

# Average Model of Boost Converter, including Parasitics, operating in Discontinuous Conduction Mode (DCM)

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**Abstract**—This paper presents the principles of state-space average modeling of DC-DC Boost converters operating in the Discontinuous Conduction Mode (DCM) which occurs in converters due to low load current operation. The switching ripples in inductor current or capacitor voltage cause polarity reversal of the applied switch current or voltage and thus a zero current mode is reached. Modern converters are often designed to operate in DCM for all loads due to their higher efficiency. In this paper, the reduced-order & full-order average models for the Boost converter are derived. Also, the output transfer function of the Boost converter is calculated which can be further utilized for designing a robust controller. Furthermore, the effects of parasitic elements and losses are included based on the state-space averaging technique. The open-loop transfer functions of the proposed models are derived and the behavior of the Boost converter is verified by analysing transient step responses.

**Index Terms**—Boost converters, Discontinuous conduction mode, Full-order model, Parasitic realization, Reduced-order model, State-space average modeling.

## NOMENCLATURE

$T_s$	Total time period for one cycle
$t_{on}$	ON-time period
$d_1$	Duty cycle
$d_1 T_s$	ON-time period
$d_2 T_s$	OFF-time period
$i_{pk}$	Peak value of inductor current
$\bar{i}_L$	Average value of inductor current
$x$	state vector
$A_1$	state (or system) matrix during ON-state
$B_1$	input matrix during ON-state
$A_2$	state (or system) matrix during OFF-state
$B_2$	input matrix during OFF-state
$A_3$	state (or system) matrix during DCM
$B_3$	input matrix during DCM
$n_L$	No. of inductors in the converter circuit
$K$	Correction matrix
$\tilde{i}_L$	Small perturbation value of inductor current
$\tilde{v}_C$	Small perturbation value of capacitor voltage
$\tilde{v}_{in}$	Small perturbation value of input voltage

$\tilde{d}$	Small perturbation value of duty cycle
$I_L$	Equilibrium point value of inductor current
$V_C$	Equilibrium point value of capacitor voltage
$V_{in}$	Equilibrium point value of input voltage
$D$	Equilibrium point value of duty cycle
$M$	Output to input voltage ratio

## I. INTRODUCTION

AMONGST the different topologies of DC-DC converters [1], the Boost converter with typical efficiencies of 70-95% [2], is the one where the output voltage is always greater than its input voltage. By comparing the Boost converter with the Buck or Buck-Boost converters, it is found that the design of the Boost converter is more difficult since the Boost converter is considered as a non-minimum phase system and also has a zero root in the right half of the s-plane. In other words, as a result of the duty cycle, which is the control input for this converter and appears in the current and voltage equations, the solution of the state-space equations of this converter is more difficult [3].

The structure of DC-DC converters consists of linear (i.e. resistor R, inductor L and capacitor C) and nonlinear (i.e. switch) components. Since these converters can be characterized as nonlinear and time-variant systems, then the small-signal model of the state-space average model is required to design a linear controller. References [4-6] show examples of the small-signal analysis and design of a linear controller in the frequency domain for DC-DC converters.

Consideration of all of the system parameters (such as conduction resistances, switch conduction voltages, switching times, and inductor- and capacitor-resistances) in the modelling procedure is an essential step towards the design of a robust controller. In [7-9] Basso, Tomescue and Towati considered the inductor- and capacitor-resistances and the output current in the modelling procedure of the Boost converter. In [10], Ben-Yaakov considered the capacitor resistance and the output current in the modelling procedure of the Boost converter. In [11, 12], a Pulse Width Modulated (PWM) converter with ideal system parameters, both in continuous and discontinuous conduction modes, is modelled and the effect of conduction resistances is included for the same model. In [13], the average model of the PWM converter

is demonstrated by considering the conduction resistances and their voltage drop in DCM.

This paper presents DC and small-signal circuit models of the DC-DC Boost converters operating in DCM. The parasitic components are included in these models to improve the modelling accuracy. The DC model is used to extract the equations for the DC voltage transfer function and the efficiency. The small-signal model is used to extract the equations for the open-loop small-signal transfer functions, such as control to output transfer function, input to output transfer function, and input impedance.

Finally, the various transfer functions (Control, Output Impedance, etc.), various Bode diagrams and transient step responses have been plotted and compared with ideal cases.

## II. BOOST REGULATOR STATE EQUATIONS FOR ON-OFF TIME SWITCHING

For modelling with the state-space technique, the desired state variables for any electric circuit are the energy storing elements (i.e. inductor current and capacitance voltage). To start applying the state-space technique to any complicated circuit, it must first be converted into a piece-wise linearized circuit(s) in which circuit laws can be applied.

In the Boost converter (Fig.1), there are two operating modes; the ON- and OFF-modes. The ON-time is defined by  $d_1T_s$ , and the OFF-time is defined by  $d_2T_s = (1-d_1)T_s$ , where  $T_s$  is the total time period for one cycle and  $d_1$  is the duty cycle which is defined as the ratio of the ON-time period to the total switching time period for one cycle i.e.  $d_1 = t_{on} / T_s$ .

The main switch S is turned ON and OFF by a pulse with a duty cycle equal to  $d_1$ . Therefore, the piece-wise linearized equivalent circuits of the system in ON- and OFF-modes with duration  $d_1T_s$  and  $(1-d_1)T_s$  are represented by Figs. 2 and 3 respectively.

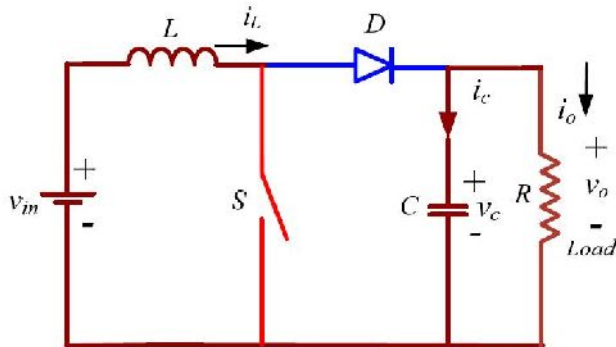


Fig.1 Boost converter circuit

Using  $i_L$  and  $v_c$  as the two state variables ( $x = [i_L \ v_c]^T$ ), and by writing the KVL and KCL for the loops of Figs. 2 and 3, the state-space equations can be derived:

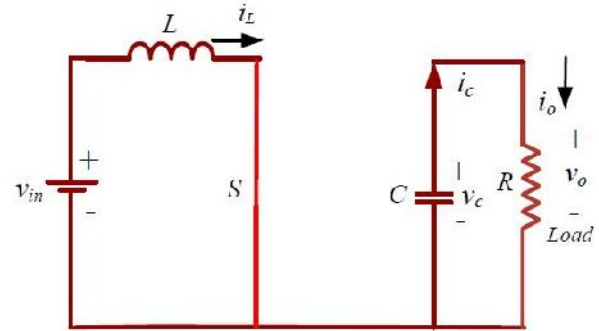


Fig.2 Equivalent circuit of Boost converter during ON-state

During the ON-state, the state-space (ss) form will be:

$$\begin{bmatrix} \frac{di_L}{dt} \\ \frac{dv_c}{dt} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -1/RC \end{bmatrix} \begin{bmatrix} i_L \\ v_c \end{bmatrix} + \begin{bmatrix} 1/L \\ 0 \end{bmatrix} v_{in} \quad (1)$$

$$v_o = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} i_L \\ v_c \end{bmatrix} \quad (2)$$

During the OFF-state, the ss form will be:

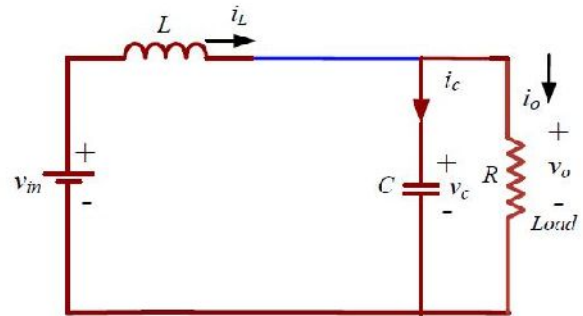


Fig.3 Equivalent circuit of Boost converter during OFF-state

$$\begin{bmatrix} \frac{di_L}{dt} \\ \frac{dv_c}{dt} \end{bmatrix} = \begin{bmatrix} 0 & -1/L \\ 1/C & -1/RC \end{bmatrix} \begin{bmatrix} i_L \\ v_c \end{bmatrix} + \begin{bmatrix} 1/L \\ 0 \end{bmatrix} v_{in} \quad (3)$$

$$v_o = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} i_L \\ v_c \end{bmatrix} \quad (4)$$

Fig.4 shows the steady-state waveforms for the Continuous Conduction Mode (CCM) where the inductor current flows continuously [ $i_L(t) > 0$ ].

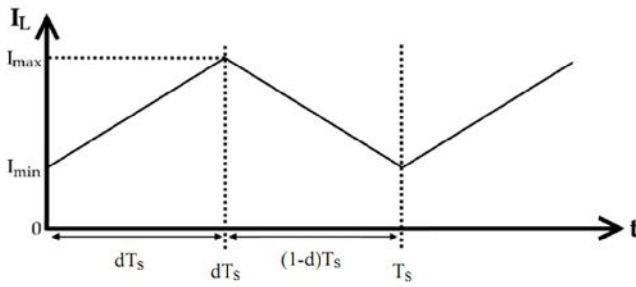


Fig.4 Steady-state waveforms for CCM

In Discontinuous Conduction Mode (DCM), in addition to two modes as in Continuous Conduction Mode (CCM), there is a third mode of operation in which capacitor voltage or inductor current is zero. For DCM operation, during first interval (i.e. ON-period) the switch is turned on and inductor current rises and reaches a peak when the switch is about to turn off, and resets to zero at the end of the OFF-period. During DCM, in ss form:

$$\begin{bmatrix} \frac{di_L}{dt} \\ \frac{dv_C}{dt} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} i_L \\ v_C \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} v_{in} \quad (5)$$

$$v_o = \begin{bmatrix} 0 & 0 \end{bmatrix} \begin{bmatrix} i_L \\ v_C \end{bmatrix}$$

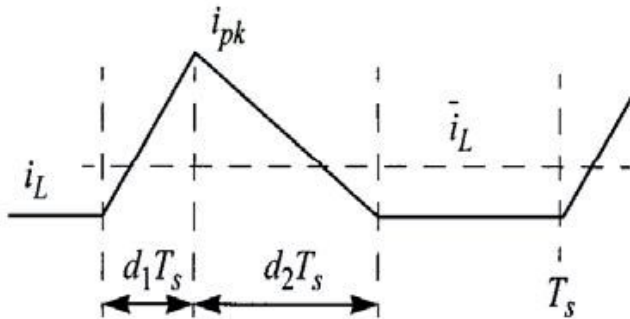


Fig.5 Inductor current waveform of DC-DC Converter in DCM

Thus,

$$\text{'ON' Mode } \dot{x} = A_1 x + B_1 v_{in} \quad \text{For } t \in [0, d_1 T_s] \quad (7)$$

$$\text{'OFF' Mode } \dot{x} = A_2 x + B_2 v_{in} \quad \text{For } t \in [d_1 T_s, (d_1 + d_2) T_s] \quad (8)$$

$$\text{'DC' Mode } \dot{x} = A_3 x + B_3 v_{in} \quad \text{For } t \in [(d_1 + d_2) T_s, T_s] \quad (9)$$

Where,

$d_1 T_s$  = ON-period time

$d_2 T_s$  = OFF-period time

$T_s$  = Total time period for one cycle

$i_{pk}$  = Peak value of inductor current

$\bar{i}_L$  = Average value of inductor current

$v_{in}$  = Input voltage

The modelling method for DCM operation comprises of three steps:

- Averaging;
- Inductor current analysis;
- Duty-ratio constraint.

State-space averaging techniques are employed to get a set of equations that describe the system over one switching period. After applying an averaging technique to equations (7)-(9), the following expression can be found:

$$\dot{x} = [A_1 d_1 + A_2 d_2 + A_3 (1 - d_1 - d_2)]x + [B_1 d_1 + B_2 d_2 + B_3 (1 - d_1 - d_2)]u \quad (10)$$

The above equation can be written as  $\dot{x} = Ax + Bu$ , where,  
 $A = [A_1 d_1 + A_2 d_2 + A_3 (1 - d_1 - d_2)]$  &  
 $B = [B_1 d_1 + B_2 d_2 + B_3 (1 - d_1 - d_2)]$ .

For state-space averaging technique in DCM, only the matrix parameters are averaged and not the state variables. Equation (10) will hold when true average of every state variable is used.

From Fig.5, it can be deduced that the average value of inductor current is:

$$\bar{i}_L = \frac{i_{pk}}{2} (d_1 + d_2) \quad (11)$$

Consider the circuit behavior when the switch is 'OFF'; the current which is delivered to the capacitor does not necessarily have the same value as the average inductor current. Since the inductor current changes slowly with time, the capacitor equation can be solved employing the 'conservation of energy' principle, and after that the averaging step is performed. The total amount of charge which the capacitor obtains from the inductor during switching cycle is:

$$Q_C = \frac{i_{pk} d_1 T_s}{2} \quad (12)$$

Thus, average charging current would be of value:

$$\frac{Q_C}{T_s} = \frac{i_{pk} d_1}{2} \quad (13)$$

When a capacitor is connected to a resistive load, then the net average charging current which is delivered to the capacitor is given by:

$$\left( \frac{i_{pk} d_1}{2} \right) - \left( \frac{v_C}{R} \right) \quad (14)$$

Hence, the average capacitor current will be given by:

$$C \frac{dv_C}{dt} = \left( \frac{i_{pk} d_1}{2} \right) - \left( \frac{v_C}{R} \right) \quad (15)$$

Note here that the above expression differs from the KCL expression of capacitor which is obtained through state-space averaging. From (10), we can define the state-space-average (SSA) charging current as the inductor current's average multiplied by the duty ratio for which the inductor is charging the capacitor. From (11), the SSA charging current can be expressed as:

$$\bar{i}_L d_1 = \frac{i_{pk} d_1}{2} (d_1 + d_2)$$

This expression is different from the actual charging current in (13). It can be implied that a 'charge conservation' law is violated in unmodified SSA as the averaging step is done on a complete model thus leading to mismatching of responses with averaged response of DC-DC converters. Thus (10) is modified by dividing the inductor current by the factor  $(d_1 + d_2)$ . The basic method is to rearrange the  $x$ , thus  $x = [i_L, v_C]^T$ , where sub-vector  $i_L$  contains all  $(n_L)$  inductor currents of the converter and defined by a matrix  $K$ , as below:

$$K = \text{diag} \left[ \underbrace{\frac{1}{d_1 + d_2}, \dots, \frac{1}{d_1 + d_2}}_{n_L}, 1, 1, 1, \dots, 1 \right]$$

With this correction matrix, the average modified model becomes

$$\dot{x} = [A_1 d_1 + A_2 d_2 + A_3 (1 - d_1 - d_2)] K x + [B_1 d_1 + B_2 d_2 + B_3 (1 - d_1 - d_2)] u$$

Where,

$$A_1 = \begin{bmatrix} 0 & 0 \\ 0 & \frac{-1}{RC} \end{bmatrix}, A_2 = \begin{bmatrix} 0 & \frac{-1}{L} \\ 1 & \frac{-1}{RC} \end{bmatrix} \text{ and } A_3 = \begin{bmatrix} 0 & 0 \\ 0 & \frac{-1}{RC} \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 1 \\ \frac{1}{L} \end{bmatrix}, B_2 = \begin{bmatrix} 1 \\ \frac{1}{L} \end{bmatrix} \text{ and } B_3 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

The SSA model for the above equation is

$$\frac{d}{dt} \begin{bmatrix} i_L \\ v_C \end{bmatrix} = \begin{bmatrix} 0 & \frac{-d_2}{L} \\ d_2 & \frac{-1}{RC} \end{bmatrix} \begin{bmatrix} i_L \\ v_C \end{bmatrix} + \begin{bmatrix} d_1 + d_2 \\ L \\ 0 \end{bmatrix} v_{in}$$

Since there is only one inductor, the  $x$  is of dimension two, the correction matrix  $K$  is simply given by

$$K = \begin{bmatrix} 1 & 0 \\ d_1 + d_2 & 1 \end{bmatrix}$$

Thus, the modified average model of Boost converter in DCM is given below:

$$\begin{aligned} \frac{d}{dt} \begin{bmatrix} i_L \\ v_C \end{bmatrix} &= \begin{bmatrix} 0 & \frac{-d_2}{L} \\ d_2 & \frac{-1}{RC} \end{bmatrix} K \begin{bmatrix} i_L \\ v_C \end{bmatrix} + \begin{bmatrix} d_1 + d_2 \\ L \\ 0 \end{bmatrix} v_{in} \\ &= \begin{bmatrix} 0 & \frac{-d_2}{L} \\ d_2 & \frac{-1}{RC} \end{bmatrix} \begin{bmatrix} i_L \\ v_C \end{bmatrix} + \begin{bmatrix} d_1 + d_2 \\ L \\ 0 \end{bmatrix} v_{in} \end{aligned} \quad (21)$$

### (16) III. REDUCED ORDER AVERAGE MODEL FOR BOOST SWITCHING REGULATOR

To complete the average model represented in (18), a duty ratio constraint is defined showing the dependency of  $d_2$  on other variables. Usually, in conventional SSA techniques, the inductor voltage balance equation is used in defining the duty-ratio constraint.

For the Boost converter topology, utilizing the volt-second balance over the switching cycle,

$$\begin{aligned} v_L &= L \frac{di}{dt} \\ \therefore di &= \frac{v_L dt}{L} \\ \text{For time } T_1 &= d_1 T_s, \\ I_{max} - 0 &= \frac{v_{in} T_1}{L} \\ \text{For time } T_2 &= d_2 T_s, \\ 0 - I_{max} &= \frac{v_{in} - v_C}{L} T_2 \end{aligned} \quad (17)$$

By removing  $I_{max}$  from above equations,

$$\begin{aligned} I_{max} &= \frac{v_{in}}{L} d_1 T_s = \frac{v_C - v_{in}}{L} d_2 T_s \\ \therefore v_{in} d_1 &= (v_C - v_{in}) d_2 \\ \therefore d_2 &= \frac{v_{in}}{v_C - v_{in}} d_1 \end{aligned} \quad (22)$$

Substituting  $d_2$  in (21), this will result in

$$\frac{di_L}{dt} = 0 \quad (23)$$

$$\frac{dv_C}{dt} = \frac{v_{in} i_L}{v_C C} - \frac{v_C}{RC} \quad (24)$$

From these calculations for a Boost converter, it can be seen that inductor current dynamics disappear, thus resulting in a degenerate model. Since inductor current is not present in the state variable in this reduced-order model, it must be replaced

by expressing it as an algebraic function of other variables, so that inductor dynamics are removed.

For a Boost converter, the peak of the inductor current is given by:

$$i_{pk} = \frac{v_{in}}{L} d_1 T_s$$

The average value of inductor current is given by:

$$\bar{i}_L = \frac{i_{pk}}{2} (d_1 + d_2) = \frac{v_{in}}{2L} d_1 (d_1 + d_2) T_s$$

$$\begin{bmatrix} \tilde{i}_L(s) \\ \tilde{v}_C(s) \end{bmatrix} = \tilde{X}(s) = (sI - A)^{-1} B U(s)$$

$$\begin{aligned} \therefore \begin{bmatrix} \tilde{i}_L(s) \\ \tilde{v}_C(s) \end{bmatrix} &= \frac{1}{\Delta} \cdot \begin{bmatrix} 0 & 0 \\ s \left( \frac{D^2 T_s}{LC(M-1)} + \frac{M}{RC(M-1)} \right) & s \left( \frac{DT_s V_{in}}{LC(M-1)} \right) \end{bmatrix} \begin{bmatrix} \tilde{v}_{in}(s) \\ \tilde{d}(s) \end{bmatrix} \end{aligned} \quad (25)$$

Where,

$$\Delta = s^2 + s \left[ \frac{1}{RC} \left( \frac{2M-1}{M-1} \right) \right]$$

Substituting by (22), the above relationship can be written as

$$\bar{i}_L = \frac{v_{in}}{2L} \left( \frac{d_1^2 T_s v_C}{v_C - v_{in}} \right)$$

(27) Hence, two transfer functions can be found as follows:

Substituting (27) in (24), this will result in the conventional average model for a Boost converter in DCM. In this model, the dependency on the average inductor current is removed.

$$\frac{\tilde{v}_C(s)}{\tilde{v}_{in}(s)} = \frac{s \left( \frac{D^2 T_s}{LC(M-1)} + \frac{M}{RC(M-1)} \right)}{\Delta} \quad (32)$$

And

$$\frac{\tilde{v}_C(s)}{\tilde{d}(s)} = \frac{s \left( \frac{DT_s V_{in}}{LC(M-1)} \right)}{\Delta} \quad (33)$$

$$\frac{dv_C}{dt} = \left( \frac{v_{in}^2}{2LC} \right) \left( \frac{d^2 T_s}{v_C - v_{in}} \right) - \left( \frac{v_C}{RC} \right)$$

Now, applying standard linearization techniques and applying small perturbations as follows to (28):

$$\begin{aligned} i_L &= I_L + \tilde{i}_L; \\ v_C &= V_C + \tilde{v}_C; \\ v_{in} &= V_{in} + \tilde{v}_{in}; \\ d &= D + \tilde{d}; \end{aligned}$$

$$\therefore \frac{d(V_C + \tilde{v}_C)}{dt} = \left( \frac{[(V_{in} + \tilde{v}_{in})]^2}{2LC} \right) \left( \frac{(D + \tilde{d})^2 T_s}{[(V_C + \tilde{v}_C) - (V_{in} + \tilde{v}_{in})]} \right) - \left( \frac{(V_C + \tilde{v}_C)}{RC} \right) \quad (29)$$

Separating terms of  $\tilde{i}_L$ ,  $\tilde{v}_C$ ,  $\tilde{v}_{in}$  and  $\tilde{d}$  and then converting it to state-space form,

$$\begin{bmatrix} \dot{\tilde{i}}_L \\ \dot{\tilde{v}}_C \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & \left( \frac{-1}{RC} \right) \left( \frac{2M-1}{M-1} \right) \end{bmatrix} \begin{bmatrix} \tilde{i}_L \\ \tilde{v}_C \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \left( \frac{D^2 T_s}{LC(M-1)} + \frac{M}{RC(M-1)} \right) & \left( \frac{DT_s V_{in}}{LC(M-1)} \right) \end{bmatrix} \begin{bmatrix} \tilde{v}_{in} \\ \tilde{d} \end{bmatrix} \quad (30)$$

Which is in the form of:

$$\frac{d}{dt} \begin{bmatrix} \tilde{i}_L \\ \tilde{v}_C \end{bmatrix} = A \begin{bmatrix} \tilde{i}_L \\ \tilde{v}_C \end{bmatrix} + B \begin{bmatrix} \tilde{v}_{in} \\ \tilde{d} \end{bmatrix}$$

Since,

$$i_{pk} = \frac{v_{in}}{L} d_1 T_s$$

Substituting this into (11), this will result in a duty constraint

$$d_2 = \frac{2L i_L}{d_1 T_s v_{in}} - d_1 \quad (34)$$

This constraint is different from the earlier one which is derived for a reduced-order model showing that it enforces correct average charging of output capacitor. Putting  $d_2$  into (21), the following relations are derived:

$$\frac{di_L}{dt} = \frac{2i_L}{d_1 T_s} \left( 1 - \frac{v_C}{v_{in}} \right) + \frac{d_1 v_C}{L} \quad (35)$$

$$\frac{dv_C}{dt} = \frac{i_L}{C} - \frac{d_1^2 T_s v_{in}}{2LC} - \frac{v_C}{RC}$$

Equating (35) and (36) to zero and finding the solutions for  $i_L$  and  $v_C$ , the DC-operating point can be obtained. Let the scalar value of M be the output to input voltage ratio. Thus,

$$M = \frac{v_C}{v_{in}} = \frac{1}{2} + \frac{1}{2} \sqrt{1 + \frac{2D^2 R}{LF_s}}$$

Now applying standard linearization techniques and applying small perturbations as follows to (35) and (36):

$$\begin{aligned} i_L &= I_L + \tilde{i}_L; \\ v_C &= V_C + \tilde{v}_C; \\ v_{in} &= V_{in} + \tilde{v}_{in}; \\ d_1 &= d = D + \tilde{d}; \end{aligned}$$

Then,

$$\frac{d(I_L + \tilde{i}_L)}{dt} = \frac{2(I_L + \tilde{i}_L)}{(D + \tilde{d})T_s} \left(1 - \frac{(V_C + \tilde{v}_C)}{(V_{in} + \tilde{v}_{in})}\right) + \frac{(D + \tilde{d})(V_C + \tilde{v}_C)}{L} \quad (36)$$

And

$$\frac{d(V_C + \tilde{v}_C)}{dt} = \frac{(I_L + \tilde{i}_L)}{C} - \frac{(D + \tilde{d})^2 T_s (V_{in} + \tilde{v}_{in})}{2LC} - \frac{(V_C + \tilde{v}_C)}{RC} \quad (37)$$

Now, the small signal model can be derived with the following equation:

$$\frac{d}{dt} \begin{bmatrix} \tilde{i}_L \\ \tilde{v}_C \end{bmatrix} = A \begin{bmatrix} \tilde{i}_L \\ \tilde{v}_C \end{bmatrix} + B \begin{bmatrix} \tilde{v}_{in} \\ \tilde{d} \end{bmatrix}$$

Where,

$$A = \begin{bmatrix} \frac{2(1-M)}{DT_s} & \frac{-D}{L(M-1)} \\ \frac{1}{C} & \frac{-1}{RC} \end{bmatrix} \text{ and } B = \begin{bmatrix} \frac{DM^2}{L(M-1)} & \frac{2MV_{in}}{L} \\ \frac{-D^2 T_s}{2LC} & \frac{-DT_s V_{in}}{LC} \end{bmatrix}$$

Since,

$$\begin{bmatrix} \tilde{i}_L(s) \\ \tilde{v}_C(s) \end{bmatrix} = \tilde{x}(s) = (sI - A)^{-1} B U(s)$$

$$\therefore \begin{bmatrix} \tilde{i}_L(s) \\ \tilde{v}_C(s) \end{bmatrix} = \frac{1}{\Delta} \cdot$$

$$\begin{bmatrix} \left( \frac{sM^2}{L(M-1)} + \frac{M^2}{RLC(M-1)} + \frac{MD}{RLC} \right) \left( \frac{2sMV_{in}}{L} + \frac{4MV_{in}}{RLC} \right) \\ \left( \frac{M^2}{LC(M-1)} + \frac{D(1-M)}{LC} + \frac{-sD^2 T_s}{2LC} \right) \left( \frac{-sDT_s V_{in}}{LC} + \frac{2V_{in}}{LC} \right) \end{bmatrix} \cdot \begin{bmatrix} \tilde{v}_{in}(s) \\ \tilde{d}(s) \end{bmatrix} \quad (40)$$

Where,

$$\Delta = s^2 + s \left( \frac{1}{RC} - \frac{2(1-M)}{DT_s} \right) + \frac{2(2M-1)}{DT_s RC}$$

Hence, the following transfer functions can be formulated from the small signal model:

$$\frac{\tilde{i}_L(s)}{\tilde{v}_{in}(s)} = \frac{1}{\Delta} \cdot \left( \frac{sM^2}{L(M-1)} + \frac{M^2}{RLC(M-1)} + \frac{MD}{RLC} \right) \quad (41)$$

$$\frac{\tilde{i}_L(s)}{\tilde{d}(s)} = \frac{1}{\Delta} \cdot \left( \frac{2sMV_{in}}{L} + \frac{4MV_{in}}{RLC} \right) \quad (42)$$

$$\frac{\tilde{v}_C(s)}{\tilde{v}_{in}(s)} = \frac{1}{\Delta} \cdot \left( \frac{M^2}{LC(M-1)} + \frac{D(1-M)}{LC} + \frac{-sD^2 T_s}{2LC} \right) \quad (43)$$

$$\frac{\tilde{v}_C(s)}{\tilde{d}(s)} = \frac{1}{\Delta} \cdot \left( \frac{-sDT_s V_{in}}{LC} + \frac{2V_{in}}{LC} \right) \quad (44)$$

## V. PARASITIC REALIZATION IN THE BOOST SWITCHING REGULATOR

Due to the difficulties faced during the modelling procedure of the Boost converter, parasitic elements such as conduction voltages, conduction resistances, inductor resistances and equivalent series resistances (ESR) of capacitors have been ignored. The idea of simply considering ideal/lossless components and leaving out parasitic elements simplifies the model development procedure and allows to understand the fundamental behaviour of the switching converter system. However, the effects of parasitic elements and losses are important for improving model accuracy, studying efficiency and dynamic performance of the system. The problem with including the parasitic elements is that they lead to nonlinear current and voltage waveforms and hence result in complications in the modelling procedure.

A Boost converter circuit, with parasitics included, will look like as shown in Fig.6.

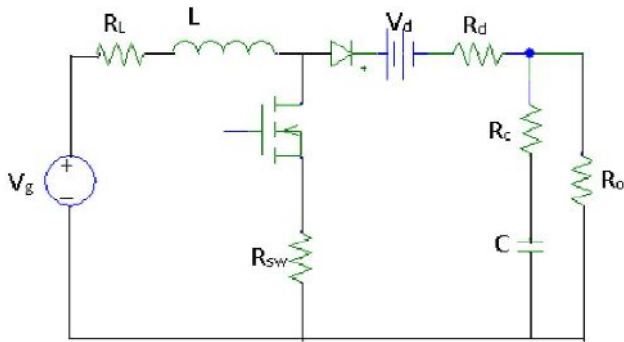


Fig.6 Boost Converter Circuit with parasitic

During the ON-state:

$$\begin{bmatrix} \frac{di_L}{dt} \\ \frac{dv_C}{dt} \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{(R_L + R_{sw})}{L} & 0 \\ 0 & \frac{-1}{C(R_o + R_c)} \end{bmatrix}}_{A_1} \begin{bmatrix} i_L \\ v_C \end{bmatrix} + \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{B_1} v_{in} \quad (45)$$

During the OFF-state:

$$\begin{bmatrix} \frac{di_L}{dt} \\ \frac{dv_C}{dt} \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{-(R_c + R_d)}{L} - \frac{R_c}{L(1 + \frac{R_c}{R_o})} & \frac{-1}{(1 + \frac{R_c}{R_o})} \\ \left(1 - \frac{R_c}{R_o(1 + \frac{R_c}{R_o})}\right) & \frac{-1}{CR_o(1 + \frac{R_c}{R_o})} \end{bmatrix}}_{A_2} \begin{bmatrix} i_L \\ v_C \end{bmatrix} + \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{B_2} v_{in} \quad (46)$$

During DCM period:

$$\begin{bmatrix} \frac{di_L}{dt} \\ \frac{dv_C}{dt} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & \frac{-1}{C(R_o + R_c)} \end{bmatrix}}_{A_3} \begin{bmatrix} i_L \\ v_C \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{B_3} v_{in} \quad (47)$$

Now, applying the averaging technique, this will result in:

$$\dot{x} = [A_1 d_1 + A_2 d_2 + A_3(1 - d_1 - d_2)]x + [B_1 d_1 + B_2 d_2 + B_3(1 - d_1 - d_2)]u$$

$$+ [B_1 d_1 + B_2 d_2 + B_3(1 - d_1 - d_2)]u$$

So, the SSA model for the above equation is

$$\frac{d}{dt} \begin{bmatrix} i_L \\ v_C \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{-d_1(R_L + R_{sw})}{L} - \frac{d_2 R_L}{L} - \frac{d_2 R_d R_c}{L(1 + \frac{R_c}{R_o})} & \frac{-d_2}{(1 + \frac{R_c}{R_o})} \\ \frac{d_2 R_o}{[C(R_o + R_c)]} & \frac{-1}{C(R_c + R_o)} \end{bmatrix}}_A \begin{bmatrix} i_L \\ v_C \end{bmatrix} + \underbrace{\begin{bmatrix} d_1 + d_2 \\ 0 \end{bmatrix}}_B v_{in} \quad (48)$$

Since there is only one inductor, the x is of dimension two, the correction matrix K is simply given by

$$K = \begin{bmatrix} 1 & 0 \\ d_1 + d_2 & 1 \end{bmatrix} \quad (49)$$

With this correction matrix, the average modified model becomes

$$\dot{x} = [A_1 d_1 + A_2 d_2 + A_3(1 - d_1 - d_2)]Kx + [B_1 d_1 + B_2 d_2 + B_3(1 - d_1 - d_2)]u$$

Hence, the modified average model of Boost converter in DCM will look like:

$$\frac{d}{dt} \begin{bmatrix} i_L \\ v_C \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{-d_1(R_L + R_{sw})}{L} - \frac{d_2 R_L}{L} - \frac{d_2 R_d R_c}{L(1 + \frac{R_c}{R_o})} & \frac{-d_2}{(1 + \frac{R_c}{R_o})} \\ \frac{d_2 R_o}{[C(R_o + R_c)]} & \frac{-1}{C(R_c + R_o)} \end{bmatrix}}_A \begin{bmatrix} i_L \\ v_C \end{bmatrix} + \underbrace{\begin{bmatrix} 1 & 0 \\ d_1 + d_2 & 1 \end{bmatrix}}_K \begin{bmatrix} i_L \\ v_C \end{bmatrix} + \underbrace{\begin{bmatrix} d_1 + d_2 \\ 0 \end{bmatrix}}_B v_{in} \quad (50)$$

In the above state-space equation, replacing  $d_2$  by the duty constraint (34), this will result in

$$\frac{di_L}{dt} = \frac{-d_1(R_L + R_{sw})i_L}{\left(\frac{2L^2 i_L}{d_1 T_s v_{in}}\right)} - \frac{\left(\frac{2L i_L}{d_1 T_s v_{in}} - d_1\right)\left(R_L + R_d + \frac{R_c R_o}{R_c + R_o}\right)i_L}{\left(\frac{2L^2 i_L}{d_1 T_s v_{in}}\right)} \quad (47)$$



$$-\frac{\left(\frac{2Li_L}{d_1T_s v_{in}} - d_1\right)R_o v_c}{(R_c + R_o)} + \frac{2i_L}{d_1T_s}$$

$$-\frac{\left(\frac{2Li_L}{d_1T_s v_{in}} - d_1\right)v_{in}}{L}$$

And

$$\frac{dv_c}{dt} = \frac{\left(\frac{2Li_L}{d_1T_s v_{in}} - d_1\right)R_o i_L}{\left(\frac{2Li_L}{d_1T_s v_{in}}\right)C(R_c + R_o)} - \frac{v_c}{C(R_c + R_o)}$$

Now applying standard linearization techniques and applying small perturbations to (51) and (52), this will result in the small-signal model as:

$$\frac{d}{dt} \begin{bmatrix} \tilde{i}_L \\ \tilde{v}_c \end{bmatrix} = A \begin{bmatrix} \tilde{i}_L \\ \tilde{v}_c \end{bmatrix} + B \begin{bmatrix} \tilde{v}_{in} \\ \tilde{d} \end{bmatrix}$$

Where,

$$A = \begin{bmatrix} \frac{A_1}{\frac{DV_{in}}{R_o}} & \frac{A_2}{\frac{DV_{in}}{-1}} \\ \frac{A_3}{C(R_c + R_o)} & \frac{A_4}{C(R_c + R_o)} \end{bmatrix} \text{ and}$$

$$B = \begin{bmatrix} \frac{A_3}{\frac{DV_{in}}{-D^2T_s R_o}} & \frac{A_4}{\frac{DV_{in}}{-DT_s V_{in} R_o}} \\ \frac{A_3}{2LC(R_c + R_o)} & \frac{A_4}{LC(R_c + R_o)} \end{bmatrix}$$

Where,

$$A_1 = \frac{DV_{in}}{L} \left( R_L + R_d + \frac{R_c R_o}{R_c + R_o} \right) - \frac{2LV_c}{T_s} \left( \frac{R_o}{R_c + R_o} \right) + \frac{2V_{in}}{T_s};$$

$$A_2 = -\frac{2LV_c}{T_s} \left( \frac{R_o}{R_c + R_o} \right) I_L + D^2 V_{in} \left( \frac{R_o}{R_c + R_o} \right);$$

$$A_3 = \frac{D^2 T_s V_{in}}{L^2} \left( R_d - R_{sw} + \frac{R_c R_o}{R_c + R_o} \right) + D^2 V_c \left( \frac{R_o}{R_c + R_o} \right) v_o = \frac{v_c + R_c i_L}{\left(1 + \frac{R_c}{R_o}\right)} = \frac{v_c}{\left(1 + \frac{R_c}{R_o}\right)} + \frac{R_c i_L}{\left(1 + \frac{R_c}{R_o}\right)}$$

$$+ \frac{2I_L}{T_s} - \frac{DI_L}{L} \left( R_c + R_d + \frac{R_c R_o}{R_c + R_o} \right);$$

$$A_4 = \frac{3D^2 T_s V_{in}^2}{2L^2} \left( R_d - R_{sw} + \frac{R_c R_o}{R_c + R_o} \right) - \frac{DI_L}{L} \left( R_c + R_d + \frac{R_c R_o}{R_c + R_o} \right).$$

Since,

$$\begin{bmatrix} \tilde{i}_L(s) \\ \tilde{v}_c(s) \end{bmatrix} = X(s) = (sI - A)^{-1} B U(s)$$

$$(51) \therefore \begin{bmatrix} \tilde{i}_L(s) \\ \tilde{v}_c(s) \end{bmatrix} = \frac{1}{\Delta} \cdot \begin{bmatrix} \left(s + \frac{1}{C(R_c + R_o)}\right) & \frac{-A_2}{DV_{in}} \\ \frac{-R_o}{C(R_c + R_o)} & \left(s - \frac{A_1}{DV_{in}}\right) \end{bmatrix}$$

$$(52) \begin{bmatrix} \frac{A_3}{\frac{DV_{in}}{DV_{in}}} & \frac{A_4}{\frac{DV_{in}}{DV_{in}}} \\ \frac{-D^2 T_s R_o}{2LC(R_c + R_o)} & \frac{-DT_s V_{in} R_o}{LC(R_c + R_o)} \end{bmatrix} \cdot \begin{bmatrix} \tilde{v}_{in}(s) \\ \tilde{d}(s) \end{bmatrix} \quad (54)$$

Where,

$$\Delta = s^2 + s \left( \frac{1}{C(R_c + R_o)} - \frac{A_1}{DV_{in}} \right) - \frac{A_1}{CDV_{in}(R_c + R_o)} + \frac{A_2 R_o}{CDV_{in}(R_c + R_o)}$$

Hence, the following transfer functions can be formulated from the small signal model:

$$\frac{\tilde{i}_L(s)}{\tilde{v}_{in}(s)} = \frac{1}{\Delta} \cdot \left( s + \frac{1}{C(R_c + R_o)} \right) \left( \frac{A_3}{DV_{in}} \right) + \frac{D^2 T_s R_o A_2}{2LCDV_{in}(R_c + R_o)}$$

$$(53) \frac{\tilde{i}_L(s)}{\tilde{d}(s)} = \frac{1}{\Delta} \cdot \left( s + \frac{1}{C(R_c + R_o)} \right) \left( \frac{A_4}{DV_{in}} \right) + \frac{-DT_s V_{in} R_o A_2}{LCDV_{in}(R_c + R_o)}$$

$$\frac{\tilde{v}_c(s)}{\tilde{v}_{in}(s)} = \frac{1}{\Delta} \cdot \left[ \frac{-R_o A_3}{CDV_{in}(R_c + R_o)} - \frac{-sD^2 T_s R_o}{2LC(R_c + R_o)} + \frac{DA_1 T_s}{2LCV_{in}(R_c + R_o)} \right]$$

$$\frac{\tilde{v}_c(s)}{\tilde{d}(s)} = \frac{1}{\Delta} \cdot \left[ \frac{-DT_s V_{in} R_o}{LC(R_c + R_o)} - \frac{sDT_s V_{in} R_o}{LC(R_c + R_o)} - \frac{R_o A_4}{CDV_{in}(R_c + R_o)} \right]$$

Since,

$$(59)$$

$$\therefore v_o = \left[ \frac{R_c}{\left(1 + \frac{R_c}{R_o}\right)} \quad \frac{1}{\left(1 + \frac{R_c}{R_o}\right)} \right] \begin{bmatrix} i_L \\ v_c \end{bmatrix} \quad (60)$$

This is in the form of:

$$Y(s) = C(s)X(s)$$

Since,



$$x(s) = (sI - A)^{-1}BU(s)$$

$$\therefore Y(s) = C(s)(sI - A)^{-1}BU(s)$$

$$\therefore Y(s) = \frac{1}{\Delta} \cdot \begin{bmatrix} R_c & 1 \\ \left(1 + \frac{R_c}{R_o}\right) & \left(1 + \frac{R_c}{R_o}\right) \end{bmatrix} \cdot \begin{bmatrix} \left(s + \frac{1}{C(R_c + R_o)}\right) & \frac{-A_2}{DV_{in}} \\ \frac{-R_o}{C[(R)_c + R_o]} & \left(s - \frac{A_1}{DV_{in}}\right) \end{bmatrix} \cdot \begin{bmatrix} \frac{A_3}{DV_{in}} & \frac{A_4}{DV_{in}} \\ \frac{-D^2 T_s R_o}{2LC[(R)_c + R_o]} & \frac{-DT_s V_{in} R_o}{LC[(R)_c + R_o]} \end{bmatrix} \cdot \begin{bmatrix} \tilde{v}_{in}(s) \\ \tilde{d}(s) \end{bmatrix} \quad (61)$$

Hence, two transfer functions can be found as follows:

$$(R)_c$$

$$(v_{io}(s))/(\tilde{d}(s)) = 1/\Delta \cdot (1/((1 + R_1 C/R_{1o}))) \cdot ((s + 1/C(R_1 C + R_{1o})) \cdot (A_4 + (DV_{in}))) + (-DT_s V_{in} R_o A_1 2)/(\Delta \cdot C(R_1 C + R_{1o})) \cdot (A_4 + (DV_{in})))$$

## VI. SIMULATION RESULTS

The simulations were done by using the following model parameters:  $L=18\mu H$ ,  $C=4.7\mu F$ ,  $V_{in}=5V$ ,  $D=0.7$ ,  $R_c=0.1$ ,  $R_d=0.15$ ,  $R_L=0.8$ ,  $R_{sw}=0.17$  and  $F_s=350$  kHz. The simulations were performed for two different cases.

Figures 7 and 8 show the Bode diagrams for the transfer function representing the relation between the output voltage and duty ratio ( $\tilde{d}$ ) (i.e. Control transfer function) for ideal and non-ideal DC-DC Boost converters in DCM, respectively.

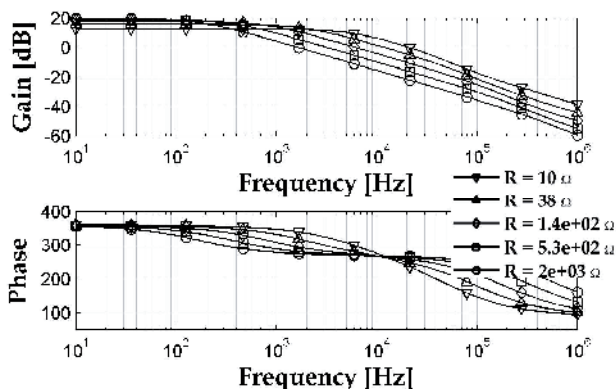


Fig.7 Bode diagram for ideal DC-DC Boost converter in DCM

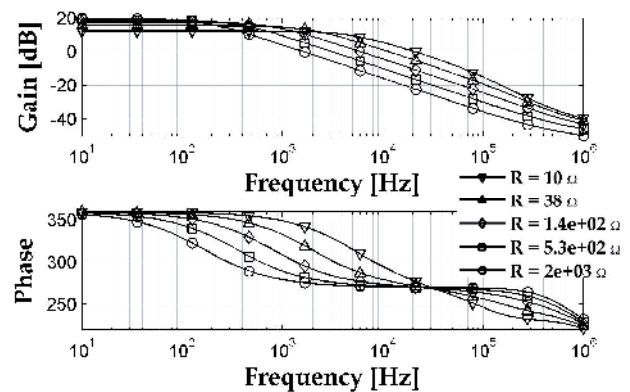


Fig.8 Bode diagram for non-ideal DC-DC Boost converter in DCM

As seen from Fig.7 and Fig.8, there is a remarkable decrease in the open loop gain as the load resistance decreases. When the load resistance goes below a certain range, the open loop gain goes under 0 dB which makes the converter ineffective without an external feedback controller.

Figures 9 and 10 show the transient step responses for ideal and non-ideal DC-DC Boost converters in DCM respectively. As indicated in Fig.9 and Fig.10, these figures were plotted by using two different load values to see the transient behavior as a function of the load value. The steady-state behavior is a function of the load value. This is the important reason behind why the closed-loop control is essential, where the duty cycle (the only controllable parameter in the circuit) must be controlled and adapted in order to preserve the required steady-state DC voltage for different load values. Moreover, by comparing Fig.9 and Fig.10 for the same values of the load resistance, it is easy to observe that the step response for the ideal DC-DC Boost converter is looking identical to the step response for the non-ideal DC-DC Boost converter. This indicates that the ideal and non-ideal transfer functions should be approximately equal; which, in turn, reveals that the loss elements found in the non-ideal DC-DC Boost converter circuit have little effect on the dynamic response of the DC-DC Boost converter operating in DCM.

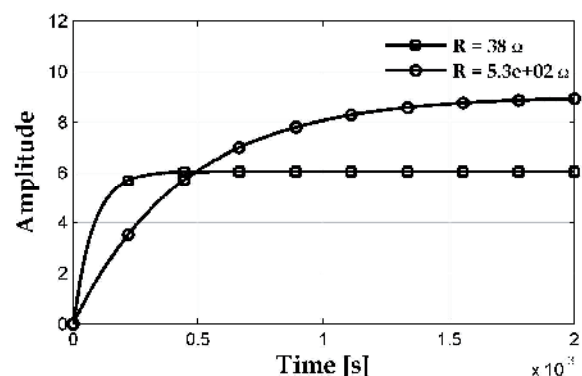


Fig.9 Transient step response for ideal DC-DC Boost converter in DCM

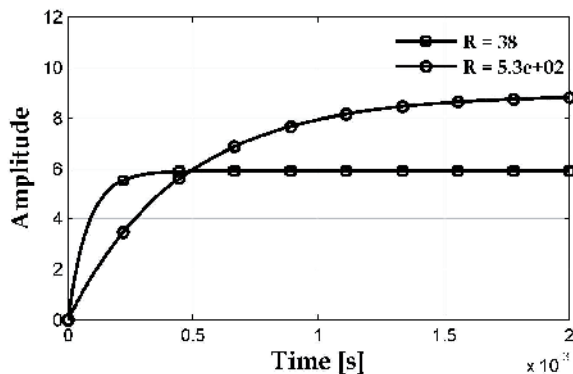


Fig.10 Transient step response for non-ideal DC-DC Boost converter in DCM

## VII. CONCLUSION

First, the various aspects of average modelling of DC-DC Boost converter operating in DCM are studied. Basically, the modelling procedure consists of three steps:

1. Averaging the matrix parameters and selection of the correction matrix (K) depending on the number of inductor currents of the converter.
2. Conversion of state-space equations into differential equations for inductor current and capacitor voltage.
3. Defining a duty ratio constraint so that the expression consists of only one duty ratio.

Second, the reduced- and full-order average models have been derived. It was found that the reduced-order model can estimate the behavior in the low frequency range but the full-order model, since dynamics of inductor are present, is more precise.

Finally, various parasitic components have been taken into consideration and a full-order model is developed. The system dynamic behavior for the DC-DC Boost converter with ideal components and DC-DC Boost converter with non-ideal components operating in DCM are compared via Bode plots and transient step responses under different values of the load resistance in order to help in designing a robust controller.

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