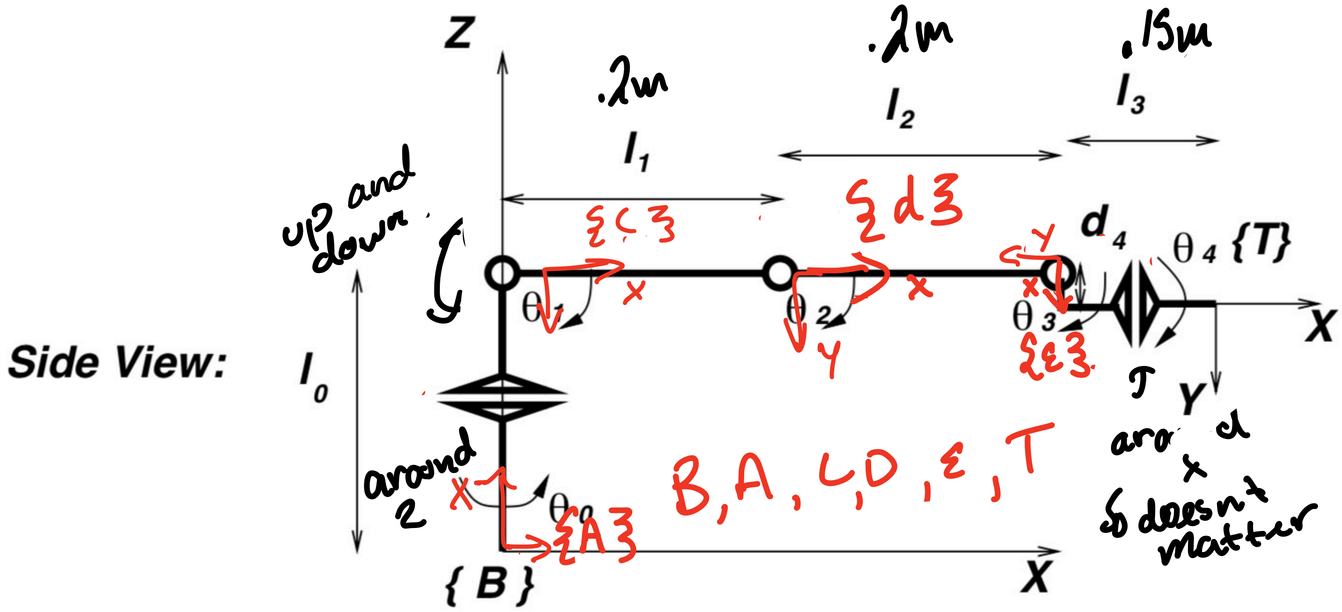
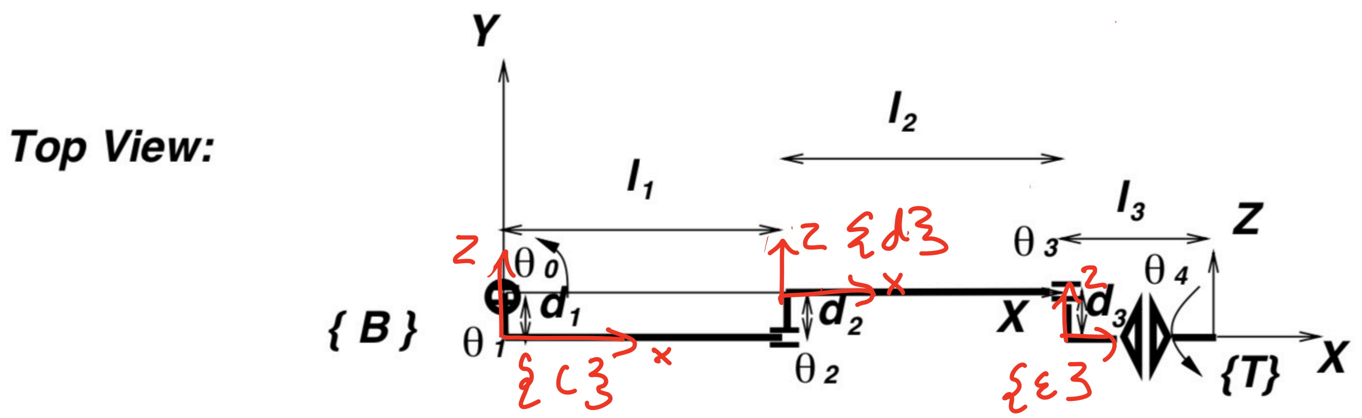
Gabriel de Sa





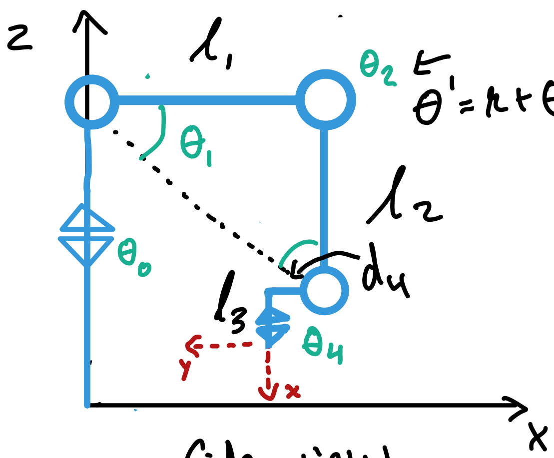
1. Forward Kinematic Derivation:

$$^B_T T = {}^B_A T \cdot {}^A_C T \cdot {}^C_D T \cdot {}^D_\varepsilon T \cdot {}^\varepsilon_T T$$

$$= R_z(\theta_0) D_z(l_0) \cdot R_y(\theta_1) D_x(l_1) D_y(d_1) \cdot R_y(\theta_2) D_x(l_2) D_y(d_2)$$
$$\cdot R_y(\theta_3) D_x(l_3) D_y(d_3) D_z(d_4) \cdot {}^\varepsilon_T T$$

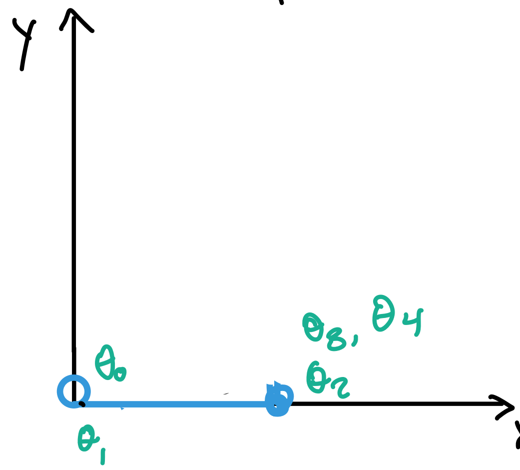
- ${}^\varepsilon_T T$ doesn't change the x, y, or z location of the tool frame. It acts like a wrist and rotates around its own axis.
- Since orientation is not required, there is no need to include it in our calculations.
- D_1 and D_2 cancel, so we can simply have them as the identity matrix and remove them from the equation.

2. Inverse Kinematic Derivation:



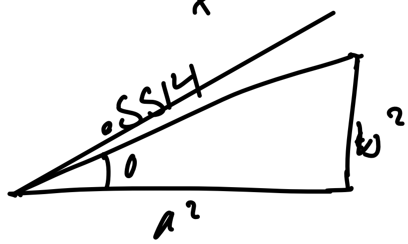
Side view

Top view



1. Solve for θ_0 in top view and remove offset

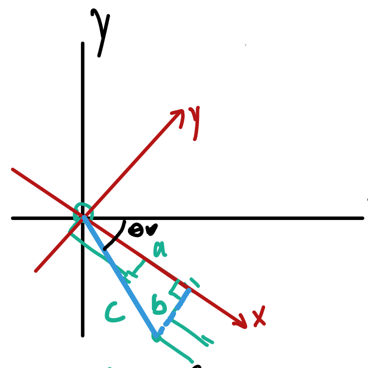
$$\theta_0 = \operatorname{atan}\frac{y}{x} - \operatorname{asin}\left(\frac{d_3}{\sqrt{x^2+y^2}}\right)$$



2. Move base frame and remove offset.

$$x' = x - l_3\cos\left(\frac{\pi}{2}\right), \quad z' = z + l_3\sin\left(\frac{\pi}{2}\right)$$

$$\begin{pmatrix} x'' \\ y'' \\ z'' \end{pmatrix} = \begin{pmatrix} \sqrt{c^2-b^2} + d_4 \\ 0 \\ z' - l_0 \end{pmatrix}$$



$a^2 = c^2 - b^2$
$c = \sqrt{x^2+y^2}$
$b = d_3$

$$\theta_2 = \pm \operatorname{acos}\left(\frac{x^2+z^2-l_1^2-l_2^2}{2 l_1 l_2}\right)$$

$$\alpha = \operatorname{acos}\left(\frac{l_1^2 + x^2 + z^2 - l_2^2}{2 l_1 \left(\sqrt{x^2+z^2}\right)}\right) \qquad \psi = \operatorname{atan}\frac{z}{x}$$

$\theta_1 = \psi + \alpha \; (-1)$ since θ_1 can only be negative on the robot

4. Solve for θ_3

$$\theta_3 = \Phi - \theta_1 - \theta_2$$
$$= \frac{\pi}{2} - \theta_1 - \theta_2$$

$\Phi = \frac{\pi}{2}$ radians because it points down