Time Series Analysis by State Space Methods

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ILLUSTRATION SAMPLE

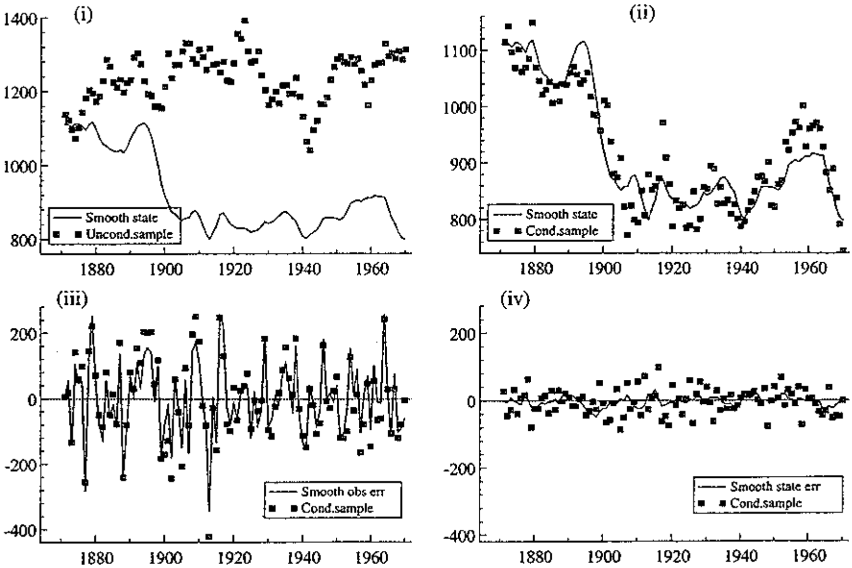

Fig. 2.4. Simulation: (i) smoothed state $\hat{\alpha}_t$ and sample $\alpha_t^{(\cdot)}$; (ii) smoothed state $\hat{\alpha}_t$ and sample $\tilde{\alpha}_t$; (iii) smoothed observation error $\hat{\varepsilon}_t$ and sample $\tilde{\varepsilon}_t$; (iv) smoothed state error $\hat{\eta}_t$ and sample $\tilde{\eta}_t$.

It is, however, easier and more transparent to proceed as follows, using the original time domain. For filtering at times $t = \tau, \ldots, \tau^* - 1$, we have

$$E(\alpha_t | Y_{t-1}) = E(\alpha_t | Y_{\tau-1}) = E\left(\alpha_\tau + \sum_{j=\tau}^{t-1} \eta_j \middle| Y_{\tau-1}\right) = a_\tau$$

and

$$\mathrm{Var}(\alpha_t | Y_{t-1}) = \mathrm{Var}(\alpha_t | Y_{\tau-1}) = \mathrm{Var}\left(\alpha_\tau + \sum_{j=\tau}^{t-1} \eta_j \middle| Y_{\tau-1}\right) = P_\tau + (t-\tau)\sigma_\eta^2.$$

giving

$$a_{t+1} = a_t, \qquad P_{t+1} = P_t + \sigma_\eta^2, \qquad t = \tau, \ldots, \tau^* - 1, \tag{2.38}$$

the remaining values a_t and P_t being given as before by (2.11) for $t = 1, \ldots, \tau$ and $t = \tau^*, \ldots, n$. The consequence is that we can use the original filter (2.11) for all t by taking $v_t = 0$ and $K_t = 0$ at the missing time points. The same procedure is used when more than one group of observations is missing. It follows that allowing for missing observations when using the Kalman filter is extremely simple.

The forecast error recursions from which we derive the smoothing recursions are given by (2.18). These error-updating equations at the missing time points