Equivalent Circuit Modeling - Several Extensions

So far, we have found the following:

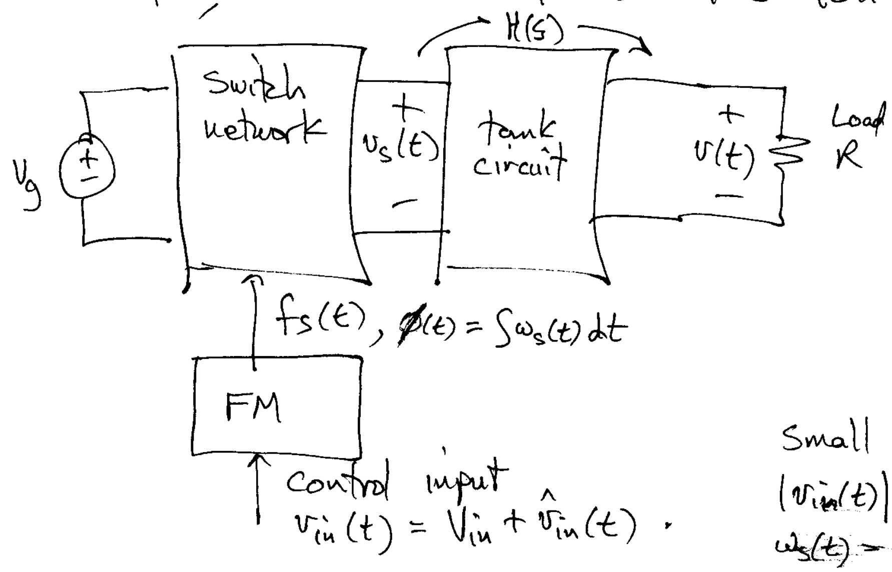


Fig. 1

Small signals:
$|\hat{v}_{in}(t)| << V_{in}$, leads to
$\omega_s(t) = f_s(t) \cdot 2\pi = \omega_{so} + \hat{\omega}_s(t)$, etc.

AC output voltage having both frequency modulation and phase modulation:

$$v(t) = \|A\| \cos(\omega_{so} t + \angle A) \qquad (1)$$

where the envelope (amplitude) of $v(t)$ is

$$\|A(t)\| = \|A_o\| + \hat{a}(t) \qquad (2)$$

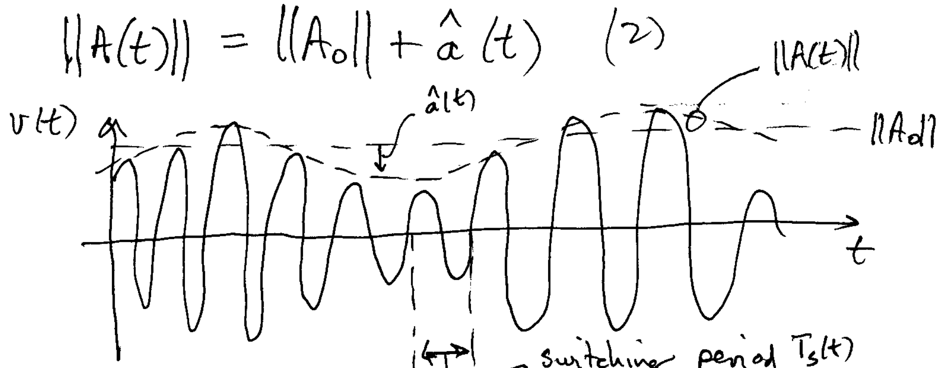


Fig. 2

where $$\|A_o\| = V_{so}\,\|H(j\omega_{so}t)\| \qquad (3)$$
is the amplitude of the ac output under quiescent operating conditions

and $$\hat{a}(s) = G_{env}(s)\cdot \hat{v}_{in}(s) \qquad (4)$$

with $G_{env}(s)$ given by Eq. (38) of the previous lecture notes:

$$G_{env}(s) = \frac{V_{s1}}{2\omega_m\,\|H(j\omega_{so})\|}\cdot\frac{H(-j\omega_{so})N(s+j\omega_{so})D(s-j\omega_{so}) - H(j\omega_{so})N(s-j\omega_{so})D(s+j\omega_{so})}{D(s+j\omega_{so})\cdot D(s-j\omega_{so})} \qquad (5)$$

and $H(j\omega)$ = the tank transfer function = $\dfrac{N(j\omega)}{D(j\omega)}$,

$H(j\omega_{so})$, $H(-j\omega_{so})$, and $\|H(j\omega_{so})\|$ depend only on the quiescent operating point ω_{so} and are not functions of s in Eq. (5) (i.e., these are treated as constants in this small-signal analysis!).

The Laplace transform variable s in Eq. (5) is taken with respect to the control input v_{in}. For example, if $v_{in}(t)$ is varied sinusoidally so that $v_{in}(t) = V_{in} + \hat{v}_{in}(t)$ with $\hat{v}_{in}(t) = \epsilon\cos(\omega_m t)$, then we can find the control-to-output-envelope sinusoidal steady-state response by letting $s = j\omega_m$ in Eq. (5). The magnitude and phase of $G_{env}(j\omega_m)$ describe the magnitude and phase of the envelope variations $\hat{a}(t)$ in Fig. 2, relative to the magnitude and phase of the control input perturbations $\hat{v}_{in}(t)$.

Another note:

The ω_m term in the denominator of Eq.(5) is similar to a pole at the origin. Indeed, if $s = j\omega_m$, then $G_{env}(s)$ contains a term of the form $1/(s/j)$. It can be shown that this term cancels similar numerator terms in Eq.(5), with the result that Eq.(5) represents a transfer function that follows all the usual rules for conventional linear time-invariant systems.

We would next like to derive equivalent circuit models for these converters, that describe the dc and small-signal ac transfer functions, including the effects of amplitude modulation and frequency modulation of the tank ac waveforms. We will address the following objectives and considerations:

- we want to make the sinusoidal approximation so that we can employ frequency-domain analysis
- we want to model how the tank circuit, through its frequency response, converts a frequency-modulated input signal into an amplitude- (plus frequency-) modulated output signal. We are interested in the amplitude (envelope) of the ac output.
- We want develop equivalent circuit models that model the physical processes in the converter, and that can be manipulated using conventional circuit analysis to gain insight.
- We want to implement the resulting models in simulation programs such as SPICE

We will address the above objectives using an extension of the conventional phasor analysis technique; this extension allows modelling of signals containing amplitude and phase modulation.

The modified phasor transformation

The objective of this transformation is to remove the switching-frequency (carrier-frequency) ω_s components of the tank waveforms, and thereby emphasize the relationships and dynamics of the waveform envelopes. The envelopes vary at the modulation frequency ω_m, and hence the complex phasor quantities of this analysis technique contain time variations that change with frequency ω_m.

Consistent with our previous analysis, we define the system phase angle as

$$\phi(t) = \int \omega_s(t)\, dt \qquad (6)$$

this definition is needed to accommodate frequency modulation of the ac carrier.

In the traditional sinusoidal-steady-state phasor analysis, a time-invariant but complex-valued quantity $\underline{x}$ is employed to represent the magnitude and phase of the sinusoid $x(t) = A\cos(\omega t + B)$. By defining the complex number $\underline{X}$ to have the same magnitude and phase as the sinusoid $x(t)$, we can develop a simplified analysis technique for ac analysis. Hence, $\underline{x} = Ae^{jB}$, and $x(t) = Re\left(\underline{x}\, e^{j\omega t}\right)$ where $Re(\cdot)$ is the real part of $(\cdot)$.

To model variations in the amplitude of the waveforms, we can let the magnitude of $\underline{x}$ vary with time: let $\underline{x}(t)$ vary, at the input modulation frequency ω_m.

To linearize the resulting model, we are going to need to perturb and linearize $\underline{x}(t)$. Hence, variations in $\underline{x}(t)$ need to be small. The problem is that frequency modulation causes the $e^{j\omega t}$ term ($e^{j\omega_s t}$?) to have a rotating component (the sidebands) whose variations are not small. We don't want this large variation to be absorbed into $\underline{x}(t)$, because then we wouldn't be able to perturb and linearize.

The above problem can be avoided if we define

$$x(t) = \mathrm{Re}\left(\underline{x}(t) \cdot e^{j\phi(t)}\right) \qquad (7)$$

with $\phi(t)$ given by Eq. (6). This equation is the definition of the modified phasor transformation. The $e^{j\phi(t)}$ term supplies the rotational, or underlying sinusoidal, component to the waveform. The complex phasor $\underline{x}(t)$ supplies the information regarding the amplitude modulation $\|\underline{x}(t)\|$ and phase modulation $\angle \underline{x}(t)$. If the amplitude and phase modulation are sufficiently small, then the small-signal assumption $\underline{x}(t) = \underline{X}_0 + \underline{\hat{x}}(t)$ is justified, with $\|\underline{X}_0\| \gg \|\underline{\hat{x}}(t)\|$.

Modified phasor transformation of an inductor

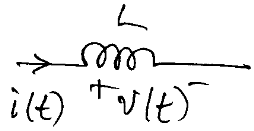


Fig. 3

For an inductor, we have the defining equation

$$v(t) = L\,\frac{d\,i(t)}{dt} \qquad (8)$$

Let's consider that the voltage $v(t)$ and current $i(t)$ are nearly sinusoidal, but contain amplitude and frequency modulation. Hence, let's represent these waveforms using modified phasors as follows:

$$\begin{aligned} v(t) &= \mathrm{Re}\left[\underline{v}(t)\, e^{j\phi(t)}\right] \\ i(t) &= \mathrm{Re}\left[\underline{i}(t)\, e^{j\phi(t)}\right] \end{aligned} \qquad (9)$$

with $\phi(t)$ as defined in Eq. (6). By plugging Eq. (9) into Eq. (8), we obtain

$$\mathrm{Re}\left[\underline{v}(t)\, e^{j\phi(t)}\right] = \mathrm{Re}\left[L\,\frac{d}{dt}\left\{\underline{i}(t)\, e^{j\phi(t)}\right\}\right] \qquad (10)$$

and hence

$$\underline{v}(t)\, e^{j\phi(t)} = L\,\frac{d\,\underline{i}(t)}{dt}\, e^{j\phi(t)} + L\,\underline{i}(t)\,\underbrace{\frac{d e^{j\phi(t)}}{dt}}_{=\,j\omega_s e^{j\phi(t)}} \qquad (11)$$

which implies that

$$\underline{v}(t) = L\frac{d\underline{i}(t)}{dt} + j\omega_s L\,\underline{i}(t) \qquad (12)$$

(it can be proven that the order of the real part and the derivative in the above equations can be interchanged). In Eq. (12), $\underline{v}(t)$ and $\underline{i}(t)$ are complex-valued "modified phasors" that represent the amplitude modulation and phase modulation of $v(t)$ and $i(t)$.

The last term in Eq. (12) includes the multiplication of the time-varying quantities $\omega_s(t)$ and $\underline{i}(t)$, which is a nonlinear operation. Hence construction of a linear model requires perturbation and linearization. Let's assume that the control input $v_{in}(t)$ can be written as

$$v_{in}(t) = V_{in} + \hat{v}_{in}(t) \qquad (13)$$
$$\text{with } |\hat{v}_{in}(t)| \ll |V_{in}|$$

In response, the frequency modulator varies the angular switching frequency according to

$$\omega_s(t) = \omega_{s0} + \hat{\omega}_s(t) \qquad (14)$$
$$\text{with } \omega_{s0} \gg |\hat{\omega}_s(t)|$$

In response to these switching frequency variations, there is amplitude and phase modulation in $v(t)$ and $i(t)$ that satisfy:

$$\underline{v}(t) = \underline{V} + \hat{\underline{v}}(t), \quad \|\hat{\underline{v}}(t)\| \ll \|\underline{V}\|$$
$$\underline{i}(t) = \underline{I} + \hat{\underline{i}}(t), \quad \|\hat{\underline{i}}(t)\| \ll \|\underline{I}\| \qquad (15)$$

Then Eq. (12) becomes

$$\underline{V} + \hat{\underline{v}}(t) = L\frac{d\underline{I}}{dt}^{\nearrow 0} + L\frac{d\hat{\underline{i}}(t)}{dt} + j(\omega_{s0} + \hat{\omega}_s(t))L(\underline{I} + \hat{\underline{i}}(t)) \qquad (16)$$

Upon elimination of the second-order nonlinear term containing $\hat{\omega}_s(t)\,\hat{\underline{i}}(t)$, we obtain;

$$\underline{V} = j\omega_{s0} L \underline{I} \qquad \text{(steady state equation)} \qquad (17)$$

and

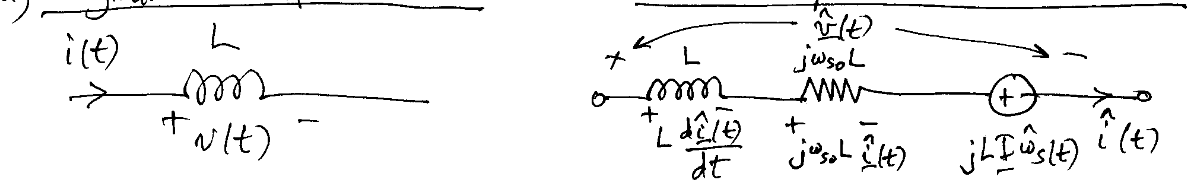
$$\hat{\underline{v}}(t) = L\frac{d\hat{\underline{i}}(t)}{dt} + j\omega_{s0} L\, \hat{\underline{i}}(t) + jL\underline{I}\,\hat{\omega}_s(t) \qquad (18)$$

Equation (17) is identical to the sinusoidal steady-state equation for the inductor under equilibrium conditions, i.e., with conventional phasor analysis. Equation (18) is the small-signal equation that shows how amplitude- and phase-modulation of $\hat{i}(t)$, and how frequency modulation of the control input (i.e., $\hat{\omega}_s(t)$) induces amplitude- and phase-modulation of the induced inductor voltage via $\hat{\underline{v}}(t)$.

We can now construct an equivalent circuit model for the inductor, as in Fig. 4.

Fig. 4

a) Original circuit element   b) Modified phasor transform



In Fig. 4(a), $v(t)$ and $i(t)$ are the original circuit waveforms, that are amplitude- and frequency-modulated sinusoids. In Fig. 4(b) the carrier has been removed from the waveforms. The complex-valued phasors $\hat{\underline{v}}(t)$ and $\hat{\underline{i}}(t)$ have magnitudes that follow the magnitudes of the sinusoids $v(t)$ and $i(t)$; i.e., the magnitudes of the phasors $\hat{\underline{v}}(t)$ and $\hat{\underline{i}}(t)$ model the amplitude modulation of $v(t)$ and $i(t)$. In addition, the phases of $\hat{\underline{v}}(t)$ and $\hat{\underline{i}}(t)$ follow the phase modulation of $v(t)$ and $i(t)$.

The inductor in Fig. 4(b) induces a term in $\hat{\underline{v}}(t)$ equal to $L\, d\hat{\underline{i}}(t)/dt$. The "resistor" in Fig. 4(b) induces a term in $\hat{\underline{v}}(t)$ that is proportional to $\hat{\underline{i}}(t)$, according to $j\omega_{so} L\, \hat{\underline{i}}(t)$. The independent source of Fig. 4(b) induces a voltage that depends on the control (frequency) modulation: $jL\underline{I}\,\hat{\omega}_s(t)$; this term directly models how variations in the switching frequency are converted into amplitude modulation by the inductor.

Modified phasor transformation of a capacitor

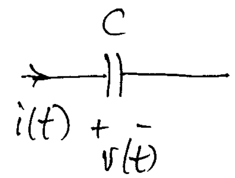


Fig. 5

For a capacitor, we have the defining relationship

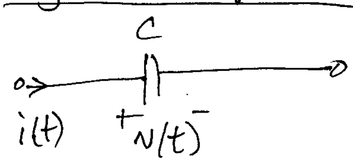
$$i(t) = C\,\frac{dv(t)}{dt} \qquad (19)$$

By analysis analogous to the previous pages, one obtains

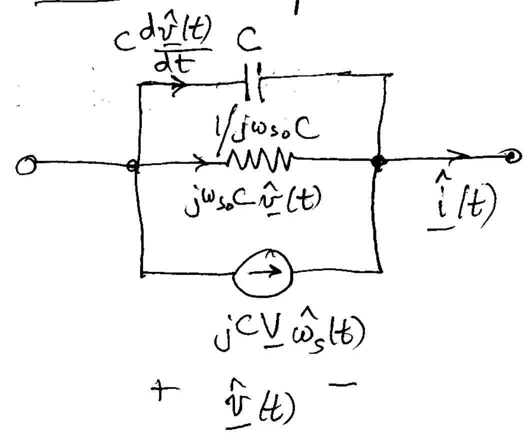
$$\underline{\hat{i}}(t) = C\,\frac{d\underline{\hat{v}}(t)}{dt} + j\omega_{s0} C\,\underline{\hat{v}}(t) + jC\underline{V}\,\hat{\omega}_s(t) \qquad (20)$$

which corresponds to the equivalent circuit model of Fig. 6(b).

(a) original circuit element



(b) modified phasor transformation



Modified phasor transformation for a resistor

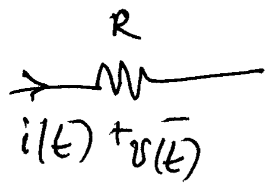


Fig. 7

For a resistor, we have

$$v(t) = i(t)\, R \qquad (21)$$

Upon substitution of Eq. (9) into Eq. (21) one obtains

$$\underline{v}(t) = \underline{i}(t)\ R \qquad (22)$$

Hence, the transformed circuit element for the resistor is a real-valued resistor as illustrated in Fig. 8.

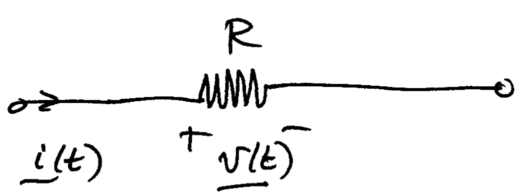


Fig. 8

Modified phasor transformation of the switch network

We have found in previous courses that the switch network can be modeled as an effective transformer, for PWM dc-dc converters. The transformer model represents the transformation of voltage and current levels, ideally with 100% efficiency.

Use of the modified phasor transformation allows the switch network to be modeled with transformers as well. Some accommodation for the fact that the switch network converts dc to ac must be made. This is possible because the steady-state ac phasors are constant quantities that represent the magnitudes and phases of the ac signals – effectively, phasor analysis allows use of dc circuit analysis techniques to solve sinusoidal steady-state circuits. Hence, the "dc transformer," having a complex (phasor), "turns ratio" is used to model the conversion of dc signals into ac sinusoidal signals.

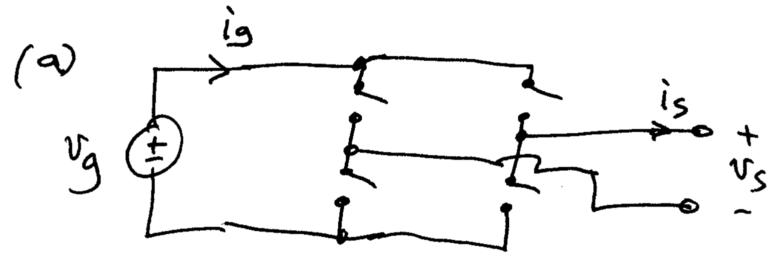
The derivation is based on the time-variable transformer (TVT) approach, in which the switch network is modelled by an effective transformer whose instantaneous turns ratio is defined as the ratio of the respective input and output quantities.

Since this ratio typically changes when the switches change their conducting states, the effective transformer has a time-varying turns ratio. There are no approximations so far, and this model represents the exact time-varying circuit equations.

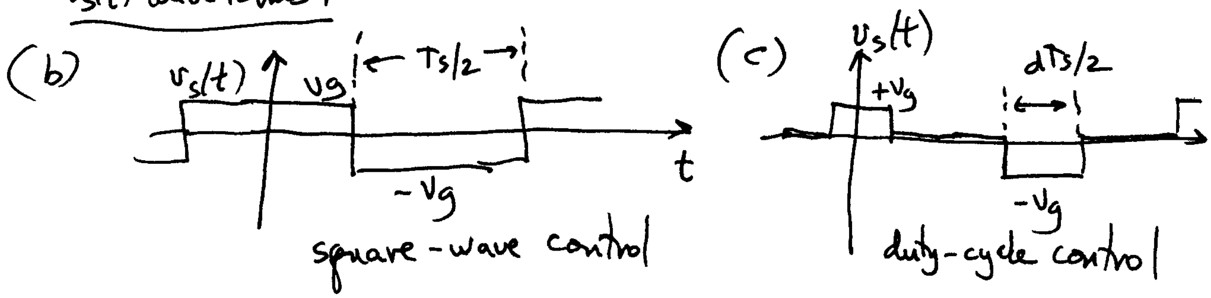
Next, we make the sinusoidal approximation, to replace the waveforms on the ac side of the transformer by their fundamental components. Switching ripples on the dc side are also averaged away. Finally, the ac-side waveforms are represented by phasors. The turns ratio then becomes the ratio of the ac-side phasor voltage to the dc-side average voltage. This turns ratio is, in general, complex-valued.

Full-bridge example

Fig. 9



$v_s(t)$ waveforms:

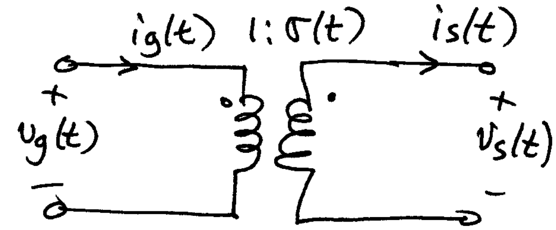


Let's define the time-varying turns ratio as

$$\sigma(t) = \frac{v_s(t)}{v_g(t)} = \frac{i_g(t)}{i_s(t)} \qquad (23)$$

which implies that the transformer is lossless: $v_g(t)\, i_g(t) = v_s(t)\, i_s(t)$.

Effective transformer
Fig. 10



For the circuit and waveforms of Fig. 9, the possible instantaneous values of $\sigma(t)$ are -1, 0, or $+1$, depending on the instantaneous switch states. For the waveform of Fig. 9(b), the waveform of $\sigma(t)$ is

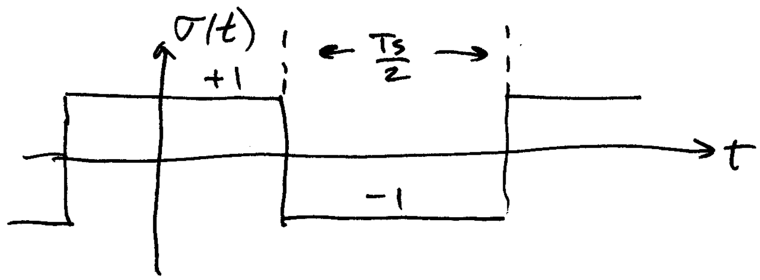


Fig. 11

with duty-cycle control, Fig. 9(c), the waveform is

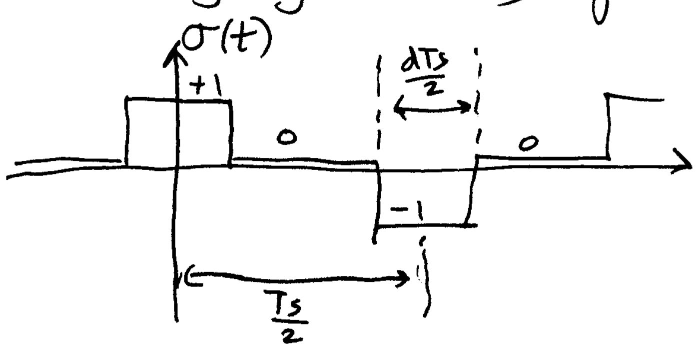


Fig. 12

The next step is to approximate the ac-side waveforms, and hence also $\sigma(t)$, by their fundamental components.

As previously described, the fundamental component of $v_s(t)$ is $v_{s1}(t) = \frac{4}{\pi} V_g \cos(\omega_s t)$. By similar analysis, the fundamental component of the square-wave $\sigma(t)$ waveform of Fig. 11 is

$$\sigma_1(t) = \frac{4}{\pi} \cos(\omega_s t) \tag{24}$$

For the case of the duty-cycle controlled waveform of Fig. 12, one obtains

$$\sigma_1(t) = \frac{4}{\pi} \sin\left(\frac{\pi d}{2}\right) \cos(\omega_s t) \tag{25}$$

with $0 \le d \le 1$; $d(t)$ is a control input that allows amplitude modulation of the switch voltage in full-bridge switch networks. Multiplication of Eq. (24) or (25) by the source voltage V_g leads to the fundamental component of the switch output voltage, $v_{s1}(t)$.

We next express $\sigma_1(t)$ as a phasor, according to

$$\sigma_1(t) = \mathrm{Re}\left(\underline{\sigma}\, e^{j\phi(t)}\right) \tag{26}$$

with $\phi(t) = \int \omega_s(t)\, dt$ for frequency modulation,
$\phi(t) = \omega_s t$ for constant-frequency operation

[$\underline{\sigma} = \frac{4}{\pi}$ for square-wave FM, or $\underline{\sigma} = \frac{4}{\pi}\sin\left(\frac{\pi d}{2}\right)$ for duty-cycle control.
** Note! see next page – it's better to use rms and define $\underline{\sigma}$ as $\frac{1}{\sqrt{2}}$ smaller.

In the phasor-transformed inverter model, dc-side waveforms are modeled by their averaged (low-frequency) components, as in PWM dc-dc averaged models. AC-side waveforms are modeled by phasors, as described on the previous pages.

The switch network is the interface between the dc and ac portions of the inverter. With the phasor-transformed modeling approach, the function of the switch network is represented by an effective transformer whose turns ratio is given by the ratio of the dc-side averaged voltage to the ac-side phasor voltage (or vice-versa, depending on the direction in which the turns ratio is defined).

In the example of the previous pages, the effective turns ratio is $1 : \underline{\sigma}$

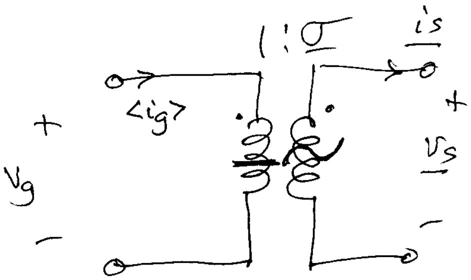


Fig. 13
The phasor-transformed switch network model

This model is intended to relate the dc-side and ac-side waveforms with the usual transformer equations as nearly as possible. It has the strange property that the turns ratio is a phasor. Can we make this work? We need to check: (i) the "voltage" ratio, (ii) the "current" ratio, (iii) conservation of average power. (It works to do this in some basic cases, but nobody to date has proven how to do this in general!)

(i) the "voltage" ratio

our model uses average voltage on the dc side and phasor-transformed voltage on the ac side; (31)

$v_{dc}(t) = v_g(t)$ $\qquad$ $v_{ac}(t)$ is represented by $\underline{v_s}(t)$

in steady state or large signals;

$v_{dc}(t) = v_g(t)$ $\qquad$ $\underline{v_s}(t) = \underline{\sigma}\, v_g(t)$ $\qquad$ (32)

if the ac voltage is represented by a phasor using the peak ac value, then $\underline{\sigma} = \frac{4}{\pi}$. If we use rms instead, then $\underline{\sigma} = \frac{2\sqrt{2}}{\pi}$, we will see in (iii) that rms works better.

(ii) the "current" ratio

we have been assuming that $i_{s1}(t) = I_{s1}\cos(\omega_s t + \varphi_s)$ and $\langle |i_g(t)| \rangle = \frac{2}{\pi} I_{s1}\cos(\varphi_s)$. If we use peak-amplitude phasors, then

$$\langle |i_g| \rangle = \frac{1}{2}\,\mathrm{Re}\left(\underline{\sigma}^* \underline{i_s}\right) \quad \text{with } \underline{\sigma} = \frac{4}{\pi} \text{ and } \underline{i_s} = I_{s1} e^{j\varphi_s} \qquad (33)$$

If instead we use rms-amplitude phasors, then

$$\langle |i_g| \rangle = \mathrm{Re}\left(\underline{\sigma}^* \underline{i_s}\right) \quad \text{with } \underline{\sigma} = \frac{2\sqrt{2}}{\pi}$$

$$\text{and } \underline{i_s} = \frac{I_{s1}}{\sqrt{2}} e^{j\varphi_s} \qquad (34)$$

(iii) the "Power" ratio – conservation of average power

$$\langle P_{in} \rangle = V_g \langle |i_g| \rangle \quad \text{and} \quad \langle P_{out} \rangle = \mathrm{Re}(\underline{v_s}\, \underline{i_s}^*) \qquad (27)$$

with the steady-state expressions of the previous page, we have

$$\langle P_{in} \rangle = V_g \, \mathrm{Re}(\underline{\sigma}^* \underline{i_s}) \qquad \text{using rms amplitudes}$$

$$\langle P_{out} \rangle = \mathrm{Re}(\underline{v_s}\, \underline{i_s}^*) = \mathrm{Re}(\underline{\sigma} V_g \underline{i_s}^*) \qquad (28)$$

$$= V_g \, \mathrm{Re}(\underline{\sigma}\, \underline{i_s}^*) = V_g \mathrm{Re}(\underline{\sigma}^* \underline{i_s}) = \langle P_{in} \rangle$$

note $\mathrm{Re}(z) = \mathrm{Re}(z^*)$ since taking the conjugate doesn't change the real part.

so $\langle P_{in} \rangle = \langle P_{out} \rangle$.

The bottom line for the transformer model of the switch network

- use rms amplitudes for ac-side waveform phasors.
- the basic equations are

$$\underline{v_{out}} = \underline{\sigma}\, v_{in} \qquad (29)$$

$$i_{in} = \mathrm{Re}(\underline{\sigma}^* \underline{i_{out}})$$

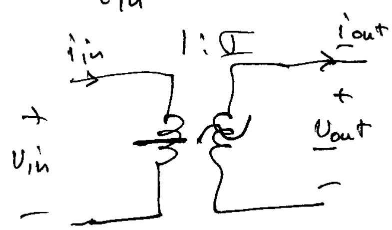


Fig. 14

series or parallel resonant inverter example: $\underline{\sigma} = \frac{2\sqrt{2}}{\pi}$ (square wave inverter)

or $\underline{\sigma} = \frac{2\sqrt{2}}{\pi} \sin\left(\frac{\pi d}{2}\right)$ (duty-cycle control)

(30)

Parallel resonant inverter example

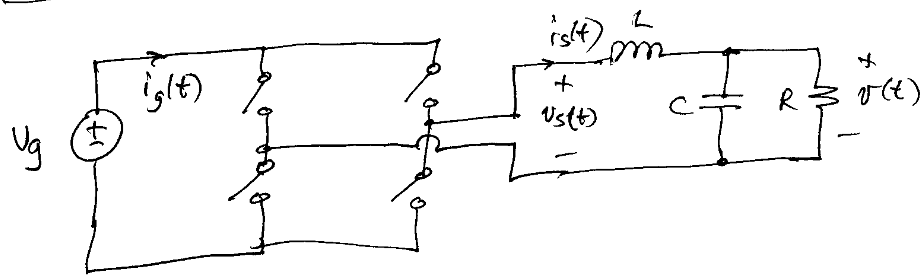


Fig. 15

model

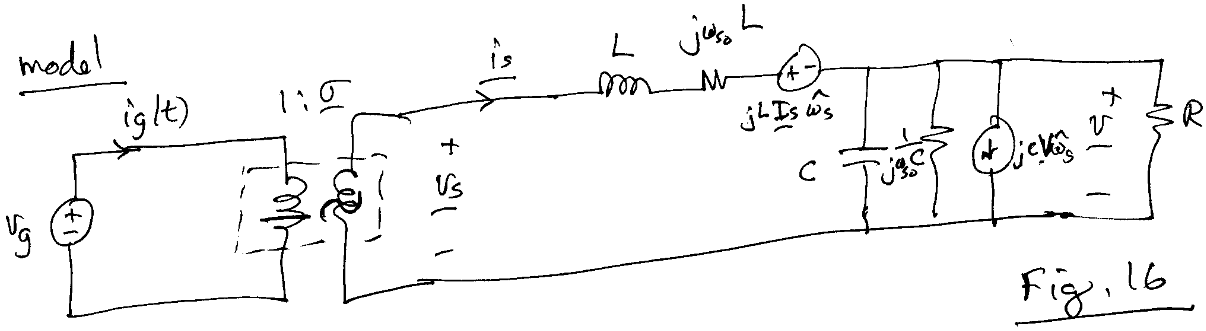


Fig. 16

steady-state solution

set $L \to$ short, $C \to$ open, $\hat{\omega}_s \to 0$

circuit reduces to

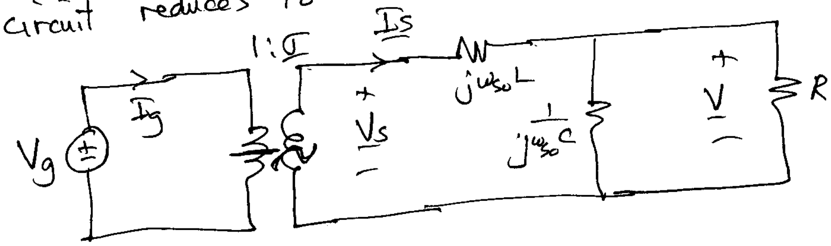


Fig. 17

The network equations: (35)

$$\underline{V_s} = \sigma V_g = \frac{2\sqrt{2}}{\pi} V_g$$

$$\underline{V} = \frac{R \| \frac{1}{j\omega_{s0}C}}{j\omega_{s0}L + R \| \frac{1}{j\omega_{s0}C}} \underline{V_s} = \frac{2\sqrt{2}\, V_g}{\pi} \frac{1}{1 + \frac{j\omega_{s0}L}{R} - \omega_{s0}^2 LC}$$

and $$\|\underline{V}\| = \frac{2\sqrt{2}\, V_g}{\pi} \frac{1}{\sqrt{(1-\omega_{s0}^2 LC)^2 + \left(\frac{\omega_{s0}L}{R}\right)^2}}$$

which agrees with previous steady-state analysis

also $I_g = \mathrm{Re}(\sigma^* \underline{I_s})$

Or, transforming an impedance through effective transformer:

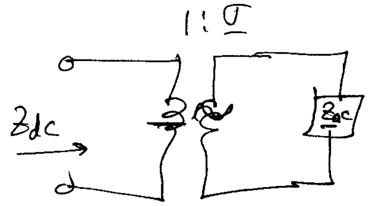


Fig. 18

$$Z_{dc} = \frac{Re(\underline{Z}_{ac})}{\underline{\sigma}\,\underline{\sigma}^*} \qquad (36)$$

so
$$I_g = \frac{V_g}{Z_{dc}} = \frac{\underline{\sigma}\,\underline{\sigma}^* V_g}{Re(\underline{Z}_{ac})} = \left(\frac{2\sqrt{2}}{\pi}\right)^2 \frac{V_g}{Re\left(j\omega_{s0}L + R \| \frac{1}{j\omega_{s0}C}\right)}$$

Small-signal ac model

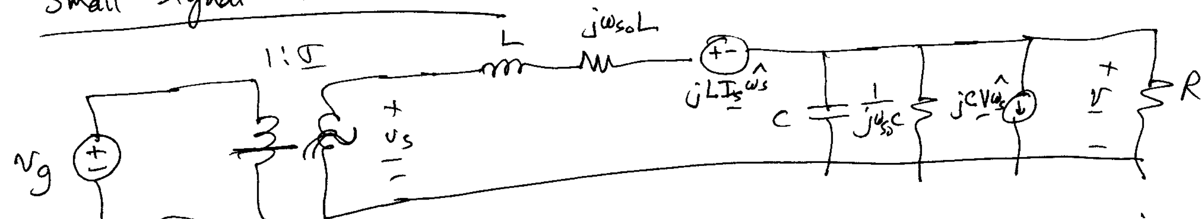


Fig. 19

$$\underline{v}_s = \underline{\sigma}\, v_g = \frac{2\sqrt{2}}{\pi} v_g$$

(could also perturb and linearize v_g: $v_g(t) = V_g + \hat{v}_g$) (37)

we get

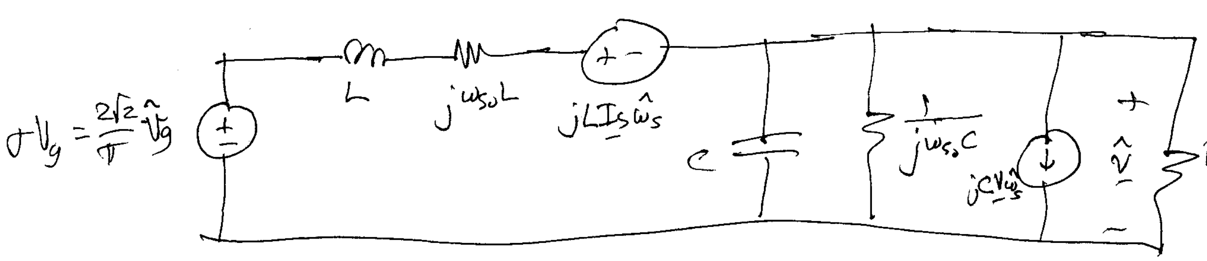


Fig. 20

Three sources: $\frac{2\sqrt{2}}{\pi} v_g$, $j\hat{\omega}_s L\underline{I}_s$, $jC\underline{V}\hat{\omega}_s$. To find $\frac{\hat{v}}{\hat{v}_g}$: set $\hat{\omega}_s = 0$.

To find $G_{vv} = \frac{\hat{v}}{\hat{\omega}_s}$. Set $\hat{v}_g = 0$.

Finding $G_{env}(s)$:

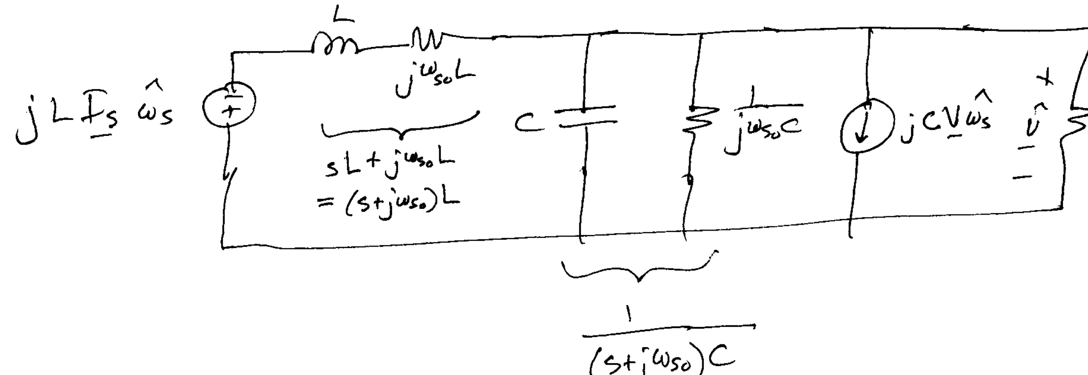


Fig. 21

$$\hat{\underline{v}} = (-jL\underline{I}_s\hat{\omega}_s)\cdot \frac{R \,\|\, \frac{1}{(s+j\omega_{so})C}}{(s+j\omega_{so})L + R \,\|\, \frac{1}{(s+j\omega_{so})C}} - (jC\underline{V}\hat{\omega}_s)\cdot R \,\|\, \frac{1}{(s+j\omega_{so})C} \,\|\, (s+j\omega_{so})L \tag{38}$$

To recover the variation in the envelope of $\underline{v}(t)$ we need to compute the magnitude of $\underline{v}(t) = \underline{V} + \hat{\underline{v}}(t)$.

We have

$$\|\underline{v}\| = \|\underline{V} + \hat{\underline{v}}\| = \sqrt{(\underline{V}+\hat{\underline{v}})(\underline{V}^* + \hat{\underline{v}}^*)} \tag{39}$$

$$= \sqrt{\underline{V}\underline{V}^* + \underline{V}^*\hat{\underline{v}} + \underline{V}\hat{\underline{v}}^* + \hat{\underline{v}}\hat{\underline{v}}^*}$$

$$= \|\underline{V}\| \cdot \sqrt{1 + \frac{\underline{V}^*\hat{\underline{v}} + \underline{V}\hat{\underline{v}}^* + \hat{\underline{v}}\hat{\underline{v}}^*}{\underline{V}\underline{V}^*}}$$

Approximate the square function (linearize):

$$\|\underline{V}\|\sqrt{1 + \frac{b(t)}{\|\underline{V}\|^2}} = \|\underline{V}\|\cdot\left(1 + \frac{1}{2}\frac{b(t)}{\|\underline{V}\|^2} + \cdots\right) \tag{40}$$

$$\approx \|\underline{V}\| + \frac{1}{2}\frac{b(t)}{\|\underline{V}\|} + \cdots \quad \text{for } \|b(t)\| \ll \|\underline{V}\|$$

(see p.18, Eq.(34) of packet 1)

So we get

$$\|\underline{v}\| \cong \underbrace{\|\underline{V}\|}_{\text{steady-state component}} + \underbrace{\frac{\underline{V}\,\hat{\underline{v}}^{*} + \underline{V}^{*}\hat{\underline{v}} + \overset{\text{small - 2nd order}}{\cancel{\hat{\underline{v}}\,\hat{\underline{v}}^{*}}}}{2\|\underline{V}\|}}_{\text{small-signal ac component}} + \cancel{\text{higher order terms}} \qquad (41)$$

The small-signal variation in the output voltage envelope is given by

$$\hat{v}_{env} = \frac{\underline{V}\,\hat{\underline{v}}^{*} + \underline{V}^{*}\hat{\underline{v}}}{2\|\underline{V}\|} \qquad (42)$$

Compare with Eqs. (35) and (36) of packet 1 — this is the basic result for the phasor transform method.

SPICE simulation

Evaluation of these transfer functions gets pretty tedious. MATLAB or a similar computer package can help. Another useful approach is to implement the above phasor equivalent circuit models in SPICE.

SPICE requires that the element values be real, so we need to do some work to adapt the above models. The approach is to simulate the real and imaginary components of the phasors separately, using separate real-valued circuits that can be implemented in SPICE. We let the phasor voltages and currents such as $\hat{\underline{v}}$ and $\hat{\underline{i}}$ be decomposed into scalar real and imaginary parts:

$$\hat{\underline{v}} = \hat{v}_1 + j\,\hat{v}_2 \qquad (43)$$
$$\hat{\underline{i}} = \hat{i}_1 + j\,\hat{i}_2$$

etc.

The inductor model, Eq. (18), becomes

$$\hat{v}_1 + j\,\hat{v}_2 = L\,\frac{d(\hat{i}_1 + j\hat{i}_2)}{dt} + j\omega_{so}L(\hat{i}_1 + j\hat{i}_2) + jL(I_1 + jI_2)\hat{\omega}_s \qquad (44)$$

We can write the real and imaginary parts of Eq. (44) as two separate equations:

$$\hat{v}_1 = L\,\frac{d\hat{i}_1}{dt} - \omega_{so}L\,\hat{i}_2 - L I_2\,\hat{\omega}_s \quad \text{(real part)}$$
$$\hat{v}_2 = L\,\frac{d\hat{i}_2}{dt} + \omega_{so}L\,\hat{i}_1 + L I_1\,\hat{\omega}_s \quad \text{(imaginary part)} \qquad (45)$$

We now construct two equivalent circuits (real and imaginary components of the signals) whose voltages and currents are scalars that can be simulated in SPICE.

real components

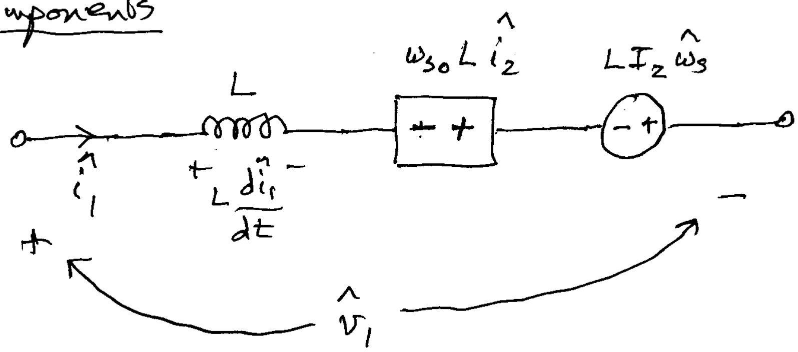
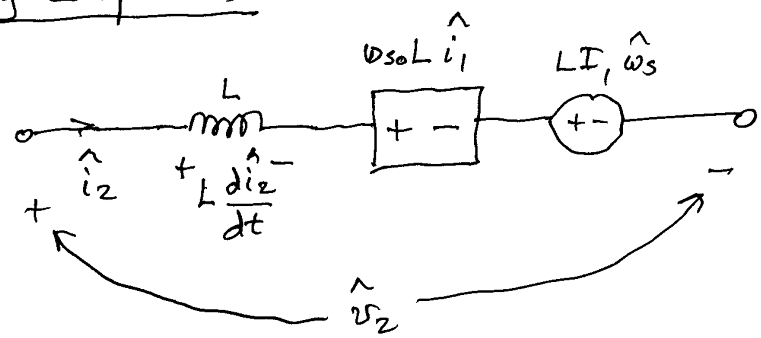


Fig. 22

imaginary components



We can treat the capacitor model, Eq. (20), in a similar manner:

$$\hat{i}_1 + j\hat{i}_2 = C\frac{d(\hat{v}_1 + j\hat{v}_2)}{dt} + j\omega_{so}C(\hat{v}_1 + j\hat{v}_2) + jC(V_1 + jV_2)\hat{\omega}_s \tag{46}$$

which can be written as two separate equations:

real parts

$$\hat{i}_1 = C\frac{d\hat{v}_1}{dt} - \omega_{so}C\hat{v}_2 - CV_2\hat{\omega}_s$$

imaginary parts (47)

$$\hat{i}_2 = C\frac{d\hat{v}_2}{dt} + \omega_{so}C\hat{v}_1 + CV_1\hat{\omega}_s$$

These equations lead to the equivalent circuits of Fig. 23.

real components

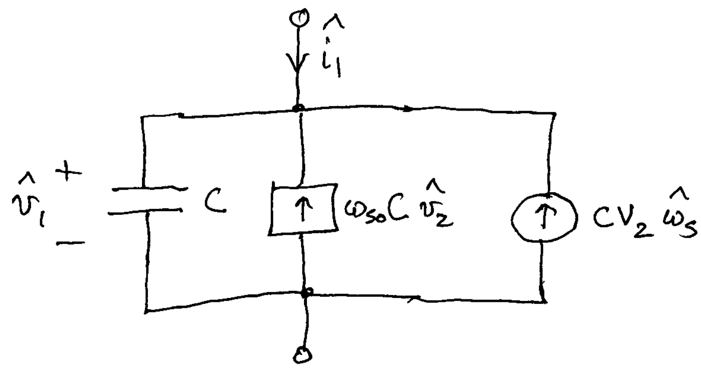
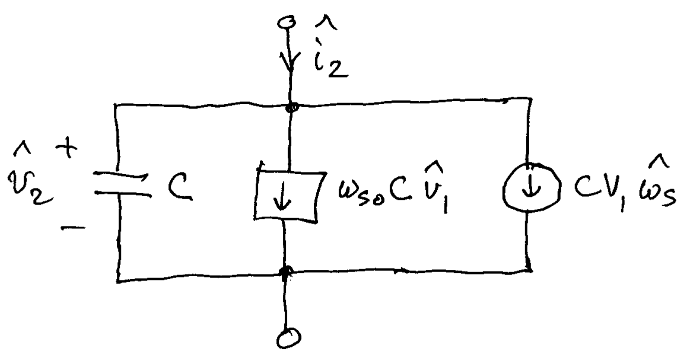


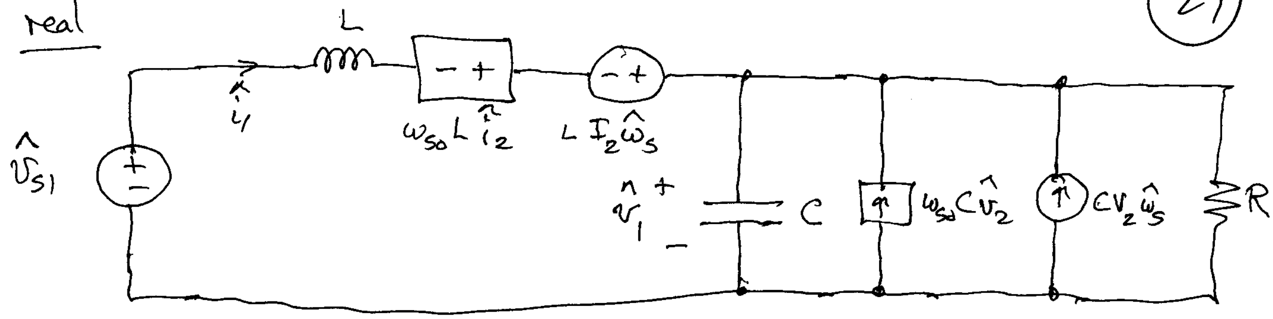
Fig. 23

imaginary components



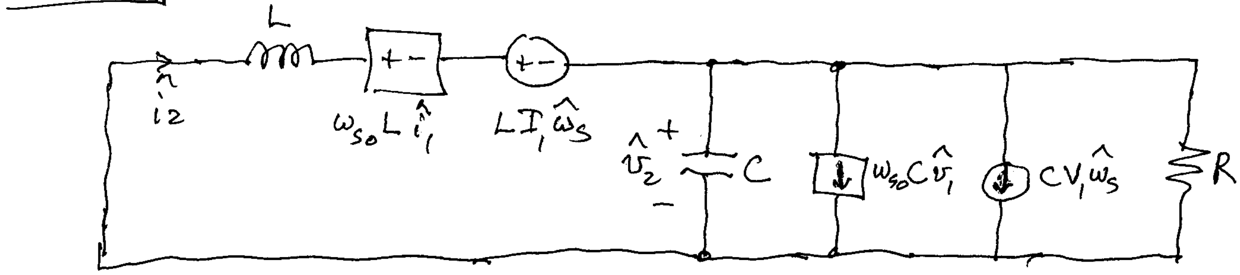
A SPICE-compatible model for the parallel-resonant inverter of Fig. 15 is illustrated in Fig. 24. The switch output voltage is taken, as usual, as the zero phase reference of the system; hence $\hat{v}_{s1} = \frac{2\sqrt{2}}{\pi} \hat{v}_g$ and $\hat{v}_{s2} = 0$ for square-wave frequency modulation.

real



imaginary

Fig. 24



To get SPICE to compute the envelope of the output voltage, we need to implement Eq. (42). If we let the output voltage phasor $\underline{v} = \underline{V} + \hat{\underline{v}}$ be equal to real and imaginary components

$$\underline{V} = V_1 + j V_2 \qquad (48)$$
$$\hat{\underline{v}} = \hat{v}_1 + j \hat{v}_2$$

(actually, we have already done this for the capacitor voltage in Fig. 24) then Eq. (42) becomes

$$\hat{v}_{env} = \frac{(V_1 + jV_2)(\hat{v}_1 - j\hat{v}_2) + (V_1 - jV_2)(\hat{v}_1 + j\hat{v}_2)}{2\sqrt{V_1^2 + V_2^2}} \qquad (49)$$

which can be simplified to

$$\hat{v}_{env} = \frac{V_1 \hat{v}_1 + V_2 \hat{v}_2}{\sqrt{V_1^2 + V_2^2}} \qquad (50)$$

A block diagram implementing Eq. (50) is shown below:

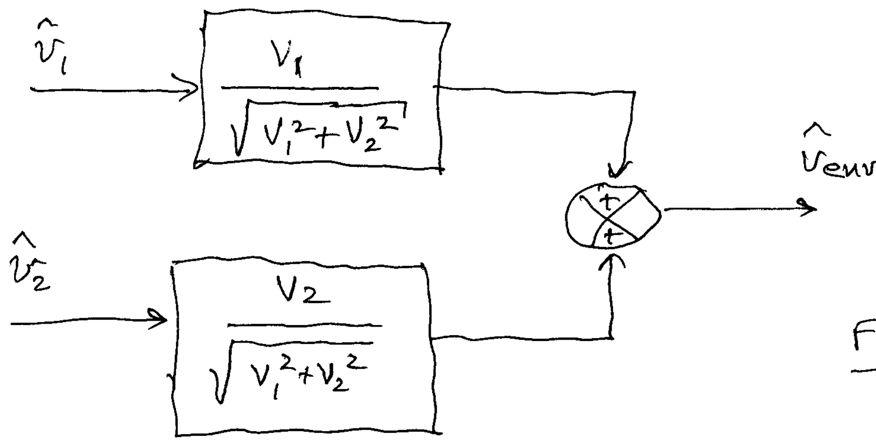


Fig. 25

When we append Fig. 25 to the circuits of Fig. 24, we obtain a model suitable for SPICE simulation that allows the small-signal transfer functions to be plotted.