Cambridge International AS & A Level

Cambridge International Examinations
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS **9709/62**

Paper 6 Probability & Statistics 1 **(S1)** **October/November 2015**

1 hour 15 minutes

Additional Materials: List of Formulae (MF9)

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READ THESE INSTRUCTIONS FIRST

An answer booklet and a graph paper booklet are provided inside this question paper. You should follow the instructions on the front cover of both booklets. If you need additional answer paper or graph paper ask the invigilator for a continuation booklet or graph paper booklet.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.

This document consists of **3** printed pages, **1** blank page and **2** inserts.

CAMBRIDGE International Examinations

1 For n values of the variable x, it is given that $\Sigma(x - 100) = 216$ and $\Sigma x = 2416$. Find the value of n. [3]

2 A committee of 6 people is to be chosen at random from 7 men and 9 women. Find the probability that there are no men on the committee. [3]

3 One plastic robot is given away free inside each packet of a certain brand of biscuits. There are four colours of plastic robot (red, yellow, blue and green) and each colour is equally likely to occur. Nick buys some packets of these biscuits. Find the probability that

(i) he gets a green robot on opening his first packet, [1]

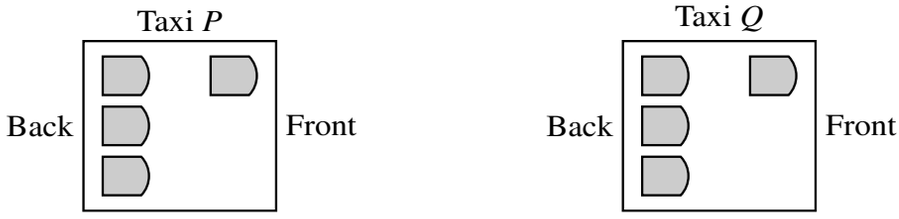
(ii) he gets his first green robot on opening his fifth packet. [2]

Nick's friend Amos is also collecting robots.

(iii) Find the probability that the first four packets Amos opens all contain different coloured robots. [3]

4 A group of 8 friends travels to the airport in two taxis, P and Q. Each taxi can take 4 passengers.

(i) The 8 friends divide themselves into two groups of 4, one group for taxi P and one group for taxi Q, with Jon and Sarah travelling in the same taxi. Find the number of different ways in which this can be done. [3]



Each taxi can take 1 passenger in the front and 3 passengers in the back (see diagram). Mark sits in the front of taxi P and Jon and Sarah sit in the back of taxi P next to each other.

(ii) Find the number of different seating arrangements that are now possible for the 8 friends. [4]

5 The weights, in kilograms, of the 15 rugby players in each of two teams, A and B, are shown below.

Team A	97	98	104	84	100	109	115	99	122	82	116	96	84	107	91
Team B	75	79	94	101	96	77	111	108	83	84	86	115	82	113	95

(i) Represent the data by drawing a back-to-back stem-and-leaf diagram with team A on the left-hand side of the diagram and team B on the right-hand side. [4]

(ii) Find the interquartile range of the weights of the players in team A. [2]

(iii) A new player joins team B as a substitute. The mean weight of the 16 players in team B is now 93.9 kg. Find the weight of the new player. [3]

6 A fair spinner A has edges numbered 1, 2, 3, 3. A fair spinner B has edges numbered -3, -2, -1, 1. Each spinner is spun. The number on the edge that the spinner comes to rest on is noted. Let X be the sum of the numbers for the two spinners.

(i) Copy and complete the table showing the possible values of X. [1]

		Spinner A			
		1	2	3	3
Spinner B	-3	-2			
	-2			1	
	-1				
	1				

(ii) Draw up a table showing the probability distribution of X. [3]

(iii) Find $\mathrm{Var}(X)$. [3]

(iv) Find the probability that X is even, given that X is positive. [2]

7 **(a)** A petrol station finds that its daily sales, in litres, are normally distributed with mean 4520 and standard deviation 560.

(i) Find on how many days of the year (365 days) the daily sales can be expected to exceed 3900 litres. [4]

The daily sales at another petrol station are X litres, where X is normally distributed with mean m and standard deviation 560. It is given that $\mathrm{P}(X > 8000) = 0.122$.

(ii) Find the value of m. [3]

(iii) Find the probability that daily sales at this petrol station exceed 8000 litres on fewer than 2 of 6 randomly chosen days. [3]

(b) The random variable Y is normally distributed with mean μ and standard deviation σ. Given that $\sigma = \frac{2}{3}\mu$, find the probability that a random value of Y is less than 2μ. [3]

1 For n values of the variable x, it is given that $\Sigma(x - 100) = 216$ and $\Sigma x = 2416$. Find the value of n. [3]

$\Sigma x - 100n = 216$

$2416 - 216 = 100n$

$2200 = 100n$

$n = 22$

2 A committee of 6 people is to be chosen at random from 7 men and 9 women. Find the probability that there are no men on the committee. [3]

$7M + 9W \quad -- \quad 16$

Possibilities

$9C_6$

Total $= {}^{16}C_6 = 8008$

No M $= 84$

0.0105

3 One plastic robot is given away free inside each packet of a certain brand of biscuits. There are four colours of plastic robot (red, yellow, blue and green) and each colour is equally likely to occur. Nick buys some packets of these biscuits. Find the probability that

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i) 0.25

ii) $(0.75)^4(0.25) = 0.791$

iii) $4! \times (0.25)^4 = \frac{3}{32}$

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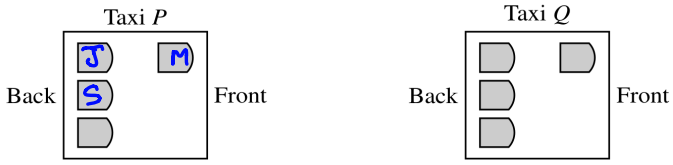

Each taxi can take 1 passenger in the front and 3 passengers in the back (see diagram). Mark sits in the front of taxi P and Jon and Sarah sit in the back of taxi P next to each other.

(ii) Find the number of different seating arrangements that are now possible for the 8 friends. [4]

i) J 2! S M N
$(^6C_2 + {}^6C_4) \times 2$

ii) $8 - 3 = 5$
P
1
1 1 –
1 Or
– 1 1
10

Q
–
– – – –
$(^4P_4)$
$24 \times 4!$

34

5 The weights, in kilograms, of the 15 rugby players in each of two teams, A and B, are shown below.

Team A	97	98	104	84	100	109	115	99	122	82	116	96	84	107	91
Team B	75	79	94	101	96	77	111	108	83	84	86	115	82	113	95

(i) Represent the data by drawing a back-to-back stem-and-leaf diagram with team A on the left-hand side of the diagram and team B on the right-hand side. [4]

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(iii) A new player joins team B as a substitute. The mean weight of the 16 players in team B is now 93.9 kg. Find the weight of the new player. [3]

i)

A		B
	7	5 7 9
4 4 2	8	2 3 4 6
9 8 7 6 1	9	4 5 6
9 7 4 0	10	1 8
6 5	11	1 3 5
2	12	

Key

4 | 10 | 1

means 104 for A

101 for B

ii) LQ = 91

UQ = 109

IQ = 18

iii) $\frac{1399 + x}{16} = 93.9$

$x = 103.4$

6 A fair spinner A has edges numbered 1, 2, 3, 3. A fair spinner B has edges numbered −3, −2, −1, 1. Each spinner is spun. The number on the edge that the spinner comes to rest on is noted. Let X be the sum of the numbers for the two spinners.

(i) Copy and complete the table showing the possible values of X. [1]

Spinner B \ Spinner A	1	2	3	3
−3	−2	−1	0	0
−2	−1	0	1	1
−1	0	1	2	2
1	2	3	4	4

(ii) Draw up a table showing the probability distribution of X. [3]

(iii) Find Var(X). [3]

(iv) Find the probability that X is even, given that X is positive. [2]

i)

x	−2	−1	0	1	2	3	4
$P(X=x)$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{4}{16}$	$\frac{3}{16}$	$\frac{3}{16}$	$\frac{1}{16}$	$\frac{2}{16}$

iii) $E(X^2) - E(X)^2$

$\frac{31}{8} - 1^2 = \frac{79}{24} = 2.875$ ✓

iv) $P(\text{even} \mid \text{Positive}) = \frac{P(\text{even} \cap \text{Positive})}{P(\text{positive})} = \frac{5/16}{9/16} = \frac{5}{9}$

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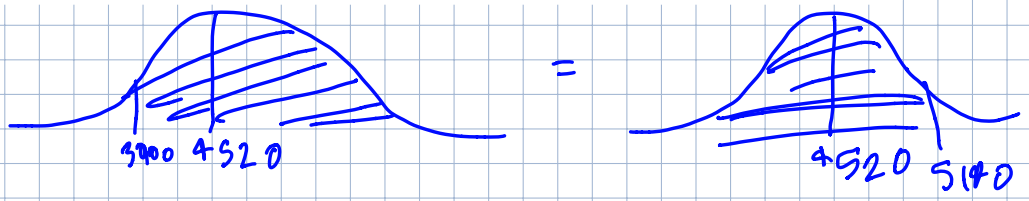
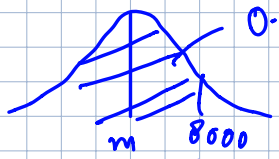
7ai)



ii) $1 - 0.878$



$\frac{620}{560} = 1.1071$

$\Phi(1.1071) = 0.8657$

0.8657×365

316

$\Phi\left(\frac{8000-m}{560}\right) = 0.878$

$\frac{8000-m}{560} = 1.165$

$m = 7347.6$

$= 7350$

iii) $P(X > 8000) = 0.122$

$X \sim B(6, 0.122)$

$P(X < 2) = P(0, 1)$

$= {}^6C_0 \times 0.122^0 \times 0.878^6 = 0.458107$

$+ {}^6C_1 \times 0.122^1 \times 0.8785 = 0.381929$

0.84

b) $\Phi\left(\frac{2\mu-\mu}{\frac{2}{3}\mu}\right) = \Phi\left(\frac{\mu}{\frac{2}{3}\mu}\right) = \Phi(1.5)$

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0.9332 0.933