Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2014**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

* 8 6 9 8 8 0 6 8 8 5 *

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **3** printed pages and **1** blank page.

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1 Solve the inequality $|3x - 1| < |2x + 5|$. [4]

2 A curve is defined for $0 < \theta < \frac{1}{2}\pi$ by the parametric equations

$$x = \tan\theta, \quad y = 2\cos^2\theta\sin\theta.$$

Show that $\frac{dy}{dx} = 6\cos^5\theta - 4\cos^3\theta$. [5]

3 The polynomial $4x^3 + ax^2 + bx - 2$, where a and b are constants, is denoted by $p(x)$. It is given that $(x + 1)$ and $(x + 2)$ are factors of $p(x)$.

(i) Find the values of a and b. [4]

(ii) When a and b have these values, find the remainder when $p(x)$ is divided by $(x^2 + 1)$. [3]

4 **(i)** Show that $\cos(\theta - 60°) + \cos(\theta + 60°) \equiv \cos\theta$. [3]

(ii) Given that $\frac{\cos(2x - 60°) + \cos(2x + 60°)}{\cos(x - 60°) + \cos(x + 60°)} = 3$, find the exact value of $\cos x$. [4]

5 The complex numbers w and z are defined by $w = 5 + 3i$ and $z = 4 + i$.

(i) Express $\frac{iw}{z}$ in the form $x + iy$, showing all your working and giving the exact values of x and y. [3]

(ii) Find wz and hence, by considering arguments, show that

$$\tan^{-1}\left(\tfrac{3}{5}\right) + \tan^{-1}\left(\tfrac{1}{4}\right) = \tfrac{1}{4}\pi.$$ [4]

6 It is given that $I = \int_0^{0.3} \left(1 + 3x^2\right)^{-2} dx$.

(i) Use the trapezium rule with 3 intervals to find an approximation to I, giving the answer correct to 3 decimal places. [3]

(ii) For small values of x, $\left(1 + 3x^2\right)^{-2} \approx 1 + ax^2 + bx^4$. Find the values of the constants a and b. Hence, by evaluating $\int_0^{0.3} (1 + ax^2 + bx^4)\,dx$, find a second approximation to I, giving the answer correct to 3 decimal places. [5]

7 The equations of two straight lines are

$$\mathbf{r} = \mathbf{i} + 4\mathbf{j} - 2\mathbf{k} + \lambda(\mathbf{i} + 3\mathbf{k}) \quad \text{and} \quad \mathbf{r} = a\mathbf{i} + 2\mathbf{j} - 2\mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} + 3a\mathbf{k}),$$

where a is a constant.

(i) Show that the lines intersect for all values of a. [4]

(ii) Given that the point of intersection is at a distance of 9 units from the origin, find the possible values of a. [4]

8 The variables x and y are related by the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \tfrac{1}{5}xy^{\frac{1}{2}}\sin\left(\tfrac{1}{3}x\right).$$

(i) Find the general solution, giving y in terms of x. [6]

(ii) Given that $y = 100$ when $x = 0$, find the value of y when $x = 25$. [3]

9 **(i)** Sketch the curve $y = \ln(x + 1)$ and hence, by sketching a second curve, show that the equation

$$x^3 + \ln(x + 1) = 40$$

has exactly one real root. State the equation of the second curve. [3]

(ii) Verify by calculation that the root lies between 3 and 4. [2]

(iii) Use the iterative formula

$$x_{n+1} = \sqrt[3]{\left(40 - \ln(x_n + 1)\right)},$$

with a suitable starting value, to find the root correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

(iv) Deduce the root of the equation

$$(\mathrm{e}^y - 1)^3 + y = 40,$$

giving the answer correct to 2 decimal places. [2]

10 By first using the substitution $u = \mathrm{e}^x$, show that

$$\int_0^{\ln 4} \frac{\mathrm{e}^{2x}}{\mathrm{e}^{2x} + 3\mathrm{e}^x + 2}\,\mathrm{d}x = \ln\left(\tfrac{8}{5}\right).$$ [10]

1 Solve the inequality $|3x-1| < |2x+5|$. [4]

$(3x-1)(3x-1) < (2x+5)(2x+5)$

$9x^2 - 6x + 1 < 4x^2 + 20x + 25$

$5x^2 - 26x - 24 < 0$

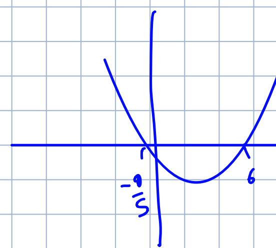
$\dfrac{-26 \pm \sqrt{26^2 - 4(5)(-24)}}{10}$

$x_{1,2}\ \dfrac{26 \pm 34}{10}$

$x_1 = 6 \qquad x_2 = -\dfrac{4}{5}$

$x > -\dfrac{4}{5} \qquad x < 6$

$-\dfrac{4}{5} < x < 6$



2 A curve is defined for $0 < \theta < \frac{1}{2}\pi$ by the parametric equations

$$x = \tan\theta, \quad y = 2\cos^2\theta\sin\theta.$$

Show that $\dfrac{dy}{dx} = 6\cos^5\theta - 4\cos^3\theta.$ [5]

$\frac{dy}{d\theta} = (2\cos^2\theta\cos\theta) + (\sin\theta)(.4\cos x\sin x)$

$= 2\cos^3 x - 4\cos x\sin^2 x$

$\frac{dx}{d\theta} = \sec^2\theta = \frac{1}{\cos^2\theta} \quad \therefore \frac{d\theta}{dx} = \cos^2$

$\frac{dy}{dx} = (2\cos^3 x - 4\cos x\sin^2 x)(\cos^2 x)$

$= 2\cos^3 x - 4\cos^3 x(1-\cos^2 x)$

$= 2\cos^5\theta - 4\cos^3\theta + 4\cos^5\theta$

$= 6\cos^5\theta - 4\cos^3\theta$

3 The polynomial $4x^3 + ax^2 + bx - 2$, where a and b are constants, is denoted by $p(x)$. It is given that $(x+1)$ and $(x+2)$ are factors of $p(x)$.

(i) Find the values of a and b. [4]

(ii) When a and b have these values, find the remainder when $p(x)$ is divided by (x^2+1). [3] 7

i) $x=-1$
$x=-2$

$-4 + a - b - 2 = 0$
$a - b = 6$
$a = b + 6$

$-32 + 4a - 2b - 2 = 0$
$4a - 2b = 34$
$4b + 24 - 2b = 34$
$2b = 10$
$b = 5$
$\therefore a = 11$

ii)

$$\begin{array}{r} 4x + 11 \\ x^2+1 \enclose{longdiv}{4x^3 + 11x^2 + 5x - 2} \\ -\ \underline{4x^3 + 4x} \\ 0 + 11x^2 + x \\ \underline{11x^2 + 11} \\ 0 + x - 13 \end{array}$$

$x - 13$

4 **(i)** Show that $\cos(\theta - 60°) + \cos(\theta + 60°) \equiv \cos\theta$. [3]

(ii) Given that $\dfrac{\cos(2x - 60°) + \cos(2x + 60°)}{\cos(x - 60°) + \cos(x + 60°)} = 3$, find the exact value of $\cos x$. [4]

i) $\cos\theta\cos 60° + \sin\theta \quad \sin 60$

$\frac{1}{2}\cos\theta + \cancel{\frac{\sqrt{3}}{2}\sin\theta} \quad + \quad \frac{1}{2}\cos\theta \quad \cancel{-\frac{\sqrt{3}}{2}\sin\theta}$

$= \cos\theta$

ii) $\dfrac{\cos 2x}{\cos x} = 3$

$\cos 2x = 3\cos x$

$2\cos^2 x - 1 - 3\cos x = 0$

$$\frac{3 \pm \sqrt{9 - 4(2)(-1)}}{4}$$

$\cos x = \dfrac{3 \pm \sqrt{17}}{4}$ ignore +

$\therefore \cos x = \dfrac{3 - \sqrt{17}}{4}$

5 The complex numbers w and z are defined by $w = 5 + 3\mathrm{i}$ and $z = 4 + \mathrm{i}$.

(i) Express $\dfrac{\mathrm{i}w}{z}$ in the form $x + \mathrm{i}y$, showing all your working and giving the exact values of x and y. [3]

(ii) Find wz and hence, by considering arguments, show that

$$\tan^{-1}\left(\tfrac{3}{5}\right) + \tan^{-1}\left(\tfrac{1}{4}\right) = \tfrac{1}{4}\pi. \qquad [4]$$

i)

$$\frac{5i - 3(4-i)}{(4+i)(4-i)}$$

$$\frac{20i + 5 - 12 + 3i}{16 + 1} = \frac{23i - 7}{17}$$

$$= \frac{-7}{17} + \frac{23}{17}i$$

ii)

$(5+3i)(4+i)$

$20 + 17i - 3$

$17 + 17i$

$\arg w = \tan^{-1}\left(\frac{3}{5}\right)$

$\text{“} \ z = \tan^{-1}\left(\frac{1}{4}\right)$

$\arg w + \arg z = \arg wz = \tan^{-1}\left(\frac{17}{17}\right)$

$= \frac{1}{4}\pi$

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where a is a constant.

(i) Show that the lines intersect for all values of a. [4]

(ii) Given that the point of intersection is at a distance of 9 units from the origin, find the possible values of a. [4]

i)

$$\begin{pmatrix} 1+\lambda \\ 4 \\ -2+3\lambda \end{pmatrix} = \begin{pmatrix} a+\mu \\ 2+2\mu \\ -2+3a\mu \end{pmatrix}$$

$4 = 2 + 2\mu$

$\mu =$

$\to \lambda = a$

① $-$ $1 + \lambda = a + 1$

② $-$ $-2 + 3\lambda = -2 + 3a$

$-2 + 3a = -2 + 3a$

$3a - 3a = -2 + 0$

$\underline{\underline{0 = 0}}$

ii)

$\sqrt{(1+\lambda)^2 + 16 + (-2+3\lambda)^2} = 9$

$(1+\lambda)(1+\lambda) + 16 + (-2+3\lambda)(-2+3\lambda) = 81$

$1 + 2\lambda + \lambda^2 + 16 + 4 - 12\lambda + 9\lambda^2 = 81$

$10\lambda^2 + 10\lambda - 60 = 0$

$\lambda^2 + \lambda - 6 = 0$

$\dfrac{-1 \pm \sqrt{1 - 4(1)(-6)}}{2}$

$\dfrac{-1 \pm 5}{2}$

$\therefore \lambda = 2$ or -3

Poi =

$\begin{pmatrix} 3 \\ 4 \\ 4 \end{pmatrix}$ or $\begin{pmatrix} -2 \\ 4 \\ -11 \end{pmatrix}$

$\therefore a = \underline{\underline{2}}$ or $\underline{\underline{-3}}$

8 The variables x and y are related by the differential equation

$$\frac{dy}{dx} = \tfrac{1}{5}xy^{\frac{1}{2}}\sin\left(\tfrac{1}{3}x\right).$$

(i) Find the general solution, giving y in terms of x. [6]

(ii) Given that $y = 100$ when $x = 0$, find the value of y when $x = 25$. [3]

i)

$$\int y^{-\frac{1}{2}}\,dy = \frac{1}{5}\int x\sin\left(\tfrac{1}{3}x\right)dx$$

$u = x \quad u' = 1 \quad v' = \sin\left(\tfrac{1}{3}x\right) \quad v = -3\cos\tfrac{1}{3}x$

$$2y^{\frac{1}{2}} = \frac{1}{5}\left[-3x\cos\tfrac{1}{3}x + 3\int\cos\tfrac{1}{3}x\,dx\right]$$

$$2y^{\frac{1}{2}} = \frac{1}{5}\left[-3x\cos\tfrac{1}{3}x + 9\sin\tfrac{1}{3}x + c\right]$$

$$2y^{\frac{1}{2}} = -\frac{3x\cos\left(\frac{1}{3}x\right)}{5} + \frac{9}{5}\sin\left(\tfrac{1}{3}x\right) + \frac{c}{5}$$

$$y^{\frac{1}{2}} = \frac{-3x\cos\left(\frac{1}{3}x\right) + 9\sin\left(\frac{1}{3}x\right) + c}{10}$$

$$y = \left(\frac{-3x\cos\left(\frac{1}{3}x\right) + 9\sin\left(\frac{1}{3}x\right)}{10} + c\right)^2$$

ii)

$100 = c^2$

$c = 10$

$$y = \left(\frac{-3(25)\cos\left(\frac{25}{3}\right) + 9\sin\frac{25}{3}}{10} + 10\right)^2$$

$y = \underline{\underline{203}}$

9 **(i)** Sketch the curve $y = \ln(x + 1)$ and hence, by sketching a second curve, show that the equation

$$x^3 + \ln(x + 1) = 40 \qquad 40 - x^3$$

has exactly one real root. State the equation of the second curve. [3]

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(iii) Use the iterative formula

$$x_{n+1} = \sqrt[3]{(40 - \ln(x_n + 1))},$$

with a suitable starting value, to find the root correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

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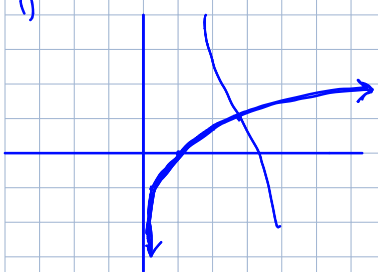
$$(e^y - 1)^3 + y = 40,$$

giving the answer correct to 2 decimal places. [2]

i)



x	y
0.37	-1
1	0
e	1
7.4	2

$x = e^y$

x	y^3
0	39
2	32
3	13
4	-24

$y = 40 - x^3$

ii)

LHS		RHS
40	>	28
40	<	65

change of sing

iv)

10 By first using the substitution $u = e^x$, show that

$$\int_0^{\ln 4} \frac{e^{2x}}{e^{2x} + 3e^x + 2}\,dx = \ln\left(\tfrac{8}{5}\right).$$

[10]

$u = e^x \quad \frac{du}{dx} = e^x \quad dx = \frac{du}{e^x} = \frac{du}{u} \quad u = e^0 = 1$

$u^2 = e^{2x} \quad u = e^{\ln 4} = 4$

$$\int_1^4 \frac{u^{\cancel{2}}}{u^2+3u+2}\,\frac{du}{\cancel{u}}$$

$$\int_1^4 \frac{u}{u^2+3u+2} \qquad \frac{u}{} = \frac{A}{(u+1)} + \frac{B}{(u+2)}$$

$u = A(u+2) + B(u+1)$

$$\int_1^4 -\frac{1}{u+1} + \frac{2}{u+2}\,du$$

$-2 = -B$

$B = 2$

$-1 = A$

$$-\frac{1}{u+1} + \frac{2}{u+2}$$

$$\Big[-\ln(u+1) + 2\ln(u+2)\Big]_1^4$$

$(-\ln 5 + 2\ln 6) - (-\ln 2 + 2\ln 3)$

$1.974 - 1.40$

$2\ln 6 - \ln 5 + \ln 2 - 2\ln 3$

$2\ln 6 - 2\ln 3 + \ln 2 - \ln 5$

$2\ln 2 + \ln\frac{2}{5}$

$\ln 2^2 + \ln\frac{2}{5}$

$\ln 4 + \ln\frac{2}{5}$

$= \ln\left(\frac{8}{5}\right)$

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