APPLYING SATISFIABILITY TO THE ANALYSIS OF CRYPTOGRAPHY

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WHY USE SAT FOR CRYPTOGRAPHY?

- Cryptographic algorithms are critical
  - Central to commerce, private communication, etc.
  - We want them to be correct
- Cryptography is hard
  - Rare expertise
  - Top experts have designed algorithms now known vulnerable
  - Design $\rightarrow$ implementation can introduce other problems
- Key primitives used in cryptography amenable to SAT
  - Can be given denotational semantics in propositional logic
  - Many interesting problems surprisingly tractable

CRYPTOGRAPHIC OPERATIONS AS SAT PROBLEMS

- Many primitives naturally made of bit vector operations
  - Block ciphers
  - Stream ciphers
  - Hash functions
  - Pseudo-random number generators (PRNGs)
- Public-key algorithms representable, but trickier
  - Lots of number theory, including multiplication
  - SMT can alleviate some of this (but tends to be slower on bit-vectory things)
- We'll focus on primitives with type $\{0,1\}^n \rightarrow \{0,1\}^m$

TRANSLATING ALGORITHMS TO PROPOSITIONAL LOGIC

- A function from the ZUC stream cipher

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uint8_t c = a + b;
if (c & 0x80) {
  c = (c & 0x7F) + 1;
}
return c;
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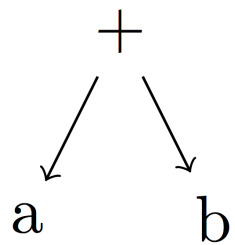

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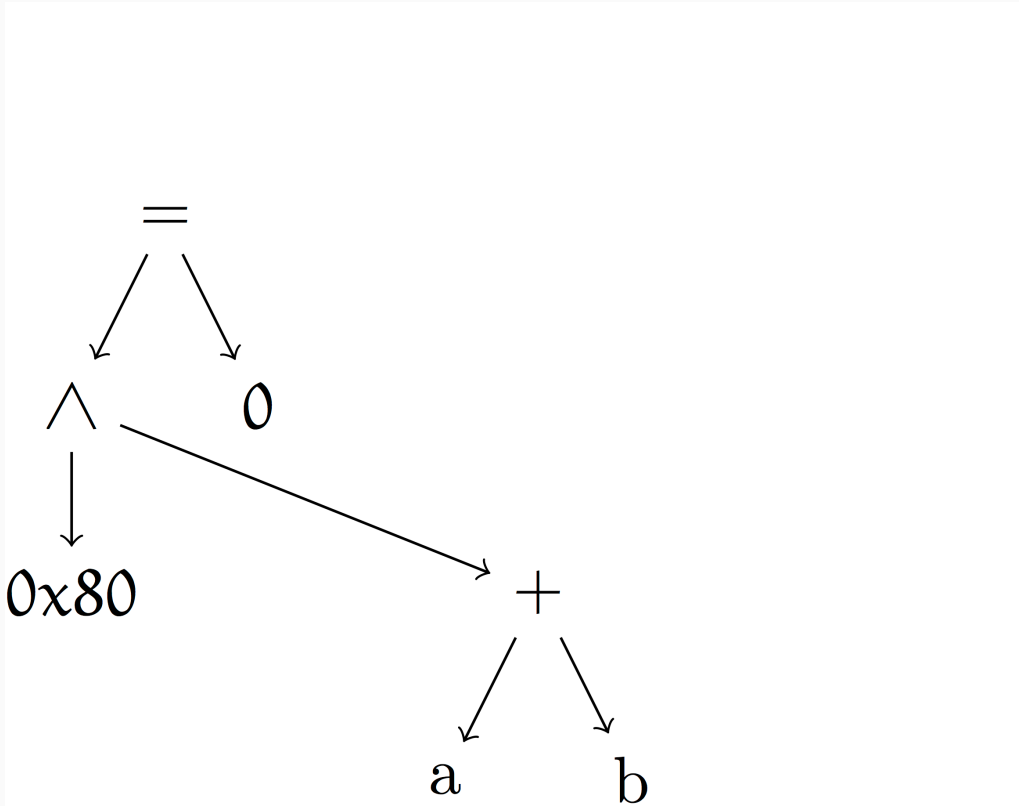

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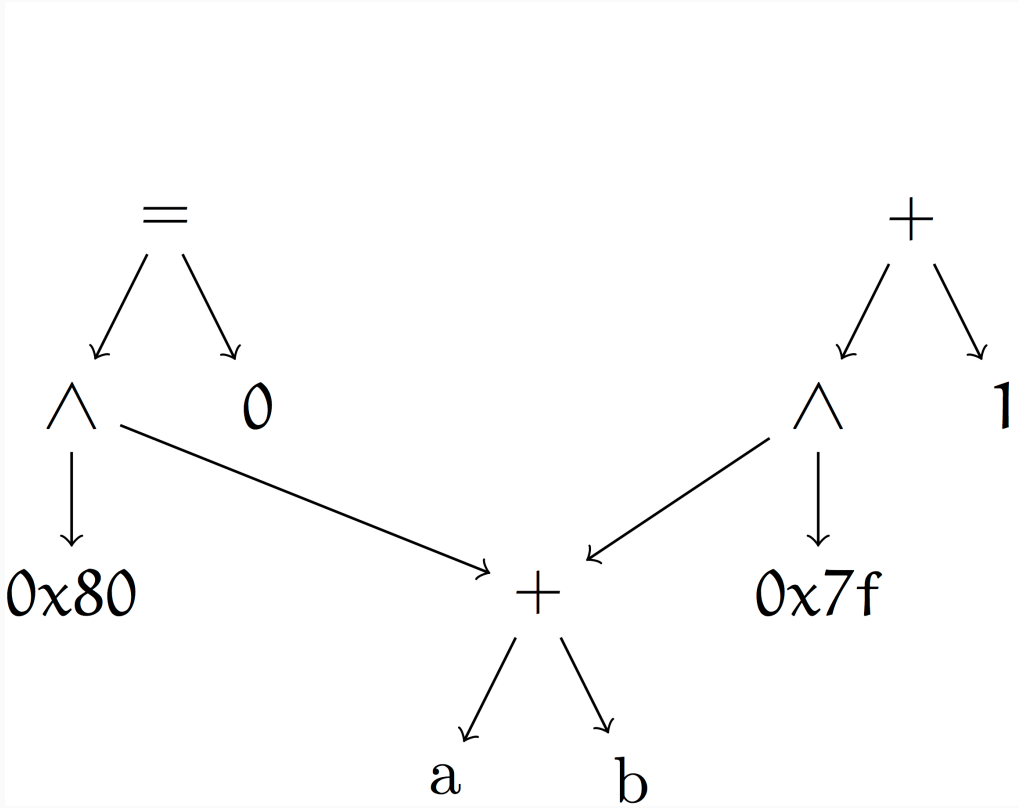

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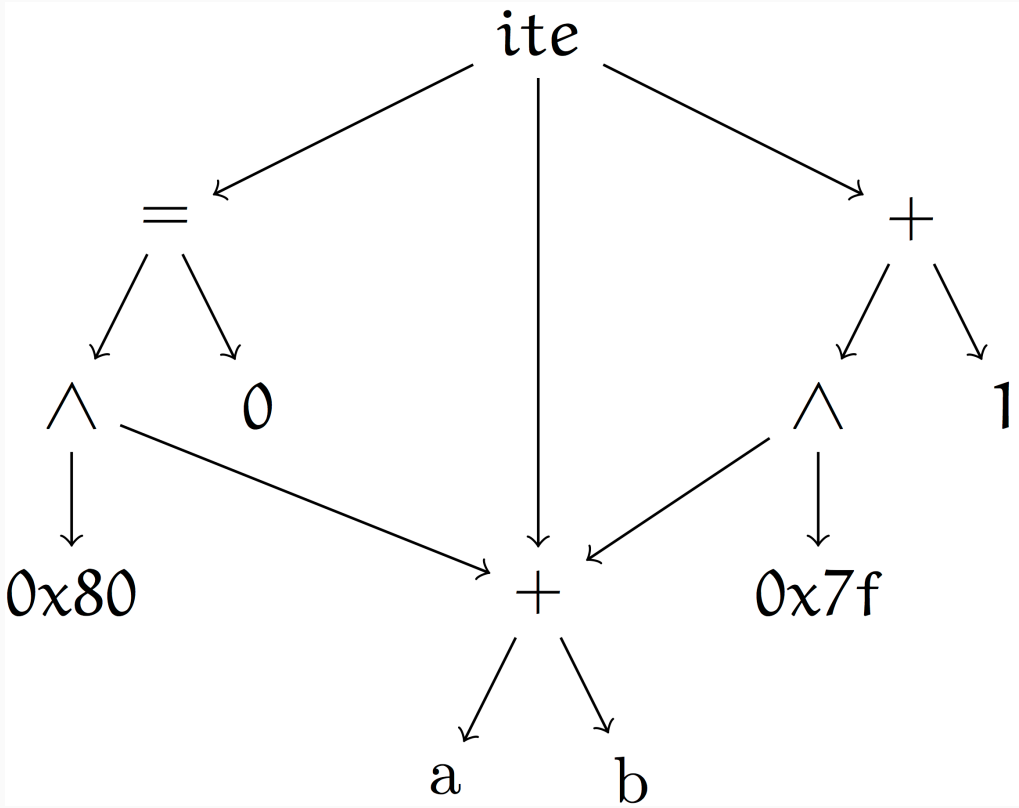
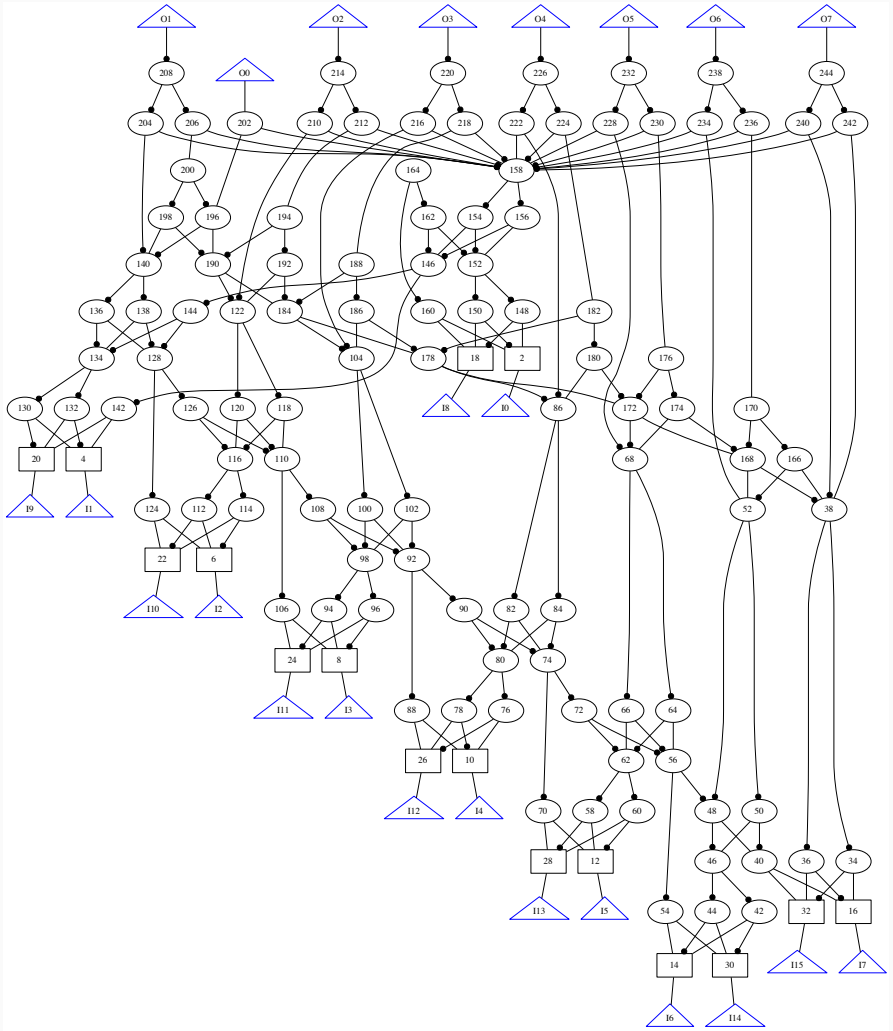


TRANSLATING ALGORITHMS TO PROPOSITIONAL LOGIC (CONT.)



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SPECIFIC TOOLS

- Transalg (Russian Academy of Sciences) [@transalg]
- Cryptol and the Software Analysis Workbench (SAW) (Galois)
  - Cryptol for describing algorithms concisely
  - SAW for proving properties about them
- TODO: others

- Each with a concise formula summarizing the SAT problem

$$\forall x.\ P \text{ or } \exists y.\ Q$$

- Cryptol specifications available online
- Analyzed using SAW + ABC
- Cryptol or SAW file names mentioned in slides (e.g. [file.saw])

`https://github.com/galoisinc/sat2015-crypto`

file.saw $\longrightarrow$ examples/file.saw

PSEUDO-RANDOM GENERATORS

- Randomness is critical to cryptography
  - Keys must be unpredictable, or they're vulnerable
- Typical structure:

  seed $\longrightarrow$ pseudo-random function $\longrightarrow$ pseudo-random value
- Seed often comes from "true" random source
- But it's critical not to lose entropy from this source!

INJECTIVITY OF PRNG SEEDING

- Bug in Android cryptographic PRNG discarded entropy
- Intended: 160 bits of entropy, from system PRNG to SHA-1

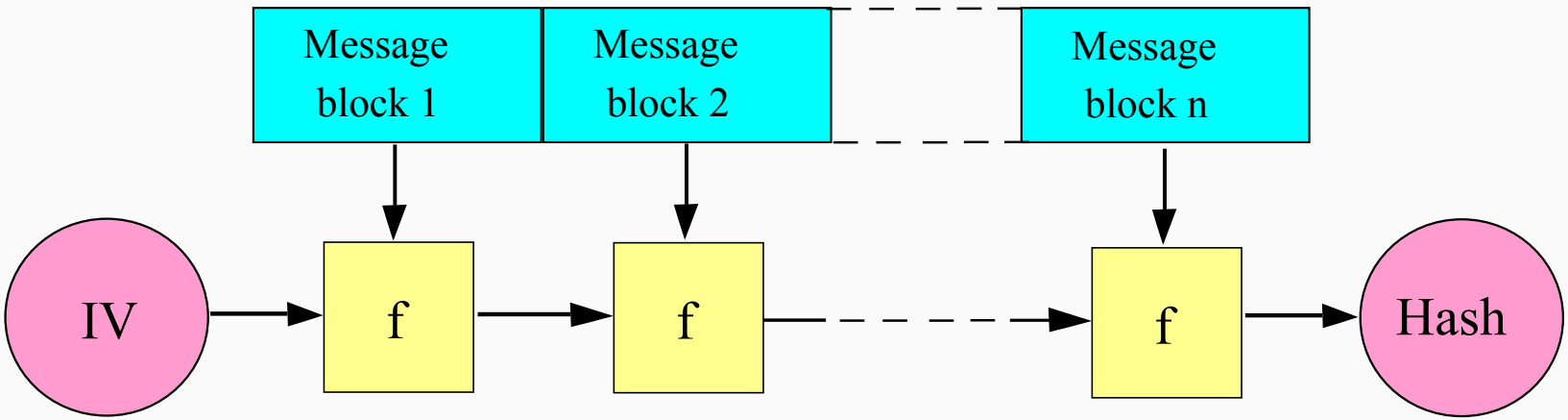
- Implemented: 64 bits of entropy survive

- Used to steal around $6k of BitCoin
- Fixed in Android 4.4 (KitKat)
- Similar weaknesses existed in Debian, FreeBSD at times

INJECTIVITY OF PRNG SEEDING (CONT.)

$$\forall x, y.\ x \neq y \Rightarrow f(x) \neq f(y)$$

- Here, x and y come from system PRNG, f is code between system PRNG and SHA-1
- Easily provable with SAT solver [android-prng.saw]
  - Symbolic execution: Java code $\rightarrow$ AIG
  - Annotation on system RNG variables, input to SHA-1
  - ABC can find collision (0.017s), prove fixed version (0.010s)
- Dörre and Klebanov used a different approach to prove fixed code [@dorre2015prng]
  - Information flow annotations on methods using a contract verification tool (KeY) and 95 manual proof steps



- Key property: hard to find two messages that have the same hash (a collision)
- Often built using Merkle–Damgård construction, iterating compression function
- Compression function f usually n iterations of simpler function g

$$\exists x, y.\ x \neq y \wedge f(x) = f(y)$$

- Discovering a collision
- Black box search in $O(2^{n/2})$

$$\exists x.\ f(x) = a \text{ (for some known value } a)$$

- Discovering message given hash value (inversion)
- Harder than finding a collision ($O(2^n)$)
- We want to know that it's hard to solve these problems

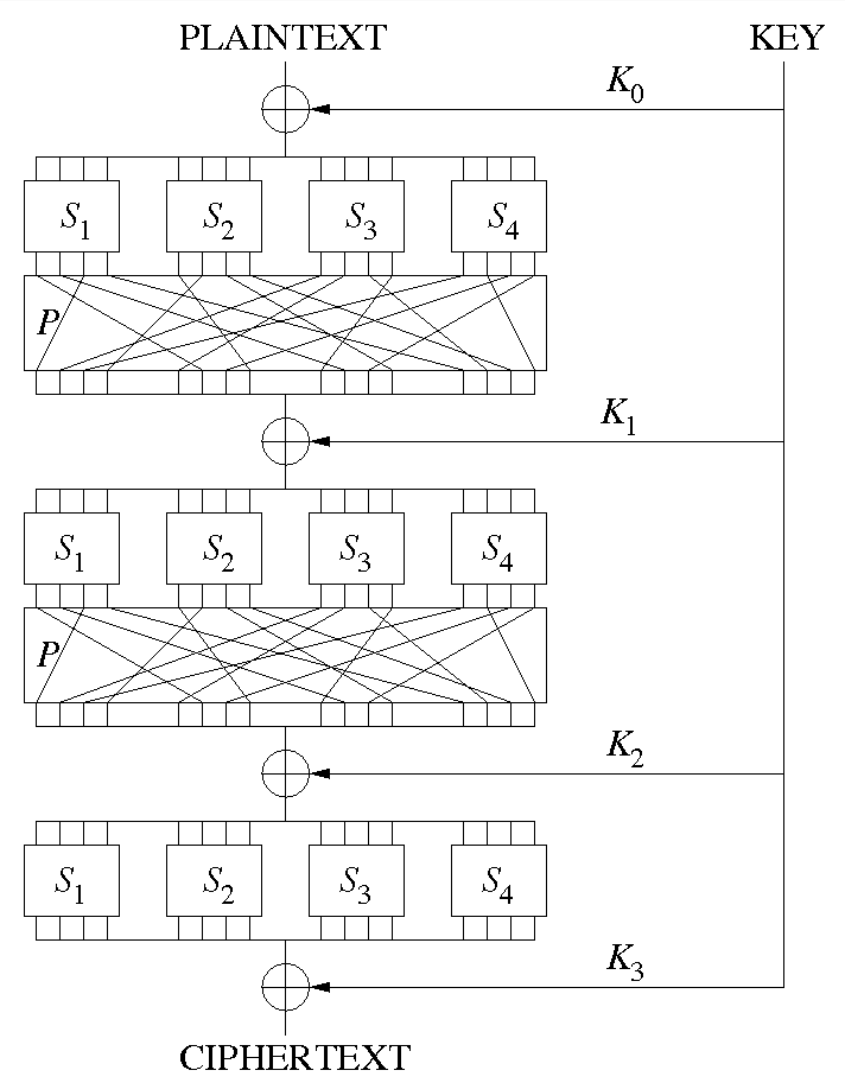
- Mironov and Zhang analyzed collisions in MD4, MD5, SHA-0
  - Direct translation of MD4 $\rightarrow 2^{22}$ solutions in $<$ 1h
  - Collision on full MD5 in around 100h
    - Reduced rounds much easier (25 in 38s with ABC) [MD5.saw]
    - Used differential path derived manually (more on this later)
  - Estimated around 3 million (2006-era) CPU hours for SHA-0
- No known collisions on SHA-1 from this, but it may be a matter of time
  - Algorithm (not SAT-based) to find collisions in 2^{63} operations

EVALUATING SHA-3 CANDIDATES

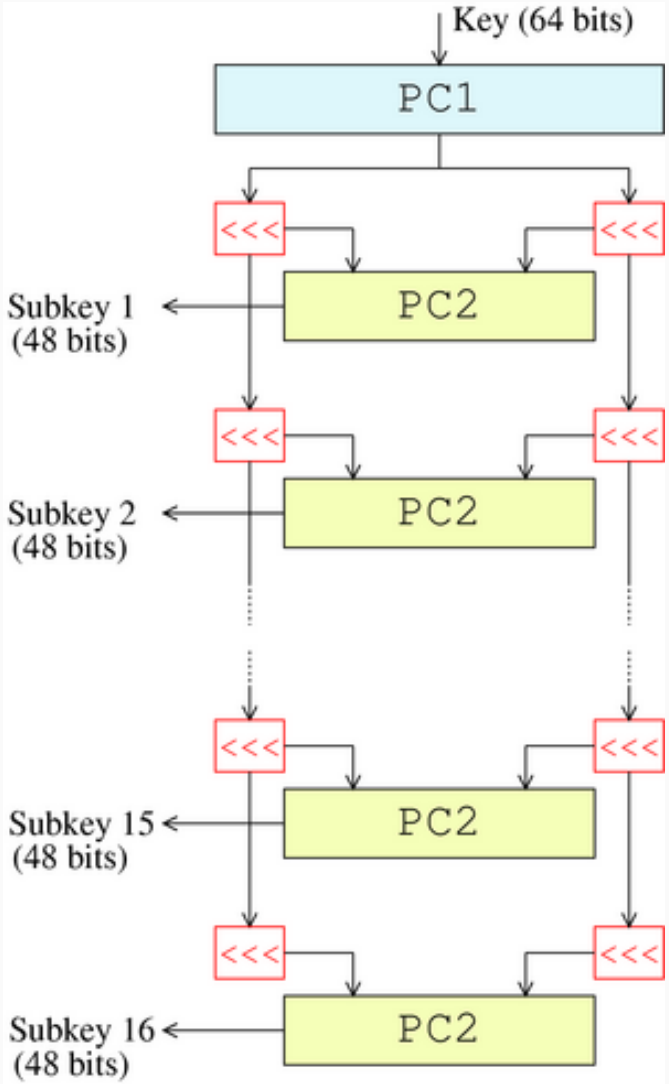
- Many candidate algorithms for the SHA-3 standard
- No preimages discoverable by SAT on full algorithms
- Homsirikamol et al. found preimages for fewer rounds
  - Direct translation, with no manual cryptanalysis

Algorithm	Rounds	Security margin	Code
SHA-1	21	74% (21/80)	SHA-0-1.saw (36s)
SHA-256	16	75% (16/64)	SHA265.saw (1.3s)
Keccak-256	2	92% (2/24)	N/A
BLAKE-256	1	93% (1/14)	Blake256.saw (17s)
Groestl-256	0.5	95% (0.5/10)	N/A
JH-256	2	96% (2/42)	N/A
Skein-512-256	1	99% (1/72)	N/A

- Given a key, a block cipher is a pseudo-random permutation
  - Therefore, invertable (for a fixed key)
- Often built as a substitution-permutation network
  - Will return to S-boxes



KEY EXPANSION



- Symmetric encryption often involves a single shared key
  - Block ciphers typically need one key per round
  - Stream ciphers need one "key" per message block
- So we need to expand the initial key in an unpredictable way
- Should preserve size of key space

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INJECTIVITY OF KEY EXPANSION

$$\forall x, y.\ x \neq y \Rightarrow f(x) \neq f(y)$$

- Provable injectivity of SIMON, Salsa20 key expansion
  - Around 10-20s with ABC [simon.saw] [Salsa20.saw]
- Provable injectivity of AES key expansion
  - A little under a minute with ABC [AES.saw]
- ZUC, a stream cipher for GSM, had a vulnerability
  - Key expansion not injective in ZUC 1.4 (0.5s) [zuc.saw]
  - Provably fixed in ZUC 1.5 (0.6s) [zuc.saw]
  - Originally shown with custom search procedure taking 3m
- Not injective for DES!
  - Known to have weak keys
  - Can show quickly with ABC (0.08s) [DES.saw]

$$\exists m.\ E(m, k) = c \text{ for known } k \text{ and } c$$

- Decrypting using encryption code
- This actually works!
  - Relatively efficient for DES (0.2s), 3DES (0.8s) [DES.saw] [3DES.saw]
  - More modern ciphers slower:
    - AES (1.5m) [AES.saw]
    - SIMON (3.6m) [simon.saw]
    - Speck (20s) [speck.saw]
- Can also run the encryption directly:
  - $\exists c.\ E(m, k) = c$ for known m and k
  - Usually takes $1/5$ to $1/2$ the time of decryption
  - Not really useful, but illustrates the flexibility of SAT

BLOCK CIPHER CONSISTENCY

$$\forall m, k.\ D(E(m, k), k) = m$$

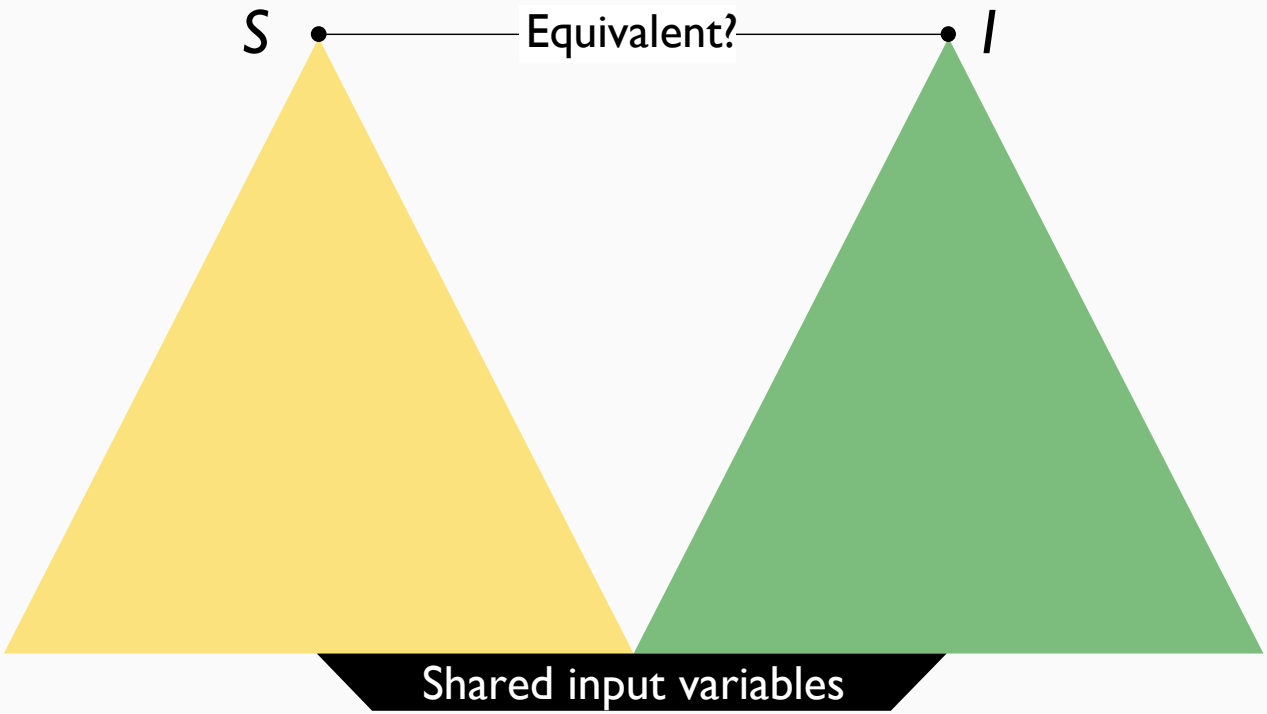
- For a block cipher with encryption function E, decryption D
- Feasible to show for many ciphers
  - DES (4s) [DES.saw]
  - 3DES (8s) [3DES.saw]
  - SIMON (128-256) (6.2m) [simon.saw]
  - Speck (2.7m) [speck.saw]
- Hard to show for AES (at least with encodings and solvers we've tried) [AES.saw]

EQUIVALENCE CHECKING

$$\forall x.\ f(x) = g(x)$$

- Two functions give equivalent output for all inputs
- AIGs give distinct benefits over direct CNF creation
  - Intuitive construction
  - Sharing subterms reduces overall expression size
  - SAT sweeping helps identify candidate equivalences
    - Especially effective for cryptography
  - Use best available SAT solver for final phase
- Works on many cryptographic primitives, including AES (~6.4m) [AES-eq.saw]

EQUIVALENCE CHECKING ILLUSTRATED



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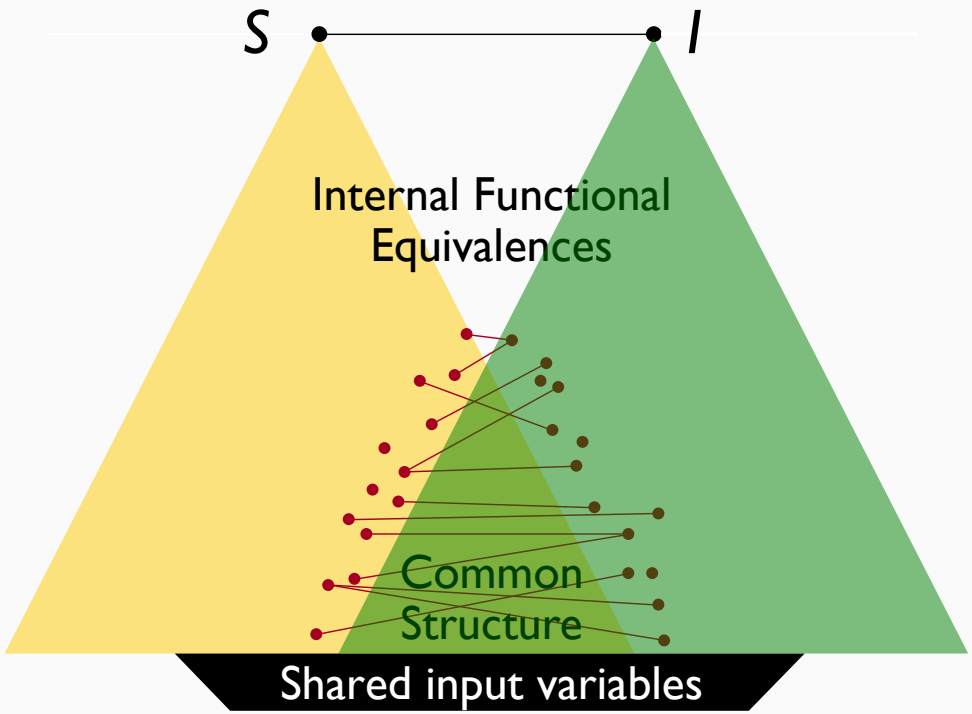

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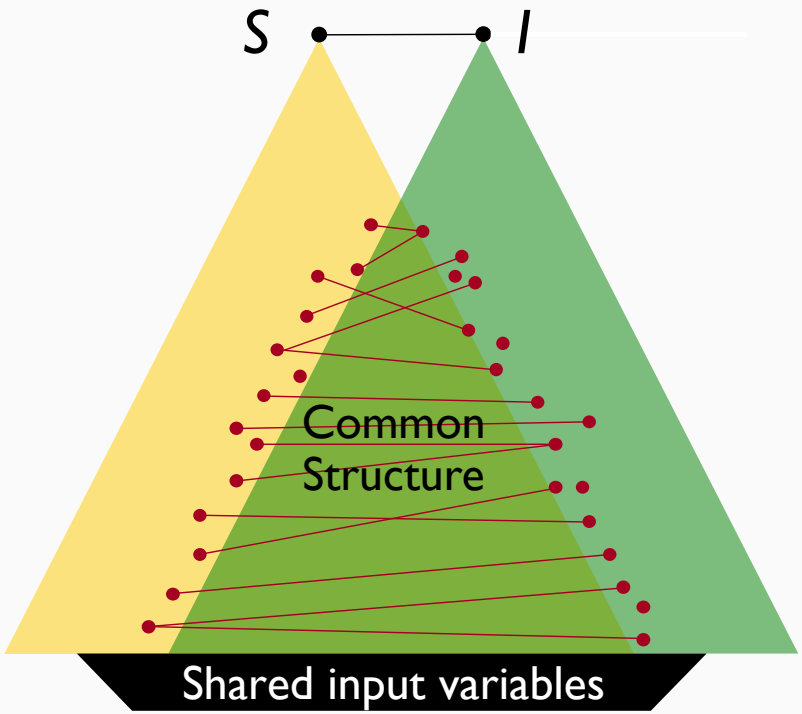

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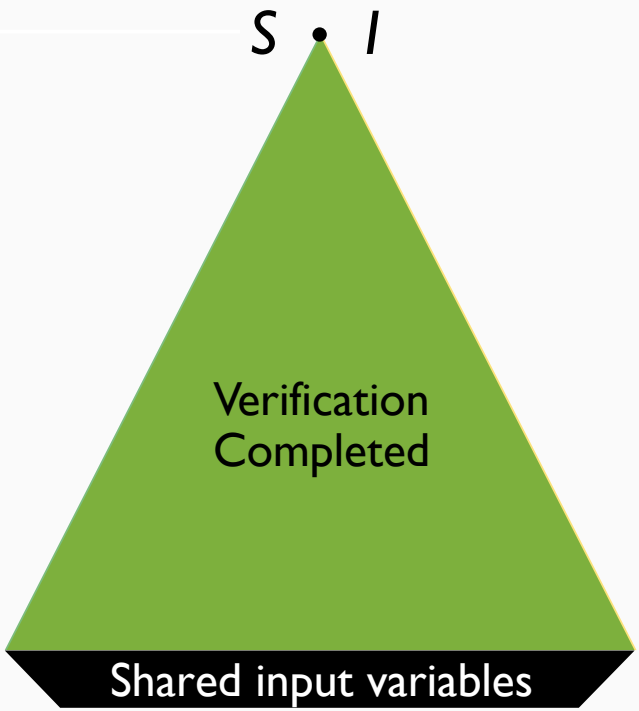
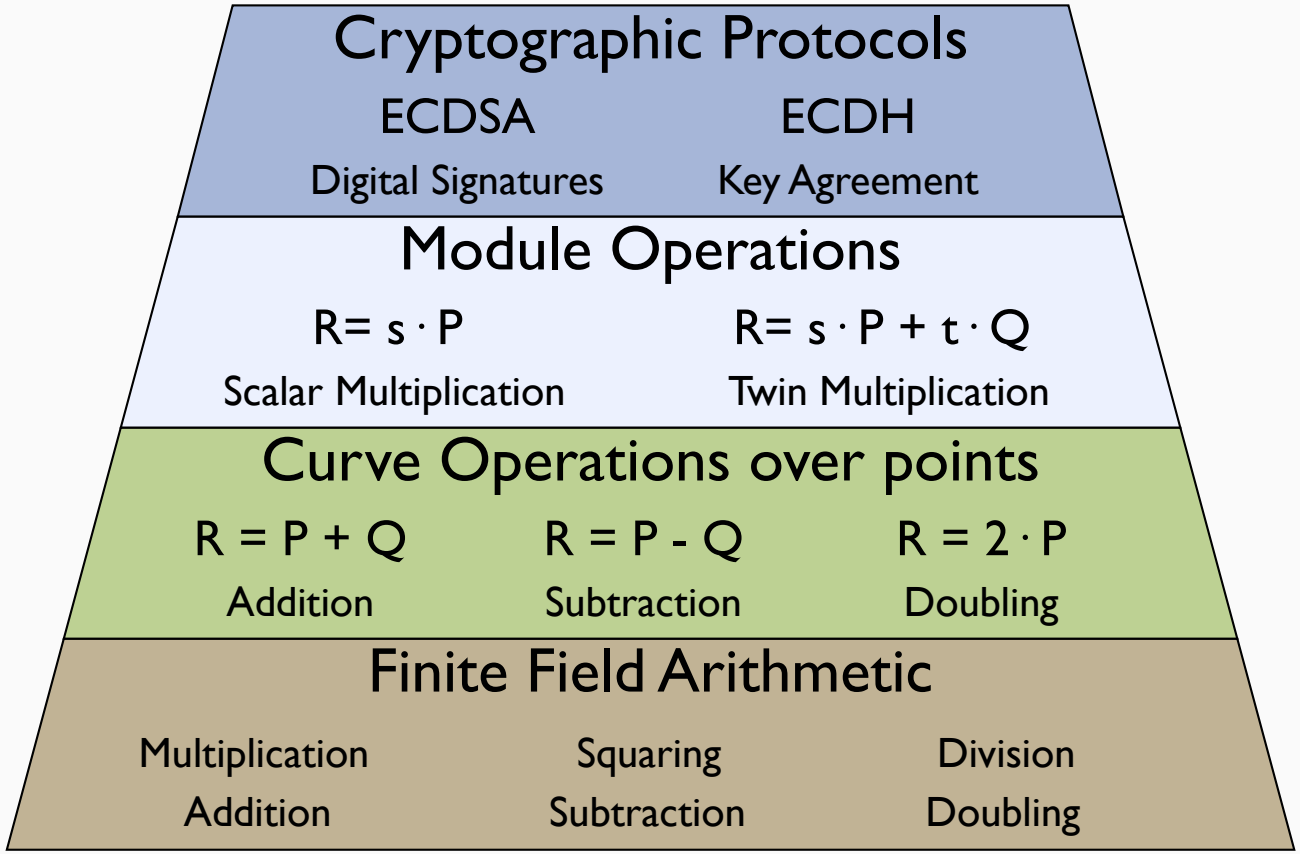


COMPOSITIONAL EQUIVALENCE CHECKING: MOTIVATION



COMPOSITIONAL EQUIVALENCE CHECKING

- Key tool: uninterpreted functions
- Symbolic execution turns imperative code into functional code
  - So procedure calls can be uninterpreted functions
  - If we know all inputs and outputs
- Used for checking equivalence between Cryptol and Java ECDSA
  - Takes around 5 minutes to run
  - Takes ~1500 lines of script (mostly I/O mapping, a few rewrite rules)
  - ABC for leaves, rewriting + Z3 for higher layers

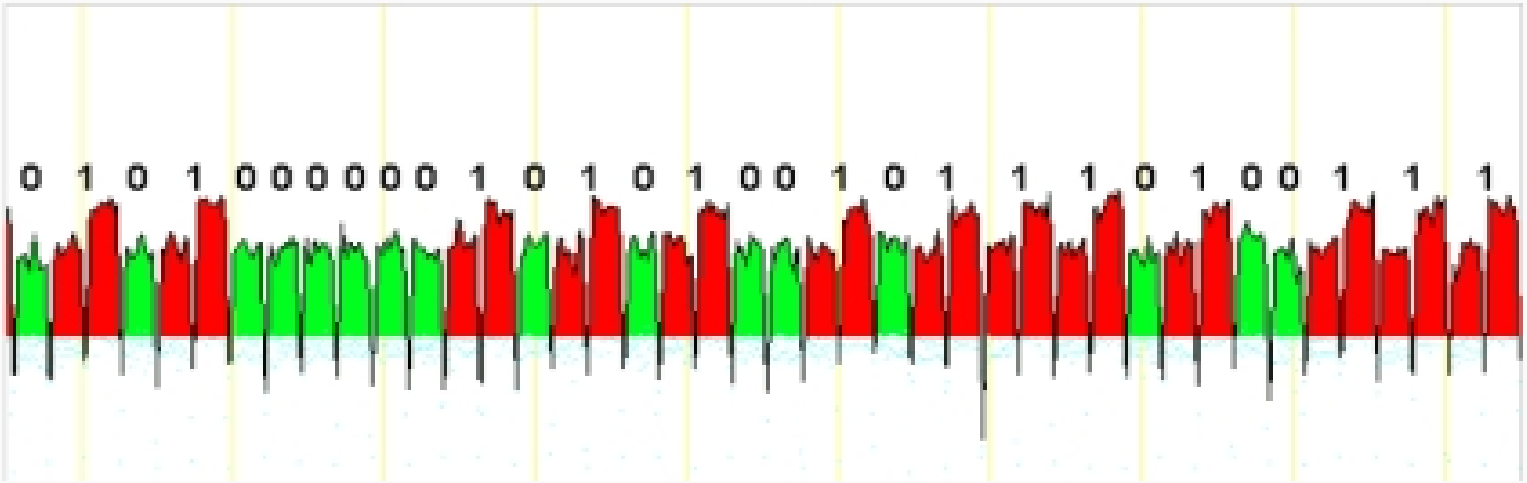
- Attack on symmetric block ciphers
- Known plaintext attack: assumes attacker has some set of (m, c) pairs
- Basic idea: can we approximate the encryption function by a linear function?
    - Where *linear* here means made up entirely of XOR operations
- Any time the encryption function agrees with a linear function too often, this can ease cryptanalysis
- Can use #SAT to count how often it behaves linearly

DIFFERENTIAL CRYPTANALYSIS

- Like linear cryptanalysis, known plaintext attack on symmetric block ciphers
- Analysis of how differences in input affect differences in output
- Disproportional effects can be exploitable
- A differential characteristic is a set of differences as they traverse a path through the algorithm
- Can use #SAT to calculuate the distributions of differential characteristics

- SIMON is a lightweight block cipher published recently
- Kölbl *et al.* used SAT for linear and differential cryptanlysis of SIMON [@kolbl2015simon] [simon-diff.saw]
- Using #SAT to calculate differential characteristic distributions
- Not direct analysis of code, but of manually-derived simplification
- Serves as an additonal tool for cryptanalysis, not push-putton
- Suggests slightly different parameters possibly preferable to published numbers

SIDE CHANNEL ANALYSIS



- Traditional side channel analysis
  - Observe specific part of last round of block cipher
  - Combine with pre-calculated formulas to recover key
- Caveat: only works if you can observe the right signals

- Example: hardware implementation
  - Encode entire algorithm (as implemented) as a boolean circuit
  - Observe any internal values possible
  - Constrain internal variables accordingly
  - Try to solve for key (or other aspects of state)
- More information than just inputs or outputs
- Can use any internal variable in the algorithm
- Hamming weights plus a handful of (m, c) pairs enough to recover AES key [@mohamed2012improved-sc]

CREATING CODE: OPTIMAL S-BOXES

- S-boxes intended to unpredictably substitute bits
  - Ideally indistinguishable from a random function
  - But in practice representable as a short program using only linear operations
- Generating optimal S-boxes:
  - Or any function on a small domain
  - $\exists p.\ [\![p]\!](x_0) = y_0 \wedge \cdots \wedge [\![p]\!](x_n) = y_n$
- Fuhs *et al.* found a 23-instruction AES S-box program [@fuhs2010sbox]
  - Less than a minute with MiniSat
  - Proving unsatisfiability of 22 instructions took 106 hours (CryptoMiniSat)

- General synthesis:
  - $\exists p.\ \forall x.\ [\![p]\!](x) = f(x)$
- Generating efficient implementations:
  - $\min p.\ \forall x.\ [\![p]\!](x) = f(x)$
- Generally, a hard problem
  - But QBF solvers are getting powerful
  - Many papers in SMT community about this problem recently

OTHER EXAMPLES

- Cryptographic protocol analysis
- Diffusion analysis
- Differential fault analysis
- TODO: more!

CURRENTLY DIFFICULT PROBLEMS

- Some problems are difficult with current solvers, but potentially tractable
  - AES equivalence in CNF (tractable using SAT sweeping!)
  - AES consistency
  - Good benchmarks for solver improvement!
- Some problems that are intrinsically difficult (we hope)
  - Multiplication difficulty seems related to hardness of factoring
  - Finding a hash collisions had better be hard
  - Finding k given m, c should be hard, too
- Synthesis is currently hard
  - But provers are getting better at this (QBF and $\exists\forall$-SMT)
- Very similar problems can be different in difficulty
  - Reversing AES vs. finding k from m, c

- Cryptol is a DSL for cryptographic code
  - Allows expression of algorithms at a high-level of abstraction
  - Built-in connection to SMT solvers
  - BSD-licensed
    - http://cryptol.net
- The Software Analysis Workbench (SAW) allows analysis of implementations
  - Symbolic execution of Cryptol, C, Java
  - Bindings to SMT and SAT solvers, including ABC
  - Freely available (with source) for non-commercial use
    - http://saw.galois.com
- All the examples from this talk are available
  - https://github.com/galoisinc/sat2015-crypto

SUMMING IT UP

- Cryptography and SAT go very nicely together
  - Easily representable as propositional formulas
  - AIGs are super handy! (not just for hardware)
- More use of SAT during algorithm development will make our crypto stronger
  - And it's happening: see SHA-3, Trivium, SIMON, etc.
- Nice source of hard benchmark problems for solvers

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REFERENCES