香港科技大學
THE HONG KONG UNIVERSITY OF SCIENCE AND TECHNOLOGY
電子及計算機工程學系
DEPARTMENT OF ELECTRONIC & COMPUTER ENGINEERING

Introduction to Aerial Robotics
Lecture 10

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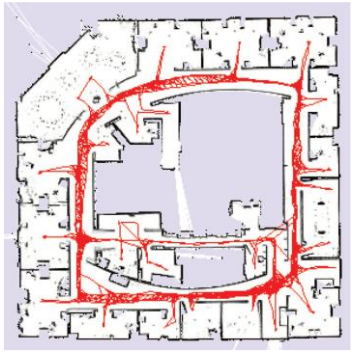
Outline

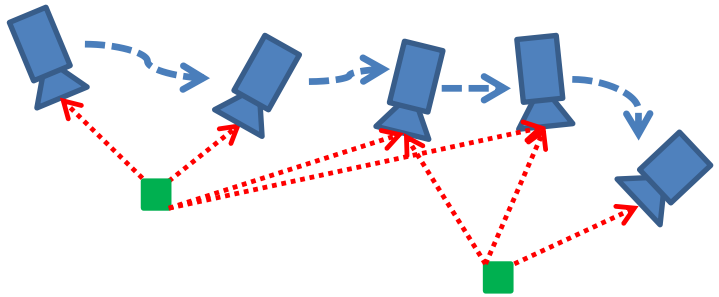
- The Basics

$$P(x \mid y, z) = \frac{P(y \mid x, z)\ P(x \mid z)}{P(y \mid z)}$$

- 2D Pose Graph SLAM



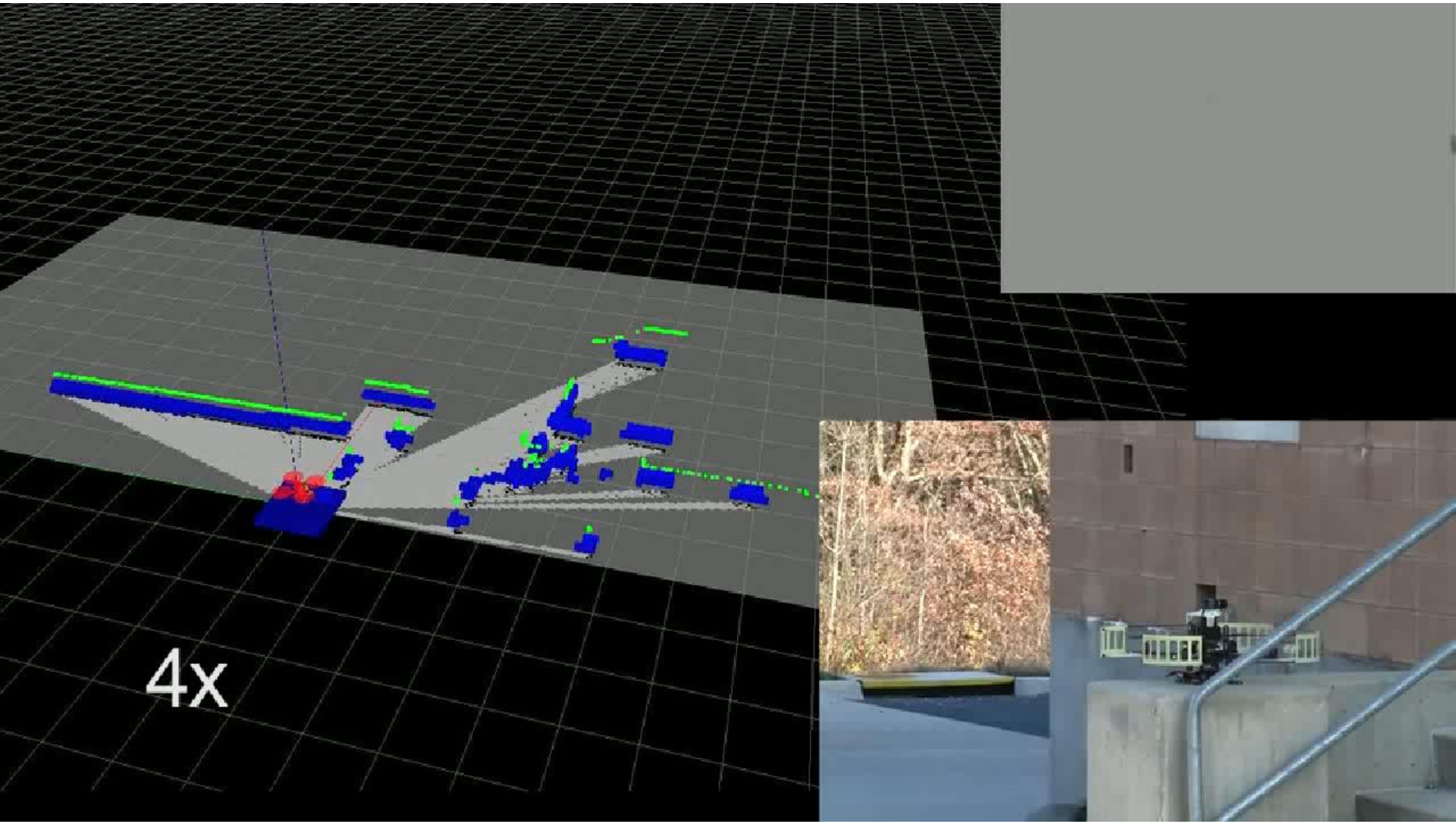
- Monocular Visual-Inertial SLAM



This is Science Fiction…

From Movie “Prometheus”

This is Real…



Shen, et al, 2013

The Localization Problem

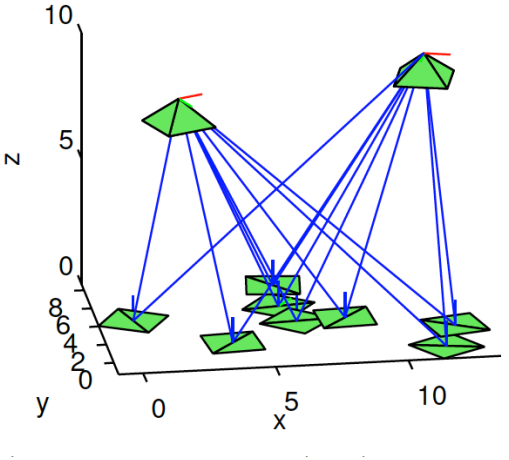
A robot is exploring a known, static environment.

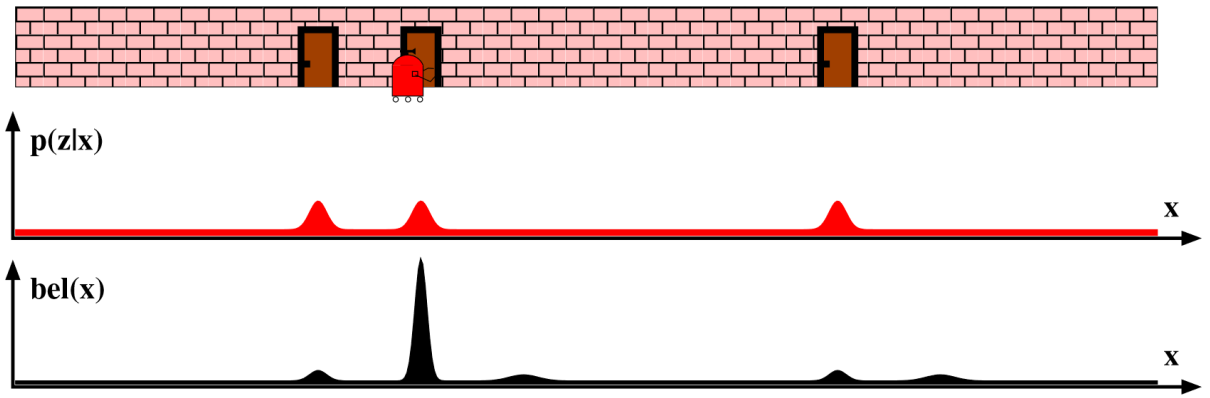
Given:

- Measurements from sensor(s)

Estimate:

- Path/trajectory of the robot





The SLAM Problem

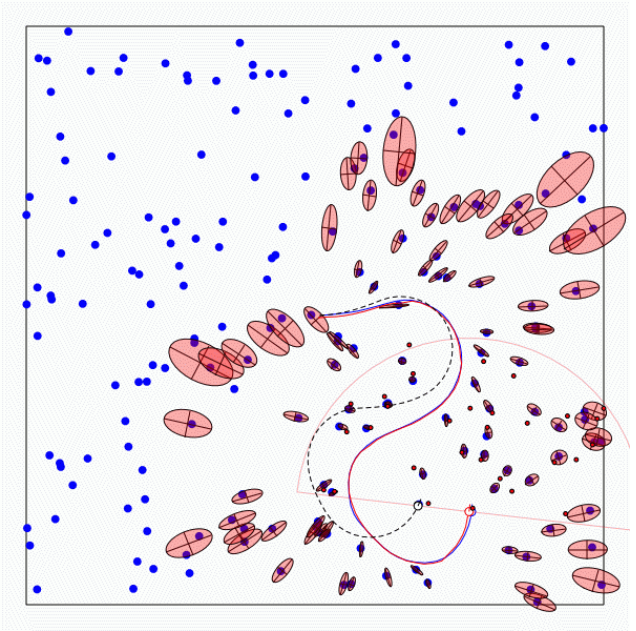
A robot is exploring an unknown, static environment.

Given:

- Measurements from sensor(s)

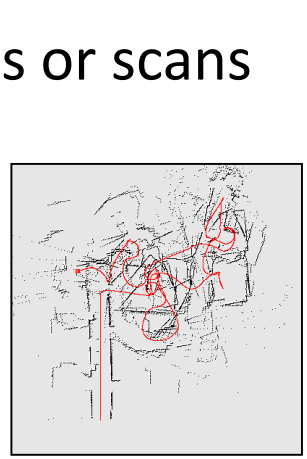
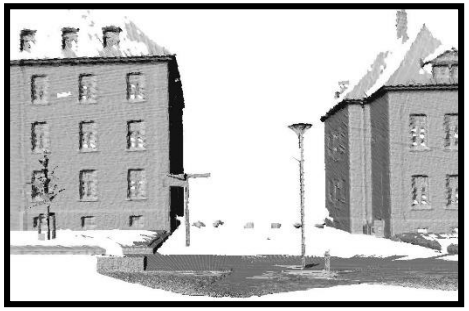
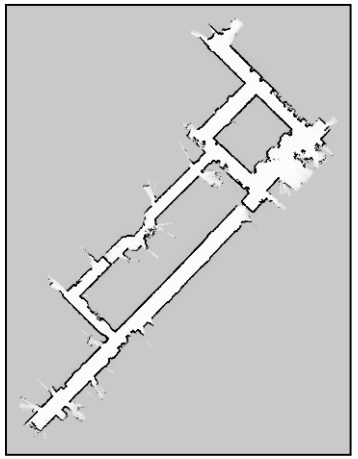
Estimate:

- The map
- Path/trajectory of the robot



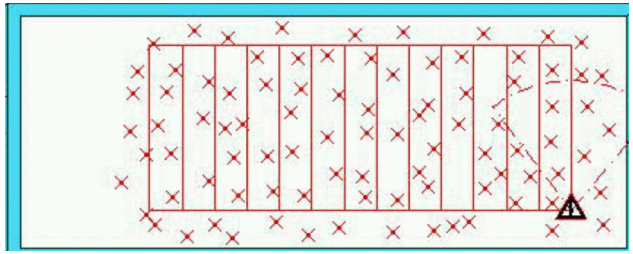
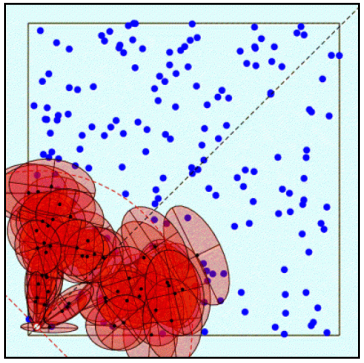
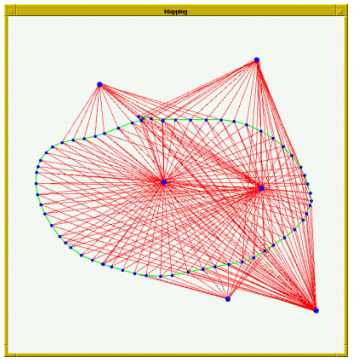
Representations

- Grid maps or scans

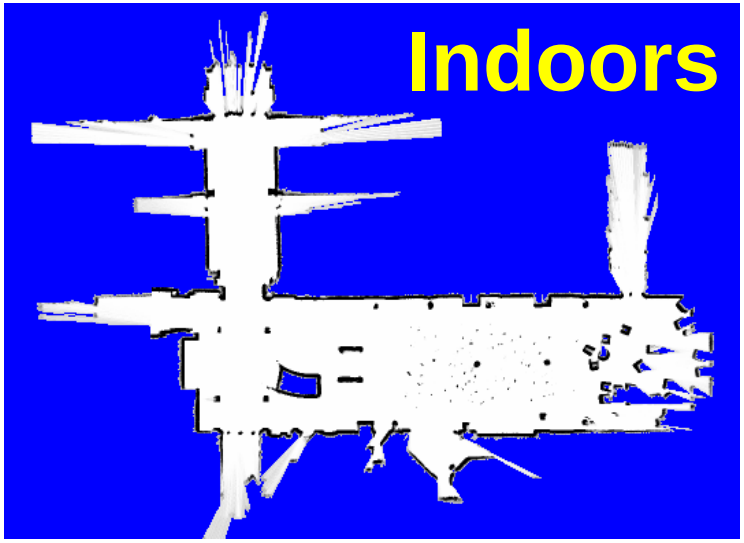
[Lu & Milios, 97; Gutmann, 98: Thrun 98; Burgard, 99; Konolige & Gutmann, 00; Thrun, 00; Arras, 99; Haehnel, 01;…]

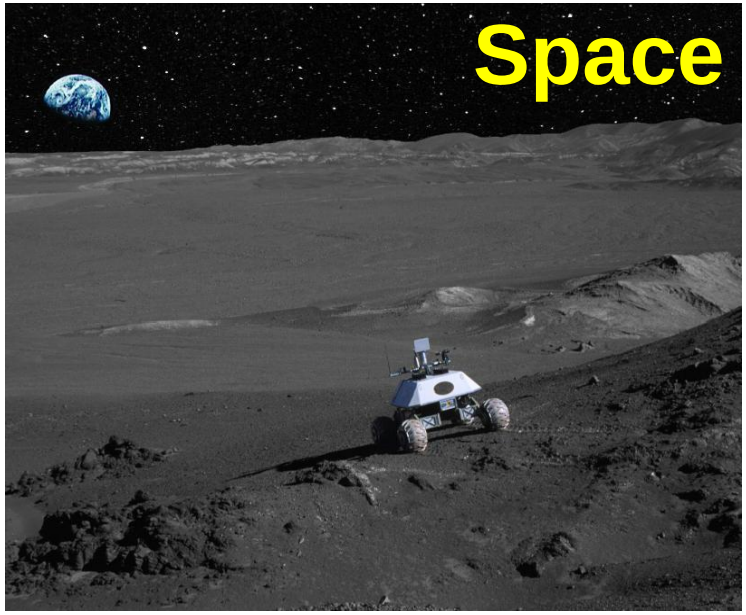
- Landmark/feature-based

[Leonard et al., 98; Castelanos et al., 99: Dissanayake et al., 2001; Montemerlo et al., 2002;…

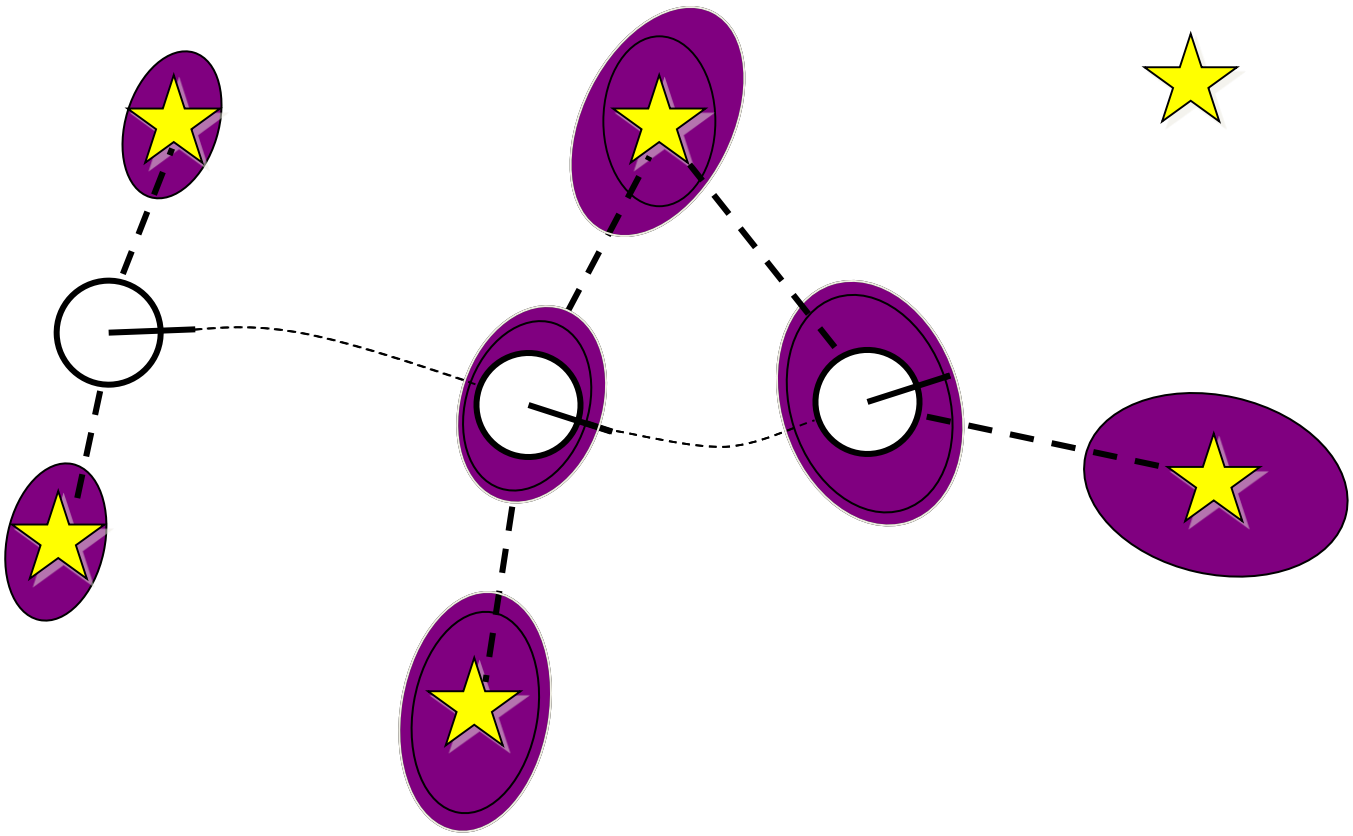
SLAM Applications





Why is SLAM a hard problem?

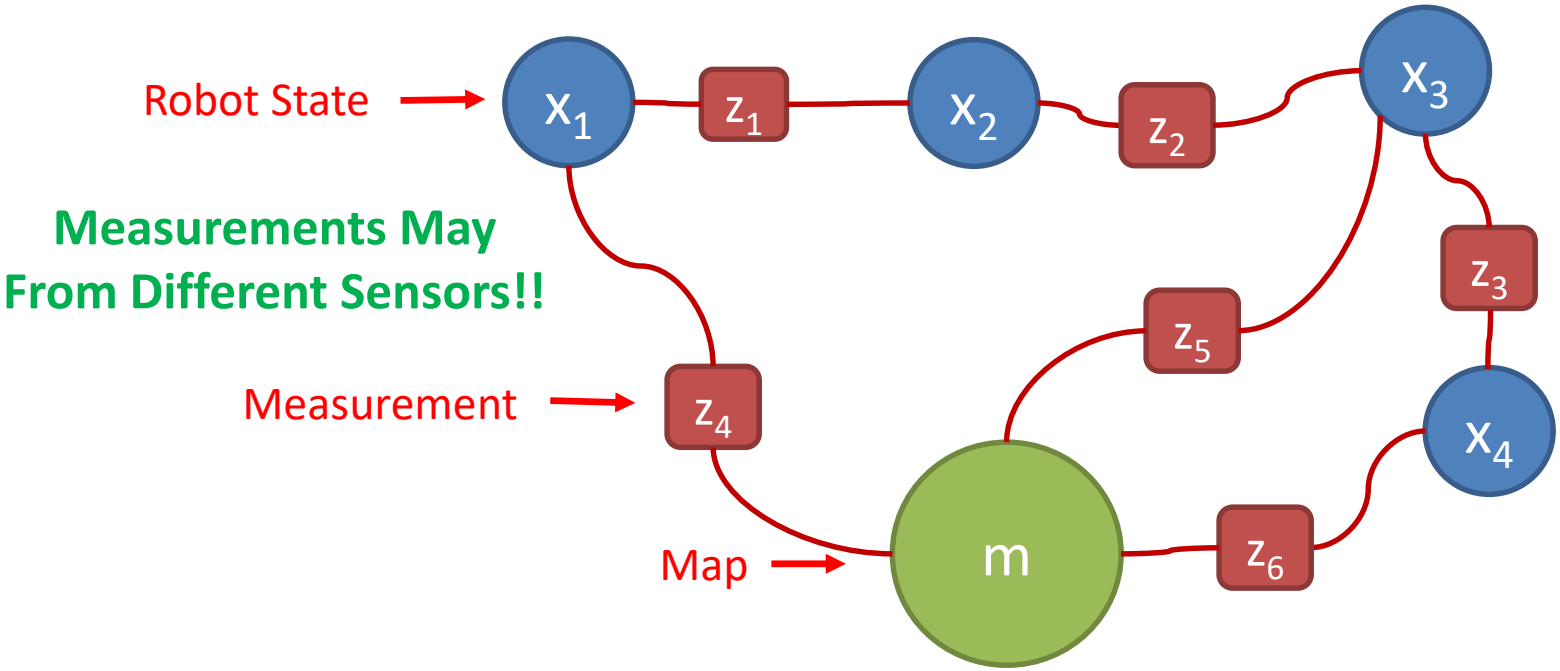
SLAM: robot path and map are both **unknown**



Robot path error correlates errors in the map

Courtesy Sebastian Thrun, Wolfram Burgard and Dieter Fox

Graphical Model of SLAM

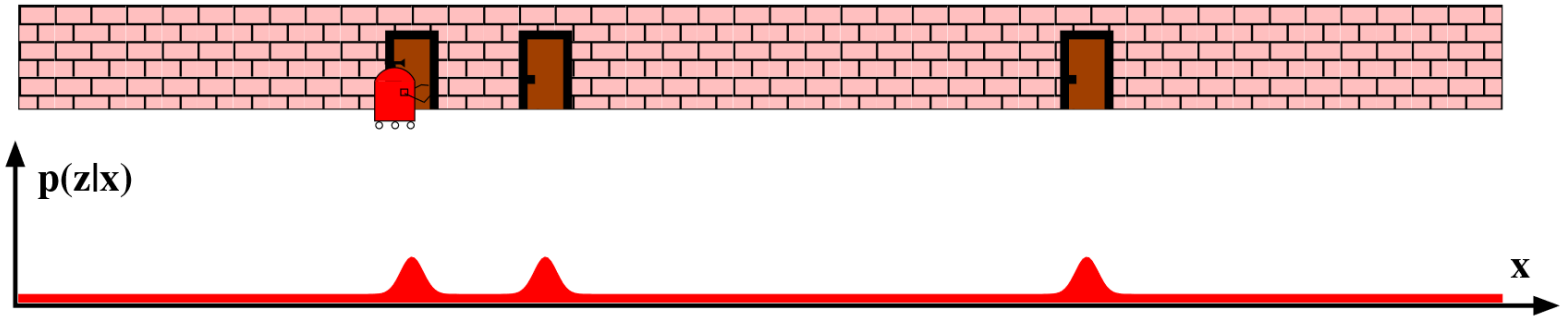


$$(\hat{x}_{1:t}, \hat{m}) = \arg\max_{x_{1:t},\ m} p(x_{1:t}, m \mid z_{1:N})$$

??????

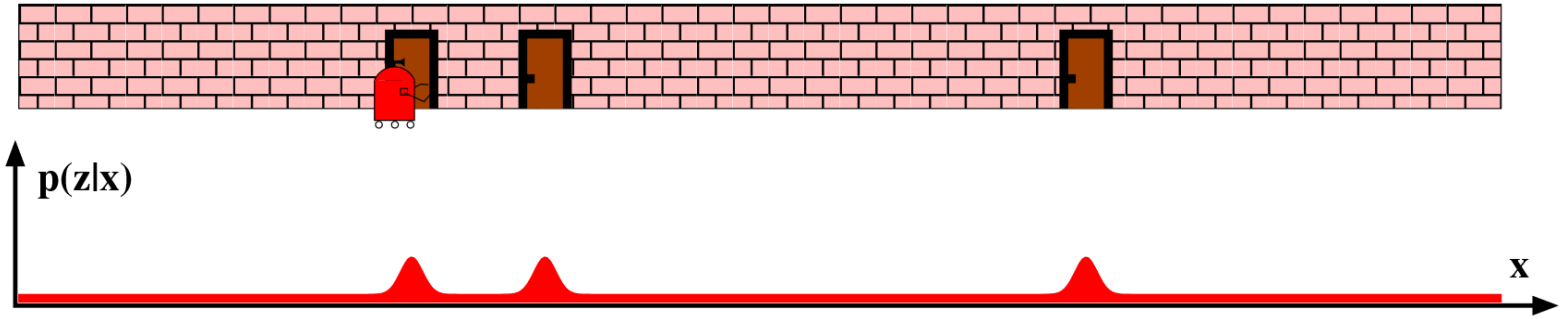
Conditional Probability

- **Definition:** Probability of an event z occurring conditioned on another event x occurring
- $p(z \mid x) = \frac{p(x,z)}{p(x)} \quad \Leftrightarrow \quad p(x,z) = p(z \mid x)\, p(x)$
- **Example:**
  - z = {there is a door next to the robot}



Conditional Independence

- **Definition:** Two random variables are ***conditionally independent*** if the outcome of one has ***no effect*** on the outcome of the other when conditioned on the outcome of a third random variable
- $p(z_1, z_2 \mid x) = p(z_1 \mid x)\, p(z_2 \mid x)$
- **Example:**
  - Let Z_1, Z_2 be two measurements taken from the same place
  - Z_1, Z_2 are conditionally independent given X



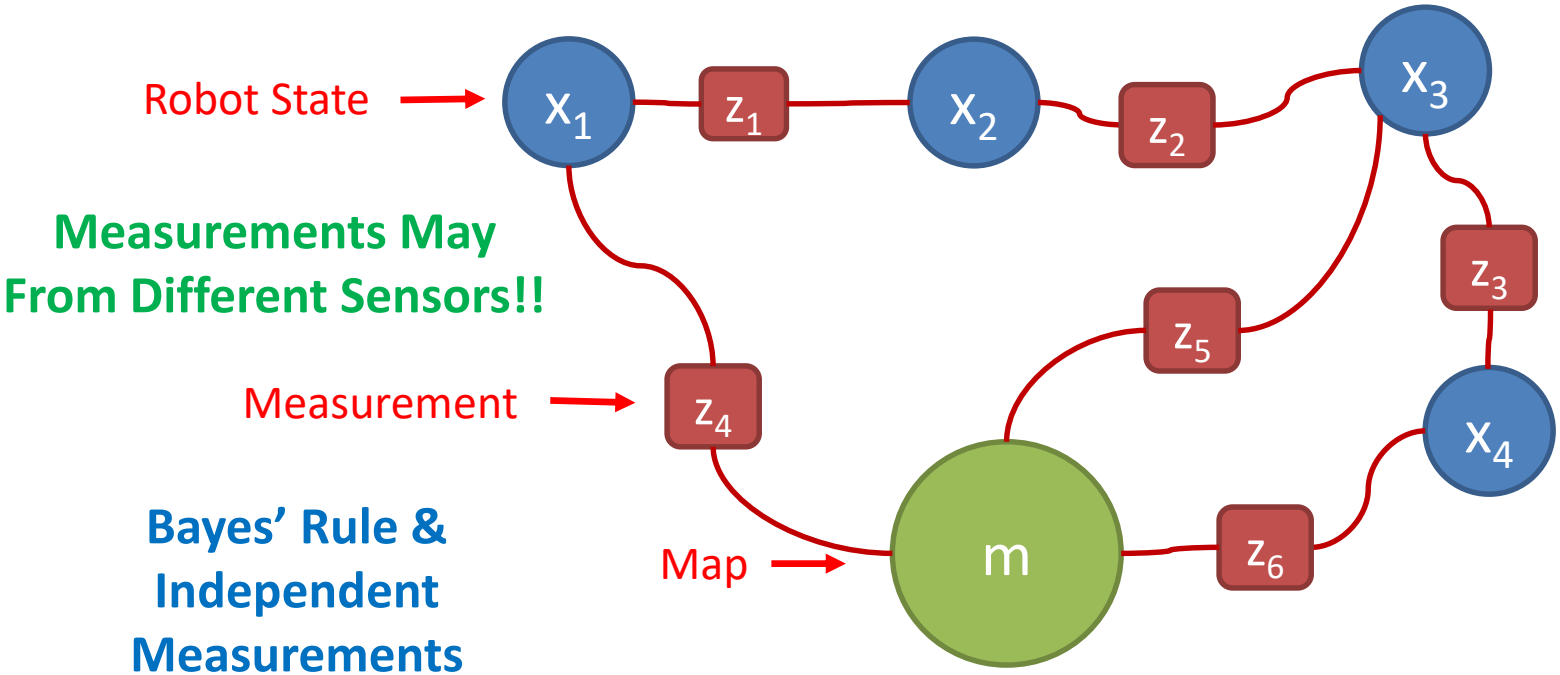
Bayes' Theorem

- $p(x \mid z) = \frac{p(z \mid x)\, p(x)}{p(z)} = \frac{p(z \mid x)\, p(x)}{\int p(z \mid x')\, p(x') dx'}$

- **Intuition:** Describes how the belief about a random variable X should change to account for the collected evidence (measurement) z

- **Derivation:**
  - $p(x, z) = p(z \mid x)\, p(x) = p(x \mid z)\, p(z)$

SLAM as Maximum A Posteriori Estimation



Bayes' Rule & Independent Measurements

$$(\hat{x}_{1:t}, \hat{m}) = \arg\max_{x_{1:t},\, m} p(x_{1:t}, m \mid z_{1:N})$$

$$= \arg\max_{x_{1:t},\, m} \prod_{i=1\ldots N} p(z_i \mid x_{1:t}, m) \cdot p(x_{1:t}, m)$$

Measurement Models

Prior

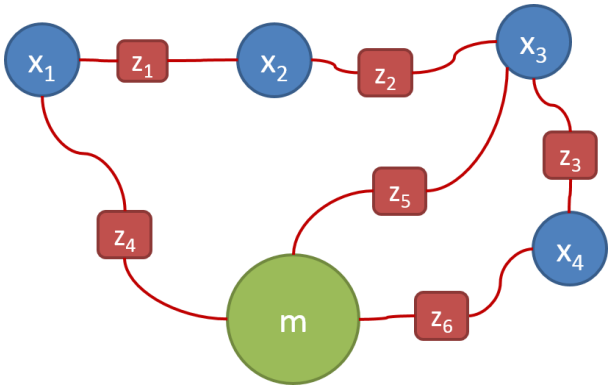
The Gaussian Case

- The multivariate Gaussian distribution

$$p(z \mid x, m) = \frac{1}{\sqrt{(2\pi)^k |\Sigma|}} e^{-\frac{1}{2}(z-h(x,m))^T \Sigma^{-1} (z-h(x,m))}$$

Measurement Model: Predicted measurement from states

Measurement Residual



- The Gaussian case for SLAM:

$$(\hat{x}_{1:t}, \hat{m}) = \arg\max_{x_{1:t},\, m} p(x_{1:t}, m \mid z_{1:N})$$

-log

$$= \arg\max_{x_{1:t},\, m} \prod_{i=1\ldots N} p(z_i \mid x_{1:t}, m) \cdot p(x_{1:t}, m)$$

Measurement Residual

$$= \arg\min_{x_{1:t},\, m} \sum_{i=1\ldots N} r_{z_i}^T \Sigma_{z_i}^{-1} r_{z_i} + r_{xm}^T \Sigma_{xm}^{-1} r_{xm}$$

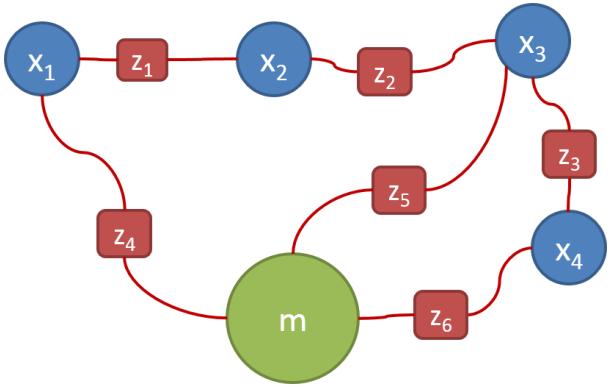
$$= \arg\min_{x_{1:t},\, m} \sum_{i=1\ldots N} \left\| r_{z_i} \right\|_{\Sigma_{z_i}}^2 + \left\| r_{xm} \right\|_{\Sigma_{xm}}^2$$

Nonlinear Least Square!

SLAM as Nonlinear Least Square

- Nonlinear least square

$$\min_{x_{1:t}} \sum_{m} \sum_{i=1\ldots N} r_{z_i}^T \Sigma_{z_i}^{-1} r_{z_i} + r_{xm}^T \Sigma_{xm}^{-1} r_{xm}$$



$$X = [x_{1:t}, m]$$

- Linearization

$$r_{z_i} \approx r_{z_i,\hat{X}} + \frac{\partial r_{z_i,\hat{X}}}{\partial X}\Big|_{\hat{X}} \delta X = r_{z_i,\hat{X}} + J_{z_i,\hat{X}} \delta X$$

Measurement Jacobians

Iterate

- Gauss-Newton Method

$$J^T \Sigma^{-1} J \delta X = -J^T \Sigma^{-1} r$$

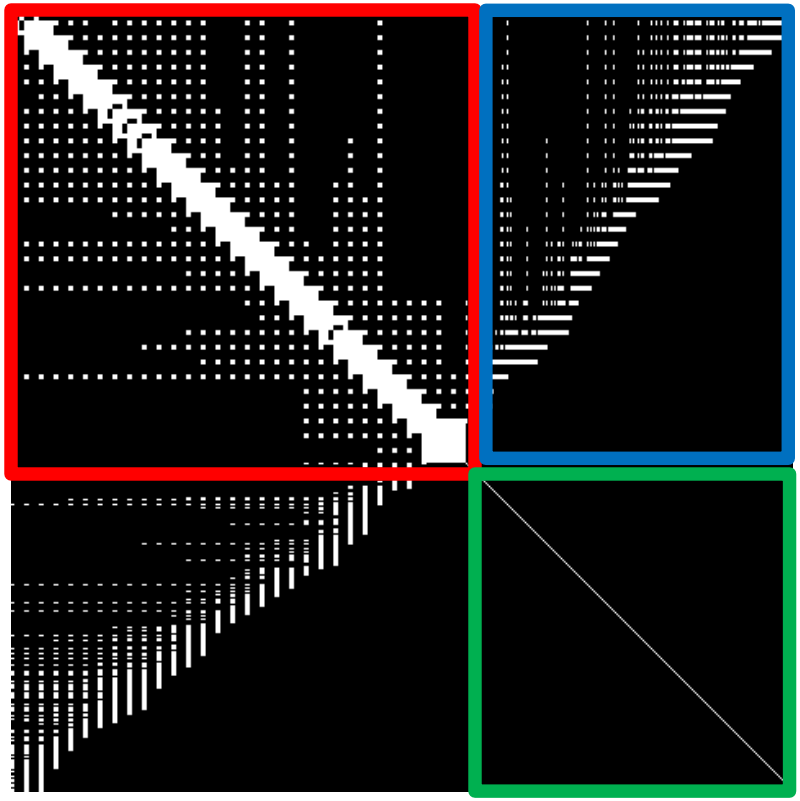
$$\hat{X}^{(k+1)} = \hat{X}^{(k)} + \delta X$$

Information matrix

Stacked Jacobian, covariance, and residual

SLAM as Nonlinear Least Square

- Linear system - the information matrix
  - Efficient solution via sparse linear equation solver

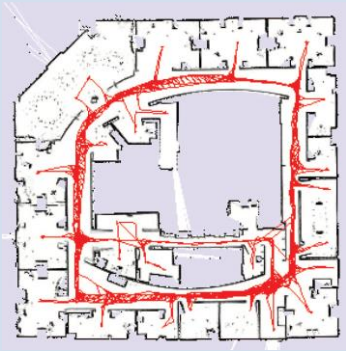


Outline

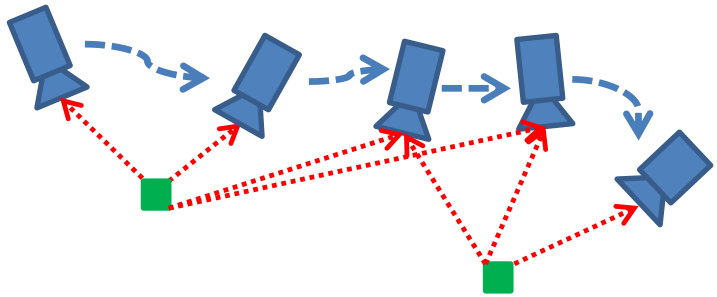
- The Basics

$$P(x \mid y, z) = \frac{P(y \mid x, z)\ P(x \mid z)}{P(y \mid z)}$$

- 2D Pose Graph SLAM

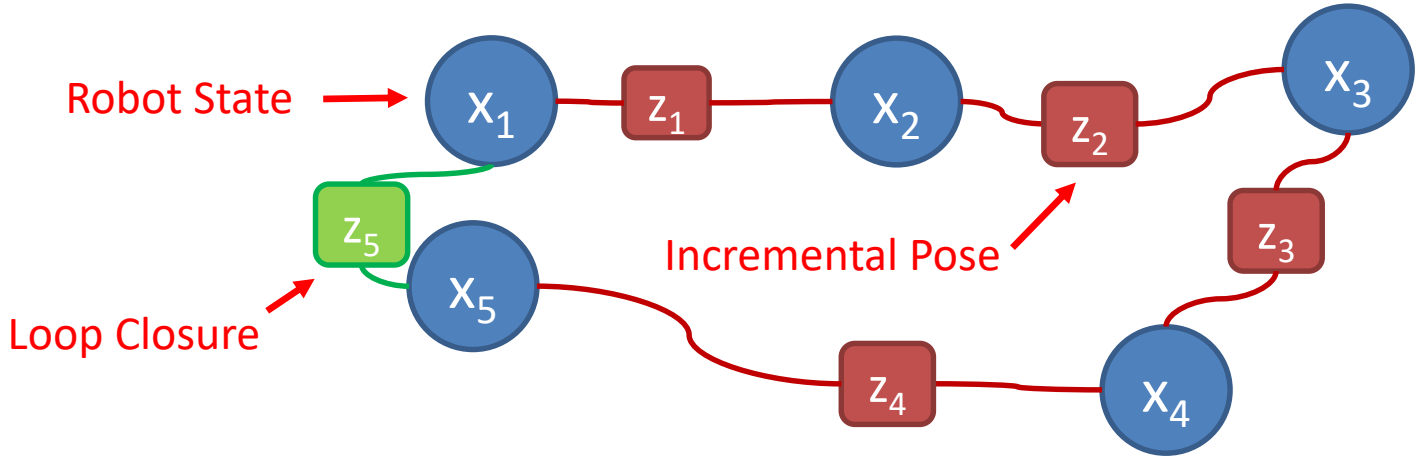


- Monocular Visual-Inertial SLAM



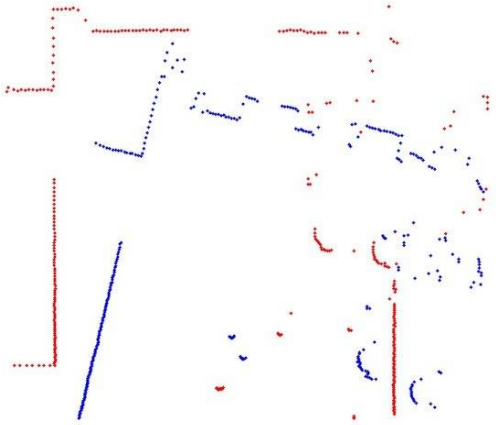
2D Pose Graph SLAM

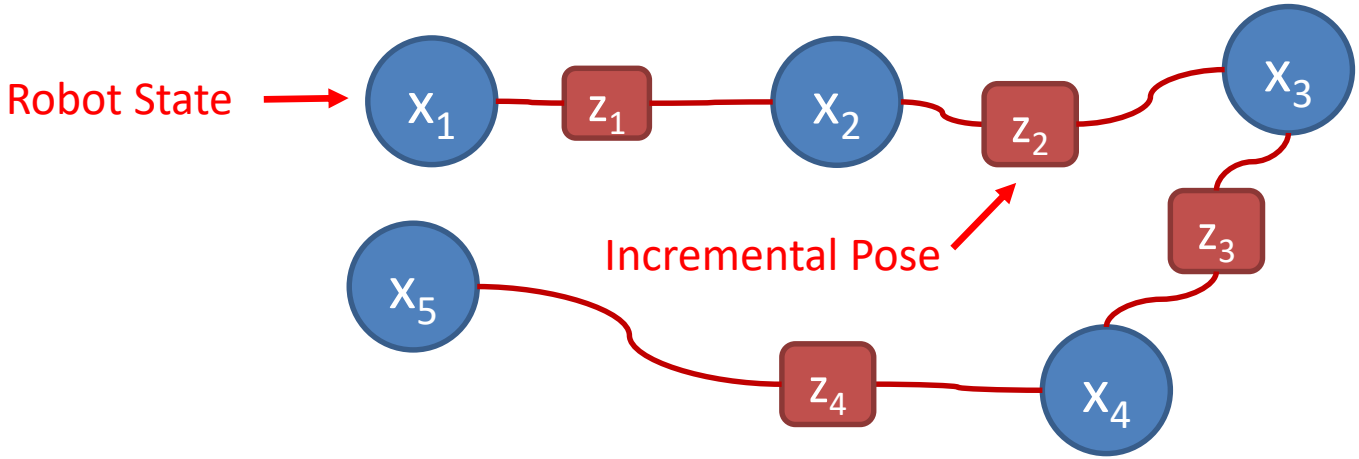
- The only type of measurement is 2D rigid body transformations between 2D robot poses
- Graphical representation
- No Map



2D Pose Graph SLAM

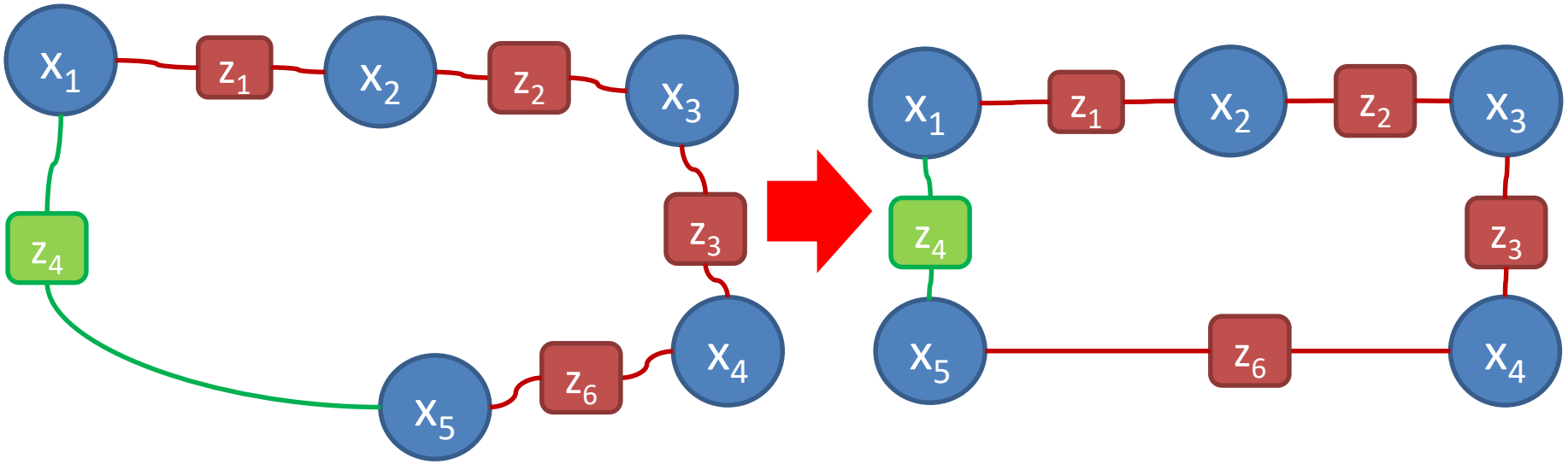
- Incremental pose transformation from scan matching





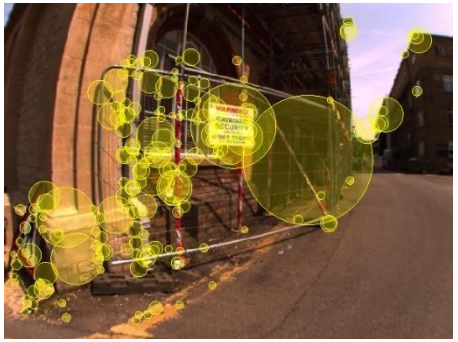
2D Pose Graph SLAM

- Global consistency enforced by loop closure

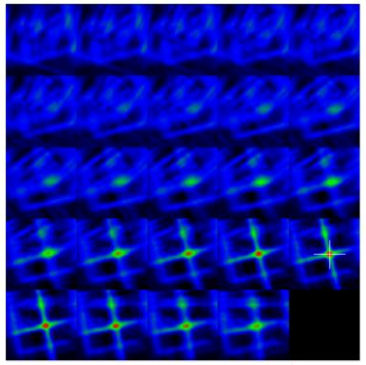


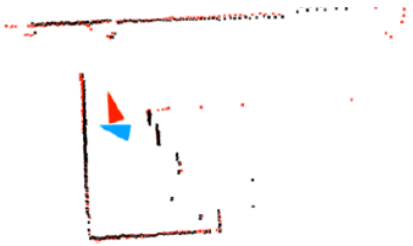
Loop Closure

- Detect re-visting of places, and find out relative poses
  - Appearance-based: FABMAP (M. Cummins and P. Newman, 2008)



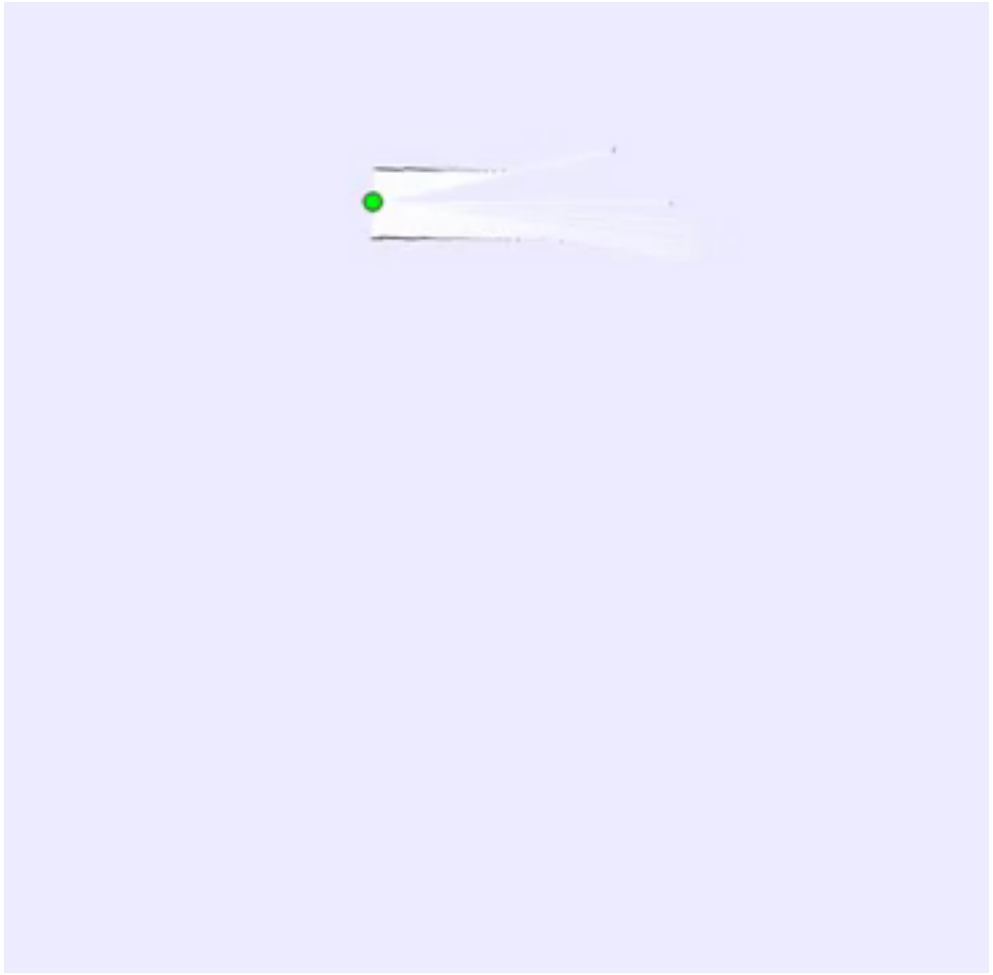
  - Scan matching (E. Olson, 2009)





Example

The map is obtained by transforming and accumulating laser scans using robot poses solved by pose graph SLAM



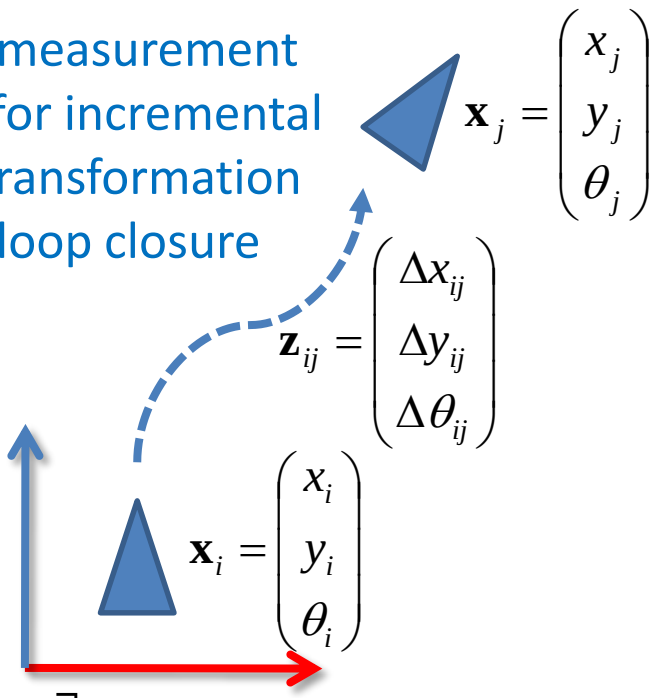
2D Pose Graph SLAM

$$\hat{\mathbf{x}}_{1:t} = \arg\max_{\mathbf{x}_{1:t}} p(\mathbf{x}_{1:t} \mid \mathbf{z}_{1:N})$$

$$= \arg\max_{\mathbf{x}_{1:t}} \prod_{i,j} p(\mathbf{z}_{ij} \mid \mathbf{x}_{1:t}) \cdot p(\mathbf{x}_1)$$

$$= \arg\min_{\mathbf{x}_{1:t}} \sum_{i,j} \mathbf{r}_{z_{ij}}^T \Sigma_{z_{ij}}^{-1} \mathbf{r}_{z_{ij}} + \mathbf{r}_{x_1}^T \Sigma_{x_1}^{-1} \mathbf{r}_{x_1}$$

Same measurement model for incremental pose transformation and loop closure



Measurement Residual

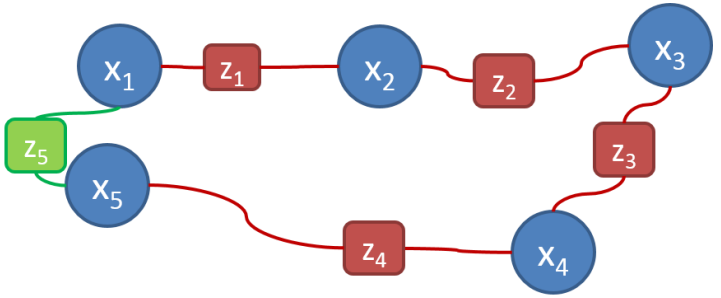
$$\mathbf{r}_{z_{ij}} = \begin{pmatrix} \Delta x_{ij} \\ \Delta y_{ij} \\ \Delta \theta_{ij} \end{pmatrix} - \begin{bmatrix} \begin{pmatrix} \cos\theta_i & -\sin\theta_i \\ \sin\theta_i & \cos\theta_i \end{pmatrix}^{-1} \begin{pmatrix} x_j - x_i \\ y_j - y_i \end{pmatrix} \\ \theta_j - \theta_i \end{bmatrix} = \mathbf{z}_{ij} - h(\mathbf{x}_i, \mathbf{x}_j)$$

$$\mathbf{r}_{z_{ij}} \sim N(0, \Sigma_{z_{ij}})$$

SLAM as Nonlinear Least Square

- Nonlinear least square

$$\hat{\mathbf{x}}_{1:t} = \arg\min_{\mathbf{x}_{1:t}} \sum_{i,j} \mathbf{r}_{z_{ij}}^T \Sigma_{z_{ij}}^{-1} \mathbf{r}_{z_{ij}} + \mathbf{r}_{x_1}^T \Sigma_{x_1}^{-1} \mathbf{r}_{x_1}$$



- Linearization

$$\mathbf{r}_{z_{ij}} \approx \mathbf{r}_{z_{ij},\hat{X}} + \frac{\partial \mathbf{r}_{z_{ij},\hat{X}}}{\partial X}\Big|_{\hat{X}} \delta X = \mathbf{r}_{z_{ij},\hat{X}} + \mathbf{J}_{z_{ij},\hat{X}} \delta X \qquad X = [\mathbf{x}_{1:t}]$$

$$\mathbf{J}_{z_{ij},\hat{X}} = \begin{bmatrix} 0 & \cdots & \begin{pmatrix} \hat{\mathbf{R}}_{\theta_i}^{-1} & \begin{pmatrix} \sin\hat{\theta}_i & -\cos\hat{\theta}_i \\ \cos\hat{\theta}_i & \sin\hat{\theta}_i \end{pmatrix} \begin{pmatrix} \hat{x}_j - \hat{x}_i \\ \hat{y}_j - \hat{y}_i \end{pmatrix} \\ 0 & -1 \end{pmatrix} & \cdots & \begin{pmatrix} -\hat{\mathbf{R}}_{\theta_i}^{-1} & 0 \\ 0 & 1 \end{pmatrix} & \cdots \end{bmatrix}$$

3x3N

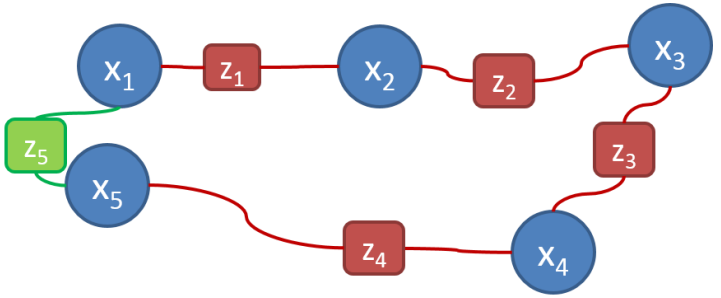
i Block

j Block

SLAM as Nonlinear Least Square

- Nonlinear least square

$$\hat{\mathbf{x}}_{1:t} = \arg\min_{\mathbf{x}_{1:t}} \sum_{i,j} \mathbf{r}_{z_{ij}}^T \Sigma_{z_{ij}}^{-1} \mathbf{r}_{z_{ij}} + \mathbf{r}_{x_1}^T \Sigma_{x_1}^{-1} \mathbf{r}_{x_1}$$



- Linearization

$$\mathbf{r}_{z_{ij}} \approx \mathbf{r}_{z_{ij},\hat{X}} + \frac{\partial \mathbf{r}_{z_{ij},\hat{X}}}{\partial X}\Big|_{\hat{X}} \delta X = \mathbf{r}_{z_{ij},\hat{X}} + \mathbf{J}_{z_{ij},\hat{X}} \delta X$$

Iterate

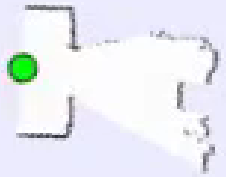
- Gauss-Newton method

$$\sum_{i,j} \mathbf{J}_{z_{ij},\hat{X}}^T \Sigma_{z_{ij}}^{-1} \mathbf{J}_{z_{ij},\hat{X}} \cdot \delta X = -\sum_{i,j} \mathbf{J}_{z_{ij},\hat{X}}^T \Sigma_{z_{ij}}^{-1} \mathbf{r}_{z_{ij},\hat{X}}$$

$$\hat{X}^{(k+1)} = \hat{X}^{(k)} + \delta X$$

More Example

The map is obtained by transforming and accumulating laser scans using robot poses solved by pose graph SLAM

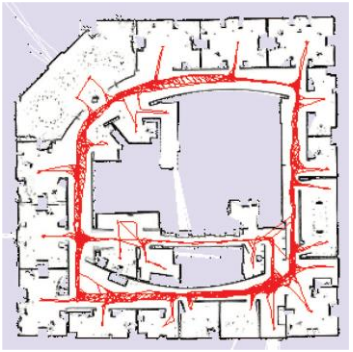


Outline

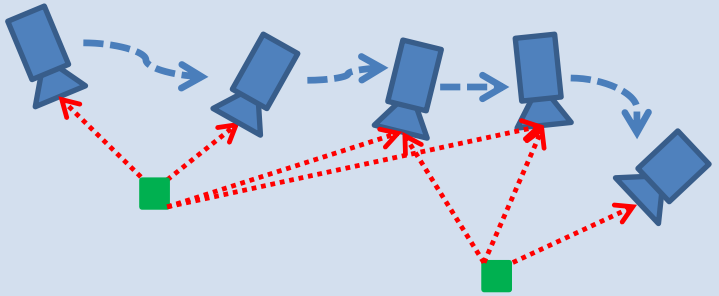
- The Basics

$$P(x \mid y, z) = \frac{P(y \mid x, z)\ P(x \mid z)}{P(y \mid z)}$$

- 2D Pose Graph SLAM

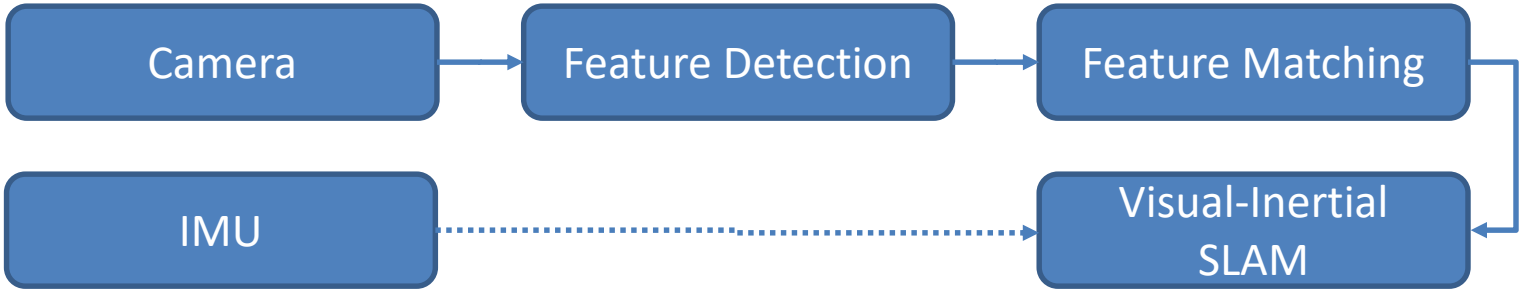


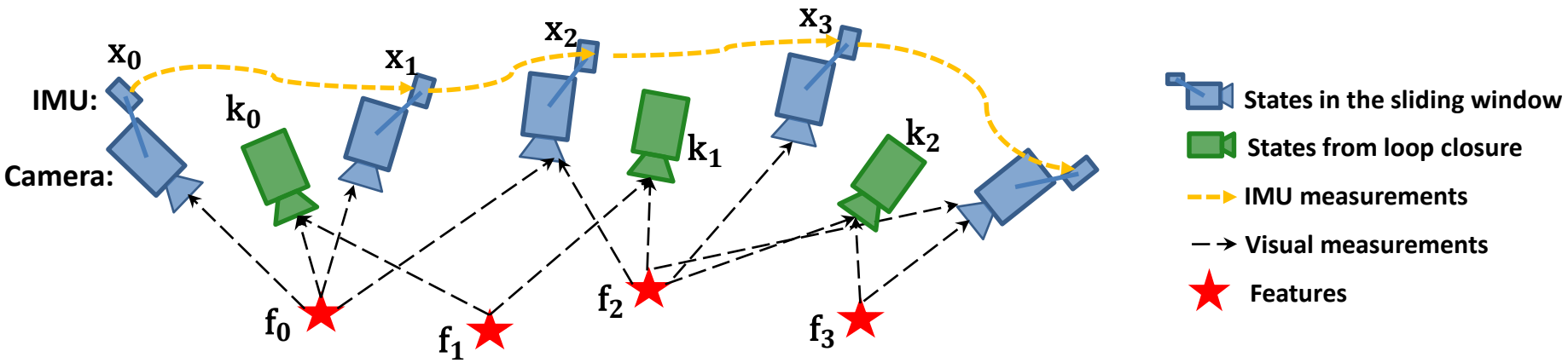
- Monocular Visual-Inertial SLAM



Visual-Inertial SLAM Pipeline

- A SLAM system that use multiple sensors
- Still a graphical representation, with different types of edges





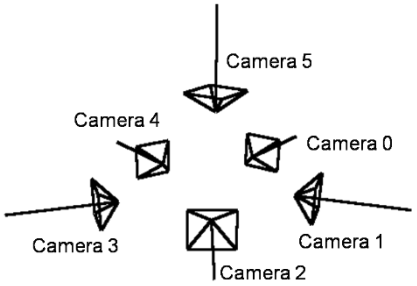
Camera Setup

- Monocular
  - Depth Unknown

- Stereo
  - Able to compute depth
  - Depth accuracy affect by baseline, resolution, and calibration

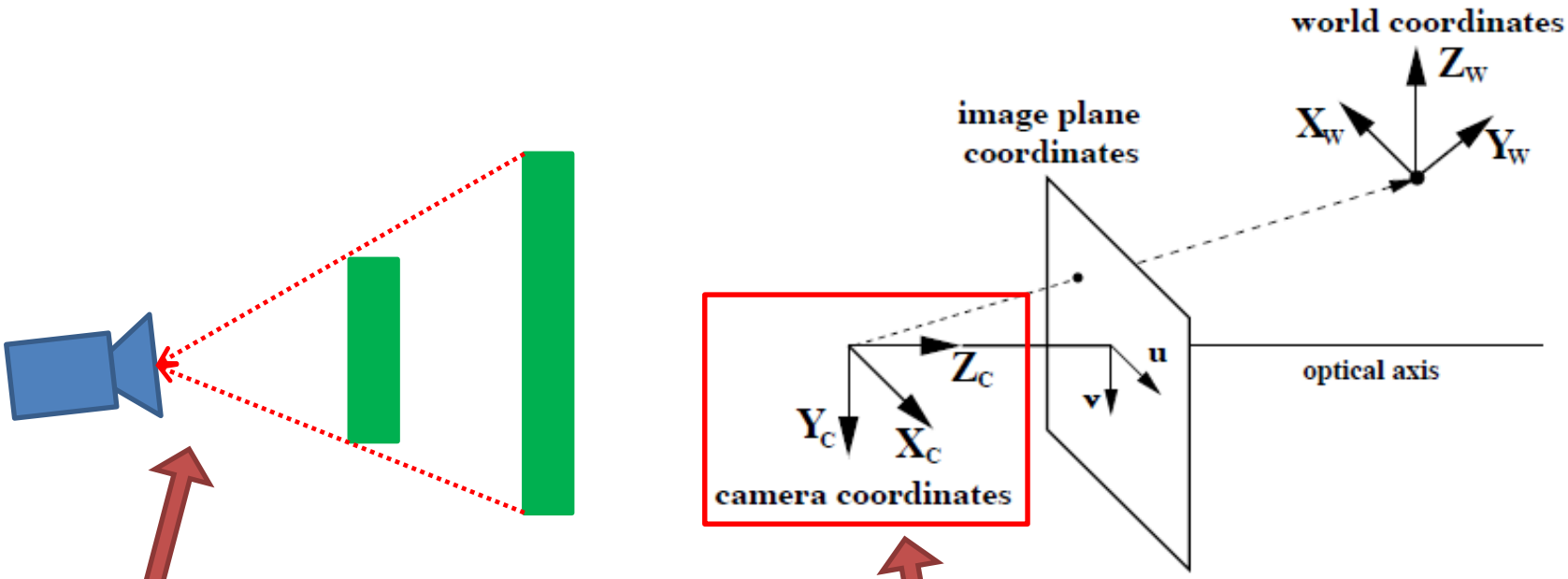
- Multi-Camera
  - Overlapping / Non-overlapping field-of-view

Omnidirectional multi-camera system Ladybug



Relative position and posture of each camera

Pin-hole Camera Model



$$\lambda \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} f & 0 & u_o \\ 0 & f & v_o \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} R & t \end{pmatrix} \begin{pmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{pmatrix} = P \begin{pmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{pmatrix}$$

Unknown Depth

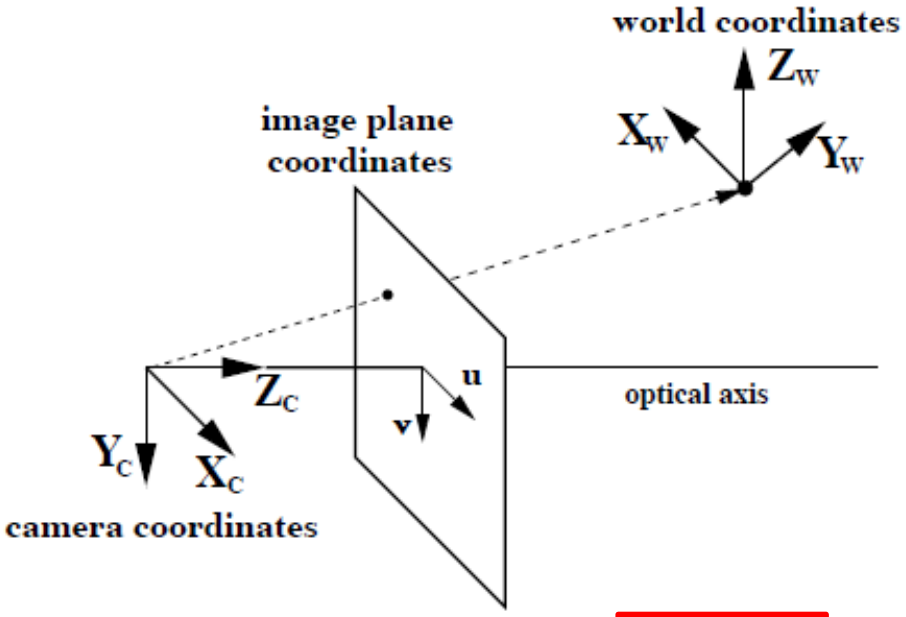
Pixel Values

Intrinsic Calibration Matrix

Camera Pose

World Point

Problem Formulation



Known

Measurement

To be Estimated

$$\lambda \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} f & 0 & u_o \\ 0 & f & v_o \end{pmatrix} \begin{pmatrix} R & t \end{pmatrix} \begin{pmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{pmatrix} = P \begin{pmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{pmatrix}$$

Pixel Values

Intrinsic Calibration Matrix

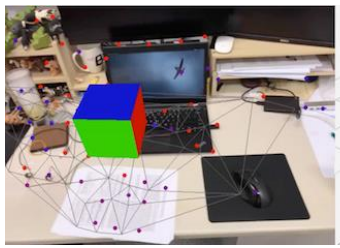
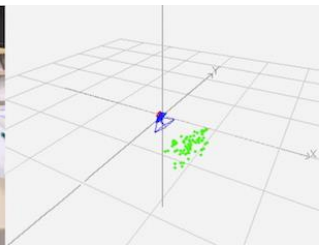
Camera Pose

World Point

Why Monocular?

- Minimum structural requirements
- Widely available sensors
- Applications:
  - State estimation for small drones
  - Mobile augmented reality



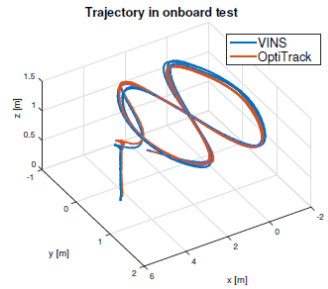




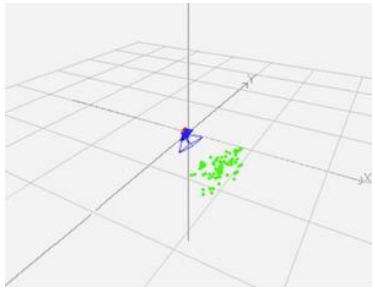
Why IMU?

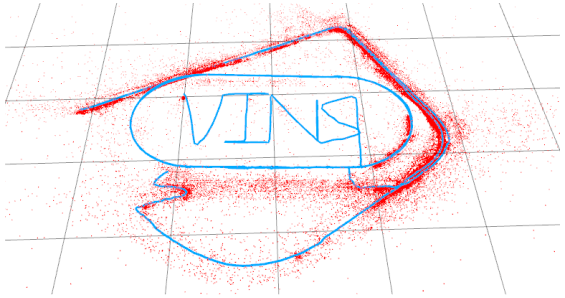

- IMU measures:
  - Linear acceleration
  - Angular velocity
- Pros:
  - Almost always available and outlier-free
  - Very high-rate measurements
  - Very mature technology, widely available at very low cost
  - Remarkable performance improvement during aggressive motions
- Cons:
  - Noisy sensor, cannot double integrate to obtain position
  - Synchronization and inter-sensor calibration requirements
  - Observability and numerical stability issues
  - Unable to operate when inertial and visual measurements are not in the same frame (e.g. on cars or trains)

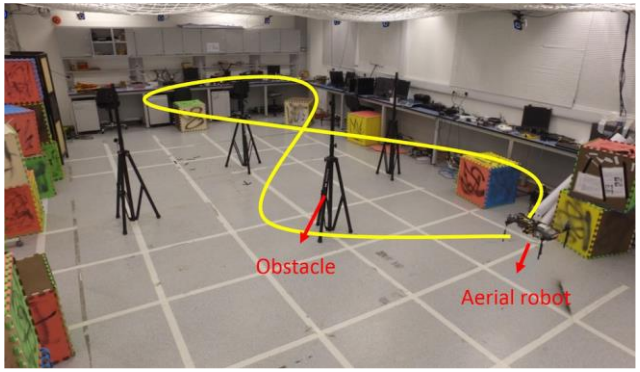
Requirements

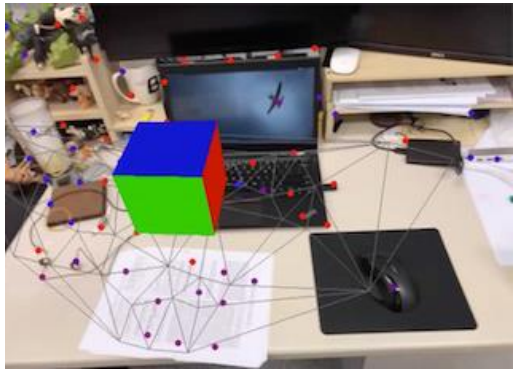
- Metric scale estimation using only one camera
- Mostly for state estimation (localization), map is sparse
- Robust and smooth odometry – local accuracy
- Loop closure – global consistency









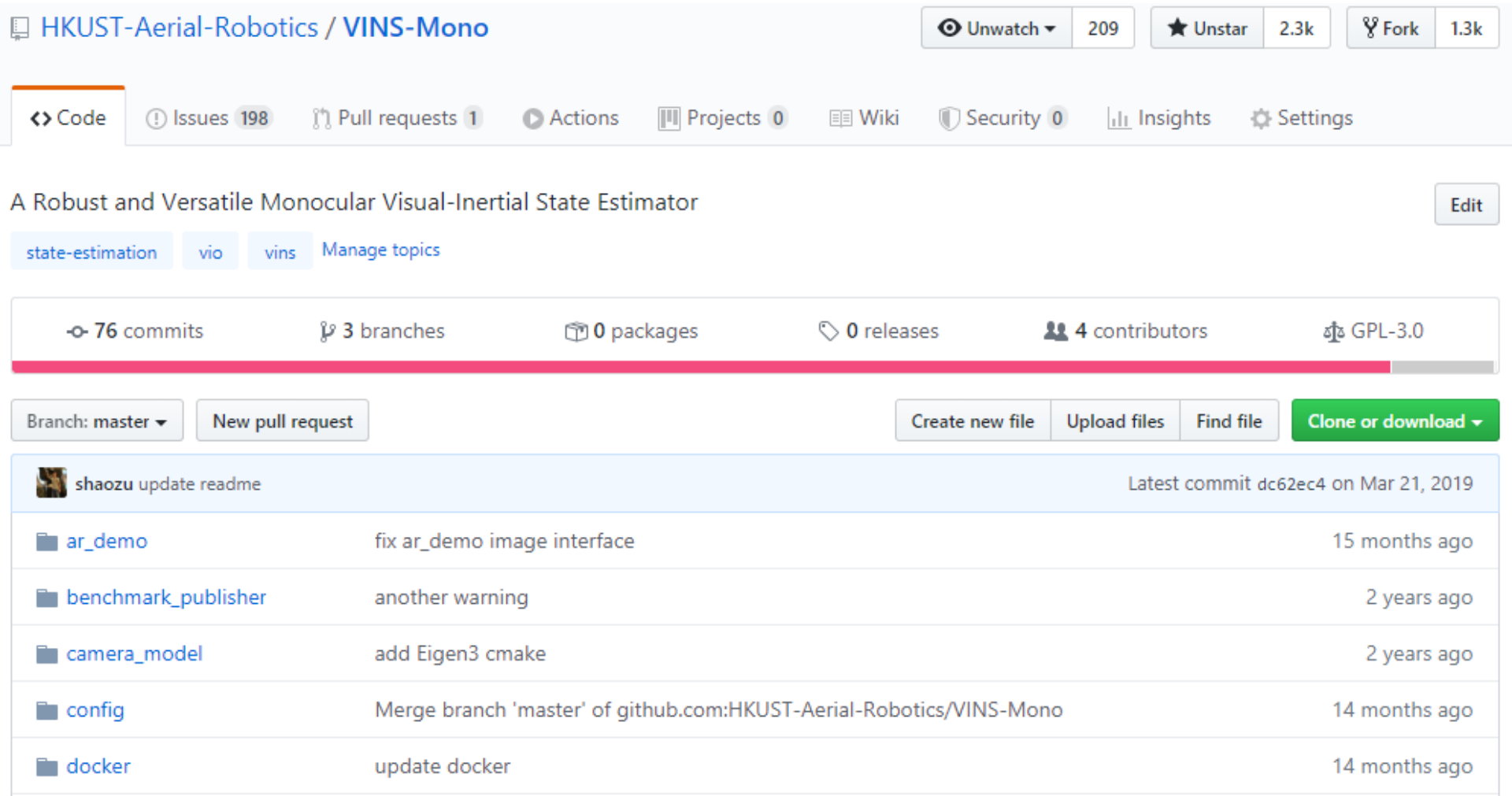


Source Code: http://github.com/HKUST-Aerial-Robotics/VINS-Mono

Related work

- MSC-KF (Mourikis and Roumeliotis, 2007)
- OKVIS (Leutenegger, et al., 2015)
  - Code: https://github.com/ethz-asl/okvis
- Visual-Inertial ORB SLAM (Mur-Artal and Tardos, 2017)
  - No official source code available yet
- Apple ARKit
- Google ARCore

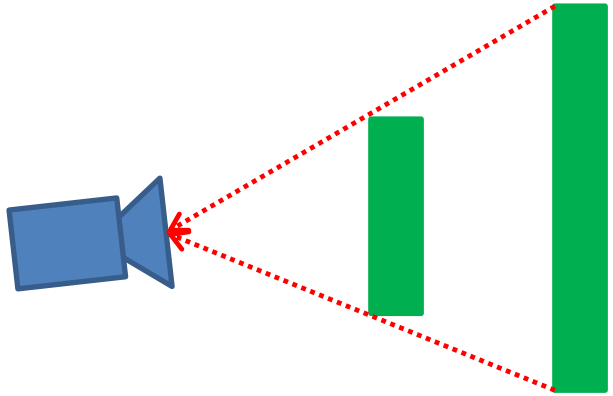
Our Solution: VINS-Mono



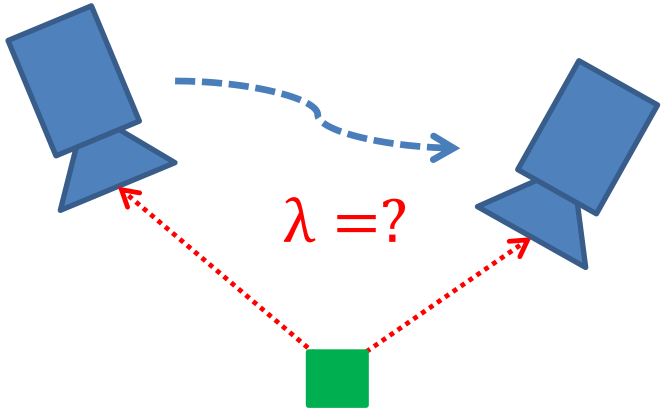
Source Code: http://github.com/HKUST-Aerial-Robotics/VINS-Mono

Challenges: Monocular Vision

- Scale ambiguity

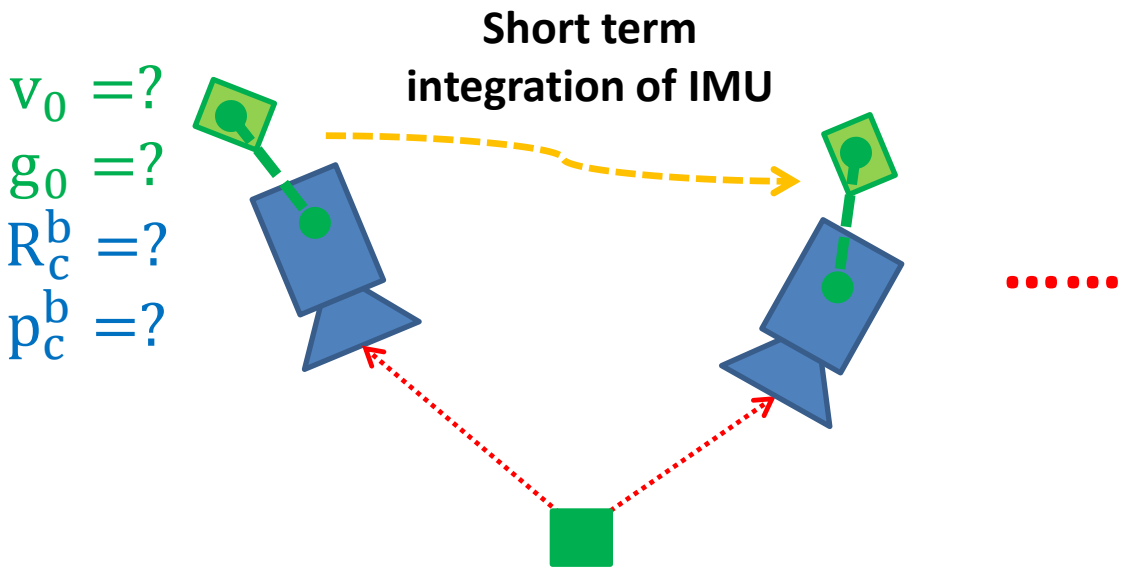


- Up-to-scale motion estimation and 3D reconstruction (Structure from Motion)



Challenges: Monocular Visual-Inertial Systems

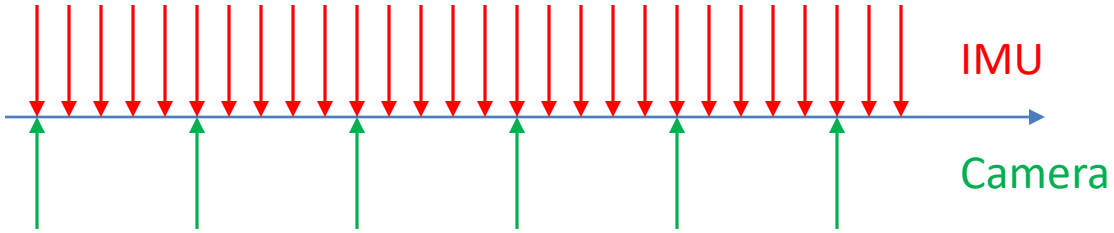
- With IMU, scale is observable (via accelerometer), but…
  - Requires recovery of initial velocity and attitude (gravity)
  - Requires online calibration camera-IMU extrinsic parameters
  - Requires multi-observation constraints



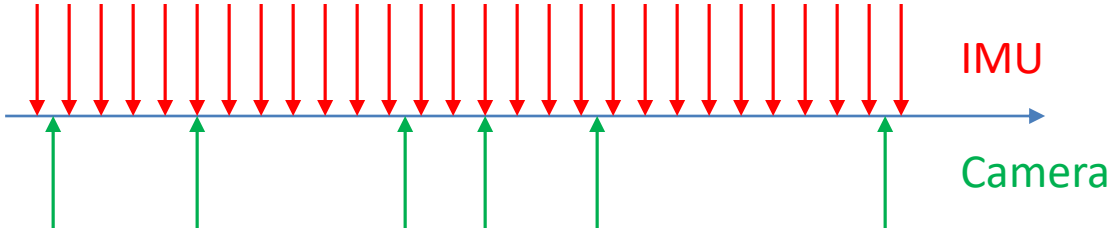
Source Code: http://github.com/HKUST-Aerial-Robotics/VINS-Mono

Challenges: Synchronization

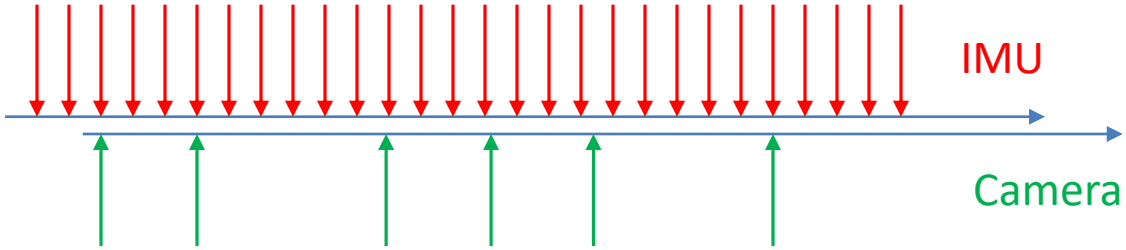
- Best: Sensors are hardware-triggered



- OK: Sensors have the same clock (e.g. running on the same system clock or have global clock correction) but capture data at different times

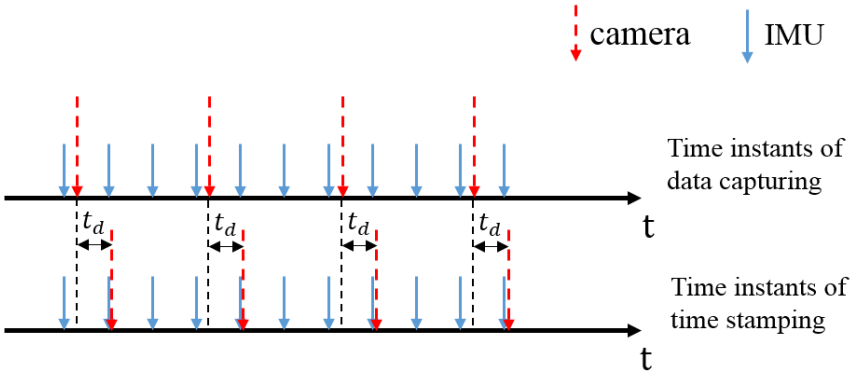


- Bad: Sensors have different clocks (e.g. each sensor has its own oscillator)

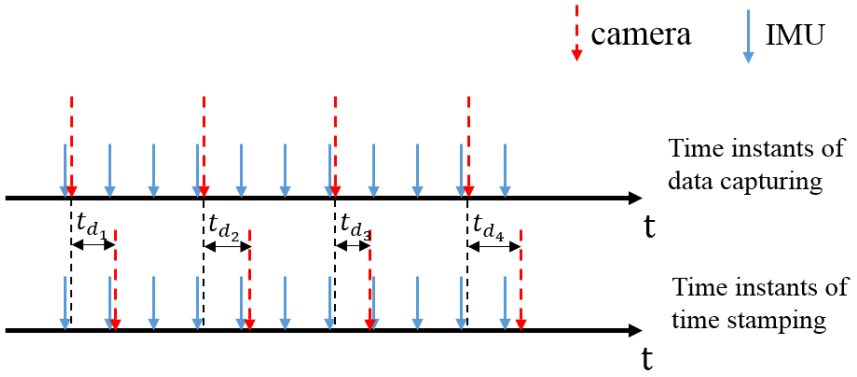


Challenges: Timestamps

- Timestamp: how the time for each sensor measurement is tagged
- Best: timestamping is done at data capture
- OK: fixed latency for time stamping
  - e.g. time is tagged on low-level hardware after some fixed-duration data processing, and will not be affected by any dynamic OS scheduling tasks
- Bad: variable latency in time stamping
  - e.g. plug two sensors into USB ports and time stamp according to the PC time. Time stamping is affected by data transmission latency from the sensor to PC



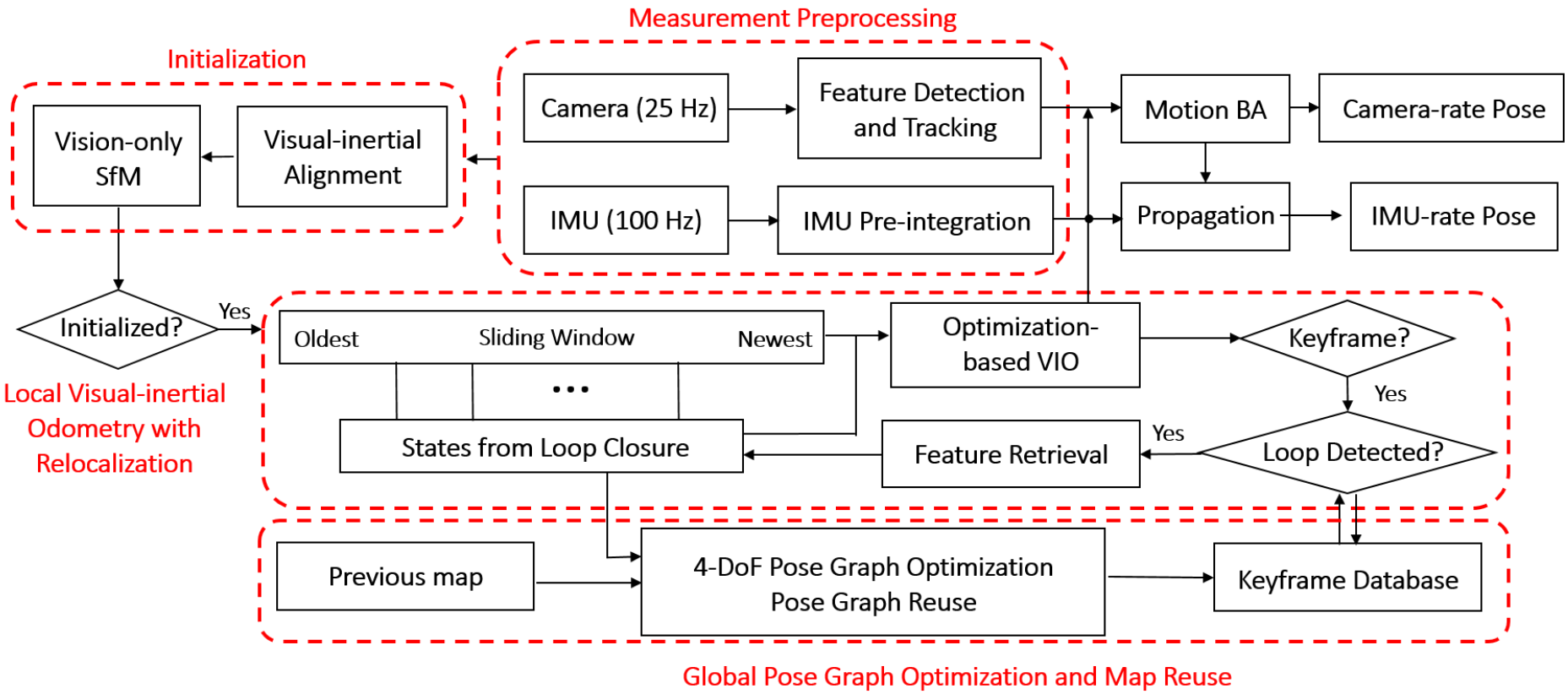
Good



Bad

Monocular Visual-Inertial SLAM

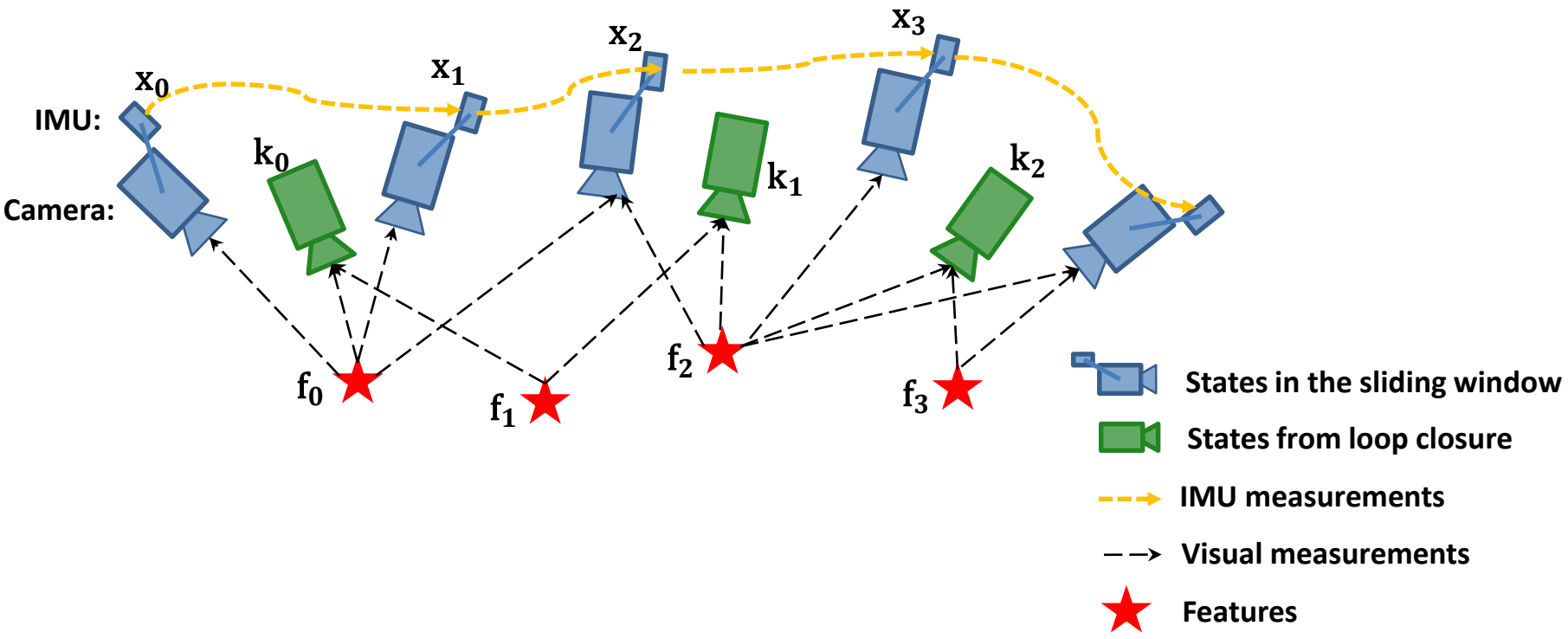
- System diagram



Source Code: http://github.com/HKUST-Aerial-Robotics/VINS-Mono

Monocular Visual-Inertial SLAM

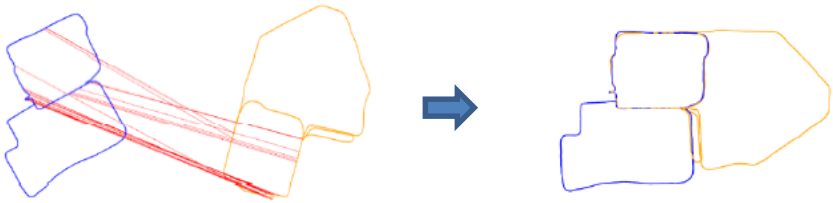
- Monocular visual-inertial odometry with relocalization
  - For local accuracy
  - Achieved via sliding window visual-inertial bundle adjustment

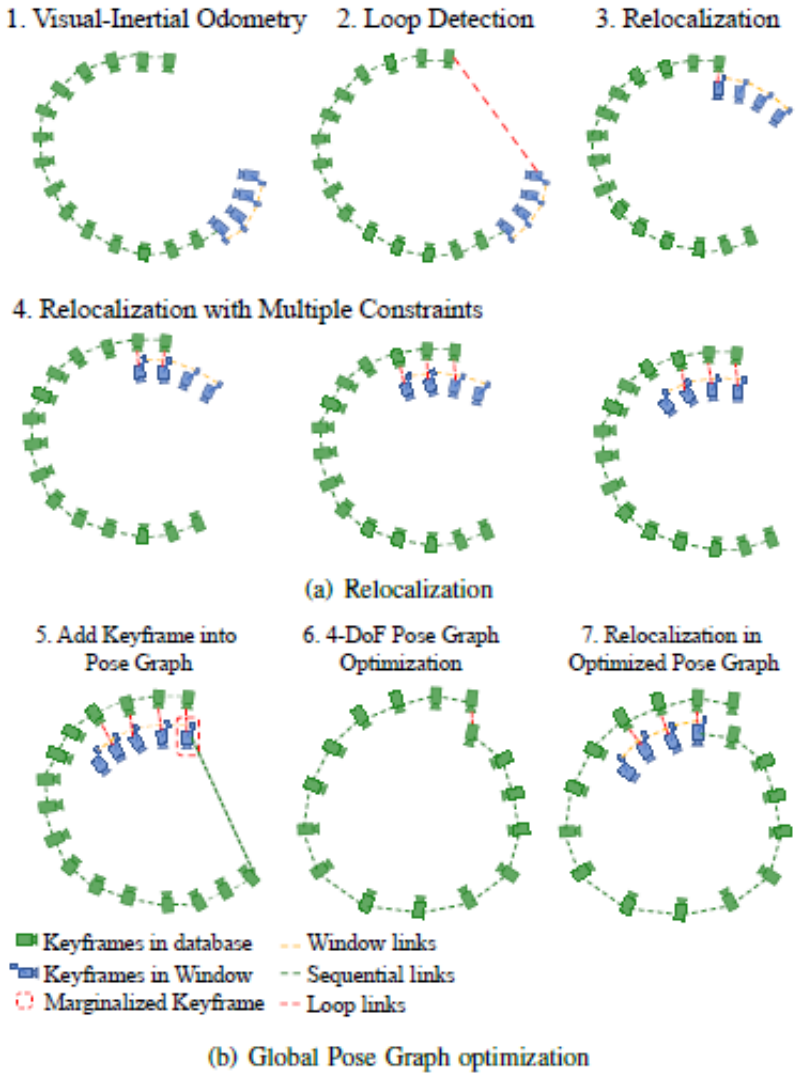


Source Code: http://github.com/HKUST-Aerial-Robotics/VINS-Mono

Monocular Visual-Inertial SLAM

- Global pose graph SLAM
  - For global consistency
  - Fully integrated with tightly-coupled re-localization
- Map reuse
  - Save map at any time
  - Load map and re-localize with respect to it
  - Pose graph merging

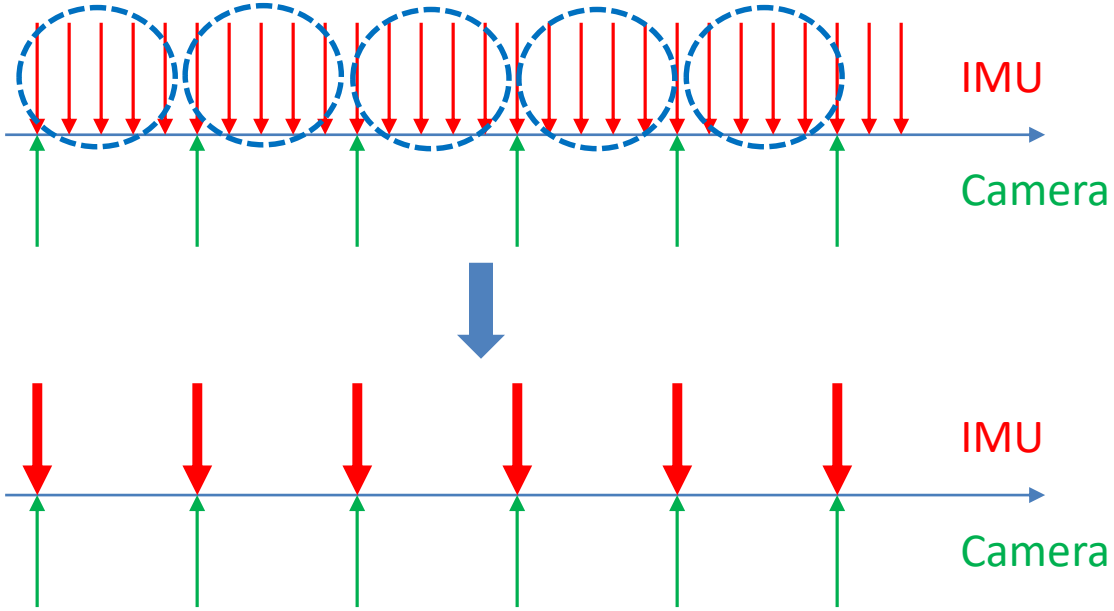




Source Code: http://github.com/HKUST-Aerial-Robotics/VINS-Mono

How to Use IMU?

- IMU integration
  - IMU has higher rate than camera
  - Cannot estimate all IMU states
  - Need to integration IMU measurements



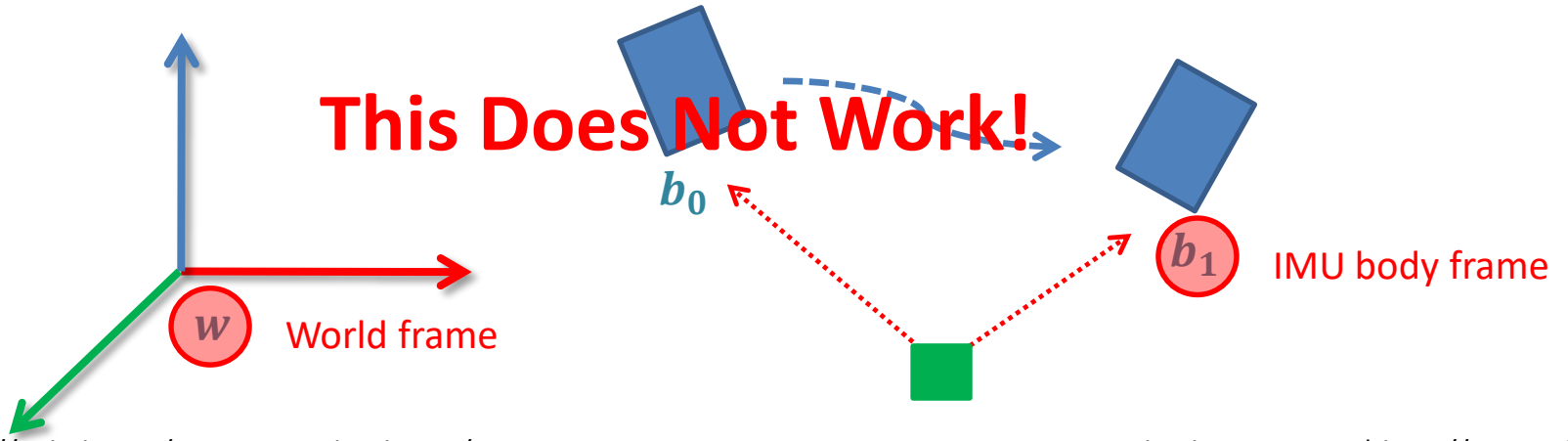
The Bad of IMU Integration in the Global Frame

- IMU integration in world frame
  - Requires global rotation at the time of integration

$$\mathbf{p}^w_{b_{k+1}} = \mathbf{p}^w_{b_k} + \mathbf{v}^w_{b_k}\Delta t_k + \iint_{t\in[t_k,t_{k+1}]} \left(\mathbf{R}^w_t(\hat{\mathbf{a}}_t - \mathbf{b}_{a_t} - \mathbf{n}_a) - \mathbf{g}^w\right) dt^2$$

$$\mathbf{v}^w_{b_{k+1}} = \mathbf{v}^w_{b_k} + \int_{t\in[t_k,t_{k+1}]} \left(\mathbf{R}^w_t(\hat{\mathbf{a}}_t - \mathbf{b}_{a_t} - \mathbf{n}_a) - \mathbf{g}^w\right) dt$$

$$\mathbf{q}^w_{b_{k+1}} = \mathbf{q}^w_{b_k} \otimes \int_{t\in[t_k,t_{k+1}]} \frac{1}{2}\boldsymbol{\Omega}(\hat{\boldsymbol{\omega}}_t - \mathbf{b}_{w_t} - \mathbf{n}_w)\mathbf{q}^{b_k}_t dt, \quad \boldsymbol{\Omega}(\boldsymbol{\omega}) = \begin{bmatrix} -\lfloor\boldsymbol{\omega}\rfloor_\times & \boldsymbol{\omega} \\ -\boldsymbol{\omega}^T & 0 \end{bmatrix}, \lfloor\boldsymbol{\omega}\rfloor_\times = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}.$$



Source Code: http://github.com/HKUST-Aerial-Robotics/VINS-Mono

IMU Pre-Integration on Manifold

- IMU integration in the body frame of first pose of interests
  - IMU Integration without initialization
  - Can use any discrete implementation for numerical integration
  - Intuitive: "position" and "velocity" changes in a "free-falling" frame

$$\mathbf{R}_w^{b_k}\mathbf{p}_{b_{k+1}}^w = \mathbf{R}_w^{b_k}\left(\mathbf{p}_{b_k}^w + \mathbf{v}_{b_k}^w\Delta t_k - \frac{1}{2}\mathbf{g}^w\Delta t_k^2\right) + \boldsymbol{\alpha}_{b_{k+1}}^{b_k}$$

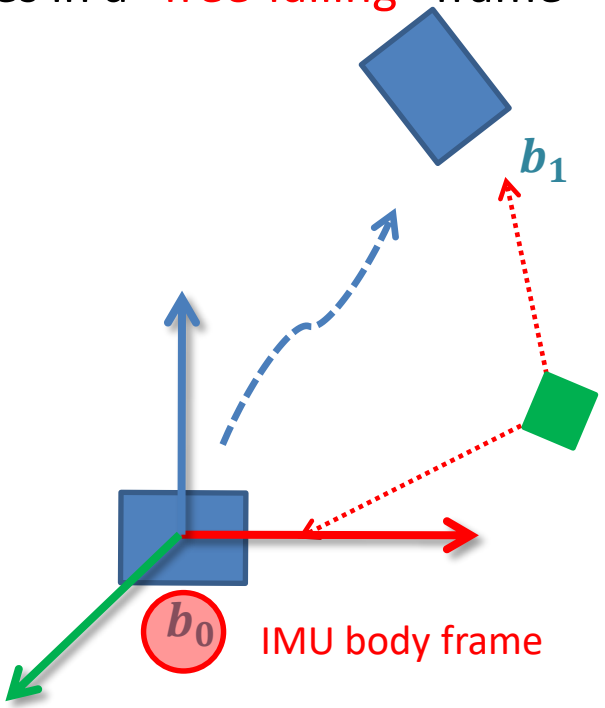
$$\mathbf{R}_w^{b_k}\mathbf{v}_{b_{k+1}}^w = \mathbf{R}_w^{b_k}\left(\mathbf{v}_{b_k}^w - \mathbf{g}^w\Delta t_k\right) + \boldsymbol{\beta}_{b_{k+1}}^{b_k}$$

$$\mathbf{q}_w^{b_k}\otimes\mathbf{q}_{b_{k+1}}^w = \boldsymbol{\gamma}_{b_{k+1}}^{b_k},$$

$$\boldsymbol{\alpha}_{b_{k+1}}^{b_k} = \iint_{t\in[t_k,t_{k+1}]}\mathbf{R}_t^{b_k}(\hat{\mathbf{a}}_t - \mathbf{b}_{a_t} - \mathbf{n}_a)dt^2$$

$$\boldsymbol{\beta}_{b_{k+1}}^{b_k} = \int_{t\in[t_k,t_{k+1}]}\mathbf{R}_t^{b_k}(\hat{\mathbf{a}}_t - \mathbf{b}_{a_t} - \mathbf{n}_a)dt$$

$$\boldsymbol{\gamma}_{b_{k+1}}^{b_k} = \int_{t\in[t_k,t_{k+1}]}\frac{1}{2}\boldsymbol{\Omega}(\hat{\boldsymbol{\omega}}_t - \mathbf{b}_{w_t} - \mathbf{n}_w)\boldsymbol{\gamma}_t^{b_k}dt.$$



Source Code: http://github.com/HKUST-Aerial-Robotics/VINS-Mono

IMU Pre-Integration on Manifold

- Uncertainty propagation on manifold
  - Derive the error state model for the IMU pre-integration dynamics

$$\begin{bmatrix} \delta\dot{\boldsymbol{\alpha}}_t^{b_k} \\ \delta\dot{\boldsymbol{\beta}}_t^{b_k} \\ \delta\dot{\boldsymbol{\theta}}_t^{b_k} \\ \delta\dot{\mathbf{b}}_{a_t} \\ \delta\dot{\mathbf{b}}_{w_t} \end{bmatrix} = \begin{bmatrix} 0 & \mathbf{I} & 0 & 0 & 0 \\ 0 & 0 & -\mathbf{R}_t^{b_k}\lfloor \hat{\mathbf{a}}_t - \mathbf{b}_{a_t} \rfloor_\times & -\mathbf{R}_t^{b_k} & 0 \\ 0 & 0 & -\lfloor \hat{\boldsymbol{\omega}}_t - \mathbf{b}_{w_t} \rfloor_\times & 0 & -\mathbf{I} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta\boldsymbol{\alpha}_t^{b_k} \\ \delta\boldsymbol{\beta}_t^{b_k} \\ \delta\boldsymbol{\theta}_t^{b_k} \\ \delta\mathbf{b}_{a_t} \\ \delta\mathbf{b}_{w_t} \end{bmatrix}$$

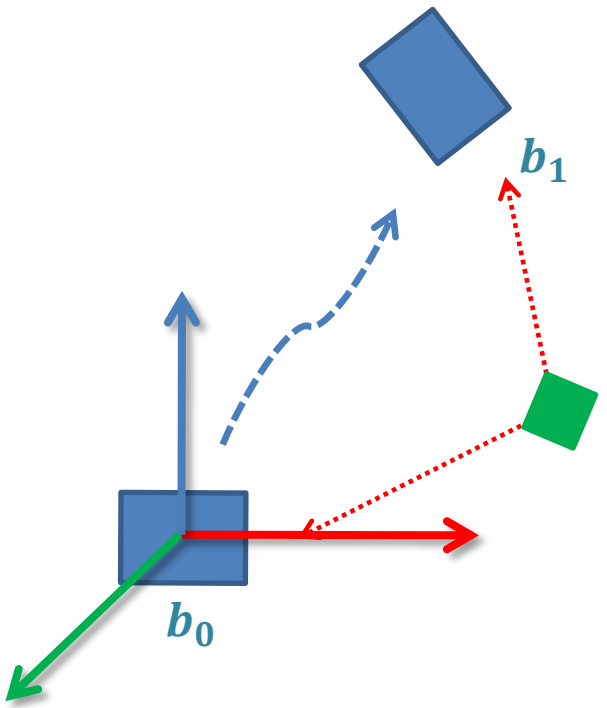
Bias uncertainty

$$+ \begin{bmatrix} 0 & 0 & 0 & 0 \\ -\mathbf{R}_t^{b_k} & 0 & 0 & 0 \\ 0 & -\mathbf{I} & 0 & 0 \\ 0 & 0 & \mathbf{I} & 0 \\ 0 & 0 & 0 & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{n}_a \\ \mathbf{n}_w \\ \mathbf{n}_{b_a} \\ \mathbf{n}_{b_w} \end{bmatrix} = \mathbf{F}_t \delta\mathbf{z}_t^{b_k} + \mathbf{G}_t \mathbf{n}_t.$$

  - Discrete-time implementation

$$\mathbf{P}_{t+\delta t}^{b_k} = (\mathbf{I} + \mathbf{F}_t\delta t)\mathbf{P}_t^{b_k}(\mathbf{I} + \mathbf{F}_t\delta t)^T + \delta t \mathbf{G}_t \mathbf{Q}_t \mathbf{G}_t{}^T, \quad t \in [k, k+1].$$

Covariance matrix for pre-integrated IMU measurements



Source Code: http://github.com/HKUST-Aerial-Robotics/VINS-Mono

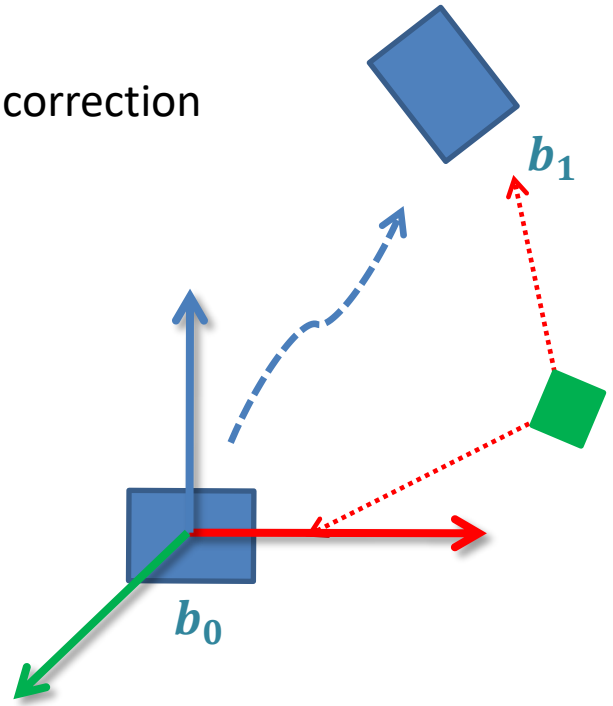
IMU Pre-Integration on Manifold

- Jacobian matrices for bias correction
  - Also derive the Jacobian of the pre-integrated measurements w.r.t. IMU bias

$$\mathbf{J}_{b_k} = \mathbf{I},$$
$$\mathbf{J}_{t+\delta t} = (\mathbf{I} + \mathbf{F}_t \delta t)\mathbf{J}_t, \quad t \in [k, k+1]$$

  - And write down the linearized model for bias correction

$$\boldsymbol{\alpha}_{b_{k+1}}^{b_k} \approx \hat{\boldsymbol{\alpha}}_{b_{k+1}}^{b_k} + \mathbf{J}_{b_a}^{\alpha} \delta \mathbf{b}_{a_k} + \mathbf{J}_{b_w}^{\alpha} \delta \mathbf{b}_{w_k}$$
$$\boldsymbol{\beta}_{b_{k+1}}^{b_k} \approx \hat{\boldsymbol{\beta}}_{b_{k+1}}^{b_k} + \mathbf{J}_{b_a}^{\beta} \delta \mathbf{b}_{a_k} + \mathbf{J}_{b_w}^{\beta} \delta \mathbf{b}_{w_k}$$
$$\boldsymbol{\gamma}_{b_{k+1}}^{b_k} \approx \hat{\boldsymbol{\gamma}}_{b_{k+1}}^{b_k} \otimes \begin{bmatrix} 1 \\ \frac{1}{2} \mathbf{J}_{b_w}^{\gamma} \delta \mathbf{b}_{w_k} \end{bmatrix}$$

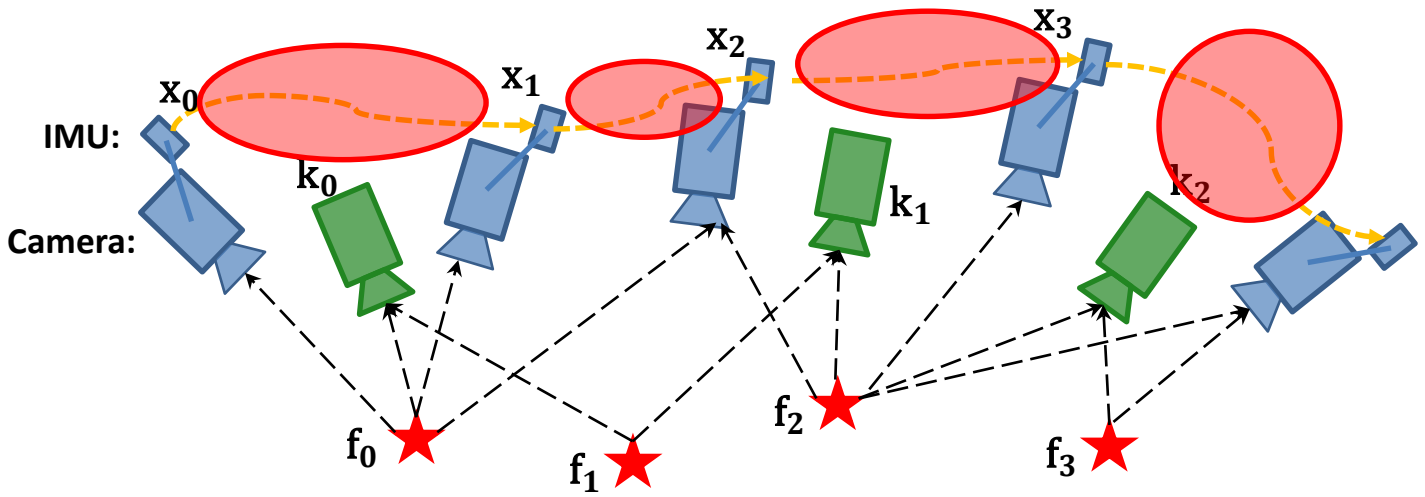


Source Code: http://github.com/HKUST-Aerial-Robotics/VINS-Mono

IMU Pre-Integration on Manifold

- Pre-integrated IMU measurement model
  - Describes the spatial and uncertainty relations between two states in the local sliding window

$$\begin{bmatrix} \hat{\alpha}_{b_{k+1}}^{b_k} \\ \hat{\beta}_{b_{k+1}}^{b_k} \\ \hat{\gamma}_{b_{k+1}}^{b_k} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \mathbf{R}_w^{b_k}(\mathbf{p}_{b_{k+1}}^w - \mathbf{p}_{b_k}^w + \frac{1}{2}\mathbf{g}^w\Delta t_k^2 - \mathbf{v}_{b_k}^w\Delta t_k) \\ \mathbf{R}_w^{b_k}(\mathbf{v}_{b_{k+1}}^w + \mathbf{g}^w\Delta t_k - \mathbf{v}_{b_k}^w) \\ \mathbf{q}_{b_k}^{w\,-1} \otimes \mathbf{q}_{b_{k+1}}^w \\ \mathbf{b}_{ab_{k+1}} - \mathbf{b}_{ab_k} \\ \mathbf{b}_{wb_{k+1}} - \mathbf{b}_{wb_k} \end{bmatrix}$$



Source Code: http://github.com/HKUST-Aerial-Robotics/VINS-Mono

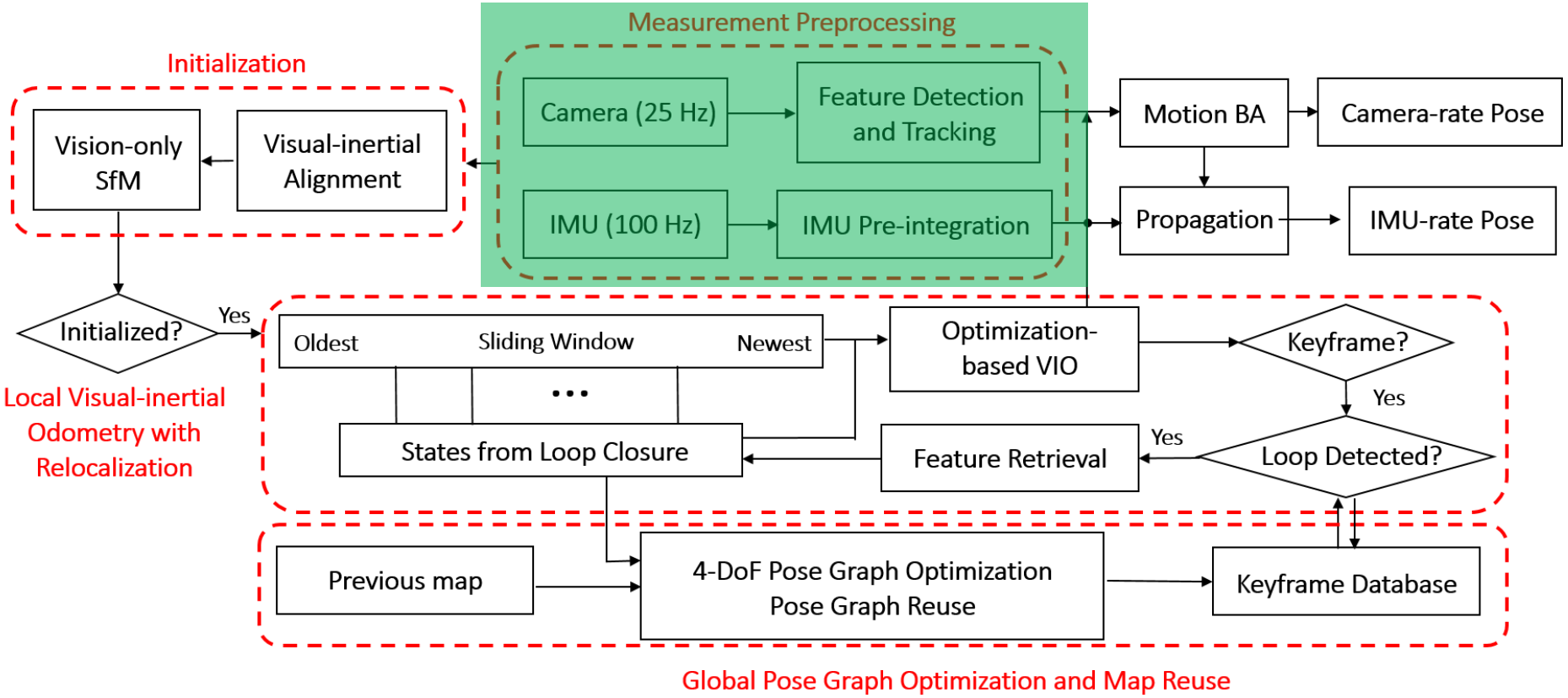
Vision Front-End

- Simple feature processing pipeline
  - Harris corners…
  - KLT tracker...
  - Track between consecutive frames
  - RANSAC for preliminary outlier removal
- Keyframe selection
  - Case 1: Rotation-compensated average feature parallax is larger than a threshold
    - Avoid numerical issues caused by poorly triangulated features
  - Case 2: Number of tracked features in the current frame is less than a threshold
    - Avoid losing tracking
  - All frames are used for optimization, but non-keyframes are removed first

Source Code: http://github.com/HKUST-Aerial-Robotics/VINS-Mono

Monocular Visual-Inertial SLAM

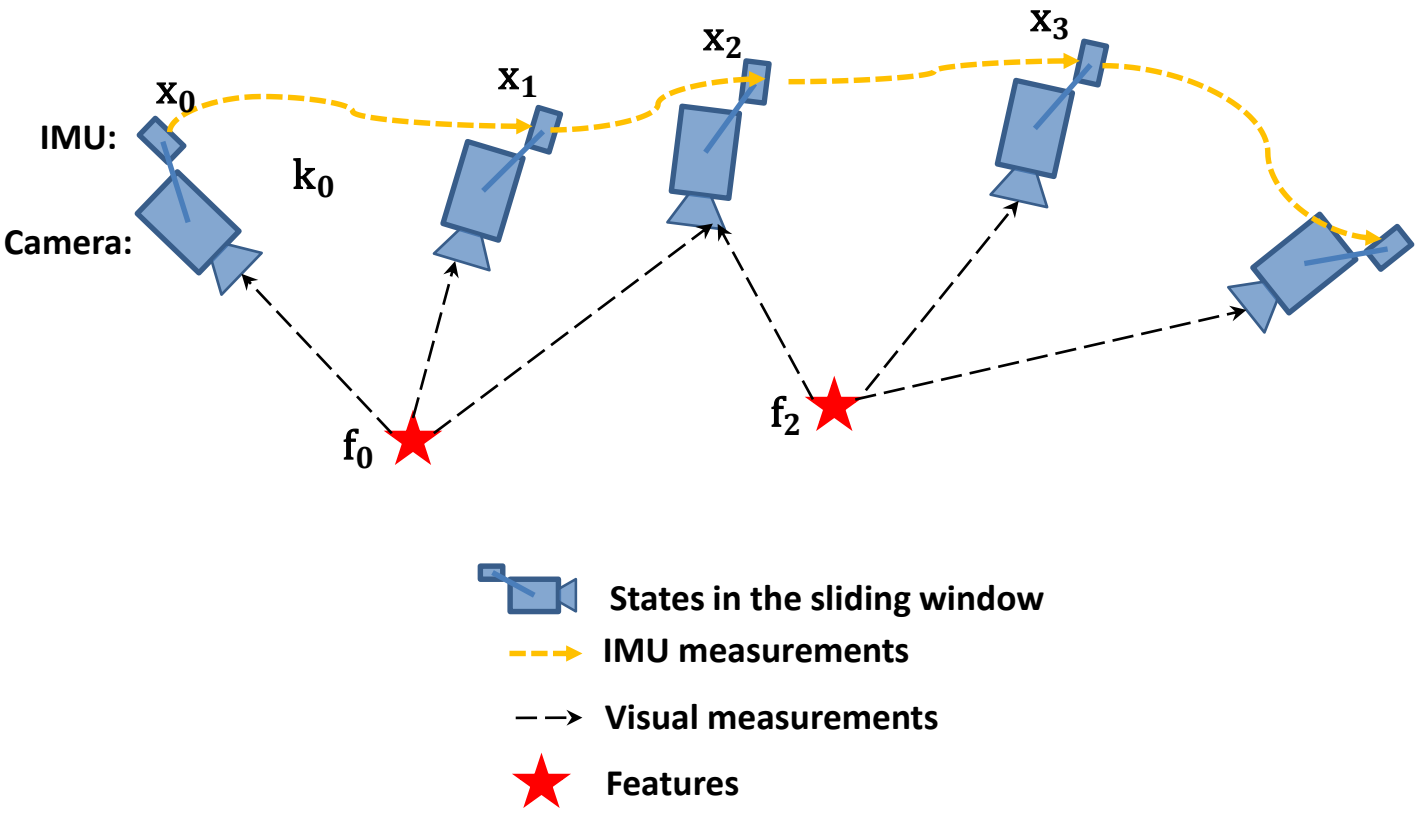
- System diagram



Source Code: http://github.com/HKUST-Aerial-Robotics/VINS-Mono

Monocular Visual-Inertial Odometry

- Nonlinear graph optimization-based, tightly-coupled, sliding window, visual-inertial bundle adjustment



Monocular Visual-Inertial Odometry

- Nonlinear graph-based optimization
  - Optimize position, velocity, rotation, IMU biases, inverse feature depth, and camera-IMU extrinsic calibration simultaneously:

$$\begin{aligned}\mathcal{X} &= [\mathbf{x}_0, \mathbf{x}_1, \cdots \mathbf{x}_n, \mathbf{x}_c^b, \lambda_0, \lambda_1, \cdots \lambda_m] \\ \mathbf{x}_k &= \left[\mathbf{p}_{b_k}^w, \mathbf{v}_{b_k}^w, \mathbf{q}_{b_k}^w, \mathbf{b}_a, \mathbf{b}_g\right], k \in [0, n] \\ \mathbf{x}_c^b &= \left[\mathbf{p}_c^b, \mathbf{q}_c^b\right],\end{aligned}$$

  - Minimize residuals from all sensors

IMU measurement residual Vision measurement residual

$$\min_{\mathcal{X}} \left\{ \|\mathbf{r}_p - \mathbf{H}_p\mathcal{X}\|^2 + \sum_{k\in\mathcal{B}} \left\| \mathbf{r}_{\mathcal{B}}(\hat{\mathbf{z}}_{b_{k+1}}^{b_k}, \mathcal{X}) \right\|^2_{\mathbf{P}_{b_{k+1}}^{b_k}} + \sum_{(l,j)\in\mathcal{C}} \left\| \mathbf{r}_{\mathcal{C}}(\hat{\mathbf{z}}_l^{c_j}, \mathcal{X}) \right\|^2_{\mathbf{P}_l^{c_j}} \right\}$$

Prior from marginalization Covariance from IMU pre-integration Pixel reprojection covariance

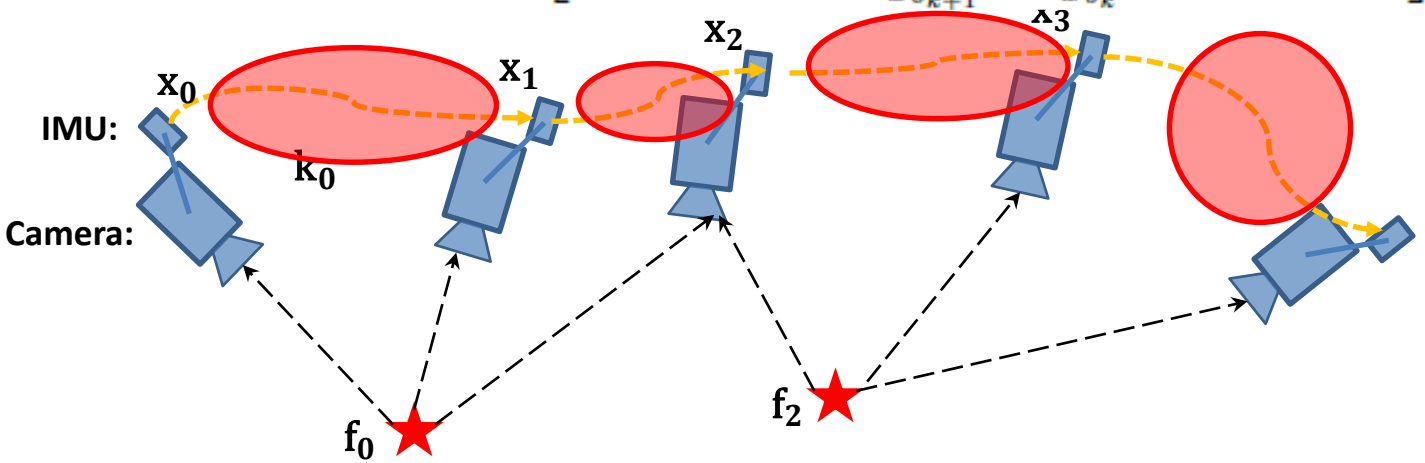
Source Code: http://github.com/HKUST-Aerial-Robotics/VINS-Mono

Monocular Visual-Inertial Odometry

- IMU measurement residual
  - Additive for "position" and "velocity" changes, and biases
  - Multiplicative for incremental rotation

IMU pre-integration "blocks"

$$\mathbf{r}_{\mathcal{B}}(\hat{\mathbf{z}}^{b_k}_{b_{k+1}}, \mathcal{X}) = \begin{bmatrix} \delta\boldsymbol{\alpha}^{b_k}_{b_{k+1}} \\ \delta\boldsymbol{\beta}^{b_k}_{b_{k+1}} \\ \delta\boldsymbol{\theta}^{b_k}_{b_{k+1}} \\ \delta\mathbf{b}_a \\ \delta\mathbf{b}_g \end{bmatrix} = \begin{bmatrix} \mathbf{R}^{b_k}_w(\mathbf{p}^w_{b_{k+1}} - \mathbf{p}^w_{b_k} + \frac{1}{2}\mathbf{g}^w\Delta t_k^2 - \mathbf{v}^w_{b_k}\Delta t_k) - \hat{\boldsymbol{\alpha}}^{b_k}_{b_{k+1}} \\ \mathbf{R}^{b_k}_w(\mathbf{v}^w_{b_{k+1}} + \mathbf{g}^w\Delta t_k - \mathbf{v}^w_{b_k}) - \hat{\boldsymbol{\beta}}^{b_k}_{b_{k+1}} \\ 2\left[\mathbf{q}^{w^{-1}}_{b_{k+1}} \otimes \mathbf{q}^w_{b_k} \otimes \hat{\boldsymbol{\gamma}}^{b_k}_{b_{k+1}}\right]_{xyz} \\ \mathbf{b}_{a b_{k+1}} - \mathbf{b}_{a b_k} \\ \mathbf{b}_{w b_{k+1}} - \mathbf{b}_{w b_k} \end{bmatrix}$$



Source Code: http://github.com/HKUST-Aerial-Robotics/VINS-Mono

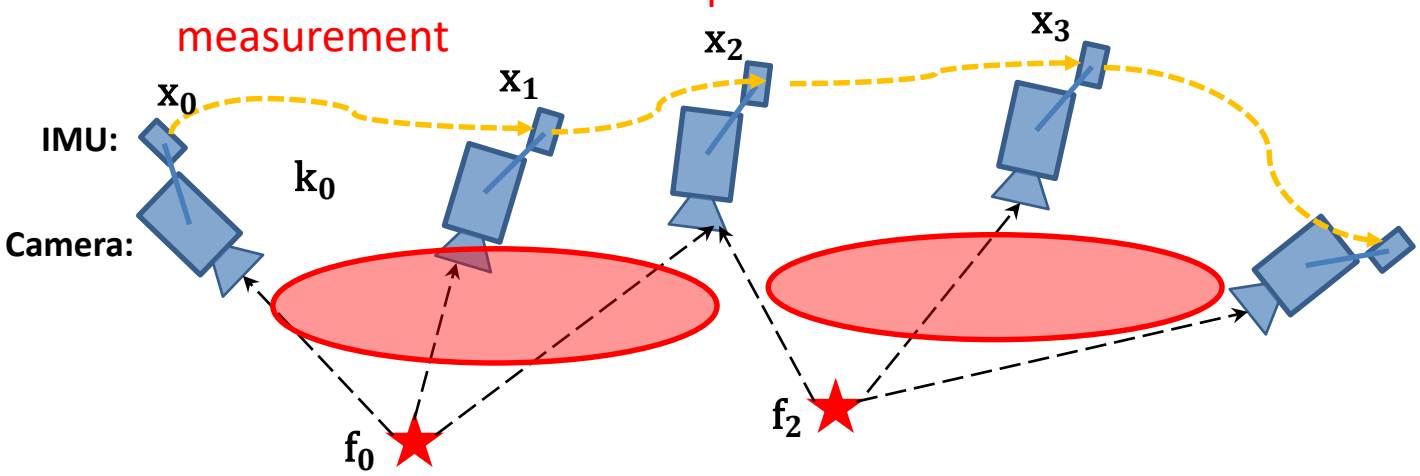
Monocular Visual-Inertial Odometry

- Vision measurement residual
  - Pixel reprojection error
  - Inverse depth model, at least 2 observations per feature, first observation to define feature direction

$$\mathbf{r}_C(\hat{\mathbf{z}}_l^{c_j}, \mathcal{X}) = \begin{bmatrix} \hat{u}_l^{c_j} \\ \hat{v}_l^{c_j} \end{bmatrix} - \pi_c\left(\mathbf{R}_b^c\left(\mathbf{R}_w^{b_j}\left(\mathbf{R}_{b_i}^w\left(\mathbf{R}_c^b \frac{1}{\lambda_l}\pi_c^{-1}\left(\begin{bmatrix} u_l^{c_i} \\ v_l^{c_i} \end{bmatrix}\right) + \mathbf{p}_c^b\right) + \mathbf{p}_{b_i}^w - \mathbf{p}_{b_j}^w\right) - \mathbf{p}_c^b\right)\right)$$

Actual feature measurement

Predicted pixel location in camera frame



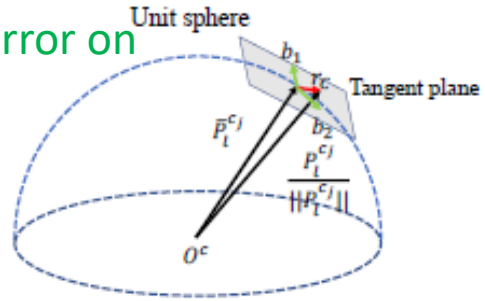
Source Code: http://github.com/HKUST-Aerial-Robotics/VINS-Mono

Monocular Visual-Inertial Odometry

- Vision measurement residual
  - Spherical camera model
  - At least 2 observations per feature

$$\mathbf{r}_{\mathcal{C}}(\hat{\mathbf{z}}_l^{c_j}, \mathcal{X}) = \begin{bmatrix} \mathbf{b}_1 & \mathbf{b}_2 \end{bmatrix}^T \cdot \left(\bar{\mathcal{P}}_l^{c_j} - \frac{\mathcal{P}_l^{c_j}}{\|\mathcal{P}_l^{c_j}\|}\right)$$

Reprojection error on tangent plane

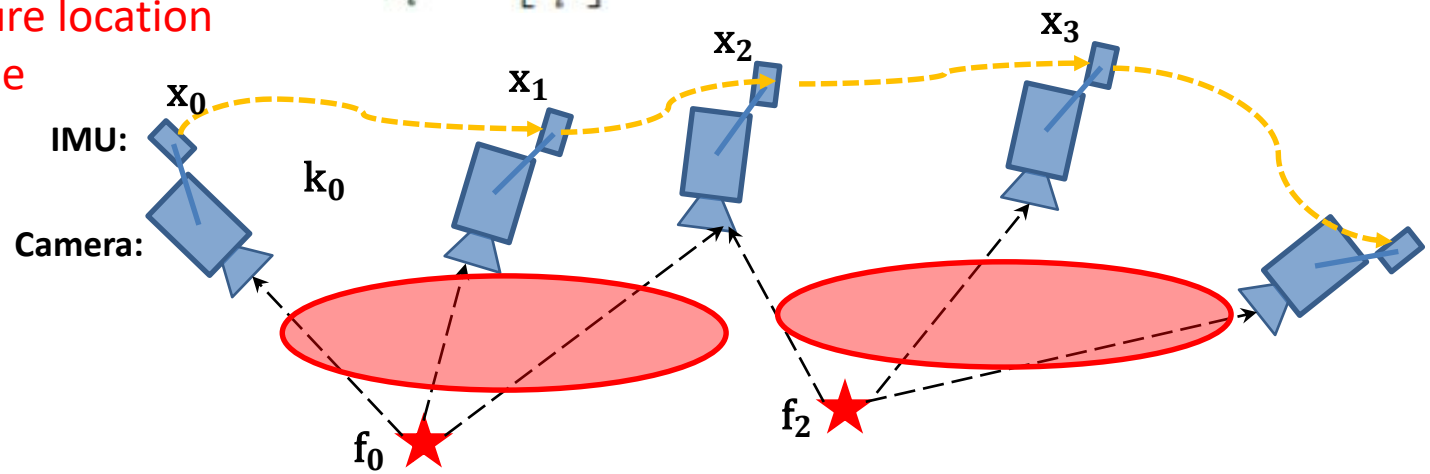


$$\bar{\mathcal{P}}_l^{c_j} = \pi_c^{-1}\left(\begin{bmatrix} \hat{u}_l^{c_j} \\ \hat{v}_l^{c_j} \end{bmatrix}\right)$$

Unit vector of feature from 2D measurement

$$\mathcal{P}_l^{c_j} = \mathbf{R}_b^c\left(\mathbf{R}_w^{b_j}\left(\mathbf{R}_{b_i}^w\left(\mathbf{R}_c^b \frac{1}{\lambda_l}\pi_c^{-1}\left(\begin{bmatrix} u_l^{c_i} \\ v_l^{c_i} \end{bmatrix}\right) + \mathbf{p}_c^b\right) + \mathbf{p}_{b_i}^w - \mathbf{p}_{b_j}^w\right) - \mathbf{p}_c^b\right)$$

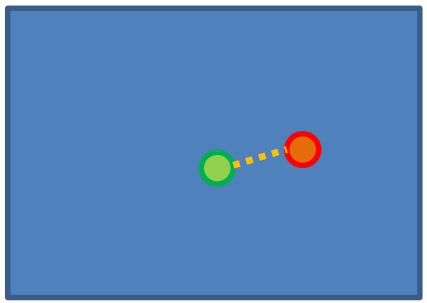
Predicted feature location in camera frame

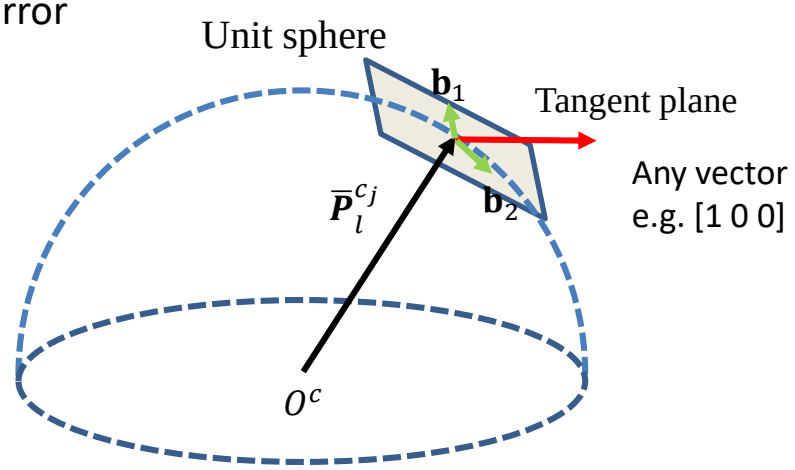


Source Code: http://github.com/HKUST-Aerial-Robotics/VINS-Mono

Monocular Visual-Inertial Odometry

- Vision measurement residual
  - Spherical camera model
  - Finding two basis vectors on the tangent plane
    - Choose any vector not parallel with $\bar{\boldsymbol{P}}_l^{c_j}$, e.g. [1 0 0]
    - $\mathbf{b}_1 = normalize(\bar{P}_l^{c_j} \times [1\ 0\ 0])$
    - $\mathbf{b}_2 = normalize(\bar{P}_l^{c_j} \times \mathbf{b}_1)$
- Spherical vs. pinhole camera models
  - Different ways to define the reprojection error
  - Able to model cameras with arbitrary FOV

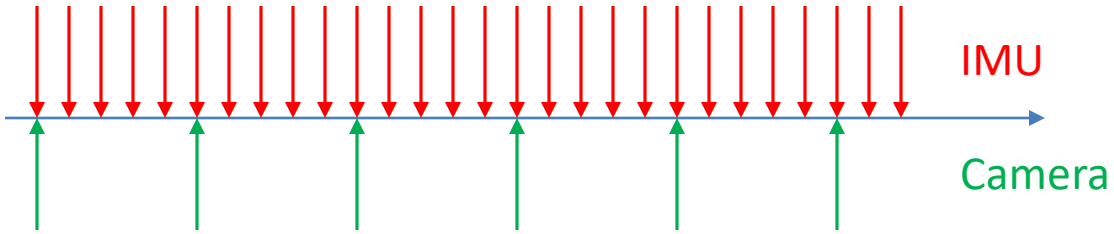




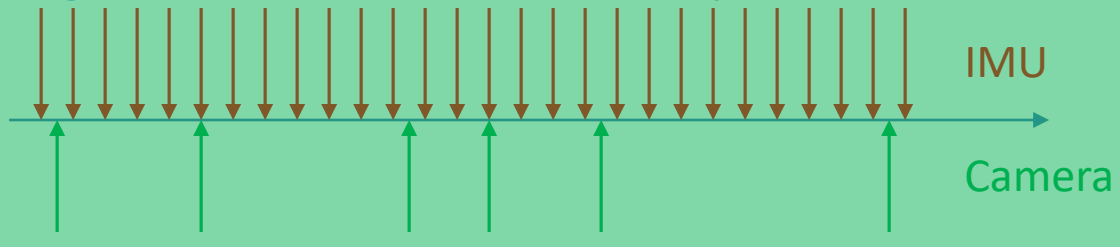
Source Code: http://github.com/HKUST-Aerial-Robotics/VINS-Mono

Review: Synchronization

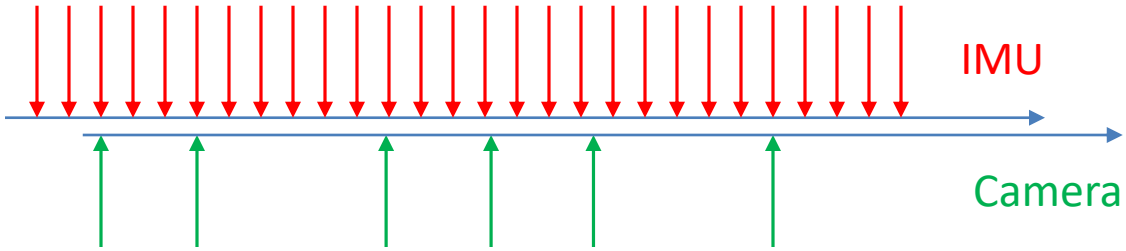
- Best: Sensors are hardware-triggered



- OK: Sensors have the same clock (e.g. running on the same system clock or have global clock correction) but capture data at different times

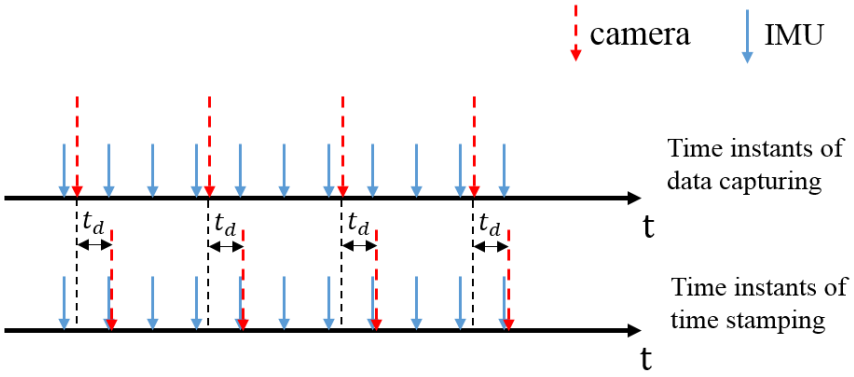


- Bad: Sensors have different clocks (e.g. each sensor has its own oscillator)

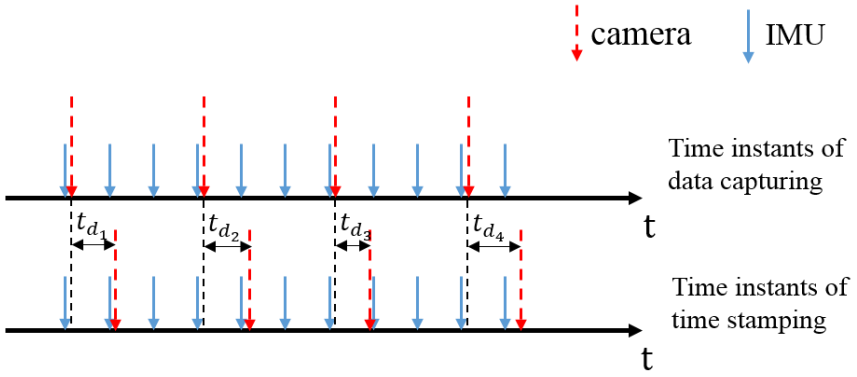


Review: Timestamps

- Timestamp: how the time for each sensor measurement is tagged
- Best: timestamping is done at data capture
- OK: fixed latency for time stamping
  - e.g. time is tagged on low-level hardware after some fixed-duration data processing, and will not be affected by any dynamic OS scheduling tasks
- Bad: variable latency in time stamping
  - e.g. plug two sensors into USB ports and time stamp according to the PC time. Time stamping is affected by data transmission latency from the sensor to PC



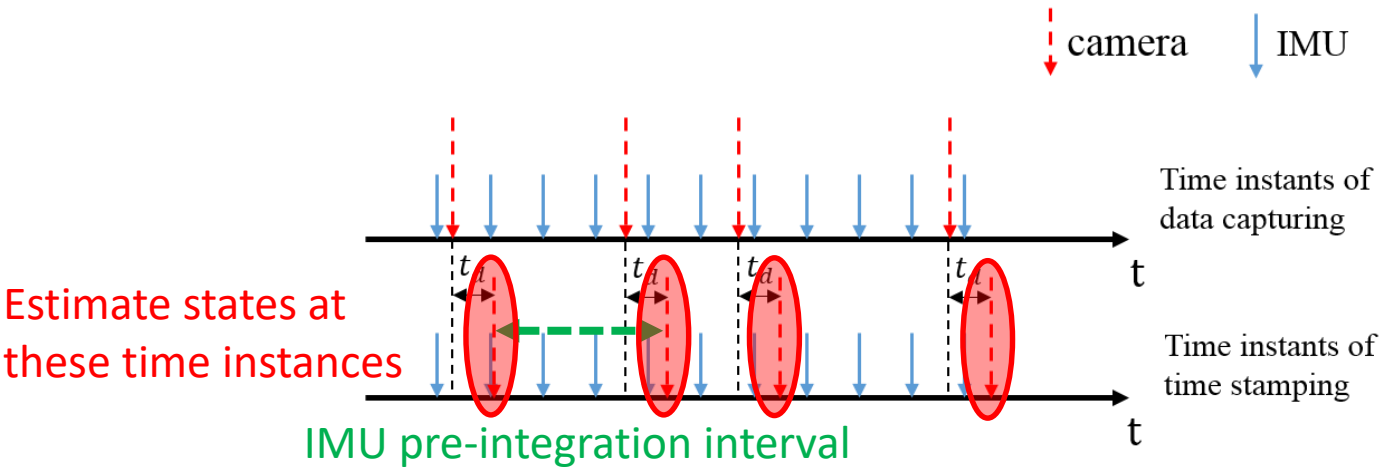
Good



Bad

Monocular Visual-Inertial Odometry

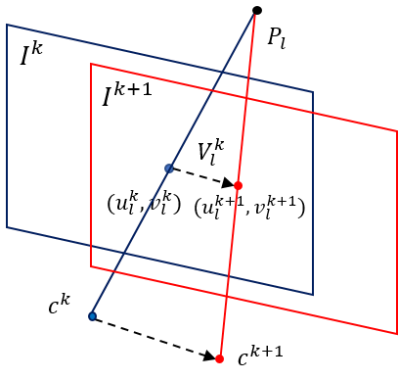
- Temporal calibration
  - Calibrate the fixed latency t_d occurred during time stamping
  - Change the IMU pre-integration interval to the interval between two image timestamps
    - Linear incorporation of IMU measurements to obtain the IMU reading at image time stamping
    - Estimates states (position, orientation, etc.) at image time stamping



Monocular Visual-Inertial Odometry

- Vision measurement residual for temporal calibration
  - Feature velocity on image plane
    - feature l moves at speed V_l^k from image k to $k+1$ in short time period $[t_k, t_{k+1}]$

$$\mathbf{V}_l^k = \left(\begin{bmatrix} u_l^{k+1} \\ v_l^{k+1} \end{bmatrix} - \begin{bmatrix} u_l^k \\ v_l^k \end{bmatrix}\right)/(t_{k+1} - t_k)$$

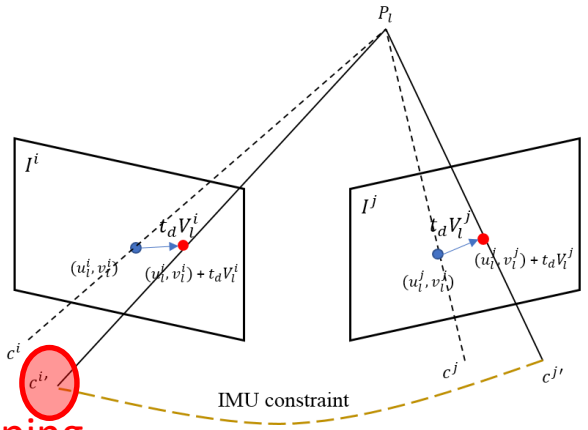


  - Visual measurement residual with time offset
    - New state variable t_d, and estimate states $(c^{i'}, c^{j'})$ at time stamping

$$\mathbf{r}_{\mathcal{C}}(\hat{\mathbf{z}}_l^{c_j}, \mathcal{X}) = \begin{bmatrix} \mathbf{b}_1 & \mathbf{b}_2 \end{bmatrix}^T \cdot \left(\bar{\mathcal{P}}_l^{c_j} - \frac{\mathcal{P}_l^{c_j}}{\|\mathcal{P}_l^{c_j}\|}\right)$$

$$\bar{\mathcal{P}}_l^{c_j} = \pi_c^{-1}\left(\begin{bmatrix} \hat{u}_l^{c_j} \\ \hat{v}_l^{c_j} \end{bmatrix} + t_d \vec{V}_l^{c_j}\right)$$

$$\mathcal{P}_l^{c_j} = \mathbf{R}_b^c\left(\mathbf{R}_w^{b_j}\left(\mathbf{R}_{b_i}^w\left(\mathbf{R}_c^b \frac{1}{\lambda_l}\pi_c^{-1}\left(\begin{bmatrix} u_l^{c_i} \\ v_l^{c_i} \end{bmatrix} + t_d \vec{V}_l^{c_i}\right)\right.\right.\right.$$
$$\left.\left.\left. + \mathbf{p}_c^b\right) + \mathbf{p}_{b_i}^w - \mathbf{p}_{b_j}^w\right) - \mathbf{p}_c^b\right)$$

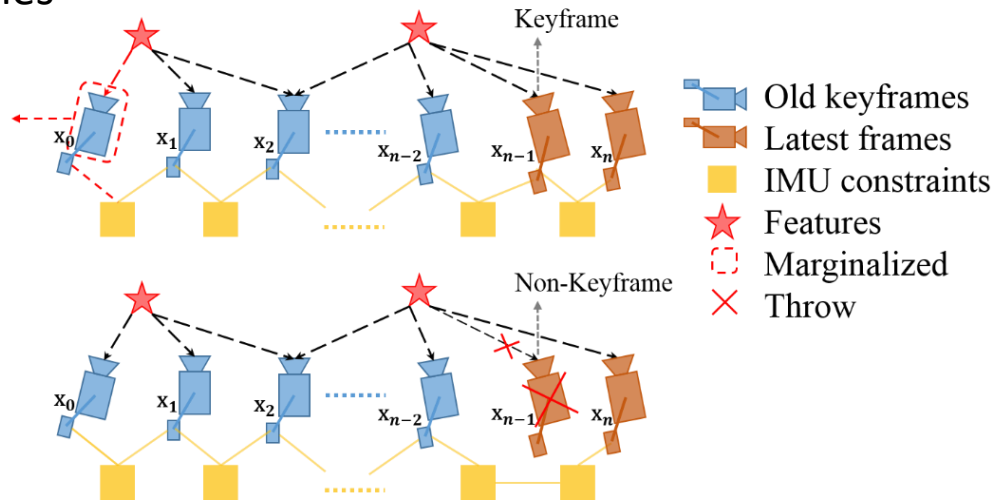


"Virtual image" at time stamping

Source Code: http://github.com/HKUST-Aerial-Robotics/VINS-Mono

Monocular Visual-Inertial Odometry

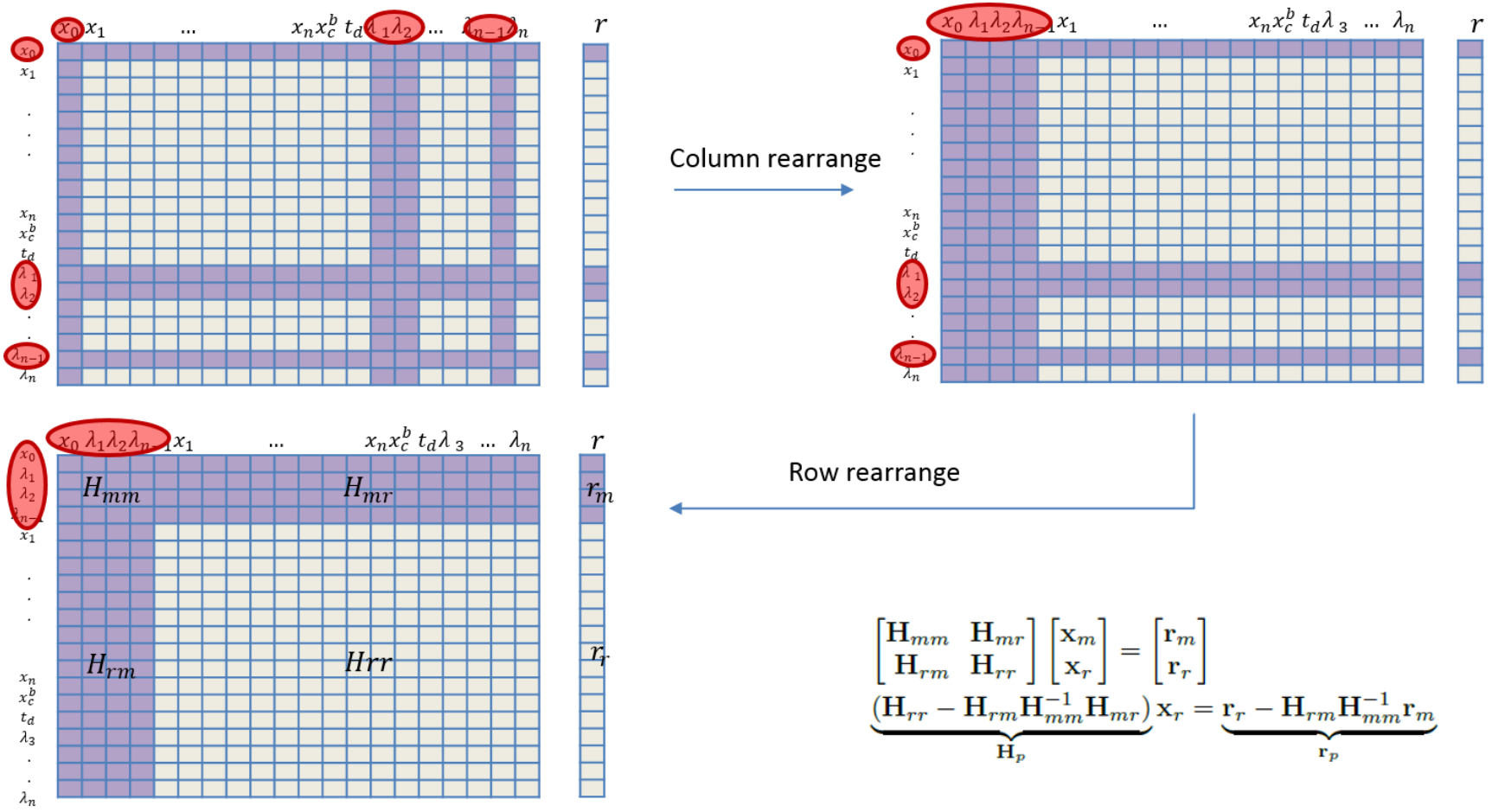
- Marginalization
  - Bound computation complexity to a sliding window of states
  - Basic principles:
    - Add all frames into the sliding window, and remove non-keyframes after the nonlinear optimization
    - keep as many keyframes with sufficient parallax as possible
    - Maintain matrix sparsity by throwing away visual measurements from non-keyframes



Source Code: http://github.com/HKUST-Aerial-Robotics/VINS-Mono

Monocular Visual-Inertial Odometry

- Marginalization via Schur complement on information matrix



$$\begin{bmatrix} \mathbf{H}_{mm} & \mathbf{H}_{mr} \\ \mathbf{H}_{rm} & \mathbf{H}_{rr} \end{bmatrix} \begin{bmatrix} \mathbf{x}_m \\ \mathbf{x}_r \end{bmatrix} = \begin{bmatrix} \mathbf{r}_m \\ \mathbf{r}_r \end{bmatrix}$$

$$\underbrace{(\mathbf{H}_{rr} - \mathbf{H}_{rm}\mathbf{H}_{mm}^{-1}\mathbf{H}_{mr})}_{\mathbf{H}_p}\mathbf{x}_r = \underbrace{\mathbf{r}_r - \mathbf{H}_{rm}\mathbf{H}_{mm}^{-1}\mathbf{r}_m}_{\mathbf{r}_p}$$

Source Code: http://github.com/HKUST-Aerial-Robotics/VINS-Mono

Monocular Visual-Inertial Odometry

- Solving the nonlinear system
  - Minimize residuals from all sensors

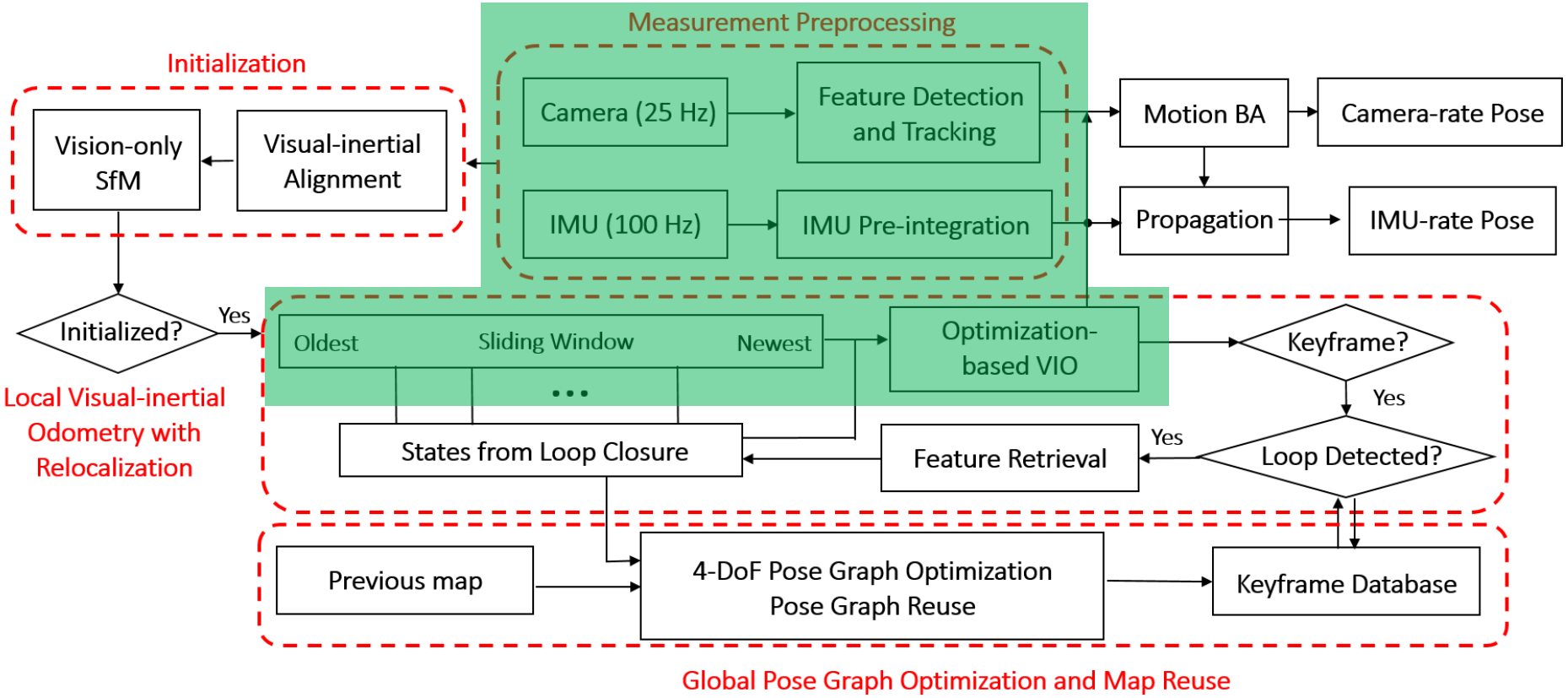
$$\min_{\mathcal{X}} \left\{ \|\mathbf{r}_p - \mathbf{H}_p \mathcal{X}\|^2 + \sum_{k \in \mathcal{B}} \left\| \mathbf{r}_{\mathcal{B}}(\hat{\mathbf{z}}_{b_{k+1}}^{b_k}, \mathcal{X}) \right\|_{\mathbf{P}_{b_{k+1}}^{b_k}}^2 + \sum_{(l,j) \in \mathcal{C}} \left\| \mathbf{r}_{\mathcal{C}}(\hat{\mathbf{z}}_l^{c_j}, \mathcal{X}) \right\|_{\mathbf{P}_l^{c_j}}^2 \right\}$$

  - Linearize (to Ax=b), solve, and iterate until time budget is reached
  - Ceres Solver (http://ceres-solver.org/)
  - Utilize sparse matrix solver
- Qualitative discussion on solution quality
  - Numerical stability issues always exist, much worse than vSLAM
    - Good: walking and aerial robots
    - Bad: ground vehicle moving in 2D
    - Failure: constant velocity or pure rotation
  - Downgraded performance in distanced scenes

Source Code: http://github.com/HKUST-Aerial-Robotics/VINS-Mono

Monocular Visual-Inertial SLAM

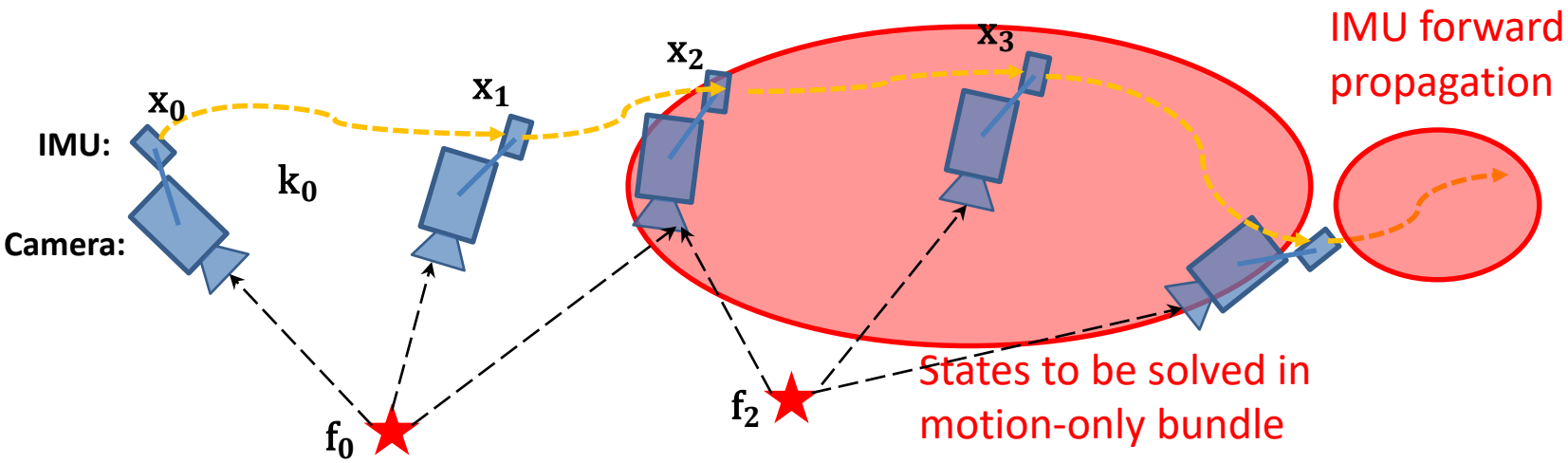
- System diagram



Source Code: http://github.com/HKUST-Aerial-Robotics/VINS-Mono

Monocular Visual-Inertial Odometry

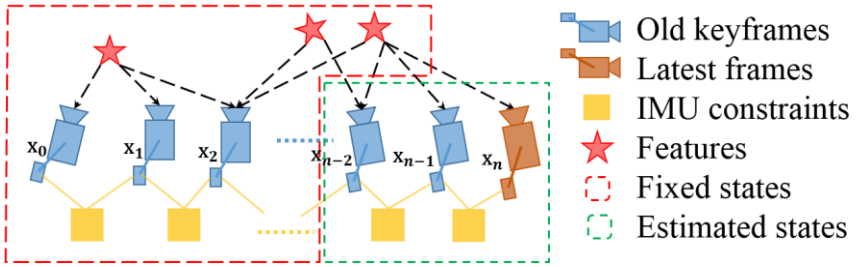
- Speeding up
  - The sliding window monocular visual-inertial bundle adjustment runs at 10Hz
  - Motion-only visual-inertial bundle adjustment to boost up the state estimation 30Hz
  - IMU forward propagation to boost to 100Hz



Monocular Visual-Inertial Odometry

- Motion-only visual-inertial bundle adjustment
  - Optimize position, velocity, rotation in a smaller windows, assuming all other quantities are fixed

$$\mathcal{X} = [\mathbf{x}_0, \mathbf{x}_1, \cdots \mathbf{x}_n, \mathbf{x}_c^b, \lambda_0, \lambda_1, \cdots \lambda_m]$$
$$\mathbf{x}_k = \left[\mathbf{p}_{b_k}^w, \mathbf{v}_{b_k}^w, \mathbf{q}_{b_k}^w, \mathbf{b}_a, \mathbf{b}_g\right], k \in [0, n]$$
$$\mathbf{x}_c^b = [\mathbf{p}_c^b, \mathbf{q}_c^b],$$



  - Prior in cost function is ignored

IMU measurement residual Vision measurement residual

$$\min_{\mathcal{X}} \left\{ \|\mathbf{r}_p - \mathbf{H}_p \mathcal{X}\|^2 + \sum_{k \in \mathcal{B}} \left\| \mathbf{r}_{\mathcal{B}}(\hat{\mathbf{z}}_{b_{k+1}}^{b_k}, \mathcal{X}) \right\|_{\mathbf{P}_{b_{k+1}}^{b_k}}^2 + \sum_{(l,j) \in \mathcal{C}} \left\| \mathbf{r}_{\mathcal{C}}(\hat{\mathbf{z}}_l^{c_j}, \mathcal{X}) \right\|_{\mathbf{P}_l^{c_j}}^2 \right\}$$

Covariance from IMU pre-integration Pixel reprojection covariance

  - Also solved using the Ceres Solver

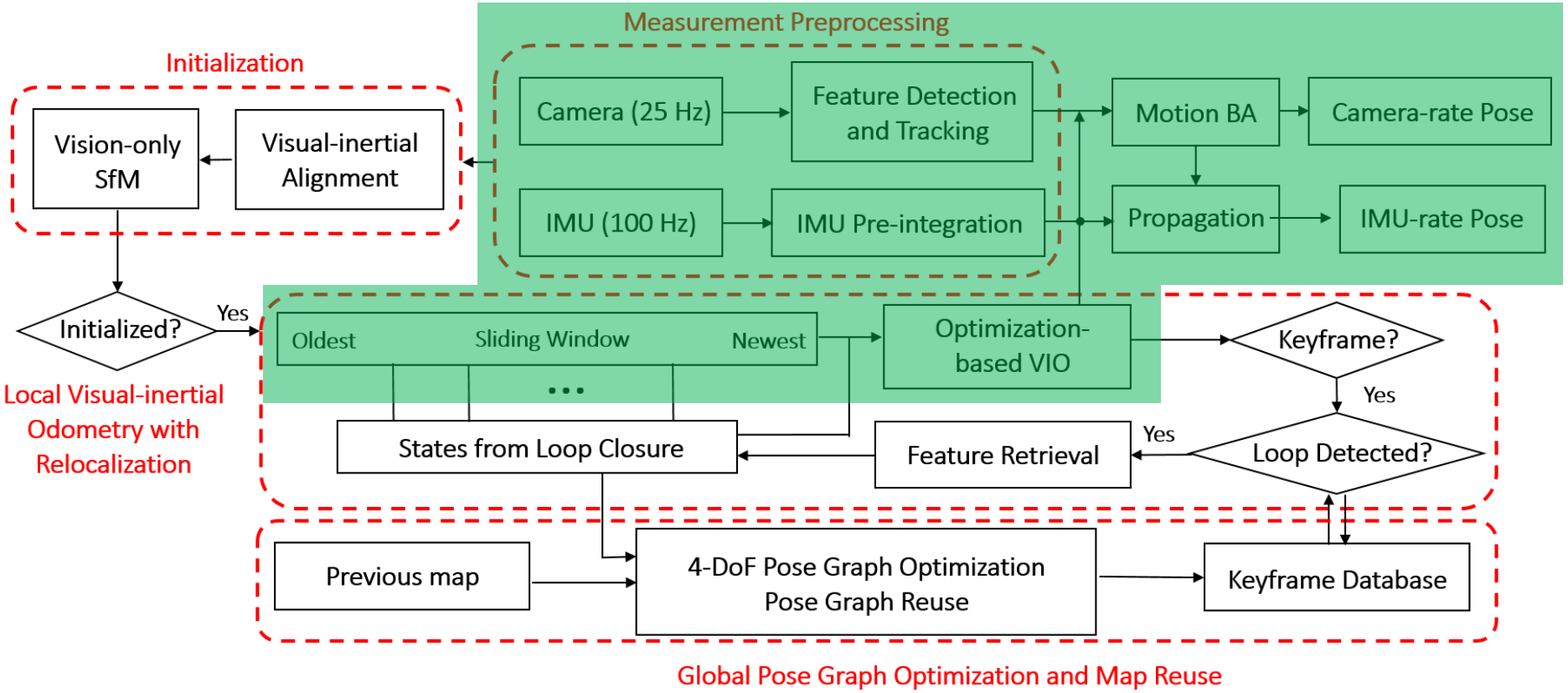
Source Code: http://github.com/HKUST-Aerial-Robotics/VINS-Mono

Monocular Visual-Inertial Odometry

- Failure detection
  - Few trackable feature in the current frame
  - Large jumps in nonlinear solver
  - Abnormal bias or extrinsic parameter calibration
  - Modeled as a standalone module, more to be added…
- Failure recovery
  - Just run the initialization again…
  - Lots of book keeping…

Monocular Visual-Inertial SLAM

- System diagram



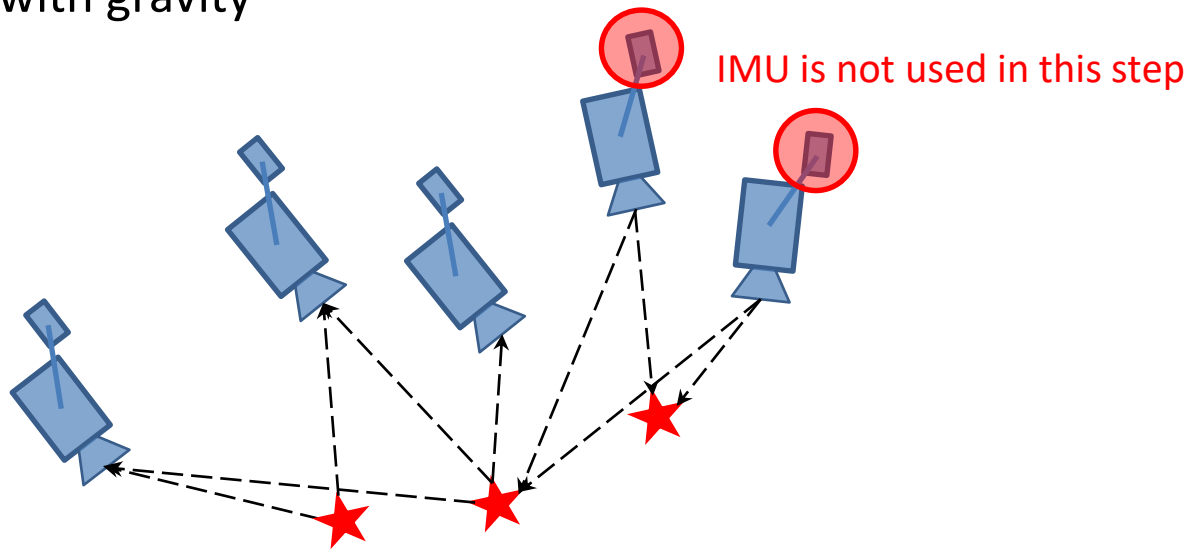
Source Code: http://github.com/HKUST-Aerial-Robotics/VINS-Mono

Estimator Initialization

- Very, very, very important for monocular visual-inertial systems
- Assumption 1: known camera-IMU extrinsic calibration during initialization
  - Does not need to be very accurate
  - Extrinsic calibration is refined in later nonlinear optimization
- Assumption 2: known accelerometer and gyroscope biases during initialization
  - Use zero values at power-up
  - Use prior values during failure recovery
  - Reasonable assumption due to slow varying nature of biases
- Pipeline
  - Monocular vision-only SFM in a local window
  - Visual-inertial alignment

Estimator Initialization

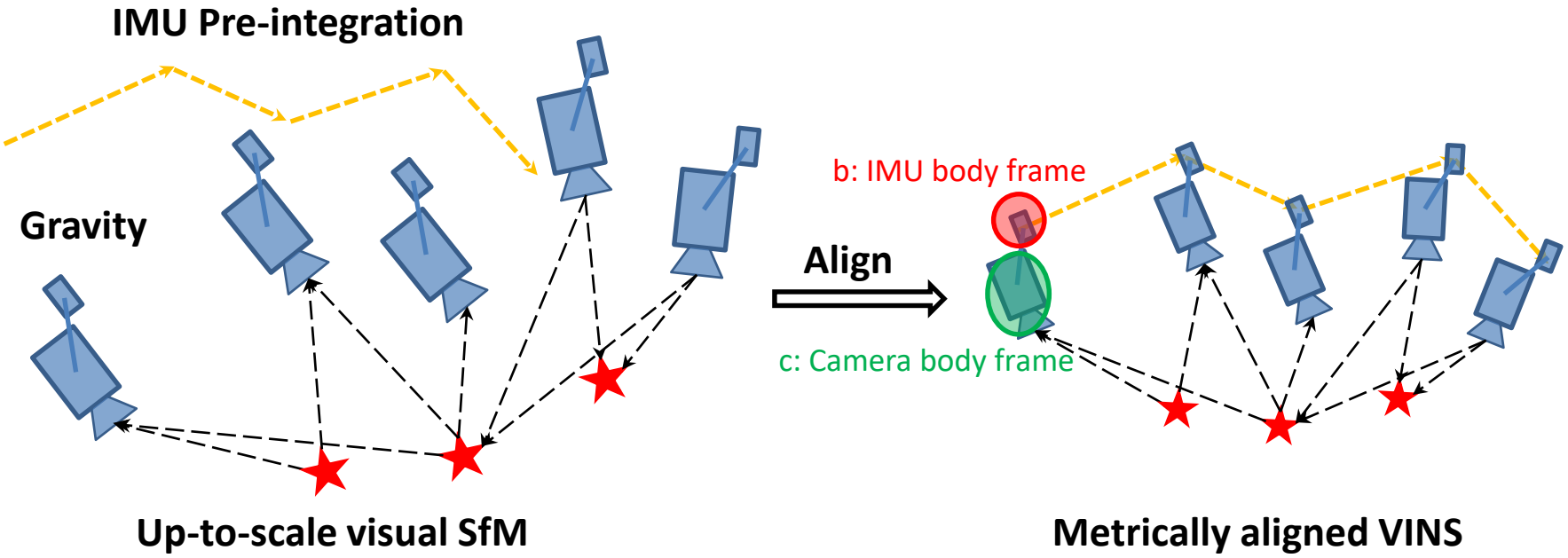
- Monocular vision-only structure-from-motion (SfM)
  - In a small window (10 frames, 1sec)
  - Up-to-scale, locally drift-free position estimates
  - Locally drift-free orientation estimates
  - Not aligned with gravity



Estimator Initialization

- Visual-inertial alignment
  - Estimates velocity of each frame, gravity vector, and scale
    - Note the coordinate frames

$$\mathcal{X}_I = [\mathbf{v}_{b_0}^{c_0}, \mathbf{v}_{b_1}^{c_0}, \cdots \mathbf{v}_{b_n}^{c_0}, \mathbf{g}^{c_0}, s]$$



Estimator Initialization

- Visual-inertial alignment
  - Linear measurement model

IMU Pre-integration

Known values from vSfM and extrinsic calibration

$$\hat{\mathbf{z}}^{b_k}_{b_{k+1}} = \begin{bmatrix} \hat{\boldsymbol{\alpha}}^{b_k}_{b_{k+1}} - \mathbf{R}^{c_0}_{c_k}\mathbf{p}^c_b + \mathbf{R}^{c_0}_{c_k}\mathbf{p}^c_b \\ \hat{\boldsymbol{\beta}}^{b_k}_{b_{k+1}} \end{bmatrix} = \mathbf{H}^{b_k}_{b_{k+1}}\mathcal{X}_I + \mathbf{n}^{b_k}_{b_{k+1}}$$

$$\approx \begin{bmatrix} -\mathbf{R}^{b_k}_{c_0}\Delta t_k & \mathbf{0} & \frac{1}{2}\mathbf{R}^{b_k}_{c_0}\Delta t_k^2 & \mathbf{R}^{b_k}_{c_0}(\bar{\mathbf{p}}^{c_0}_{c_{k+1}} - \bar{\mathbf{p}}^{c_0}_{c_k}) \\ -\mathbf{R}^{b_k}_{c_0} & \mathbf{R}^{b_k}_{c_0} & \mathbf{R}^{b_k}_{c_0}\Delta t_k & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{v}^{c_0}_{b_k} \\ \mathbf{v}^{c_0}_{b_{k+1}} \\ \mathbf{g}^{c_0} \\ s \end{bmatrix}$$

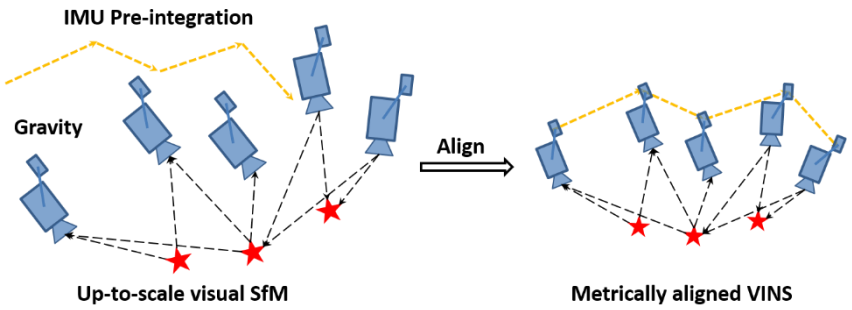
States to be initialized

Up-to-scale translation from vSfM

  - Solve a linear system
    - Scale and rotate the vSfM

$$\mathcal{X}_I = [\mathbf{v}^{c_0}_{b_0}, \mathbf{v}^{c_0}_{b_1}, \cdots \mathbf{v}^{c_0}_{b_n}, \mathbf{g}^{c_0}, s]$$

$$\min_{\mathcal{X}_I} \sum_{k\in\mathcal{B}} \left\| \hat{\mathbf{z}}^{b_k}_{b_{k+1}} - \mathbf{H}^{b_k}_{b_{k+1}}\mathcal{X}_I \right\|^2$$



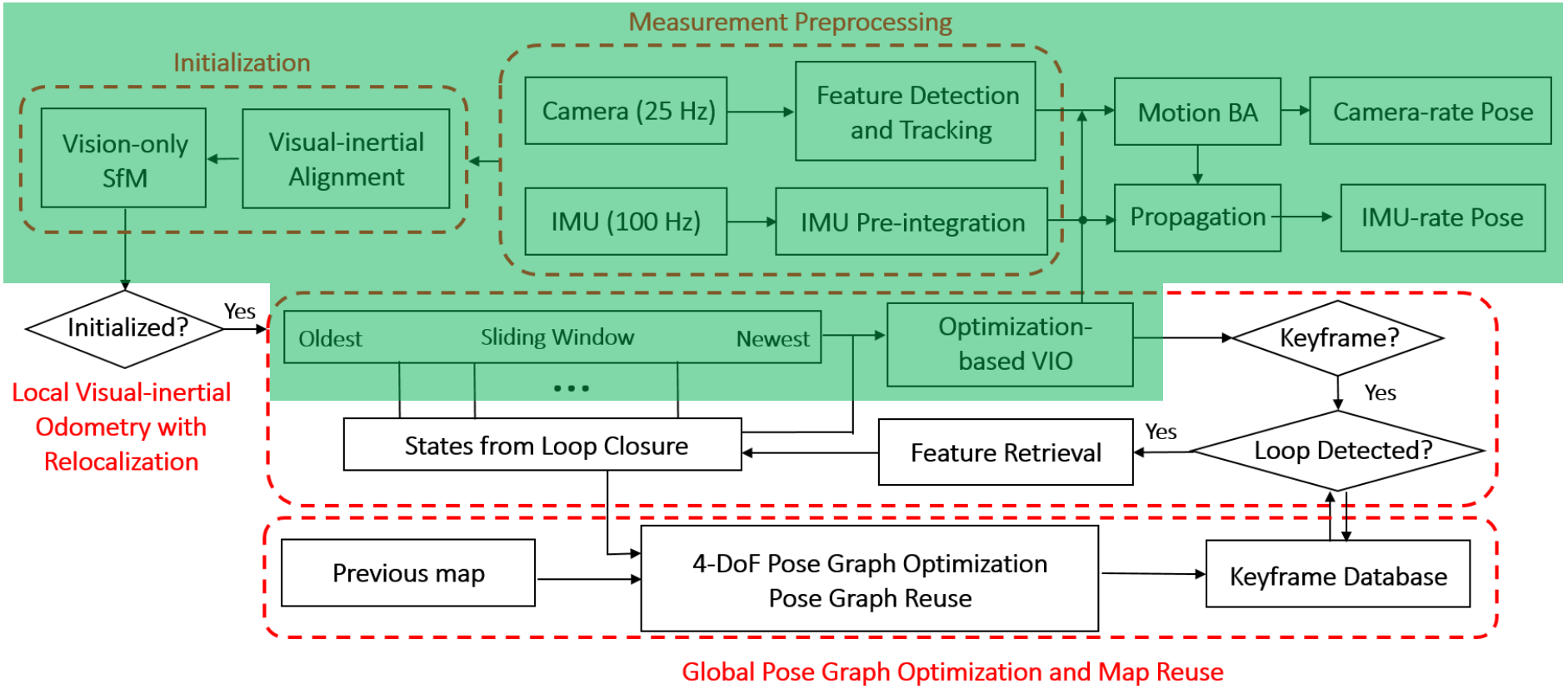
Source Code: http://github.com/HKUST-Aerial-Robotics/VINS-Mono

Estimator Initialization

- Current issues:
  - IMU biases are not initialized
    - Gyroscope: obtained from stationary measurements
    - Accelerometer: problematic…
  - May fail at high altitude scenes due to excessive IMU integration time
    - Solution: Spline-based initialization, use derivatives instead of integration
    - T. Liu and S. Shen. High altitude monocular visual-inertial state estimation: initialization and sensor fusion. In Proc. of the IEEE International Conference on Robotics and Automation (ICRA), Singapore, May 2017

Monocular Visual-Inertial SLAM

- System diagram



Source Code: http://github.com/HKUST-Aerial-Robotics/VINS-Mono

香港科技大學
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DEPARTMENT OF ELECTRONIC & COMPUTER ENGINEERING

香港科大機器人研究所
ROBOTICS INSTITUTE

Visual-Inertial SLAM for Autonomous Drone

Monoculor Visual-Inertial System (VINS-Mono) on MAV Platform for Automous Flight

Tong Qin, Peiliang Li, Zhenfei Yang and Shaojie Shen

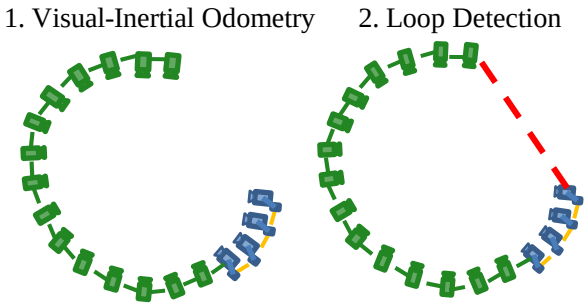
香港科技大學
THE HONG KONG UNIVERSITY OF SCIENCE AND TECHNOLOGY

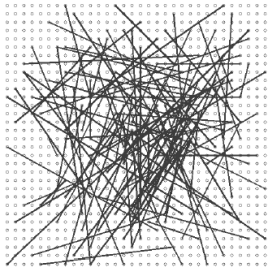
HKUST
Aerial Robotics Group

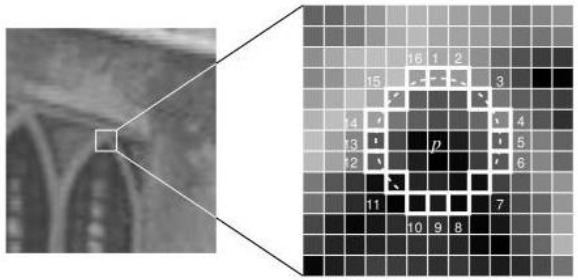
Open source: https://github.com/HKUST-Aerial-Robotics/VINS-Mono

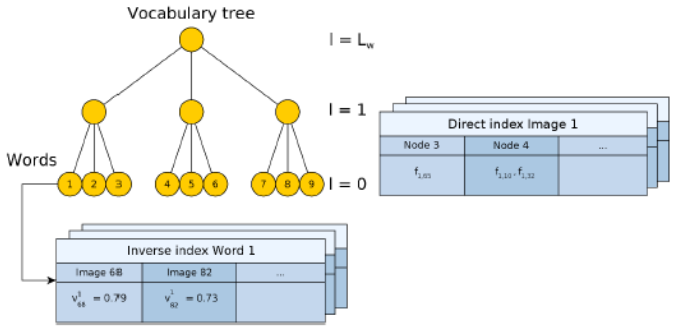
Loop Closure

- Loop detection
  - Describe features by BRIEF
    - Features that we use in the VIO (200, not enough for loop detection)
    - Extract new FAST features (500, only use for loop detection)
  - Query Bag-of-Word (DBoW2)
    - Return loop candidates







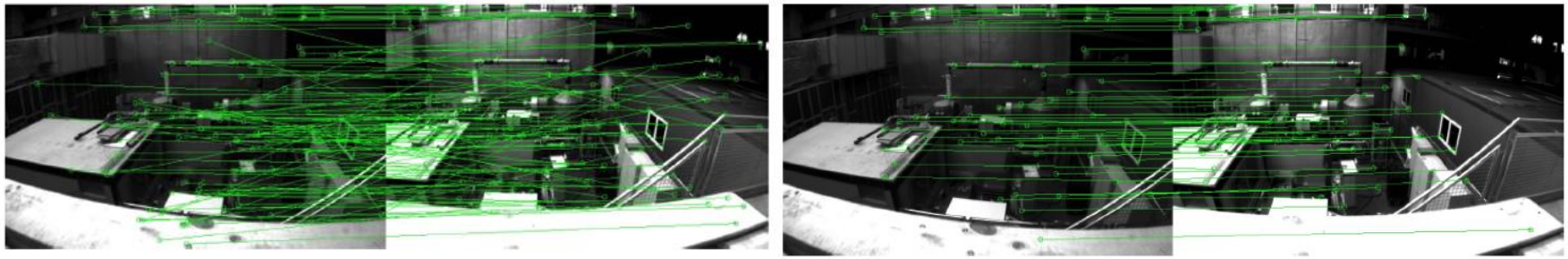


Calonder, Michael, et al. "Brief: Binary robust independent elementary features." *Computer Vision–ECCV 2010* (2010): 778-792.
Gálvez-López, Dorian, and Juan D. Tardos. "Bags of binary words for fast place recognition in image sequences." *IEEE Transactions on Robotics* 28.5 (2012): 1188-1197.

Source Code: http://github.com/HKUST-Aerial-Robotics/VINS-Mono

Loop Closure

- Feature Retrieving
  - Try to retrieve matches for features (200) that are used in the VIO
  - BRIEF descriptor match
  - Geometric check
    - Fundamental matrix test with RANSAC
    - At least 30 inliers
- Output:
  - Loop closure frames with known pose
  - Feature matches between VIO frames and loop closure frames

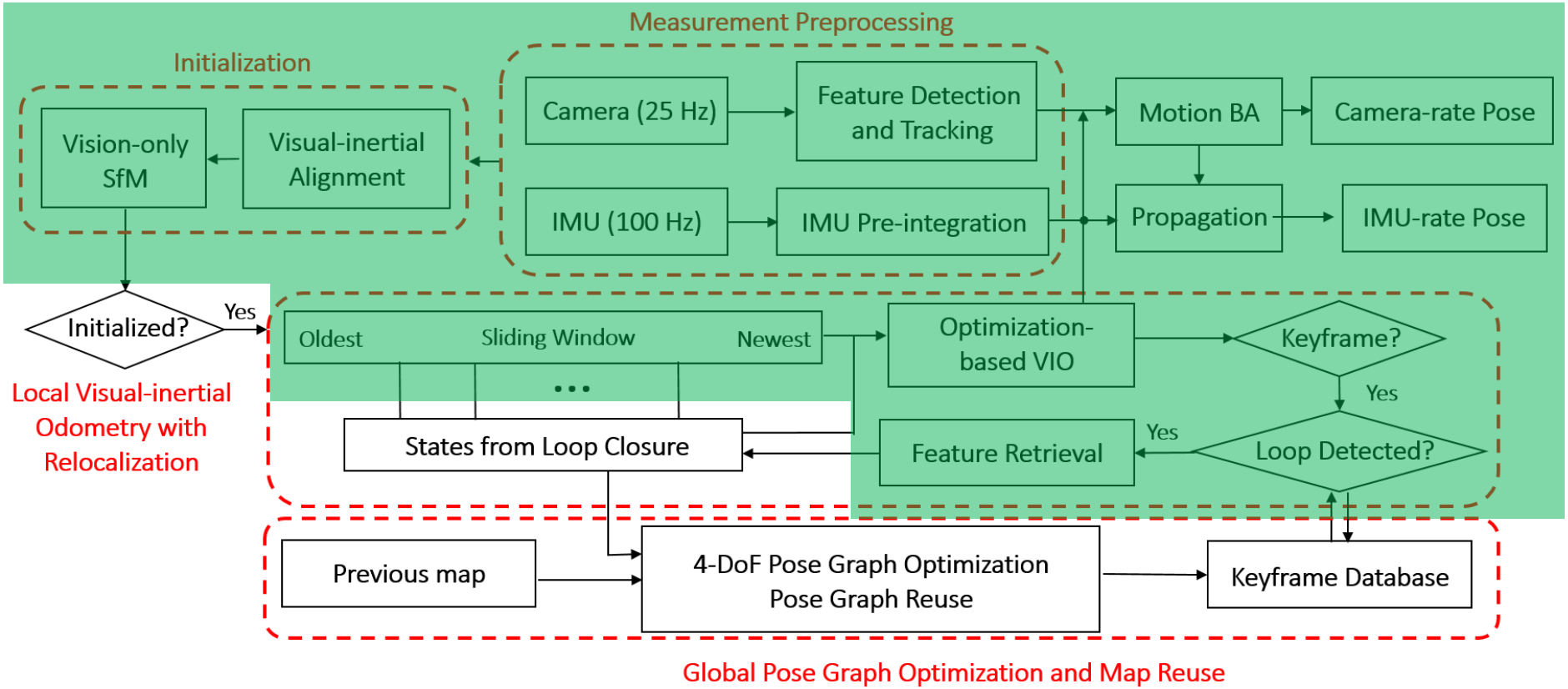


(a) (b)

Source Code: http://github.com/HKUST-Aerial-Robotics/VINS-Mono

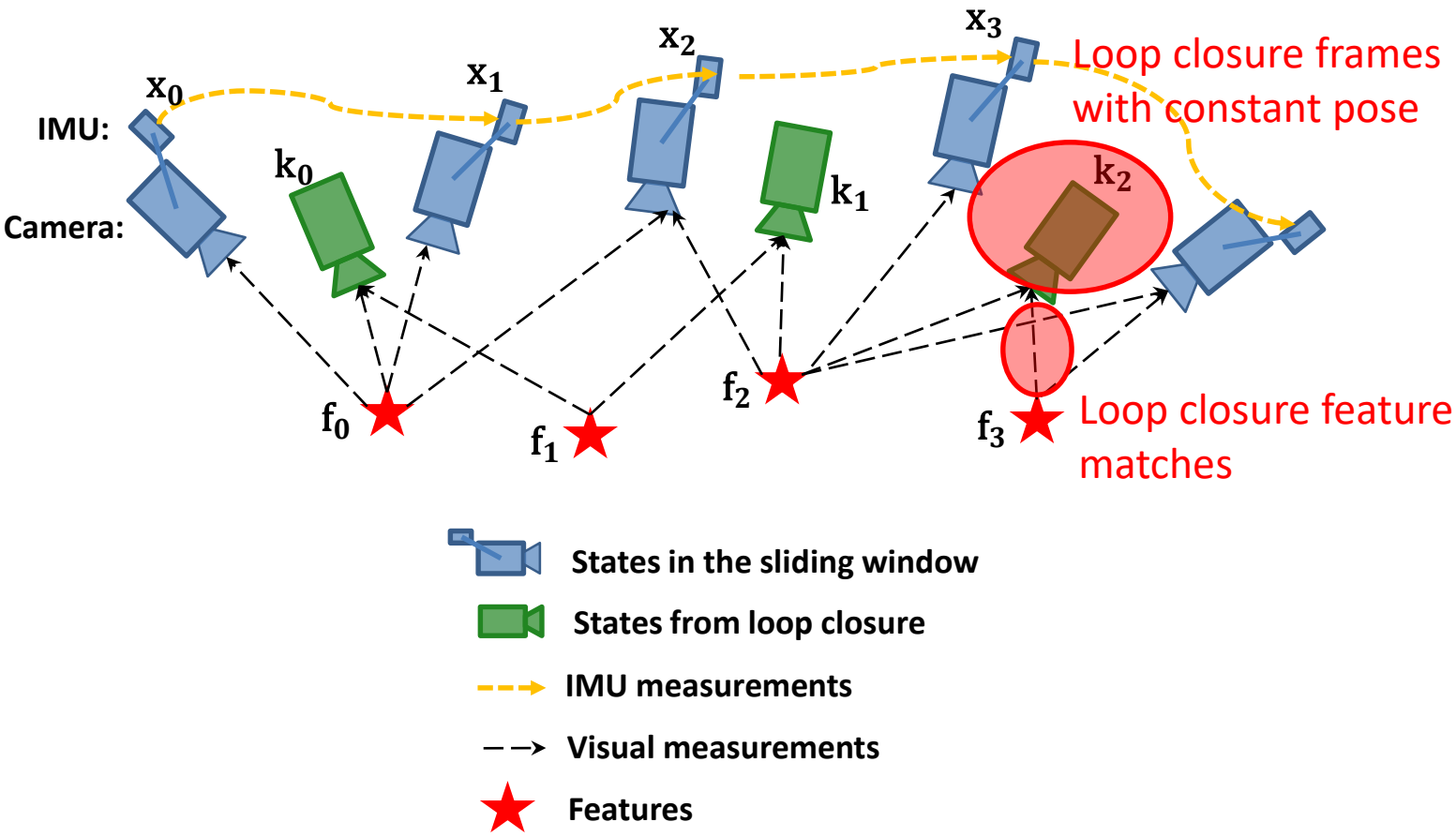
Monocular Visual-Inertial SLAM

- System diagram



Source Code: http://github.com/HKUST-Aerial-Robotics/VINS-Mono

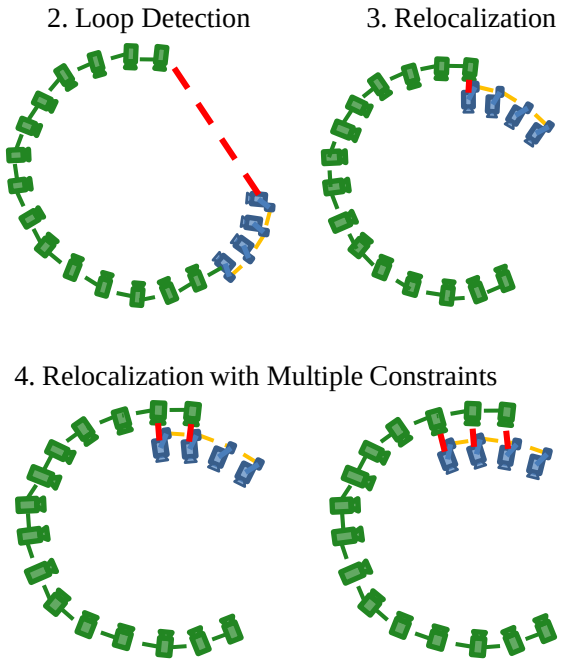
Monocular Visual-Inertial Odometry with Relozalization



Source Code: http://github.com/HKUST-Aerial-Robotics/VINS-Mono

Monocular Visual-Inertial Odometry with Relozalization

- Relocalization
  - Visual measurements for tightly-coupled relocalization
    - Observation of retrieved features in loop closure frames
    - Poses of loop closure frames are constant
    - No increase in state vector dimension for relocalization
    - Allows multi-constraint relocalization



$$\min_{\mathcal{X}} \left\{ \|\mathbf{r}_p - \mathbf{H}_p\mathcal{X}\|^2 + \sum_{k\in\mathcal{B}} \left\| \mathbf{r}_{\mathcal{B}}(\hat{\mathbf{z}}^{b_k}_{b_{k+1}}, \mathcal{X}) \right\|^2_{\mathbf{P}^{b_k}_{b_{k+1}}} + \sum_{(l,j)\in\mathcal{C}} \left\| \mathbf{r}_{\mathcal{C}}(\hat{\mathbf{z}}^{c_j}_l, \mathcal{X}) \right\|^2_{\mathbf{P}^{c_j}_l} + \sum_{(l,v)\in\mathcal{L}} \left\| \mathbf{r}_{\mathcal{C}}(\hat{\mathbf{z}}^{v}_l, \mathcal{X}, \hat{\mathbf{q}}^w_v, \hat{\mathbf{p}}^w_v) \right\|^2_{\mathbf{P}^{v}_l} \right\}, \tag{22}$$

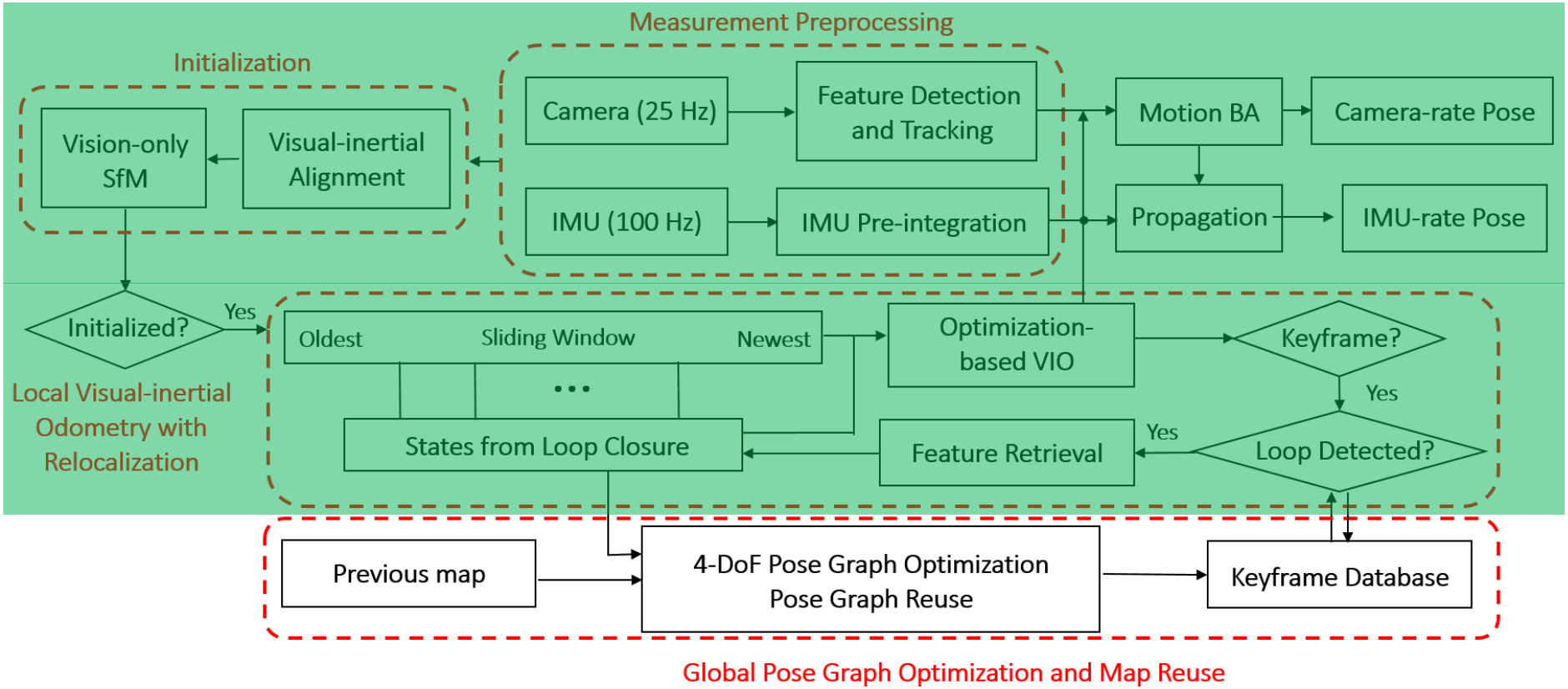
VIO residuals

Loop closure vision measurement residual

Source Code: http://github.com/HKUST-Aerial-Robotics/VINS-Mono

Monocular Visual-Inertial SLAM

- System diagram



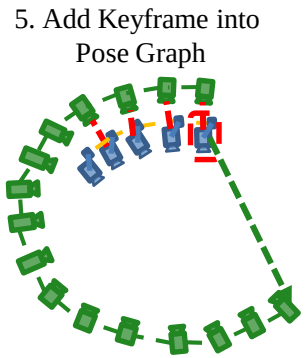
Source Code: http://github.com/HKUST-Aerial-Robotics/VINS-Mono

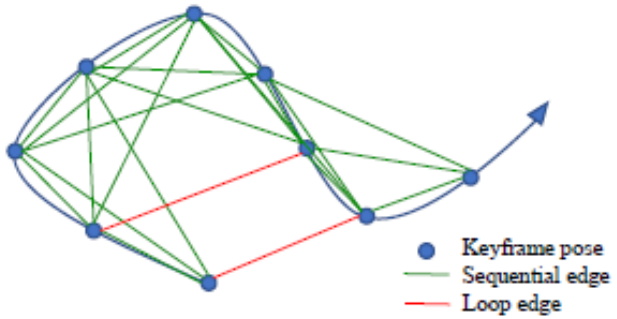
Global Pose Graph SLAM

- 4-DOF pose graph
  - Roll and pitch are observable from VIO
- Adding keyframes into pose graph

$$\hat{\mathbf{p}}_{ij}^{i} = \hat{\mathbf{R}}_{i}^{w-1}(\hat{\mathbf{p}}_{j}^{w} - \hat{\mathbf{p}}_{i}^{w})$$
$$\hat{\psi}_{ij} = \hat{\psi}_{j} - \hat{\psi}_{i}$$

  - Sequential edges from VIO
    - Connected with 4 previous keyframes
  - Loop closure edges
    - Only added when a keyframe is marginalized out from the sliding window VIO
    - Multi-constraint relocalization helps eliminating false loop closures





Global Pose Graph SLAM

- 4-DOF relative pose residual:

Observable attitude from VIO

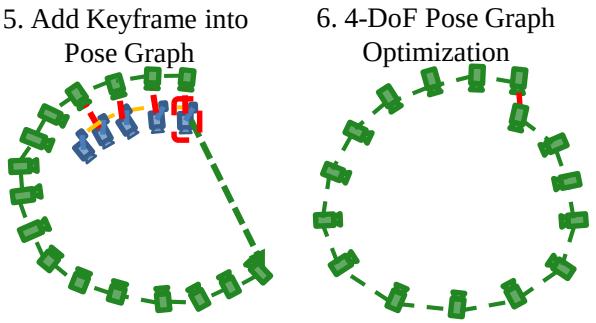
$$\mathbf{r}_{i,j}(\mathbf{p}_i^w, \psi_i, \mathbf{p}_j^w, \psi_j) = \begin{bmatrix} \mathbf{R}(\hat{\phi}_i, \hat{\theta}_i, \psi_i)^{-1}(\mathbf{p}_j^w - \mathbf{p}_i^w) - \hat{\mathbf{p}}_{ij}^i \\ \psi_j - \psi_i - \hat{\psi}_{ij} \end{bmatrix}$$

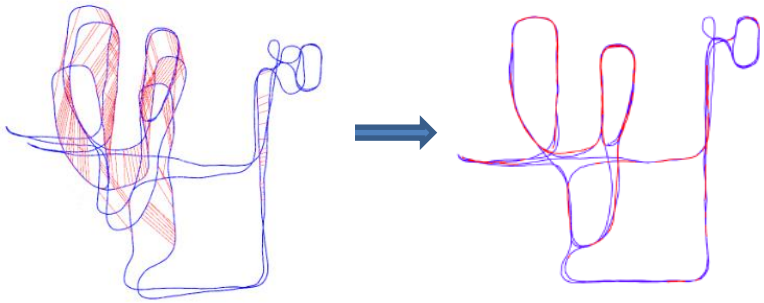
- Minimize the following cost function
  - Sequential edge from VIO
  - Loop closure edges
    - Huber norm for rejection of wrong loops

Sequential edges

$$\min_{\mathbf{p},\psi} \left\{ \sum_{(i,j)\in\mathcal{S}} \|\mathbf{r}_{i,j}\|^2 + \sum_{(i,j)\in\mathcal{L}} h(\|\mathbf{r}_{i,j}\|) \right\}$$

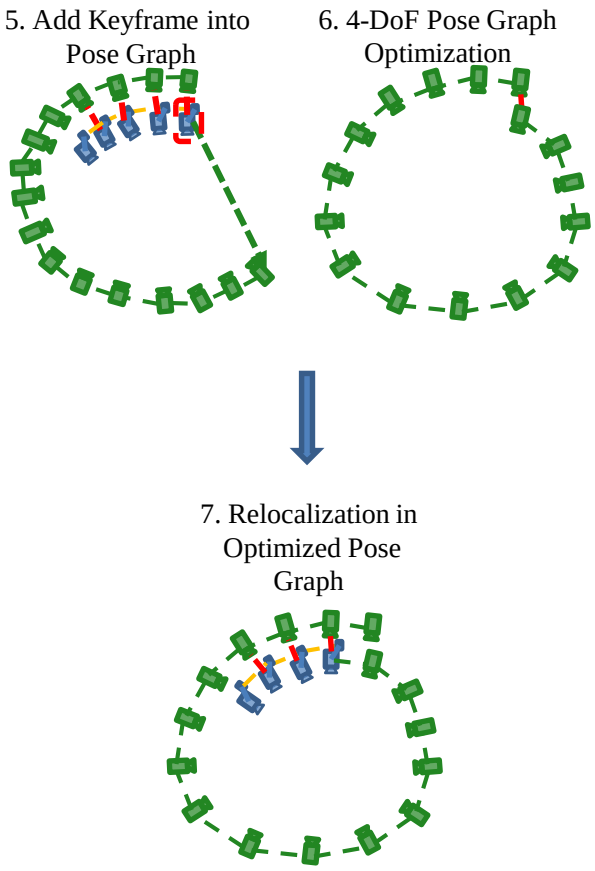
Loop closure edges





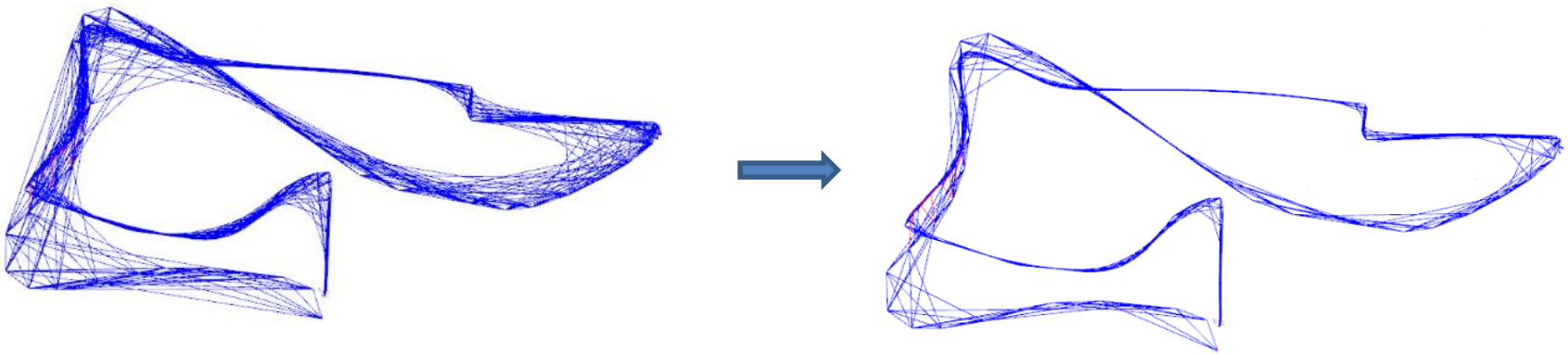
Global Pose Graph SLAM

- More on relocalization
  - Relocalization continued on the optimized pose graph
  - Relocalization and pose graph optimization run in different threads and in different rate
  - Pose graph optimization can be very slow for large-scale environments



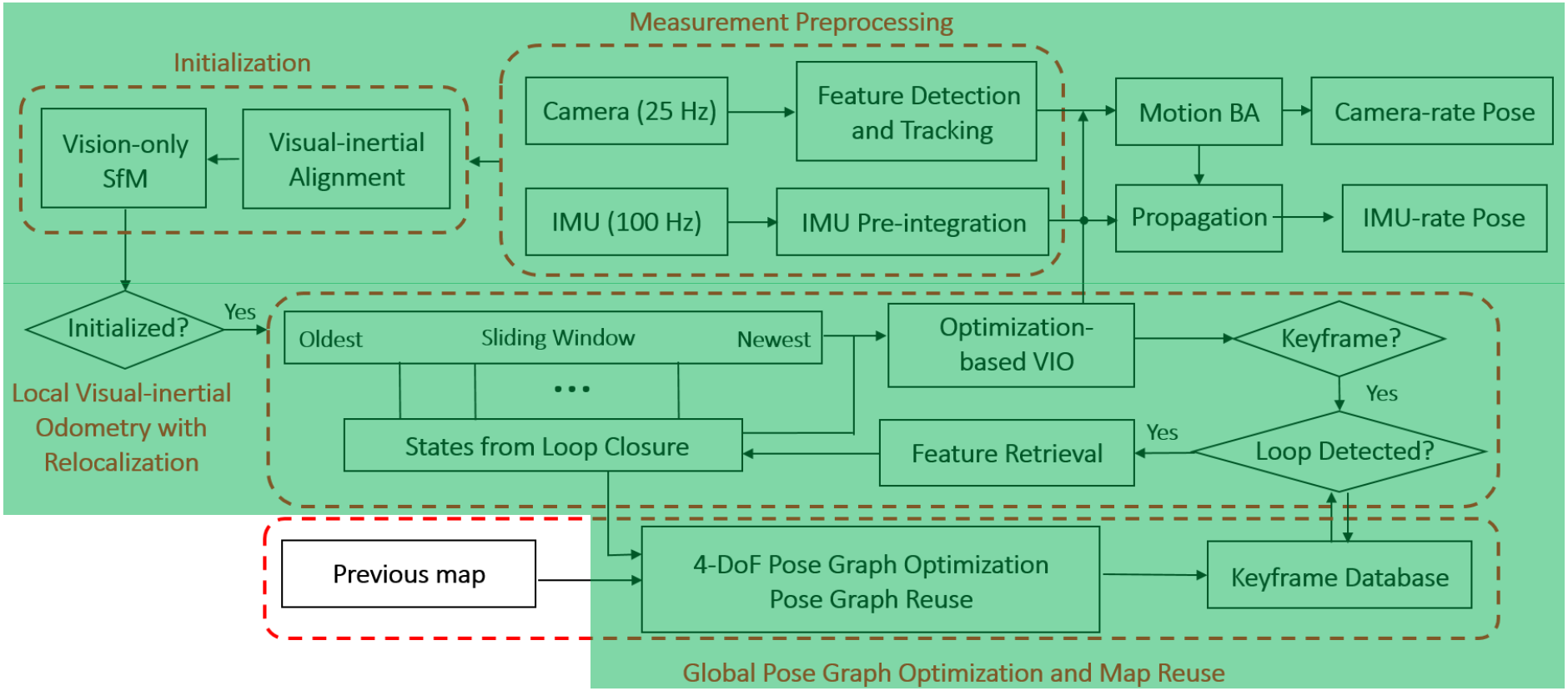
Global Pose Graph SLAM

- Simple strategy for pose graph sparsification
  - All keyframes with loop closure constraints will be kept
  - Other keyframes that are either too close to its neighbors or have very similar orientations will be removed



Monocular Visual-Inertial SLAM

- System diagram



Source Code: http://github.com/HKUST-Aerial-Robotics/VINS-Mono

Visual-Inertial SLAM in Large-Scale Environment

II. Go out laboratory

Single camera: mvBluefox
IMU: DJI A3

Pose Graph Reuse

- Pose graph saving
  - Every Keyframe
    - Index i, position $\hat{\boldsymbol{p}}_i^w$, orientation $\hat{\boldsymbol{q}}_i^w$, features' 2D location and descriptor $D(u, v, des)$
    - If i loops with v, we also save loop index v, relative translation $\hat{p}_{iv}^i$, relative yaw angle $\hat{\varphi}_{iv}$

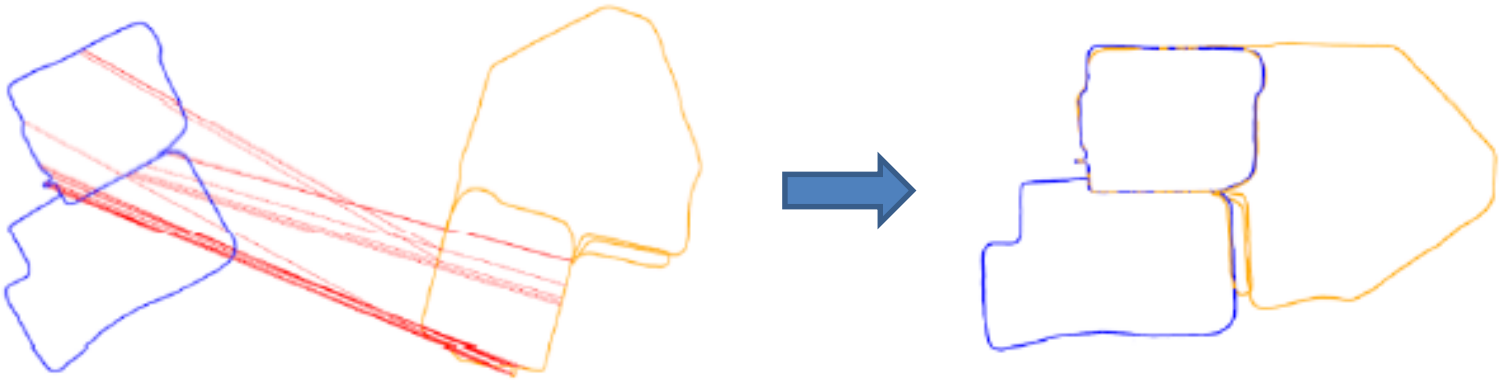
$$[i, \hat{\mathbf{p}}_i^w, \hat{\mathbf{q}}_i^w, v, \hat{\mathbf{p}}_{iv}^i, \hat{\psi}_{iv}, \mathbf{D}(u, v, des)]$$

- Pose graph loading
  - Build sequential edges
    - Connected with 4 previous keyframes
  - Build loop closure edges
    - According to loop index v, relative translation $\hat{\boldsymbol{p}}_{iv}^i$ and yaw angle $\hat{\varphi}_{iv}$

Source Code: http://github.com/HKUST-Aerial-Robotics/VINS-Mono

Pose Graph Reuse

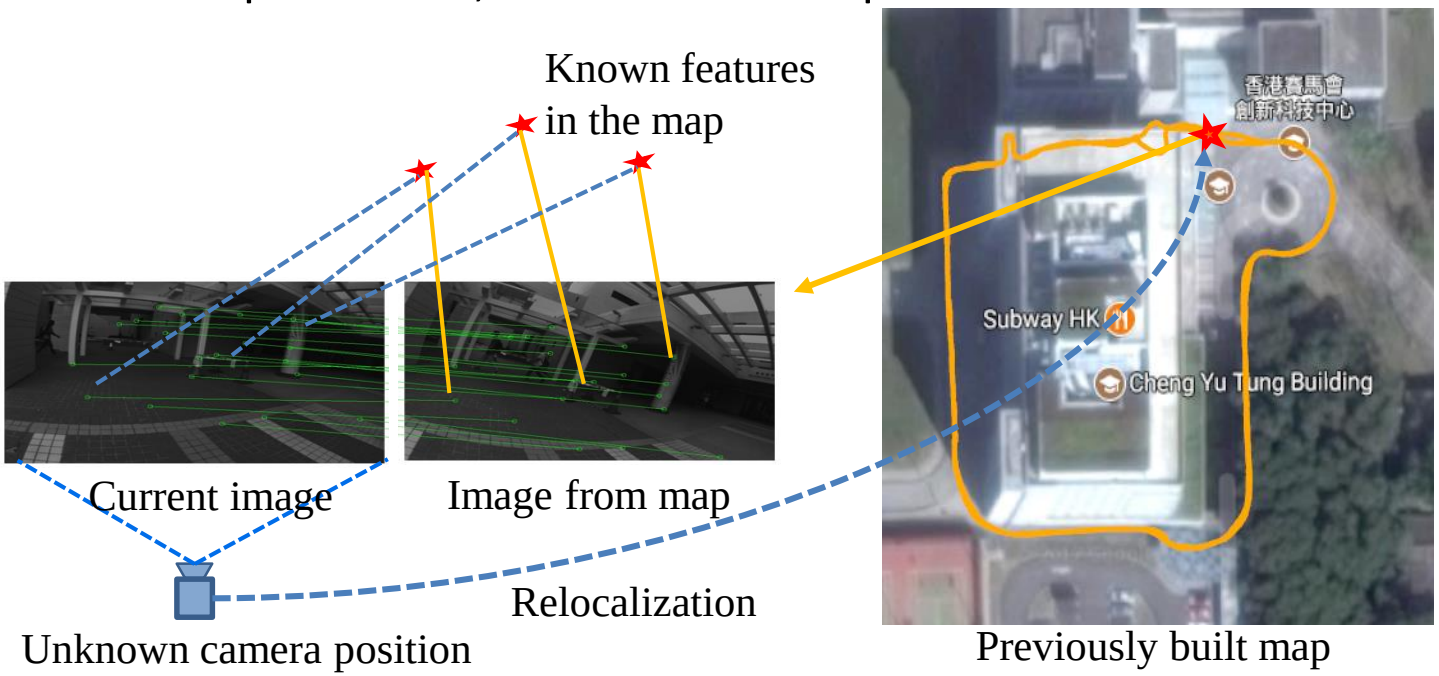
- Pose graph merging
  - Load a previous-built map
  - Build a new map
  - Detect loop connections between two maps
  - Merge two map by pose graph optimization



Source Code: http://github.com/HKUST-Aerial-Robotics/VINS-Mono

Pose Graph Reuse

- Relocalization
  - Load previous-built map (aligned with Google Map)
  - The camera starts at an unknown position
  - Detect similar image view in the map
  - Once loop detected, relocate camera pose



Source Code: http://github.com/HKUST-Aerial-Robotics/VINS-Mono

Pose Graph Reuse

Relocalization, Global Optimization and Map Merging for Monocular Visual-Inertial SLAM

Tong Qin, Peiliang Li, and Shaojie Shen

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THE HONG KONG UNIVERSITY OF SCIENCE AND TECHNOLOGY

香港科技大學－
大疆創新科技聯合實驗室
HKUST-DJI JOINT INNOVATION LABORATORY

Open source: https://github.com/HKUST-Aerial-Robotics/VINS-Mono

Remarks on Monocular Visual-Inertial SLAM

- Important factors
  - Access to raw camera data (especially for rolling shutter cameras)
  - Sensor synchronization and timestamps
  - Camera-IMU rotation
  - Estimator initialization
- Not-so-important factors
  - Camera-IMU translation
  - Types of features (we use the simplest corner+KLT)
  - Quality of feature tracking (outlier is acceptable)
- Failures – need more engineering treatment
  - Long range scenes (aerial vehicles)
  - Constant velocity (ground vehicle)
  - Pure rotation (augmented reality)
- Be aware of computational power requirement

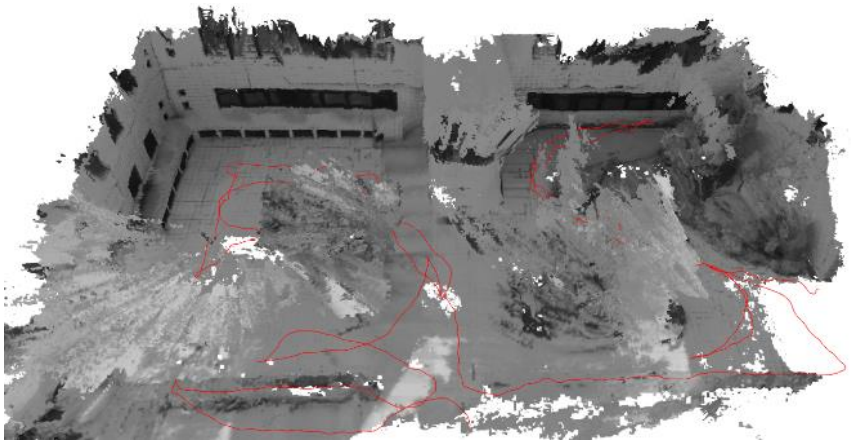
Source Code: http://github.com/HKUST-Aerial-Robotics/VINS-Mono

Remarks on Monocular Visual-Inertial SLAM

- IMU is great!!!
- Feature-based visual-inertial SLAM is very close to done
  - Some research work remains:
    - Online observability analysis
    - Large-scale, long duration operations
    - Extreme environments
    - Extreme motions
  - Big engineering challenges towards mass deployment on different devices (Android phones?)
    - Intrinsic and extrinsic calibration of IMU, rolling shutter, etc.
    - Synchronization issues
    - Poor sensors and manufacturing variations
    - Insufficient computing power
  - Big players are moving in

Remarks on Monocular Visual-Inertial SLAM

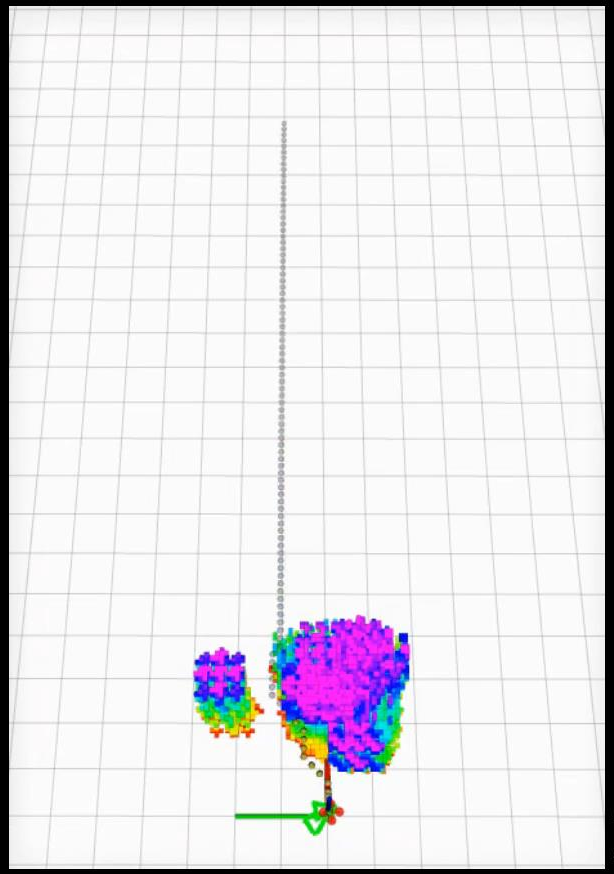
- Real-time dense mapping is interesting
  - Very few working implementations
  - How to reduce computation?
  - Parallel implementation on GPU
  - Joint optimization or alternating estimation?
  - Textureless and repetitive patterns?
  - Combination of learning and geometric-based methods
  - Efficient map representation for large-scale environments



Source Code: http://github.com/HKUST-Aerial-Robotics/VINS-Mono

Dense Mapping, Trajectory Planning, and Navigation

Indoor Experiment 2:



2x

Trajectory length: 18.6m
Total number of replans: 125
Average computing time: 43ms
Average snap: 1.21m/s^4

2x

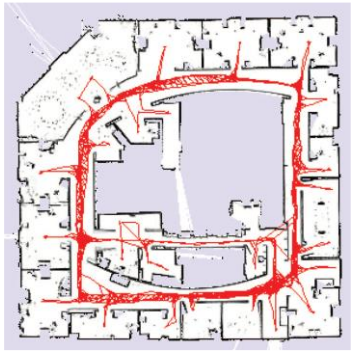
Y. Lin et al, JFR 2017; W. Ding et al, ICRA2018

Summary

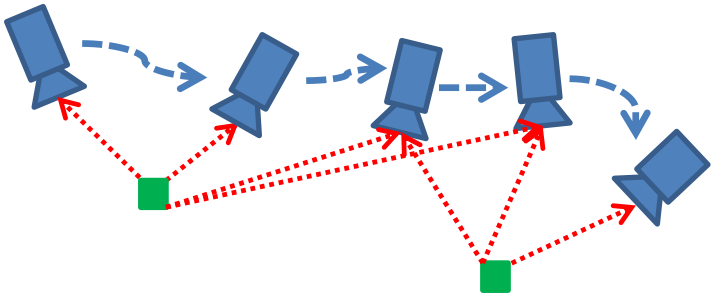
- The Basics

$$P(x \mid y, z) = \frac{P(y \mid x, z)\ P(x \mid z)}{P(y \mid z)}$$

- 2D Graph-Based SLAM



- Monocular Visual-Inertial SLAM



Logistics

- Project 3 Phase 1 due today.
- Project 3 Phase 2 is released, due in two weeks (4 May).
- Give a try to VINS-Mono after completing all your projects.
- This is the last (of my) lecture.
- Thanks to all of you for overcoming this special period of time.
- Please help with the course evaluation.
- For those of you choose to do the physical lab, the hard time comes…

Thanks for taking ELEC5660

and enjoy your projects ☺

Appendix: Quaternion

Definitions of Quaternion

- Quaternion

$$\boldsymbol{Q} \overset{\text{def}}{=} q_0 + q_1\boldsymbol{i} + q_2\boldsymbol{j} + q_3\boldsymbol{k} \overset{\text{def}}{=} q_0 + \boldsymbol{q} \overset{\text{def}}{=} \begin{bmatrix} q_0 \\ \boldsymbol{q} \end{bmatrix}$$

where $\boldsymbol{i} \circ \boldsymbol{i} = \boldsymbol{j} \circ \boldsymbol{j} = \boldsymbol{k} \circ \boldsymbol{k} \overset{\text{def}}{=} -1; \boldsymbol{i} \circ \boldsymbol{j} \overset{\text{def}}{=} \boldsymbol{k}; \boldsymbol{j} \circ \boldsymbol{k} \overset{\text{def}}{=} \boldsymbol{i}; \boldsymbol{k} \circ \boldsymbol{i} \overset{\text{def}}{=} \boldsymbol{j}$

- Conjugate

$$\bar{\boldsymbol{Q}} \overset{\text{def}}{=} q_0 - q_1\boldsymbol{i} - q_2\boldsymbol{j} - q_3\boldsymbol{k} \overset{\text{def}}{=} q_0 - \boldsymbol{q} \overset{\text{def}}{=} \begin{bmatrix} q_0 \\ -\boldsymbol{q} \end{bmatrix}$$

- Norm $\|\boldsymbol{Q}\| \overset{\text{def}}{=} \boldsymbol{Q} \circ \bar{\boldsymbol{Q}}$

- Inverse $\boldsymbol{Q}^{-1} \overset{\text{def}}{=} \dfrac{\bar{\boldsymbol{Q}}}{\|\boldsymbol{Q}\|}$

Properties of Quaternion

- Property 1

$$\boldsymbol{A} = a_0 + \boldsymbol{a}, \quad \boldsymbol{B} = b_0 + \boldsymbol{b} \qquad \text{particularly}$$

$$\boldsymbol{A} \circ \boldsymbol{B} = (a_0 b_0 - \boldsymbol{a} \cdot \boldsymbol{b}) + (a_0 \boldsymbol{b} + \boldsymbol{a} b_0 + \boldsymbol{a} \times \boldsymbol{b})$$

$$(0 + \boldsymbol{a}) \circ (0 + \boldsymbol{b}) = -\boldsymbol{a} \cdot \boldsymbol{b} + \boldsymbol{a} \times \boldsymbol{b}$$

- Property 2 $(\boldsymbol{A} \circ \boldsymbol{B}) \circ \boldsymbol{C} = \boldsymbol{A} \circ (\boldsymbol{B} \circ \boldsymbol{C})$
- Property 3 $\overline{\boldsymbol{A} \circ \boldsymbol{B}} = \overline{\boldsymbol{B}} \circ \overline{\boldsymbol{A}}$
- Property 4 $\boldsymbol{A} = a_0 + a_1 \boldsymbol{i} + a_2 \boldsymbol{j} + a_3 \boldsymbol{k}$
  $\boldsymbol{B} = b_0 + b_1 \boldsymbol{i} + b_2 \boldsymbol{j} + b_3 \boldsymbol{k}$

Exactly what we need for representing rotations!!

$$\boldsymbol{A} \circ \boldsymbol{B} = \underbrace{\begin{bmatrix} a_0 & -a_1 & -a_2 & -a_3 \\ a_1 & a_0 & -a_3 & a_2 \\ a_2 & a_3 & a_0 & -a_1 \\ a_3 & -a_2 & a_1 & a_0 \end{bmatrix}}_{\mathcal{L}(\boldsymbol{A})} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{bmatrix} = \underbrace{\begin{bmatrix} b_0 & -b_1 & -b_2 & -b_3 \\ b_1 & b_0 & b_3 & -b_2 \\ b_2 & -b_3 & b_0 & b_1 \\ b_3 & b_2 & -b_1 & b_0 \end{bmatrix}}_{\mathcal{R}(\boldsymbol{B})} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

Properties of Quaternion

- Product of Quaternions $\boldsymbol{A} \circ \boldsymbol{B} = \mathcal{L}(\boldsymbol{A}) \begin{bmatrix} b_0 \\ \boldsymbol{b} \end{bmatrix} = \mathcal{R}(\boldsymbol{B}) \begin{bmatrix} a_0 \\ \boldsymbol{a} \end{bmatrix}$

$$\mathcal{L}(\boldsymbol{A}) = \begin{bmatrix} a_0 & -a_1 & -a_2 & -a_3 \\ a_1 & a_0 & -a_3 & a_2 \\ a_2 & a_3 & a_0 & -a_1 \\ a_3 & -a_2 & a_1 & a_0 \end{bmatrix} = \begin{bmatrix} a_0 & -\boldsymbol{a}^T \\ \boldsymbol{a} & a_0 \boldsymbol{I}_3 + \widehat{\boldsymbol{a}} \end{bmatrix}$$

$$\mathcal{R}(\boldsymbol{B}) = \begin{bmatrix} b_0 & -b_1 & -b_2 & -b_3 \\ b_1 & b_0 & b_3 & -b_2 \\ b_2 & -b_3 & b_0 & b_1 \\ b_3 & b_2 & -b_1 & b_0 \end{bmatrix} = \begin{bmatrix} b_0 & -\boldsymbol{b}^T \\ \boldsymbol{b} & b_0 \boldsymbol{I}_3 - \widehat{\boldsymbol{a}} \end{bmatrix}$$

Unit Quaternion and Rotation

Recall

$$\underbrace{\begin{bmatrix}0\\ \boldsymbol{p}'\end{bmatrix}}_{\boldsymbol{P}'} = \underbrace{\begin{bmatrix}\cos\frac{\theta}{2}\\ \sin\frac{\theta}{2}\boldsymbol{u}\end{bmatrix}}_{\boldsymbol{Q}} \circ \underbrace{\begin{bmatrix}0\\ \boldsymbol{p}\end{bmatrix}}_{\boldsymbol{P}} \circ \underbrace{\begin{bmatrix}\cos\frac{\theta}{2}\\ -\sin\frac{\theta}{2}\boldsymbol{u}\end{bmatrix}}_{\overline{\boldsymbol{Q}}}$$

$$\boldsymbol{P}' \overset{\text{def}}{=} 0 + \boldsymbol{p}'; \boldsymbol{P} \overset{\text{def}}{=} 0 + \boldsymbol{p}; \qquad \boxed{\boldsymbol{Q} \overset{\text{def}}{=} \cos\frac{\theta}{2} + \sin\frac{\theta}{2}\boldsymbol{u}}$$

If using quaternion Q to represent a rotation:

Basic principle: $\boldsymbol{P}' = \boldsymbol{Q} \circ \boldsymbol{P} \circ \overline{\boldsymbol{Q}}$

- $\boldsymbol{P}', \boldsymbol{P}$ are respectively the coordinates of the post-rotation vector and prior-rotation vector, both in the same frame.
- $\boldsymbol{Q}$ is the rotation quaternion associated with the rotation

Unit Quaternion and Rotation

Basic principle: $\boldsymbol{P'} = \boldsymbol{Q} \circ \boldsymbol{P} \circ \overline{\boldsymbol{Q}}$

Property 1 Constant vector: $\boldsymbol{P}^b = \overline{\boldsymbol{Q}} \circ \boldsymbol{P} \circ \boldsymbol{R}$

- $\boldsymbol{P}^b, \boldsymbol{P}$ are respectively the coordinates in the body frame and the world frame, of the same vector
- Can be interpreted as that the vector is rotating in the opposite direction, then call the first result

Unit Quaternion and Rotation

Basic principle: $\boldsymbol{P'} = \boldsymbol{Q} \circ \boldsymbol{P} \circ \overline{\boldsymbol{Q}}$

Property 2 Two sequent rotations

- Case 1

  $\boldsymbol{Q}_1, \boldsymbol{Q}_2$ are the two rotation quaternions where the rotation axes are both represented in the initial frame

$$\left.\begin{aligned} \boldsymbol{P'} &= \boldsymbol{Q}_1 \circ \boldsymbol{P} \circ \overline{\boldsymbol{Q}}_1 \\ \boldsymbol{P''} &= \boldsymbol{Q}_2 \circ \boldsymbol{P'} \circ \overline{\boldsymbol{Q}}_2 = \underbrace{\boldsymbol{Q}_2 \circ \boldsymbol{Q}_1}_{\boldsymbol{Q}} \circ \boldsymbol{P} \circ \overline{\boldsymbol{Q}}_1 \circ \overline{\boldsymbol{Q}}_2 \end{aligned}\right\} \Rightarrow \boldsymbol{Q} = \boldsymbol{Q}_2 \circ \boldsymbol{Q}_1$$

- Case 2

  $\boldsymbol{Q}_1, \boldsymbol{Q}_2$ are the two rotation quaternions where the rotation axis of $\boldsymbol{Q}_2$ is represented in the frame obtained by performing $\boldsymbol{Q}_1$.

$$\boldsymbol{P'} = \boldsymbol{Q}_1 \circ \boldsymbol{P} \circ \overline{\boldsymbol{Q}}_1$$

$$\boldsymbol{P''} = (\boldsymbol{Q}_1 \circ \boldsymbol{Q}_2 \circ \overline{\boldsymbol{Q}}_1) \circ \boldsymbol{P'} \circ \overline{\boldsymbol{Q}_1 \circ \boldsymbol{Q}_2 \circ \overline{\boldsymbol{Q}}_1} = (\boldsymbol{Q}_1 \circ \boldsymbol{Q}_2 \circ \overline{\boldsymbol{Q}}_1) \circ \boldsymbol{Q}_1 \circ \boldsymbol{P} \circ \overline{\boldsymbol{Q}}_1 \circ \overline{\boldsymbol{Q}_1 \circ \boldsymbol{Q}_2 \circ \overline{\boldsymbol{Q}}_1} = \underbrace{\boldsymbol{Q}_1 \circ \boldsymbol{Q}_2}_{\boldsymbol{Q}} \circ \boldsymbol{P} \circ \overline{\boldsymbol{Q}_1 \circ \boldsymbol{Q}_2}$$

$$\Rightarrow \boldsymbol{Q} = \boldsymbol{Q}_1 \circ \boldsymbol{Q}_2$$

Quaternion Kinematics

Recall $\boldsymbol{P'} = \boldsymbol{Q} \circ \boldsymbol{P} \circ \overline{\boldsymbol{Q}}$

Taking derivative yields

$$\dot{\boldsymbol{P}}' = \dot{\boldsymbol{Q}} \circ \boldsymbol{P} \circ \overline{\boldsymbol{Q}} + \boldsymbol{Q} \circ \boldsymbol{P} \circ \dot{\overline{\boldsymbol{Q}}}$$

$$= \dot{\boldsymbol{Q}} \circ \underbrace{\overline{\boldsymbol{Q}} \circ \boldsymbol{P}' \circ \boldsymbol{Q}}_{\boldsymbol{P}} \circ \overline{\boldsymbol{Q}} + \boldsymbol{Q} \circ \underbrace{\overline{\boldsymbol{Q}} \circ \boldsymbol{P}' \circ \boldsymbol{Q}}_{\boldsymbol{P}} \circ \dot{\overline{\boldsymbol{Q}}}$$

$$= \dot{\boldsymbol{Q}} \circ \overline{\boldsymbol{Q}} \circ \boldsymbol{P}' + \boldsymbol{P}' \circ \boldsymbol{Q} \circ \dot{\overline{\boldsymbol{Q}}}$$

As $\boldsymbol{Q} \circ \overline{\boldsymbol{Q}} = 1$

Taking derivative yields

$$\dot{\boldsymbol{Q}} \circ \overline{\boldsymbol{Q}} + \boldsymbol{Q} \circ \dot{\overline{\boldsymbol{Q}}} = 0 \Rightarrow \begin{cases} \boldsymbol{Q} \circ \dot{\overline{\boldsymbol{Q}}} = -\dot{\boldsymbol{Q}} \circ \overline{\boldsymbol{Q}} \\ \overline{\boldsymbol{Q} \circ \dot{\overline{\boldsymbol{Q}}}} = \overline{\dot{\boldsymbol{Q}} \circ \overline{\boldsymbol{Q}}} \end{cases} \Rightarrow \dot{\boldsymbol{Q}} \circ \overline{\boldsymbol{Q}} = 0 + \boldsymbol{a}; \boldsymbol{Q} \circ \dot{\overline{\boldsymbol{Q}}} = 0 - \boldsymbol{a}$$

Notice

$$\dot{\boldsymbol{P}}' = (0 + \boldsymbol{a}) \circ \boldsymbol{P}' + \boldsymbol{P}' \circ (0 - \boldsymbol{a})$$

Quaternion Kinematics

$$\dot{\boldsymbol{P}}' = (0 + \boldsymbol{a}) \circ \boldsymbol{P}' + \boldsymbol{P}' \circ (0 - \boldsymbol{a})$$

$$= \begin{bmatrix} 0 \\ \boldsymbol{a} \end{bmatrix} \circ \begin{bmatrix} 0 \\ \boldsymbol{p}' \end{bmatrix} + \begin{bmatrix} 0 \\ \boldsymbol{p}' \end{bmatrix} \circ \begin{bmatrix} 0 \\ -\boldsymbol{a} \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ \boldsymbol{a} \times \boldsymbol{p}' \end{bmatrix} - \boldsymbol{a} \cdot \boldsymbol{p}' + \begin{bmatrix} 0 \\ \boldsymbol{p}' \times (-\boldsymbol{a}) \end{bmatrix} - \boldsymbol{p}' \cdot (-\boldsymbol{a})$$

$$\boxed{\begin{bmatrix} 0 \\ \dot{\boldsymbol{p}}' \end{bmatrix} = \begin{bmatrix} 0 \\ 2\boldsymbol{a} \times \boldsymbol{p}' \end{bmatrix}}$$

Recall $\dot{\boldsymbol{p}}' = \boldsymbol{\omega} \times \boldsymbol{p}'$, $\boldsymbol{\omega}$ is angular velocity vector represented in the world frame

Then $2\boldsymbol{a} = \boldsymbol{\omega} \Rightarrow \boldsymbol{a} = \frac{1}{2}\boldsymbol{\omega}$

$$\Rightarrow \dot{\boldsymbol{Q}} \circ \overline{\boldsymbol{Q}} = \begin{bmatrix} 0 \\ \boldsymbol{a} \end{bmatrix} = \frac{1}{2} \underbrace{\begin{bmatrix} 0 \\ \boldsymbol{\omega} \end{bmatrix}}_{\boldsymbol{\Omega}} \quad \boxed{\Rightarrow \dot{\boldsymbol{Q}} = \frac{1}{2}\boldsymbol{\Omega} \circ \boldsymbol{Q}}$$

$$\boldsymbol{Q} = \begin{bmatrix} q_0 \\ \boldsymbol{q} \end{bmatrix} \Rightarrow \dot{\boldsymbol{Q}} = \frac{1}{2}\boldsymbol{\Omega} \circ \boldsymbol{Q} = \mathcal{L}(\boldsymbol{\Omega})\boldsymbol{Q} = \frac{1}{2}\begin{bmatrix} 0 & -\boldsymbol{\omega}^T \\ \boldsymbol{\omega} & \widehat{\boldsymbol{\omega}} \end{bmatrix}\boldsymbol{Q}$$

Quaternion Kinematics

$$\dot{\boldsymbol{Q}} = \frac{1}{2}\boldsymbol{\Omega} \circ \boldsymbol{Q}$$

$$\boldsymbol{\Omega} = \begin{bmatrix} 0 \\ \boldsymbol{\omega} \end{bmatrix}$$

$\boldsymbol{\omega}$ is angular velocity represented in the world frame

– What if $\boldsymbol{\omega}$ is represented in the body frame (i.e. $\boldsymbol{\omega}^b$)?

$\boldsymbol{\Omega}^b = \begin{bmatrix} 0 \\ \boldsymbol{\omega}^b \end{bmatrix}$ is represented in body frame

$$\Rightarrow \quad \boldsymbol{\Omega}^b = \overline{\boldsymbol{Q}} \circ \boldsymbol{\Omega} \circ \boldsymbol{Q}$$

$$\Rightarrow \quad \boxed{\dot{\boldsymbol{Q}} = \frac{1}{2}\boldsymbol{Q} \circ \boldsymbol{\Omega}^b}$$

$$\boldsymbol{Q} = \begin{bmatrix} q_0 \\ \boldsymbol{q} \end{bmatrix}$$

$$\Rightarrow \dot{\boldsymbol{Q}} = \frac{1}{2}\boldsymbol{Q} \circ \boldsymbol{\Omega}^b = \mathcal{R}(\boldsymbol{\Omega}^b)\boldsymbol{Q} = \frac{1}{2}\begin{bmatrix} 0 & -\boldsymbol{\omega}^{b\,T} \\ \boldsymbol{\omega}^b & -\widehat{\boldsymbol{\omega}^b} \end{bmatrix}\boldsymbol{Q}$$

Unit Quaternion and Rotation

- Rotation to quaternion: $\boldsymbol{Q} = \begin{bmatrix} \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2}\boldsymbol{u} \end{bmatrix}$, where $\boldsymbol{R} = e^{\hat{\boldsymbol{u}}\theta}$
- Quaternion to rotation
- $\boldsymbol{R} = \begin{bmatrix} q_0^2 + q_1^2 - q_2^2 - q_3^2 & 2(q_1q_2 - q_0q_3) & 2(q_0q_2 + q_1q_3) \\ 2(q_1q_2 + q_0q_3) & q_0^2 - q_1^2 + q_2^2 - q_3^2 & 2(-q_0q_1 + q_2q_3) \\ 2(-q_0q_2 + q_1q_3) & 2(q_0q_1 + q_2q_3) & q_0^2 - q_1^2 - q_2^2 + q_3^2 \end{bmatrix}$
- Rotating a vector: $\boldsymbol{P'} = \boldsymbol{Q} \circ \boldsymbol{P} \circ \overline{\boldsymbol{Q}}$
- Rotating a frame: $\boldsymbol{P}^b = \overline{\boldsymbol{Q}} \circ \boldsymbol{P} \circ \boldsymbol{Q}$
- Sequential rotation (extrinsic): $\boldsymbol{Q} = \boldsymbol{Q}_n \ldots \circ \boldsymbol{Q}_2 \circ \boldsymbol{Q}_1$
- Sequential rotation (intrinsic): $\boldsymbol{Q} = \boldsymbol{Q}_1 \circ \boldsymbol{Q}_2 \ldots \circ \boldsymbol{Q}_n$
- Kinematics under spatial frame: $\dot{\boldsymbol{Q}} = \frac{1}{2}\boldsymbol{\Omega} \circ \boldsymbol{Q} = \frac{1}{2}\begin{bmatrix} 0 & -\boldsymbol{\omega}^T \\ \boldsymbol{\omega} & \hat{\boldsymbol{\omega}} \end{bmatrix}\boldsymbol{Q}$
- Kinematics under body frame: $\dot{\boldsymbol{Q}} = \frac{1}{2}\boldsymbol{Q} \circ \boldsymbol{\Omega} = \frac{1}{2}\begin{bmatrix} 0 & -\boldsymbol{\omega}^T \\ \boldsymbol{\omega} & -\hat{\boldsymbol{\omega}} \end{bmatrix}\boldsymbol{Q}$