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Ask Shorp

Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER

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MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2018**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **20** printed pages.

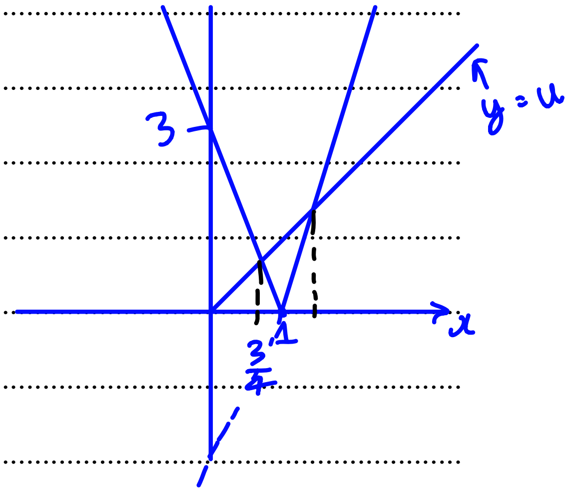
JC18 06_9709_32/RP

CAMBRIDGE International Examinations

[Turn over

1 Showing all necessary working, solve the equation $3|2^x - 1| = 2^x$, giving your answers correct to 3 significant figures. [4]

let $u = 2^x$ $\quad 3|u-1| = u$



① $y = -3u + 3 = u$

$3 = 4u$

$u = \frac{3}{4}$

② $y = 3u - 3 = u$

$2u = 3$

$u = \frac{3}{2}$

$\therefore \quad 2^x = \frac{3}{4} \quad$ or $\quad 2^x = \frac{3}{2}$

$x\ln 2 = \ln\frac{3}{4} \quad$ or $\quad x\ln 2 = \frac{3}{2}$

$x = -0.415 \quad$ or $\quad x = 0.585$

2 Showing all necessary working, solve the equation $\cot\theta + \cot(\theta + 45°) = 2$, for $0° < \theta < 180°$. [5]

$$\frac{\cos\theta}{\sin\theta} + \frac{\cos(\theta+45)}{\sin(\theta+45)} = 2$$

$$\frac{\cos\theta}{\sin\theta} + \frac{\cos\theta\cos45 - \sin\theta\sin45}{\sin\theta\cos45 + \cos\theta\sin45} = 2$$

$$\frac{\cos\theta}{\sin\theta} + \frac{\cancel{\frac{\sqrt{2}}{2}}(\cos\theta - \sin\theta)}{\cancel{\frac{\sqrt{2}}{2}}(\sin\theta + \cos\theta)} = 2$$

$$\frac{\cos\theta(\sin\theta+\cos\theta) + \sin\theta(\cos\theta - \sin\theta)}{\sin\theta(\sin\theta+\cos\theta)} = 2$$

$$\frac{\sin\theta\cos\theta + \cos^2\theta + \sin\theta\cos\theta - \sin^2\theta}{\sin^2\theta + \sin\theta\cos\theta} = 2$$

$$\cancel{2\sin\theta\cos\theta} + \cos 2\theta = 2\sin^2\theta + \cancel{2\sin\theta\cos\theta}$$

$$1 - 2\sin^2\theta = 2\sin^2\theta$$

$$4\sin^2\theta = 1$$

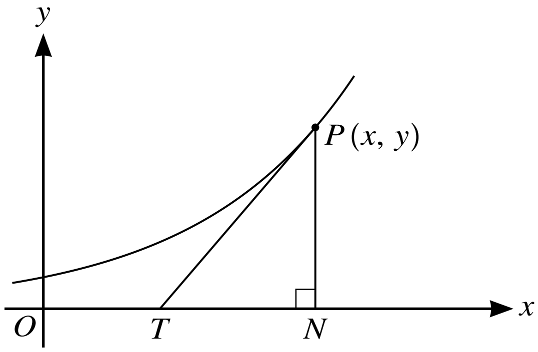
$$\sin\theta = \pm\sqrt{\frac{1}{4}}$$

$\sin\theta = \sqrt{\frac{1}{4}}$ or $\sin\theta = -\sqrt{\frac{1}{4}}$

$\theta = 30°$ or $150°$

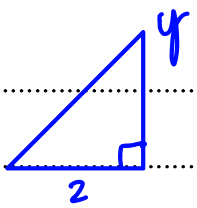
$\theta =$ ✗ more than 180°

3



In the diagram, the tangent to a curve at the point P with coordinates (x, y) meets the x-axis at T. The point N is the foot of the perpendicular from P to the x-axis. The curve is such that, for all values of x, the gradient of the curve is positive and $TN = 2$.

(i) Show that the differential equation satisfied by x and y is $\frac{dy}{dx} = \frac{1}{2}y$. [1]



$y = mx + C$

$m = \frac{y - 0}{2} = \frac{dy}{dx} = \frac{1}{2}y$

The point with coordinates $(4, 3)$ lies on the curve.

(ii) Solve the differential equation to obtain the equation of the curve, expressing y in terms of x. [5]

$\int \frac{1}{y}\,dy = \frac{1}{2}\int dx$

$\ln y = \frac{1}{2}x + C$

$\ln 3 = 2 + C$

$C = \ln(3) - 2$

$\therefore \ln y = \frac{1}{2}x - 2 + \ln 3$

$y = \frac{e^{\frac{1}{2}x}}{e^2} \times e^{\ln 3}$

$y = \frac{e^{\frac{1}{2}x}}{e^2} \times 3$

$y = 3e^{\frac{1}{2}x-2}$

4 **(i)** Show that $\dfrac{2\sin x - \sin 2x}{1 - \cos 2x} \equiv \dfrac{\sin x}{1 + \cos x}$. [4]

$$\frac{2\sin x - 2\sin x\cos x}{1 - \cos^2 x + 1}$$

$$= \frac{2\sin x - 2\sin x\cos x}{2 - 2\cos^2 x}$$

$$= \frac{\cancel{2}(\sin x - \sin x\cos x)}{\cancel{2}(1 - \cos^2 x}$$

$$\frac{\sin x - \sin x\cos x}{1 - \cos^2 x}$$

$(1-\cos x)(1+\cos x)$

$1 - \cos^2 x$

$$\frac{\sin x\cancel{(1-\cos x)}}{\cancel{(1-\cos x)}(1+\cos x)}$$

$$\frac{\sin x}{1+\cos x}$$

(ii) Hence, showing all necessary working, find $\int_{\frac{1}{3}\pi}^{\frac{1}{2}\pi} \frac{2\sin x - \sin 2x}{1 - \cos 2x}\, dx$, giving your answer in the form $\ln k$. [4]

$\int \frac{\sin x}{1+\cos x}\, dx \quad \longrightarrow \frac{d}{dx} = -\sin x$

$-1 \int_{\frac{1}{3}\pi}^{\frac{1}{2}\pi} \frac{-\sin x}{1+\cos x}\, dx$

let $f(x) = -\ln|1+\cos x|\, dx$

$f(\tfrac{1}{2}\pi) = -\ln(1) = 0$

$f(\tfrac{1}{3}\pi) = -\ln(1+\tfrac{1}{2}) = -\ln\tfrac{3}{2}$

$\therefore \quad 0 - (-\ln\tfrac{3}{2})$

$\ln\tfrac{3}{2}$

5 The equation of a curve is $x^2(x + 3y) - y^3 = 3$.

(i) Show that $\dfrac{dy}{dx} = \dfrac{x^2 + 2xy}{y^2 - x^2}$. [4]

$x^3 + 3x^2y - y^3 = 3$

$3x^2 + 3x^2\frac{dy}{dx} + 6xy - 3y^2\frac{dy}{dx} = 3$

$\cancel{3}(x^2 + 2xy) = \cancel{3}(y^2 - x^2)\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{x^2 + 2xy}{y^2 - x^2}$

(ii) Hence find the exact coordinates of the two points on the curve at which the gradient of the normal is 1. [4]

if m of normal = 1 $\therefore$ m of curve = -1

$$\frac{x^2+2xy}{y^2-x^2} = -1$$

$$\cancel{x^2}+2xy = \cancel{x^2} - y^2$$

$$2xy + y^2 = 0$$

$y^2 = -2x\cancel{y}$

$y = -2x$

$y(2x+y) = 0$

$y = 0$

$x^3+3x^2y-y^3 = 3$

$x^3+3x^2(-2x)-(-2x)^3 = 3$

$x^3-6x^3+8x^3 = 3$

$3x^3 = 3$

$x^3 = 1$

$x = 1$

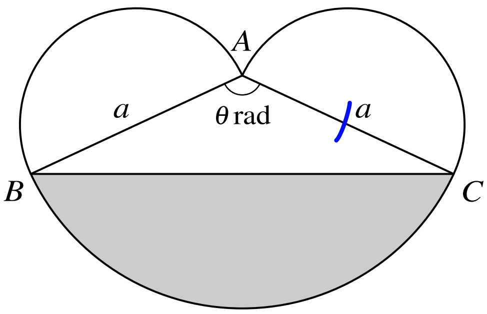
$x^3+3x^2(0)-0 = 3$

$x^3 = 3$

$x = \sqrt[3]{3}$

$\therefore$ $(1,-2)$ and $\rightarrow$ $(\sqrt[3]{3}, 0)$

6



The diagram shows a triangle ABC in which $AB = AC = a$ and angle $BAC = \theta$ radians. Semicircles are drawn outside the triangle with AB and AC as diameters. A circular arc with centre A joins B and C. The area of the shaded segment is equal to the sum of the areas of the semicircles.

(i) Show that $\theta = \frac{1}{2}\pi + \sin\theta$. [3]

shaded area $= \frac{1}{2}a^2\theta - \frac{1}{2}a^2\sin\theta$

$= \frac{1}{2}a^2(\theta - \sin\theta)$

area of 1 semicircle $= \frac{\pi\left(\frac{a}{2}\right)^2}{2} = \frac{\pi a^2}{4} \times \frac{1}{2} = \frac{\pi a^2}{8}$

$\therefore$ area of 2 '' $= \frac{\pi a^2}{4}$

$\frac{1}{2}\cancel{a^2}(\theta - \sin\theta) = \frac{\pi \cancel{a^2}}{4}$

$\theta - \sin\theta = \frac{1}{2}\pi$

$\theta = \frac{1}{2}\pi + \sin\theta$

(ii) Verify by calculation that θ lies between 2.2 and 2.4. [2]

let $f(x) = \frac{1}{2}\pi - \theta + \sin\theta$

$f(2.2) = 0.179$

$f(2.4) = -0.1537$

change of sign

(iii) Use an iterative formula based on the equation in part **(i)** to determine θ correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

$\theta = \frac{1}{2}\pi + \sin\theta$

let $\theta_1 = 2.3$

$\therefore \theta_2 = \frac{1}{2}\pi + \sin 2.3 = 2.3165$

$\theta_3 = 2.3054$ $\quad\therefore \theta = 2.31$

$\theta_4 = 2.3129$

$\theta_5 = 2.3079$

$\theta_6 = 2.3112$

$\theta_7 = 2.3089$

$\theta_8 = 2.3105$

7 Throughout this question the use of a calculator is not permitted.

The complex numbers $-3\sqrt{3} + \mathrm{i}$ and $\sqrt{3} + 2\mathrm{i}$ are denoted by u and v respectively.

(i) Find, in the form $x + \mathrm{i}y$, where x and y are real and exact, the complex numbers uv and $\frac{u}{v}$. [5]

$u \cdot v$

$(-3\sqrt{3} + i)(\sqrt{3} + 2i)$

$-3(3) - 6\sqrt{3}i + \sqrt{3}i + 2(-1)$

$-9 - 5\sqrt{3}i - 2$

$-11 - 5\sqrt{3}i$

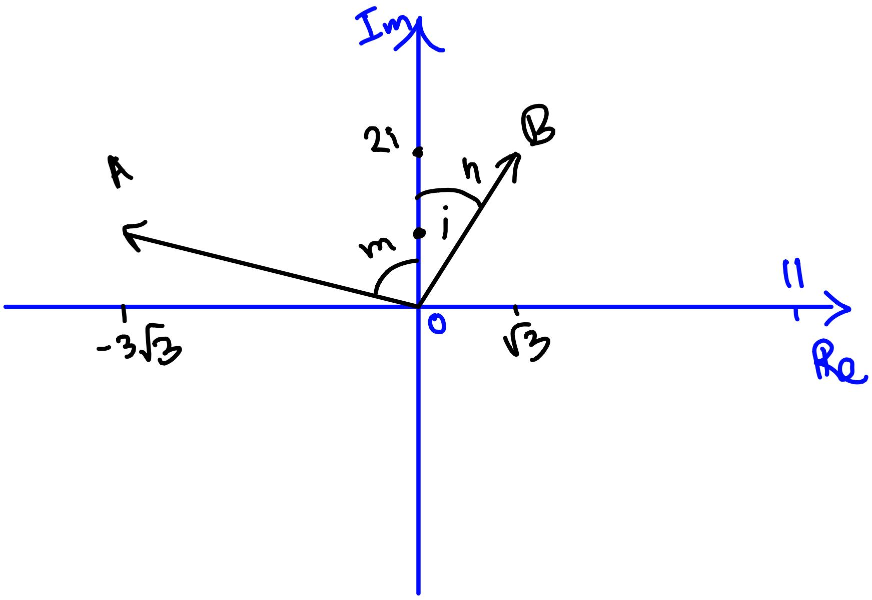
$$\frac{u}{v} = \frac{-3\sqrt{3} + i \ (\sqrt{3} - 2i)}{\sqrt{3} + 2i \ (\sqrt{3} - 2i)}$$

$$= \frac{-9 + 6\sqrt{3}i + \sqrt{3}i - 2(-1)}{3 - 4(-1)}$$

$$= \frac{-7 + 7\sqrt{3}i}{7}$$

$$= -1 + \sqrt{3}i$$

(ii) On a sketch of an Argand diagram with origin O, show the points A and B representing the complex numbers u and v respectively. Prove that angle $AOB = \frac{2}{3}\pi$. [3]



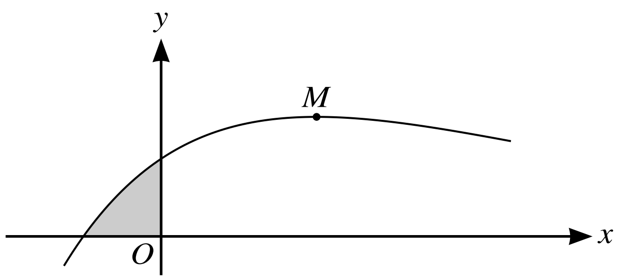
$AOB = \angle m + \angle n$

$\angle m = \tan^{-1}\left(\frac{3\sqrt{3}}{1}\right)$

$\angle n = \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\tan^{-1}\left(3\sqrt{3}\right) + \tan^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{2}{3}\pi$

8



The diagram shows the curve $y = (x+1)e^{-\frac{1}{3}x}$ and its maximum point M.

(i) Find the x-coordinate of M. [4]

$y = (x+1)\, e^{-\frac{1}{3}x}$

$u \qquad v$

$\frac{dy}{dx} = 0 = (x+1)\left(-\frac{1}{3}e^{-\frac{1}{3}x}\right) + \left(e^{-\frac{1}{3}x}\right)(1) = 0$

$-\frac{xe^{-\frac{1}{3}x}}{3} - \frac{1}{3}e^{-\frac{1}{3}x} + e^{-\frac{1}{3}x} = 0$

$-\frac{1}{3}xe^{-\frac{1}{3}x} + \frac{2}{3}e^{-\frac{1}{3}x} = 0$

$e^{-\frac{1}{3}x}\left(-\frac{1}{3}x + \frac{2}{3}\right) = 0$

$\frac{1}{3}x = \frac{2}{3}$

$x = 2$

(ii) Find the area of the shaded region enclosed by the curve and the axes, giving your answer in terms of e. [5]

$\int_{-1}^{0} (x+1)\, e^{-\frac{1}{3}x} \, dx$ $\quad u \quad v'$

when $x = 0$, $y = 1$ | $y = 1$, $x = -1$

2.34

$u = x+1 \qquad u' = 1$

$V = -3e^{-\frac{1}{3}x} \qquad V' = e^{-\frac{1}{3}x}$

$(x+1)(-3e^{-\frac{1}{3}x}) - \int -3e^{-\frac{1}{3}x} \, dx$

$-3xe^{-\frac{1}{3}x} - 3e^{-\frac{1}{3}x} + 3\int e^{-\frac{1}{3}x} dx$

$-3xe^{-\frac{1}{3}x} - 3e^{-\frac{1}{3}x} + 3(-3)e^{-\frac{1}{3}x}$

$-3xe^{-\frac{1}{3}x} - 12e^{-\frac{1}{3}x}$

let $f(x) = -3e^{-\frac{1}{3}x}(x+4)$

$f(0) = -3(1)(4) = -12$

$f(-1) = -3e^{\frac{1}{3}}(3) = -9e^{\frac{1}{3}}$

$\therefore -12 + 9e^{\frac{1}{3}}$

$9e^{\frac{1}{3}} - 12$

9 Let $f(x) = \dfrac{x - 4x^2}{(3-x)(2+x^2)}$.

(i) Express $f(x)$ in the form $\dfrac{A}{3-x} + \dfrac{Bx+C}{2+x^2}$. [4]

$(x - 4x^2) = A(2+x^2) + (Bx+C)(3-x)$

$2A + Ax^2 + 3Bx - Bx^2 + 3C - Cx$

let $x = 3$

$3 - 4(3)^2 = A(2+9) + (C)(0)$

$-33 = 11A$

$A = -3$

$-3(2+x^2)$

$-6 - 3x^2$

$-x^2 + x + 6 = 3Bx - Bx^2 + 3C - Cx$

$-Bx^2 = -\cancel{x^2}$

$B = 1$

$\cancel{-x^2} + x + 6 = 3x - \cancel{x^2} + 3C - Cx$

$-2x + 6 = 3C - Cx$

$Cx = 2x$

$C = 2$

(ii) Hence obtain the expansion of $\mathrm{f}(x)$ in ascending powers of x, up to and including the term in x^3. [5]

$-3(3-x)^{-1} + (x+2)(2+x^2)^{-1}$

$-3(3)^{-1}\left(1-\frac{x}{3}\right)^{-1}$

$-1\left[1+(-1)\left(-\frac{x}{3}\right)+\frac{(-1)(-2)\left(-\frac{x}{3}\right)^2}{2!}+\frac{(-1)(-2)(-3)\left(-\frac{x}{3}\right)^3}{3!}\right]$

$-1\left(1+\frac{x}{3}+\frac{1}{9}x^2+\frac{1}{27}x^3\right)$

$-1-\frac{x}{3}-\frac{x^2}{9}-\frac{x^3}{27}$

$(x+2)\times 2^{-1}\left(1+\frac{x^2}{2}\right)^{-1}$

$(x+2)\left(\frac{1}{2}\right)\left(1+(-1)\left(\frac{x^2}{2}\right)\right)$

$(x+2)\left(\frac{1}{2}-\frac{x^2}{4}\right)$

$\frac{1}{2}x-\frac{x^3}{4}+1-\frac{1}{2}x^2$

$\cancel{-1}-\frac{x}{3}-\frac{x^2}{9}-\frac{x^3}{27}+\frac{1}{2}x-\frac{x^3}{4}+\cancel{1}-\frac{1}{2}x^2$

$\frac{1}{6}x-\frac{11x^2}{18}-\frac{31}{108}x^3$

10 Two lines l and m have equations $\mathbf{r} = 2\mathbf{i} - \mathbf{j} + \mathbf{k} + s(2\mathbf{i} + 3\mathbf{j} - \mathbf{k})$ and $\mathbf{r} = \mathbf{i} + 3\mathbf{j} + 4\mathbf{k} + t(\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ respectively.

(i) Show that the lines are skew. [4]

$$\begin{pmatrix} 2+2s \\ -1+3s \\ 1-s \end{pmatrix} = \begin{pmatrix} 1+t \\ 3+2t \\ 4+t \end{pmatrix}$$

$2+2s = 1+t$

(1) $2s - t = -1$

$-1+3s = 3+2t$

(2) $3s - 2t = 4$

(2) − 2×(1)

(1)×2 = $4s - 2t = -2$

$3s - 2t = 4$

$-\ 4s - 2t = -2$

$-s + 0 = 6$

$s = -6$

$\therefore t = 2s + 1$

$= -12 + 1 = -11$

$t = -11$

$1 - (-6) = 4 + (-11)$

$7 \neq -7$

$\therefore$ skew

A plane p is parallel to the lines l and m.

(ii) Find a vector that is normal to p. [3]

(iii) Given that p is equidistant from the lines l and m, find the equation of p. Give your answer in the form $ax + by + cz = d$. [3]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.