

# Parsing & Error Recovery

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COS 320

# Error Recovery

- What should happen when your parser finds an error in the user's input?
  - stop immediately and signal an error
  - record the error but try to continue
- In the first case, the user must recompile from scratch after possibly a trivial fix
- In the second case, the user might be overwhelmed by a whole series of error messages, all caused by essentially the same problem
- We will talk about how to do error recovery in a principled way

# Error Recovery

- **Error recovery:**
  - process of adjusting input stream so that the parser can continue after unexpected input
- **Possible adjustments:**
  - delete tokens
  - insert tokens
  - substitute tokens
- **Classes of recovery:**
  - **local recovery:** adjust input at the point where error was detected (and also possibly immediately after)
  - **global recovery:** adjust input before point where error was detected.
- Error recovery is possible in both top-down and bottom-up parsers

# Local Bottom-up Error Recovery

exp : NUM	()	exps : exp	()
exp PLUS exp	()	exps ; exp	()
LPAR exp RPAR	()		

- general strategy for both bottom-up and top-down:
  - look for a **synchronizing token**

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LPAR exp RPAR	()		

- general strategy for both bottom-up and top-down:
  - look for a **synchronizing token**
- accomplished in bottom-up parsers by adding error rules to grammar:

exp : LPAR **error** RPAR   ()

exps : exp                   ()  
      | **error** ; exp       ()

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- general strategy for both bottom-up and top-down:
  - look for a **synchronizing token**
- accomplished in bottom-up parsers by adding error rules to grammar:

exp : LPAR **error** RPAR    ()

exps : exp                    ()  
      | **error** ; exp        ()

- in general, follow error with a synchronizing token. Recovery steps:
  - **Pop stack** (if necessary) until a state is reached in which the action for the error token is shift
  - **Shift** the error token
  - **Discard input** symbols (if necessary) until a state is reached that has a non-error action
  - **Resume** normal parsing

# Local Bottom-up Error Recovery

exp : NUM                   ()  
     | exp PLUS exp       ()  
     | ( exp )            ()

exps : exp                   ()  
      | exps ; exp           ()

exp : ( error )            ()

exps : exp                   ()  
      | error ; exp           ()

input:       NUM PLUS ( NUM PLUS @#\$ PLUS NUM ) PLUS NUM  
stack:       exp PLUS ( exp PLUS

yet to read

@#\$ is an unexpected token!

# Local Bottom-up Error Recovery

```
exp : NUM          ()
     | exp PLUS exp ()
     | ( exp )      ()
```

$$\begin{array}{l} \text{exprs} : \text{exp} \quad () \\ \quad | \text{exprs} ; \text{exp} \quad () \end{array}$$

exp : ( error ) ( )

```
exps : exp      ()
      | error ; exp  ()
```

input: NUM PLUS ( NUM PLUS @\$ PLUS NUM ) PLUS NUM

stack:      exp PLUS (

pop stack until shifting “error” can result in correct parse



# Local Bottom-up Error Recovery

```
exp : NUM          ()
     | exp PLUS exp ()
     | ( exp )      ()
```

$$\begin{array}{l} \text{exps : exp} \quad () \\ \quad | \text{exps ; exp} \quad () \end{array}$$
$$\text{exp} : ( \text{error} ) \quad ()$$

```

exps : exp      ()
    | error ; exp  ()

```

input: NUM PLUS ( NUM PLUS @\$ PLUS NUM ) PLUS NUM

stack:      exp PLUS ( error

## shift “error”

# Local Bottom-up Error Recovery

exp : NUM                   ()  
     | exp PLUS exp       ()  
     | ( exp )            ()

exps : exp                   ()  
      | exps ; exp         ()

exp : ( error )            ()

exps : exp                   ()  
      | error ; exp         ()

input:       NUM PLUS ( NUM PLUS @\$ PLUS NUM ) PLUS NUM

yet to read

stack:       exp PLUS ( error

discard input until we can legally  
shift or reduce

# Local Bottom-up Error Recovery

```
exp : NUM          ()
     | exp PLUS exp ()
     | ( exp )      ()
```

$$\begin{array}{l} \text{exps : exp} \quad () \\ \quad | \text{exps ; exp} \quad () \end{array}$$

exp : ( error ) ( )

```
exps : exp      ()
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```

input: NUM PLUS ( NUM PLUS @\$ PLUS NUM ) PLUS NUM

yet to read

stack:      exp PLUS ( error )

SHIFT )

# Local Bottom-up Error Recovery

exp : NUM                   ()  
     | exp PLUS exp       ()  
     | ( exp )            ()

exps : exp                   ()  
      | exps ; exp           ()

exp : ( error )            ()

exps : exp                   ()  
      | error ; exp           ()

input:           NUM PLUS ( NUM PLUS @\$ PLUS NUM ) PLUS NUM

yet to read

stack:           exp PLUS exp

REDUCE using exp ::= ( error )

# Local Bottom-up Error Recovery

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exp : NUM          ()
     | exp PLUS exp ()
     | ( exp )      ()
```

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$$\text{exp} : ( \text{error} ) \quad ()$$

```
exps : exp      ()
    | error ; exp  ()
```

input: NUM PLUS ( NUM PLUS @\$ PLUS NUM ) PLUS NUM

stack:      exp PLUS exp

continue parsing...

# Top-down Local Error Recovery

- also possible to use synchronizing tokens
- here, a synchronizing token for non terminal  $X$  is a member of  $\text{Follow}(X)$ 
  - when parsing  $X$  and an error is found; eat input stream until you get to a member of  $\text{Follow}(X)$

non-terminals: S, E, L

terminals: NUM, IF, THEN, ELSE, BEGIN, END, PRINT, ,, =

rules:

1. S ::= IF E THEN S ELSE S

2.     | BEGIN S L

3.     | PRINT E

4. L ::= END

5.     | ; S L

6. E ::= NUM = NUM

```
val tok = ref (getToken ())
fun advance () = tok := getToken ()
fun eat t = if (! tok = t) then advance () else error ()
```

```
fun skipto toks =
  if member(!tok, toks) then ()
  else
    eat(!tok); skipto toks
```

```
fun S () = case !tok of
  IF => ... | BEGIN => ... | PRINT => ...
```

```
and L () = case !tok of
  END   => eat END
  | SEMI => eat SEMI; S (); L ()
```

```
and E () = case !tok of
  NUM => eat NUM; eat EQ; eat NUM
```

non-terminals: S, E, L

terminals: NUM, IF, THEN, ELSE, BEGIN, END, PRINT, ,, =

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1. S ::= IF E THEN S ELSE S

2.     | BEGIN S L

3.     | PRINT E

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    eat(!tok); skipto toks
```

```
fun S () = case !tok of
  IF => ... | BEGIN => ... | PRINT => ...
  | _ => skipto [ELSE,END,SEMI]
```

```
and L () = case !tok of
  END   => eat END
  | SEMI => eat SEMI; S (); L ()
  | _    =>
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  else
    eat(!tok); skipto toks
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  IF => ... | BEGIN => ... | PRINT => ...
  | _ => skipto [ELSE,END,SEMI]
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```
and L () = case !tok of
  END   => eat END
  | SEMI => eat SEMI; S (); L ()
  | _    => skipto [ELSE, END,SEMI]
```

```
and E () = case !tok of
  NUM => eat NUM; eat EQ; eat NUM
  | _  => skipto [THEN,ELSE,END,SEMI]
```

# global error recovery

- **global error recovery** determines the smallest set of insertions, deletions or replacements that will allow a correct parse, even if those insertions, etc. occur before an error would have been detected
- ML-Yacc uses **Burke-Fisher error repair**
  - tries **every possible single-token insertion**, deletion or replacement at every point in the input up to K tokens before the error is detected
    - eg: K = 20; parser gets stuck at token 500; all possible repairs between token 480-500 tried
    - best repair = longest successful parse

# global error recovery

- Consider Burke-Fisher with
  - K-token window
  - N different token types
- Total number of repairs =  $K + 2K*N$ 
  - deletions (K) +
  - insertions  $(K + 1)*N$  +
  - replacements  $(K)(N-1)$
- Affordable in the uncommon case when there is an error

# global error recovery

- Parser must be able to back up K tokens and reparse
- Mechanics:
  - parser maintains old stack and new stack

K-token window  
maintained in queue  
by parser

input: ID := NUM ; ID := ID + ( ID := NUM + ...

new stack: S ; ID := E + (

old stack: ID := NUM

# global error recovery

- Parser must be able to back up K tokens and reparse
- Mechanics:
  - parser maintains old stack and new stack

K-token window  
maintained in queue  
by parser

input: ID := NUM ; ID := ID + ( ID := NUM + ...

*K-token window* (under ID := ID + ( )  
*yet to read* (under ID := NUM + ...)

new stack: S ; ID := E + (

old stack: ID := NUM

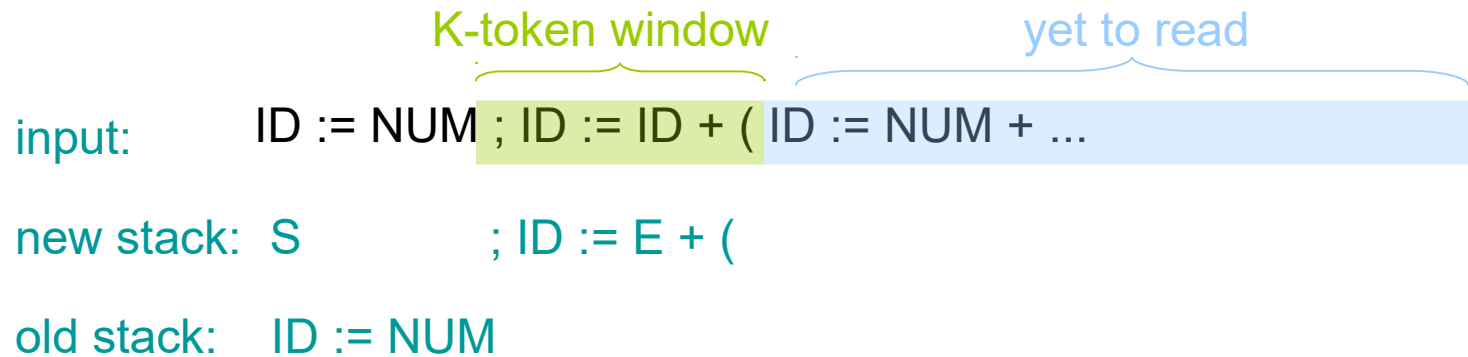
old stack lags the new stack by K=6 tokens

Reductions ( $E ::= \text{NUM}$ ) and ( $S ::= \text{ID} := \text{NUM}$ ) applied to old stack in turn

# global error recovery

- Parser must be able to back up K tokens and reparse
- Mechanics:
  - parser maintains old stack and new stack

K-token window  
maintained in queue  
by parser



semantic actions performed once when reduction is “committed” on the old stack

# Burke-Fisher in ML-Yacc

- ML-Yacc provides additional support for Burke-Fisher
  - to continue parsing, we need semantics values for inserted tokens
  - some multiple-token insertions & deletions are common, but it is too expensive for ML-Yacc to try every 2,3,4- token insertion, deletion

```
%value ID {make_id "bogus"}  
%value INT {0}  
%value STRING {""}
```

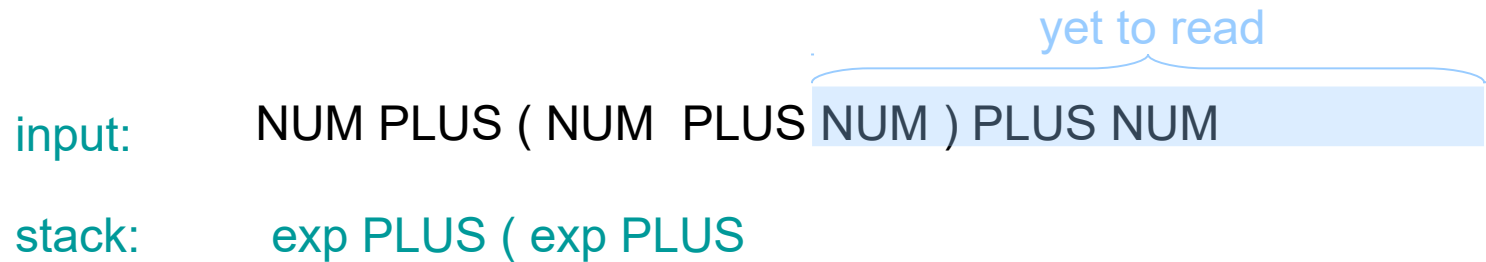
```
%change EQ -> ASSIGN  
| SEMI ELSE -> ELSE  
| -> IN INT END
```

ML-Yacc  
would do this  
anyway but by  
specifying,  
it tries it first



# finally the magic: how to construct an LR parser table

input:        NUM PLUS ( NUM PLUS NUM ) PLUS NUM  
stack:        exp PLUS ( exp PLUS



- At every point in the parse, the LR parser table tells us what to do next
  - shift, reduce, error or accept
- To do so, the LR parser keeps track of the parse “state” ==> a state in a finite automaton

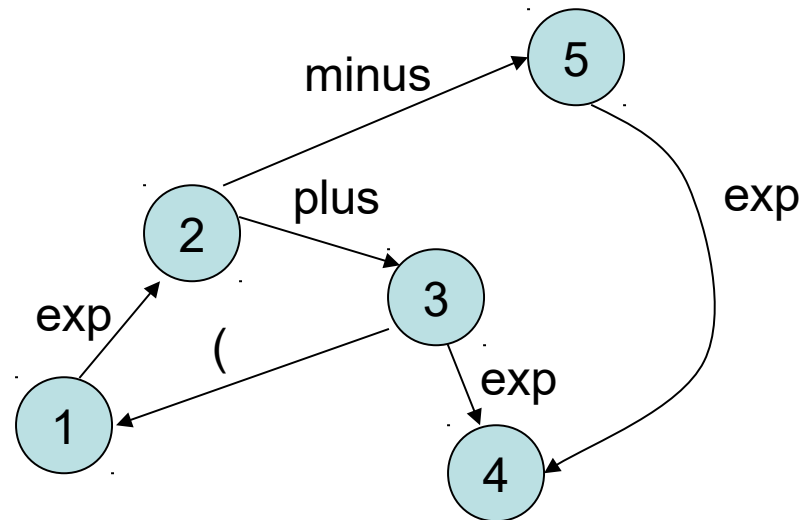
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input: NUM PLUS ( NUM PLUS NUM ) PLUS NUM

stack: exp PLUS ( exp PLUS

yet to read

finite automaton;  
terminals and  
non terminals  
label edges



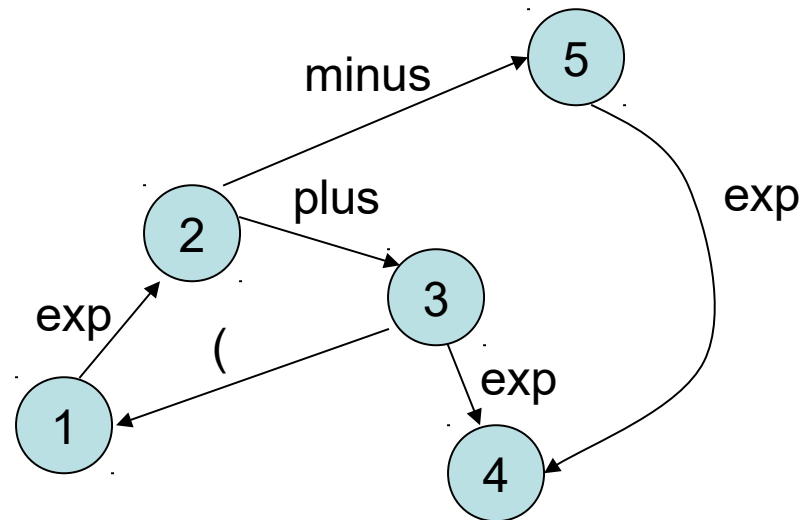
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state-annotated stack: 1

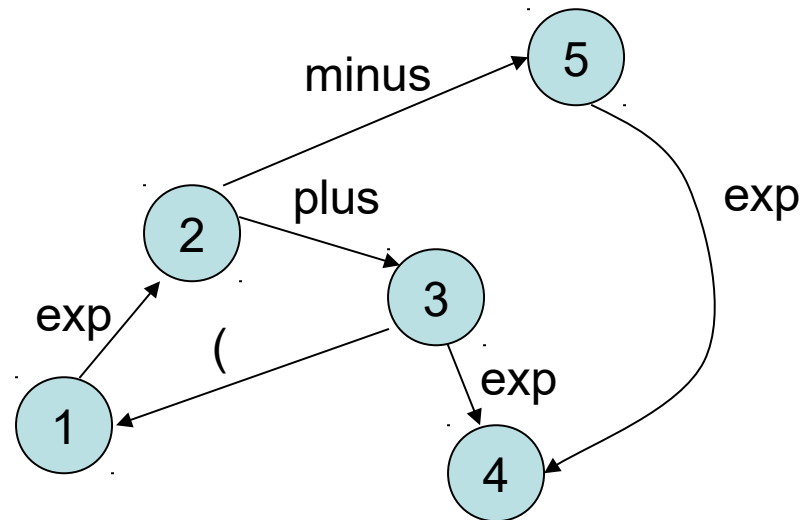
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state-annotated stack: 1 exp 2

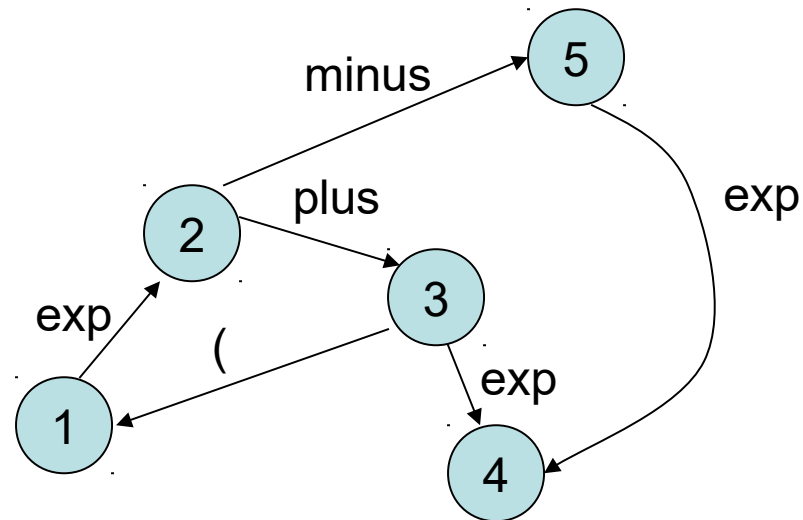
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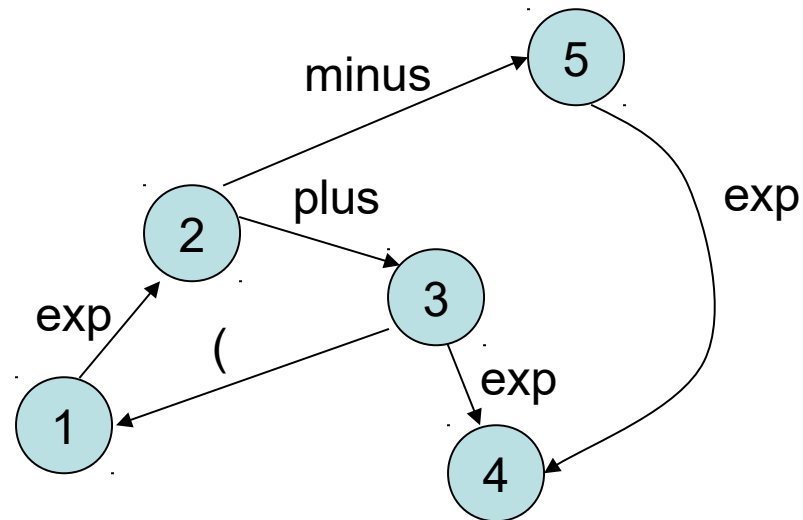
state-annotated stack: 1 exp 2 PLUS 3

# finally the magic: how to construct an LR parser table

input: NUM PLUS ( NUM PLUS NUM ) PLUS NUM  
stack: exp PLUS ( exp PLUS

yet to read

finite automaton;  
terminals and  
non terminals  
label edges



state-annotated stack: 1 exp 2 PLUS 3 ( 1 exp 2 PLUS 3

this state  
and input  
tell us what  
to do next

# The Parse Table

- At every point in the parse, the LR parser table tells us what to do next according to the automaton state at the top of the stack
  - shift, reduce, error or accept

states	Terminal seen next ID, NUM, := ...
1	
2	sn = shift & goto state n
3	rk = reduce by rule k
...	a = accept
n	= error

# The Parse Table

- Reducing by rule k is broken into two steps:
  - current stack is:  
A 8 B 3 C ..... 7 RHS 12
  - rewrite the stack according to  $X ::= \text{RHS}$ :  
A 8 B 3 C ..... 7 X
  - figure out state on top of stack (ie: goto 13)  
A 8 B 3 C ..... 7 X 13

states	Terminal seen next ID, NUM, := ...	Non-terminals X,Y,Z ...
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n	= error	



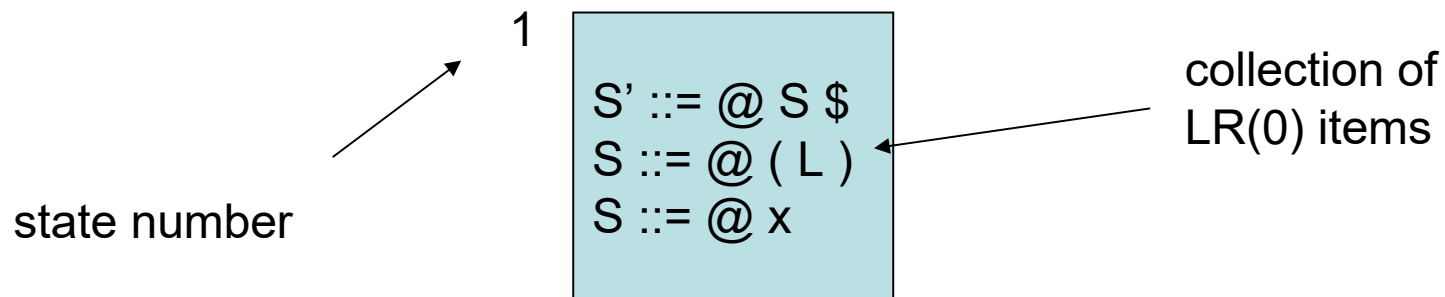
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# LR(0) parsing

- each state in the automaton represents a collection of LR(0) **items**:
  - an **item** is a rule from the grammar combined with “@” to indicate where the parser currently is in the input
    - eg:  $S' ::= @ S \$$  indicates that the parser is just beginning to parse this rule and it expects to be able to parse  $S$  then  $\$$  next
- A whole automaton state looks like this:



- LR(1) states look very similar, it is just that the items contain some look-ahead info

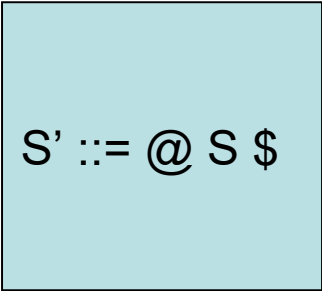
# LR(0) parsing

- To construct states, we begin with a particular LR(0) item and construct its **closure**
  - the closure adds more items to a set when the “@” appears to the left of a non-terminal
  - if the state includes  $X ::= s @ Y s'$  and  $Y ::= t$  is a rule then the state also includes  $Y ::= @ t$

Grammar:

- 0.  $S' ::= S \$$ 
  - $S ::= ( L )$
  - $S ::= x$
  - $L ::= S$
  - $L ::= L , S$

1



$S' ::= @ S \$$

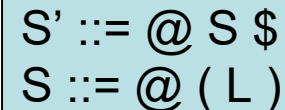
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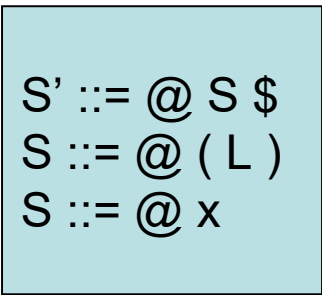
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$S' ::= @ S \$$   
 $S ::= @ ( L )$   
 $S ::= @ x$

} Full  
Closure

# LR(0) parsing

- To construct an LR(0) automaton:
  - start with start rule & compute initial state with closure
  - pick one of the items from the state and move “@” to the right one symbol (as if you have just parsed the symbol)
    - this creates a new item ...
    - ... and a new state when you compute the closure of the new item
    - mark the edge between the two states with:
      - a terminal T, if you moved “@” over T
      - a non-terminal X, if you moved “@” over X
  - continue until there are no further ways to move “@” across items and generate new states or new edges in the automaton

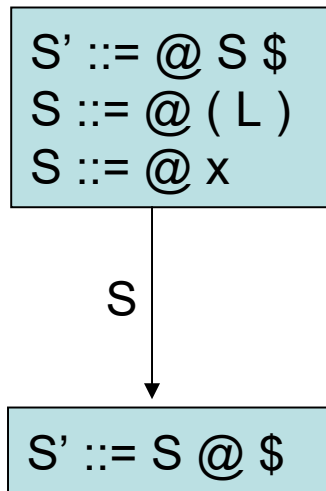
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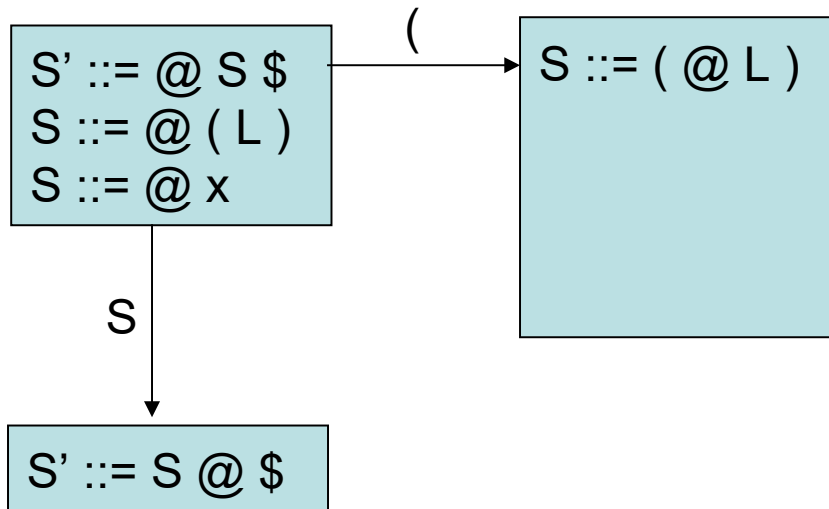
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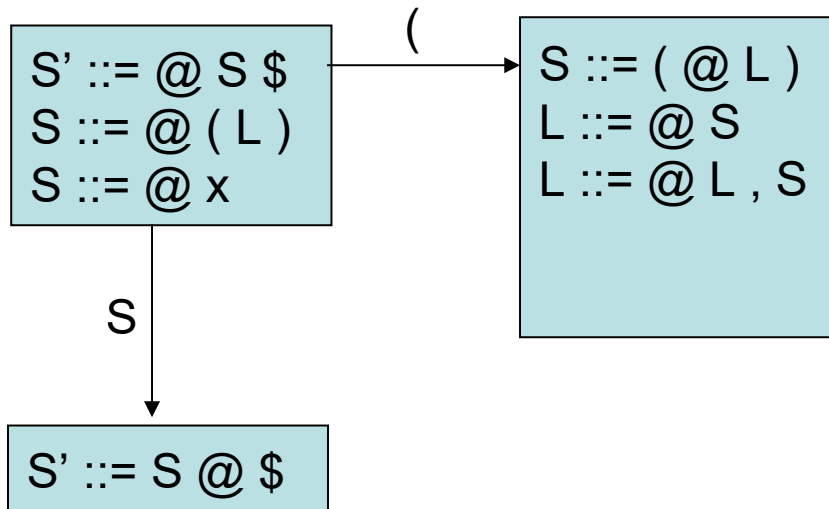
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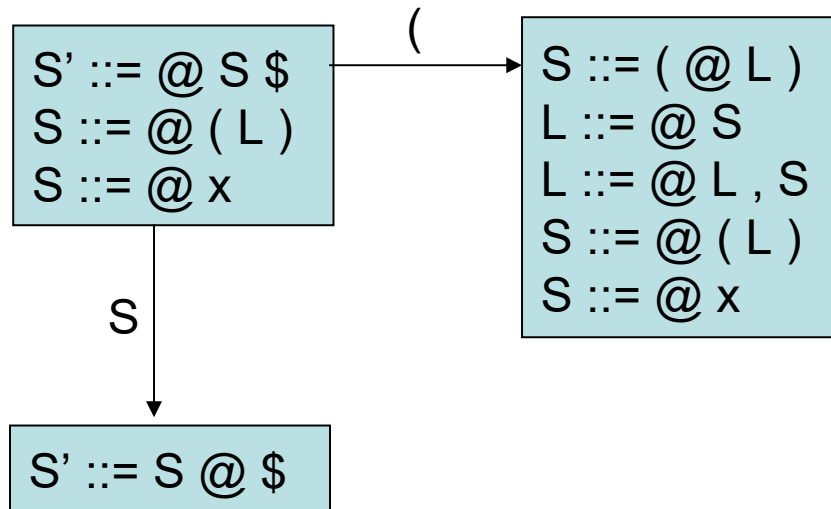
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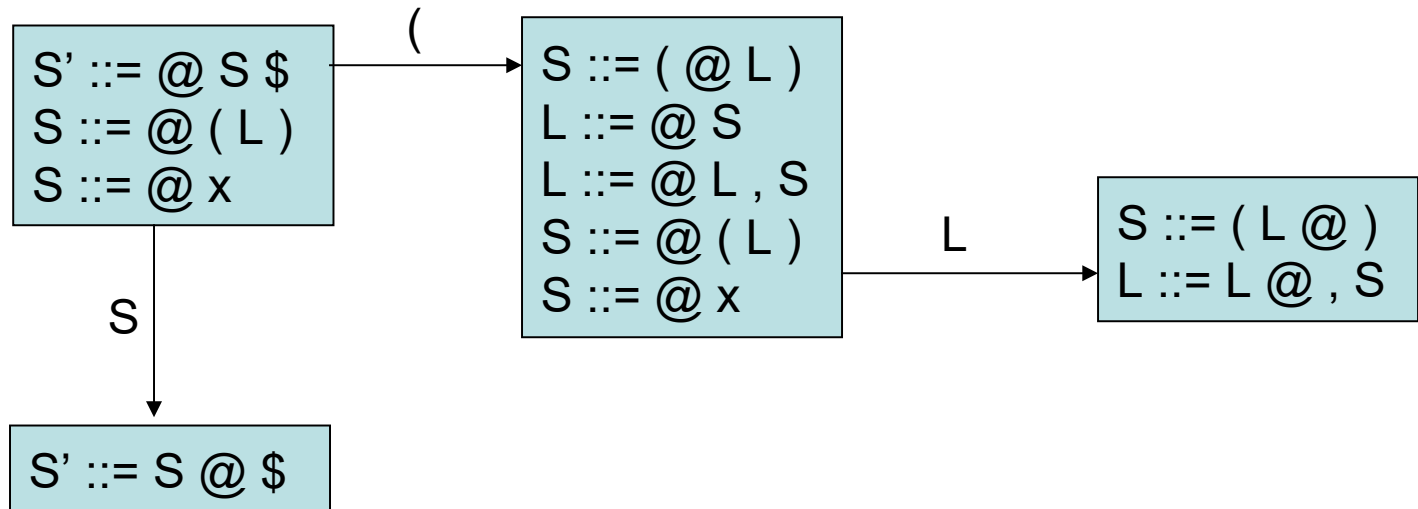
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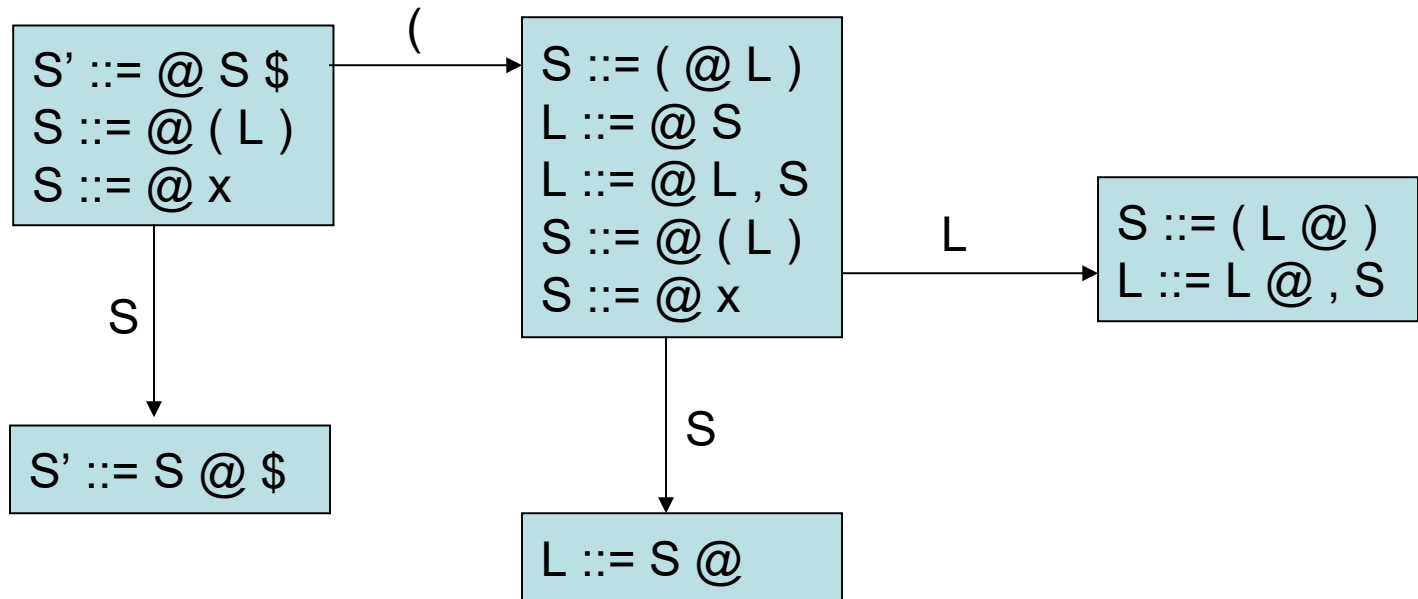
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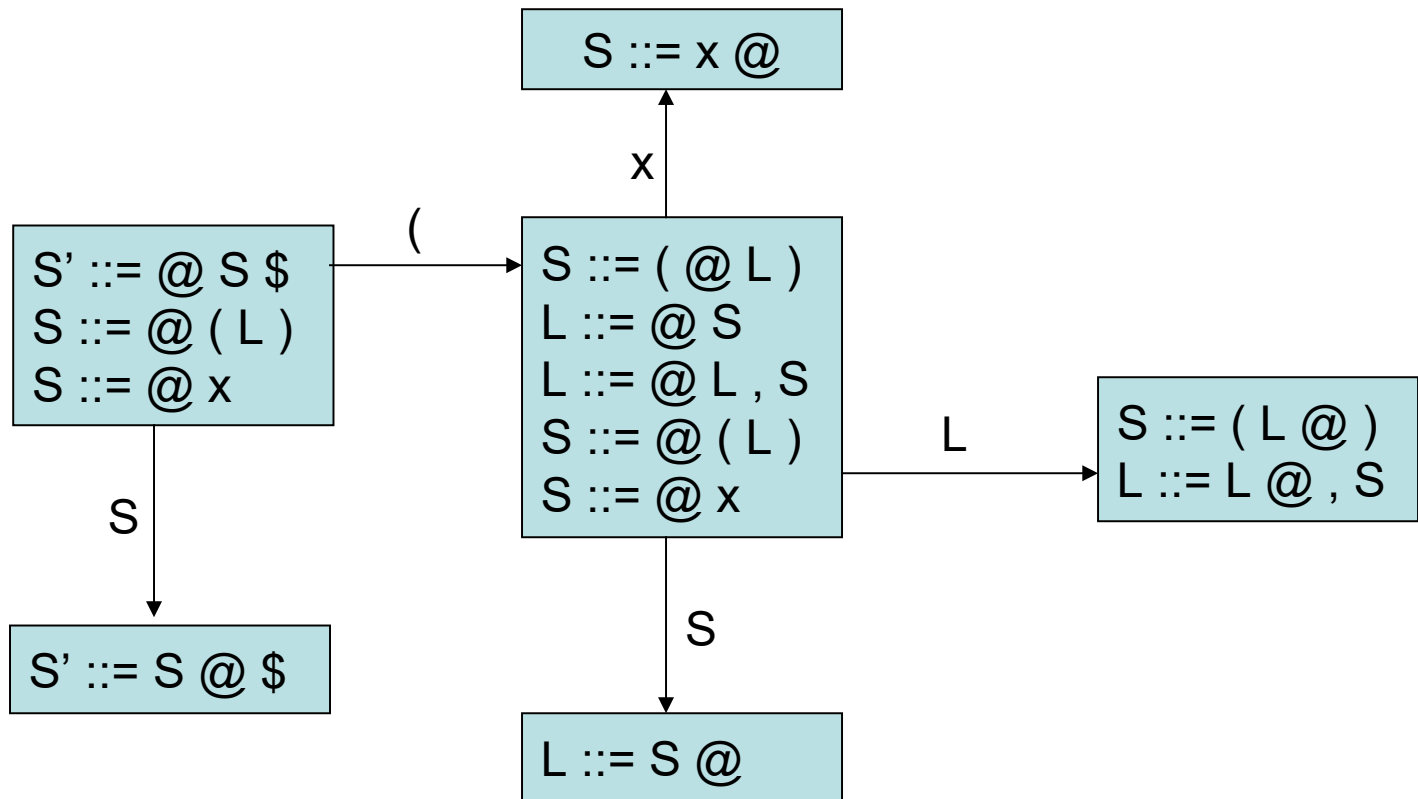
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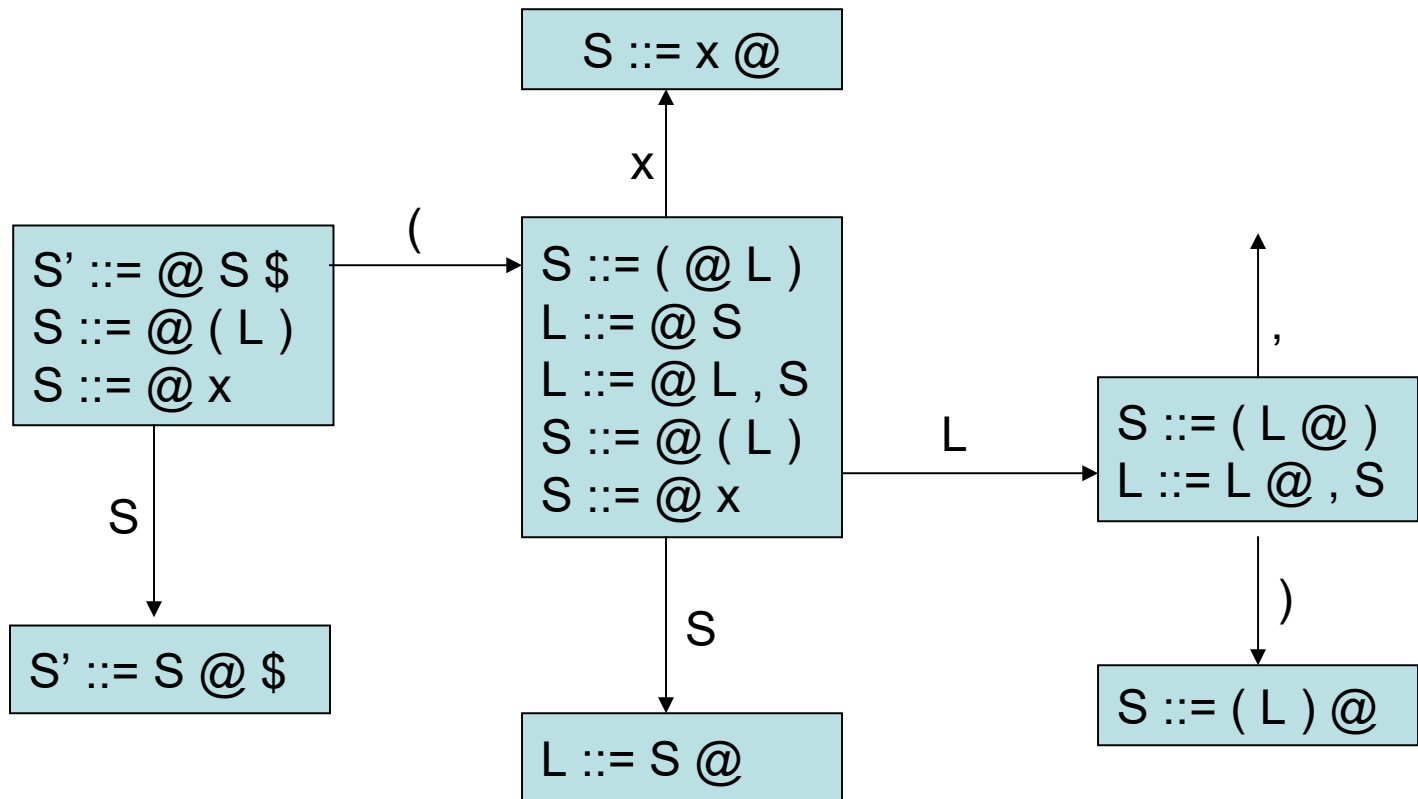
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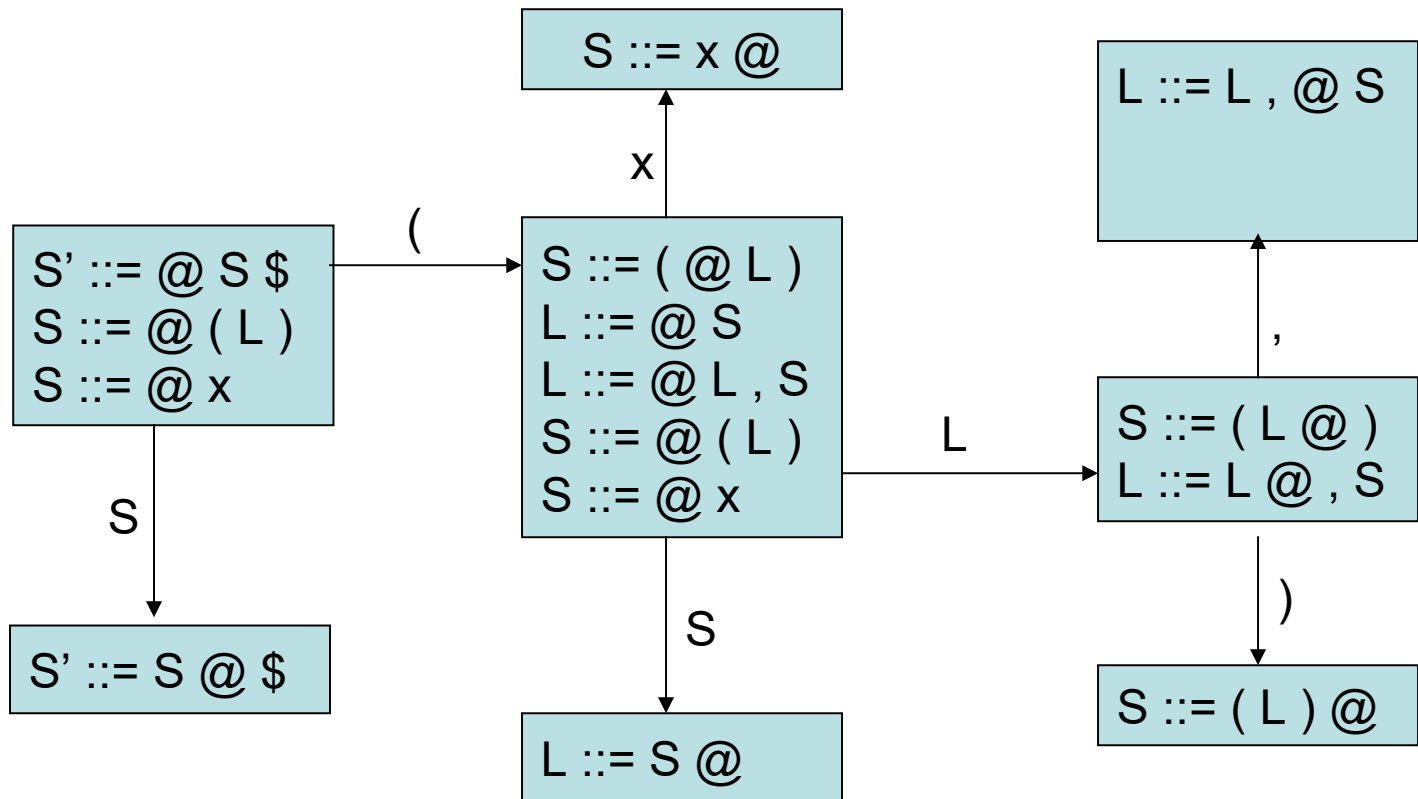
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- $L ::= L , S$



Grammar:

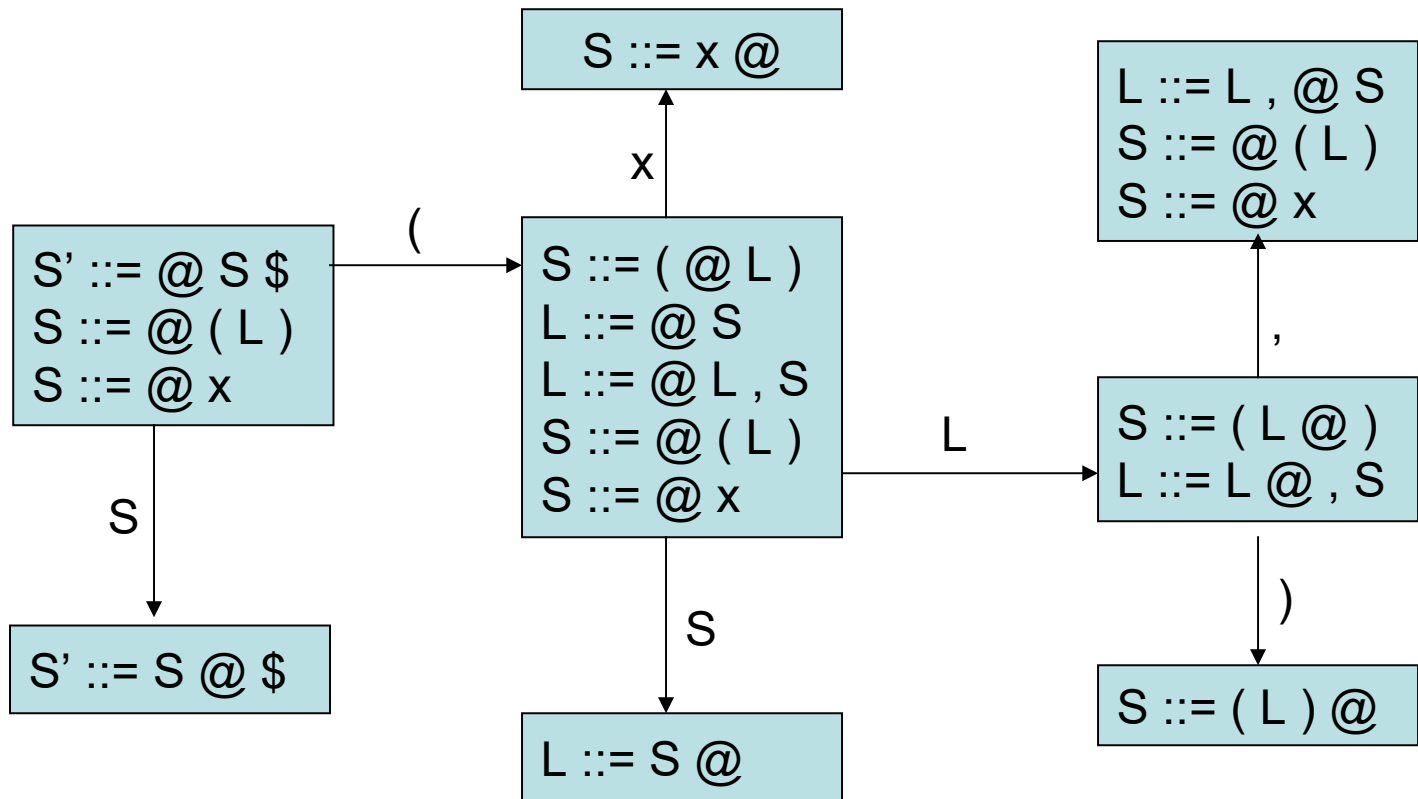
0.  $S' ::= S \$$
- $S ::= ( L )$
  - $S ::= x$
  - $L ::= S$
  - $L ::= L , S$





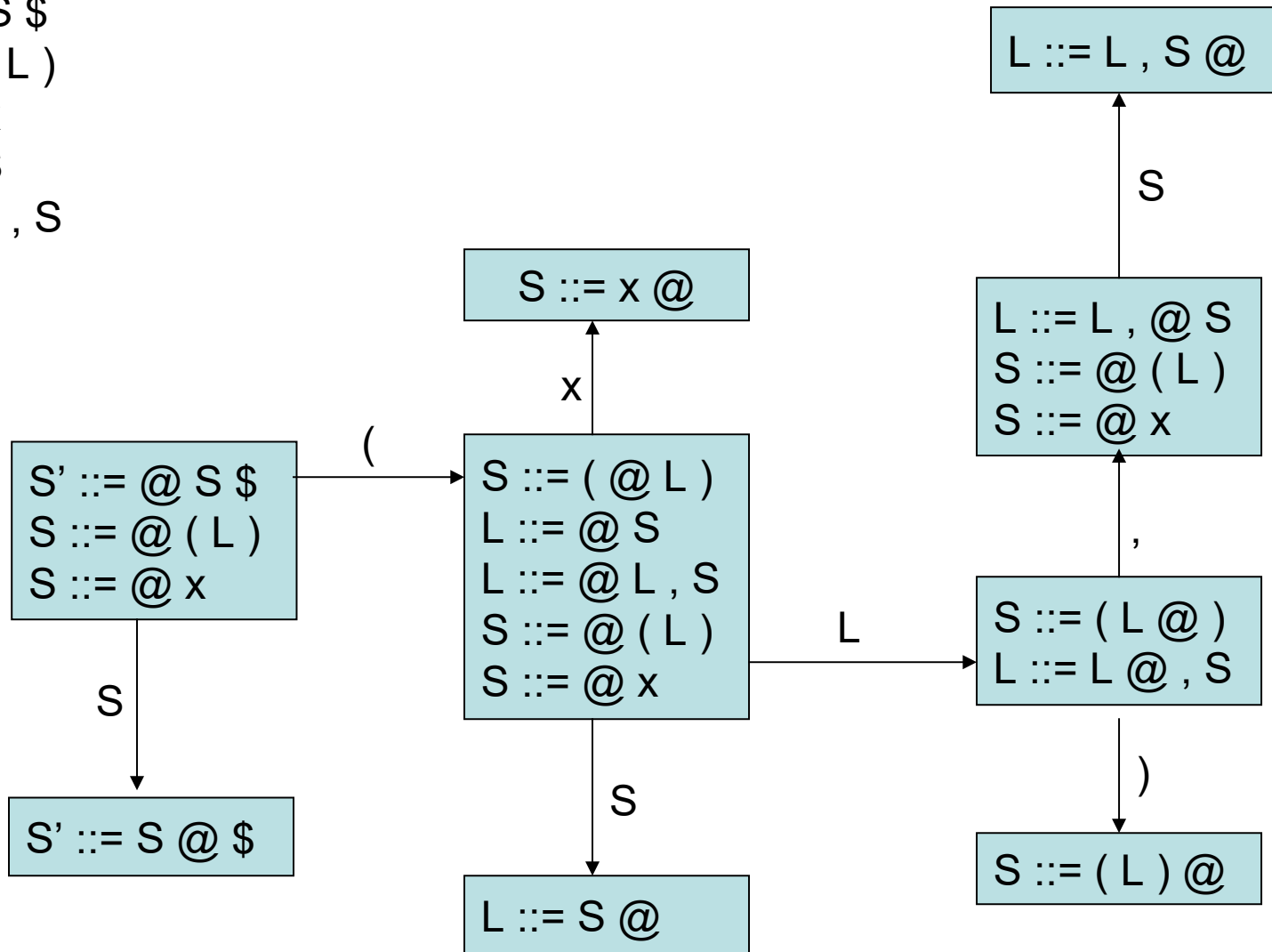
# Grammar:

0.  $S' ::= S \$$
- $S ::= ( L )$
- $S ::= x$
- $L ::= S$
- $L ::= L , S$



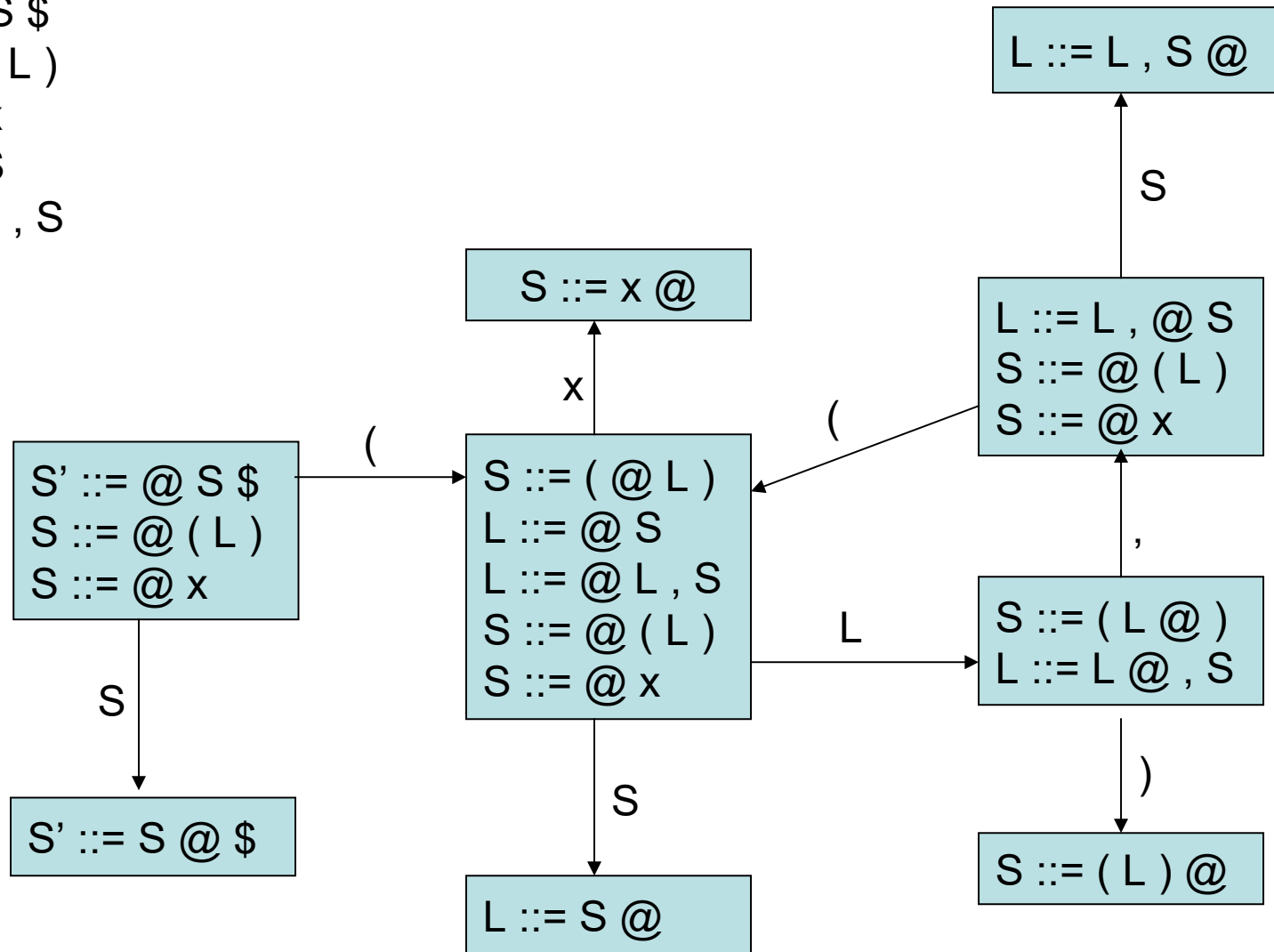
# Grammar:

0.  $S' ::= S \$$
- $S ::= ( L )$
  - $S ::= x$
  - $L ::= S$
  - $L ::= L , S$



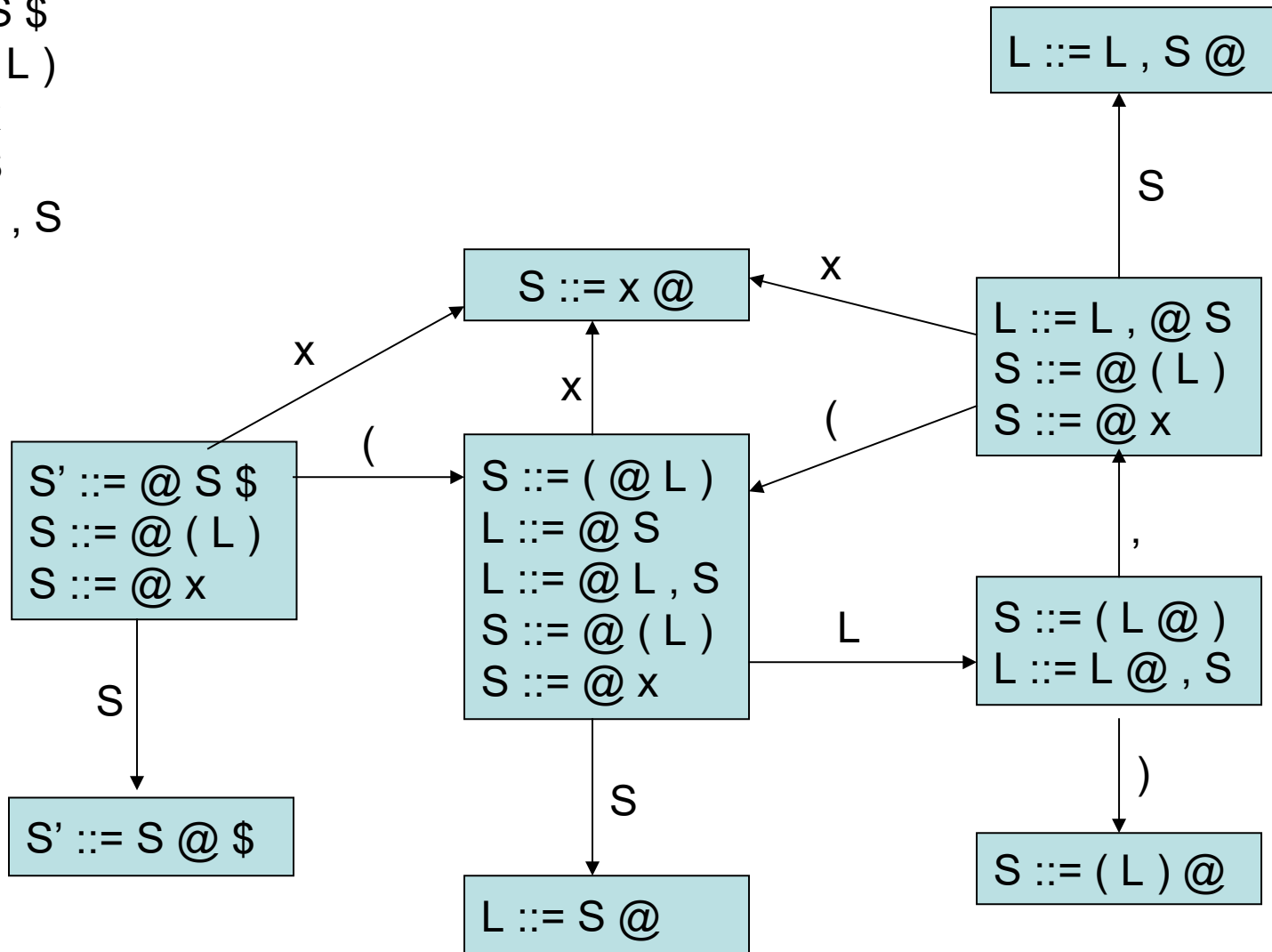
# Grammar:

0.  $S' ::= S \$$
- $S ::= ( L )$
  - $S ::= x$
  - $L ::= S$
  - $L ::= L , S$



Grammar:

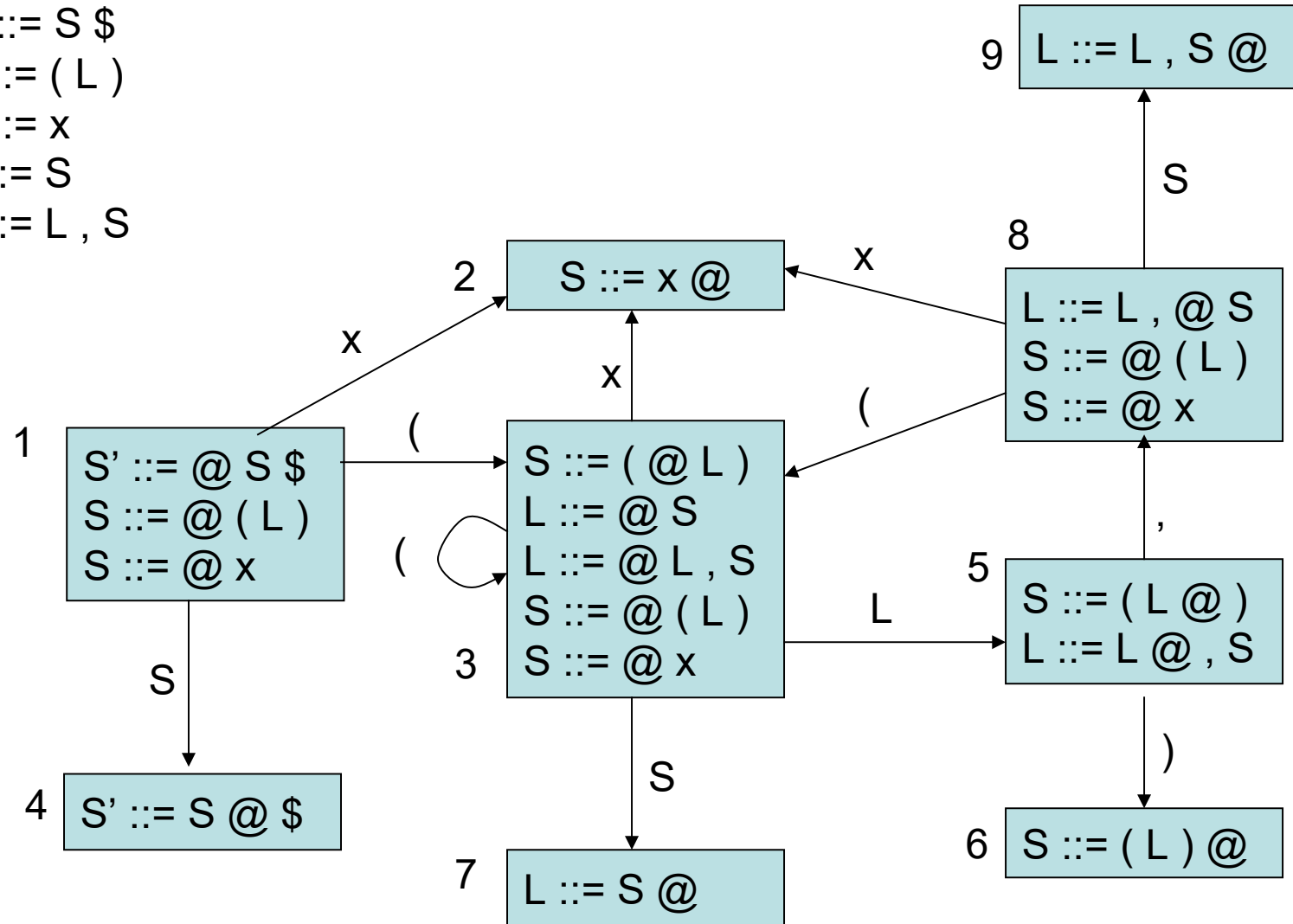
0.  $S' ::= S \$$
- $S ::= ( L )$
  - $S ::= x$
  - $L ::= S$
  - $L ::= L , S$



Grammar:

Assigning numbers to states:

0.  $S' ::= S \$$
- $S ::= ( L )$
  - $S ::= x$
  - $L ::= S$
  - $L ::= L , S$

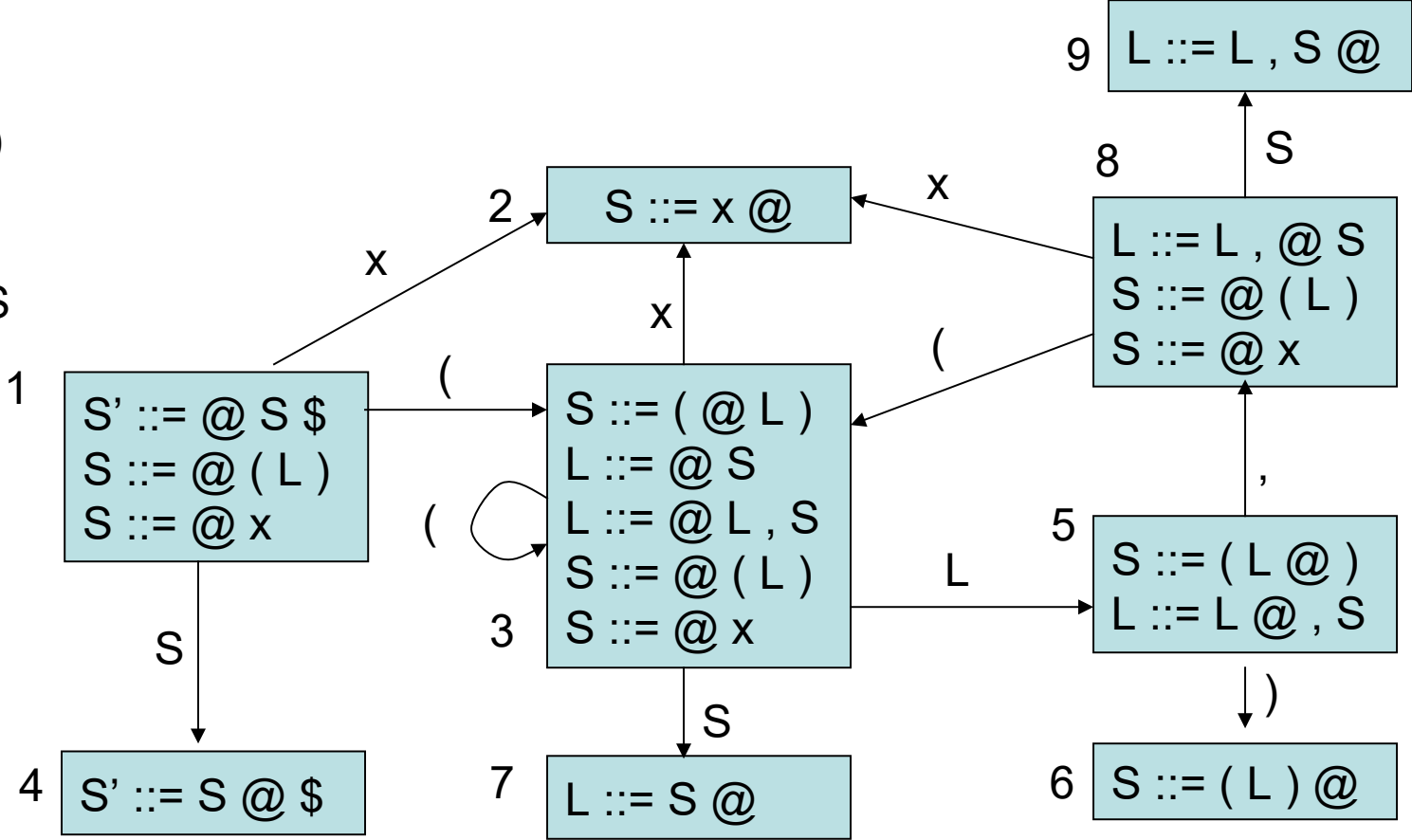


# computing parse table

- State  $i$  contains  $X ::= s @ \$ \implies \text{table}[i, \$] = a$
- State  $i$  contains rule  $k$ :  $X ::= s @ \implies \text{table}[i, T] = rk$  for all terminals  $T$
- Transition from  $i$  to  $j$  marked with  $T \implies \text{table}[i, T] = sj$
- Transition from  $i$  to  $j$  marked with  $X \implies \text{table}[i, X] = gj$

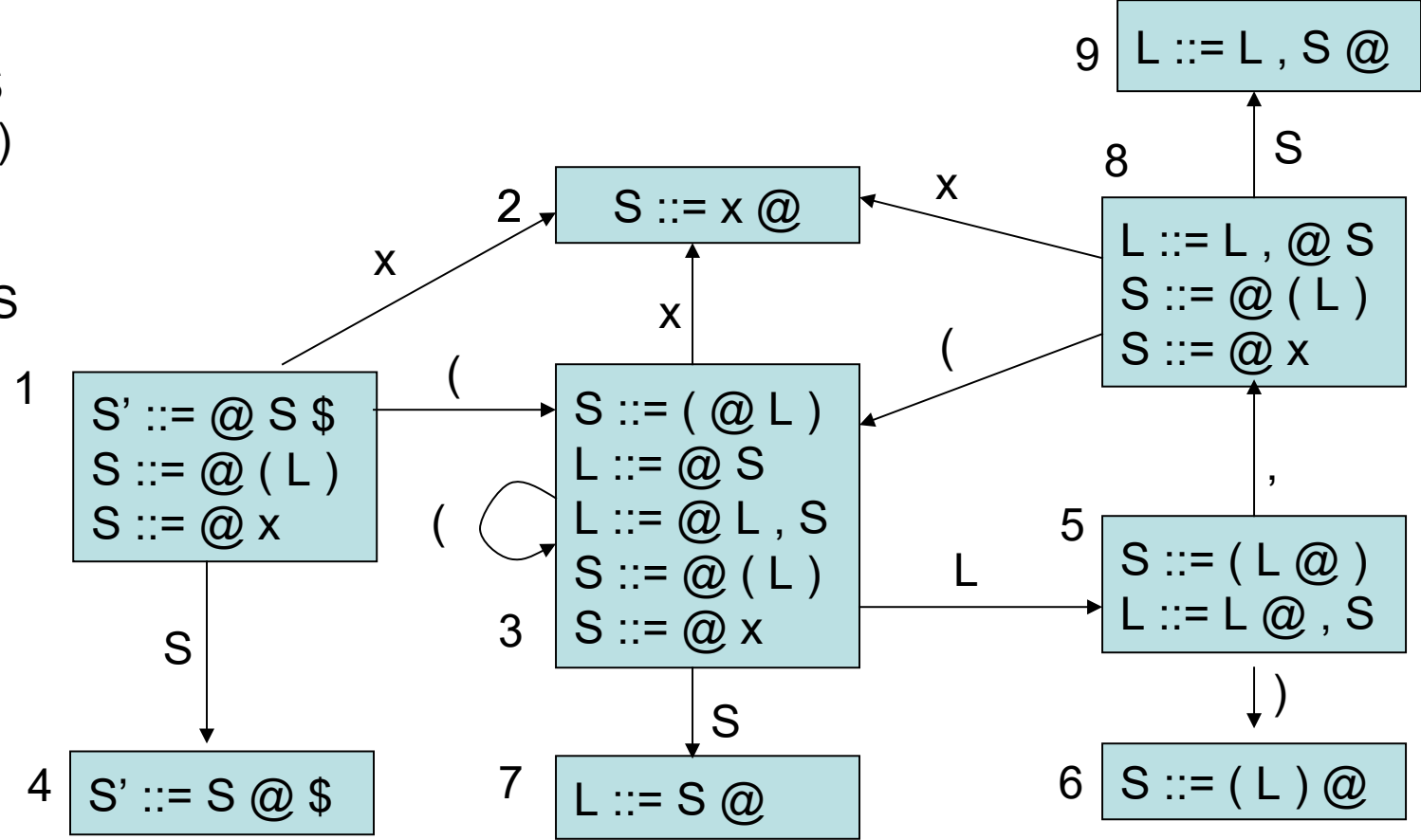
states	Terminal seen next ID, NUM, := ...	Non-terminals X,Y,Z ...
1		
2	sn = shift & goto state n	gn = goto state n
3	rk = reduce by rule k	
...	a = accept	
n	= error	

- 0.  $S' ::= S \$$
- $S ::= ( L )$
- $S ::= x$
- $L ::= S$
- $L ::= L , S$



states	(	)	x	,	\$	S	L
1							
2							
3							
4							
...							

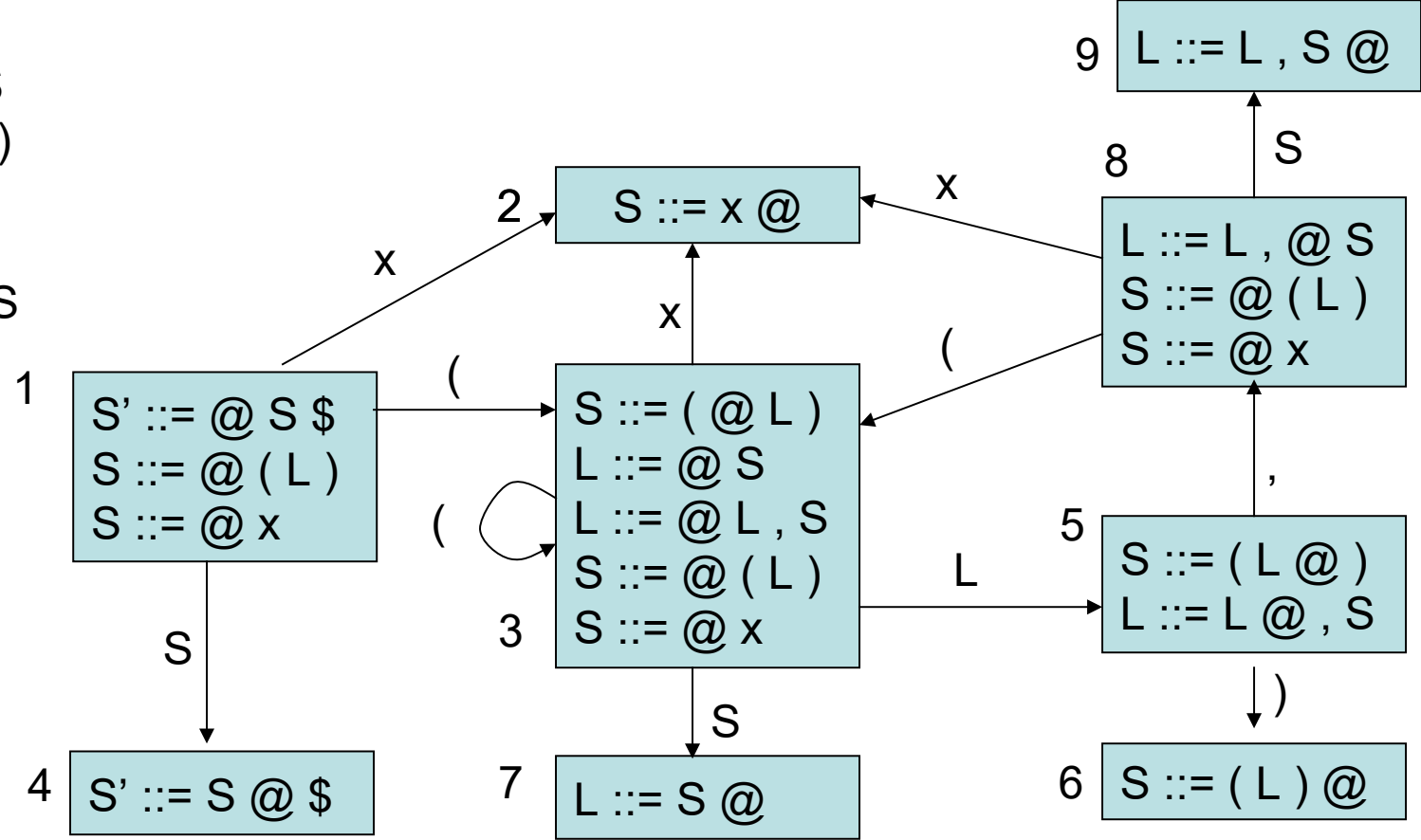
- 0.  $S' ::= S \$$
- $S ::= ( L )$
- $S ::= x$
- $L ::= S$
- $L ::= L , S$



states	(	)	x	,	\$	S	L
1	s3						
2							
3							
4							
...							

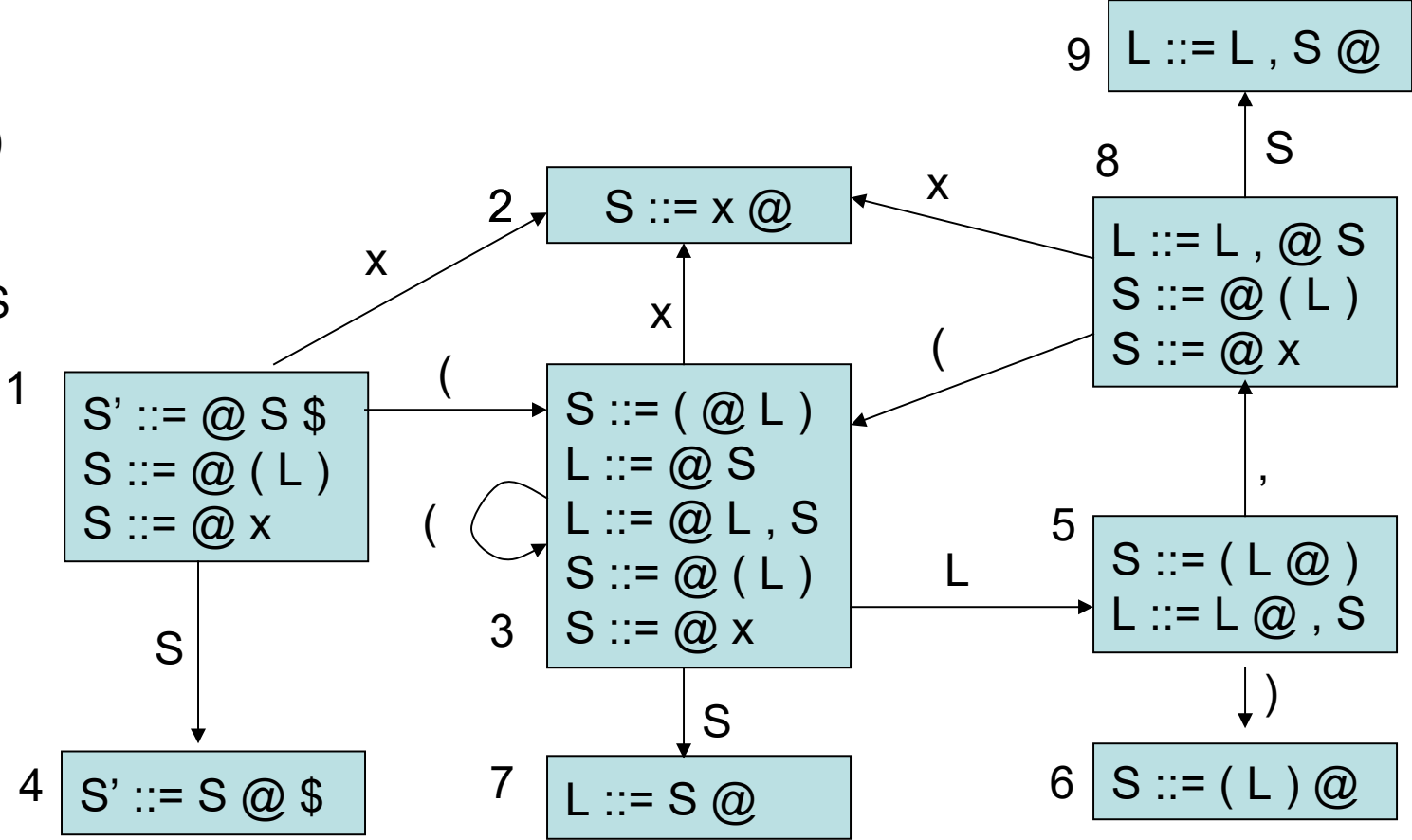


0.  $S' ::= S \$$
- $S ::= ( L )$
  - $S ::= x$
  - $L ::= S$
  - $L ::= L , S$



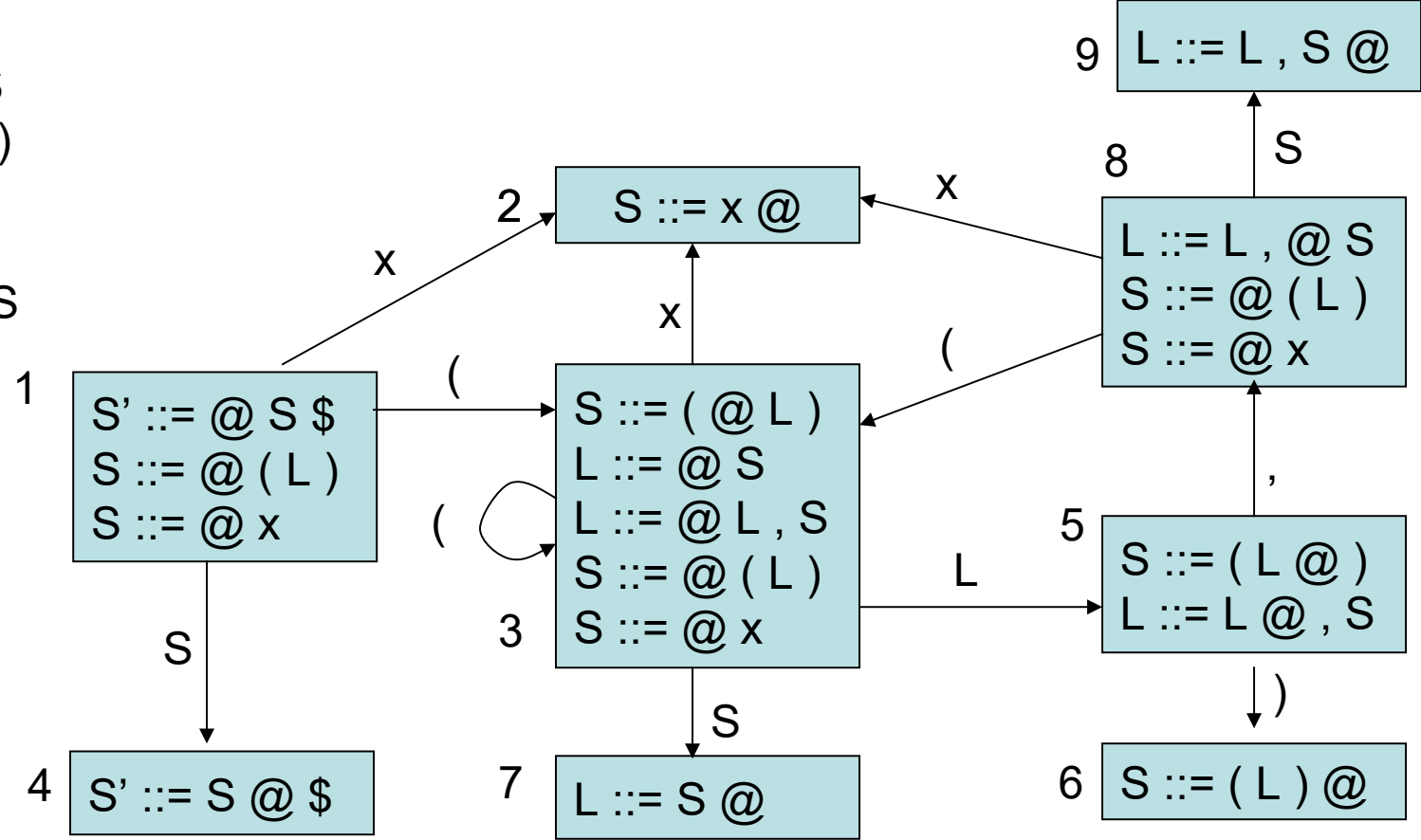
states	(	)	x	,	\$	S	L
1	s3		s2				
2							
3							
4							
...							

- 0.  $S' ::= S \$$
- $S ::= ( L )$
- $S ::= x$
- $L ::= S$
- $L ::= L , S$



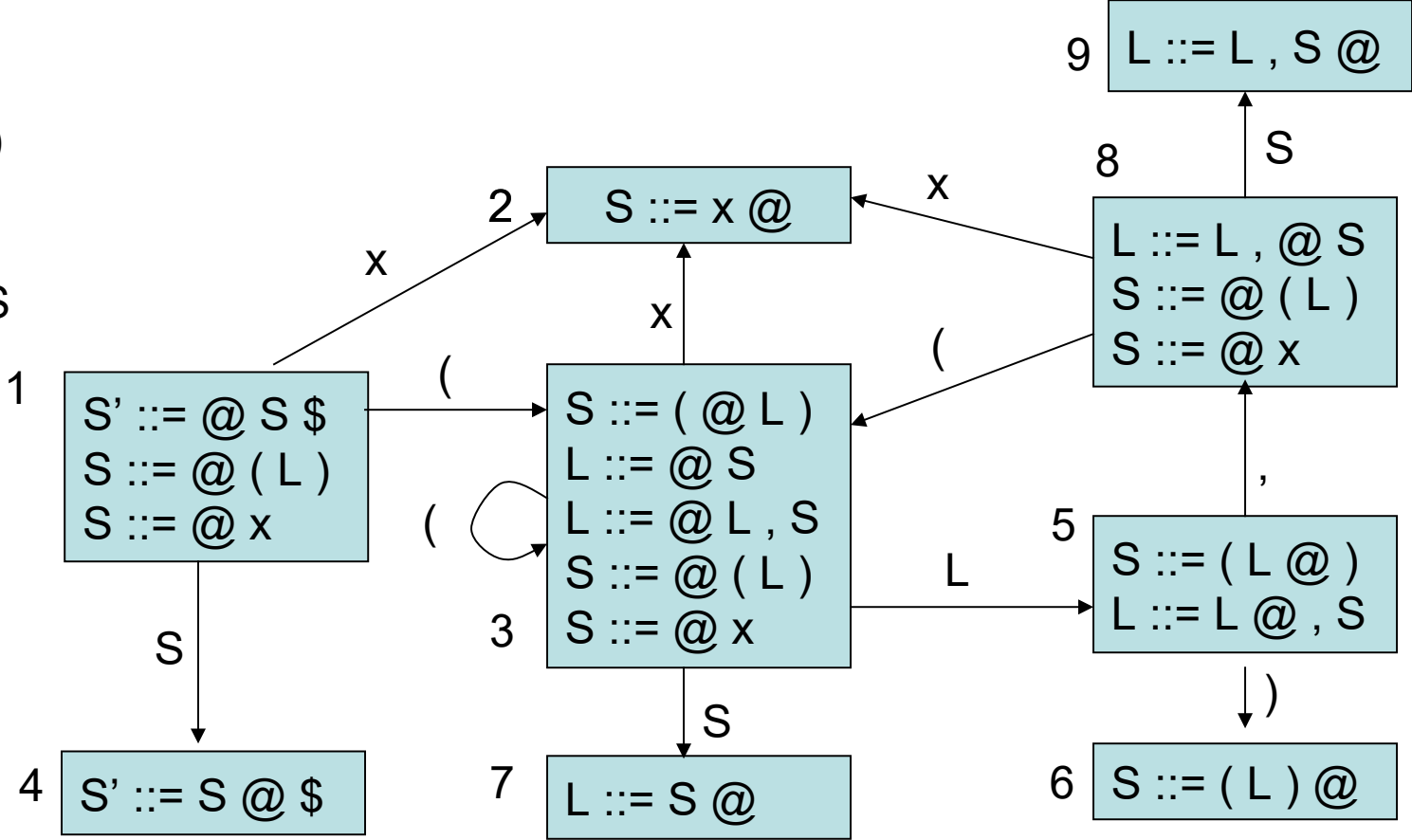
states	(	)	x	,	\$	S	L
1	s3		s2			g4	
2							
3							
4							
...							

- 0.  $S' ::= S \$$
- $S ::= ( L )$
- $S ::= x$
- $L ::= S$
- $L ::= L , S$



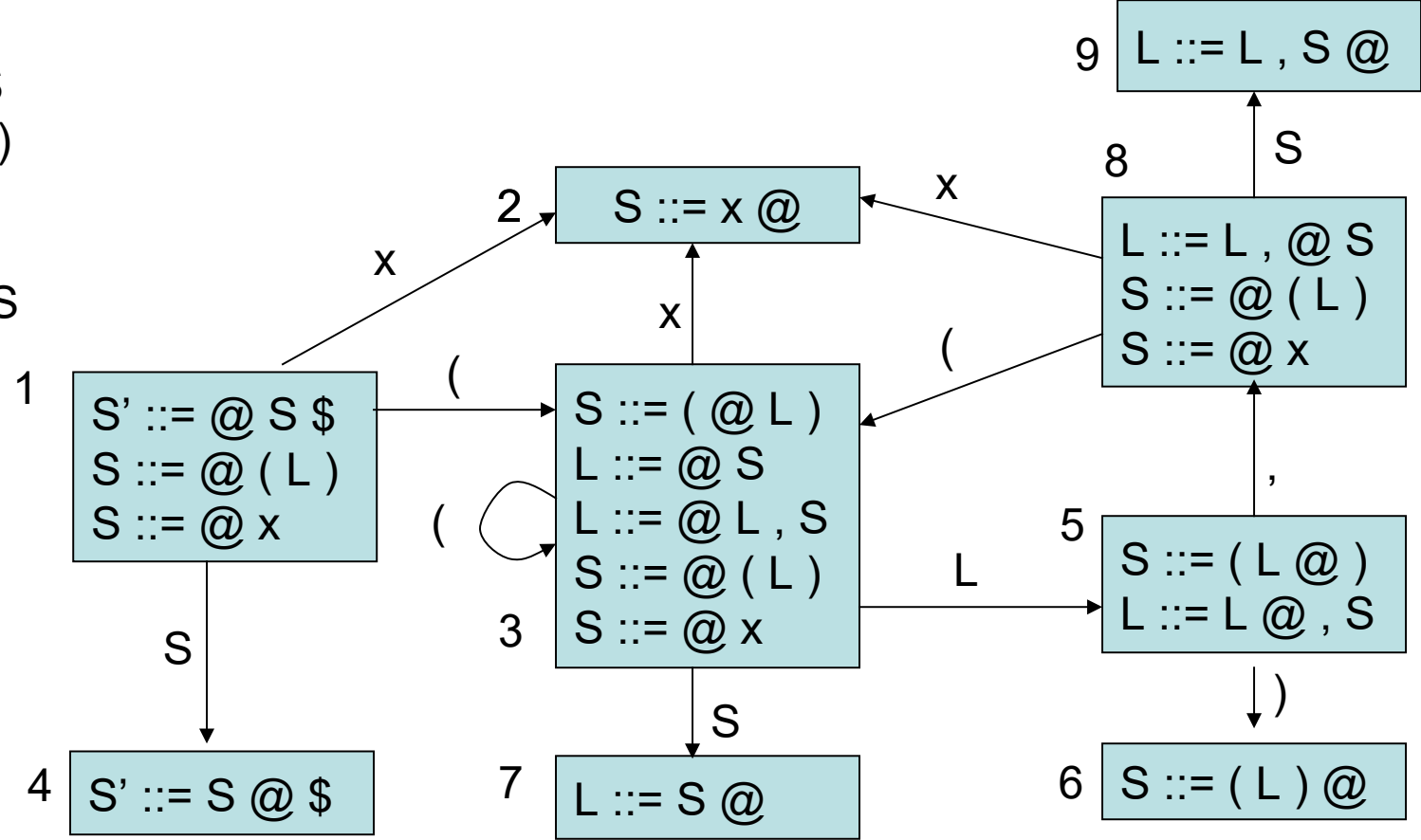
states	(	)	x	,	\$	S	L
1	s3		s2			g4	
2	r2	r2	r2	r2	r2		
3							
4							
...							

- 0.  $S' ::= S \$$
- $S ::= ( L )$
- $S ::= x$
- $L ::= S$
- $L ::= L , S$



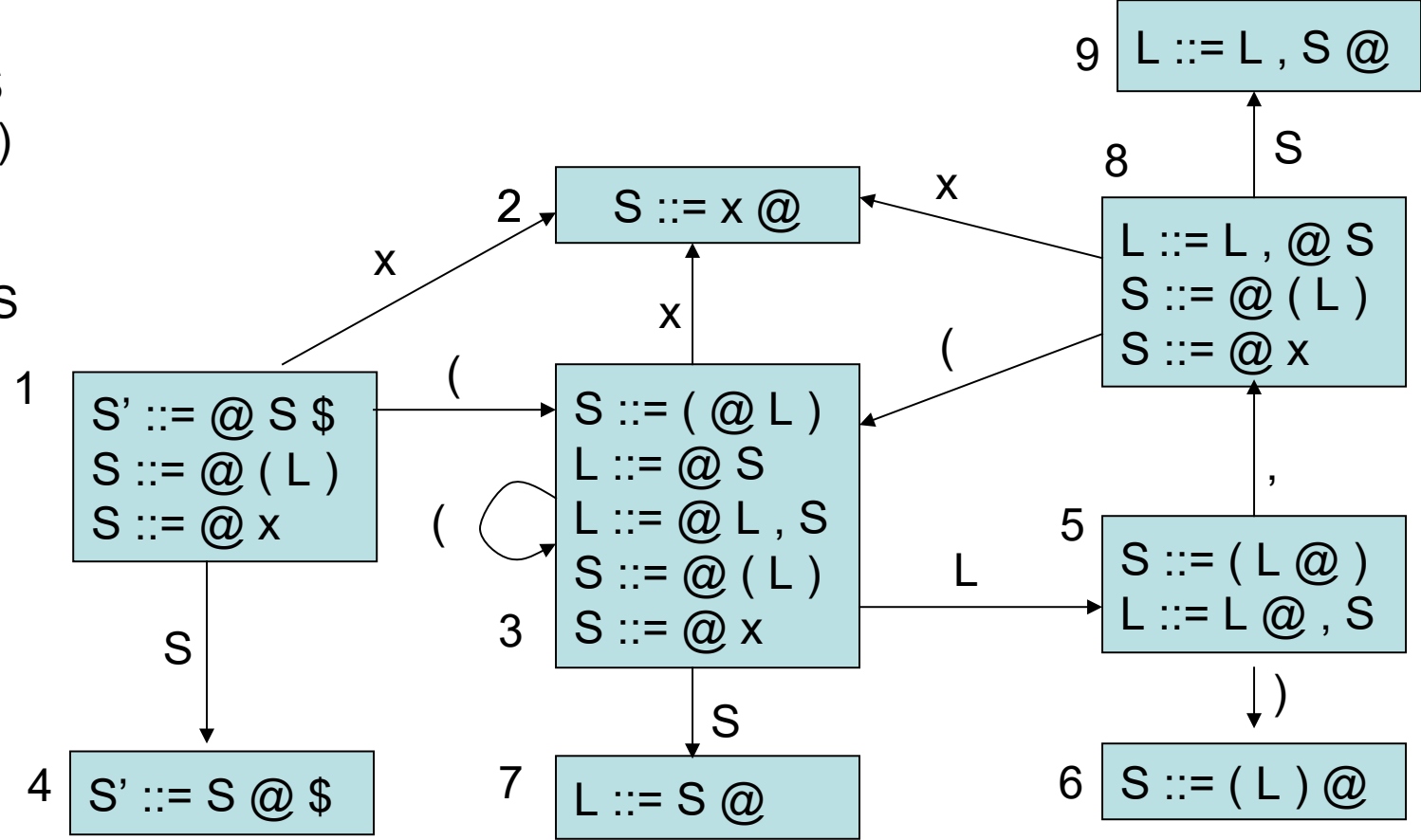
states	(	)	x	,	\$	S	L
1	s3		s2			g4	
2	r2	r2	r2	r2	r2		
3	s3		s2				
4							
...							

- 0.  $S' ::= S \$$
- $S ::= ( L )$
- $S ::= x$
- $L ::= S$
- $L ::= L , S$



states	(	)	x	,	\$	S	L
1	s3		s2			g4	
2	r2	r2	r2	r2	r2		
3	s3		s2			g7	g5
4							
...							

- 0.  $S' ::= S \$$
- $S ::= ( L )$
- $S ::= x$
- $L ::= S$
- $L ::= L , S$



states	(	)	x	,	\$	S	L
1	s3		s2			g4	
2	r2	r2	r2	r2	r2		
3	s3		s2			g7	g5
4					a		
...							

states	(	)	x	,	\$	S	L
1	s3		s2			g4	
2	r2	r2	r2	r2	r2		
3	s3		s2			g7	g5
4					a		
5		s6		s8			
6	r1	r1	r1	r1	r1		
7	r3	r3	r3	r3	r3		
8	s3		s2			g9	
9	r4	r4	r4	r4	r4		

0.  $S' ::= S \$$
- $S ::= ( L )$
  - $S ::= x$
  - $L ::= S$
  - $L ::= L , S$

input: ( x , x ) \$

stack: 1

states	(	)	x	,	\$	S	L
1	s3		s2			g4	
2	r2	r2	r2	r2	r2		
3	s3		s2			g7	g5
4					a		
5		s6		s8			
6	r1	r1	r1	r1	r1		
7	r3	r3	r3	r3	r3		
8	s3		s2			g9	
9	r4	r4	r4	r4	r4		

0.  $S' ::= S \$$
- $S ::= ( L )$
  - $S ::= x$
  - $L ::= S$
  - $L ::= L , S$

input: ( x , x ) \$

stack: 1 ( 3



states	(	)	x	,	\$	S	L
1	s3		s2			g4	
2	r2	r2	r2	r2	r2		
3	s3		s2			g7	g5
4					a		
5		s6		s8			
6	r1	r1	r1	r1	r1		
7	r3	r3	r3	r3	r3		
8	s3		s2			g9	
9	r4	r4	r4	r4	r4		

1.  $S' ::= S \$$
2.  $S ::= ( L )$
3.  $S ::= x$
4.  $L ::= S$
5.  $L ::= L , S$

input:      ( x , x ) \$

stack:    1 ( 3 x 2

states	(	)	x	,	\$	S	L
1	s3		s2			g4	
2	r2	r2	r2	r2	r2		
3	s3		s2			g7	g5
4					a		
5		s6		s8			
6	r1	r1	r1	r1	r1		
7	r3	r3	r3	r3	r3		
8	s3		s2			g9	
9	r4	r4	r4	r4	r4		

0.  $S' ::= S \$$
- $S ::= ( L )$
  - $S ::= x$
  - $L ::= S$
  - $L ::= L , S$

input:      ( x , x ) \$

. yet to read

stack:    1 ( 3 S

states	(	)	x	,	\$	S	L
1	s3		s2			g4	
2	r2	r2	r2	r2	r2		
3	s3		s2			g7	g5
4					a		
5		s6		s8			
6	r1	r1	r1	r1	r1		
7	r3	r3	r3	r3	r3		
8	s3		s2			g9	
9	r4	r4	r4	r4	r4		

0.  $S' ::= S \$$
- $S ::= ( L )$
  - $S ::= x$
  - $L ::= S$
  - $L ::= L , S$

input:      ( x , x ) \$

. yet to read

stack:    1 ( 3 S 7

states	(	)	x	,	\$	S	L
1	s3		s2			g4	
2	r2	r2	r2	r2	r2		
3	s3		s2			g7	g5
4					a		
5		s6		s8			
6	r1	r1	r1	r1	r1		
7	r3	r3	r3	r3	r3		
8	s3		s2			g9	
9	r4	r4	r4	r4	r4		

0.  $S' ::= S \$$
- $S ::= ( L )$
  - $S ::= x$
  - $L ::= S$
  - $L ::= L , S$

input:      ( x , x ) \$

stack:    1 ( 3 L

states	(	)	x	,	\$	S	L
1	s3		s2			g4	
2	r2	r2	r2	r2	r2		
3	s3		s2			g7	g5
4					a		
5		s6		s8			
6	r1	r1	r1	r1	r1		
7	r3	r3	r3	r3	r3		
8	s3		s2			g9	
9	r4	r4	r4	r4	r4		

0.  $S' ::= S \$$
- $S ::= ( L )$
  - $S ::= x$
  - $L ::= S$
  - $L ::= L , S$

input:      ( x , x ) \$

stack:    1 ( 3 L 5

states	(	)	x	,	\$	S	L
1	s3		s2			g4	
2	r2	r2	r2	r2	r2		
3	s3		s2			g7	g5
4					a		
5		s6		s8			
6	r1	r1	r1	r1	r1		
7	r3	r3	r3	r3	r3		
8	s3		s2			g9	
9	r4	r4	r4	r4	r4		

0.  $S' ::= S \$$
- $S ::= ( L )$
  - $S ::= x$
  - $L ::= S$
  - $L ::= L , S$

input:      ( x , x ) \$

. yet to read

stack:    1 ( 3 L 5 , 8

states	(	)	x	,	\$	S	L
1	s3		s2			g4	
2	r2	r2	r2	r2	r2		
3	s3		s2			g7	g5
4					a		
5		s6		s8			
6	r1	r1	r1	r1	r1		
7	r3	r3	r3	r3	r3		
8	s3		s2			g9	
9	r4	r4	r4	r4	r4		

0.  $S' ::= S \$$
- $S ::= ( L )$
  - $S ::= x$
  - $L ::= S$
  - $L ::= L , S$

input:      ( x , x ) \$

. yet to read

stack:    1 ( 3 L 5 , 8 x 2

states	(	)	x	,	\$	S	L
1	s3		s2			g4	
2	r2	r2	r2	r2	r2		
3	s3		s2			g7	g5
4					a		
5		s6		s8			
6	r1	r1	r1	r1	r1		
7	r3	r3	r3	r3	r3		
8	s3		s2			g9	
9	r4	r4	r4	r4	r4		

0.  $S' ::= S \$$
- $S ::= ( L )$
  - $S ::= x$
  - $L ::= S$
  - $L ::= L , S$

input:      ( x , x ) \$

. yet to read

stack:    1 ( 3 L 5 , 8 S



states	(	)	x	,	\$	S	L
1	s3		s2			g4	
2	r2	r2	r2	r2	r2		
3	s3		s2			g7	g5
4					a		
5		s6		s8			
6	r1	r1	r1	r1	r1		
7	r3	r3	r3	r3	r3		
8	s3		s2			g9	
9	r4	r4	r4	r4	r4		

0.  $S' ::= S \$$
- $S ::= ( L )$
  - $S ::= x$
  - $L ::= S$
  - $L ::= L , S$

input:      ( x , x ) \$

. yet to read

stack:    1 ( 3 L 5 , 8 S 9

states	(	)	x	,	\$	S	L
1	s3		s2			g4	
2	r2	r2	r2	r2	r2		
3	s3		s2			g7	g5
4					a		
5		s6		s8			
6	r1	r1	r1	r1	r1		
7	r3	r3	r3	r3	r3		
8	s3		s2			g9	
9	r4	r4	r4	r4	r4		

0.  $S' ::= S \$$
- $S ::= ( L )$
  - $S ::= x$
  - $L ::= S$
  - $L ::= L , S$

input:      ( x , x ) \$

yet to read

stack:    1 ( 3 L

states	(	)	x	,	\$	S	L
1	s3		s2			g4	
2	r2	r2	r2	r2	r2		
3	s3		s2			g7	g5
4					a		
5		s6		s8			
6	r1	r1	r1	r1	r1		
7	r3	r3	r3	r3	r3		
8	s3		s2			g9	
9	r4	r4	r4	r4	r4		

0.  $S' ::= S \$$
- $S ::= ( L )$
  - $S ::= x$
  - $L ::= S$
  - $L ::= L , S$

input:      ( x , x ) \$

yet to read

stack:    1 ( 3 L 5

etc .....

# LR(0)

- Even though we are doing LR(0) parsing we are using some look ahead (there is a column for each non-terminal)
- however, we only use the terminal to figure out which state to go to next, not to decide whether to shift or reduce

states	(	)	x	,	\$	S	L
1	s3		s2			g4	
2	r2	r2	r2	r2	r2		
3	s3		s2			g7	g5

# LR(0)

- Even though we are doing LR(0) parsing we are using some look ahead (there is a column for each non-terminal)
- however, we only use the terminal to figure out which state to go to next, not to decide whether to shift or reduce

states	(	)	x	,	\$	S	L
1	s3		s2			g4	
2	r2	r2	r2	r2	r2		
3	s3		s2			g7	g5



ignore next automaton state

states	no look-ahead	S	L
1	shift	g4	
2	reduce 2		
3	shift	g7	g5

# LR(0)

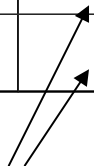
- Even though we are doing LR(0) parsing we are using some look ahead (there is a column for each non-terminal)
- however, we only use the terminal to figure out which state to go to next, not to decide whether to shift or reduce
- If the same row contains both shift and reduce, we will have a conflict ==> the grammar is not LR(0)
- Likewise if the same row contains reduce by two different rules

states	no look-ahead	S	L
1	shift, reduce 5	g4	
2	reduce 2, reduce 7		
3	shift	g7	g5

# SLR


- SLR (simple LR) is a variant of LR(0) that reduces the number of conflicts in LR(0) tables by using a tiny bit of look ahead
- To determine when to reduce, 1 symbol of look ahead is used.
- Only put reduce by rule  $(X ::= \text{RHS})$  in column T if T is in  $\text{Follow}(X)$

states	(	)	x	,	\$	S	L
1	s3		s2			g4	
2	r2	s5	r2				
3	r1		r1	r5	r5	g7	g5



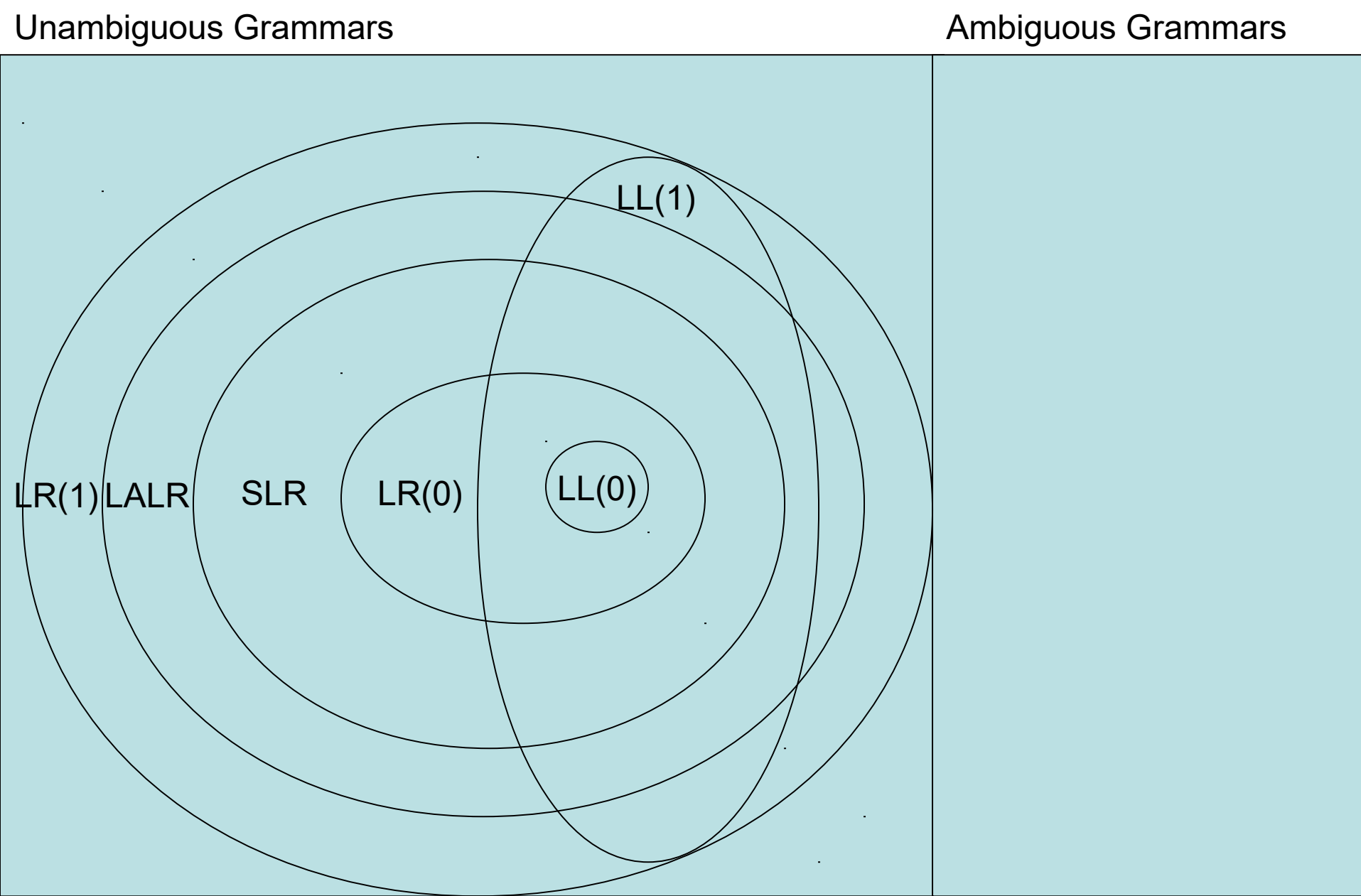
cuts down the number of rk slots & therefore cuts down conflicts

# LR(1) & LALR

- LR(1) automata are identical to LR(0) except for the “items” that make up the states
- LR(0) items:  
 $X ::= s1 @ s2$
- LR(1) items  
 $X ::= s1 @ s2, T$   look-ahead symbol added
  - Idea: sequence  $s1$  is on stack; input stream is  $s2 T$
- Find closure with respect to  $X ::= s1 @ Y s2, T$  by adding all items  $Y ::= s3, U$  when  $Y ::= s3$  is a rule and  $U$  is in  $First(s2 T)$
- Two states are different if they contain the same rules but the rules have different look-ahead symbols
  - Leads to many states
  - LALR(1) = LR(1) where states that are identical aside from look-ahead symbols have been merged
  - ML-Yacc & most parser generators use LALR
- READ: Appel 3.3 (and also all of the rest of chapter 3)



# Grammar Relationships



# summary

- LR parsing is more powerful than LL parsing, given the same look ahead
- to construct an LR parser, it is necessary to compute an LR parser table
- the LR parser table represents a finite automaton that walks over the parser stack
- ML-Yacc uses LALR, a compact variant of LR(1)