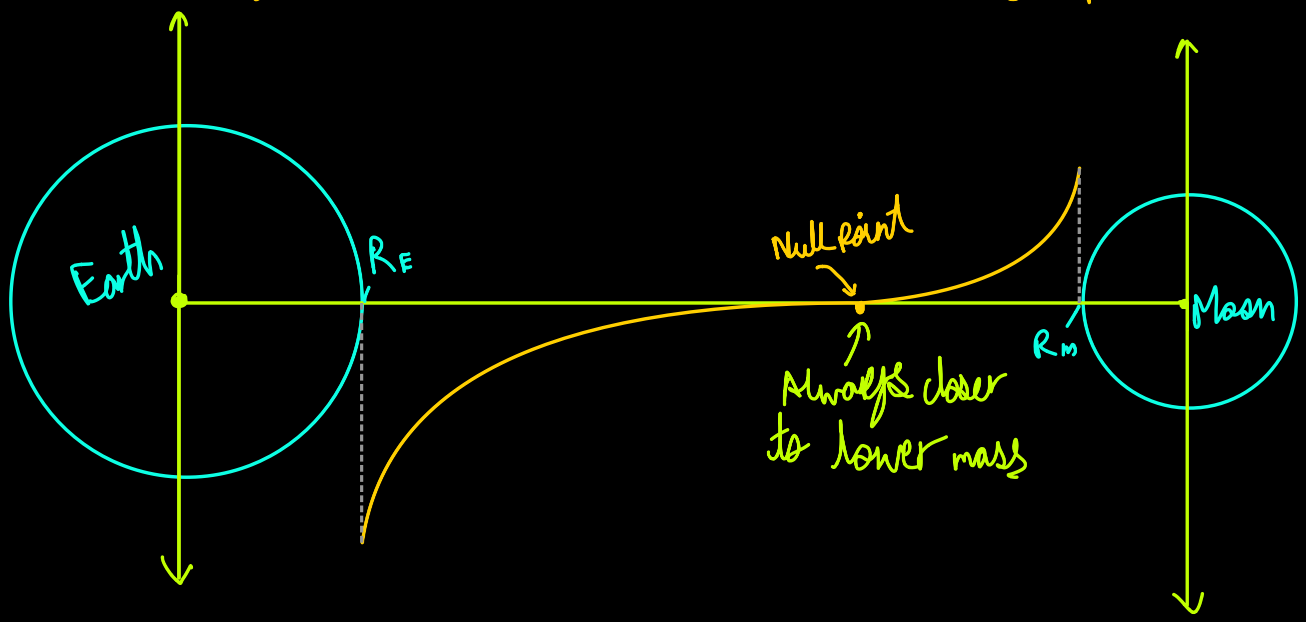
$\frac{4\pi r^3}{3}\rho = M$

$M = \frac{gr^2}{G}$

$$\frac{4\pi r^{\cancel{3}}}{3}\rho = \frac{g\cancel{r^{2}}}{G}$$

$$g \propto r$$

How g (Gravitational field strength) varies between 2 planets



Important things to keep in mind when drawing two object's masses

1) check the direction of field lines, if both are opposite to each other there will a null point, and an arrow pointing towards right shall be plot on positive y axis and an arrow pointing in left shall be plot on -ve y axis

2) check where null point will lie

3) Check that higher mass has higher g as compared to lower mass

Questions

Q1. Why does centripetal acceleration exists even if the speed is always constant in uniform circular motion?

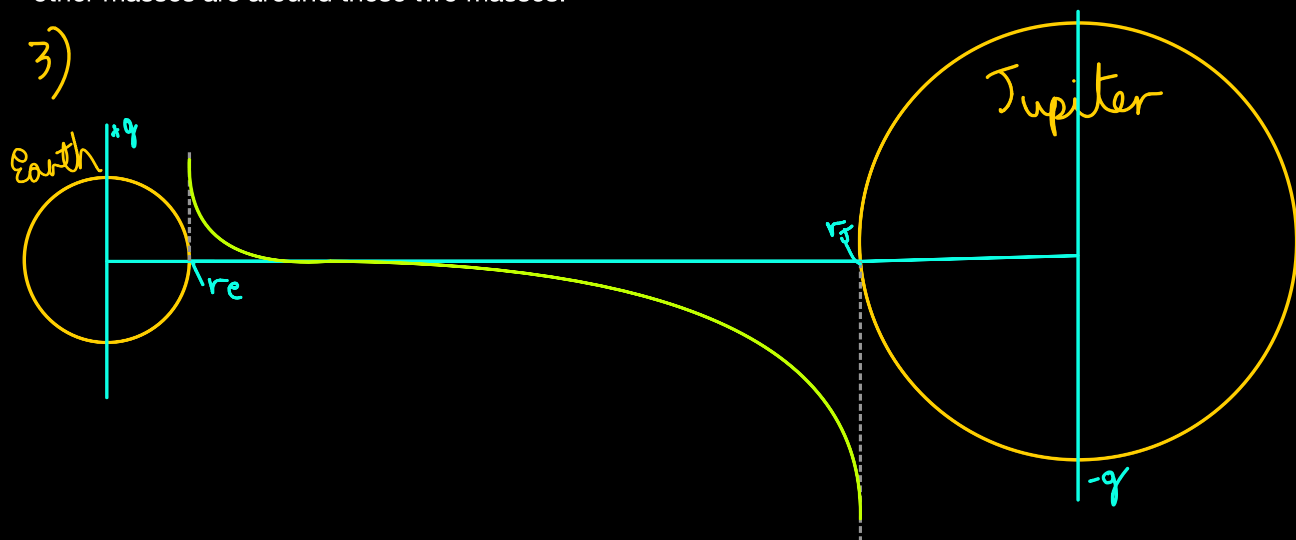
1) because acceleration is the change in velocity (not speed) per unit time, and even tho the magnitude of velocity is constant, its direction keeps changing, thus velocity is changing all the time

Q2. In any uniform circular motion, at a particular instant if you want to make equation of centripetal force at that point what would you do?

2) we would resolve all the forces and then find the resultant force in direction of centeripetal force

Q3. Sketch the graph of gravitational field strength vs distance between Jupiter and Earth. Assume no other masses are around these two masses.

3)



Q4. A rocket of mass m travels from Planet A of mass Ma to Planet B of mass Mb in straight line.

(i) Name the point at which gravitational influence of A will be over and B will dominate.

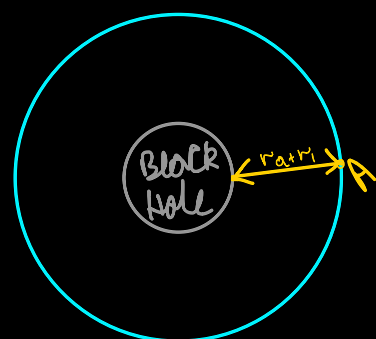
(ii) How would u calculate minimum speed of rocket such that it reaches the surface of planet B.

4i) Null Point

ii) We would calc the ΔV required to reach the Null point, because after that B would pull the rocket

Q5. Two neutron stars A and B are orbiting a black hole whose mass is M. It is given that radius of A is ra, distance between centre of of black hole to the surface of A is r1 and the time periods of A and B are T1 and 3T1 respectively. Find the distance between centre of B and Black hole.

5)



$$\frac{T_a^2}{r_a^3} = \frac{T_b^2}{r_b^3}$$

$$\frac{1^2}{(r_a+r_1)^3} = \frac{3^2}{r_b^3}$$

$$\therefore \quad r_b^3 = 9(r_a+r_1)^3$$

$$r_b = \sqrt[3]{9(r_a+r_1)}$$

Q6. In which condition we should we use ½mv^2 and when should we use GMm/2R for Kinetic Energy?

6) $\frac{1}{2}mv^2$ when finding K.E of an object when it's not in any orbit

$\frac{GMm}{2R}$ when finding K.E of a sattelite orbiting a planet

Q7. It is given that a satellite is orbiting a planet whose density is ρ. Prove the given formula of tangential velocity of satellite:
v = √(4/3)Gρπr^2

$$F_g = F_c$$

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$M = \rho V$$
$$V = \frac{4}{3}\pi r^3$$

$$v^2 = \frac{GM}{r}$$

$$\therefore \quad v^2 = \frac{G\rho \times \frac{4}{3}\pi r^{\cancel{3}2}}{\cancel{r}}$$

$$v = \sqrt{\left(\frac{4}{3}\right) G\rho\pi r^2}$$

Rowe papers

Q13) a) WD/M in bringing a test mass from infinity to a point in the g field

b) i) $\frac{-GM}{r}$

$$6.2 \times 10^{7} = \frac{6.67 \times 10^{-11} \times M}{6.4 \times 10^{6}}$$

$$M = 5.945 \times 10^{24}$$
$$= 6.0 \times 10^{24} \text{ Kg}$$

ii) Ke =

iii) 1) would bend towards the mass.

2) greater speed.

33) a) $g = \frac{GM}{r^2}$

$= \frac{6.67 \times 10^{-11} \times 6.4 \times 10^{23}}{\left(\frac{6.8 \times 10^6}{2}\right)^2}$

$= 3.695 \approx 3.7\ N\,kg^{-1}$

b) change $= m g h_f$

$= 2.4 \times 3.695 \times 1800$

$= 1.59 \times 10^4$

$\approx 1.6 \times 10^4$

c) $4D = 8r$ $\qquad r = \dfrac{6.8 \times 10^6}{2}$

$$\frac{1}{2}\cancel{m}v^2 = \frac{GM\cancel{m}}{r}$$

$$v = \sqrt{\frac{2GM}{r}}$$

$$v = \sqrt{\frac{2(6.67 \times 10^{-11})(6.4 \times 10^{23})}{8\left(\frac{6.8 \times 10^6}{2}\right)}}$$

$$v = 1.77 \times 10^3$$
$$v \approx 1.8 \times 10^3$$

25bi)