THEORY AND APPLICATIONS OF ELECTRON TUBES

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THEORY AND APPLICATIONS
OF
ELECTRON TUBES

BY

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This book is dedicated
to the many scientists and engineers
upon whose work it is based

PREFACE TO THE SECOND EDITION

The preparation of the second edition of "Theory and Applications of Electron Tubes" was prompted by two principal considerations. The first, and probably more important, of these was the necessity of bringing the book up to date as regards the principal new developments of the past five years in the field which it covers. Although the press of other work connected with the defense and war efforts has made it impossible for the author to make an exhaustive search of the literature of this period, he believes that the most important subjects have been adequately treated.

The second factor that led to the preparation of the new edition was the desirability of incorporating improvements in presentation that have been suggested or that have suggested themselves during five years of use of the book in college courses. Much of the material has been rearranged, and the chapter on Modulation and Detection and that on Oscillators have been to a considerable extent rewritten. A few changes in symbols or definitions have appeared to be desirable. In response to many requests, problem answers are included in the new edition.

The author of a book in a field that develops as rapidly as that covered by this book is faced with the difficulty of keeping the book up to date without increasing the size unduly. Probably the best judges of what material should be eliminated or added are the instructors in the courses in which the book is used as a text. The author will welcome suggestions and criticisms.

HERBERT J. REICH.

CAMBRIDGE, MASS.,
August, 1944.

PREFACE TO THE FIRST EDITION

Electron tubes, which have made possible the rapid development of radio to its present state of refinement, have been assuming an increasing importance in power control and transmission, in manufacturing, in the home, and in the various branches of engineering and scientific research. The rapidly growing field of application of electronic devices has necessitated the addition of courses in theoretical and applied electronics to engineering and scientific curriculums. The need for a single book to assemble and coordinate our present knowledge of the theory and application of electron tubes led to the writing of this book.

The book is intended to give the student a sufficiently thorough grounding in the fundamental principles of electron tubes and associated circuits to enable him to apply electron tubes to the solution of new problems. The author has not attempted to discuss all applications of tubes to special problems but rather to cover basic principles and typical applications. Since it was not his purpose to write a treatise on the subject of applications of electron tubes, Class C amplification and the design of radio transmitters and receivers, which are adequately treated in books on radio engineering, have not been taken up. The basic principles that are presented, however, are applicable to radio engineering problems, as well as to industrial electronics, power control, electrical measurements, and other fields of use of tubes. Although written primarily as a text for college students, it is hoped that it will also prove to be of value to practicing engineers as a reference book.

The book is based upon mimeographed notes that have been used in the author's courses on electron tubes during the past five years. These notes have been kept up to date and have been revised as use in the classroom has indicated where improvement could be made.

A problem encountered in the preparation of the manuscript was the choice of symbols. In the main, the symbols used are those which have been standardized by the Standards Committee of the Institute of Radio Engineers. Although the use of the symbols e and E for the voltage of tube electrodes is in agreement with the practice of most writers on the subject of electron tubes in the United States during the past twenty-five years, it is not in agreement with the symbols standardized by the American Institute of Electrical Engineers nor with those used in England. Since the basic symbols that are used in this book have already been standardized by the Institute of Radio Engineers, the author feels that it would be a mistake to set up new symbols now. Because of the very large number of symbols that must be used in the analysis of tubes, difficulties are invariably encountered, regardless of the system of nomenclature that is adopted.

The series expansion for electrode currents has been made the basis of the analysis of the operation of high-vacuum tubes and associated circuits. In order to justify the use of the series expansion and several other very useful equations, the outlines of their derivations are included. The student will not be seriously handicapped by the omission of these derivations. Because the author believes that a thorough understanding of the principles of detection and modulation is of great value in the study of distortion in amplifiers, the chapter on detection and modulation precedes those on amplification. The arrangement of subject matter is such, however, that little difficulty will be experienced by the student if this chapter is studied after those on amplifiers. Equivalent-circuit and graphical methods of analysis are stressed throughout the book.

H. J. REICH.

URBANA, ILL.,
November, 1938.

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THEORY AND APPLICATIONS OF ELECTRON TUBES

CHAPTER 1

PHYSICAL CONCEPTS

An *electron tube* is a device consisting of a number of electrodes contained within a totally or partly evacuated enclosure. The usefulness of such a device arises from its capacity to pass current, the magnitude of which may be controlled by the voltages of the electrodes. As suggested by the term *electron tube,* the operation of all types of electron tubes is dependent upon the movement of *electrons* within the tubes. The electron, which is the smallest known particle, has a mass of 9.03×10^{-28} g and a negative charge ε of 16×10^{-20} coulomb or 4.8×10^{-10} e.s.u. The operation of many types of electron tubes also depends upon the separation and motion of other elementary particles of which matter is composed. For this reason a brief discussion of the fundamental processes governing the behavior of these elementary particles is of value in the study of electron tubes and their applications.

1-1. Excitation, Ionization, and Radiation.—The mass of the electron is greatly exceeded by that of an atom, the lightest atom, hydrogen, having a mass approximately eighteen hundred times that of the electron. The major portion of the mass of an atom is accounted for by the *nucleus,* a stable assemblage of charged and neutral particles having a net positive charge equal to the atomic number of the element. The atom also contains relatively loosely bound electrons, normally equal in number to its atomic number. The normal atom is therefore neutral.

Experiments show that, in addition to possessing kinetic energy, an atom is capable of absorbing energy internally. The internal energy appears to be associated with the configuration of the particles of which the atom is composed. The internal energy can be altered only in discrete quantities, called *quanta,* and hence the atom can exist only in definite stable *states,* which are characterized by the internal energy content. Under ordinary conditions an atom is most likely to be in that state in which the internal energy is a minimum, known as the *normal state.* If the internal energy of the atom exceeds that of its normal state, it is said to be *excited.* Excitation may be caused in a number of ways, among which is collision of the atom with rapidly moving positive or negative particles, which may give up some or all of their kinetic energy to the atom during the collision. A limiting case of excitation is *ionization,* in which the energy absorbed by the atom is sufficient to allow a loosely bound electron to leave the atom against the electrostatic force which tends to hold it within the atom. An atom that has lost one or more electrons is said to be *ionized* and is one type of *positive ion.* It is possible for excitation or ionization to take place in successive steps by the absorption of two or more quanta of energy.

The return of an excited atom to a state of lower energy content is usually accompanied by electromagnetic radiation. Since the energy of the atom can have only discrete values, the radiated electromagnetic energy corresponding to a change from one given energy state to another of lower energy is always associated with the releasing of a definite quantum of electromagnetic energy, called a *photon.* The frequency of the radiated energy is determined by the relation $W_1 - W_2 = h\nu$, in which W_1 and W_2 are the values of the internal energy of the atom in the initial and final states; h is a universal constant called *Planck's constant,* 6.55×10^{-27} erg-sec; and ν is the frequency of the radiated energy in cycles per second. From the form of this relation it is seen that the quantity $h\nu$ is the energy of the emitted photon. Each line of the emission spectrum of an element represents the transition of atoms of the element from some energy state to another of lower internal energy.

In interacting with atoms and molecules, photons exhibit some of the characteristics of material particles, a photon behaving as though it were a bundle of energy. The collision of an atom with a photon whose energy is equal to the required change of internal energy may result in excitation of the atom. If the energy of the colliding photon is equal to or greater than that necessary to remove an electron from the atom, the collision may result in ionization of the atom.

A number of attempts have been made to give a picture of the structure of atoms that will account for the phenomena of excitation, ionization, and radiation. The most successful of these, proposed by Bohr, is based upon the assumption that one or more electrons move about the central nucleus of an atom in a manner similar to the motion of the planets about the sun. To account for the observed definite values of internal energy, it was assumed that the electrons can move only in certain orbits and that the internal energy of the atom is increased when one or more electrons are abruptly displaced from given orbits to others at a greater distance from the nucleus. Radiation was assumed to result when one or more electrons jump from given orbits to others nearer to the nucleus. Any such picture is valuable principally in its ability to explain observed phenomena and to predict others. The complexity of atoms containing more than two orbital electrons limited the usefulness of the Bohr picture of the atom to the hydrogen and helium atoms.

The phenomena of excitation, ionization, and radiation are also observed in molecules. Because of the greater complexity of molecules, and the fact that their internal energy is partly associated with the vibrational and rotational motion of the atoms of which they are composed, a molecule has many more stable states than an atom of the same element.

1-2. Electron Volt.—Since the energy that a charged particle acquires in free space when accelerated by an electric field is equal to the product of the charge by the difference of potential between the initial and final positions, the difference in potential may be used as a measure of the gain in kinetic energy. Any quantity of energy may, in fact, be expressed in "electron volts." An *electron volt* is the amount of energy gained by an electron when accelerated in free space through a difference of potential of 1 volt. It is often convenient to express in electron volts the energy required to ionize or excite atoms or molecules.

1-3. Excitation and Ionization Potentials.—An *excitation potential* is the energy, expressed in electron volts, that must be given to an atom or molecule in order to cause a transition from a given state to one of higher internal energy. An *ionization potential* is the least energy, expressed in electron volts, that must be supplied to a normal or an ionized atom or molecule in order to remove an electron from the atom or molecule. Inasmuch as all atoms but hydrogen contain more than one electron, an atom or molecule may, in general, have more than one ionization potential. The first ionization potential applies to the removal of an electron from a normal atom or molecule; the second ionization potential applies to the removal of a second electron from an atom or molecule that has already lost one electron; etc. A less likely type of ionization is the simultaneous removal of two or more electrons. Table 1-I lists the first ionization potentials of some of the elements that are used in electron tubes. It should be noted that when ionization or excitation potentials are expressed in electron volts they indicate the minimum voltage that must be applied between two electrodes in order to cause ionization as the result of acceleration of electrons or other singly charged particles by the resulting field between the electrodes.

TABLE 1-I.—FIRST IONIZATION POTENTIALS IN ELECTRON VOLTS

Argon	15.69	Nitrogen	14.48	Sodium	5.12
Neon	21.47	Carbon dioxide	14.4	Rubidium	4.16
Helium	24.46	Mercury	10.38	Cesium	3.87
Hydrogen	13.53	Lithium	5.37	Magnesium	7.61
Oxygen	13.55	Potassium	4.32	Barium	5.19

1-4. Ionization.—The positively charged mass resulting from the removal of one or more electrons from an atom is only one of many types of ions. In general, an *ion* is an elementary particle of matter or a small group of such particles having a net positive or negative charge. Atoms or molecules that have lost one or more electrons, or that have picked up one or more extra electrons, and simple or complex groups of a number of atoms or molecules bearing excess positive or negative charge, are special examples of ions. This definition of an ion also includes such relatively simple particles as the electron and other elementary charged particles of which atomic nuclei are composed. The process of ionization, broadly defined, is the production of ions in gases, liquids, or solids. It may result from a number of causes, among which are

1. Collision of atoms or molecules with
   a. Electrons.
   b. Positive or negative ions of atomic or molecular mass.
   c. Excited atoms or molecules.
2. Collision of atoms or molecules with photons (photoelectric effect).
3. Cosmic radiation.
4. High temperatures in gases or vapors.
5. Chemical action.

1-5. Ionization by Moving Electrons.—One of the most important causes of ionization in electron tubes is the collision of rapidly moving electrons with atoms or molecules. In order that a single moving electron may ionize an atom or molecule it is necessary that the kinetic energy of the electron be at least equal to the first ionization potential of the atom or molecule. It is sometimes observed, however, that ions appear in a gas or vapor when the bombarding electrons have energy corresponding to the first excitation potential. The explanation of this is that the atom may be ionized in steps, each successive impact by an electron supplying sufficient energy to cause a transition to a state of higher energy. Thus, although the first ionization potential of mercury is 10.38 electron volts, ionization of mercury vapor by moving electrons begins when the colliding electrons have energy corresponding to the lowest excitation potential, 4.68 electron volts. Other more complicated processes may produce similar results. Precise measurements show that the voltages at which moving electrons begin to ionize a gas or vapor are very sharply defined.

1-6. Ionization by Positive Ions.—Ionization by positive ions is a more complicated phenomenon than ionization by electrons. One reason for this is that a positive ion which strikes a neutral atom or molecule is able to surrender not only its kinetic energy but also some or all of its own energy of ionization, thus reverting to a state of lower internal energy. A collision in which an ionized or excited atom or molecule transfers all or part of its energy of excitation to another atom or molecule is known as a *collision of the second kind.* The transferred energy may be used to excite or ionize the unexcited particle or may be converted into kinetic energy of one or both particles. In a mixture of gases a bombarding positive ion of one of the gases may ionize a neutral atom or molecule of the other, the difference in energies of ionization being supplied by or added to the kinetic energies of the two particles. Because of conservation of momentum, the large mass of the positive ion also complicates the process of ionization by positive ions. Ionization by bombardment of positive ions requires higher accelerating potentials than by electrons, and the potentials at which ionization begins are not sharply defined.

1-7. Amount of Ionization.—The ionization potential is a measure only of the minimum kinetic energy, below which a moving ion cannot ionize a normal atom or molecule by a single collision. It does not follow that every electron that acquires this amount of energy will necessarily ionize a gas through which it moves. The likelihood of ionization, which differs for different gases, is a function of the energy of the bombarding particles. The amount of ionization produced in a gas or vapor by charges that are accelerated by electric fields in the gas may be most conveniently specified by the *ionization coefficient.* The ionization coefficient is defined as the number of ionizing collisions made by an ion per centimeter of advance through the gas. It differs for different gases and for different types of ions and is a function of the gas pressure and the electric field strength. The ionization factor varies with electric field strength because increase of field strength increases the energy acquired by an ion between collisions. It varies with gas pressure because increase of gas density increases the likelihood that an ion will strike a gas particle in a given distance, but decreases the distance it moves and hence the energy it acquires between successive collisions with gas particles. Because of these conflicting effects, the ionization factor passes through a maximum value as the pressure is raised from a low value.

1-8. Photoionization.—In its narrower sense, the photo-electric effect is the release of electrons from the surface of a solid by light or other electromagnetic radiation. In its broader sense the photoelectric effect is the ionization of an atom or molecule by collision with a photon and may take place not only at the surface of a solid, but throughout a gas, liquid, or solid. The photoelectric effect and its applications will be discussed in detail in Chap. 13. The principles that govern ionization by collision also apply to photoelectric ionization. The energy of a single incident photon $h\nu$ must be at least equal to the first ionization potential of the atom. Impact of photons with atoms results not only in ionization but also in excitation. Ionization may occur in successive steps by a rapid sequence of impacts of successive photons. Excitation by photons is the inverse process to radiation, just as collision of the second kind is the inverse phenomenon to excitation or ionization by collision. Absorption spectra are an indication of the conversion of radiant energy into energy of excitation or ionization.

The frequencies of photons capable of ionizing most elements lie outside of the visible spectrum. The exceptions to this rule are the alkali metals, which are therefore used in light-sensitive electron tubes.

1-9. Ionization by Cosmic Rays.—The exact constitution of cosmic rays is still the basis of much scientific controversy. All experiments seem to indicate, however, that the primary source is outside of the earth and its atmosphere. They appear to be either electromagnetic radiation, of very short wave length, or charged particles that move with extremely high velocity. Ionization by cosmic rays at the surface of the earth results mainly from secondary rays formed in the outer atmosphere and appears to consist of both photoelectric ionization and ionization by collision. It is of importance in connection with glow and arc discharges because it is one of the sources of initial ionization that is necessary to the formation of glows in cold-cathode discharge tubes.

1-10. Space Charge and Space Current.—A group of free charges in space is called *space charge*. If the charge is of one sign only or if the density of charge of one sign in a given volume exceeds that of the other sign, the charge will give rise to an electrostatic field. The relation between the net charge within a volume and the resulting electrostatic flux is given by Gauss's law, which states that the electric flux through any surface enclosing free charges is equal to 4π times the sum of the enclosed charges.[1]

The movement of free charges in space constitutes a current, called *space current*. The current per unit area is equal to the product of the charge density and the velocity normal to the area. The conventional direction of current is in the direction of the motion of positive charges and opposite in direction to the motion of negative charges.

1-11. Deionization.—The rapid disappearance of the products of ionization is necessary for the proper functioning of certain types of electron tubes containing gas or vapor. The removal of ions from a volume of gas or vapor takes place in four ways:

1. Volume recombination.
2. Surface recombination.
3. Action of electric fields.
4. Diffusion.

[1] PAGE, L., and ADAMS, N. I., "Principles of Electricity," pp. 18, 41, D. Van Nostrand Company, Inc., New York, 1931.

There is good experimental evidence that the direct recombination of electrons with positive ions is of relatively rare occurrence in electrical discharges. Volume recombination results mainly from the attachment of electrons to neutral gas molecules to form the heavier and slower-moving negative ions, which subsequently combine with positive ions. The likelihood of attachment of electrons to neutral molecules decreases with increase in temperature, decrease in pressure, and increase in field strength. The rate of recombination of positive and negative ions is proportional to the product of the two ion densities, the constant of proportionality being called the *coefficient of recombination.* The coefficient of recombination is different for different gases.

Surface recombination occurs at conducting walls of the tube that contains the ionized gas, at the electrodes, and at surfaces of any other conducting solids that project into the ionized gas. Recombination takes place as the result of charges of opposite sign that are induced on conducting surfaces or pulled out by strong fields at the surfaces. On insulating surfaces or on conductors that are electrically isolated, charges may also accumulate and build up an electric field that repels charges of the same sign and attracts those of opposite sign. Surface recombination is one of the main factors affecting deionization in glow- and arc-discharge tubes and in circuit breakers of the "deion" type.

The electric field in the vicinity of the electrodes carries ions to the electrode surfaces, where surface recombination may take place. Within the main part of the gas, the electric field can change the ion density in a given small volume only if the ion density is not uniform throughout the tube, or at least in the vicinity of the volume under consideration. If the ion density is uniform, the field will sweep as many new ions into one side of the volume as it removes from the other. If an ion density gradient does exist, the action of the field may either increase or decrease the density in the given volume. An example of deionization by electric fields is the removal of ions in glow- and arc-discharge tubes when applied potentials are reduced below the value necessary to maintain a glow or arc.

Diffusion of ions results from the fact that ions, like gas molecules, have a random motion which carries them from point to point. If the ion density in an element of volume is greater than in adjacent elements, then on the average more ions leave the given element than enter it; if, on the other hand, the ion density is uniform throughout the region under consideration, then on the average just as many ions enter a given element of volume in unit time as leave it. Thus, deionization by diffusion, like deionization by electric fields, is dependent upon nonuniform density. Because of surface recombination and nonuniform production of ions throughout the tube, ion density differences are set up in glow- and arc-discharge tubes which enable diffusion and electric fields to be effective deionizing agents.

1-12. Free Electrons in Metals. Electron Affinity.—Some of the electrons of metallic atoms are very loosely bound to the atoms. When groups of such atoms are massed together into solids or liquids, the loosely bound electrons can readily pass from atom to atom. Because of their random temperature velocity these *free electrons* are constantly moving about within the mass, and, although at some particular instant they may be loosely bound to particular atoms, on the average they experience no force in any particular direction. It is these electrons that make metallic conduction possible and that play indispensable roles in thermionic emission.

An electron that happens to pass through the surface of the metal in the course of its random motion will, while it is still close to the surface, induce a positive charge on the surface of the metal. This induced charge results in a force that tends to return the electron into the metal, the so-called *image force.* In order to escape from the metal the electron must give up a certain amount of kinetic energy. The kinetic energy that an electron loses in passing through the surface of a metal and far enough away to be beyond the range of the image forces is called the *electron affinity* of that metal and is represented by the symbol w.

The electron affinity is different for different materials and varies greatly with the condition of the surface and with impurities contained in the substance, particularly at the surface. Schottky has shown theoretically that the electron affinity should be reduced by strong electric fields at the surface and that it should vary from point to point on a given surface because of microscopic irregularities, being smaller at projections and larger in hollows.[1] Table 1-II lists representative values of electron affinity for a number of metals commonly used in electron tubes.

TABLE 1-II.—ELECTRON AFFINITY w IN ELECTRON VOLTS

Tungsten	4.52	Magnesium	2.7
Platinum	5.0	Nickel	4.0
Tantalum	4.1	Sodium	1.9
Molybdenum	4.3	Mercury	4.4
Carbon	4.5	Calcium	2.5
Copper	4.0	Barium	2.0
Thorium	3.0	Thorium on tungsten	2.63
Oxide-coated nickel	0.5 to 1.5		

1-13. Contact Difference of Potential.[2]—A potential difference, called *contact difference of potential,* exists between the surfaces of two

[1] SCHOTTKY, W., *Physik. Z.*, **12,** 872 (1914), **20,** 220 (1919); *Ann. Physik,* **44,** 1011 (1914); *Z. Physik,* **14,** 63 (1923).

[2] An interesting discussion of contact potential and other small effective electrode voltages is given by R. M. Bowie, *Proc. I.R.E.*, **24,** 1501 (1936).

different metals that are in contact or are connected through an external circuit, as shown in Fig. 1-1. Contact difference of potential results from the fact that the electron affinities of the two metals differ. It is very nearly equal to the difference between the electron affinities divided by the charge of an electron. To prove this, suppose that an electron starts from any point a close to the surface of one of the metals, passes into this surface, thence around the circuit, and out through the surface of the second metal to any point b close to its surface. In passing into the first metal the electron does work equal to $-w_1$, the electron affinity of that metal. In crossing the junction between the metals it does a small amount of work because of the Peltier potential difference, which always exists at the junction of two different metals and which is also a function of the electron affinities of the metals. In passing out of the surface of the second metal it does work equal to w_2, the electron affinity of that metal. If the small amount of energy that the electron loses as the result of impacts with atoms and molecules in the metals (I^2R loss) is neglected and the Peltier potential difference is called V_P, the total amount of work done is $\mathcal{E}V_P + w_2 - w_1$, where $\mathcal{E}$ is the charge of an electron. $\mathcal{E}V_P$ may be shown to be small in comparison with $w_2 - w_1$, and so the work done is approximately $w_2 - w_1$. Since the potential difference between two points may be defined as the work done in moving a unit charge between the points, it follows that a difference of potential approximately equal to $(w_2 - w_1)/\mathcal{E}$ exists between any two points a and b close to the surfaces of the two metals. If the connection between the metals is made through an external circuit containing one or more additional metals, the proof is altered only in respect to the additional Peltier potential differences, which are negligible.

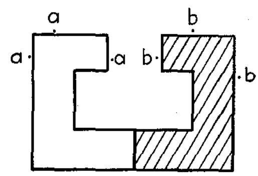

FIG. 1-1.—Production of contact difference of potential by metals of unequal electron affinities.

Inasmuch as the various electrodes in electron tubes may be made of different metals, contact potentials may exist. The contact potential differences may be of the order of 1 to 4 volts, as seen from Table 1-II, and must sometimes be taken into account when applied voltages are small or when high accuracy is necessary in the analysis of electron tube behavior.

1-14. Emission of Electrons and Other Ions from Solids.—The presence of ions in a given volume may result not only from ionization processes in that volume but also from the introduction of ions produced or existing elsewhere. The mechanism by which ions are introduced may be diffusion, action of electric fields, or the emission of electrons or positive ions from the surfaces of solids or liquids. There are five ways in which ions may be emitted from the surfaces of solids or liquids:

1. Thermionic emission.
2. Photoelectric emission.
3. Secondary emission.
4. Field emission.
5. Radioactive disintegration.

Thermionic and photoelectric emission will be discussed in detail in later chapters and therefore need no further consideration at this point.

Secondary Emission.—When ions or excited atoms impinge upon the surface of a solid, electrons may be ejected from the surface. These are called *secondary electrons*, and the phenomenon is termed *secondary emission.* Some secondary emission as the result of electron bombardment appears to take place when the energy of the impinging electrons is less than the electron affinity of the emitter. For this reason it seems likely that the energy that makes possible the escape of a secondary electron is obtained not only from the impinging electron, but also from the thermal energy of the emitter. The number of secondary electrons ejected per primary electron increases with the velocity of the primary electrons and may become great enough to have an appreciable effect upon the behavior of electron tubes at accelerating voltages as low as 5 to 10 volts. At several hundred volts the emission passes through a maximum and then continues to fall with further increase of accelerating voltage. This may be because the primary electrons penetrate farther into the solid and transfer most of their energy to electrons that are so far from the surface that they collide with atoms or molecules before reaching it. The number of secondary electrons emitted per primary electron is in general higher for surfaces having a low electron affinity. It also depends upon the condition of the emitting surface, being reduced by degassing the emitter and by carbonizing the surface. A single primary electron may eject as many as 8 or 10 secondary electrons. The primary electrons may be absorbed, reflected, or scattered by the surface. The number of secondary electrons released by a single impinging particle is less for positive-ion bombardment than for electron bombardment, and emission does not appear to take place unless the energy of the impinging ions exceeds the electron affinity of the emitter. The phenomenon of secondary emission is an important one in all types of electron tubes.

Field Emission.—Field emission is the emission of electrons as the result of intense electric fields at a surface. It is probably an important factor in the operation of certain types of arc discharges. The field strengths required to pull electrons through surfaces at ordinary temperatures are so great that the phenomenon is difficult to produce by direct means.

Emission Resulting from Radioactive Disintegration.—The emission of positive or negative ions in the disintegration of radioactive substances is of importance in glow- and arc-discharge tubes because the presence of minute traces of radioactive materials in the tube walls and electrodes may thus produce a small amount of residual ionization, which makes possible the initial flow of current.

1-15. Electron Dynamics.—The motion of ions, including electrons, is governed by the same laws as the motion of larger masses that can be observed directly. Analyses of the dynamics of masses are based primarily upon Newton's second law, which may be stated symbolically by the equation

$$f = ma \tag{1-1}$$

in which f is the force in dynes acting upon a mass of m grams and a is the resulting acceleration in centimeters per second per second. Aside from forces resulting from the collision of charged particles with other charged or uncharged masses, the forces acting upon charged particles are electrostatic and electromagnetic. An *electric* or a *magnetic* field is said to exist in a region in which electric or magnetic forces, respectively, act. The *electric intensity* (*electric force*) at any point is a vector quantity which is given, both in magnitude and in direction, by the force (mechanical) per unit positive charge which would act on a charged particle placed at this point. The *magnetic intensity* (*magnetic force*) at a point may be defined as the vector quantity which is measured in magnitude and direction by the force (mechanical) that would be exerted on a unit magnetic pole placed at the point.

1-16. Motion of an Electron with Zero Initial Velocity in a Uniform Electric Field.—In the simplest case commonly encountered in electron tubes, the electrostatic force acting upon an ion results from the application of a potential difference to two parallel plane electrodes whose area is large in comparison with their separation. Except near the edges, the electric intensity between such plates is constant throughout the space between them. Since difference of potential between two points may be defined as the work done in moving a unit charge between the points, it follows that the potential difference is equal to

$$E = \int_0^d F\,dx \qquad \text{e.s.u. (statvolts)} = 300 \int_0^d F\,dx \qquad \text{volts} \tag{1-2}$$

where F is the electric intensity in e.s.u. and d is the electrode spacing in centimeters. Since F is constant,

$$E = F \int_0^d dx = Fd \qquad \text{e.s.u.} \tag{1-3}$$

and

$$F = \frac{E}{d} \qquad \text{e.s.u.} \tag{1-4}$$

The force exerted upon an electron between two such plates is equal to the force exerted upon a unit charge times the charge of the electron:

$$f_e = \frac{E\mathcal{E}}{d} \qquad \text{dynes} \tag{1-5}$$

where E is expressed in e.s.u. (statvolts), $\mathcal{E}$ in e.s.u. (statcoulombs), and d in centimeters. It follows from Eqs. (1-1) and (1-5) that the acceleration of the electron is

$$a_e = \frac{f_e}{m_e} = \frac{E\mathcal{E}}{m_e d} \qquad \text{cm/sec}^2 \tag{1-6}$$

in which m_e is the mass of the electron in grams. Since, by definition, acceleration is the rate of change of velocity, the velocity at any instant after the electron starts moving under the influence of the field is

$$v_e = \int_0^t a_e\, dt = \frac{E\mathcal{E}}{m_e d} \int_0^t dt = \frac{E\mathcal{E}t}{m_e d} \qquad \text{cm/sec} \tag{1-7}$$

The distance moved by the electron in the time t is

$$s = \int_0^t v_e\, dt = \frac{E\mathcal{E}}{m_e d} \int_0^t t\, dt = \frac{E\mathcal{E}}{2m_e d}\, t^2 \qquad \text{cm} \tag{1-8}$$

From Eq. (1-8) it follows that the time taken for the electron to move from one electrode to the other under the sole influence of the electric field is

$$t_d = d\sqrt{\frac{2m_e}{E\mathcal{E}}} \qquad \text{sec} \tag{1-9}$$

The velocity with which it strikes the positive plate is

$$v_d = \frac{E\mathcal{E}t_d}{m_e d} = \sqrt{\frac{2E\mathcal{E}}{m_e}} \qquad \text{cm/sec} \tag{1-10}$$

The energy with which it strikes and which is converted into heat in the positive electrode is

$$K.E. = \tfrac{1}{2}m_e v_d^2 = E\mathcal{E} \qquad \text{ergs} \tag{1-11}$$

Equation (1-11) might have been obtained directly, since the difference of potential between the electrodes is the energy acquired by a unit charge moved under the sole influence of the field and, if the charge does not collide with other particles on the way, this energy can appear only in kinetic form.

1-17. Motion of an Electron in a Uniform Electric Field. Initial Velocity Parallel to the Field.—If an electron leaves one of the plates with initial velocity v_o parallel to the field, the velocity at any instant thereafter is

$$v_e = v_o \pm a_e t \qquad \text{cm/sec} \tag{1-12}$$

in which the minus sign is used if the direction of the electric force is such as to reduce the initial velocity. The distance moved in the time t is

$$s = \int_0^t (v_o \pm a_e t)dt = v_o t \pm \tfrac{1}{2}a_e t^2 \qquad \text{cm} \tag{1-13}$$

If the field reduces the initial velocity, the maximum distance moved can be found by differentiating Eq. (1-13). The time taken for the electron to move through this distance is found by equating ds/dt to zero.

$$0 = \frac{ds}{dt} = v_o - a_e t_{\max} \tag{1-14}$$

$$t_{\max} = \frac{v_o}{a_e} \qquad \text{sec} \tag{1-15}$$

$$s_{\max} = \frac{v_o^2}{a_e} - \frac{v_o^2}{2a_e} = \frac{v_o^2}{2a_e} \qquad \text{cm} \tag{1-16}$$

The electron will reach the second electrode if

$$s_{\max} \equiv \frac{v_o^2}{2a_e} \equiv \frac{m_e d}{2E\mathcal{E}} v_o^2 \geqq d \tag{1-17}$$

that is, if

$$\tfrac{1}{2}m_e v_o^2 \geqq E\mathcal{E} \tag{1-18}$$

Equation (1-18) follows directly from the law of conservation of energy, since the electron can reach the second electrode only if its initial kinetic energy $\frac{1}{2}m_e v_o^2$ equals or exceeds the energy it would lose in moving between the electrodes, *i.e.*, $E\mathcal{E}$.

It also follows from the law of energy conservation that energy gained by a charged particle while moving in vacuum under the action of an electric field must be supplied by the source of potential applied to the electrodes. Conversely, energy given up by an ion in moving in an electric field is returned to the source, or converted into I^2R loss in conductors joining the electrodes to the source, or into heat of impact if the electron strikes an electrode or other surface.

1-18. Motion of Electrons in Nonuniform Electric Fields.—In general, the electric intensity in electron tubes is not uniform, and so the integration of Eqs. (1-2), (1-7), (1-8), and (1-13) is less simple. When space charge becomes appreciable, the motion of individual electrons is also affected by the fields set up by other electrons. This important phenomenon will be discussed in detail in Chap. 2.

Inspection of the methods used in deriving Eqs. (1-5) to (1-18) shows that the equations may be applied to other ions than electrons by making suitable changes in the values of charge and mass.

1-19. Importance of Transit Time.—The time taken for electrons and other ions to move between electrodes is so small that it may be neglected in many analyses of the operation of electron tubes (see Probs. 1-1 and 1-2). It cannot be neglected, however, when the electrode voltages alternate at frequencies so high that the time of transit is of the same order of magnitude as the periods of the voltages. It is also of importance in the study of the deionization of gas- or vapor-filled tubes.

1-20. Electric Field Normal to Initial Direction of Motion.—Up to this point it has been assumed that the initial velocity of the electron is

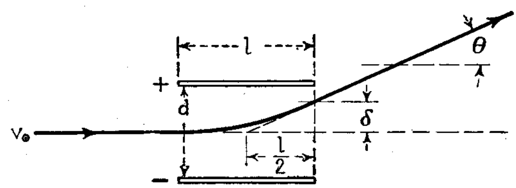

FIG. 1-2.—Deflection of electron beam by electric field.

parallel to the electric field. Under this assumption there is no change in the direction of motion in vacuum. In cathode-ray oscillograph and television tubes, electron beams are deflected by electric fields perpendicular to the initial direction of motion. The arrangement is essentially that shown in Fig. 1-2. Electrons enter the space between the deflecting electrodes with a velocity v_o parallel to the electrode surfaces, and are deflected by the electric field between the electrodes, which have a length l, a separation d, and a potential difference E.

The acceleration produced by the field is normal to the initial velocity and its magnitude is given by Eq. (1-6) as $E\mathcal{E}/m_e d$ cm/sec². Since the velocity normal to the electric field remains constant, the time taken for an electron to move through the deflecting field, which is assumed to be uniform between the plates and zero on either side, is l/v_o. The vertical displacement at the point where the electron leaves the plates is given by Eq. (1-8). It is

$$\delta = \frac{E\mathcal{E}}{2m_e d}t^2 = \frac{E\mathcal{E}l^2}{2v_o^2 m_e d} \qquad \text{cm} \tag{1-19}$$

The vertical velocity when the electron leaves the deflecting plates is given by Eq. (1-7)

$$v_y = \frac{E\mathcal{E}t}{m_e d} = \frac{E\mathcal{E}l}{v_o m_e d} \qquad \text{cm/sec} \tag{1-20}$$

The horizontal velocity, which is unaffected by the field, is still v_o. Therefore, the final direction of motion makes an angle with the initial direction given by the relation

$$\tan\theta = \frac{v_y}{v_o} = \frac{E\mathcal{E}l}{v_o^2 m_e d} \tag{1-21}$$

But by Eq. (1-19),

$$\frac{\delta}{\frac{1}{2}l} = \frac{E\mathcal{E}l}{v_o^2 m_e d} = \tan\theta \tag{1-22}$$

Examination of Fig. 1-2 shows, therefore, that after deflection the electrons move as though they had passed through a point midway between and midway along the deflecting plates.

1-21. Motion of an Electron in a Magnetic Field Normal to the Initial Velocity.—Like a current-carrying conductor, an electron moving normal to a magnetic field experiences a force perpendicular to the field and to the direction of motion of the charge. The magnitude of the or ce is given by the relation

$$f_h = B\mathcal{E}'v \qquad \text{dynes} \tag{1-23}$$

in which B is the flux density in gauss, $\mathcal{E}'$ is the charge of an electron in *electromagnetic* units, and v is the velocity in centimeters per second.

If the electronic charge is expressed in e.s.u., Eq. (1-23) may also be written

$$f_h = \frac{B\mathcal{E}v}{3 \times 10^{10}} \qquad \text{dynes} \tag{1-24}$$

The action of a magnetic field differs from that of an electric field in that the force on a moving charge in an electric field is always parallel to the field, whereas the force on a moving charge in a magnetic field is normal to the field and to the instantaneous velocity. If an electron enters a uniform magnetic field with an initial velocity v_o normal to the field, it will be deflected at right angles to the field by the force $B\mathcal{E}'v_o$ dynes. Since the acceleration is normal to the velocity, the speed remains constant. If ρ is the instantaneous radius of curvature, the radial acceleration is v_o^2/ρ. Therefore, by Eq. (1-1),

$$B\mathcal{E}'v_o = \frac{m_e v_o^2}{\rho} \tag{1-25}$$

and

$$\rho = \frac{v_o m_e}{B\mathcal{E}'} = 3 \times 10^{10}\,\frac{v_o m_e}{B\mathcal{E}} \qquad \text{cm} \tag{1-26}$$

Since B is assumed to be constant, the electron moves with constant speed along a path of constant radius of curvature, *i.e.*, along a circular path, in a plane perpendicular to the field. The dependence of the radius of curvature upon the velocity and upon the mass makes possible the separation of charged particles of different velocities or masses.

If the initial velocity also has a component parallel to the field, the electron will describe a spiral path around an axis parallel to the field. A similar spiral motion results if the initial velocity is parallel to the field but some other force, such as repulsion between two or more electrons, produces an acceleration normal to the field. This principle may be used in preventing the spreading of and in focusing a beam of electrons. Electron beams may also be focused by the use of nonuniform electric fields such as exist between adjacent cylinders of unequal diameter when a difference of potential exists between them.[1] These methods are used in focusing electron beams in cathode-ray oscillograph and television tubes and in electron microscopes (see Secs. 2-4 and 15-17).

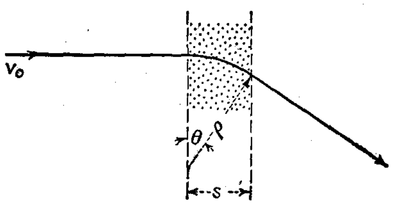

FIG. 1-3.—Deflection of electron beam by magnetic field.

In cathode-ray oscilloscope and television tubes, electron beams may be deflected by magnetic fields normal to the initial velocity of the electrons that comprise the beams. Although a general analysis is complicated, a useful expression for the beam deflection angle may be readily derived under the assumption that the field is uniform throughout a distance s and zero elsewhere, as shown in Fig. 1-3. Inspection of Fig. 1-3 shows that

$$\sin \theta = \frac{s}{\rho} \tag{1-27}$$

in which ρ, the radius of curvature of the circular path within the field, is given by Eq. (1-26). Substitution of Eq. (1-26) in Eq. (1-27) gives the relation:

$$\sin \theta = \frac{sB\mathcal{E}}{3 \times 10^{10} v_o m_e} \tag{1-28}$$

For small deflection angles, Eq. (1-28) simplifies to

$$\theta = \frac{sB\mathcal{E}}{3 \times 10^{10} v_o m_e} \tag{1-29}$$

In the range of θ in which Eq. (1-29) is valid, the path of the electrons

[1] See, for instance, I. G. MALOFF, and D. W. EPSTEIN, "Electron Optics in Television," McGraw-Hill Book Company, Inc., New York, 1939.

after deflection by the magnetic field is the same as though they had originated at the center of the region in which the field acts.

1-22. Crossed Electric and Magnetic Fields.—If an electron is sent through electric and magnetic fields that are perpendicular to each other and to the initial velocity of the electron, as shown in Fig. 1-4, the electron is acted upon by a force $\mathcal{E}F$ dynes caused by the electric field and a force $B\mathcal{E}v_o/(3 \times 10^{10})$ dynes caused by the magnetic field. These forces are both normal to the surfaces of the deflecting plates. If they are equal in magnitude and opposite in direction, the electron is undeflected. This is true if

$$\mathcal{E}F = \frac{B\mathcal{E}v_o}{3 \times 10^{10}} \tag{1-30}$$

or

$$v_o = 3 \times 10^{10} \frac{F}{B} \quad \text{cm/sec} \tag{1-31}$$

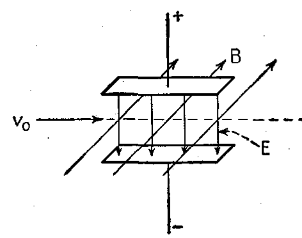

FIG. 1-4.—Path of an electron through balanced electric and magnetic fields.

in which F is measured in e.s.u. and B is measured in gauss. This phenomenon may obviously be used to measure the speed of electrons or other charged particles.

Problems

1-1. *a.* Find the time of transit of an electron between parallel plane electrodes having a separation of 0.2 cm and a potential difference of 250 volts.

$$\mathcal{E} = 4.8 \times 10^{-10} \text{ e.s.u.}$$

b. Find the energy delivered to the positive electrode by the electron.

1-2. *a.* An electron at the surface of a plane electrode is accelerated toward a second parallel plane electrode by a 200-volt battery, the polarity of which is reversed without loss of time 10^{-9} sec after the circuit is closed. If the electrode separation is 1.5 cm, on which electrode will the electron terminate its flight?

b. What will become of the kinetic energy that it acquires during its acceleration?

1-3. *a.* An electron having initial kinetic energy of 10^{-9} erg at the surface of one of two parallel plane electrodes and moving normal to the surface is slowed down by the retarding field caused by a 400-volt potential applied between the electrodes. Will the electron reach the second electrode?

b. What will become of its initial energy?

1-4. The electrons that comprise a beam of cathode rays are given their velocity v_o by means of a potential difference of 500 volts impressed between the electron source and an accelerating anode. Determine the difference of potential that must be impressed between two deflecting electrodes 3 cm long and 1 cm apart in order to deflect the beam through an angle of 20 degrees.

1-5. The electrons that comprise a beam of cathode rays are given their velocity v_o by means of a potential difference of 1500 volts impressed between the electron source and an accelerating anode. Determine the flux density that must exist throughout a distance of 2 cm in order to deflect the beam through an angle of 10 degrees.

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CHAPTER 2

THERMIONIC EMISSION; THE HIGH-VACUUM THERMIONIC DIODE

The operation of most electron tubes is dependent upon thermionic emission. The theory of thermionic emission is, therefore, of great importance in the study of electron tubes. It is the purpose of this chapter to discuss the basic principles of thermionic emission, the construction of practical emitters, and the flow of electron space current in high-vacuum tubes containing two electrodes.

2-1. Theory of Thermionic Emission.—Richardson's theory of the emission of electrons from hot bodies is in many respects analogous to the kinetic theory of vaporization.[1] Heat possessed by a metal is believed to be stored not only in the kinetic energy of random motion of atoms and molecules, but also in the kinetic energy of free electrons. As a result of collisions between electrons or between electrons and atoms or molecules, the speed and direction of motion of a given electron are constantly changing. The random motion of the electrons causes some of them to strike the inner surface of the metal. Electrons that arrive at the surface will escape from the metal if they have a component of velocity toward the surface equal to or greater than u_w, corresponding to a kinetic energy $\frac{1}{2}m_e u_w^2$ equal to the electron affinity w. The number of electrons that reach the surface in unit time with a normal component of velocity equal to or greater than u_w is proportional to the fraction of all the free electrons throughout the metal that have such velocities. At room temperatures this fraction is extremely small, and so no thermionic emission is detectable. As the temperature of the emitter is increased, however, the average velocity of the free electrons increases, and so the number having velocities equal to or greater than u_w is increased. This can be seen from the velocity-distribution curves of Fig. 2-1. These are theoretical curves so constructed that the area under a given curve between any two velocities u and $u + \Delta u$ is proportional to the fraction of the free electrons having velocities lying within this range at the temperature for which the curve is constructed. The area lying to the right of a given positive value of u, such as u_w, is proportional to the fraction of the electrons having velocities in excess of this value. Figure 2-1

[1] Richardson, O. W., *Proc. Cambridge Phil. Soc.*, **11**, 286 (1901); "Emission of Electricity from Hot Bodies," rev. ed., Longmans, Green & Company, New York, 1921.

shows that the fraction of the electrons having velocities toward the surface equal to or greater than u_w increases with temperature. Hence the rate at which electrons escape from the metal, *i.e.*, the emission current, increases with temperature. Observable emission currents are obtained at temperatures in excess of 1000°K. Emission current is also increased by reduction of electron affinity, since reduction of electron affinity lowers the velocity u_w and therefore increases the number of electrons whose velocity toward the surface exceeds u_w. This is clearly shown by Fig. 2-1, since reduction of work function w means that u_w is moved to the left and the area under the curve to the right of u_w is increased.

Electrons that escape will have resultant velocities made up of the excess perpendicular to the surface, plus the original components parallel to the surface, which are not altered by the surface forces. If the emitted electrons are not drawn away by an external field, they will form a

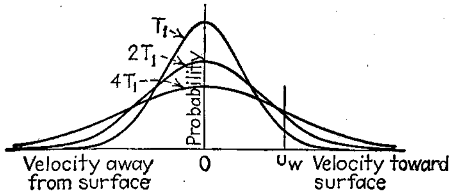


FIG. 2-1.—Maxwellian distribution curves of velocities normal to emitter surface at three temperatures.

space charge, the individual particles of which are moving about with random velocities. Because the initial average normal velocity of the electrons after emission is away from the surface and because of the mutual repulsion of like charges, electrons drift away from the surface. Collisions between electrons cause some of them to acquire velocity components toward the emitter, where they may reenter the surface with a gain of kinetic energy equal to the electron affinity. Another factor responsible for the return of electrons to the emitter is the electrostatic field set up by the negative space charge and, if the emitter is insulated, by the positive charge that it acquires as the result of loss of electrons. This field increases with the density of space charge, and equilibrium is established when only enough electrons can move away from the surface to supply the loss by diffusion of the space charge. If diffusion can then be prevented, just as many electrons return to the metal in unit time as leave it. Figure 2-2 gives a rough picture of the electron distribution under equilibrium conditions.

If a second, cold electrode is placed near the emitting surface in vacuum and connected to the emitter through a galvanometer, as shown in Fig. 2-3, the meter will indicate the small current resulting from the drift of electrons from the emitter to the second electrode. These electrons return to the emitter through the galvanometer and prevent the emitter from becoming positively charged. This phenomenon, first observed by Edison, is called the *Edison effect.* When the second electrode is made positive with respect to the emitter by the addition of a battery, as shown in Fig. 2-4, the current is increased. As the voltage is gradually raised, it is found that at any emitter temperature there is a more or less definite voltage beyond which the current is nearly constant, all emitted electrons being drawn to the collector. This current is called the *saturation current,* and the corresponding voltage, the *saturation voltage.* The lack of increase of current beyond saturation voltage is

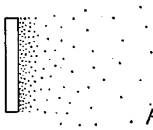


FIG. 2-2.—Distribution of electrons near an emitting surface.

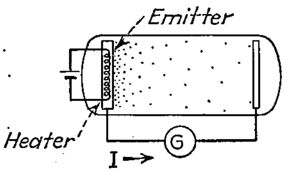


FIG. 2-3.—Flow of anode current as the result of diffusion of electrons from cathode to anode without the application of anode voltage (Edison effect).

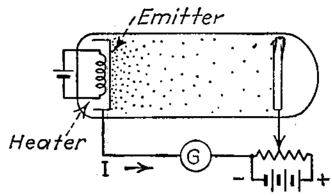


FIG. 2-4.—Use of anode voltage to increase anode current.

spoken of as *voltage saturation.* Saturation current varies with the temperature and electron affinity of the emitter. The negative emitter in Fig. 2-4 is called the *cathode;* and the positive collector, the *anode* or *plate.* An electron tube containing only a cathode and an anode is called a *diode.* Although the electrons move from cathode to anode, the current, according to convention, is said to flow from anode to cathode within the tube.

2-2. Richardson's Equation.—By means of classical kinetic theory and by thermodynamic theory, Richardson derived two slightly different equations for saturation current as a function of temperature.[1] It is not possible experimentally to determine which of Richardson's equations is correct, but this was later done theoretically by M. v. Laue, S. Dushman, and A. Sommerfeld. The equation that is now believed to be correct is

$$I_s = AT^2\epsilon^{-w/kT} \tag{2-1}$$

in which I_s is the saturation current per unit area of emitter, T is the absolute temperature, w is the electron affinity of the emitter, k is Boltzmann's universal gas constant, and A is a constant, probably universal for pure metals. The value of k is 8.63×10^{-5} electron volt/degree and the

[1] RICHARDSON, *loc. cit.*

theoretical value of A for pure metals is 60.2. The form of the curve that represents Richardson's equation, shown in Fig. 2-5*a*, is determined practically entirely by the exponential factor.

It is important to note that Richardson's equation holds only for the saturation current and that the anode voltage must, therefore, be high enough at all times so that all emitted electrons are drawn to the anode. If the anode voltage is fixed at some value E_1, while the temperature is raised, then at some temperature the current will begin to be limited by space charge in a manner similar to that when there is no accelerating voltage. Further increase of temperature will then have no effect upon the current. This temperature is called the *saturation temperature*, and the failure of the current to increase at higher temperature is spoken of as *temperature saturation*. If the emitter were homogeneous and the electrostatic field constant over the surface of the emitter, the advent

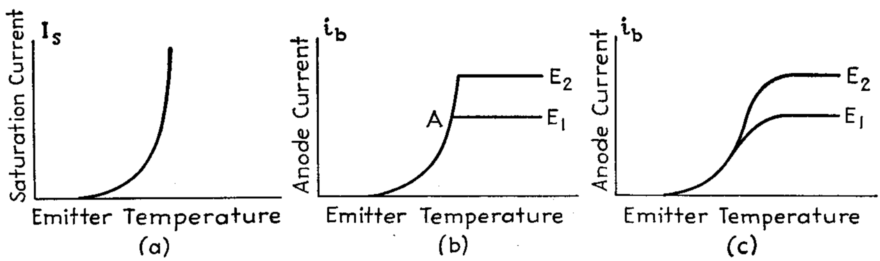


FIG. 2-5.—Curves of anode current *vs.* emitter temperature: (*a*) saturation emission current; (*b*) theoretical curves at two values of anode voltage; (*c*) experimental curves at two values of anode voltage.

of saturation would be abrupt, as indicated at point A of Fig. 2-5*b*. Actually, because of variations of temperature and electron affinity and of electrostatic field, saturation does not take place over the whole cathode surface at the same temperature, and so experimentally determined curves bend over gradually, as shown by the curves of Fig. 2-5*c*. At higher anode voltage E_2, saturation occurs at a higher temperature. If the voltage is increased with temperature, the current will continue to rise with temperature, as in Fig. 2-5*a*, until the temperature becomes sufficiently high to vaporize the emitter.

Since only those electrons which have relatively high energies can escape from the metal, thermionic emission necessarily results in the reduction of the average energy of the remaining electrons and molecules, and hence of the temperature of the emitter. Heat must be supplied continuously to the emitter in order to prevent its temperature from falling as the result of emission. The cooling effect of emission current is plainly visible in filaments in which the emission current is comparable with the heating current, as in the type 30 tube.

Richardson's equation shows that the emission current which can be obtained at any temperature varies inversely with the electron affinity. The exponential form of the equation shows that small changes in temperature or electron affinity may result in large changes of emission current. A 15 per cent reduction of electron affinity produces an eight- or tenfold increase of emission in the working range of temperature. The ratio of the emission current in milliamperes per square centimeter to the heating power in watts per square centimeter is called the *emission efficiency*. Emission efficiency increases with decrease of electron affinity.

A satisfactory practical source of thermionic emission must satisfy two requirements: it must have a high emission efficiency and it must have a long life. Thermal losses can be reduced by proper design of the emitter and by reduction of emitter temperature. (Cathodes of special design, which give very low thermal losses, can be used in gaseous discharge tubes. These will be discussed in Sec. 12-16.) The life of an emitter increases with the difference between the normal operating temperature and the vaporization or melting temperatures of the metal or metals of which it is constructed. Since low operating temperature is made possible by low electron affinity, it is evident that the choice of emitters of low electron affinity is favorable to long life, as well as to high efficiency.

2-3. Pure Metallic Emitters.—Pure metals having low electron affinities, such as the alkali metals or calcium, cannot be used as emitters in electron tubes because they vaporize excessively at temperatures at which appreciable emission is obtained. Only two pure metals, tantalum and tungsten, are suitable for use as practical emitters. Although the electron affinity of tantalum is lower than that of tungsten, tantalum is more sensitive to the action of residual gases and has lower vaporization temperature. Tantalum is therefore seldom used. Pure metallic emitters are now used only in large high-voltage (above 3500 volts) power tubes, in which they are found to have longer life than the special emitters that are used successfully in small tubes.

2-4. Thoriated Tungsten Emitters.—The presence of impurities in a metal may produce a marked change in the value of its electron affinity. This is usually attributed to the formation of thin layers of these impurities at the surface. Such a layer may produce very high fields at the surface by virtue of the fact that it may be electropositive or electronegative relative to the main metal. Thus the presence of an absorbed layer of oxygen, which is electronegative with respect to tungsten, results in a field that opposes the emission of electrons and therefore increases the electron affinity of tungsten. The presence, on the other hand, of a monatomic layer of thorium atoms or ions on the surface of tungsten reduces its electron affinity remarkably. It is of interest to note that the electron affinity of thoriated tungsten may be even lower than that of pure thorium (see Table 1-II).

The reduction of the electron affinity of tungsten as the result of introduction of small amounts of thorium was first observed by Langmuir in 1914 in the course of a study of the properties of tungsten filaments.[1] Thorium oxide is introduced into the tungsten in the course of its manufacture. Subsequent high temperature converts a portion of the thorium oxide into metallic thorium, which diffuses to the surface. Investigations by Dushman, Becker, and others[2] indicated that the lowest value of w and the highest value of the constant A in Richardson's equation are obtained when the tungsten is completely, or perhaps very nearly, covered with a single layer of thorium atoms.

Thoriated tungsten shows no increase of emission over that of pure tungsten until it is *activated*. The activation process is performed after evacuation of the tube. It consists first in "flashing" the emitter for a few moments at a temperature of 2500 to 2800°K. This high temperature reduces some of the thorium oxide to thorium. The temperature is then kept for some minutes at about 2200°K, which allows the metallic thorium to diffuse to the surface. The best value of diffusing temperature is determined by the rates of diffusion of thorium to the surface, and of evaporation from the surface. If the temperature is too high, the evaporation exceeds the diffusion, resulting in deactivation. The emitter is normally operated at temperatures that do not exceed 2000°K, which is sufficiently low so that evaporation of thorium from the surface is negligible. If the emitter is accidentally operated at such a high temperature that the whole supply of thorium diffuses to the surface and evaporates, it can be reactivated by repeating the original activation process. This may be done several times before all the thorium oxide is used up.

A useful tool in the study of the phenomenon of activation is the "electron microscope."[3] This consists of the emitter and means for accelerating the electrons and for focusing them upon a screen[4] which fluoresces under the impact of electrons. It has been shown that the

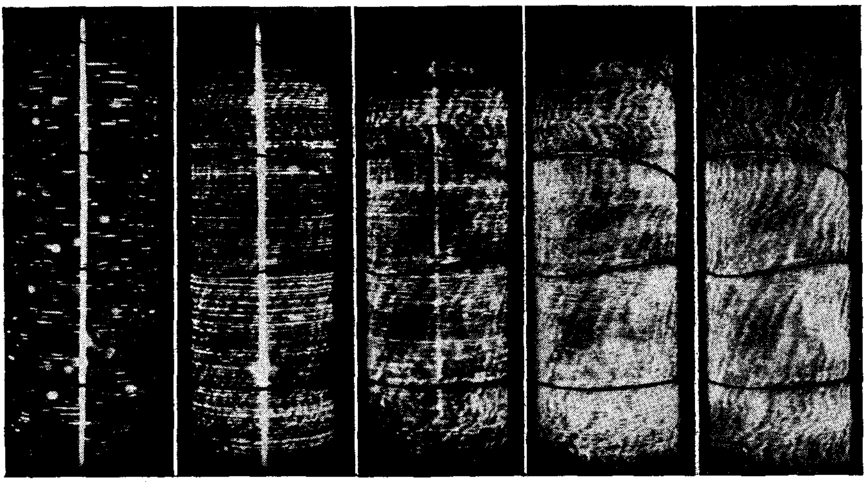
[1] LANGMUIR, I., *Phys. Rev.*, **4**, 544 (1914).

[2] DUSHMAN, S., and EWALD, J., *Phys. Rev.*, **29**, 857 (1927); BECKER, J. A., *Trans. Am. Electrochem. Soc.*, **55**, 153 (1929).

[3] KNOLL, M., and RUSKA, E., *Ann. Physik*, **12**, 607 (1932). The electron microscope has many applications besides that mentioned here. Recent instruments may be used in place of ordinary microscopes in the study of matter and give higher magnification than can be attained with light. For a survey and a bibliography of this subject, see R. P. Johnson, *J. Applied Physics*, **9**, 508 (1938).

[4] See, for instance, I. G. MALOFF and D. W. EPSTEIN, "Electron Optics in Television," McGraw-Hill Book Company, Inc., New York, 1938.

action of nonuniform electromagnetic and electrostatic fields upon electron beams is similar to the action of lenses upon light.[1] Thus it is possible to obtain on the screen a sharp enlarged image which shows clearly the individual points of emission of the cathode.[2] Similar results are achieved by use of a straight filament at the axis of a cylindrical glass tube, the inner surface of which is covered with a fluorescent material.[3] The coated surface, which acts as the anode, is maintained at a positive



(a) (b) (c) (d) (e)

FIG. 2-6.—Typical activation behavior of thoriated tungsten. Image (a) immediately after 10 sec. at 2800°K, (b) after additional 4 min. at 1850°K, (c) after 20 min. at 1850°K, (d) after 30 min. at 1850°K, (e) after 70 min. at 1850°K. All pictures were made at 1200°K. The decreasing exposure time is evidenced by reduction of apparent brilliance of the filament. (*Courtesy of R. P. Johnson.*)

potential of several thousand volts by means of a wire helix coiled inside the tube in contact with the coating. Electrons emitted by the filament are attracted radially toward the anode coating, where they produce a magnified image of the electron emission at the surface of the filament. Figure 2-6 gives a series of photographs of the screen of such a tube, showing the activation of thoriated tungsten. (The bright vertical line is caused by light from the filament, and the dark lines by the shadow of the helix.)

[1] BUSCH, H., *Ann. Physik*, **81**, 974 (1926); MALOFF, I. G., and EPSTEIN, D. W., *Proc. I.R.E.*, **22**, 1386 (1934); EPSTEIN, D. W., *Proc. I.R.E.*, **24**, 1095 (1936); MALOFF and EPSTEIN, "Electron Optics in Television," *op. cit.*

[2] See, for instance, E. BRÜCHE and H. JOHANNSON, *Ann. Physik*, **15**, 145 (1932); M. KNOLL, *Electronics*, September, 1933, p. 243.

[3] JOHNSON, R. P., and SHOCKLEY, W., *Phys. Rev.*, **49**, 436 (1936). See also *Electronics*, January, 1936, p. 10, March, 1937, p. 23.

The presence of even small amounts of gas has a very destructive effect upon a thoriated tungsten emitter. This may result either from direct chemical action, such as oxidation, or from the removal of thorium from the surface by the bombardment of positive ions. The sensitiveness of thoriated emitters to the action of gases and the rate of evaporation of thorium may be greatly reduced by heating the emitter in an atmosphere of hydrocarbon vapor, which causes the formation of a shell of tungsten carbide. Because of the reduction of the rate of evaporation of the thorium, a carbonized emitter can be operated at a much higher temperature, with consequent increase of emission current and efficiency. At this higher temperature the increase of diffusion makes possible the con-

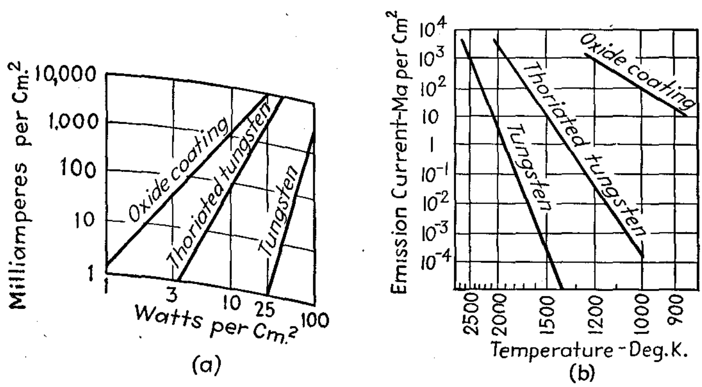


FIG. 2-7.—(a) Typical curves of emission current *vs.* heating power; (b) typical curves of emission current *vs.* emitter temperature.

tinuous replacement of thorium removed from the surface by the action of gas molecules or ions. Figure 2-7 shows that the emission efficiency of thoriated tungsten is much higher than that of pure tungsten and that a given emission may be obtained at a much lower temperature. Because of the lower electron affinity, higher emission efficiency, and longer life of oxide-coated emitters, thoriated tungsten emitters are now used very little in receiving tubes.

2-5. Oxide-coated Emitters.—By far the most widely used emitters in small high-vacuum tubes are oxide-coated emitters, first used by Wehnelt.[1] Although the process of manufacture of oxide-coated cathodes varies considerably, it consists, in general, in coating a core metal, usually nickel or alloys of nickel and other metals, with one or more layers of a mixture of barium and strontium carbonates (approximately 60 per cent barium carbonate and 40 per cent strontium carbonate). The carbonates

[1] WEHNELT, A., *Ann. Physik*, **14**, 425 (1904). For an excellent review of the subject of oxide-coated emitters see J. P. Blewett, *J. Applied Physics*, **10**, 668 and 831 (1939).

may be suspended in water, although a binder such as collodion or a mixture of one part of Zapon varnish in 20 parts of amyl acetate is usually used. The mixture may be applied to the core by spraying or by dipping or dragging the core through the mixture. When a thick coating is desired, the mixture is preferably applied in the form of several thin coatings heated sufficiently between applications to burn out the binder. After application of the carbonate coating, the emitter is mounted in the tube, which is then evacuated, and the emitter is heated electrically to a temperature of about 1400°K. The high temperature reduces the carbonates to oxides, the liberated carbon dioxide being removed by the pumps. The temperature is then lowered somewhat and voltage is applied to the anode for some time, during which the emission builds up to its proper value. The normal operating temperature ranges from 1000 to 1300°K.

Many experiments have been performed to determine what takes place during the activation process and from which part of the emitter electrons are emitted. Reduction of the oxides to pure metal may result from chemical reaction, from electrolysis of the oxides, or from the bombardment of positive ions formed in the gas between the anode and the emitter by electrons accelerated by the applied field. Perhaps all three of these processes occur. Free metal formed throughout the oxide diffuses toward the surface. Although particles of free metal are distributed throughout the coating in a completed emitter, most recent evidence appears to indicate that the emission takes place at the outer surface.

The electron affinity, saturation emission current, and emission efficiency of oxide-coated emitters are greatly dependent upon the manufacturing processes. The electron affinity ranges from 0.5 to 1.5 electron volts. Typical curves of saturation current *vs.* temperature and emission efficiency *vs.* heating power are shown in Fig. 2-7. Examination of these curves shows that the saturation current and emission efficiency are considerably higher than those of thoriated tungsten. The relatively low temperature at which oxide-coated cathodes can be operated is a distinct advantage in most applications.

The emission from oxide-coated cathodes is reduced or destroyed by the presence of oxygen, due to oxidation of the active metal or to removal of the active metal or even the complete coating by positive-ion bombardment. Another cause of damage to oxide-coated cathodes is the development of hot spots. Because of nonuniform activation of the emitter, emission is not uniform over the surface. The flow of emission current through the oxide coating, which has high resistance, raises its temperature. Since the temperature rise is greatest at points of the cathode at which the emission is high, the emission increases still more at these points. If the current is not limited by space charge, the action may become cumulative and the current and temperature increase to such an extent that the coating is removed. In filamentary cathodes the local rise in temperature may be so great as to melt the filament. Hot spots are most likely to occur at high anode voltages. The tendency toward the formation of hot spots is also increased by deactivation caused by the flow of excessive space current.

When full emission current is drawn from an oxide-coated emitter, the current first falls rapidly and then slowly approaches a steady value. This decay of current is thought to be caused by electrolytic removal of barium from the surface or by electrolytic deposition of oxygen on the surface. The initial emission is recovered if the emitter is heated without the flow of space current. The useful life of oxide-coated emitters, which is several thousand hours, is terminated by a rather sudden decay in emission to a very low value. This may be caused by evaporation of free barium and of the supply of barium oxide that furnishes free barium during the active life of the emitter. The useful life of a vacuum tube containing an oxide-coated cathode may also be terminated by the liberation of gas from the emitter. Schade has pointed out that the peak current that can be obtained from an oxide-coated cathodes greatly exceeds the steady current.[1] Transient peak currents of 25 amp/cm^2 have been observed from well-activated cathodes. The stable peak emission over an extended period is usually less than one-third of this value.

Formation of hot spots and the likelihood of ionization of gas emitted during the life of oxide-coated emitters make them unsuitable for use in high-voltage transmitting tubes.

2-6. Cesiated Tungsten Emitters.—A fourth type of emitter, not used commercially in thermionic tubes, is produced by depositing a monatomic layer of cesium on tungsten. Because the ionizing potential of cesium vapor is less than the electron affinity of tungsten, the tungsten removes an electron from a cesium atom which strikes it, leaving a positive ion which is held to the tungsten surface by the resulting electrostatic field. The force of adhesion is even greater if the tungsten is first covered with a monatomic layer of oxygen, which is electronegative with regard to tungsten. The strong electrostatic field between the cesium ions and the tungsten or oxygen reduces the electron affinity to the comparatively low value of 0.7 electron volt or less. Because cesium melts at a temperature only slightly above room temperature, the cesium vapor is obtained by merely introducing a small amount of cesium into the evacuated tube, the subsequent vaporization being sufficient to coat the filament.

[1] SCHADE, O. H., *Proc. I.R.E.*, **31**, 341 (1943).

The low electron affinity of the tungsten-oxygen-cesium emitter makes possible high emission currents at a temperature of only 1000°K. This type of emitter has several disadvantages, however, which make it impractical for use in commercial tubes. As the result of the high vapor pressure of cesium at operating temperatures of the tube, the characteristics of the tube are influenced by tube temperature. Too high temperature vaporizes the cesium, causing temporary reduction in emission, or even removal of the oxygen layer with permanent reduction of emission. Except at very low anode voltages, ionization of the cesium vapor occurs, resulting in fluctuations of anode current. The presence of positive ions is also detrimental to the action of amplifier tubes for other reasons, which will be discussed (Secs. 2-8, 6-7, 13-16).

2-7. Mechanical Construction of Cathodes.—Cathodes used in high-vacuum thermionic tubes are divided into two general classes: fila-

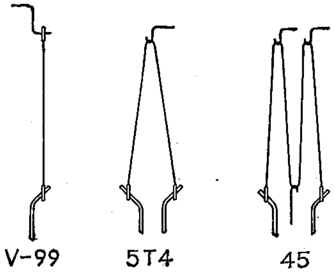


FIG. 2-8.—Typical filamentary cathodes. (*Courtesy of Radio Corporation of America.*)

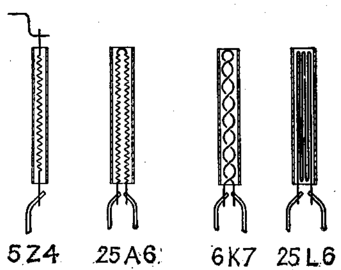


FIG. 2-9.—Structure of typical heater-type cathodes. (*Courtesy of Radio Corporation of America.*)

mentary and indirectly heated. Figure 2-8 shows the form of typical filamentary cathodes. Early vacuum tubes used only filamentary cathodes. When filamentary cathodes are operated on alternating current, the stray alternating electrostatic field and the alternating voltage across the filament cause an alternating component of plate current that may be objectionable. This difficulty led to the development of the indirectly heated, or *heater-type*, cathode.

Indirectly heated cathodes used in receiving tubes consist of an oxide-coated cylindrical sleeve, usually of nickel, within which is some form of heater. The most common types of heaters are illustrated in Fig. 2-9. The 5Z4 and 25A6 heater coils are helically wound. That of the 6K7 type of cathode is wound in a reverse helix. After being wound and formed, the coils are coated with a refractory insulating material and inserted into the sleeve. The heater of the 25L6 type of cathode is covered with a refractory insulating coating of sufficient adherence to permit the wire to be bent into the desired shape after it has been coated. Because of the small magnetic field produced by the

6K7 type of heater, there is very little 60-cycle plate-current variation, or "hum," when the heater is operated on alternating current. The loop type of helical heater exemplified by the 25A6 heater and the folded type of heater used in the 25L6 cathode make possible the use of enough wire for 25-volt operation. The advantage of the 25L6 construction is its low cost. More complicated cathode structures used in arc-discharge tubes will be discussed in Chap. 12.

The advantages of heater-type cathodes over filamentary cathodes are the much lower alternating component of plate current resulting from a-c operation of the heaters, and the possibility of using a single source of power to heat a number of cathodes between which a difference of potential exists. The small effect of the alternating heating current upon the plate current results in part from the fact that the indirectly heated cathode is a unipotential surface, and in part from the small magnitude of the stray alternating fields caused by the flow of heater current. A disadvantage of indirectly heated cathodes is their much longer heating time. Because oxide-coated emitters do not stand up in high-voltage tubes and because indirect heating cannot be used with pure tungsten or thoriated tungsten emitters, heater-type cathodes are not used in tubes that require high plate voltage.

2-8. Effects of Gas upon Emission and Space Currents.—The deleterious effects of gas upon emitters of various types as the result of chemical action, adsorption of thin layers of gas upon the surface, and positive-ion bombardment have already been mentioned. When the anode voltage is sufficiently high to produce ionization of the gas, other effects become apparent. If anode current is at first limited by space charge, the appearance of positive ions tends to neutralize the negative space charge surrounding the filament, thus increasing the anode current. If the anode voltage is high enough to give saturation current initially, increase of current occurs because the electrons and ions produced by bombardment of neutral gas molecules by the emitted electrons add to the current. Unfortunately this increase of current is likely to be accompanied by a number of undesirable effects. Currents through ionized gases usually fluctuate, resulting in "noise" in tubes used for amplification. The relatively low velocity of positive and negative ions produces a lag in the response of current to changes of voltage. Variations of gas pressure resulting from changes of temperature or from the absorption or emission of gas from the walls and electrodes may cause the characteristics of the tube to vary. Finally, in a gassy tube, positive-ion current flows to an electrode to which a negative voltage is applied. When the anode current is controlled by means of a negative voltage applied to an electrode through a high resistance, the flow of current through the resistance may cause an objectionable voltage drop and, under certain circumstances, may even result in damage to the tube (see Secs. 6-7, 12-20, and 13-16).

In the manufacture of high-vacuum tubes, many precautions are taken to ensure the removal of gas from walls and electrodes. The electrodes are thoroughly cleaned and are then heated for several minutes in an atmosphere of hydrogen, which removes oxygen and water vapor. After the tube is assembled and connected to the pumps, the electrodes are heated to about 800 or 1000°C by high-frequency induction in order to remove other occluded gases. Residual gas is removed by the use of *getters*, which are active chemical substances such as barium, magnesium, aluminum, and tantalum, having the property of combining with gases when they are vaporized. In glass tubes a small amount of the getter is mounted in such a position that it will be heated and vaporized or "flashed" during the inductive heating of the elements. By proper location of the getter, the vapor can be prevented from condensing in places where it might cause electrical leakage or undesirable primary or secondary emission. The effectiveness of the getter results not only from its chemical combination with gases during flashing, but also from subsequent absorption of gases by getter that has condensed on the walls of the tube. The action of the getter during flashing is increased by ionization of the gas by means of voltages applied between the electrodes or by the radio-frequency field of the induction heater. To ensure the removal of gas from the walls, tubes are baked during the process of manufacture. On machines that exhaust and seal the tubes separately, the bulbs are heated in ovens during exhaustion. On "Sealex" machines, the tubes are sealed and exhausted on the same machine, the heat from sealing being used to drive gases from the bulbs during exhaustion.

In tubes with metal envelopes, the shielding action of the shell makes it impossible to heat the electrodes and the getter by induction. The electrodes may be heated by radiation from the shell, which is heated by gas flames. Although the getter may be fastened to the inside of the shell and vaporized by heating the shell locally, another method has been developed that requires less critical control.[1] A short length of tantalum ribbon, which connects the shell to its terminal pin in the base, is coated with a mixture of barium and strontium carbonates. While the tube is on the pumps, the temperature of the tantalum wire is raised electrically to about 1100°C. This converts the carbonates into oxides. After the tube has been sealed off, the tantalum wire is heated to a temperature in excess of 1200°C. This causes the tantalum to reduce the oxides to pure metallic barium and strontium, which vaporize. Since the vapor moves in straight lines, it can be directed as desired by means of shields

[1] LEDERER, E. A., and WAMSLEY, D. H., *RCA Rev.*, **2**, 117 (1937). This article also discusses gettering methods used in glass tubes.

and by proper location of the tantalum wire. This type of getter is called *batalum* [see (11) in Fig. 3-11*b*].

2-9. Limitation of Anode Current by Space Charge.—The effect of space charge in limiting space current and the increase of anode current resulting from an accelerating anode potential have already been mentioned in connection with the theory of thermionic emission. Before proceeding to a discussion of the quantitative relation between anode current and anode potential in a two-element tube, it is of interest to discuss further the physical picture underlying the phenomenon. The behavior of the emitted thermionic electrons is complicated by their initial velocities. For this reason it is best first to formulate a theory on the assumption that the initial velocities are zero and then, when they are taken into consideration, to see in what manner the results should be altered. For the present, therefore, initial velocities will be assumed to be zero. It will be further assumed that both cathode and anode are homogeneous, constant-potential, parallel planes of large area, and hence that the electric field over the surface of the cathode may be assumed to be uniform.

Electrons that leave the cathode constitute a space charge that exerts a retarding field at the cathode. The net field at the surface of the cathode is the difference between this retarding field and the accelerating field produced by the positive voltage of the anode. The number of electrons in the space, and hence the retarding component of field at the cathode, increase with the anode current. When the positive anode voltage is applied, the anode current builds up with great rapidity to such a value that the average retarding field at the cathode caused by the space charge is equal to the accelerating field caused by the anode voltage, making the average field zero at the cathode. Increase of emission then does not raise the anode current, as the additional emitted electrons merely reenter the cathode. If it were possible in some manner to increase the density of space charge by increasing the current beyond this equilibrium value, or if the anode voltage were reduced slightly, then the net field at the cathode would be a retarding one. For an instant, all emitted electrons would be prevented from moving away from the cathode, and the current and space-charge density would be automatically reduced to a value that would again make the average field at the cathode zero. An increase of anode voltage causes the accelerating field to exceed the retarding field. The number of electrons moving to the anode then increases until the retarding field again equals the accelerating field.

At first thought it may not seem plausible that there can be a steady flow of electrons to the anode when both the velocities of emitted electrons and the average electrostatic field are zero at the cathode. It is only the *time average* field, however, that is zero at any point on the cathode. The *instantaneous* field at any point may vary in a random manner between positive and negative values. Immediately after one or more electrons have entered some point of the anode, the net field at a corresponding point of the cathode may be positive, causing one or more electrons to move away from the cathode. These electrons produce a retarding field behind them, which prevents the departure of more electrons from that point until the entrance of other electrons into the anode again results in an accelerating field. Many electrons are entering and leaving the space at any instant, so that the field fluctuations are rapid and haphazard.

2-10. Child's Law.—The foregoing descriptive explanation shows that, if an ample supply of electrons is available at the cathode, the anode current in a diode varies with the voltage applied between the anode and cathode. A mathematical analysis of this phenomenon was first made by Child.[1] The equation relating the anode current and voltage is called *Child's law.* The general derivation for electrodes of any size and shape is too difficult to yield a useful equation, and so only the relatively simple cases such as those applying to plane parallel electrodes of large area and to long concentric cylinders are ordinarily considered. In deriving Child's law for plane parallel electrodes the following assumptions are made:

1. The cathode temperature is high enough at all points so that more electrons are emitted than are drawn to the anode; *i.e.*, the current is limited by space charge.
2. The cathode and anode are parallel plates whose area is large as compared to their spacing; *i.e.*, the electrostatic field is normal to the electrode surface and uniform over the surface of any plane parallel to the electrodes.
3. The surfaces of the anode and cathode are equipotential surfaces.
4. The space between the cathode and the anode is sufficiently free of gas so that electrons do not lose energy by collision with gas molecules in moving from the cathode to the anode.
5. Emitted electrons have zero initial velocity after emission.

Under these assumptions the following three equations may be written:

$$\frac{d^2V}{dx^2} = -4\pi\rho \qquad (2\text{-}2)$$

$$\mathcal{E}V = \tfrac{1}{2}m_e v^2 \qquad (2\text{-}3)$$

$$i_b = -\rho v A \qquad (2\text{-}4)$$

in which V is the potential, relative to the cathode, at a distance x from the cathode; ρ is the (negative) density of electron space charge at a

[1] CHILD, C. D., *Phys. Rev.*, **32**, 498 (1911).

distance x from the cathode; $\mathcal{E}$ and m_e are the charge and the mass, respectively, of an electron; v is the velocity dx/dt of an electron at a distance x from the cathode; i_b is the anode current; and A is the area of the electrodes.

Equation (2-2) combines in symbolic form the definitions of potential difference and electric field. It is a special form of Poisson's equation, one of the most important fundamental laws of electrostatics, and may be derived directly from Gauss's law[1] (see Sec. 1-10). Equation (2-3) states that the energy gained by an electron in moving from the cathode to a distance x from the cathode under the influence of the electric field appears entirely in the form of kinetic energy. Equation (2-4) is a symbolic formulation of the definition of the magnitude of an electric current as the rate of flow of charge.

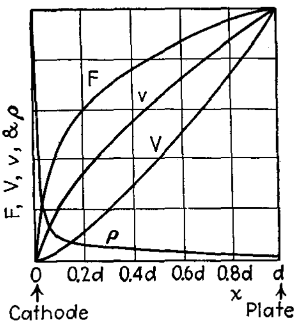


FIG. 2-10.—Variation of field strength F, potential V, electron velocity v, and space-charge density ρ with distance x from plane cathode. Zero initial velocity. Arbitrary units.

In the solution of the simultaneous differential Eqs. (2-2), (2-3), and (2-4), the following boundary conditions must be applied: At the cathode, the potential V, the average electric field $\partial V/\partial x$, and the velocity v are zero. At the anode, where x is equal to the cathode-to-anode spacing d, the potential is equal to e_b, the applied anode voltage. Solution of the equations and substitution of numerical values of $\mathcal{E}$ and m_e give the following equation for the anode current of a diode:[2]

$$i_b = 2.34 \times 10^{-6} \frac{Ae_b^{3/2}}{d^2} \qquad \text{amp} \tag{2-5}$$

By combining Eq. (2-5) with Eqs. (2-2), (2-3), and (2-4), theoretical expressions may be derived for density of space charge, electron velocity, electric field strength, and potential as functions of distance from the cathode. Curves derived from these are shown in Fig. 2-10.

Child's law for concentric cylinders whose length is large as compared to their spacing is

$$i_b = 14.68 \times 10^{-6} \frac{he_b^{3/2}}{br} \qquad \text{amp} \tag{2-6}$$

in which r is the radius of the anode, h is the length of the electrodes, and b is a factor whose value depends upon the ratio of the radius of the anode to that of the cathode. b has the approximate value $\frac{1}{4}$ for a ratio 2, $\frac{1}{2}$ for a ratio 3, and 0.9 for a ratio 8. If the plate diameter is large as compared

[1] PAGE, L., and ADAMS, N. I., "Principles of Electricity," p. 83, D. Van Nostrand Company, Inc., New York, 1931.

[2] PAGE and ADAMS, *op. cit.*, p. 297.

to that of the cathode, Eq. (2-6) reduces to the approximate form

$$i_b = 14.7 \times 10^{-6} \frac{he_b^{3/2}}{r} \qquad \text{amp} \tag{2-7}$$

Equations (2-5) to (2-7) show the importance of close spacing between cathode and anode if large currents are desired at small anode voltages.

2-11. Deviations from Child's Law Observed in Practical Diodes.—Deviations from Child's law result from the failure of practical diodes to satisfy the assumptions made in its derivation. Since the temperature of the cathode is fixed by considerations of emission efficiency and life, there is always a saturation voltage above which the current is not limited by space charge but by filament emission. If other assumptions were satisfied, the saturation voltage would be quite definite and the current-voltage curve would be as shown by the dotted lines of Fig. 2-11. Because of variations in temperature, electron affinity, and field strength over the cathode surface, the anode voltage at which voltage saturation takes place is not the same for all points of the cathode. The curve of anode current *vs.* anode voltage therefore bends over gradually, as shown by the full line of Fig. 2-11. Above saturation the current is not entirely constant but continues to rise somewhat with anode voltage. This is explained by reduction of electron affinity with increase of external field (see Sec. 1-12), and lack of homogeneity of the surface of the emitter. The effect is particularly noticeable with oxide-coated cathodes.

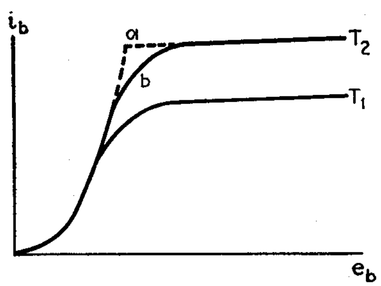


FIG. 2-11.—Curves of anode current i_b *vs.* anode voltage e_b at two values of emitter temperature.

The assumptions of uniform field and equipotential cathode are satisfied fairly closely in heater-type diodes with cylindrical plates. The voltage drop in filamentary cathodes may be shown to change Child's $\frac{3}{2}$-power law into a $\frac{5}{2}$-power law at anode voltages relative to the negative end of the filament that are less than the voltage of the positive end of the filament. This tends to make the lower part of the i_b-e_b curve steeper. The exact effect upon the curve of failure to satisfy the assumption of uniform field is complicated and impossible to predict completely.

2-12. Effect of Initial Velocities of Emitted Electrons.—Modified forms of Child's law which take into consideration the initial velocities of emitted electrons have been derived by Schottky, Langmuir, and others.[1] For the purpose of this book, a qualitative explanation of the effect of initial velocities is sufficient. Let it first be supposed that

[1] SCHOTTKY, W., *Physik. Z.*, **15**, 526 (1914); *Ann. Physik*, **44**, 1011 (1914); LANGMUIR, I., *Phys. Rev.*, **21**, 419 (1923); DAVISSON, C., *Phys. Rev.*, **25**, 808 (1925).

the electrons emerge with zero velocity. Under equilibrium conditions the space current and the space-charge density assume such values that the *average* field at the cathode is zero (see Sec. 2-9). The field and potential distributions in the interelectrode space are as shown in Fig. 2-10. Now let the emitted electrons suddenly have initial velocities which, for the sake of simplicity, are assumed to be the same for all electrons. Electrons that, without initial velocity, would have reentered the cathode now move toward the anode in spite of the fact that the average field is zero. As a result, the current and the space-charge density increase. The retarding field of the space charge now exceeds the accelerating field of the anode, giving a net retarding field at the cathode surface which slows up the electrons in the vicinity of the cathode. Equilibrium results when the retarding field in the vicinity of the cathode is sufficiently high so that the electrons are brought to rest in a plane a short distance s from the cathode. The average field in this plane is zero, but instantaneous fluctuations allow just enough electrons to pass to give the required anode current. The behavior is similar to that which would obtain if the initial velocity were zero and the cathode were moved toward the anode by the distance s. Because of the random distribution of electron velocities the phenomenon is actually more complicated than this simplified picture indicates. The simple theory shows, however, that the effect of initial velocities is to increase the anode current corresponding to any anode voltage and is, therefore, equivalent to that of a small increase of anode voltage. Because the electrons emerge with velocities corresponding to kinetic energy of the order of a volt or less, the effect is appreciable only for low anode voltages. The field and potential distributions throughout the interelectrode space are plotted in Fig. 2-12. That the potential must pass through a minimum where the field is zero follows from the fact that the field at any point may be expressed as the space derivative of the potential at that point.

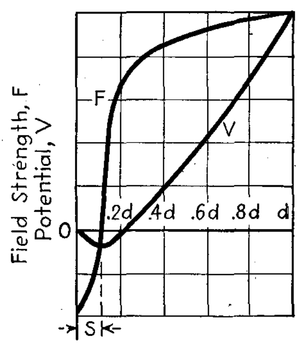


FIG. 2-12.—Variation of field strength and potential with distance from cathode for plane parallel electrodes of separation d. Initial velocities of emitted electrons considered.

The lower part of the i_b-e_b curve of Fig. 2-11 is raised slightly as the result of initial velocities of emitted electrons, and a small negative voltage must be applied in order to reduce the anode current to zero. Theoretical equations relating anode current and voltage at negative anode voltages have been derived by Schottky[1] and Davisson.[2] Because

[1] SCHOTTKY, *loc. cit.;* LANGMUIR, *loc. cit.*

[2] DAVISSON, *loc. cit.*

of their complicated form and because of failure to satisfy in practice the assumptions made in their derivation, these are seldom of great practical value. At negative anode voltages that are high enough to reduce the anode current to the order of 50 μa or less, the anode current of diodes with unipotential cathodes follows closely the empirical relation

$$i_b = k_1 \epsilon^{k_2 e_b} \tag{2-8}$$

in which k_1 and k_2 are constants for a given tube. The current departs materially from this exponential law as the negative anode voltage is reduced in magnitude in the vicinity of zero voltage, particularly at high cathode temperatures. Experimental curves corresponding to Eq. (2-8) were first obtained by Germer.[1]

Unless the plate of a highly evacuated diode becomes hot enough to emit electrons, increase of negative anode voltage beyond the value that reduces the current to zero has no further effect upon the anode current. The fact that anode current flows in one direction only is employed in the application of diodes to detection and to power rectification, which will be discussed in Chaps. 9 and 14.

2-13. Relation of Richardson's and Child's Laws.—It should be noted that Richardson's equation and Child's law apply to two different conditions of operation of two-element tubes. Richardson's equation holds only under voltage saturation, whereas Child's law applies only under temperature saturation. In most applications, vacuum tubes are used in such a manner that temperature saturation prevails.

2-14. Shot Effect.—The random motion of electrons causes rapid variations of the number of electrons that pass from the cathode to the anode in unit time, and thus produces fluctuations of anode current. This phenomenon, which may be readily detected by the use of sufficient amplification, is called the *shot effect.* It is one of the factors that limit amplification by vacuum tubes (see Sec. 6-28).

Heating of the Plate.—The kinetic energy acquired by electrons in moving from the cathode to the plate is converted into heat when the electrons strike the plate. The average current that a vacuum tube can pass is limited by the temperature of the plate at which absorbed gas is driven out of the plate or electron emission takes place from the plate. The energy that is converted into heat at the plate is equal to the time integral of the product of the plate current and plate voltage. Radiation of heat from the plate is increased by blackening its outer surface.

2-15. Classification of Tubes.—Electron tubes may be classified in a number of ways. These classifications include those based upon the process involved in the emission of electrons from the cathode, the degree of evacuation, the number of electrodes, and the type of application.

[1] GERMER, L. H., *Phys. Rev.*, **25**, 795 (1925).

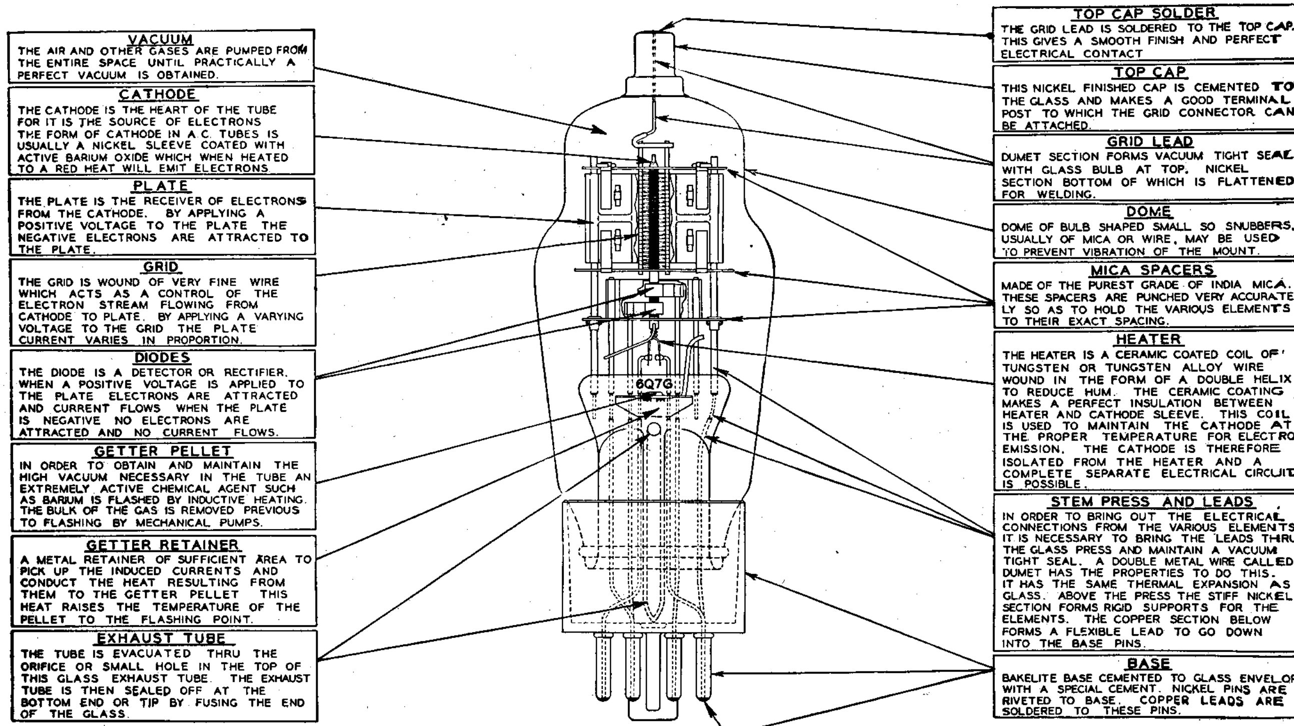


FIG. 2-13.—Structure of typical glass receiving tube. (*Courtesy of Ken-rad Tube and Lamp Corp.*)

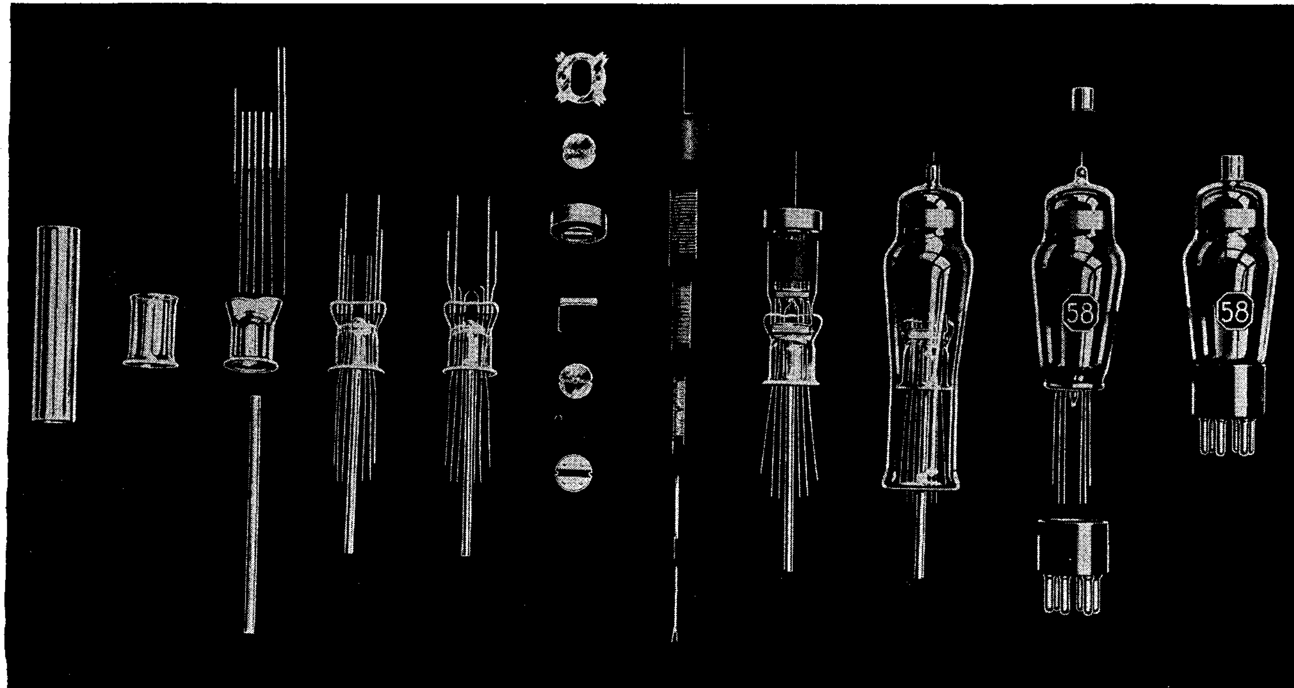


FIG. 2-14.—Assembly of typical glass receiving tube. (*Courtesy of Radio Corporation of America.*)

Included in the first classification are the thermionic tube and the phototube. A *thermionic tube* is an electron tube in which the electron or ion

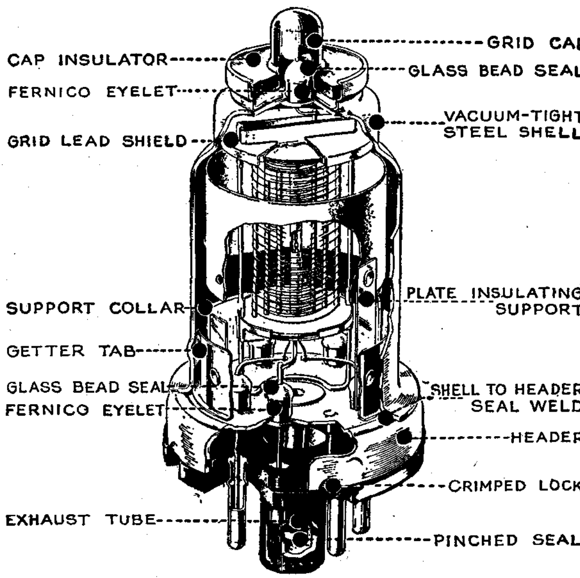


FIG. 2-15.—Structure of typical metal receiving tube.

emission is produced by the heating of an electrode. A *phototube* is an electron tube in which electron emission is produced directly by radiation falling upon an electrode. According to degree of evacuation, electron

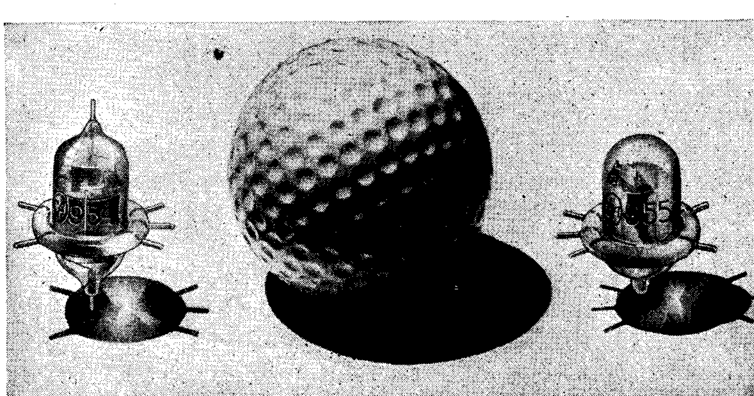


FIG. 2-16.—Size of typical "acorn" tubes shown in comparison with a golf ball. (*Courtesy of Radio Corporation of America.*)

tubes are classified as high-vacuum tubes and gas- or vapor-filled tubes. A *high-vacuum tube* (*vacuum tube* or *pliotron*) is an electron tube evacuated to such a degree that its electrical characteristics are essentially unaffected by gaseous ionization. A *gas-filled* or *vapor-filled tube* (*gas tube*) is an electron tube in which the pressure of the gas or vapor is such as to affect appreciably the electrical characteristics of the tube. According to the number of electrodes, tubes are classified as *diodes*, *triodes*, *pentodes*, etc. For convenience or economy, or for reduction of space or weight, two or more sets of elements may be enclosed in a single envelope. Thus, there are duplex (double) triodes, duplex-diode pentodes, triode pentodes, etc. The diverse classification of electron tubes according to application will be made in later chapters.

2-16. Structure of Tubes.—Tubes are made with both glass and metal envelopes.[1] The principal advantages of metal tubes lie in their greater mechanical strength and in the fact that the electrodes are permanently and completely shielded without the use of an external shield. Furthermore, they do not require on the inside of the envelope the conducting coating that must be used in glass tubes to prevent the wall from acquiring a positive charge as the result of secondary emission caused by the impact of electrons that pass around the plate. A disadvantage of metal tubes is that the shells become so hot in operation that they cannot be conveniently handled. This is of importance in the routine factory testing of radio receivers. Another minor disadvantage is the impossibility of determining visually whether the heater is in operation. Glass tubes appear to be somewhat more reliable. In rectifiers, particularly, metal tubes are likely to give difficulty as the result of short circuits. Figures 2-13 and 2-14 show the construction of a glass receiving tube; Figs. 2-15 and 3-11*b* show typical metal receiving tubes.

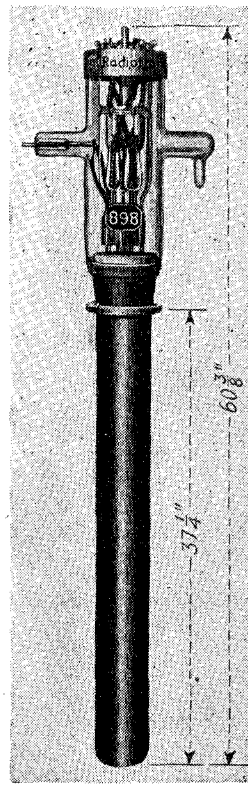


FIG. 2-17.—One-hundred-kilowatt water-cooled transmitting tube. (*Courtesy of Radio Corporation of America.*)

The great range in size of vacuum tubes is illustrated by Figs. 2-16 and 2-17. Figure 2-16 shows a typical *acorn* tube, developed for use at very high frequencies, at which it is essential to keep lead capacitance and inductance as small as possible.[2] No base is used on the acorn type of tube, connections being made directly to the electrode leads. Figure

[1] PIKE, O. W., and METCALF, G. F., *Electronics*, October, 1934, p. 312. See also *Electronics*, September, 1935, p. 31.

[2] SALZBERG, B., and BURNSIDE, D. G., *Proc. I.R.E.*, **23**, 1142 (1935).

2-17 shows a 100-kw water-cooled transmitting tube of a type that is used in large broadcasting stations.

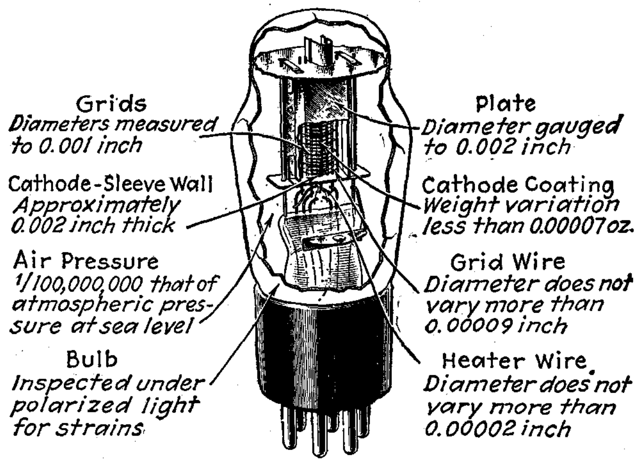


FIG. 2-18.—Materials used in typical radio receiving tubes. The complex nature of the structure of the modern vacuum tube and of the manufacturing processes are well illustrated by a consideration of the materials that are used.

Gases.—Argon, carbon dioxide, chlorine, helium, hydrogen, illuminating gas, neon, nitrogen, and oxygen.

Metals and Compounds.—Alumina, aluminum, ammonium chloride, arsenic trioxide, barium, barium carbonate, barium nitrate, borax, boron, cesium, calcium, calcium aluminum fluoride, calcium carbonate, calcium oxide, carbon, chromium, cobalt, cobalt oxide, copper, iridium, iron, lead, lead acetate, lead oxide, magnesia, magnesium, mercury, misch metal, molybdenum, monel, nickel, phosphorus, platinum, potassium, potassium carbonate, silica, silicon, silver, silver oxide, sodium, sodium carbonate, sodium nitrate, tantalum, thorium, thorium nitrate, tin, titanium, tungsten, zinc, zinc chloride, and zinc oxide.

Accessories.—Bakelite, ethyl alcohol, glass, glycerine, isolantite, lava, malachite green, marble dust, mica, nigrosine, petroleum jelly, porcelain, rosin, shellac, synthetic resin, and wood fiber.

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CHAPTER 3

GRID-CONTROLLED HIGH-VACUUM TUBES

3-1. The greatest single advance in the development of vacuum tubes undoubtedly came with the introduction by De Forest of a control electrode between the cathode and the plate of the diode.[1] The principal value of such a control electrode arises from the fact that relatively large plate current and power may be controlled by small variations of voltage of the control electrode relative to the cathode without the expenditure of appreciable power in the control circuit. A three-electrode vacuum tube containing an anode, a cathode, and a control electrode is called a *triode*. Figure 3-1 shows the cross sections of typical high-vacuum triodes.

The form of the control electrode in early tubes led to the use of the term *grid* for this electrode. A grid is now defined more broadly as an electrode that contains openings through which electrons or ions may pass. Many vacuum tubes contain two or more grids. In numerous applications of multigrid tubes the voltages of all but one grid are kept constant. This grid, called the *control grid*, serves to vary the plate (or other electrode) current by means of changes of voltage applied to it. The behavior of multigrid tubes with all grid voltages but that of the control grid fixed is in many respects similar to that of a triode.

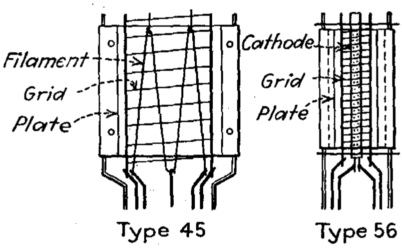


FIG. 3-1.—Electrode structure of typical filamentary and heater-type triode receiving tubes.

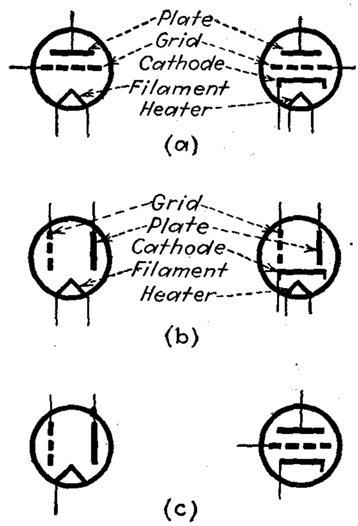


FIG. 3-2.—Symbols for filamentary and heater-type high-vacuum triodes.

Standard symbols for filamentary and heater-type triodes are shown

[1] DE FOREST, LEE, U. S. Patent 841387 (1907); U. S. Patent 879532 (1908).

in Fig. 3-2*a*.[1] Often circuit diagrams can be simplified by the use of the modified forms of Fig. 3-2*b*. In most circuit diagrams the filament and heater connections are of secondary importance and may be omitted. Whenever possible, therefore, simplified symbols such as those of Fig. 3-2*c* will be used. Except for the omission of the grid, diode symbols are the same as those used for triodes. Figure 14-20 contains the symbol used for a diode with two anodes.

3-2. Theory of Grid Action in Triodes. Equations for Plate and Grid Currents.—Electrodynamic analyses of the action of the grid in

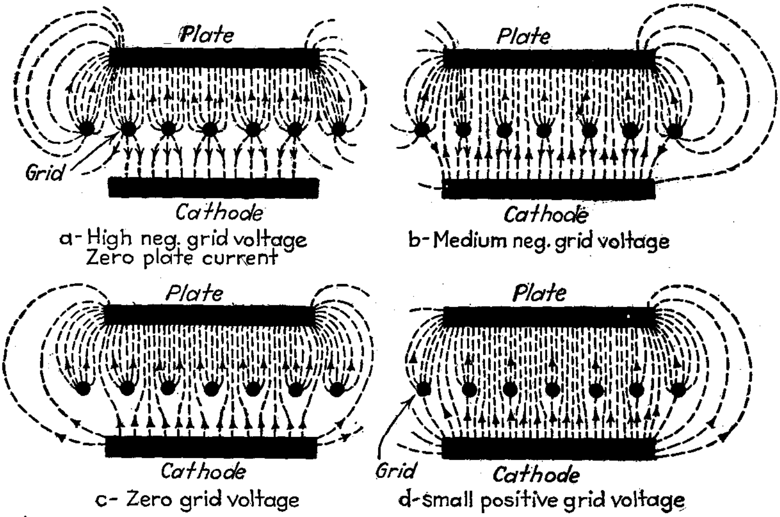


FIG. 3-3.—Approximate field distribution resulting from applied voltages in a triode with plane parallel electrodes for fixed positive plate voltage and four values of grid voltage. Arrows indicate the direction of the force on an electron.

controlling the plate current of a triode have been developed.[2] For the purposes of this book it will be better to present a brief qualitative discussion of the phenomenon of grid control and to base subsequent derivations upon empirically determined facts.

In Fig. 3-3 is shown the approximate field distribution resulting from applied electrode voltages in a triode with plane cathode and anode. Lines are used in the customary manner to indicate the electrostatic field, but, contrary to convention, the arrows indicate the direction in which electrons are urged by the field.[3] The plate voltage is assumed to be posi-

[1] "Standards on Electronics," p. 15, Institute of Radio Engineers, New York, 1938.

[2] See, for instance, E. L. CHAFFEE, "Theory of Thermionic Vacuum Tubes," Chap. VII, McGraw-Hill Book Company, Inc., New York, 1933.

[3] No special effort has been made in Fig. 3-3 to depict the field distribution accurately. Figure 3-3 is derived from more complete and carefully constructed diagrams shown on pp. 175 and 176 of "Theory of Thermionic Vacuum Tubes," by E. L. Chaffee.

tive and constant in value. If the grid is made sufficiently negative with respect to the cathode, all lines of force terminate on the grid, and no field exists directly between the plate and the cathode. This is illustrated in Fig. 3-3*a*. The field at all points of the cathode is in the direction to return emitted electrons to the cathode and so the plate current is zero. If the grid is slightly less negative, some field will extend directly from the anode to the cathode, as shown in Fig. 3-3*b*, and there will be a force tending to carry electrons from certain points of the cathode to the anode. Further decrease of negative grid potential increases the areas of the cathode over which the field tends to remove electrons and strengthens the average field over these areas. As long as the grid is negative with respect to the cathode, electrons can reach the grid only if they have sufficient kinetic energy to overcome the retarding field terminating on the grid. Because the initial velocity of emission of the electrons is small, electron current to the grid is zero unless the negative grid voltage is appreciably less than a volt. If the tube contains traces of gas, there may be a small positive-ion current to the grid when the grid is negative. At zero grid potential, illustrated by Fig. 3-3*c*, no point of the cathode experiences a retarding field. Initial velocities cause some electrons to strike the grid, giving a small grid current. When the grid is positive, as in Fig. 3-3*d*, there is an accelerating field over the whole surface of the cathode. A portion of this field terminates on the grid, causing appreciable grid current to flow.

As in the diode, the net field at the cathode is actually the resultant of the retarding field produced by the space charge and the field caused by the electrode voltages. Equilibrium is established when the average net field is zero at a short distance from the cathode. Increase of applied accelerating field causes the net field to become positive and thus allows more electrons to go to the plate. The space current and space charge increase until the average net field is again zero a short distance from the cathode.

Since the field at the cathode depends upon the potential both of the grid and of the plate, the plate current is a function of both grid and plate voltages. This may be stated symbolically by the functional equation

$$i_b = f(e_b, e_c) \tag{3-1}$$

in which i_b is the plate current, e_b is the plate voltage, and e_c is the grid voltage. Because of the screening action of the grid, only a portion of the field from the plate extends directly to the cathode, whereas there is nothing to intercept the field between the grid and the cathode. One is led to guess, therefore, that the plate current is affected more by changes of grid voltage than of plate voltage, *i.e.*, that the grid voltage is μ times as effective as the plate voltage in controlling the plate current, μ

being a factor greater than unity. μ is not necessarily constant. The effect of initial velocity of emitted electrons is the same as though a small increase were made in either grid or plate potential. The contact potentials may either increase or decrease the effective field at the cathode. It is convenient to combine the effects of initial velocity and contact potentials into a single quantity ϵ, an equivalent voltage that would produce the same effect upon plate current as the initial velocity plus the contact potentials. ϵ is ordinarily so small in comparison with externally applied potentials that it may be neglected. This analysis leads to the assumption that the plate current is a function of $(e_b + \mu e_c + \epsilon)$. This fact may be expressed by the functional equation

$$i_b = F(e_b + \mu e_c + \epsilon) \qquad (T_f = \text{const.}) \tag{3-2}$$

where T_f is the temperature of the cathode.

The use of Eq. (3-2) is justified by the fact that it is verified experimentally. Sometimes it is possible to make use of the more explicit approximate law

$$i_b = A(e_b + \mu e_c)^n \tag{3-3}$$

in which A is a constant. The exponent n varies considerably with grid and plate voltages, the values ranging roughly between 1.2 and 2.5 for negative values of e_c. When either grid or plate voltage is maintained constant, however, the variation of n is often so small that n may be assumed to be constant over certain ranges. In some analyses, n is assumed to be equal to 1.5 when $e_c = 0$, although actual values may depart appreciably from this value.

μ is called the *plate amplification factor*, or simply the *amplification factor* of the tube. It is a measure of the relative effectiveness of the grid and plate voltages in controlling the plate current. The amplification factor will be shown to be related to certain of the characteristic curves of a triode, and it will be defined mathematically on the basis of this relationship (see Sec. 3-5). The value of μ depends upon the shape and spacing of the electrodes,[1] and to some extent upon the plate current; it may also be made to vary with electrode voltages (see Sec. 3-7). Electrodynamic analysis, based upon the assumption that the electrodes are parallel and of infinite extent, that the grid wire spacing is large compared to the diameter of the grid wire, and that the space charge between electrodes is zero, yields the following approximate formula for μ:[2]

$$\mu = \frac{2\pi p}{a \log_\epsilon \left(2 \sin \dfrac{\pi r}{a}\right)} \tag{3-4}$$

[1] KUSUNOSE, Y., *Proc. I.R.E.*, **17**, 1706 (1929).

[2] SCHOTTKY, W., *Arch. Elektrotech.*, **8**, 21 (1919); SALZBERG, B., *Proc. I.R.E.*, **30**, 134 (1942).

in which p is the distance between the planes of the grid and the plate, a is the distance between adjacent grid wires, and r is the radius of the grid wire. The following approximate expression for μ applies to an electrode structure in which the anode and the cathode are long cylinders and the grid is helical.[1]

$$\mu = \frac{2\pi \, \rho_g/\rho_p \, (\rho_p - \rho_g)}{a \log_\epsilon a/2\pi r} \tag{3-5}$$

in which ρ_p and ρ_g are the radii of the anode and grid, respectively, and a is the spacing of the grid wires. Because of factors neglected in the derivation of Eqs. (3-4) and (3-5) and also because modern tube design is based largely upon empirical methods, the value of these equations lies mainly in the indication that they give of the general effects of triode tube dimensions upon the amplification factor.

The effect of space charge is to cause a variation of amplification factor with plate current or electrode voltages. Except in the case of "variable-mu" tubes (see Sec. 3-7), in which the grid wires are not equally spaced, the variation of μ with electrode voltages is sufficiently small so that μ may often be assumed to be constant over the working range of current and voltages. At positive grid voltages, however, diversion of current from the plate to the grid lowers μ.

The grid current of a triode is also a function of the grid and plate voltages. This fact may be expressed by the functional equation

$$i_c = G(e_c + \mu_g e_b + \epsilon) \qquad (T_f = \text{const.}) \tag{3-6}$$

in which μ_g, the *grid amplification factor*, is less than unity.

3-3. Time of Transit of Electrons.—Because of its small mass, the acceleration of an electron is so rapid that the time taken for electrons to pass from the cathode to the plate may usually be neglected and the response of electrode currents to changes of electrode voltages considered to be instantaneous. At the very high frequencies corresponding to wave lengths of the order of a few meters or less, however, time of transit must be taken into consideration.[2] Since the operation of vacuum tubes at ultrahigh frequencies will not be discussed in detail in this book, the time of transit of electrons will usually be neglected.

3-4. Static and Dynamic Characteristics.—Theoretical and practical studies of the performance of vacuum tubes and vacuum-tube circuits are

[1] ABRAHAM, H., *Arch. Elektrotech.*, **8**, 42 (1919); KING, R. W., *Phys. Rev.*, **15**, 256 (1920).

[2] BENHAM, W. E., *Phil. Mag.*, **5**, 641 (1928); LLEWELLYN, F. B., *Proc. I.R.E.;* **21**, 1532 (1933); **22**, 947 (1934); **23**, 112 (1935); CHAFFEE, J. G., *Proc. I.R.E.*, **22**, 1009 (1934); FERRIS, W. R., *Proc. I.R.E.*, **24**, 82, 105 (1936); NORTH, D. O., *Proc. I.R.E.*, **24**, 108 (1936); BRAINERD, J. G., KOEHLER, G., REICH, H. J., and WOODRUFF, L. F., "Ultra-high-frequency Techniques," D. Van Nostrand Company, Inc., New York, 1942.

greatly facilitated by the use of curves relating the electrode currents and voltages, called *characteristics.* A *static electrode characteristic* is a relation, usually shown by a graph, between the voltage and the current of that electrode, other electrode voltages being maintained constant. A *static transfer characteristic* is a relation, usually shown by a graph, between the voltage of one electrode and the current of another electrode, all other voltages being maintained constant. Unless otherwise specified, the term *transfer characteristic* is understood to apply to characteristics relating control-grid voltage and plate current, which are the most frequently used transfer characteristics.

Strictly, a *static* characteristic is one obtained with steady voltages, whereas a *dynamic* characteristic is one obtained with alternating voltages. Inasmuch as all voltages but one are specified to be constant in the above definitions, the characteristics obtained with alternating voltages differ from those obtained with direct voltages only when the frequency is so high that tube capacitances and electron transit time cause appreciable out-of-phase components of current. The term *dynamic transfer characteristic* has come to be applied to a transfer characteristic obtained with alternating control-grid voltage when the electrode current under consideration passes through an external impedance, called the *load impedance,* the supply voltage for that current being maintained constant. Voltage drops in the load impedance cause the electrode voltage to differ from the supply voltage, and the electrode voltage to vary with current. In general, *IR* drop in the load causes the transfer characteristic to be affected by load even when the characteristic is derived by using steady voltages. Extension of the term *dynamic transfer characteristic* to include such a characteristic may be justified by considering it to be a limiting curve obtained as the frequency of alternating voltage is made to approach zero.

There are four sets of static characteristics of triodes. They are the plate characteristics i_b *vs.* e_b at constant values of e_c; the grid characteristics i_c *vs.* e_c at constant values of e_b; the grid-plate transfer characteristics i_b *vs.* e_c at constant values of e_b; and the plate-grid transfer characteristics i_c *vs.* e_b at constant values of e_c.[1] The behavior of a triode is completely specified by either the plate and grid families of characteristics or the two families of transfer characteristics.

[1] Since the static characteristics are constructed by plotting corresponding values of direct voltages and currents, the letters used in representing these voltages and currents should be capitals to be entirely in accord with the system of nomenclature used in this book (Sec. 3-17). In most applications of the characteristic curves, however, the currents and voltages are assumed to vary, and so must be represented by lower-case symbols. For this reason, lower-case symbols have been used in Eqs. (3-1) to (3-6) and will be used for all characteristic curves.

In Figs. 3-4*a* and 3-4*b* are shown typical characteristics for a triode. The bending of the i_b-e_c curves of Fig. 3-4*b* at positive values of e_c, particularly noticeable for low values of plate voltage, is caused by diver-

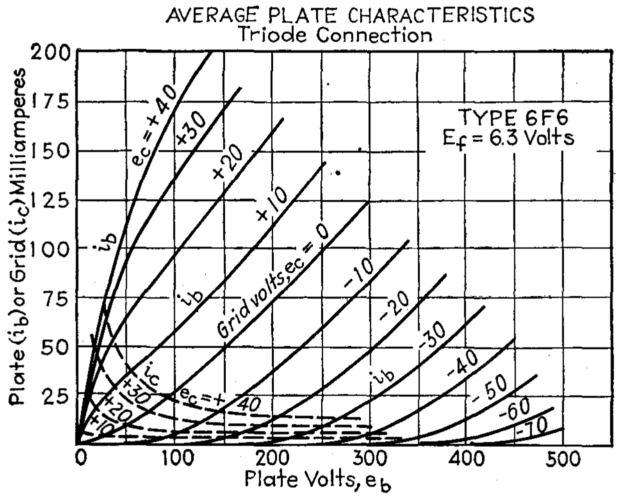


FIG. 3-4*a*.—Typical triode plate characteristics and plate-grid transfer characteristics.

sion to the grid of electrons emitted by the cathode, and by secondary emission. Secondary electrons emitted by the plate when the grid voltage exceeds the plate voltage are drawn to the grid and constitute a current opposite in direction to the normal plate current. Typical curves of i_b and i_c *vs.* e_b at large positive grid voltages are shown in Fig. 3-4*c*. The reversal of curvature of the plate characteristics of Fig. 3-4*c* between the plate voltages of 50 and 100 for grid voltages of 45 and above is also the result of secondary emission.

The points at which the characteristic curves intercept the voltage axes are called *cutoff points*, and the corresponding voltages, the *cutoff voltages*. An approximate relation between grid and plate voltages at plate current cutoff can be derived from Eq. (3-3). When $i_b = 0$, $(e_b + \mu e_c)^n = 0$. This can be true only if $e_b = -\mu e_c$. Because μ is never strictly constant, the accuracy of this relation depends upon the voltages at which μ is evaluated. Considerable error may result when the usual published value of μ is used, but the relation is often useful in making a

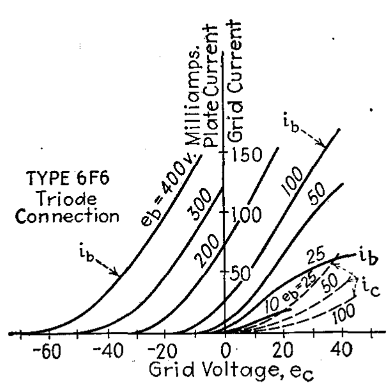


FIG. 3-4*b*.—Typical triode grid-plate transfer characteristics and grid characteristics (same tube as for Fig. 3-4*a*).

rapid approximate determination of cutoff voltages. Because of lack of homogeneity of the emitter, voltage drops in the cathode, variation of the electric field at various points of the cathode, and distribution of initial velocities of emission, the intersections of the characteristic curves with the voltage axis at cutoff are not sharp. Although some grid current usually flows at negative grid voltages lower than half a volt to a volt, the simplifying assumption is often made that grid current cutoff occurs at zero grid voltage.

The transfer characteristics of Fig. 3-4*b* may be derived from the plate characteristics of Fig. 3-4*a*, and vice versa. In Chap. 4 it will be shown that much essential information concerning the performance of

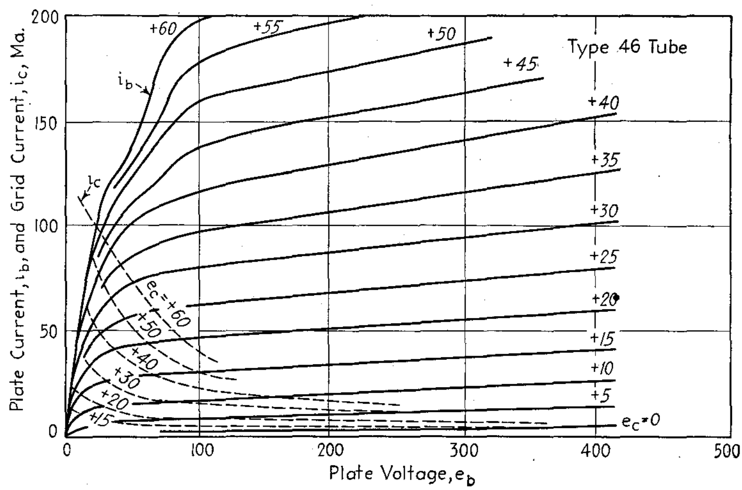


FIG. 3-4*c*.—Typical triode plate characteristics and plate-grid transfer characteristics at high positive grid voltages.

tubes in circuits may be obtained from the plate characteristics. It is important to note that in tubes with filamentary cathodes the grid and plate voltages are measured with respect to the negative end of the filament.

The curves of plate and grid current against cathode temperature of a triode are similar to those of plate current against cathode temperature of a diode, but voltage at which saturation becomes apparent depends upon both plate voltage and grid voltage. Since triodes are almost always operated above temperature saturation, these curves are of comparatively little practical value in connection with ordinary applications.

3-5. Tube Factors.—The mathematical and graphical analyses of the operation of vacuum tubes and vacuum-tube circuits require the use of certain tube *factors* whose numerical values are dependent upon the construction of the tube and upon the electrode voltages and currents, and which serve as indices of the ability of given tubes to perform specific functions. Although only triodes have been discussed so far; it will be convenient also to present at this point general definitions that apply to tubes with more than three electrodes.

Mu-factor is the ratio of the change in one electrode voltage to the change in another electrode voltage, under the conditions that a specified current remains unchanged and that all other electrode voltages are maintained constant. It is a measure of the relative effect of the voltages of two electrodes upon the current in the circuit of any specified electrode. As most precisely used, the term refers to infinitesimal changes. Symbolically, mu-factor is defined by the equation

$$\mu_{jkl} = -\frac{\partial e_j}{\partial e_k} \qquad (i_l,\ e_l,\ e_m,\ \text{etc.},\ = \text{const.}) \tag{3-7}$$ [1]

The most important mu-factor is the control-grid–plate mu-factor, or *amplification factor.* Amplification factor is the ratio of the change in plate voltage to a change in control-grid voltage under the conditions that the plate current remains unchanged and that all other electrode voltages are maintained constant. It is a measure of the effectiveness of the control-grid voltage, relative to that of the plate voltage, upon the plate current. The sign is taken as positive when the voltage changes of the two electrodes must be of opposite sign. As most precisely used, the term refers to infinitesimal changes. Symbolically, amplification factor is defined by the equation

$$\mu = -\frac{\partial e_b}{\partial e_c} \qquad (i_b = \text{const.}) \tag{3-8}$$ [2]

It must be proved that the factor μ which appears in Eqs. (3-2) and (3-3) is the same as that defined by Eq. (3-8). Examination of Eq. (3-2) shows that, unless the plate current is independent of electrode voltages, which is true in practice only when the current is zero or saturated, i_b is constant only when $e_b + \mu e_c + \epsilon$ is constant. Since tubes are seldom used continuously either with zero plate current or with saturation plate

[1] Although the partial derivative implies that other variables are held constant, for the sake of emphasis it seems advisable in this and following equations to indicate the constant parameters in parentheses.

[2] The symbolic definitions of tube factors are usually written in terms of the alternating components of electrode voltages and currents, rather than in terms of the total values. Since the difference between the instantaneous value of the alternating component of a varying quantity and the instantaneous total value of the quantity is equal to the average value, the derivative of which is zero, derivatives of the alternating component and of the total quantity are equivalent. In order to show more closely the relation of the tube factors to the characteristic curves, and to simplify derivations based upon Eq. (3-2), the tube factors will be defined in terms of the total values of currents and voltages (see Secs. 3-16 and 3-17 for symbols).

current, i_b is constant when

$$e_b + \mu e_c + \epsilon = \text{const.} \tag{3-9}$$

Differentiation of Eq. (3-9) shows that

$$-\left.\frac{de_b}{de_c}\right]_{i_b = \text{const.}} = \mu \tag{3-10}$$

or

$$\mu = -\frac{\partial e_b}{\partial e_c} \tag{3-11}$$

Since Eq. (3-11) is identical with Eq. (3-8), the factor μ of Eq. (3-2) is the same as that of Eq. (3-8).

(*A-c*) *electrode conductance* is the ratio of the change in the current in the circuit of an electrode to a change in the voltage of the same electrode, all other electrode voltages being maintained constant. As most precisely used, the term refers to infinitesimal changes as indicated by the defining equation[1]

$$g_j = \frac{\partial i_j}{\partial e_j} \qquad (e_k,\ e_l,\ \text{etc.} = \text{const.}) \tag{3-12}$$

(See also Sec. 3-26.)

(*A-c*) *electrode resistance* r_j is the reciprocal of electrode conductance.

The electrode conductance that is used most frequently in the analysis of vacuum tubes and vacuum-tube circuits is the *plate conductance*[1]

$$g_p = \frac{\partial i_b}{\partial e_b} \qquad (e_c = \text{const.}) \tag{3-13}$$

(*A-c*) *plate resistance* is the reciprocal of plate conductance.[1]

$$r_p \equiv \frac{1}{g_p} = \frac{\partial e_b}{\partial i_b} \qquad (e_c = \text{const.}) \tag{3-14}$$

(See also Sec. 3-26.)

Transconductance is the ratio of the change in the current in the circuit of an electrode to the change in the voltage of another electrode, under the condition that all other voltages remain unchanged. As most

[1] The a-c electrode conductance defined by Eq. (3-12) must not be confused with the d-c electrode conductance, which is defined as the ratio of the total or direct electrode current to the total or direct electrode voltage. Similarly, the a-c plate resistance defined by Eq. (3-14) must not be confused with the d-c plate resistance, e_b/i_b. D-c electrode conductances and resistances are rarely of value and never essential in the analysis of vacuum tubes and associated circuits. Hence the a-c conductances and resistances are usually referred to simply as conductances and resistances, the adjective "a-c" being omitted.

precisely used, the term refers to infinitesimal changes as indicated by the defining equation

$$g_{jk} = \frac{\partial i_j}{\partial e_k} \qquad (e_j,\ e_l,\ \text{etc.} = \text{const.}) \tag{3-15}$$

The transconductance most frequently used in the analysis of vacuum tubes and vacuum-tube circuits is the *grid-plate transconductance* (*mutual conductance*), which is defined symbolically as

$$g_m = \frac{\partial i_b}{\partial e_c} \qquad (e_b = \text{const.}) \tag{3-16}$$

Unless otherwise specified, the term *transconductance* usually refers to control-grid–plate transconductance and will be so used in the remainder of this book.

Grid Factors.—In many applications of vacuum tubes the operating voltages are such that no conduction current flows to the control grid and all electrode voltages except those of the control grid and the plate are kept constant. Under these conditions, vacuum-tube problems and derivations can be treated by the use of only three of the factors that have been defined: μ, r_p, and g_m. If conduction current flows to the control grid, it may be necessary to make use of the corresponding control-grid factors, which are defined symbolically as follows:

$$\textit{Grid amplification factor } \mu_g = -\frac{\partial e_c}{\partial e_b} \qquad (i_c = \text{const.}) \tag{3-17}$$

$$\textit{Grid conductance } g_g = \frac{\partial i_c}{\partial e_c} \qquad (e_b = \text{const.}) \tag{3-18}$$

$$\textit{Plate-grid transconductance } g_n = \frac{\partial i_c}{\partial e_b} \qquad (e_c = \text{const.}) \tag{3-19}$$

Because of the effect of space charge and because of division of the total cathode current between the grid and the plate, μ_g is not in general the reciprocal of μ.

Proof That $g_m = \mu/r_p$.—Only two of the plate factors are independent. This may be shown by taking the partial derivatives of the plate current, as expressed by Eq. (3-2).

$$g_m = \frac{\partial i_b}{\partial e_c} = \mu F'(e_b + \mu e_c + \epsilon) \tag{3-20}$$

$$g_p = \frac{\partial i_b}{\partial e_b} = F'(e_b + \mu e_c + \epsilon) \tag{3-21}$$

Dividing Eq. (3-20) by Eq. (3-21) gives

$$\frac{g_m}{g_p} = \mu \qquad \text{or} \qquad g_m = \frac{\mu}{r_p} \tag{3-22}$$

A similar derivation, based on Eq. (3-6), shows that

$$g_n = \frac{\mu_g}{r_g} \tag{3-23}$$

3-6. Relation of Tube Factors to Characteristic Curves.—The definitions state that g_p, g_m, and μ are the slopes of the i_b-e_b, i_b-e_c, and e_b-e_c curves, respectively, at points corresponding to the given voltages. Values of these factors may, therefore, be determined accurately by measuring the slopes of the static characteristics at points corresponding to the electrode voltages, and approximately by taking the ratios of small increments of current and voltage corresponding to points on the characteristics. All three factors may be determined from a single family of characteristics. The most accurate method of obtaining the three plate

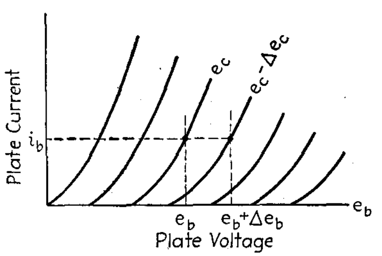


FIG. 3-5.—Method of determining triode amplification factor from the family of plate characteristics.

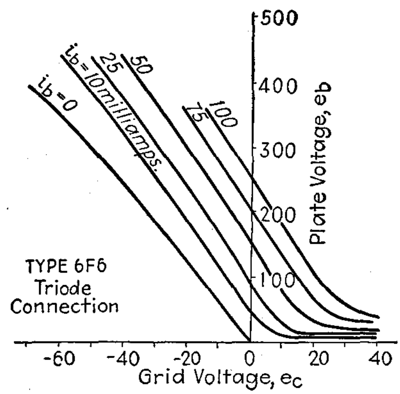


FIG. 3-6.—Typical triode e_b-e_c characteristics. (Derived from Fig. 3-4a.)

factors of a triode directly from the plate family of characteristics is to find r_p from the reciprocal of the slope of the tangent to the i_b-e_b curve at the point corresponding to the given electrode voltages, μ from the ratio of Δe_b to Δe_c at the constant current of the point, as shown in Fig. 3-5, and g_m from the ratio of μ to r_p. Curves of e_b *vs.* e_c at constant i_b for a typical triode are shown in Fig. 3-6.[1] The practically constant slope of the curves except at low plate voltages and high positive grid voltages indicates that over the normal operating range the amplification factor is nearly constant. Except in tubes designed to have variable amplification factor (Sec. 3-7), the variation of μ over the normal range of voltages does not exceed 10 to 15 per cent in triodes. Because of the small variation of μ at negative grid voltages, fairly large voltage incre-

[1] e_b-e_c curves are not usually used in the solution of vacuum-tube problems and are shown here only to point out the nearly constant value of the amplification factor in the normal range of voltages.

ments may be used without great error in determining its value from the plate characteristics. Some increase in accuracy is gained by using a grid-voltage increment such that the point at which μ is desired is at the center of the increment, rather than at one side as in Fig. 3-5.

3-7. Sharp-cutoff and Remote-cutoff Grids. Variable-mu Tubes.—Thus far it has been assumed that the grid-wire spacing and diameter and the spacing of the grid from the cathode are uniform throughout the length of the grid. When this is true the field strength does not vary greatly over the cathode surface and so the negative grid voltage necessary to prevent electrons from going to the plate at any value of plate voltage is very nearly the same for all points of the cathode. The static transfer characteristic therefore approaches the grid-voltage axis relatively sharply. For this reason a grid of uniform structure is called a *sharp-cutoff grid.* If some dimension of the grid, such as the spacing between the wires, varies along the grid, on the other hand, the field at the cathode varies correspondingly at the cathode surface. A greater negative grid voltage is required to prevent electrons from leaving the cathode at points corresponding to portions of the grid where the spacing is large than to portions where the spacing is small. Cutoff consequently takes place at different values of grid voltage at different parts of the cathode and so the static transfer characteristic approaches the axis gradually. Such a grid is known as a *gradual-cutoff* or *remote-cutoff* (*supercontrol*) *grid.* Because the mu-factor corresponding to an elementary length of the grid varies along the grid and because the mu-factor of the entire grid varies greatly with electrode voltages, such a grid is also termed a *variable-mu* grid. A tube that has a variable-mu control grid is called a variable-mu tube.

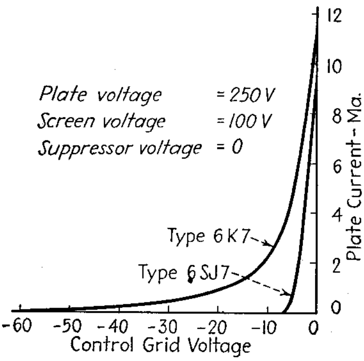


FIG. 3-7.—Comparison of transfer characteristics of similar tubes having remote-cutoff (6K7) and sharp-cutoff (6SJ7) grids.

Figure 3-7 shows the static transfer characteristics of two comparable tubes, one of which has a sharp-cutoff control grid, and the other a remote-cutoff control grid. Multigrid tubes may contain both one or more sharp-cutoff grids and one or more remote-cutoff grids. The advantages of remote-cutoff grids and tubes are discussed in Secs. 6-19 and 6-25 in connection with their use in voltage amplifiers.

3-8. Multigrid Tubes.—Many desirable characteristics can be attained in vacuum tubes by the use of more than one grid. The most common types of multigrid tubes are the tetrode and the pentode. A *tetrode* is a four-electrode type of thermionic tube containing an anode, a cathode, a control electrode, and an additional electrode, which is ordinarily a grid. A *pentode* is a five-electrode type of thermionic tube containing an anode, a cathode, a control electrode, and two additional electrodes, which are ordinarily grids.

The symbols for tetrodes and pentodes are similar to those for triodes, the various grids being shown in the relative positions that they occupy in the tubes. A special symbol, shown in Fig. 3-8*b*, is often used for screen-grid tetrodes.

The Screen-grid Tetrode.—One stimulus to the development of multigrid tubes was the necessity of reducing the capacitance between the grid and plate of the triode. If a vacuum tube used in a voltage amplifier has high grid-plate capacitance, the relatively large variations of plate voltage may induce appreciable variations of grid voltage. If the phase relations are correct, this induced grid voltage may add to the impressed alternating voltage in such a manner as to cause the amplifier to oscillate (see Sec. 10-28). This difficulty imposes a limit upon the amplification that can be attained in radio-frequency amplifiers. For some years the problem was solved by "neutralizing." Neutralization consists in connecting the grid through a small variable condenser to a point in the output circuit whose voltage is opposite in phase to that of the plate. The condenser may be adjusted so that it balances out, or neutralizes, the effect of the grid plate capacitance. Difficulties of adjustment, circuit complications, and the cost of patent royalties made it advantageous to solve the problem by removing the cause, rather than by counteracting it. This was accomplished by introducing between the control grid and the plate another grid, the *screen grid*, the purpose of which is to shield the grid from the plate, and thus reduce the grid-to-plate capacitance.[1] Further reduction in capacitance between the grid and the plate was attained by placing the control-grid terminal at the top of the tube, instead of on the base. The screen-grid tetrode proved to have other characteristics which are fully as important as its low grid-to-plate capacitance.

The general construction of the elements of a type 24A screen-grid tetrode is shown in Fig. 3-8*a*. The screen grid consists of two cylinders of fine-mesh screening, one of which is between the plate and the control grid and the other outside of the plate. These two cylinders are joined at the top by an annular disk, which completes the shielding. The

[1] SCHOTTKY, W., *Arch. Elektrotech.*, **8**, 299 (1919); U. S. Patent 1537708; BARKHAUSEN, H., *Jahrb. drahtl. Tel. u. Tel.*, **14**, 43 (1919); HOWE, G. W. O., *Radio Rev.*, **2**, 337 (1921); HULL, A. W., and WILLIAMS, N. H., *Phys. Rev.*, **27**, 432 (1926); HULL, A. W., *Phys. Rev.*, **27**, 439 (1926); WARNER, J. C., *Proc. I.R.E.*, **16**, 424 (1928) (with 22 references); PRINCE, D. C., *Proc. I.R.E.*, **16**, 805 (1928); WILLIAMS, N. H., *Proc. I.R.E.*, **16**, 840 (1928); PIDGEON, H. A., and MCNALLY, J. O., *Proc. I.R.E.*, **18**, 266 (1930).

potential of the screen is normally intermediate between the quiescent potentials of the cathode and the plate. The positive voltage of the screen draws the electrons away from the cathode. Some of these electrons strike the screen and result in a screen current which usually performs no useful function; the rest pass through the screen grid and into the field of the plate, which causes them to be drawn to the plate. Since the electrostatic field of the plate terminates almost completely on the screen, the capacitance between the plate and the grid is very small. Furthermore, variations of plate voltage have little effect on the plate current. The control-grid voltage, on the other hand, is just as effective as in the triode. The change in plate current resulting from a change in plate voltage at constant grid voltage is small, and the ratio of the change in plate voltage to the change in grid voltage, necessary to produce a given change in plate current, is very high. It follows from the definitions of plate resistance and amplification factor that the screen-grid tetrode has high plate resistance and high amplification factor. By proper choice of control-grid structure and spacing of electrodes the transconductance can also be made high. A screen-grid tetrode can therefore be designed to have the same transconductance as that of an equivalent triode and very much higher amplification factor and plate resistance.

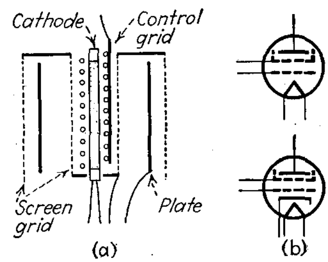


FIG. 3-8.—(a) Electrode structure of heater-type screen-grid tetrode. (b) Tube symbols for filamentary and heater-type screen-grid tetrodes.

In Fig. 3-9 is shown a family of plate characteristics for a typical screen-grid tetrode, the type 24A. The negative slope of the characteristics at plate voltages lower than the screen voltage is the result of secondary emission from the plate. At zero plate voltage there is a small plate current which results from those electrons which pass through the screen with sufficient velocity to reach the plate. As the plate voltage is raised, more and more electrons are drawn to the plate after passing through the screen. The velocity with which they strike the plate increases with the plate voltage and, when e_b is about 10 volts, becomes sufficiently high to produce appreciable secondary emission from the plate. Because the screen is at a higher voltage than the plate, these secondary electrons are drawn to the screen. Since the secondary electrons move in the direction opposite to that of the primary electrons, they reduce the net plate current. If the plate is not treated to reduce secondary emission, the number of secondary electrons leaving the plate may exceed the number of primary electrons that strike the plate, and so the plate current may reverse in direction. This is shown by the dashed curve of Fig. 3-9, which is for the old type 24A tube, with untreated plate.

It is to be expected that all secondary electrons emitted by the plate will return to the plate when the plate voltage is higher than the screen voltage. The rise in plate current starting at voltages considerably lower than the screen voltage shows, however, that many secondary electrons return to the plate while the screen is still positive relative to the plate. The reason for this is the retarding field at the plate produced by electron space charge between the screen and the plate. (It will be explained in Sec. 3-11 how tubes may be designed so that this space charge prevents secondary electrons emitted by the plate from going to the screen even at plate voltages much lower than the screen voltage.)

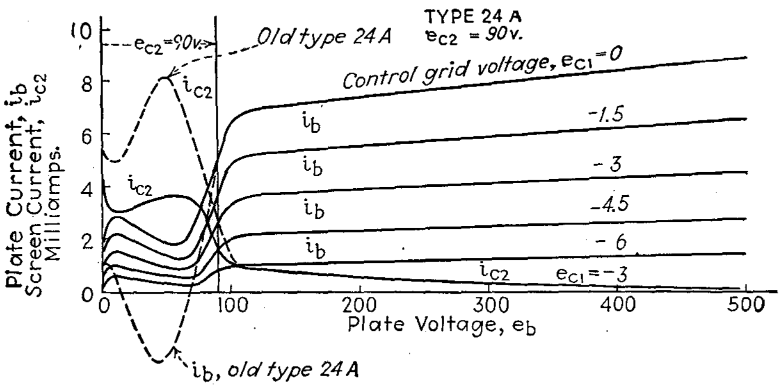


FIG. 3-9.—Typical screen-grid tetrode plate characteristics at 90-volt screen voltage, e_{c2}.

As the plate voltage approaches the screen voltage, the field at the plate produced by the screen voltage becomes less than the retarding field of the space charge and so the slower-moving secondary electrons are returned to the plate. At voltages higher than that at which all secondary electrons are returned to the plate, secondary emission from the plate has no effect upon the plate current, which is then determined almost entirely by the screen- and control-grid voltages. Since very little of the plate field penetrates to the cathode, further increase of plate voltage has only a small effect upon the plate current. The increase of plate current at plate voltages higher than the screen voltage is accounted for partly by increase in the number of secondary electrons from the screen that are drawn to the plate.

3-9. The Space-charge Tetrode.—Instead of using the inner grid of a tetrode as the control electrode and applying a positive voltage to the second grid, it is possible to operate the tube by applying a small positive voltage to the inner grid and using the second grid as the control electrode.[1] The positive voltage on the first grid overcomes the effect of the space charge in the vicinity of the cathode, and thus increases the plate current and the transconductance. Some of the electrons are drawn to the positive inner space-charge grid, but the remainder pass through this grid and into the region controlled by the second grid and the plate. The effect is in some respects the same as though the cathode were placed much closer to the control grid in a triode. A high negative voltage on the second grid prevents the electrons from passing to the plate and returns them to the positive space-charge grid. As the negative control-grid voltage is reduced, more electrons pass to the plate and fewer to the space-charge grid. Thus, the plate current increases, and the space-charge–grid current decreases with decrease of negative voltage on the control grid. Figure 3-10 shows typical curves of plate current and of first-grid current as a function of second-grid voltage.

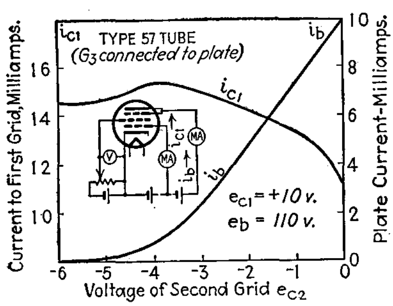


FIG. 3-10.—Characteristics showing first-grid current i_{c1} and plate current i_b of a space-charge tetrode as a function of second-grid voltage e_{c2}.

Although the transconductance of a space-charge tetrode is greater than that of a triode with a similar cathode, the relatively high current to the space-charge grid results in a less efficient use of the cathode current. Because more recently developed pentodes have much better characteristics than space-charge tetrodes, space-charge tetrodes are now used only in special applications, some of which will be discussed in later chapters.

3-10. The Pentode.—For most applications the curved portions of the characteristic curves of screen-grid tetrodes at plate voltages lower than the screen voltage are undesirable. In amplifiers, excessive distortion results if the tube is operated in this region and, if the circuit contains inductance and capacitance, oscillation may occur (see Sec. 10-17). Restriction of operation to the region to the right of the plate-current dip reduces the output voltage or power that can be obtained at a given value of operating plate voltage.

By the use of a ribbed plate and special treatment to reduce secondary emission, it is possible to design tetrodes whose characteristic curves do not have portions with negative slope. The type 48 tetrode is an example of such a tube. The effects of secondary emission can also be reduced or eliminated by preventing the secondary electrons emitted by

[1] ARDENNE, M. VON, *Hochfrequenztech. u. Elektroakustik*, **42**, 149 (1933). See also *Wireless Eng.*, **11**, 93 (1934) (abstr.); I. LANGMUIR, U. S. Patent 1558437, filed Oct. 29, 1913; WARNER, *loc. cit.*

the plate from going to the screen. This can be done by placing between the screen and the plate a third grid, called the *suppressor grid.* Figure 3-11*a* shows the arrangement of the electrodes of a typical suppressor pentode of the voltage amplifier type, the 57. The purpose of the shield in the dome of the tube is to shield the control-grid lead and terminal from the plate. This shield is connected internally to the cathode and shaped so as to act as the continuation of an external shield which may be placed around the tube. In voltage pentodes of more recent design, such as that shown in Fig. 3-11*b*, special precautions in placing and shielding the leads have made it possible to connect the control grid to a base pin instead of to a terminal at the top of the tube.[1] In power pentodes the control-grid terminal is in the base of the tube.

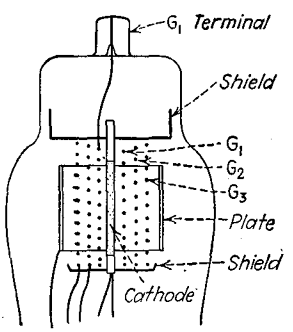


FIG. 3-11*a*.—Electrode structure of the type 57 pentode. G_1G_2, and G_3 are normally the control, screen, and suppressor grids, respectively.

Figure 3-12 shows a series of plate characteristics of the 57 tube at constant control-grid and screen-grid voltages for a number of suppressor-grid voltages. It can be seen that the secondary-emission dips move to the left and become less pronounced as the suppressor-grid voltage is reduced below the screen voltage. When the suppressor voltage is zero, *i.e.*, when the suppressor is connected to the cathode, the secondary-emission effects are almost entirely absent.

The explanation of the action of the suppressor is simple. When the suppressor is connected to the cathode, the field between the plate and the suppressor is always such as to move electrons toward the plate. The secondary electrons removed from the plate have sufficiently low velocity of emission so that even at low plate voltages few can permanently leave the plate against this retarding field. Velocity acquired by the primary electrons in the space between the cathode and the screen carries most of them through the screen and suppressor and thence into the field beyond the suppressor, which draws them to the plate.

The additional shielding effect of the suppressor grid results in grid-plate capacitance that is even lower than that of tetrodes and in plate resistance and amplification factor that are even greater than those of tetrodes. It can be seen from Fig. 3-12 that the plate resistance of the suppressor pentode can be varied by means of negative suppressor voltage.

In some pentodes the suppressor is permanently connected to the cathode internally; in others, on the other hand, all three grids may have

[1] KELLY, R. L., and MILLER, J. F., *Electronics*, September, 1938, p. 26.

external connections in order that the grids may be used in various ways. When the second and third grids are connected to the plate, the tube has ordinary triode characteristics. When the first and second grids are used together as the control grid and the third is connected to the plate, the tube acts as a triode with very high amplification factor and low plate current. Other special applications of pentodes will be dis-

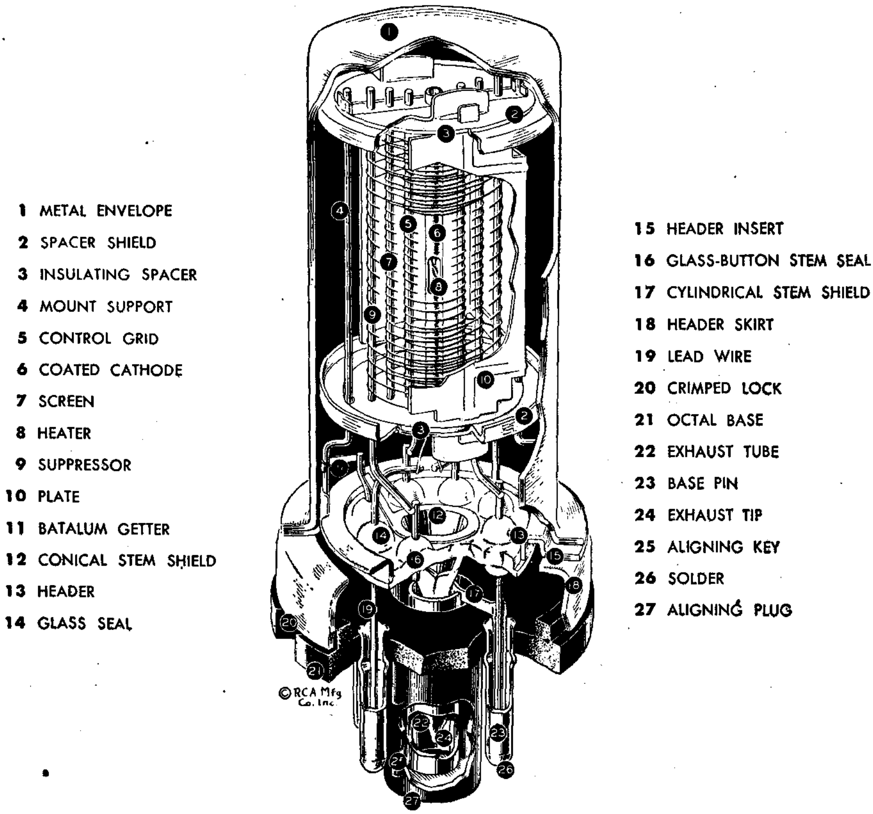


FIG. 3-11*b*.—Typical metal voltage-amplifier pentode in which the control-grid connection is brought out through the base. Note the conical stem shield 12. (*Courtesy of Radio Corporation of America.*)

cussed in later chapters (see end of Sec. 6-1 and Secs. 10-3, 10-4, 10-12, 10-23, 10-39, 12-6, and 12-33).

Figures 3-13 and 3-14 show the plate characteristics for two types of suppressor pentodes, the 57 and the 2A5 (see also Figs. A-13 to 15 and A-19, pages 677–678). It will become apparent from material to be presented in this and later chapters that ideal characteristics for amplifier tubes would be straight, parallel, and equidistant for all values of plate voltage. The gradual bending of the characteristics at low plate voltages, particularly

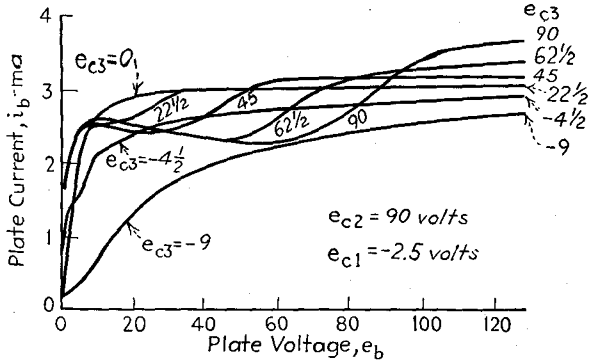


FIG. 3-12.—Plate characteristics of the type 57 pentode, showing the effect of variation of suppressor voltage e_{c3} at 90-volt screen voltage e_{c2}.

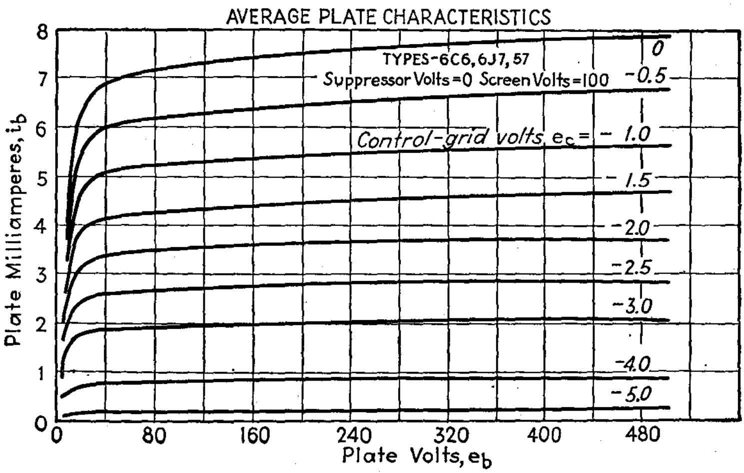


FIG. 3-13.—Plate characteristics of a typical voltage pentode.

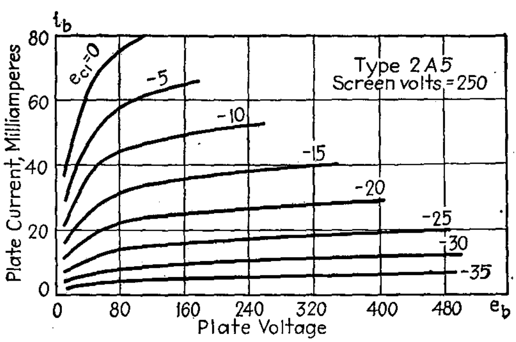


FIG. 3-14.—Plate characteristics of typical power pentode.

noticeable in the characteristics of pentodes designed to give large power output, such as the type 2A5, causes distortion in amplifiers and is, therefore, objectionable. It is the result of nonuniformity of the field in the plane of the suppressor grid. Because of the shielding action of the grids, the plate voltage has relatively little effect upon the number of electrons arriving in the plane of the suppressor grid. As the plate voltage is raised, however, more and more electrons arriving at the suppressor are drawn to the plate. The comparatively constant "saturation" plate current is obtained when all electrons arriving at the plane of the suppressor are drawn to the plate. Because of nonuniformity of field in the plane of the suppressor, saturation is attained at different plate voltages for various points in this plane, and so the characteristics bend over gradually and have broad knees. This difficulty is avoided in the *beam power pentode*.[1]

3-11. Beam Pentodes.—In the beam tube the secondary electrons are returned to the plate by the repulsion of negative space charge between the screen and the plate. This space charge is accentuated by the retarding field of the plate when the plate potential is lower than the screen potential. By proper design the space charge may be made so dense as to cause the formation of a virtual cathode [*i.e.*, a plane of zero average field and zero electron velocity (see Sec. 2-12)] near the plate at low plate voltages. For all values of plate voltage less than the screen voltage a potential minimum is formed which may be kept sufficiently lower than the plate potential so that secondary electrons from the plate are returned to the plate. The action is similar to that of a suppressor grid; but, if the density of space charge and the electron velocity at the virtual cathode are uniform and the distance of the virtual cathode from the plate is everywhere the same, saturation takes place simultaneously at all points in the plane of the virtual cathode, and the knee of the plate characteristic is sharp. The virtual cathode and the plate act in a manner similar to a diode, the plate current being limited by space charge at low plate voltages. As in a diode, saturation at low plate voltage, *i.e.*, a low-voltage knee, requires that the virtual cathode shall be close to the plate.

In the beam power tube, of which the 6L6 is a typical example, the required electron density is achieved by confining the electrons to beams. The homogeneity of space charge and electron velocity is attained by proper design of the contours of the cathode, grids, and plate and by correct choice of the ratio of screen-plate to screen-cathode spacing (2.9)

[1] SCHADE, O. H., *Proc. I.R.E.*, **26**, 137 (1938). See also F. BELOW, *Z. Fernmeldetech.* **9**, 113 (1928); R. S. BURNAP, *RCA Rev.*, **1**, 101 (1936); J. F. DREYER, JR., *Electronics*, April, 1936, p. 18; J. H. O. HARRIES, *Electronics*, May, 1936, p. 33; B. SALZBERG and A. V. HAEFF, *RCA Rev.*, **2**, 336 (1938).

and beam angle (approximately 60 degrees). The electrons are confined to beams by means of beam-forming plates, as shown in Fig. 3-15, which are at cathode potential. The flattened cathode gives a larger effective area than a round cathode and so results in a higher transconductance.

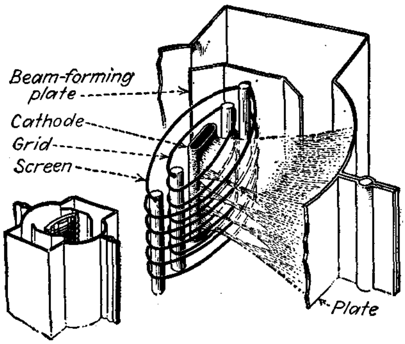


FIG. 3-15.—Electrode structure of the type 6L6 beam power pentode. (*Courtesy of Radio Corporation of America.*)

The screen current of beam power tubes is much lower than that of suppressor pentodes. The screen and control grid have equal pitch and are proportioned and assembled so that the screen grid is hidden from the cathode by the control grid, and the individual beam sheets formed by the control grid are focused in the plane of the screen. Very few, therefore, of the electrons moving toward the plate can strike the screen directly. Furthermore, because of the sharpness of the beams and the uniform fields, few electrons acquire tangential velocity at the expense of velocity normal to the electrode planes. The number of electrons that miss the plate at low plate voltage and return to the screen is therefore also small. Low screen current results in a number of

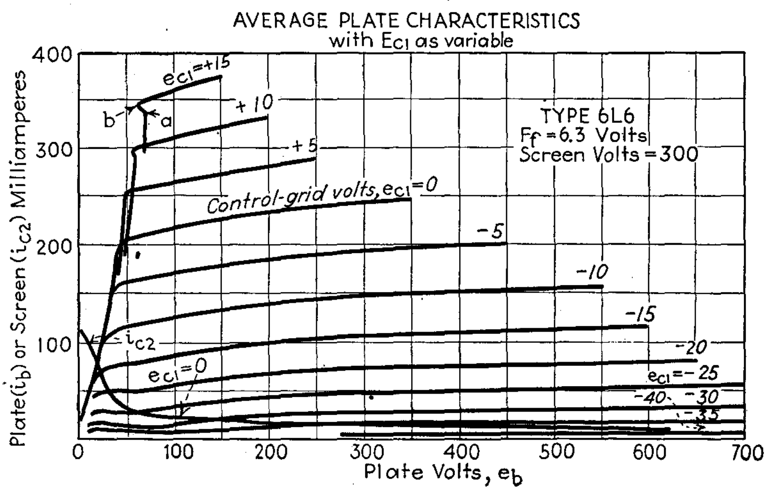


FIG. 3-16.—Plate characteristics of the type 6L6 beam power pentode.

advantages, among which are low screen dissipation and consequent larger power rating without danger of electron emission from the screen because of high screen temperature, and the possibility of using low-power resistors in the voltage divider that supplies the screen voltage.

Figure 3-16 shows the plate characteristics of the 6L6. These characteristics are straight and parallel over a much greater range of plate voltage than those of suppressor pentodes, and the knees are sharper. The peculiar shape of the knees of the curves for positive control-grid voltages is the result of two possible distributions of space charge at the same value of plate voltage.[1] As the plate voltage is gradually increased from a low value, the plate current increases continually until the point *a* is reached, at which it jumps abruptly to the higher value corresponding to this voltage. When the plate voltage is then gradually reduced, the current decreases continuously until the point *b* is reached, at which it falls abruptly to the lower value corresponding to this plate voltage. The abrupt changes in current are accompanied by changes in space-charge distribution.

3-12. Equations for Plate Current of Multigrid Tubes.—Plate-current equations similar to Eqs. (3-1) and (3-2) may be written for tetrodes and pentodes. In most circuits, however, only the control-grid and plate voltages are varied, and Eqs. (3-1) and (3-2) may be applied.

Determination of Tetrode and Pentode Factors from Characteristics. The plate characteristics of tetrodes and pentodes are not ordinarily shown for sufficiently small increments of control-grid voltage to make possible the direct determination of μ from the plate characteristics. Furthermore, the plate resistance of voltage-amplifier tetrodes and pentodes, such as the 24A (Fig. 3-9) and the 6J7 (Fig. 3-13), is so high that the plate characteristics are nearly horizontal throughout the working range of voltages and the plate resistance can be determined only approximately from the tangent to the plate characteristic at the point corresponding to the given voltages. The plate resistance of power suppressor pentodes such as the 2A5 (Fig. 3-14), however, and of beam pentodes such as the 6L6 (Fig. 3-16) is sufficiently low so that it can be determined with fair accuracy from the tangent to the plate characteristic.

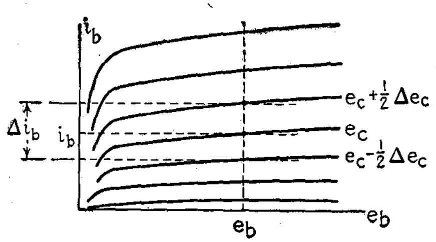


FIG. 3-17.—Graphical determination of transconductance of pentodes.

An approximate value of g_m can be found from increments of plate current and grid voltage at constant plate voltage, as determined from the plate characteristics. Since g_m is not constant, accuracy in the values of g_m found by this method is dependent upon the use of very small increments. Accuracy also makes it desirable to use an increment of

[1] SALZBERG and HAEFF, *loc. cit.*

grid voltage such that the point at which g_m is desired lies at the center of the increment, as shown in Fig. 3-17. If the grid voltage intervals corresponding to the available plate characteristics are large, it may be advisable to construct the static transfer characteristic corresponding to the given plate voltage and to find g_m accurately from the slope of this characteristic at the given plate voltage. Equation (3-22) may be used to find an approximate value of μ from g_m in conjunction with the approximate value of r_p determined from the plate characteristic.

3-13. Duplex Tubes and Tubes with More than Three Grids.—The behavior of the individual units of duplex tubes is no different than when these units are enclosed in separate envelopes. For this reason duplex tubes require no further discussion. Special tubes having more than three grids will be treated in later chapters in connection with their applications (see Secs. 6-18, 6-19, 9-12*A*, and 9-22).

3-14. Applied Voltages in Grid and Plate Circuits.—In most applications of vacuum tubes, one or more alternating voltages are applied to the control-grid and plate circuits, in addition to the steady voltages, as shown in Fig. 3-18. The steady applied voltages E_{cc}, E_{bb}, and E_{ff} are called the *grid supply voltage* or *C-supply voltage*, the *plate supply voltage* or *B-supply voltage*, and the *filament* or *heater supply voltage*, respectively. v_g and v_p, the applied alternating voltages in the grid and plate circuits, are called the *grid excitation voltage* and *plate excitation voltage*, respectively. Because of impedance drops resulting from the flow of electrode currents or other circuit currents, the electrode voltages in general differ from the voltages impressed upon the electrode circuits. This will be discussed in detail in Secs. 3-17 and 4-1.

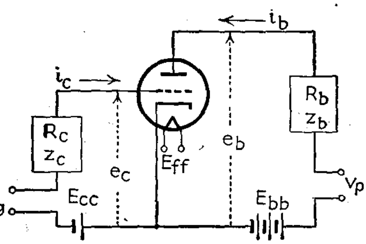


FIG. 3-18.—Triode grid and plate circuits, showing supply voltages, impressed alternating voltages, and electrode voltages.

All electrode voltages are measured with respect to the cathode, and an electrode voltage is said to be positive if the electrode is positive relative to the cathode. Similarly, supply voltages, instantaneous values of exciting voltages, and instantaneous values of voltages across load impedances are ordinarily measured relative to the cathode side of these voltages and are called positive if they tend to make the electrode positive.

3-15. Form of Alternating Plate-current Wave.—Figure 3-19 shows the manner in which a sinusoidal grid voltage causes the plate current to vary when the load is nonreactive. The wave of plate current is constructed by projecting from the wave of grid voltage to the transfer characteristic at various instants throughout the cycle. The significance of the *time axis* of the wave of varying plate current and of the current I_{bt} corresponding to this axis should be noted. I_{bt} is the current assumed at the instants in the cycle when the alternating grid voltage is zero. The reason why I_{bt} differs from the steady current I_{bo}, assumed when the excitation is zero, will be explained in detail in Chap. 4 (Secs. 4-8 to 4-10).

Curvature of the transfer characteristic in general causes the wave of varying plate current to be asymmetrical even though the exciting voltage is sinusoidal. It will be proved in later sections that the wave of alternating plate current, measured relative to the time axis, in general contains not only a fundamental component of the same frequency as the grid excitation voltage, but also harmonics of that frequency, and a steady component. When the load is nonreactive, the wave of plate voltage is of the same form as the wave of plate current.

Usually it is most convenient to measure the instantaneous value of the alternating component of plate or grid current or voltage relative to the time axis of the wave. Occasionally, however, it is necessary to measure the instantaneous value with respect to the average value. An instantaneous value measured relative to the axis differs from the value measured relative to the average in that the former contains the steady component of the alternating current or voltage, whereas the latter does not. Unless otherwise specifically stated, the terms *alternating plate current* and *alternating grid current*, and the corresponding terms for voltages, will be understood to refer to values measured relative to the time axes.

In some applications of vacuum tubes the curvature of the characteristic or the amplitude of the alternating plate current, or both, are small enough so that the alternating plate current is essentially sinusoidal, as shown in Fig. 3-20. The difficulty in making a rigorous analysis sometimes necessitates the assumption that the alternating plate current is sinusoidal, even though it is known to be distorted.

Most circuit elements dealt with in the power field of electrical engineering are linear elements, *i.e.*, elements in which the currents flowing through the elements are proportional to the voltages across the elements. The currents that flow as the result of simultaneous application of direct and alternating voltages or of two or more alternating voltages of different frequencies are entirely independent. Hence, when several voltages are applied simultaneously, the net result may be determined by making relatively simple analyses involving only direct currents or alternating currents of only one frequency at a time and superimposing the individual results. Comparatively few symbols suffice in dealing with such circuits. Vacuum tubes, on the other hand, are nonlinear circuit elements. The direct currents that flow in a circuit containing a nonlinear element depend not only upon the impressed direct voltages but also upon the impressed alternating voltages. Similarly, the alternating currents depend upon both the alternating and the direct impressed voltages. Furthermore, in addition to currents having the same frequencies as the impressed voltages, alternating currents will in general flow whose frequencies differ from those of the impressed alternating voltages. It is apparent that the analysis of vacuum-tube circuits may be considerably more complicated than that of power circuits containing only linear circuit elements.

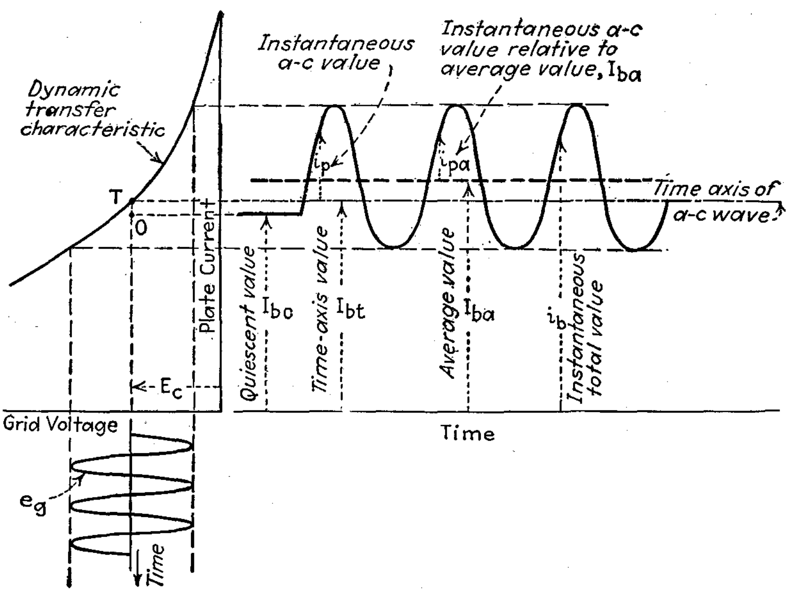


FIG. 3-19.—Plate-current relations for asymmetrical wave of plate current.

3-16. Symbols.—Because it is often necessary to consider simultaneously the various direct and alternating voltages and currents in vacuum-tube circuits, a large number of symbols must be used. This is unfortunate but usually unavoidable. The symbols used in this book are based upon those standardized by the Institute of Radio Engineers.

The student will find it helpful to learn the following rules, according to which these symbols are formed:

1. The subscripts $_c$ and $_g$ refer to the grids or to the grid circuits, and the subscripts $_b$ and $_p$ to the plate or to the plate circuit.

2. Lower-case letters indicate instantaneous values of varying quantities, and capital letters indicate steady (direct) values and average, r-m-s, and crest values of varying quantities (see Figs. 3-19 and 3-20).

3. Lower-case letters with subscripts $_c$ and $_b$ indicate *total* instantaneous values of varying quantities (see Figs. 3-19 and 3-20).

4. Lower-case letters with subscripts $_g$ and $_p$ indicate instantaneous values *of the alternating components* of varying quantities. If the wave form of the varying quantity is asymmetrical, second subscripts $_a$ and $_t$ are added to differentiate between an instantaneous value measured relative to the average value and an instantaneous value measured relative to the value corresponding to the axis of the wave (see Fig. 3-19).

5. Capital letters with subscripts $_c$ and $_b$ indicate direct or average values. Second subscripts $_o$, $_a$, and $_t$ are added to differentiate between quiescent, average, and time-axis values of an asymmetrical wave of total varying plate current or voltage (see Fig. 3-19).

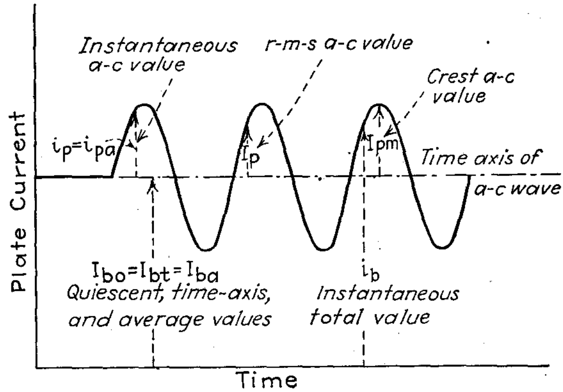


FIG. 3-20.—Plate-current relations for sinusoidal wave of plate current.

6. Capital letters with subscripts $_g$ and $_p$ indicate r-m-s or crest values of alternating quantities. Crest values of sinusoidal alternating quantities are distinguished from r-m-s values by the addition of the second subscript $_m$ (see Fig. 3-20).

In tetrodes and pentodes and in tubes having more than three grids, it is necessary to distinguish between the voltages and between the currents of the various grids. This is done by means of the addition of a number in the subscript to indicate the number of the grid. Thus e_{c1}, e_{c2}, e_{c3}, etc., indicate the instantaneous total voltages of the first, second, and third, etc., grids, grid 1 being nearest the cathode. Usually the first grid serves as the control electrode, and e_{c1} indicates the control-grid voltage. In many analyses it is necessary to speak only of the control-grid voltage. It is then convenient to omit the number in the subscript, even though the tube contains more than one grid. In the work that follows, therefore, it will be understood that, when no number appears in the subscript, reference is to the control grid and that the term *grid* refers to the control grid.

The symbols are as follows:

Control-grid supply voltage . . . . E_{cc1} or E_{cc}
Screen-grid supply voltage . . . . E_{cc2}
Plate supply voltage . . . . E_{bb}
Filament or heater supply voltage . . . . E_{ff}
Instantaneous value of grid excitation voltage . . . . v_g
Instantaneous value of plate excitation voltage . . . . v_p
Instantaneous total grid voltage . . . . e_c
Instantaneous total plate voltage . . . . e_b
Instantaneous total grid current . . . . i_c
Instantaneous total plate current . . . . i_b
Quiescent (zero excitation) value of grid voltage . . . . E_{co}
Quiescent (zero excitation) value of plate voltage . . . . E_{bo}
Quiescent (zero excitation) value of grid current . . . . I_{co}
Quiescent (zero excitation) value of plate current . . . . I_{bo}
Voltage corresponding to time axis of wave of varying plate voltage . . . . . E_{bt}
Current corresponding to time axis of wave of varying plate current . . . . . I_{bt}
Average value of grid voltage (grid bias) . . . . E_c (or E_{ca})
Average value of plate voltage . . . . E_{ba}
Average value of grid current . . . . I_{ca}
Average value of plate current . . . . I_{ba}
Instantaneous value of alternating component of grid voltage . . . . e_g
Instantaneous value of alternating component of plate voltage, measured relative to the time axis of the wave of alternating plate voltage . . . . . . e_p
Instantaneous value of alternating component of plate voltage, measured relative to average plate voltage . . . . e_{pa}
Instantaneous value of alternating component of grid current . . . . i_g
Instantaneous value of alternating component of plate current, measured relative to the time axis of the wave of alternating plate current (see Fig. 3-19) . . . . i_p
Instantaneous value of alternating component of plate current, measured relative to average plate current (see Fig. 3-19) . . . . i_{pa}
Effective value of alternating component of grid voltage . . . . E_g
Effective value of alternating component of plate voltage . . . . E_p
Effective value of alternating component of grid current . . . . I_g
Effective value of alternating component of plate current . . . . I_p
Crest value of sinusoidal alternating component of grid voltage (grid swing) . . . . E_{gm}
Crest value of sinusoidal alternating component of plate voltage . . . . E_{pm}
Crest value of sinusoidal alternating component of grid current . . . . I_{gm}
Crest value of sinusoidal alternating component of plate current . . . . I_{pm}

Impedance of the grid circuit at angular frequency ω . . . . . . $\begin{cases} (z_c)_\omega = (r_c)_\omega + j(x_c)_\omega \\ \text{or} \quad z_c = r_c + jx_c \end{cases}$

Impedance of the plate circuit (load impedance) at angular frequency ω . . . . $\begin{cases} (z_b)_\omega = (r_b)_\omega + j(x_b)_\omega \\ \text{or} \quad z_b = r_b + jx_b \end{cases}$

Admittance of the grid load at angular frequency ω . . . $\begin{cases} (y_c)_\omega \equiv \dfrac{1}{(z_c)_\omega} = (g_c)_\omega - j(b_c)_\omega \\ \text{or } y_c \equiv \dfrac{1}{z_c} = g_c - jb_c \end{cases}$

Admittance of the plate load at angular frequency ω. $\begin{cases} (y_b)_\omega \equiv \dfrac{1}{(z_b)_\omega} = (g_b)_\omega - j(b_b)_\omega \\ \text{or } y_b \equiv \dfrac{1}{z_b} = g_b - jb_b \end{cases}$

D-c resistance of the grid load . . . . R_c
D-c resistance of the plate load . . . . R_b
Effective value of the alternating voltage drop in the plate circuit impedance . . . . E_{zb}

3-17. Current and Voltage Relations in the Grid and Plate Circuits.—The following relations are apparent from Fig. 3-18 and from Fig. 3-19 and the similar wave of plate voltage:

$$i_b = I_{bt} + i_p = I_{ba} + i_{pa} \tag{3-24}$$
$$e_b = E_{bt} + e_p = E_{ba} + e_{pa} \tag{3-25}$$
$$E_{ba} = E_{bb} - I_{ba}R_b \tag{3-26}$$
$$e_p = v_p - \left(L_b \frac{di_p}{dt} + r_b i_p + \frac{1}{C_b}\int i_p\,dt\right) \tag{3-27}$$

in which L_b, r_b, and C_b are the equivalent series inductance, resistance, and capacitance of the plate load. For the special case in which the wave of alternating plate current is symmetrical, as illustrated by the sinusoidal current wave of Fig. 3-20, I_{ba} and I_{bt} are identical with I_{bo}, and E_{ba} and E_{bt} are identical with E_{bo}. i_p and i_{pa} are then also identical, as are e_p and e_{pa}. For a symmetrical wave, therefore, Eqs. (3-24), (3-25), and (3-26) reduce to the following:

$$i_b = I_{bo} + i_p \tag{3-28}$$
$$e_b = E_{bo} + e_p \tag{3-29}$$
$$E_{bo} = E_{bb} - I_{bo}R_b \tag{3-30}$$

Equation (3-30) also holds when the excitation voltage is zero. When the plate current is sinusoidal, Eq. (3-27) may be written more conveniently in terms of effective values, as follows:

$$E_p = V_p - I_p z_b \tag{3-31}$$

In many circuits the plate excitation voltage V_p is zero, and Eq. (3-31) reduces to

$$E_p = -I_p z_b \tag{3-32}$$

The following equations are apparent from Fig. 3-18 and from a diagram analogous to that of Fig. 3-19:

$$i_c = I_{ct} + i_g = I_{ca} + i_{ga} \tag{3-33}$$
$$e_c = E_{ct} + e_g = E_c + e_{ga} \tag{3-34}$$
$$E_c \text{ (or } E_{ca}) = E_{cc} - I_{ca}R_c \tag{3-35}$$
$$e_g = v_g - \left(L_c \frac{di_g}{dt} + r_c i_g + \frac{1}{C_c}\int i_g\,dt\right) \tag{3-36}$$

in which L_c, r_c, and C_c are the equivalent series inductance, resistance, and capacitance of the grid load. In many vacuum-tube circuits, currents other than grid current flow through impedances contained in the grid circuit (see Sec. 4-1). The expression for e_g given by Eq. (3-36) must then be replaced by one including the alternating voltages resulting from the flow of these additional currents.

In many vacuum-tube circuits the control grid is maintained sufficiently negative so that grid current does not flow. Unless some other direct current flows through a resistance contained in the grid circuit, Eq. (3-35) then simplifies to

$$E_c(\text{or } E_{ca}) = E_{cc} \tag{3-37}$$

and, unless some other alternating current flows through an impedance contained in the grid circuit, Eq. (3-36) becomes

$$e_g = v_g \tag{3-38}$$

which may also be written in terms of effective values as follows:

$$E_g = V_g \tag{3-39}$$

The method of evaluating E_g, when E_g results wholly or in part from the flow of alternating currents through impedances contained in the grid circuit, will be explained in detail in Sec. 4-1.

3-18. Static and Dynamic Operating Points.—The steady values of electrode voltages and currents assumed when the excitation voltages are zero are called the *quiescent* or *static operating* voltages and currents. The point on the static characteristics corresponding to given static operating voltages and currents is termed the *static operating point* (or *quiescent point*). The static operating point will be indicated on the characteristic curves by the letter O and will sometimes be referred to as the O-point. The average values of electrode voltages and currents assumed with excitation are called the *dynamic operating* voltages and currents. The corresponding point on the plate characteristics is termed the *dynamic operating point* and will be indicated by the letter A. The dynamic and static operating points coincide when the wave of plate current is symmetrical. Static and dynamic operating points will be discussed in detail in Secs. 4-7 to 4-13.

The amplitude of the alternating control-grid voltage E_{gm} is called the *grid swing*. The direct component of control-grid voltage E_c is called the *grid bias* or *C bias*. Because the symbol E_c has long been used to represent the grid bias, this symbol, rather than the alternative E_{ca}, will be used throughout the remainder of this book. Grid bias may be obtained by the use of a battery or other source of fixed voltage, or it may be produced by the flow of cathode current through a resistance, as in Figs. 5-7 and 5-8. The former is called *fixed bias,* the latter *self-bias.* The term *bias* is also sometimes applied to the direct component of voltage of electrodes other than the control grid.

3-19. Generation of Harmonics.—If a sine wave of excitation voltage is applied to an electrode of a vacuum tube, it is found that the plate current in general contains not only an alternating component of the same frequency as that of the applied voltage but also components whose frequencies are equal to harmonics of the impressed frequency. Usually there is also a change in average plate current. If two or more sinusoidal voltages are impressed simultaneously the wave form of the alternating plate current is even more complicated, containing the applied frequencies

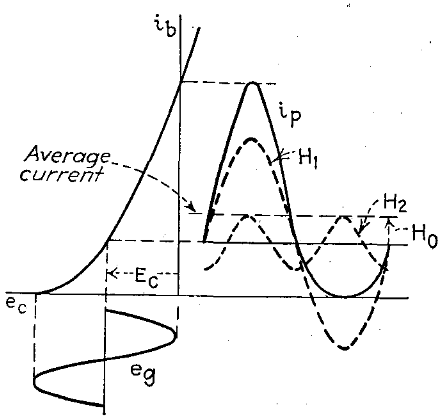


FIG. 3-21.—Generation of second-harmonic and steady components of plate current by a tube with a parabolic transfer characteristic.

and their harmonics and also frequencies equal to the sums and differences of the applied frequencies and their integral multiples.[1]

The generation of new frequencies by a vacuum tube is associated with the fact that it is not a linear circuit element, *i.e.,* that the characteristic curves are not linear. The appearance of plate-current components of frequencies different from the impressed frequency as the result of curvature of the transfer characteristic can be readily demonstrated graphically. The transfer characteristic of Fig. 3-21 is parabolic, and the exciting voltage sinusoidal, as indicated by the curve of e_g. The wave of i_p is constructed by finding from the transfer characteristic the values of instantaneous plate current corresponding to instantaneous grid voltages at various instants of the cycle. The dotted curves show

[1] It is evident, therefore, that it is not in general possible to apply to the vacuum tube the superposition theorem, which states that the current that flows through a linear circuit element as the result of a number of simultaneously impressed voltages of different frequencies is equal to the sum of the currents that would flow if the various voltage components were applied individually.

the fundamental, steady, and second-harmonic components into which the alternating plate current i_p may be resolved.

3-20. Series Expansion for Alternating Plate Current.—Theoretically, it should be possible to predict from Eq. (3-2) and corresponding tetrode and pentode equations the form of the alternating plate current corresponding to an exciting voltage of known wave form. Practically, however, $F(e_b + \mu e_c + \epsilon)$ is so complicated in form that Eq. (3-2) is of little or no value for this purpose. The behavior of a nonlinear circuit element can in general be analyzed mathematically most readily by expressing the alternating current in the form of an infinite power series. For a two-terminal element, in which the current depends upon only one voltage, the series involves only the impressed voltage, the element impedance and its derivatives at the operating point, and the circuit impedance. It may be derived by the application of Taylor's expansion for a function of one variable to the functional equation of current. Because the plate current of a vacuum tube depends upon all the electrode voltages, a complete series expansion applicable to a tube with three or more electrodes must involve all electrode voltages, as well as various tube factors and their derivatives at the operating point.[1] The general series expansion for tetrodes and pentodes is complicated, therefore, particularly if control-grid current is assumed to flow and the control-grid circuit contains an impedance. When all electrode voltages except those of the control grid and the plate are constant, however, as is usually true, the action of multigrid tubes is similar to that of triodes, and the general form of the series reduces to that for a triode. The triode expansion may consequently be applied to any tube in which only the plate and control-grid voltages vary. Furthermore, most problems in which the series expansion is of value can be adequately treated by the use of the form derived under the assumption that control-grid current does not flow. A further simplification results from the assumption that μ is constant, which is approximately true in many tubes. For the general forms of the series expansion for plate current of multigrid tubes, and for the similar expansions for currents to other electrodes, the student should refer to the work of Llewellyn[2] and others.[3]

3-21. Series Expansion. *Resistance Load.*—The series expansion for the plate current of a triode with negative grid voltage, constant amplification factor, and nonreactive load has the following form:

$$i_p = a_1 e + a_2 e^2 + a_3 e^3 + \cdots \tag{3-40}$$

[1] CARSON, J. R., *Proc. I.R.E.*, **7**, 187 (1919).

[2] LLEWELLYN, F. B., *Bell System Tech. J.*, **5**, 433 (1926).

[3] BRAINERD, J. G., *Proc. I.R.E.*, **17**, 1006 (1929); CAPORALE, P., *Proc. I.R.E.*, **18**, 1593 (1930); BONER, M. O., *Phys. Rev.*, **39**, 863 (1932); BENNETT, W. R., *Bell System Tech. J.*, **12**, 228 (1933); ESPLEY, D. C., *Proc. I.R.E.*, **22**, 781 (1934); BARROW, W. L., *Proc. I.R.E.*, **22**, 964 (1934).

where

$$\left.\begin{aligned} a_1 &= \frac{\mu}{r_p + r_b} \\ a_2 &= -\frac{\mu^2 r_p}{2(r_p + r_b)^3}\frac{\partial r_p}{\partial e_b} \\ a_3 &= \frac{\mu^3 r_p}{6(r_p + r_b)^5}\left[(2r_p - r_b)\left(\frac{\partial r_p}{\partial e_b}\right)^2 - r_p(r_p + r_b)\frac{\partial^2 r_p}{\partial e_b^2}\right] \end{aligned}\right\} \tag{3-41}$$

and

$$e = e_g + \frac{v_p}{\mu} \tag{3-42}$$

For common high-vacuum triodes, Eq. (3-40) converges rapidly enough so that the required accuracy is usually obtained with only a few terms of the series if the current amplitude is not too great. $\partial r_p/\partial e_b$ may be evaluated by plotting a curve of r_p *vs.* e_b determined from the static plate characteristic corresponding to the given operating grid bias. $\partial r_p/\partial e_b$ is the slope of this r_p-e_b curve at the point corresponding to the operating plate voltage. Other higher-order derivatives may be evaluated in a similar manner from curves of derivatives of the next lower order, but the accuracy rapidly decreases with increase of order of the derivative. The value of the series expansion lies not so much in the direct solution of numerical problems, as in the general analysis of the operation of vacuum tubes. In later chapters the series expansion will be applied to the analysis of the operation of vacuum tubes as amplifiers, detectors, modulators, and oscillators.

3-22. Derivation of Series Expansion.[1]—Equations (3-40) and (3-41) are derived by applying Taylor's theorem for two variables to Eq. (3-2). This gives the following equation for alternating plate current:

$$i_p = \frac{1}{1!}\left(\frac{\partial i_b}{\partial e_c}e_g + \frac{\partial i_b}{\partial e_b}e_p\right) + \frac{1}{2!}\left(\frac{\partial i_b}{\partial e_c}e_g + \frac{\partial i_b}{\partial e_b}e_p\right)^2 + \frac{1}{3!}\left(\frac{\partial i_b}{\partial e_c}e_g + \frac{\partial i_b}{\partial e_b}e_p\right)^3 + \cdots \tag{3-43}$$

in which a term of the form $\left(\frac{\partial i_b}{\partial e_c}e_g + \frac{\partial i_b}{\partial e_b}e_p\right)^n$ is a convenient symbolical representation of the terms[2]

[1] The following outline of the derivation of Eqs. (3-40) and (3-41) is included in order to explain the general method and justify the use of these equations throughout the remainder of the book. The student will not be seriously handicapped by omitting Sec. 3-22.

[2] OSGOOD, W. F., "Advanced Calculus," p. 172, The Macmillan Company, New York, 1925.

$$\frac{\partial^n i_b}{\partial e_c^n} e_g^n + \frac{n}{1!} \frac{\partial^n i_b}{\partial e_c^{n-1} \partial e_b} e_g^{n-1} e_p + \frac{n(n-1)}{2!} \frac{\partial^n i_b}{\partial e_c^{n-2} \partial e_b^2} e_g^{n-2} e_p^2 + \frac{n(n-1)(n-2)}{3!} \frac{\partial^n i_b}{\partial e_c^{n-3} \partial e_b^3} e_g^{n-3} e_p^3 + \cdots \tag{3-44}$$

and all derivatives are evaluated at the static operating point.

Although the amplification factor varies with operating voltages, for many purposes sufficient accuracy is obtained by assuming it to be constant. Under this assumption a term of the form $\partial^n i_b / \partial e_c^{n-m} \partial e_b^m$ becomes $\mu^{n-m} \dfrac{\partial^n i_b}{\partial e_b^n}$,* and Eq. (3-43) reduces to

$$i_p = (e_p + \mu e_g) \frac{\partial i_b}{\partial e_b} + \frac{(e_p + \mu e_g)^2}{2!} \frac{\partial^2 i_b}{\partial e_b^2} + \frac{(e_p + \mu e_g)^3}{3!} \frac{\partial^3 i_b}{\partial e_b^3} + \cdots \tag{3-45}$$

But, at the operating point,

$$\left.\begin{aligned} \frac{\partial i_b}{\partial e_b} &= \frac{1}{r_p} \\ \frac{\partial^2 i_b}{\partial e_b^2} &= \frac{\partial}{\partial e_b}\left(\frac{1}{r_p}\right) = -\frac{1}{r_p^2} \frac{\partial r_p}{\partial e_b} \\ \frac{\partial^3 i_b}{\partial e_b^3} &= \frac{1}{r_p^3}\left[2\left(\frac{\partial r_p}{\partial e_b}\right)^2 - r_p \frac{\partial^2 r_p}{\partial e_b^2}\right] \end{aligned}\right\} \tag{3-46}$$

Therefore,

$$i_p = \frac{e_p + \mu e_g}{1! r_p} - \frac{(e_p + \mu e_g)^2}{2! r_p^2} \frac{\partial r_p}{\partial e_b} + \frac{(e_p + \mu e_g)^3}{3! r_p^3}\left[2\left(\frac{\partial r_p}{\partial e_b}\right)^2 - r_p \frac{\partial^2 r_p}{\partial e_b^2}\right] + \cdots \tag{3-47}$$

The plate resistance r_p and its derivatives in Eq. (3-47) are evaluated at the operating point.

Vacuum tubes are usually used with some form of load impedance in the plate circuit. It is convenient to consider first the relatively simple case in which the load is a resistance. For resistance load, Eq. (3-27) is

$$e_p = v_p - i_p r_b \tag{3-48}$$

and

$$e_p + \mu e_g = v_p + \mu e_g - i_p r_b = \mu\left(e_g + \frac{v_p}{\mu}\right) - i_p r_b \tag{3-49}$$

or

$$e_p + \mu e_g = \mu e - i_p r_b \tag{3-50}$$

* This may be shown by applying $\partial^n i_b / \partial e_c^{n-m} \partial e_b^m$ and $\partial^n i_b / \partial e_b^n$ to Eq. (3-2).

Substituting Eq. (3-50) in Eq. (3-47) gives

$$i_p = \frac{\mu e - i_p r_b}{1!\, r_p} - \frac{(\mu e - i_p r_b)^2}{2!\, r_p^2}\frac{\partial r_p}{\partial e_b} + \frac{(\mu e - i_p r_b)^3}{3!\, r_p^3}\left[2\left(\frac{\partial r_p}{\partial e_b}\right)^2 - r_p \frac{\partial^2 r_p}{\partial e_b^2}\right] + \cdots \tag{3-51}$$

Equation (3-51) cannot be conveniently used, because i_p appears in an involved manner on the right side of the equation. An explicit expression for i_p may be obtained by assuming that i_p is of the form of Eq. (3-40)

$$i_p = a_1 e + a_2 e^2 + a_3 e^3 + \cdots \tag{3-40}$$

The coefficients of Eq. (3-40) may be evaluated by substituting (3-40) in (3-51), giving

$$\sum_{n=1}^{\infty} a_n e^n = \frac{\mu e - r_b \sum\limits_{n=1}^{\infty} a_n e^n}{1!\, r_p} - \frac{\left(\mu e - r_b \sum\limits_{n=1}^{\infty} a_n e^n\right)^2}{2!\, r_p^2}\frac{\partial r_p}{\partial e_b} + \frac{\left(\mu e - r_b \sum\limits_{n=1}^{\infty} a_n e^n\right)^3}{3!\, r_p^3}\left[2\left(\frac{\partial r_p}{\partial e_b}\right)^2 - r_p \frac{\partial^2 r_p}{\partial e_b^2}\right] + \cdots \tag{3-52}$$

Equation (3-52) is an identity, which must hold for all values of e. This can be true only if for every value of n the summation of terms in e^n on the left side of the identity is equal to the summation of terms of e^n on the right side. Terms in e on the right side arise only from the first term of Eq. (3-52). Therefore,

$$a_1 e = \left(\frac{\mu}{r_p} - \frac{a_1 r_b}{r_p}\right) e \tag{3-53}$$

Terms in e^2 arise from the first two terms of the right side of Eq. (3-50). Therefore,

$$a_2 e^2 = -\left[\frac{a_2 r_b}{r_p} + \frac{(\mu - a_1 r_b)^2}{2 r_p^2}\frac{\partial r_p}{\partial e_b}\right] e^2 \tag{3-54}$$

By the solution of Eqs. (3-53) and (3-54) and a similar equation in e^3, the first three coefficients of Eq. (3-40) may be shown to have the values given by Eqs. (3-41).

3-23. Harmonic Generation and Intermodulation.—Before proceeding to a discussion of the more general problem of a tube with impedance load it is necessary to show that the presence of the second- and higher-order terms of the series is associated with the production of components of alternating plate current of frequencies other than those which are contained in the applied signal. Consider, for instance, the simple case in which the excitation voltage has only a single frequency. Then

$$\begin{aligned} e &= E_m \sin \omega t \\ e^2 &= E_m{}^2 \sin^2 \omega t = \tfrac{1}{2}E_m{}^2 - \tfrac{1}{2}E_m{}^2 \cos 2\omega t \\ e^3 &= \tfrac{3}{4}E_m{}^3 \sin \omega t - \tfrac{1}{4}E_m{}^3 \sin 3\omega t \end{aligned} \tag{3-55}$$

Thus the second-order term of the series gives rise to a steady component and to a second-harmonic component in the alternating plate current. The third-order term gives rise to a fundamental and a third-harmonic component of alternating plate current. The production of harmonics in the plate current has been shown graphically in Fig. 3-21 for the simple case in which the static characteristics are assumed to obey a parabolic law, $i_b = A(e_b + \mu e_c)^2$, and z_b is assumed to be zero. If r_b is zero, a_1, a_2, and a_3, as given by Eqs. (3-41), reduce to $\mu \dfrac{\partial i_b}{\partial e_b}$, $\dfrac{\mu^2}{2!}\dfrac{\partial^2 i_b}{\partial e_b{}^2}$, and $\dfrac{\mu^3}{3!}\dfrac{\partial^3 i_b}{\partial e_b{}^3}$, respectively. Similarly, any coefficient a_n reduces to $\dfrac{\mu^n}{n!}\dfrac{\partial^n i_b}{\partial e_b{}^n}$. Since $\dfrac{\partial^3 i_b}{\partial e_b{}^3}$ and higher-order derivatives of plate current with respect to plate voltage are zero when the static characteristics obey a parabolic law, the plate-current series contains only the first- and second-order terms. Consequently the alternating plate current should consist of fundamental, steady, and second-harmonic components when the excitation is sinusoidal. This is in agreement with Fig. 3-21.

If the excitation voltage contains more than one frequency, the plate current contains not only the impressed frequencies and their harmonics, but also frequencies equal to the sums and differences of the impressed frequencies and their integral multiples, as may be shown by expanding

$$(E_1 \sin \omega_1 t + E_2 \sin \omega_2 t + E_3 \sin \omega_3 t + \cdots)^n.$$

These are called *intermodulation* frequencies. *Intermodulation* is defined as the production, in a nonlinear circuit element, of frequencies equal to the sums and differences of integral (1, 2, 3, etc.) multiples of two or more frequencies which are transmitted to that element. It should be noted that the harmonic and intermodulation frequencies contained in the output are not present in the impressed excitation but are generated by the nonlinear circuit element. It is also important to note that the fact that the intermodulation frequencies are equal to the sums and differences of multiples of the impressed frequencies does not imply that these sum and difference frequencies are the result of interaction of the generated harmonics. The sum and difference frequencies and the harmonics are generated simultaneously by the nonlinear element. It will be shown in later chapters that intermodulation in vacuum-tube circuits is sometimes desirable and sometimes objectionable.

The production of intermodulation frequencies can be demonstrated in a striking manner by a simple laboratory experiment. The voltages from two audio-frequency oscillators are filtered to remove harmonics and are applied in series to the grid circuit of a vacuum tube. The voltage developed across a plate load resistance (preferably considerably smaller than the plate resistance) is applied to the input of a low-pass filter (0 to 3000 cps, for instance), the output of which goes to headphones or, through an amplifier, to a loud-speaker. The oscillator frequencies are made high enough so that the fundamental components of the output voltage cannot pass through the filter. Various combinations of oscillator frequencies can be found at which one or more frequencies are heard in the phones, the pitch of which varies with the tuning of either oscillator. The frequencies are always found to be equal to the difference between one oscillator frequency or one of its multiples and the other oscillator frequency or one of its multiples. Since the application of oscillator harmonics to the grid is prevented by filtering the oscillator voltages, the frequencies that are heard in the output are generated by the tube. A similar experiment, performed with a high-pass filter in the output, demonstrates the production of intermodulation frequencies equal to the sums of the oscillator frequencies and their multiples.

3-24. Series Expansion. *Impedance Load and Variable Amplification Factor.*—The derivation of the series expansion for the more general case of impedance load, although similar in form to that for resistance load, is considerably more complicated. Since the coefficients of the series involve the load impedance, which depends upon the frequency, there must be a coefficient a_n for each frequency arising from the expansion of $e^n = (E_h \sin \omega_h t + E_k \sin \omega_k t + \cdots)^n$, instead of a single coefficient a_n, as with resistance load. This fact may be indicated by writing Eq. (3-40) in the form

$$i_p = \Sigma a_1 e + \Sigma a_2 e^2 + \Sigma a_3 e^3 + \cdots \tag{3-56}$$

An excellent development of the series expansion for the general case of impedance load and variable amplification factor has been given by Llewellyn, who has derived the first two coefficients.[1] These are

$$\left.\begin{aligned} (a_1)_h &= \frac{\mu}{r_p + (z_b)_h} \\ (a_2)_{h-k} &= \\ &\frac{\mu \dfrac{\partial \mu}{\partial e_b}[r_p^2 - (z_b)_h(z_b)_k] + \dfrac{\partial \mu}{\partial e_c}[r_p + (z_b)_h][r_p + (\bar{z}_b)_k] - \mu^2 r_p \dfrac{\partial r_p}{\partial e_b}}{2[r_p + (z_b)_h][r_p + (\bar{z}_b)_k][r_p + (z_b)_{h-k}]} \end{aligned}\right\} \tag{3-57}$$

[1] LLEWELLYN, *loc. cit.*

in which

$$(z_b)_h = r_b + j(x_b)_h \qquad (\bar{z}_b)_k = r_b - j(x_b)_k$$
$$(z_b)_{h-k} = r_b + j(x_b)_{h-k}$$

and r_p and μ and their derivatives are evaluated at the operating point. The subscript $_h$ in the symbol $(x_b)_h$ indicates that the reactance of the load is to be evaluated at the frequency $2\pi h$. The coefficient $(a_2)_{h+k}$ is similar to $(a_2)_{h-k}$ except that $(\bar{z}_b)_k$ and $(z_b)_{h-k}$ are replaced by $(z_b)_k$ and $(z_b)_{h+k}$, respectively. The coefficients for the second-harmonic and steady components may be written in the forms $(a_2)_{h+h}$, $(a_2)_{k+k}$, $(a_2)_{h-h}$, $(a_2)_{k-k}$, etc., and found from the formulas for $(a_2)_{h+k}$ and $(a_2)_{h-k}$ by replacing k by h wherever it appears in these formulas.

Because the plate current contains components having different frequencies, the series expansion cannot be stated in terms of effective values of currents and voltages. Since an imaginary component of instantaneous current has no apparent meaning, some question arises as to the significance of the use of complex coefficients in conjunction with instantaneous values of voltage. No difficulty arises, however, if the coefficients are considered as a form of shorthand notation. The magnitude of each coefficient can be used to determine the amplitude of the particular component of plate current, and the angle can be used to determine the phase angle of that component of current. Consider, for instance, the simple case in which e is of the form $E \sin \omega t$. The first coefficient a_1 is $\mu/[r_p + r_b + j(x_b)_\omega]$, and the first term in Eq. (3-56) is $\mu e/[r_p + r_b + j(x_b)_\omega]$, which should be interpreted as a convenient shorthand method of expressing the instantaneous value of the fundamental component of plate current,

$$\frac{\mu E}{\sqrt{(r_p + r_b)^2 + (x_b)_\omega{}^2}} \sin \left[\omega t - \tan^{-1} \frac{(x_b)_\omega}{r_p + r_b}\right].$$

The second term of the series consists of a steady component and a second-harmonic component. The latter is of the complex form

$$\frac{b}{c + jd} e^2,$$

which should be interpreted as $\dfrac{bE^2}{\sqrt{c^2 + d^2}} \sin \left(2\omega t - \tan^{-1} \dfrac{d}{c}\right)$. If the exciting voltage contains two frequencies f_h and f_k whose values are, by way of example, 60 and 100 cps, respectively, the exciting voltage is of the form $E_h \sin 120\pi t + E_k \sin (2000\pi t + \theta)$. The first term of Eq. (3-56) then becomes[1] $\mu e_h/[r_p + (z_b)_{120\pi}] + \mu e_k/[r_p + (z_b)_{2000\pi}]$, which

[1] The symbols $(z_b)_{120\pi}$ and $(x_b)_{120\pi}$ indicate that the impedance and the reactance of the load are to be evaluated at the frequency 120 cps.

should be interpreted as

$$\frac{\mu E_k}{\sqrt{(r_p + r_b)^2 + (x_b)_{120\pi}{}^2}} \sin\left[120\pi t - \tan^{-1}\frac{(x_b)_{120\pi}}{r_p + r_b}\right]$$
$$+ \frac{\mu E_k}{\sqrt{(r_p + r_b)^2 + (x_b)_{2000\pi}{}^2}} \sin\left[2000\pi t + \theta - \tan^{-1}\frac{(x_b)_{2000\pi}}{r_p + r_b}\right].$$

The second term of the series gives rise to a steady component, a 120-cycle component, a 2000-cycle component, a 1060-cycle component, and a 940-cycle component. The amplitude of the latter component can be determined, for example, by multiplying the product E_hE_k by the magnitude of the coefficient $(a_2)_{940}$, which is found by substituting $(z_b)_{2000\pi}$, $(\bar{z}_b)_{120\pi}$, and $(z_b)_{1880\pi}$ in Eq. (3-59).[1] The amplitudes of the other components can be found in a similar manner.

When only the first term of the series expansion is taken into consideration, *i.e.*, when the tube is assumed to be a linear circuit element, the plate current contains only the impressed frequencies and Eq. (3-56) reduces to

$$i_p = \frac{\mu e}{r_p + z_b} \tag{3-58}$$

for each frequency contained in the excitation voltage e. Effective values of current and voltage may then be used and Eq. (3-58) written in the form

$$I_p = \frac{\mu E}{r_p + z_b} = \frac{\mu E_g + V_p}{r_p + z_b} \tag{3-59}$$

No step in the derivation of Eqs. (3-40) and (3-41) restricts these equations to alternating exciting voltage. They apply equally well when e is a small change of applied grid or plate voltage. Equations (3-56) and (3-57), on the other hand, involve impedances that are evaluated at specific frequencies. They cannot, therefore, be used when e is not a periodic function of time. The sudden application of grid or plate excitation or of direct voltages will in general result in the production of transient components of plate current when the grid or plate circuits contain reactance. The theory of the transient behavior of tube circuits is beyond the scope of this book.[2]

3-25. Relation of Series Coefficients to Dynamic Tube Characteristics.—A study of Eqs. (3-41) and (3-57) shows that the introduction of load impedance into the plate circuit reduces the ratios of the coefficients

[1] The amplitude of a plate-current component whose frequency is $nf_h \pm mf_k$ is equal to $E_h{}^nE_k{}^m$ multiplied by the magnitude of the coefficient $(a_{n-m})_{h\pm k}$.

[2] JACKSON, W., *Phil. Mag.*, **13**, 143, 735 (1932); SCHLESINGER, K., *E. N. T.*, **38**, 144 (1931).

of the higher-order terms of the series to the first-order coefficient. This can be shown most readily for the special case of resistance load. For resistance load, Eqs. (3-57) give the following expression for the ratio of the coefficient of the second term to the coefficient of the first term of the series:

$$\frac{a_2}{a_1} = \frac{\dfrac{\partial \mu}{\partial e_b}(r_p^2 - r_b^2) + \dfrac{\partial \mu}{\partial e_c}\dfrac{(r_p + r_b)^2}{\mu} - \mu r_p \dfrac{\partial r_p}{\partial e_b}}{2(r_p + r_b)^2} \tag{3-60}$$

As r_b is increased, the ratio is reduced and approaches the limiting value, $\frac{1}{2}\left(\frac{1}{\mu}\frac{\partial \mu}{\partial e_c} - \frac{\partial \mu}{\partial e_b}\right)$. If μ is practically constant, as is true over certain ranges in many tubes, the ratio of a_2 to a_1 decreases rapidly and becomes negligibly small as r_p becomes large. If μ is constant, the introduction into the plate circuit of a load resistance equal to twice the plate resistance reduces a_1 to $\frac{1}{3}$ its value for zero load resistance and a_2 to $\frac{1}{27}$ its value for zero load resistance. The ratios of a_3 and higher-order coefficients to a_1 decrease even more rapidly with increase of load resistance than that of a_2 to a_1. A similar analysis for the more general case of impedance load, although not quite so simple, also shows that in general the amplitudes of the second- and higher-order terms of the series are decreased relative to the amplitude of the first term by an increase of load impedance and, if μ were constant, could be reduced to any desired degree by making z_b/r_p sufficiently high.

Equations (3-40) and (3-41) show that for resistance load the dynamic transfer characteristic would be a straight line if the series contained only the first term and that the presence of the higher-order terms is associated with curvature of the dynamic transfer characteristic. If only the first term were present, a change of grid voltage Δe_c would result in a proportional change of plate current Δi_b, indicating a linear relation between i_b and e_c. The higher-order terms of the series destroy this linearity, since contributions to Δi_b from the higher-order terms are not proportional to Δe_c. Reduction of the higher-order terms of the series by increase of load resistance is, therefore, accompanied by reduction of the curvature of the dynamic transfer characteristic.

That the transfer characteristic of a triode is straightened in the negative range of grid voltage by the introduction of plate load resistance and, hence, that the coefficients of the second and higher terms of the series are reduced with respect to the first term can also be shown by constructing dynamic transfer characteristics for different values of load resistance. Although the dynamic transfer characteristics can be most readily obtained from the plate family of characteristics in a manner that will be explained in Chap. 4, it is instructive at this point to derive one from the static transfer characteristics. O represents the static operating point in Fig. 3-22. With no external resistance in the plate circuit a change of grid voltage from E_c to $E_c + \Delta e_c$ would cause the current to change to i_b', corresponding to a new point a on the same static characteristic. With resistance in the plate circuit, however, the increase of plate current accompanying the change in grid voltage increases the IR drop in the plate circuit and thus reduces the voltage of the plate to a new value $E_{bo} - R_b\Delta i_b$, corresponding to point a'. Similarly, if the grid voltage is changed to $E_c - \Delta e_c$, the point shifts to b'. By plotting corresponding values of i_b and e_c it is possible to obtain the complete dynamic transfer characteristic, shown by the dashed line. Figure 3-23 shows the dynamic transfer characteristics of a type 56 triode for various values of resistance load, the plate supply voltage being increased with R_b in such a manner as to maintain the same static operating point. Figure 3-24 shows similar curves for constant plate supply voltage.

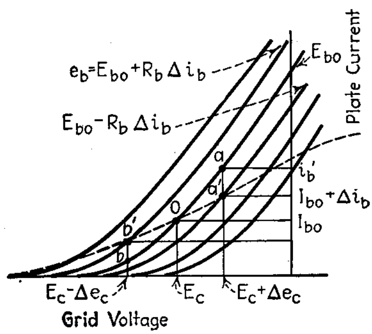


FIG. 3-22.—Construction of a dynamic transfer characteristic from the family of static transfer characteristics.

In the positive range of grid voltage, the introduction of load resistance into the plate circuit may increase the curvature of the transfer

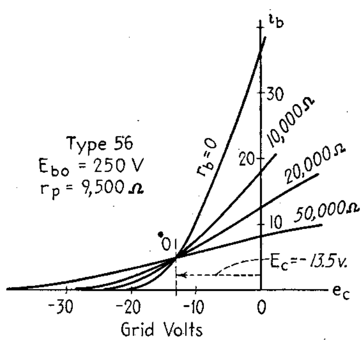


FIG. 3-23.—Dynamic transfer characteristics of the type 56 triode for four values of load resistance at fixed operating voltages. $r_b = R_b$.

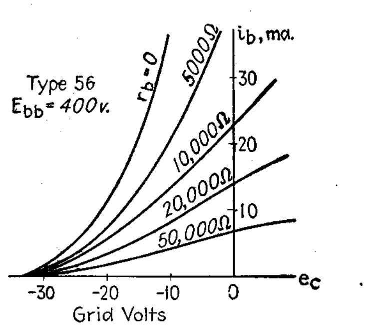


FIG. 3-24.—Dynamic transfer characteristics of the type 56 triode for five values of load resistance at fixed plate supply voltage.

characteristic. The reason for this is made apparent by reference to Fig. 3-6. Since the amplification factor is equal to the slope of the e_b-e_c characteristics, it is evident from Fig. 3-6 that the amplification factor varies greatly in the positive region of grid voltage. Hence the ratio

a_2/a_1 given by Eq. (3-60) does not approach zero as r_b is increased, but increases with r_b at large values of r_b. The increase of curvature with increase of load resistance can also be explained readily physically. As pointed out on page 49, the bending over of the transfer characteristics at positive grid voltages results in part from diversion of electrons to the grid. At a given positive grid voltage, the number of electrons diverted to the grid increases as the plate voltage is decreased. The increase of *IR* drop resulting from an increase of load resistance decreases the plate voltage. More electrons are diverted to the grid, and so the plate current is decreased. Increase of load resistance therefore increases the bending over of the transfer characteristics at positive grid voltages and may even cause them to have maxima.

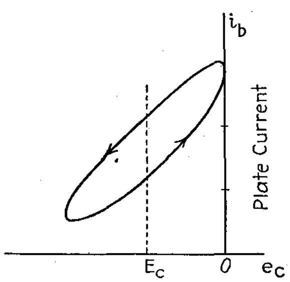


FIG. 3-25.—Typical dynamic transfer characteristic for inductive load.

At large values of grid excitation voltage, the amplification factor of pentodes is not constant throughout the negative-grid range of operation and, although the curvature of the transfer characteristic first decreases with increase of load resistance, a value is reached beyond which curvature again increases (see Sec. 6-25).

When the load contains reactance, as well as resistance, the behavior is complicated by the phase difference between the plate voltage and plate current. If the impedance is sufficiently high, the dynamic transfer characteristic obtained with sinusoidal excitation is nearly elliptical. For lower values of impedance the dynamic transfer characteristic resembles an ellipse but has a curved axis, as shown for inductive reactance in Fig. 3-25. As the reactance is reduced, the path of operation gradually changes into the curve obtained for pure resistance (see Sec. 4-6).

3-26. Dynamic Plate Resistance.—The *dynamic plate resistance* is the quotient of the alternating plate voltage by the inphase component of the alternating plate current, all other electrode voltages being maintained constant. The quadrature component of current, which results from electrode capacitances and, at ultrahigh frequencies, from electron transit time, is negligible at low audio frequency. The inphase component of plate current corresponding to a given alternating plate voltage when other electrode voltages are constant may be found from Eqs. (3-40) to (3-42) by making e_g and r_b zero. When r_b is zero $v_p = e_p$, and Eq. (3-40) becomes

$$i_p = \frac{e_p}{r_p} - \frac{e_p^2}{2r_p^2}\frac{\partial r_p}{\partial e_b} + \frac{e_p^3}{6r_p^3}\left(2\frac{\partial r_p}{\partial e_b} - r_p\frac{\partial^2 r_p}{\partial e_b^2}\right) + \cdots \qquad (3\text{-}61)$$

Because of curvature of the static plate characteristics, the derivatives of the plate resistance are not zero. When the excitation voltage is high, therefore, the second- and higher-order terms of Eq. (3-61) are not negligible. Contributions of the third- and other odd-order terms to the fundamental component of plate current cause the ratio of the alternating plate voltage to the fundamental component of alternating plate current to differ from r_p, the static plate resistance. As e_p is reduced, however, the ratio of the higher-order terms to the first term decreases, and e_p/i_p approaches r_p. If the amplitude of the plate excitation voltage is small enough, the dynamic plate resistance approximates the value determined from the slope of the static characteristic at the operating point (at frequencies low enough so that the electron transit time is negligible). The static plate resistance may, therefore, be found dynamically by using a sufficiently small plate excitation voltage. In a similar manner it may be shown that the dynamic measurement of transconductance and of variable amplification factor requires the use of small excitation voltages.

Problems

3-1. From the static plate characteristics of the type 6J5 tube find the values of μ, r_p, and g_m at the point $E_b = 200$ volts, $E_c = -8$ volts.

3-2. From the static plate characteristics of the type 6SJ7 tube, find the values of μ, r_p, and g_m at the point $E_b = 200$ volts, $E_c = -2$ volts.

3-3. *a.* From the following data find approximate values of μ, r_p, and g_m at the point $E_b = 180$ volts, $E_c = -12.5$ volts. (Note that only two of these factors can be found directly from the data given.)

E_b, volts	E_c, volts	I_b, ma
180	−12.5	7.5
160	−10.0	7.5
180	−12.3	7.84

b. From the following data find approximate values of μ, r_p, and g_m at the point $E_b = 250$ volts, $E_c = -16$ volts.

E_b, volts	E_c, volts	I_b, ma
250	−16	2.0
220	−14	2.0
260	−16	3.0

3-4. Determine the frequencies of all components of alternating plate current associated with the first three terms of the plate-current series when the frequencies 60, 100, and 900 cps are simultaneously impressed upon the grid of a vacuum tube.

3-5. Equation (3-2) suggests that it should be possible to express the alternating plate current by the following series, in which μ is assumed to be constant:

$$i_p = k_1(e_p + \mu e_g) + k_2(e_p + \mu e_g)^2 + k_3(e_p + \mu e_g)^3 + \cdots \tag{3-62}$$

By differentiating both sides of this equation successively and evaluating each derivative at the operating point, show that $k_n = \left.\frac{\partial^n i_b}{\partial e_b{}^n}\right]_0 \times \frac{1}{n!}$, and thus derive Eq. (3-45).

3-6. *a.* Show that, if the amplification factor is constant and the load impedance zero, the coefficients a_n of the series expansion for i_p reduce to $\frac{\mu^n}{n!}\frac{\partial^n i_b}{\partial e_b{}^n}$ [see Eq. (3-45)].

b. Show that, if the static characteristics are assumed to be parabolic, all coefficients of terms of the series expansion for i_p of higher order than the second are zero.

c. By means of the series expansion show that for parabolic characteristics, constant amplification factor, and zero load impedance the amplitudes of the steady and second-harmonic components of plate current are one-fourth the amplitude of the fundamental component when the grid bias E_c is one-half the cutoff value and $e = e_g = E_c \sin \omega t$.

3-7. Show that Eqs. (3-57) reduce to Eqs. (3-41) if μ is constant and the load is nonreactive.

3-8. Evaluate a_1 and a_2 graphically for a type 6J5 triode with resistance load at rated operating voltages when (*a*) $r_b = 0$; (*b*) $r_b = r_p$; (*c*) $r_b = 2r_p$.

The functions of the various portions of the circuit are indicated by the wave forms of the voltages at various points.

A much longer time base can be obtained by the use of a circular or spiral sweep. A circular or eliptical sweep is obtained by applying to the two sets of deflection plates voltages that are 90 degrees out of phase, obtained from a series combination of resistance and capacitance or inductance, as shown in Fig. 15-53. The voltage under observation

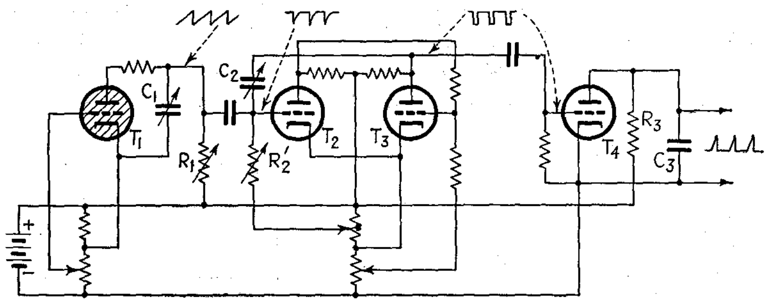


FIG. 15-52.—Expanded-sweep circuit for the observation of a small portion of the cycle of voltage impressed upon the oscilloscope.

is made to produce a radial displacement of the spot proportional to the instantaneous voltage by superimposing it upon the direct voltage of the second anode,[1] or by modulating the input voltage of the circuit of Fig. 15-53 in accordance with the voltage under observation, by means of a linear modulator. A spiral sweep is obtained by modulating the input voltage of the circuit of Fig. 15-53 in accordance with a saw-tooth voltage.

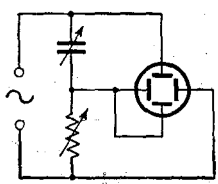


FIG. 15-53.—Circuit for producing a circular sweep.

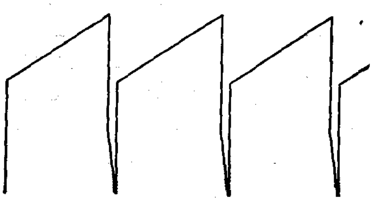


FIG. 15-54.—Form of exciting voltage required to produce linear electromagnetic sweep.

The voltage under observation is superimposed upon the saw-tooth voltage in modulating the input to the phase-splitting circuit.

In order to obtain a linear sweep by electromagnetic means, the current through the deflecting coils must be of saw-tooth form. The linear rise in current through the coils requires a linear rise in voltage across the coil resistance and a constant voltage across the coil inductance.

[1] Variation of second anode voltage varies the deflection sensitivity, and hence the radius of the circular sweep. See N. V. KIPPING, *Wireless World*, **13**, 705 (1924).

The periodic wave of voltage across the coil must therefore be the sum of a saw-tooth wave and a rectangular wave. The desired result may be obtained by impressing a wave of the form of Fig. 15-54 upon the input of the circuit of Fig. 15-55. The function of the diode, condenser, and resistance in the circuit of Fig. 15-55 is to damp out transient oscillations set up in the transformer and deflection coils by the discontinuities of current. Voltage of the form of Fig. 15-54 may be obtained from a saw-tooth-wave oscillator by the addition of resistance in series with the condenser, as shown in Fig. 15-56. The constant condenser charging current produces a constant voltage across R' and a linearly rising voltage across the condenser. R' is varied until the current wave has the correct wave form.

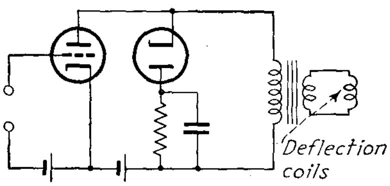


FIG. 15-55.—Amplifier for producing linear electromagnetic sweep.

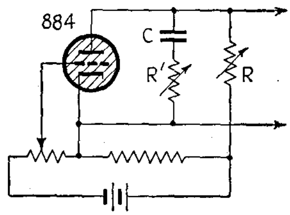


FIG. 15-56.—Oscillator for the generation of voltages of the form of Fig. 15-54.

15-21. Oscilloscope Amplifiers.—Oscilloscope amplifiers serve the dual purpose of providing sufficient voltage to give the desired deflection, and of making possible the variation of deflection voltages without drawing appreciable current from the source under observation or from the sweep oscillator. Sweep amplifiers must be capable of amplifying frequencies ranging from the fundamental frequency up to approximately the tenth harmonic. In order to cover the sweep frequency range of 50 to 50,000 cps, therefore, the sweep amplifier must have negligible frequency and phase distortion in the range from 50 to 500,000 cps. A similar frequency range is desirable in the Y-deflection amplifier in the study of wave form, particularly when the wave contains near-discontinuities. Compensated amplifiers embodying cathode-follower stages, similar to the amplifier of Fig. 6-24, are used in the best oscilloscopes.

Figure 15-57 shows the usual manner of coupling the deflection plates to the output of a single-sided amplifier. The voltage divider P changes the direct voltage between the plates and thus displaces the image on the screen. Since the second anode and one deflection plate of each pair may be connected within the tube, the use of this circuit makes possible a simplified tube structure. It has the disadvantage, however, that the variation of the potential of only one plate relative to the second anode and the cathode produces a corresponding variation of the effective accelerating voltage and thus destroys the linearity between deflection voltage and deflection.[1] This difficulty may be avoided by using a push-pull amplifier to excite the deflection plates. The potential of one plate relative to the second anode then falls by the same amount that the other rises and the average potential of the deflection plates relative to the cathode remains constant. The cathode-phase-inverter circuit of Fig. 5-16 may be conveniently used for this purpose. Figure 15-58 shows a form of this circuit that makes it possible to displace the image on the screen. Increasing the magnitude of the bias of T_1 above that of T_2 by means of the potentiometer P decreases the plate current of T_1 and makes the voltage drop across R_{b1} less than

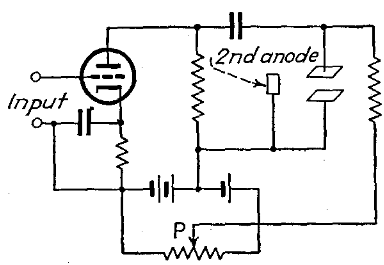


FIG. 15-57.—Single-sided oscilloscope amplifier with a control for displacing the pattern.

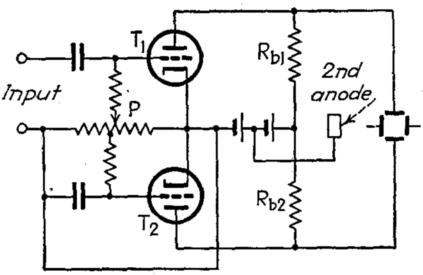


FIG. 15-58.—Push-pull oscilloscope amplifier.

that across R_{b2}. A direct difference of potential is thus established between the deflecting plates, causing a steady deflection of the image.

The complete circuit diagram of a typical oscilloscope is shown in Fig. 15-59 (see Prob. 15-1).

15-22. Electronic Switches for the Observation of Two Waves.—Most cathode-ray tubes that are now on the market contain only one set of elements and can therefore be used directly for the observation of only one voltage as a function of time. Two or more waves can be observed simultaneously by the use of a commutator,[2] but this method is not very convenient. A number of electronic circuits have been developed that make possible the observation of two waves.[3] In these circuits the two voltages to be observed are applied to two amplifiers, the outputs of which are impressed upon the Y deflecting plates. Some form

[1] DUMONT, A. B., *Electronics*, January, 1935, p. 16.

[2] BEDELL, and REICH, *loc. cit.*

[3] SEWIG, R., *Z. f. tech. Physik.*, **14**, 152 (1933); BROWN, C. B., *Electronics*, **6**, 170 (June, 1933); DAVIDSON, I. B., *J. Sci. Instruments*, **11**, 359 (1934); GARCEAU, L., *Rev. Sci. Instruments*, **6**, 171 (1935); DUMONT, A., *Electronics*, March, 1935, p. 101; WOODRUFF, L. F., *Elec. Eng.*, **54**, 1045 (1935); GEORGE, R. H., HEIM, H. J., and ROYS, C. S., *Elec. Eng.*, **54**, 1095 (1935); HUGHES, H. K., *Rev. Sci. Instruments*, **7**, 89 (1936); SHUMARD, C. C., *Elec. Eng.*, **57**, 209 (1938); REICH, H. J., *Rev. Sci. Instruments*, **12**, 191 (1941).

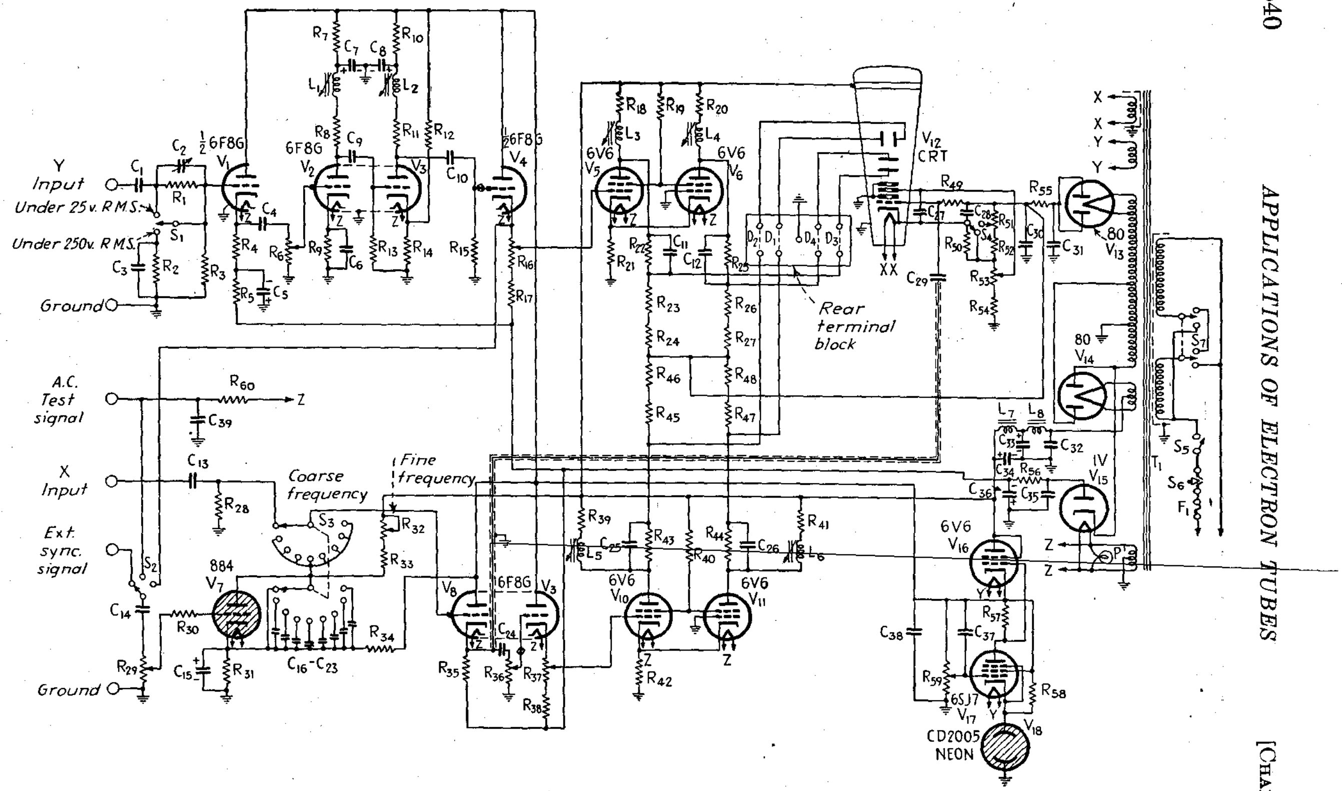


FIG. 15-59.—Complete circuit of a typical oscilloscope. (*Courtesy of Allen B. DuMont Laboratories, Inc.*)

of trigger switching circuit, such as those of Fig. 10-8, 10-9, or 12-34, synchronized to the sweep voltage, is used to control the control-grid, screen, or suppressor voltages of the amplifiers in such a manner that the amplifiers amplify during alternate sweeps. The images of the two waves are formed on the fluorescent screen during alternate sweeps. Because of phosphorescence and persistence of vision, they appear to be seen simultaneously.

A typical switching circuit is shown in Fig. 15-60.[1] This circuit uses pentode amplifier tubes controlled by a parallel thyratron switching circuit of the form of Fig. 12-34*a*. The screens of the amplifier tubes T_3 and T_4 are connected to the anodes of the thyratrons T_1 and T_2,

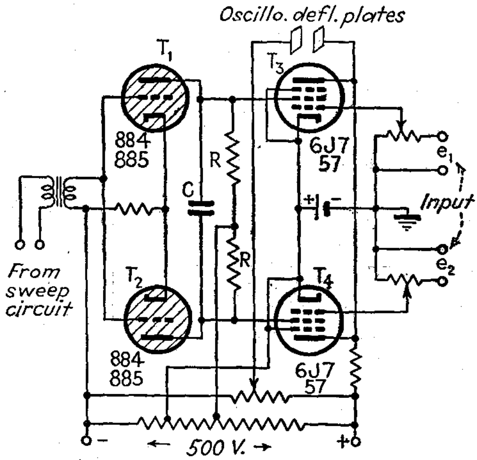


FIG. 15-60.—Electronic switch for the simultaneous observation of two waves with a single-element cathode-ray tube.

respectively. The amplifier cathodes are positive relative to the thyratron cathodes by a voltage that exceeds the thyratron tube drop. Hence, when either thyratron fires and the voltage of its anode relative to its cathode falls to a value equal to the tube drop, the screen voltage of the corresponding amplifier tube is made slightly negative relative to its cathode, so that its plate current is reduced to zero, and the tube ceases to amplify.

15-23. Electronic Transient Visualizers.—The periodic contactor circuits mentioned on pages 457 and 491 are of value in the study of transient voltages and currents by means of the cathode-ray oscillograph.[2] The sweep oscillator voltage, in addition to providing time deflection of the fluorescent spot, varies the grid voltage of an amplifier tube, the plate circuit of which contains a relay. The relay is used to open and

[1] GEORGE, HEIM, and ROYS, *ibid.*

[2] REICH, H. J., *Rev. Sci. Instruments*, **5**, 7 (1934).

close the circuit in which the transient is produced, the adjustment being such that the transient is initiated at the beginning of the timing sweep. Since the relay operates at the same instant in each sweep, the transient is observed on the screen as a stationary figure. Limitations imposed by relay inertia and chattering are overcome by the use of an 884 thyratron tube in place of the mechanical relay, as shown in Fig. 15-61.[1] Extinction of the 884 switching tube T_4 is accomplished by virtue of the ability of the 884 grid to interrupt anode current of 50 ma or less when the grid is made sufficiently negative. The saw-tooth voltage of the sweep oscillator is applied to the grid of T_4 with proper polarity so that the sudden change in voltage when the sweep oscillator tube T_2 fires causes the negative bias of T_4 to increase enough to cut

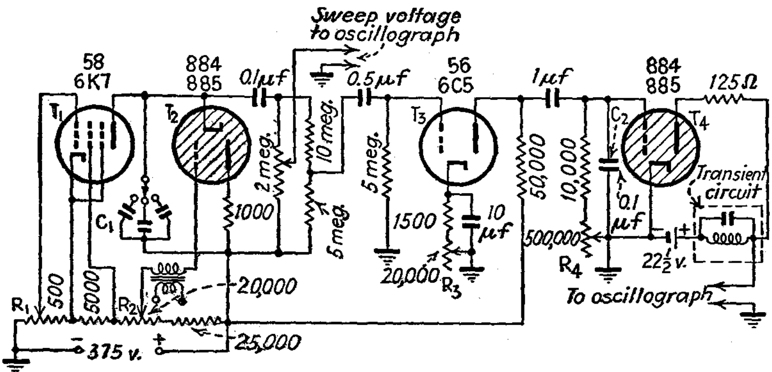


FIG. 15-61.—Electronic transient visualizer.

off the anode current. Since this change in voltage is very rapid, the extinction time of T_4 may be made as small as 10 μsec. As the condenser C_1 charges, the negative grid voltage of T_4 decreases and at some time in the cycle becomes so small that T_4 again fires. If the plate circuit of T_4 contains only resistance, the wave of anode current, and the voltage across this resistance, are observed to be square topped, as they should be if firing and extinction of T_4 are rapid. The condenser C_2 of Fig. 15-61 serves to delay the extinction of T_4, so that a transient initiated by the extinction of T_4 will not start during the return sweep of the fluorescent spot. The frequency of repetition is controlled by C_1, R_1, and R_2. R_3 and R_4 control the portion of the cycle during which T_4 conducts. Figure 15-62 shows how this device may be used in the oscillographic study of transient oscillations in coupled oscillatory circuits and in lines.

Another type of visualizer, in which the transient is initiated by a voltage impulse, is based upon the relaxation circuit of Fig. 12-43.[2] The

[1] BENNETT, J. A., master's thesis, Cornell University, June, 1935; REICH, H. J., *Elec. Eng.*, **55**, 1314 (1936), *Trans. Am. Inst. Elec. Eng.*, **56**, 873 (1937).

[2] REICH, *ibid.*

voltage that causes the transient is preferably taken from a resistance in series with C_1, either directly or through an amplifier, and the circuit is tripped by means of voltage from the sweep oscillator. A similar circuit of somewhat reduced flexibility is obtained by taking the transient exciting voltage pulse from a resistance in series with the sweep oscillator condenser, through a suitable amplifier.[1]

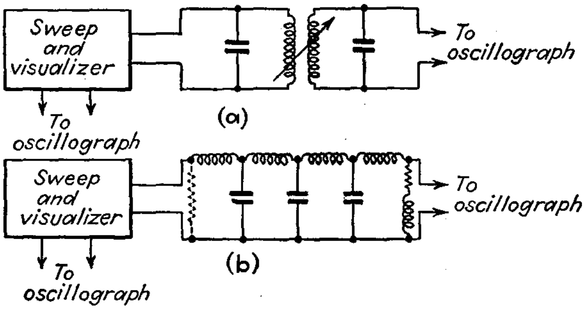


FIG. 15-62.—Circuits for using transient visualizer in the study of transient oscillations in (a) coupled circuits and (b) lines.

A third type of transient visualizer is illustrated in Fig. 15-63.[2] The action of this circuit is as follows: During one-half of the cycle of alternating supply voltage, the condenser C_1 is charged from the secondary of the transformer through the rectifier T_2. At the positive crest of the following half cycle the thyratron T_1 fires, and C_4 begins to charge,

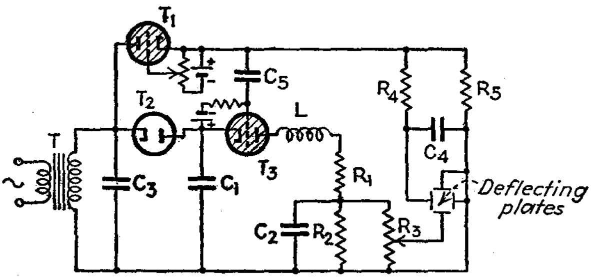


FIG. 15-63.—Transient visualizer that initiates the transient by the discharge of a condenser through a thyratron.

causing a horizontal deflection of the fluorescent spot. Firing of T_1 also applies a positive impulse to the grid of T_3, causing it to fire. C_1 discharges through T_3, producing a voltage pulse across R_3, the shape of which depends upon the parameters C_1, C_2, L, R_1, and R_2. This voltage, or a portion of it, may be applied to the circuit in which it is

[1] Unpublished paper by P. W. Ryburn, Univ. of Illinois, 1938.

[2] ROHATS, N., *Gen. Elec. Rev.*, **39**, 146 (1936); *Trans. Am. Inst. Elec. Eng.*, **56**, 873 (1937).

desired to initiate a transient. The transient is thus repeated in synchronism with the sweep at the frequency of the alternating supply voltage.

15-24. Oscillographic Comparison of Frequencies.—The cathode-ray oscillograph can be used to advantage in the calibration of oscillators by comparison with known frequencies. The simplest method is by the use of Lissajous figures. The output of the oscillator that is to be calibrated is applied to one pair of deflecting plates and the standard frequency to the other pair. Either frequency is adjusted to give a stationary pattern. The frequency ratio is equal to the ratio of the number of horizontal and vertical points of tangency of a rectangle that encloses the pattern.

A second method of frequency comparison was developed by N. V. Kipping.[1] In Kipping's circuit, shown in Fig. 15-64, the lower frequency is applied to a series combination of resistance and capacitance. The voltage across the resistance is applied to one set of deflecting plates, and the voltage across the condenser to the other. Because these voltages are 90 degrees out of phase, a circular pattern is obtained when the resistance is adjusted so as to make their amplitudes equal. The higher frequency is introduced into the anode circuit. Variation of anode voltage changes the electron velocity and, therefore, the deflection sensitivity. This causes the diameter of the circular pattern to change, the effect being equivalent to a radial displacement of the spot relative to the circle. If the ratio of the two frequencies is rational, a stationary pattern is obtained. The frequency ratio is equal to the number of whole waves superimposed on the circle, divided by the number of revolutions that the spot makes in completing the pattern. With modern tubes, which incorporate a control grid, the higher frequency may also be introduced into the grid circuit to produce a variation of intensity along the circle. The latter method is not satisfactory for other than integral frequency ratios, however, because of the difficulty of determining how many times the spot moves about the circle in completing the pattern.

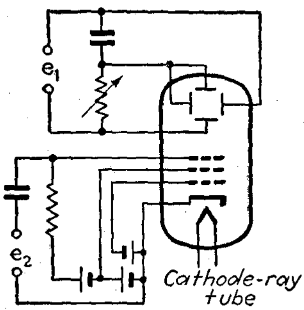


FIG. 15-64.—Circuit for the oscillographic comparison of frequency.

A third method[2] makes use of the circuit of Fig. 15-65. The two voltages e_1 and e_2, of frequency f_1 and f_2, are first applied individually, and the resistances and capacitances are adjusted so that each voltage

[1] KIPPING, N. V., *Wireless World*, **13**, 705 (1924).

[2] RANGACHARI, T. S., *Wireless Eng.*, **5**, 264 (1928), **6**, 184 (1929); KURRELMEYER, BERNHARD, *Rev. Sci. Instruments*, **7**, 200 (1936), **8**, 348 (1937).

produces a circular pattern. When the two voltages are applied simultaneously, the pattern is some form of roulette, of which the cycloids are examples.[1] If the amplitude of the higher frequency is somewhat smaller than that of the lower frequency, the pattern will in general have one or more cusps or loops. A stationary pattern is obtained if the ratio of the higher to the lower frequency f_2/f_1, reduced to its simplest form, is N_2/N_1, in which N_2 and N_1 are integers. For the circuit of Fig. 15-65, there are $N_2 + N_1$ loops or cusps, pointed outward. In generating the pattern, the fluorescent spot moves from one cusp or loop to the N_1th next one. Thus N_1 may be determined by adding one to the number of cusps skipped by the fluorescent spot. If N is the total number of cusps, $N = N_2 + N_1$. If the ratio of the frequencies cannot be expressed as the ratio of two integers, the pattern will have $N_2 + N_1$ cusps, where N_2/N_1 is the rational number that most nearly approximates the frequency ratio f_2/f_1, but will rotate. If f_2/f_1 is less than N_2/N_1, the pattern will rotate in the same direction as the spot moves when the lower frequency f_1 is applied alone, which may be termed the positive direction. The frequency ratio is

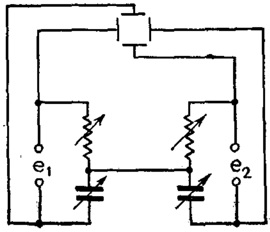


FIG. 15-65.—Circuit for the oscillographic comparison of frequency.

$$\frac{f_2}{f_1} = \frac{(1 - f_3/f_1)N_2}{N_1} \tag{15-50}$$

in which f_3 is the number of revolutions of the pattern per second. If f_2/f_1 is greater than N_2/N_1, on the other hand, the direction of rotation is negative, and the numerical value of f_3 should be taken as negative in Eq. (15-50). The positive direction of rotation can be readily determined by increasing f_1 or decreasing f_2 slightly. The rotation is positive if the speed of rotation increases.

If the position of one condenser and its associated resistor is interchanged in the circuit of Fig. 15-65, there are $N_2 - N_1$ cusps or loops, pointed inward. For frequency ratios for which N_1 exceeds $N_2 - N_1$, interpretation of the patterns is more difficult with this modified circuit since the fluorescent spot makes more than one complete trip around the center between two successively generated cusps, and the patterns are intricate. Figure 15-66 shows typical patterns obtained for several frequency ratios. All patterns are for the circuit of Fig. 15-65 with the exception of the lower right pattern, which was obtained with the transposed circuit.

[1] REYNOLDS, J. B., and WEIDA, F. M., "Analytic Geometry and Elements of Calculus," p. 246, Prentice-Hall, Inc., New York, 1930.

The circuit of Fig. 15-65 cannot be used with an oscillograph in which there is a common connection between one horizontal and one vertical deflecting plate. With this type of oscillograph the circuit of

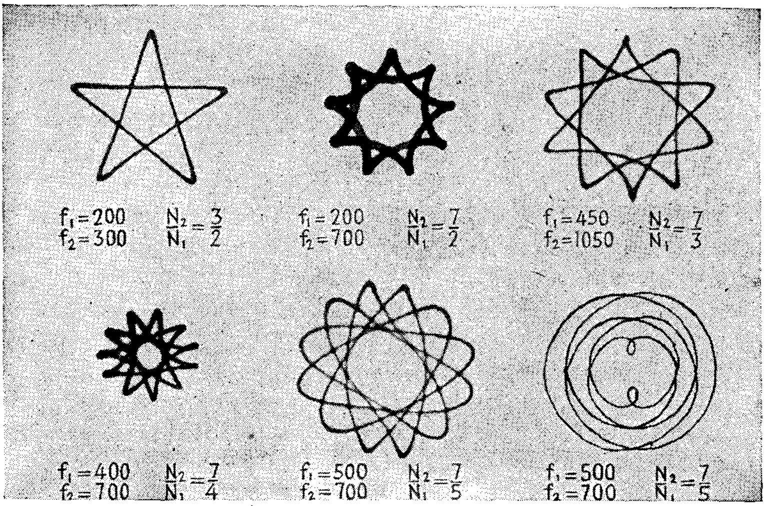


FIG. 15-66.—Oscillograms obtained with the circuit of Fig. 15-65 for various frequency ratios.

Fig. 15-67 may be used.[1] The impedance of the isolating transformer T should be high in comparison with the resistance R_1. For frequencies in the audible range an ordinary interstage coupling transformer and a

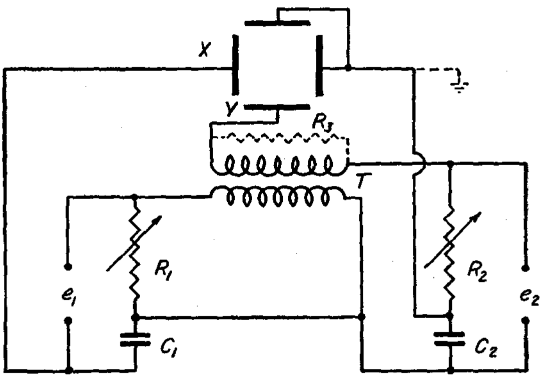


FIG. 15-67.—Modified form of the circuit of Fig. 15-65 that makes possible the use of a tube with a common connection between two deflecting plates.

5000-ohm variable resistance may be used. At frequencies above 5000 cps, the distributed capacitance of the transformer tends to prevent the formation of a circular pattern for the voltage e_1. For this reason e_1

[1] RANGACHARI, *loc. cit.;* REICH, H. J., *Rev. Sci. Instruments,* **8**, 348 (1937).

should be the lower frequency. Resistance or capacitance between the Y plates or leads produces a phase shift which makes it difficult to obtain a circular pattern for the voltage e_2 at very high frequencies. No difficulty is encountered, however, even above 10,000 cps, if the Y plates are not shunted by a leak and if the connection from the ungrounded Y plate to the transformer is short. A high resistance R_3 across the secondary is sometimes of help at high frequencies. If the primary and secondary of Fig. 15-67 are wound in the same direction, the cusps or loops of the pattern will point inward. The pattern may be transformed into one in which the cusps point outward by transposing the primary or secondary connections or by interchanging the position of either resistor and its associated condenser. Provision should be made for the variation of the amplitudes of e_1 and e_2 both in the circuit of Fig. 15-65 and in that of Fig. 15-67.

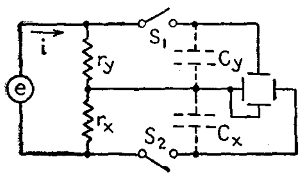


FIG. 15-68.—Circuit in which the capacitance of the deflecting plates may affect the form of the oscillogram.

15-25. Effect of Capacitance of Deflecting Plates.—Special precautions must be taken in the use of the cathode-ray oscillograph in the study of small voltages and currents. The capacitances of deflecting plates and their leads, although small, may under certain circumstances produce surprising results. In Fig. 15-68, r_x and r_y are two resistances through which a common alternating current flows. If r_x and r_y are high, it is found that when the voltages across them are applied to the cathode-ray oscillograph by closing the switches S_1 and S_2, the image may be an ellipse, instead of the straight line that would be expected with voltages that are in phase. If only one resistor were known to be

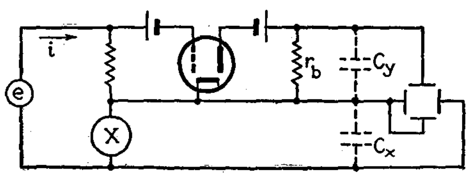


FIG. 15-69.—Circuit for the oscillographic determination of current-voltage characteristics of the element X. Errors may result from deflecting-plate capacitances C_x and C_y.

nonreactive, the incautious observer might conclude that the phase difference of the voltages results from reactance in the other resistor. Actually, the difference in phase results from inequality of the electrode capacitances C_x and C_y, or of the resistances r_x and r_y.

Although the simple circuit of Fig. 15-68 is not likely to be used, modified forms of it may be. Thus, the circuit of Fig. 15-69 may, for example, be used to determine the current-voltage characteristic of the element X. The load resistor r_b may be of the order of $\frac{1}{2}$ megohm, so that the capacitance of the deflecting plates and amplifier tube may produce appreciable shift in the phase of the voltage across r_b. Similarly, if the resistance of X is high, C_x may appreciably affect the phase of the voltage across it.

15-26. Electron-ray Tube.—Another type of cathode-ray tube is the *electron-ray tube* or *magic eye* which is used as a tuning indicator in radio

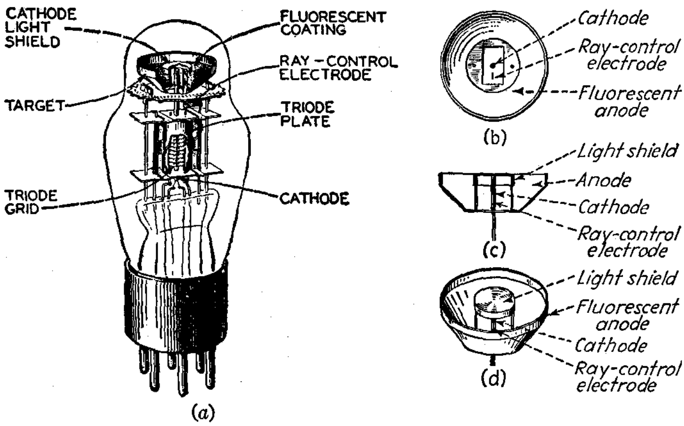


FIG. 15-70.—Structure of the type 6AB5 electron-ray tube.

receivers.[1] This tube, the construction of which is shown in Fig. 15-70, contains two sets of elements, one of which is a triode amplifier and the other a cathode-ray indicator. The latter consists of a cathode, a fluorescent anode (target), and a control electrode which controls the portion of the fluorescent anode upon which the electrons strike. The ray-control electrode is connected to the plate of the amplifier section, which is connected to the supply voltage through a high resistance, as shown in Fig. 15-71. Variation of the amplifier grid voltage changes the voltage of the ray-control electrode, and thus the portion of the target that fluoresces. The fluorescence is observed as an annular sector of varying angular width. The tube may be calibrated for use as a voltmeter where rough measurements suffice. When the electron-ray tube is used as a tuning indicator, the a-v-c biasing voltage, which increases with amplifier output, is used as the control voltage.

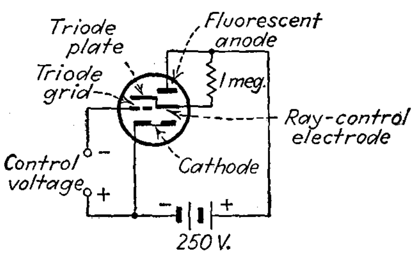


FIG. 15-71.—Electron-ray-tube circuit.

[1] WALLER, L. C., and RICHARDS, P. A., *Radio Retailing*, December, 1935, p. 47; THOMPSON, H. C., *Proc. I.R.E.*, **24**, 1276 (1936); WALLER, L. C., *QST*, October, 1936, p. 35, November, 1936, p. 23; WALLER, L. C., *RCA Rev.*, **1**, 111 (1937).

15-27. Determination of Static Tube Characteristics.—It would seem at first thought that the determination of static tube characteristics by means of direct voltages and d-c meters is such a simple procedure that it requires no discussion. Unless certain precautions are observed, however, erratic readings of plate current may be observed. These can usually be traced to high-frequency oscillations in the battery leads, the necessary inductance and capacitance being furnished by the meters, leads, and electrodes. A type 56 triode, for instance, will oscillate readily with a plate voltage of only 45 volts if the grid and plate leads happen to be of correct lengths to bring the grid and plate circuits into resonance. The difficulty may be prevented by the use of by-pass condensers from grid and plate to cathode or a resistance in series with the grid, if grid current does not flow.

15-28. Oscillographic Determination of Tube Characteristics.—The ordinary static method of obtaining static triode characteristics cannot be satisfactorily applied in certain ranges of current and voltage because of excessive plate and grid dissipation. The flow of high grid and plate currents for a sufficient time to take meter readings may result in temporary changes in characteristics as the result of change of cathode temperature or in permanent damage to the tube as the result of gas emission from the plate. A method for overcoming this difficulty by the use of oscillographic recording was devised by Kalin.[1] The oscillograph used by Kalin was electromagnetic and incorporated a rotating drum for photographic recording. The oscillograph element was placed in the plate circuit, so that the vertical deflection of the light spot was proportional to the plate current. The grid voltage was made to vary linearly with time by means of a rotary voltage divider. Since the recording drum rotated at constant speed, the grid voltage was proportional to the horizontal displacement of the spot on the film. The oscillogram consequently represented a plot of plate current against grid voltage. By this method the average plate dissipation was kept sufficiently low so that the complete family of transfer characteristics could be obtained. The substitution of a linear sweep oscillator in place of the rotating voltage divider improves Kalin's method. The sweep oscillator may be synchronized to the drum motor.

In the positive grid region the average plate and grid dissipations may be too great, even when the grid voltage is periodically varied as a linear function of time. Kozanowski and Mouromtseff[2] reduced the average dissipation to a very low value by using for the grid excitation a single pulse of voltage from the discharge of a large condenser through

[1] KALIN, ALBERT, *Univ. Wash. Eng. Expt. Sta. Bull.* 30, (1924); SCHNEIDER, W. A., *Proc. I.R.E.*, **16,** 674 (1928).

[2] KOZANOWSKI, H. N., and MOUROMTSEFF, I. E., *Proc. I.R.E.*, **21,** 1082 (1933).

a resistance. Although the instantaneous values of dissipation may be very high, the currents flow for such short intervals that the electrode temperatures do not rise so high as to damage the tube. Peak input powers of twenty to thirty times the nominal rating can be recorded by this method. The grid and plate characteristics are derived from oscillograms of grid and plate voltages and current, recorded simultaneously by a four-element moving-mirror oscillograph. The circuit is shown in Fig. 15-72. All resistances used in this circuit are non-inductive. The oscillograph elements must, of course, be accurately calibrated. The condenser is charged from the voltage supply preparatory to the taking of an oscillogram, and the discharge of the condenser is initiated automatically by the closing of the switch in synchronism with the opening of the oscillograph shutter.

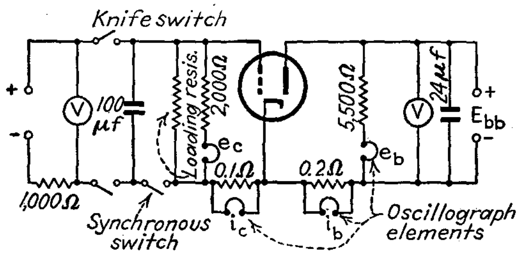


FIG. 15-72.—Circuit for the determination of tube characteristics by means of an electromagnetic oscillograph.

A cathode-ray oscillograph may also be used in the determination of static transfer characteristics, but its use involves difficulties. If electrostatic deflection is used, the plate current must be passed through a resistance in order to provide deflecting voltage. A resistance that is high enough to provide adequate voltage may appreciably alter the characteristic. If an amplifier is used in conjunction with a low resistance, the amplifier must have negligible nonlinear, frequency, and phase distortion over a wide frequency range in order to ensure distortionless amplification. The use of an inverse feedback amplifier is recommended for this purpose. Because of the impedance of the deflecting coils, difficulties are also encountered when the plate current is used to produce magnetic deflection.

Chaffee has described a cathode-ray circuit for the determination of static characteristics in which the grid of the tube under observation is excited by a voltage pulse produced by the discharge of a thyratron.[1]

15-29. Dynamic Measurement of Tube Factors.—Bridge circuits for the dynamic measurement of tube factors were first described by

[1] CHAFFEE, E. L., *Electronics*, July, 1937, p. 30; see also H. F. MAYER, *Electronics*, April, 1938, p. 14.

J. M. Miller.[1] Other circuits have since been developed. Before proceeding to a discussion of tube-factor bridges it is advisable to mention general limitations of these bridges and precautions that should be observed in their use. The equations that must be employed in the applications of these bridges are derived by means of equivalent tube circuits, which are strictly applicable only at small signal amplitudes. The accuracy of the experimentally determined values therefore increases as the signal amplitude is reduced. For this reason it is advisable to use small voltages in exciting tube-factor bridges and to employ an amplifier between the bridge and the telephone receivers if necessary. To prevent direct voltage drops that reduce operating voltages it is important to provide direct-plate-current paths of low resistance by shunting the phones with low-resistance chokes. This is particularly necessary in the circuits of Figs. 15-73, 15-74, 15-76, 15-78, 15-79, and 15-80. Transformers having low primary resistance may, of course, be used in place of chokes. In the circuits of Figs. 15-73 to 15-76, and 15-80, direct plate current also flows through the secondary of the exciting voltage input transformer, which should consequently have low d-c resistance. The circuits should be set up so that stray capacitances between leads and batteries are as small as possible. For the sake of simplicity, the tube is shown as a triode in the following circuits, multigrid tubes merely requiring the addition of other supply voltages.

15-30. Amplification-factor Bridge.—The basic bridge circuit generally used for the measurement of amplification factor is that of Fig. 15-73.[1] By constructing the equivalent plate circuit and assuming that $V_p = -(r_2/r_1)V_g$ and that grid current does not flow, the student may readily show that the alternating plate current is zero when

$$\mu = \frac{r_2}{r_1} \tag{15-51}$$

Actually the effect of tube capacitances prevents V_p and V_g from being exactly 180 degrees out of phase at high frequencies. They can be made so by the use of the balancing condenser C. The voltages V_p and V_g are opposite in phase when the ratio of the reactances of the condensers shunting r_1 and r_2 is equal to r_1/r_2, or

$$\frac{C + C_{gk}}{C_{pk}} = \frac{r_2}{r_1} \tag{15-52}$$

The voltages can also be made 180 degrees out of phase by means of a small mutual inductance M, as in Fig. 15-74. As measurements are

[1] MILLER, J. M., *Proc. I.R.E.*, **7**, 112 (1919).

usually made at 1000 cps or lower, the balancing condenser or mutual inductance is not ordinarily necessary.

r_1 should preferably be small enough so that the direct voltage drop in r_1 will not greatly alter the operating grid voltage. For accurate measurements this voltage drop should be taken into account in adjusting the tube voltages.

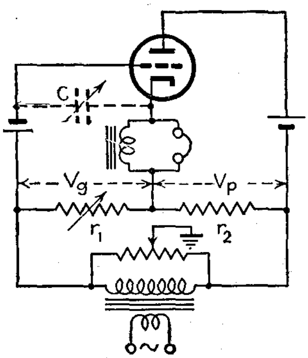


FIG. 15-73.—Bridge for the measurement of amplification factor.

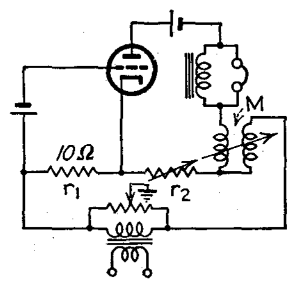


FIG. 15-74.—Bridge for the measurement of amplification factor.

15-31. Plate-resistance Bridges.—The plate resistance of triodes or other low-plate-resistance tubes may be readily measured by means of the simple four-arm bridge of Fig. 15-75.[1] When the bridge is balanced,

$$r_p = r_3 \frac{r_1}{r_2} \tag{15-53}$$

and

$$C = (C_{pk} + C_{gp}) \frac{r_1}{r_2} \tag{15-54}$$

The condenser C may be omitted if measurements are made at 1000 cps or lower. To avoid excessive direct-voltage drop in r_1, r_1 should preferably be small (10 or 100 ohms). The voltage drop should be taken into account in adjusting the operating plate voltage.

A second bridge for the measurement of plate resistance is formed by the addition of resistances r_3 and r_4 to the amplification-factor bridge of Fig. 15-74, as shown in Fig. 15-76.[2] The bridge is first balanced by means of r_2 and M, with S_1 closed and S_2 open. Then S_1 is opened and S_2 closed, and the bridge is again balanced. The student may show that the plate resistance is then given by the relation

$$r_p = 100r_4 \tag{15-55}$$

[1] BALLANTINE, S., *Proc. I.R.E.*, **7**, 134 (1919).
[2] MILLER, *loc. cit.*

It is difficult to measure the high plate resistance of tetrodes and pentodes accurately with the circuits of Figs. 15-75 and 15-76. High values of plate resistance can be conveniently measured with the circuit of Fig. 15-77.[1] If r_1 is a calibrated resistance, r_p is found by adjusting r_1 so that the voltage across r_2 is the same for the two positions of the

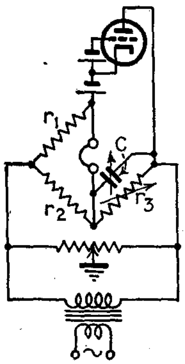


FIG. 15-75.—Bridge for the measurement of plate resistance.

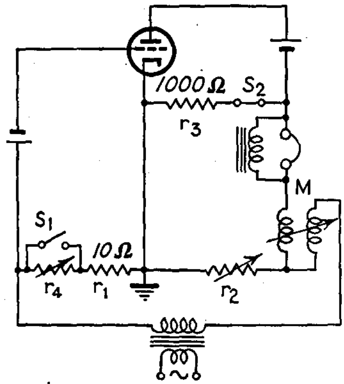


FIG. 15-76.—Bridge for the measurement of plate resistance.

switch S. If the applied voltage V is kept constant, the vacuum-tube voltmeter may be calibrated to read resistance directly, r_1 being used for calibration. A third method of measurement is to observe the voltage V necessary to give a definite deflection of the vacuum-tube voltmeter. For high accuracy r_2 should be small in comparison with r_p. This necessitates the use of a vacuum-tube voltmeter of high sensitivity.

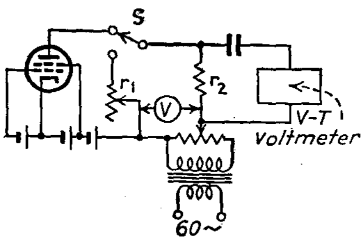


FIG. 15-77.—Bridge for the measurement of high plate resistances.

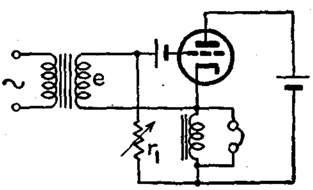


FIG. 15-78.—Bridge for the measurement of transconductance of tubes with low plate resistance.

The direct voltage drop in r_2 must be taken into consideration in adjusting the plate voltage.

15-32. Transconductance Bridges.—The transconductance of tubes with low plate resistance can be readily measured by the simple circuit of Fig. 15-78, which is a modified form of a circuit first suggested by

[1] Standards on Electronics, p. 27, Institute of Radio Eng., New York, 1938.

Appleton.[1] Examination of this bridge shows that the alternating current through the phones is made up of two components, one of which flows through r_1 and the source e, and the other through the plate of the tube, and that these two currents are opposite in phase if the electrode capacitances are small. When the circuit is balanced, these two currents are equal and, furthermore, there is no voltage drop through the phones. Therefore,

$$\frac{e}{r_1} = \frac{\mu e}{r_p} \tag{15-56}$$

or

$$g_m = \frac{1}{r_1} \tag{15-57}$$

For greater accuracy the unbalancing caused by the interelectrode

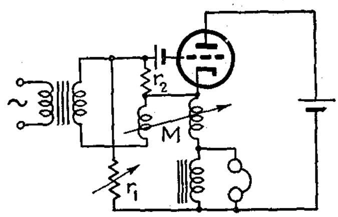


FIG. 15-79.—Transconductance bridge of Fig. 15-78, modified to incorporate reactance balance.

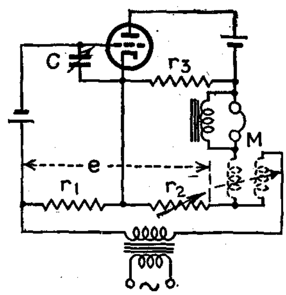


FIG. 15-80.—Bridge for the measurement of transconductance of tubes with high plate resistance. Either C or M is used for reactive balance.

capacitance at high frequency can be corrected by the addition of a mutual inductance, as in Fig. 15-79.

For tubes with high plate resistance the circuit of Fig. 15-80 is more satisfactory.[2] In this circuit the alternating voltage across r_2 is balanced against that across r_3. The bridge is balanced reactively by the condenser C, which may be replaced by a mutual inductance M, shown dotted in Fig. 15-80. For the circuit in which C is used, the voltages across r_3 and r_2 are opposite in phase and equal in magnitude when the bridge is balanced. Since no alternating current flows through the phones and the reactance of the interelectrode capacitances is high in comparison with r_3, the load impedance is practically equal to r_3. If grid current does not flow,

[1] APPLETON, E. V., *Wireless World*, **6**, 458 (1918).
[2] BALLANTINE, *loc. cit.*

$$\frac{er_2}{r_1 + r_2} = \frac{\mu e_g r_3}{r_p + r_3}$$
$$= \frac{\mu e r_3}{r_p + r_3} \cdot \frac{r_1}{r_1 + r_2} \tag{15-58}$$

or

$$g_m = \frac{r_2}{r_1 r_3} \cdot \frac{r_p + r_3}{r_p} \tag{15-59}$$

The plate resistance of tetrodes and pentodes is so high that the ratio of r_p to r_3 may be made large enough so that Eq. (15-59) reduces to

$$g_m = \frac{r_2}{r_1 r_3} \tag{15-60}$$

Equations (15-59) and (15-60) may also be shown to hold when M is used in place of C.

15-33. Bridge for the Measurement of μ, r_p, and g_m.—Inspection of Figs. 15-74, 15-76, and 15-80 shows that these three bridges may be very readily combined into a single instrument for the measurement of μ, r_p, and g_m.

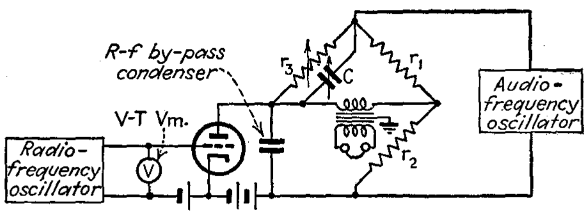


FIG. 15-81.—Bridge for the measurement of detection plate resistance.

Detection-plate-resistance Bridge.—Figure 15-81 shows a bridge for the measurement of detection plate resistance r_p'.[1] The action of this bridge is similar to that of the plate-resistance bridge of Fig. 15-75. When balance is obtained,

$$r_p' = \frac{r_2 r_3}{r_1} \tag{15-61}$$

In the measurement of detection plate resistance of diodes, the radio-frequency excitation is applied in series with the plate.

Measurement of Other Tube Factors.—The bridges discussed above may be adapted to the measurement of other tube factors of multigrid tubes.[2] Bridges have also been designed for the measurement of negative tube factors.[3] Circuits for the determination of inter-

[1] BALLANTINE, S., *Proc. I.R.E.*, **17**, 1164 (1929).

[2] Standards on Electronics, Institute of Radio Engineering, New York, 1938.

[3] CHAFFEE, E. LEON, "Theory of Thermionic Vacuum Tubes," Chap. IX, McGraw-Hill Book Company, Inc., New York, 1933; HICKMAN, R. W., and HUNT, F. C., *Rev. Sci. Instruments*, **6**, 268 (1935).

electrode capacitances, leakage conductances, tube characteristics, and other quantities are discussed in the 1938 report on electronics of the Standards Committee of the Institute of Radio Engineers.

15-34. Negative-resistance Bridges.—A bridge for the measurement of negative resistance is shown in Fig. 15-82.[1] The condition for balance is

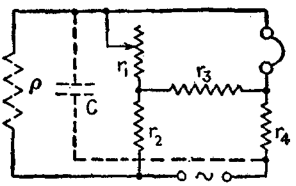


FIG. 15-82.—Bridge for the measurement of negative resistance.

$$\rho = \left(\frac{r_2}{r_3} + 1\right) r_1 + r_2 \tag{15-62}$$

The resistances of Fig. 15-82 must be taken into account in determining the operating voltages of the negative-resistance element. Proper choice of resistances simplifies the criterion for balance. If $r_2 = 99$ ohms and $r_3 = 1$ ohm, Eq. (15-62) reduces to

$$\rho = 100r_1 + 99 \tag{15-63}$$

If the negative-resistance element has appreciable capacitance, the balancing condenser C should be added.

Three modifications of Dingley's bridge that are capable of measuring a greater range of negative resistance were developed by Terman.[2] These circuits and the criteria for balance are shown in Fig. 15-83. r

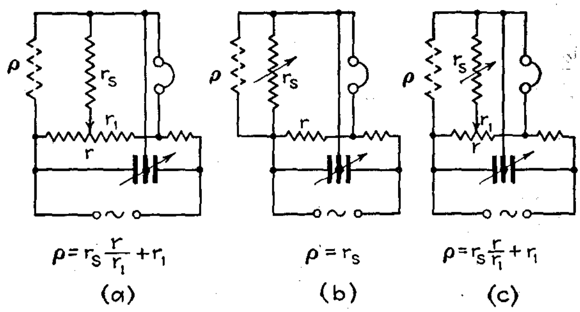


FIG. 15-83.—Three modifications of the negative-resistance bridge of Fig. 15-82.

should not exceed several hundred ohms, and r_s should be much larger than r_1. Ten thousand ohms is suggested for r_s in circuits b and c. The capacitance balance can be constructed from a two-gang variable condenser.

15-35. Tuttle Tube-factor Bridge.—The circuits that have been discussed employ a single source of excitation voltage. A different type of tube-factor bridge, in which currents caused by three separate

[1] DINGLEY, E. N., JR., *Proc. I.R.E.*, **19**, 1948 (1931).
[2] TERMAN, F. E., *Electronics*, December, 1933, p. 340.

voltages are balanced, has been developed by W. N. Tuttle.[1] The three forms of Tuttle's circuit used for the measurement of amplification factor, electrode resistance, and transconductance are shown in Figs. 15-84, 15-85, and 15-87. The transformers and attenuators in these

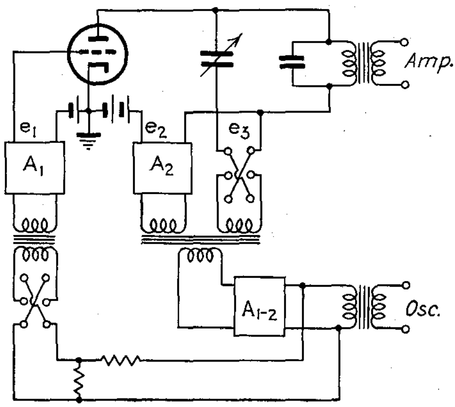


FIG. 15-84.—Bridge for the measurement of mu-factor.

circuits have been designed so that the three voltages e_1, e_2, and e_3 are exactly in phase (or 180 degrees out of phase). The function of e_3 is to furnish a reactive component of current to balance the reactive current caused by electrode and circuit capacitances.

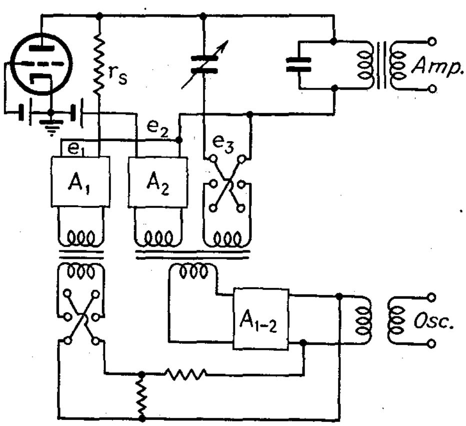


FIG. 15-85.—Bridge for the measurement of electrode resistance.

The principle of the circuit of Fig. 15-84, which measures amplification factor, is the same as that of the more common amplification-factor bridge of Fig. 15-73. The plate current resulting from e_2 in the plate circuit is balanced against the current resulting from e_1 in the grid circuit. The net

[1] TUTTLE, W. N., *Proc. I.R.E.*, **21**, 844 (1933).

current is zero when $e_2 = \mu e_1$. The voltage ratio is read directly on the attenuators.

The circuit of Fig. 15-85 measures plate resistance. An exact analysis of the operation of the circuit can be made by constructing the complete equivalent circuit and determining the conditions necessary to reduce the current in the output branch to zero. For the purpose of explaining the basic principle of operation, it is instructive to neglect the tube capacitances and to consider only the nonreactive components of current. Under the assumption that the capacitances are zero, e_3 is also zero, and the equivalent circuit is of the form shown in Fig. 15-86, in which z_0 is the impedance of the output transformer and condenser. The network equations are

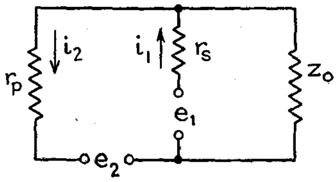


FIG. 15-86.—Equivalent circuit for the bridge of Fig. 15-85.

$$e_1 = i_1(r_s + z_o) - i_2 z_o \tag{15-64}$$
$$e_2 = -i_1 z_o + i_2(r_p + z_o) \tag{15-65}$$

Balance is obtained when $i_1 = i_2$, so that the current in the output branch is zero. Setting the currents equal and dividing Eq. (15-65) by Eq. (15-64) gives the condition for balance.

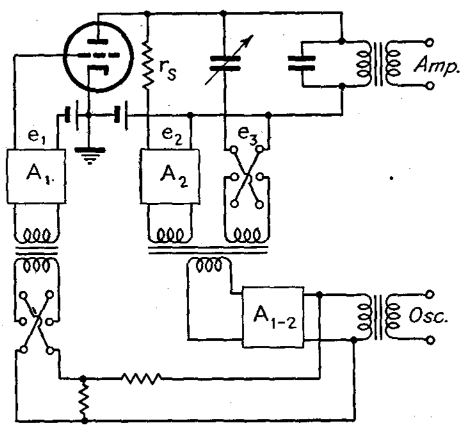


FIG. 15-87.—Bridge for the measurement of transconductance.

$$r_p = \frac{r_s e_2}{e_1} \tag{15-66}$$

The attenuators may be calibrated to read r_p directly.

The circuit of Fig. 15-87 is used to measure transconductance. The equivalent circuit, neglecting tube capacitances and grid current, is similar to that of Fig. 15-86; but since e_1 is applied in the grid circuit,

e_1 must be replaced by μe_2, and e_2 by e_1, in the equivalent circuit. The condition for balance is

$$\frac{\mu e_1}{r_p} = \frac{e_2}{r_s} \quad \text{or} \quad g_m = \frac{e_2}{e_1 r_s} \tag{15-67}$$

To measure negative factors, the phase of e_1 is reversed by means of the double-throw switch.

By using Tuttle's circuits, all tube factors may be read with ease with a single bridge. Another advantage results from the fact that all batteries may be kept at ground potential. The construction of a reliable portable instrument of this type necessitates great care in transformer design and in the location of component parts.

15-36. Harmonic Analyzers.—Harmonic analyzers for the direct measurement of harmonic amplitudes by electrical means are of four types: tuned circuit, heterodyne, dynamometer, and fundamental-suppression. Indirect measurements may also be made by taking oscillograms of the current or voltage and analyzing the oscillograms by mechanical or electrical analyzers or by graphical or selected-ordinate methods. The oscillographic method requires more time and is not so accurate as some of the direct methods.

15-37. Analyzers Using Tuned Circuits.—The use of tuned circuits for harmonic analysis was first suggested by Pupin in 1893 and was

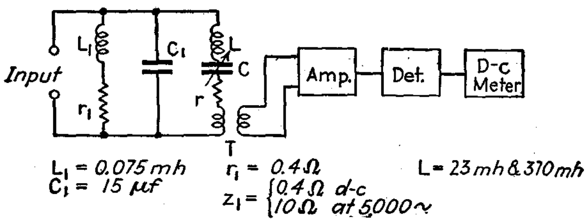


FIG. 15-88.—Tuned-filter type of harmonic analyzer.

subsequently used by him in the study of electrical apparatus.[1] In 1912, R. Beattie made an analysis to determine the best form of circuit for the tuned-circuit type of analyzer.[2] A portable analyzer based upon the tuned-filter principle was designed by Wegel and Moore in 1924.[3] The general features of their circuit are shown by Fig. 15-88. The voltage to be analyzed is applied across the series resonant circuit $LCrT$, and the voltage induced across the secondary of the transformer T is applied to the amplifier. The rectified output of the amplifier is read by

[1] PUPIN, M., *Am. J. Science*, **45**, 429 (1893); *Trans. Am. Inst. Elec. Eng.*, **11**, 523 (1894).

[2] BEATTIE, R., *Electrician*, **69**, 63 (1912).

[3] WEGEL, R. L., and MOORE, C. R., *Bell System Tech. J.*, **3**, 299 (1924); *Trans. Am. Inst. Elec. Eng.*, **43**, 457 (1924).

means of a d-c meter. The harmonic content of the applied voltage is determined by tuning the filter successively to the various harmonic frequencies and reading the output meter. The purpose of the parallel resonant circuit made up of L, r, and C_1 is to correct for the variation, with frequency, of a-c resistance of the series circuit and of the amplifier gain, so that the sensitivity remains essentially constant over the frequency range for which the instrument is designed.

Instead of taking the output of the filter from the transformer, as in Fig. 15-88, it is possible to use the voltage developed across L, C, or r. Beattie[1] and Morgan[2] have shown that it is usually best to take the voltage from the inductance. Since inductive reactance is directly proportional to frequency, the voltage across the inductance at a given current increases with frequency. This discrimination against the lower components of the input wave is desirable if the amplitudes of the lower frequency components of the input are greater than the higher frequency

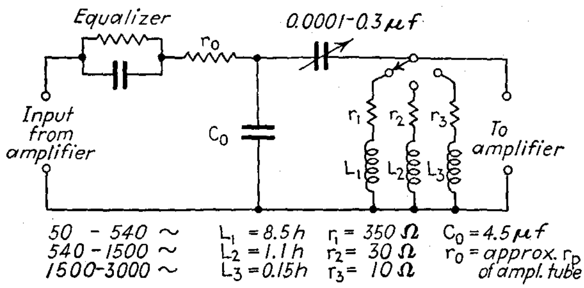


FIG. 15-89.—Amplifier-coupling filter for tuned-filter harmonic analyzer.

components, which is usually true. It helps to make possible the measurement of a small second harmonic in the presence of a large fundamental.

A singly tuned filter does not tune sharply enough to make possible measurement of the amplitude of a small-amplitude component whose frequency does not differ greatly from that of a large-amplitude component. To overcome this difficulty McCurdy and Blye[3] designed an analyzer which uses two tuned filters as coupling elements between amplifier tubes. The circuit of one filter stage is shown in Fig. 15-89. The first amplifier is preceded by a filter for suppressing the fundamental frequency. The output is read by means of a thermocouple meter.

15-38. Heterodyne Harmonic Analyzers.—In addition to the problem of obtaining sufficiently sharp tuning, the design of the tuned-filter type

[1] BEATTIE, *loc. cit.*

[2] MORGAN, F., *J. Inst. Elec. Eng.*, **71,** 819 (1932) (contains bibliography of 40 items).

[3] MCCURDY, R. G., and BLYE, P. W., *Trans. Am. Inst. Elec. Eng.*, **48,** 1167 (1929).

of analyzer is complicated by the variation of filter resistance with frequency. The audible frequency band cannot be readily covered with a single filter inductance. It is necessary to use a number of inductances and associated equalizers, each of which covers a portion of the band. These problems can be readily solved by using a fixed-frequency filter and heterodyning the output of a variable-frequency oscillator with the input voltage so as to produce a sum or difference frequency equal to the filter frequency. The fixed filter frequency makes it possible to use a highly selective filter.

A block diagram of a typical heterodyne harmonic analyzer is shown in Fig. 15-90. The frequency of the filter is higher than that of the highest input signal to be measured. The use of a balanced modulator to convert the impressed frequency to the frequency of the filter affords

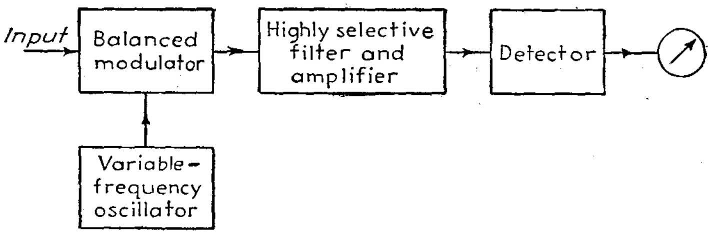


FIG. 15-90.—Block diagram of a heterodyne harmonic analyzer.

a simple means of eliminating the impressed frequency and at the same time ensures lower nonlinear distortion than could be obtained with a single-tube frequency converter. Some heterodyne analyzers may be calibrated to read directly; in others the voltage of the impressed signal is determined by measuring the voltage of a reference signal, the amplitude of which is made equal to that of the impressed signal. The various heterodyne analyzers that have been developed differ mainly as to the type of filter used. Early circuits made use of mechanical filters[1] and of filters tuned to a frequency lower than that of the input voltage.[2] In more recent analyzers, however, the filter consists of quartz crystals[3] or of two or more stages of inverse-feedback filters such as those of Fig. 6-45.[4] An important advantage of inverse-feedback filters is that they make possible the variation of selectivity at constant amplification by adjustment of feedback.

A heterodyne analyzer incorporating a two-section 50-kc quartz-crystal filter is shown in Fig. 15-91. The oscillator is tuned so that the

[1] MOORE, C. R., and CURTIS, A. S., *Bell System Tech. J.*, **6**, 217 (1927).

[2] LANDEEN, A. G., *Bell System Tech. J.*, **6**, 230 (1927).

[3] CASTNER, T. G., *Bell Laboratories Record*, **13**, 258 (1935).

[4] TERMAN, F. E., BUSS, R. R., HEWLETT, W. R., and CAHILL, F. C., *Proc. I.R.E.*, **27**, 649 (1939).

sum of the input frequency and the oscillator frequency is equal to 50 kc. The resistance R and the capacitance C correct for any slight unbalance of the balanced modulator. They are adjusted so as to eliminate the oscillator fundamental frequency from the output of the modulator. The output stage is a vacuum-tube voltmeter with a balancing circuit for eliminating the zero-signal current from the meter. The tuned-plate oscillator is designed to give constant amplitude throughout its range of 35 to 50 kc.

The instrument is calibrated by adjusting the amplifier gain to give a 2-volt reading of the output meter when 1 volt of direct voltage, obtained from the filament battery, is applied between the grids of the

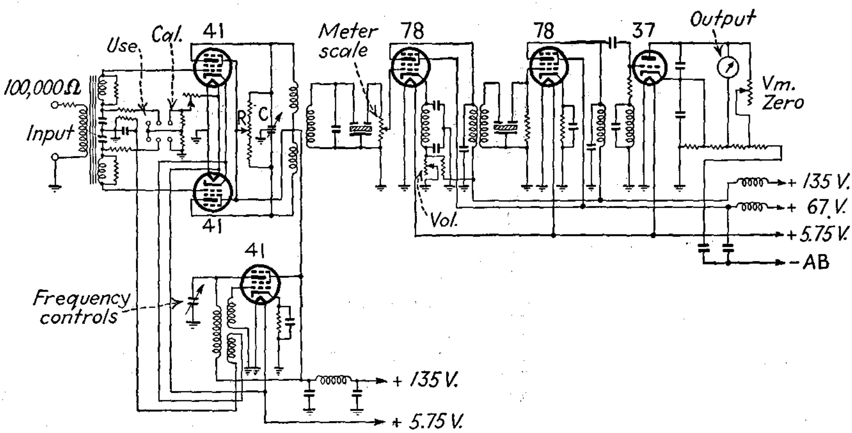


FIG. 15-91.—Heterodyne wave analyzer using a quartz-crystal filter. (*Courtesy of General Radio Co.*)

balanced modulator and the oscillator is tuned to the filter frequency. The justification for this method of calibration follows from the analysis of the balanced modulator. Equation (9-13) demonstrates that two components of the output voltage of a balanced modulator are $2Aa_2E_1E_2 \cos 2\pi(f_1 + f_2)t$ and $2Aa_2E_1E_2 \cos 2\pi(f_1 - f_2)t$ when the input voltage is made up of two components $E_1 \sin \omega_1 t$ and $E_2 \sin \omega_2 t$. As f_2 approaches zero, the sum and difference frequencies become more and more nearly equal and, in the limit, when f_2 is zero, are both equal to f_1. When one voltage is direct, therefore, the output voltage is $4Aa_2E_1E_2 \cos 2\pi f_1 t$, which has twice the amplitude of the sum- and difference-frequency components obtained when both input voltages are alternating. The frequency range of the General Radio wave analyzer is 20 to 15,000 cps, and the voltage range 200 μv to 200 volts.

In another type of heterodyne analyzer, devised by C. G. Suits, the unknown voltage and the voltage from the oscillator are applied simultaneously to the grid circuit of a square-law detector.[1] The plate current contains the impressed frequencies and their harmonics, steady components, and intermodulation components. A d-c milliammeter in the plate circuit responds to the steady components and to any intermodulation frequencies that do not greatly exceed the resonance frequency of the meter movement. By adjusting the oscillator frequency to differ only slightly from the frequency of a component of the unknown voltage, the needle can be caused to oscillate at the resulting difference frequency. Ordinarily only one difference frequency at a time will be low enough to affect the meter. According to principles set forth in Chaps. 3 and 9, the change in reading of the meter with application of the signal voltage will then be

$$\Delta I = a_2[\tfrac{1}{2}E_o^2 + \Sigma\tfrac{1}{2}E_n^2 + E_oE_x \cos 2\pi(f_x - f_o)t] \qquad (15\text{-}68)$$

where f_o and E_o are the frequency and amplitude of the oscillator voltage, f_n and E_n are the frequency and amplitude of any component of the input voltage, and f_x and E_x are the frequency and amplitude of the particular component of the input voltage that is being measured. The milliammeter needle will oscillate through a current range that is twice the amplitude of the difference-frequency component of the plate current. This fact may be expressed by the equation

$$\Delta I_x = 2a_2E_oE_x \qquad (15\text{-}69)$$

If the oscillator voltage does not vary with frequency, the amplitude of oscillation of the meter is proportional only to the amplitude of the unknown voltage, and the instrument may be calibrated by means of a single input voltage of known amplitude. In the determination of per cent harmonic relative to the fundamental, only relative values are required, and the instrument need not be calibrated if the oscillator voltage does not vary with frequency. If the oscillator is not designed to give constant output at all frequencies, the analyzer may be calibrated by applying inputs having known voltage and frequency.

Errors in reading result from two sources: oscillator harmonics and departure of the detector characteristic from a true square law. The first type of error is small if the harmonic content of the oscillator is below 1 or 2 per cent. The second results mainly from the presence of the third-order term in the series expansion for the detector plate current and may be minimized by keeping the oscillator and input voltages small. It may be practically eliminated by the use of a balance detector, similar to that of Fig. 15-2.[2]

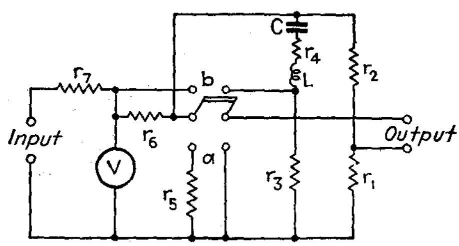
[1] SUITS, C. G., *Proc. I.R.E.*, **18**, 178 (1930). See also R. R. CHILTON, A Practical Wave Analyzer for Distortion Measurements, *Technical Bull., Inst. Rad. Eng. (Australia)*, 1941.

[2] GREENWOOD, W., *Wireless Eng.*, **9**, 310 (1932).

15-39. Dynamometer-type Harmonic Analyzers.—In the dynamometer type of analyzer the unknown voltage is applied, through an amplifier, to one coil of the dynamometer and the output of an oscillator to the other.[1] If the oscillator frequency is adjusted so that it differs only slightly from the frequency of the component of the unknown voltage to be measured, the meter needle will oscillate at the difference frequency. The amplitude of oscillation is proportional to the product of the amplitudes of the currents in the two coils. If the current from the oscillator is kept constant with the help of an additional meter, the amplitude of oscillation of the dynamometer needle is proportional only to the amplitude of the component of the unknown voltage. The instrument may, therefore, be calibrated to read the input amplitude directly.

It is also possible to construct dynamometer-type analyzers that give steady deflection.[2] The difficulty of obtaining an oscillator current of good wave form which remains exactly in phase with the voltage that is being measured makes it impractical to use this type of analyzer over the frequency range required for the testing of audio-frequency amplifiers.

15-40. Fundamental-suppression Harmonic Analyzers.—In the fundamental-suppression type of analyzer the fundamental component of the



$$\frac{r_1}{r_2} = \frac{r_3}{r_4} = k$$
$$r_1 + r_2 = r_5 + r_6$$
$$r_1 + r_2 < r_3 + r_4$$

FIG. 15-92.—Practical form of fundamental-suppression type of harmonic analyzer.

unknown voltage is removed by some form of bridge. The remaining r-m-s output from the bridge is proportional to the square root of the sum of the squares of all the harmonics. From the values of the total harmonic content and of the complete input, the fundamental component can be determined. The resonance bridge of Fig. 15-41*a* may be used for this purpose.[3] Figure 15-92 shows this bridge in convenient form for

[1] NICHOLSON, M. G., and PERKINS, W. M., *Proc. I.R.E.*, **20,** 734 (1932).

[2] COCKROFT, J. D., COE, R. T., TYACKE, J. A., and WALKER, M., *J. Inst. Elec. Eng.*, **63,** 69 (1925); *Proc. Phys. Soc. London*, **40,** 228 (1928).

[3] BELFILS, G., *Rev. gén. élec.*, **19,** 523 (1926).

harmonic measurement.[1] When the switch is in position *a*, the output is approximately

$$(\text{total input}) \times \frac{k}{k+1} \times \frac{r_5}{r_5 + r_6}$$

When the switch is in position *b*, the bridge output is equal to

$$(\text{vector sum of the harmonics}) \times \frac{k}{k+1}.$$

The bridge output is measured by means of a vacuum-tube voltmeter. The input voltage to the bridge may change with adjustment of the bridge and should be kept constant. Then if the voltmeter reads the same for the two switch positions, small distortion factors are equal to R_5/R_6. The parallel resonance bridge of Fig. 15-41*b* may be used in place of the series resonance bridge.[2] High- and low-pass filters may also be used for fundamental suppression.[3]

The fundamental-suppression type of bridge is particularly useful in determining the distortion factor δ (see Sec. 4-18). The fundamental-suppression bridge may often be used advantageously in combination with other methods of analysis. If the fundamental is removed by means of a bridge, for instance, the harmonic content may be analyzed with considerable accuracy by oscillographic and graphical methods.[4] Fundamental suppression is often advisable in connection with tuned-circuit and heterodyne analyzers because it greatly reduces the production of harmonics in the analyzer; it also reduces the selectivity requirements of the analyzer. Reduced selectivity is essential when the frequency of the unknown voltage drifts appreciably during the time required to make a measurement.

15-41. Measurement of Voltage Amplification.—Figure 15-93 shows the circuit that is commonly used for the measurement of voltage amplification of an amplifier. When the attenuation of the attenuator is equal to the gain of the amplifier, the deflection of the vacuum-tube voltmeter will not change when the position of the switch is changed. If the attenuator is calibrated in decibels, it gives a direct reading of the amplifier gain in decibels. The purpose of the resistor R is to provide the proper terminating impedance for the attenuator. If a calibrated attenuator is not available, it may be replaced by a voltage divider, as in Fig. 15-94. The voltage amplification A is equal to the ratio r_2/r_1

[1] WOLFF, IRVING, *J. Opt. Soc. Am. and Rev. Sci. Instruments*, **15,** 163 (1927).

[2] WAGNER, H. M., *Proc. I.R.E.*, **23,** 85 (1935); see also H. H. SCOTT, *Proc. I.R.E.*, **26,** 226 (1938).

[3] MCCURDY and BLYE, *loc. cit.*

[4] BROWN, S. L., *Phys. Rev.*, **31,** 302 (1928); PIDDINGTON, J. H., *Proc. I.R.E.*, **24,** 591 (1936).

when the same voltmeter reading is obtained with the switch in the two positions. Although affording a less accurate means of comparing the input and output voltages, a cathode-ray oscilloscope or an electron-ray tube may be used in place of the vacuum-tube voltmeter in the circuits of Figs. 15-93 and 15-94.

The response curve of an amplifier or other network may be observed directly by means of a cathode-ray oscilloscope.[1] This is accomplished by modulating the frequency of the amplifier excitation voltage in accordance with the sweep voltage and applying the amplifier output voltage to the vertical deflection plates of the oscilloscope. If the excitation amplitude is independent of frequency, the envelope of the pattern

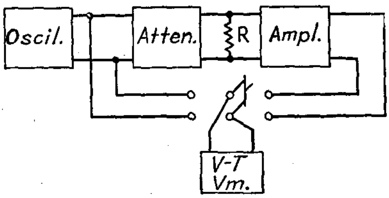


FIG. 15-93.—Circuit for the measurement of voltage amplification.

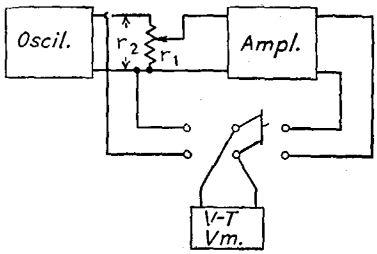


FIG. 15-94.—Circuit for the measurement of voltage amplification.

obtained on the screen is the response curve, plotted on a uniform frequency scale.

15-42. The Use of Triangular and Rectangular Waves in Amplifier Analysis.—In order to amplify a triangular or rectangular wave of voltage without observable distortion, an amplifier must have negligible frequency and phase distortion over a frequency range covering the fundamental frequency and approximately the first ten harmonics. This fact may be used to advantage in checking amplifiers oscillographically for frequency distortion. The distortion resulting from insufficient amplification of the lower-frequency components, and of the higher-frequency components, is shown by waves *a* and *b*, respectively, of Fig. 15-95. Transient oscillations resulting from resonance of transformer inductance and distributed capacitance result in an output wave of the form of wave *c*. The ratio of the resonance frequency to the frequency of the rectangular wave may be readily determined from such a wave. Rectangular waves may be used in the determination of amplifier response by applying the output of a variable-frequency rectangular-wave generator to the input of the amplifier and observing the output of the

[1] DIAMOND, H., and WEBB, J. S., *Proc. I.R.E.*, **15**, 767 (1927).

amplifier by means of an oscilloscope.[1] Negligible departure of the output wave from rectangular form over any range of frequency is an indication that the amplifier will give essentially undistorted amplification of complicated waves throughout this frequency range.

15-43. Measurement of Power Output. The power output of an amplifier with known load resistance may be determined by measuring the r-m-s voltage across the load or the r-m-s current through the load. A thermal meter or a copper oxide rectifier meter is usually used for the purpose. Figure 15-96 shows the circuit diagram of the General Radio output meter, by means of which power may be quickly measured at various values of effective load resistance. The meter consists of a variable-ratio transformer compensated by means of resistances so that a constant percentage of the power is expended in the secondary load. The load is a constant-resistance network which also serves the function of a four-range multiplier for the voltmeter. The indicating instrument is a copper oxide rectifier

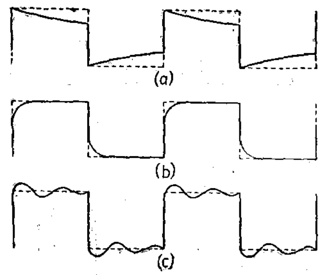


FIG. 15-95.—Distortion of rectangular wave caused by: (a) poor low-frequency response; (b) poor high-frequency response; (c) transient oscillations in coupling transformers.

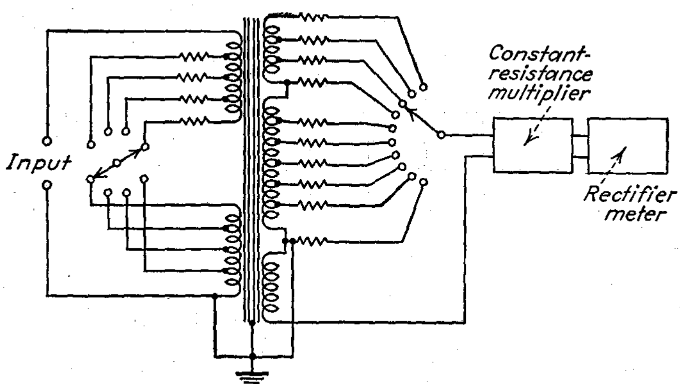


FIG. 15-96.—Circuit diagram of General Radio power-output meter.

voltmeter. The effective load may be varied in 40 steps from 2.5 to 20,000 ohms. The multiplier provides 5-, 50-, 500-, and 5000-mw ranges of power. The scale is also calibrated to read decibel power level relative to 1 mw zero level.

[1] REICH, H. J., *Proc. I.R.E.*, **19,** 401 (1931); STOCKER, A. C., *Proc. I.R.E.*, **25,** 1012 (1937); SWIFT, G., *Communications,* February, 1939, p. 22; BEDFORD, A. V., and FREDENDAHLE, G. L., *Proc. I.R.E.*, **27,** 277 (1939); ARGUIMBAU, L. B., *Gen. Rad. Expt.*, **14,** December, 1939, p. 1; WAIDELICH, D. L., *Proc. I.R.E.*, **32,** 339 (1944).

15-44. Determination of Optimum Power Output and Optimum Load.[1]—The determination of optimum power output and optimum load

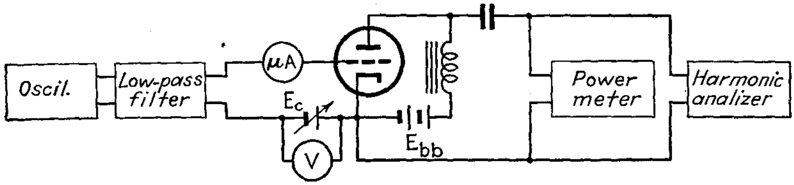


FIG. 15-97.—Circuit for the determination of optimum power output and optimum load resistance.

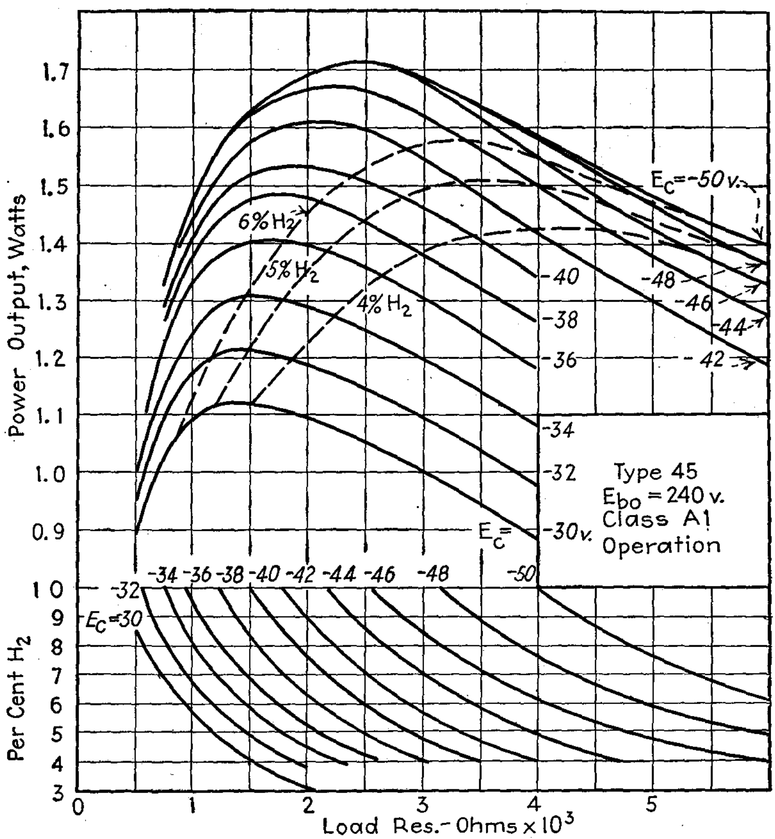


FIG. 15-98.—Experimentally determined curves of power output and harmonic content of a type 45 tube. The dashed curves show power output at constant percentage of second harmonic.

requires a series of measurements. A suitable circuit is shown in Fig. 15-97. For each load impedance there is a limiting value of bias and

[1] KELLOGG, E. W., *J. Am. Inst. Elec. Eng.*, **40,** 490 (1925); WARNER, J. C., and LOUGHREN, A. V., *Proc. I.R.E.*, **14,** 735 (1928); HANNA, C. R., SUTHERLIN, L., and UPP, C. B., *Proc. I.R.E.*, **16,** 462 (1928).

signal amplitude beyond which the harmonic content exceeds the allowable values. Curves of power output and harmonic content as a function of load impedance must be constructed for various values of bias. The simplest laboratory procedure is to set the bias at fixed values and to take readings of power output and harmonic content at five or six values of load impedance. If operation is to be restricted to the region in which no grid current flows, the signal must be adjusted for each value of bias and load so that grid current does not quite flow. The signal amplitude at which grid current commences can be determined by a galvanometer or headphones in series with the grid.

Figure 15-98 shows curves of power and harmonic content for a type 45 tube. The load impedance at which the harmonic content is just equal to the maximum allowable value at each value of bias may be determined from the curves of harmonic content. These values of impedance and bias then determine points on the power curves corresponding to the given harmonic content. The dashed curves of Fig. 15-98 show the power output at 4, 5, and 6 per cent second harmonic as a function of load resistance. From these the optimum load and power output at the given value of distortion may be read. Readings of third harmonic taken simultaneously with second harmonic showed the former to be negligible in comparison with the latter.

Problem

15-1. Carefully study the circuit of Fig. 15-59 and explain the functions of all tubes and circuit elements.

APPENDIX

A-1. Parallel Equivalent Circuits.—Figure A-2 shows the series equivalent plate circuit for the general form of triode circuit of Fig. A-1, in which the load z_b may include one or more impressed voltages. The voltage between A and B is $\mu E_g - I_p r_p$, which may be written in the form $r_p(g_m E_g - I_p)$. It is evident, therefore, that the

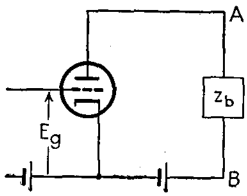


FIG. A-1.

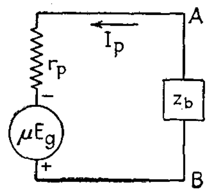


FIG. A-2.

voltage between A and B and, hence, all voltages and currents in the load, are unaltered if the equivalent voltage μE_g is removed and an additional current $g_m E_g$ is sent through r_p in the direction opposite to that in which μE_g tends to send current. The series equivalent circuit of Fig. A-2 may therefore be replaced by the parallel equivalent circuit of Fig. A-3. The current $g_m E_g$ may be assumed to be caused to flow by a constant-current generator, as shown in Fig. A-4.

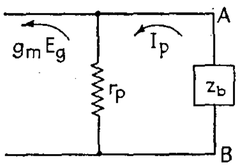


FIG. A-3.

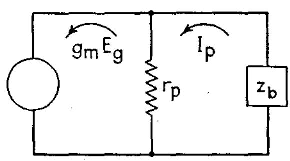


FIG. A-4.

In the series equivalent circuit, in which the equivalent generator is assumed to supply a constant voltage μE_g, the current must vary with load and plate impedance. In the parallel equivalent circuit, on the other hand, in which the generator is assumed to supply a constant current $g_m E_g$, the voltage across the load and generator must vary with load and plate impedance. It should be noted that the polarity of the voltage across the constant-current generator is determined solely by the voltage across the load. If the load contains an alternating e.m.f., the upper terminal may conceivably be positive, *i.e.*, the voltage across the generator may oppose the current through the generator. For this reason the polarity of the constant-current generator is not indicated.

Although the positive directions of most of the circuit currents and voltages may be chosen at random, the direction of the equivalent current must be properly chosen relative to the alternating grid voltage. If the instantaneous grid voltage is assumed to be positive when it makes the grid positive relative to the cathode, the equivalent current $g_m E_g$ should be indicated as flowing through the constant-current generator from the plate terminal to the cathode terminal as shown in Fig. A-4.

The procedure to be followed in the formation of the parallel equivalent circuit differs from that given in Sec. 4-2 for the series equivalent circuit only in the first step, which is as follows:

1. Connect the plate resistance and an equivalent constant-current generator in parallel between the plate and the cathode of the tube. The current supplied by the generator is $g_m E_g$ and should be indicated as flowing through the generator from the plate terminal to the cathode terminal. If alternating voltage is impressed upon more than one grid, there is a similar component of current for each additional grid that is excited.

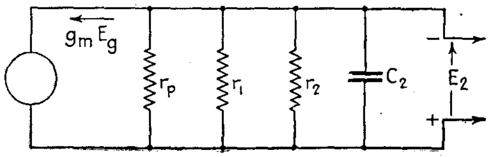


FIG. A-5.

The parallel equivalent circuit is particularly useful when the load consists of a number of parallel branches which do not contain an e.m.f., as, for example, in the resistance-capacitance–coupled amplifier at high frequencies, the parallel equivalent circuit of which is shown in Fig. A-5. It can be seen that the output voltage E_{g2} is minus the product of the current $g_m E_{g1}$ by the resultant impedance of the four parallel branches. Dividing this product by E_{g1} at once gives the voltage amplification, Eq. (6-15). In this example the parallel equivalent circuit clearly affords a more direct method of determining the voltage amplification than does the series equivalent circuit, used in Sec. 6-5. The parallel equivalent circuit does not, however, ordinarily simplify the analysis when voltages are impressed in both the grid and the plate circuits and simultaneous network equations must be solved.

A-2. Power Relations in Vacuum-tube Plate Circuits.—The power supplied to the plate circuit is

$$P_i = \frac{1}{T}\int_0^T E_{bb} i_b \, dt = E_{bb}\frac{1}{T}\int_0^T i_b \, dt = E_{bb} I_{ba} \qquad \text{(A-1)}$$

where T is the period of the fundamental component of plate current. But

$$i_b = I_{ba} + i_{pa} \qquad \text{(A-2)}$$

where i_{pa} is the instantaneous alternating plate current, measured relative to the average value I_{ba}. Furthermore,

$$E_{bb} = E_{ba} + I_{ba}R_b + e_{pa} + e_{zb} \qquad \text{(A-3)}$$

where e_{pa} is the instantaneous alternating plate voltage, measured relative to the average value E_{ba}, and e_{zb} is the instantaneous alternating voltage across the load, measured relative to the average value $I_{ba}R_b$.

Equation (A-1) may, therefore, be written in the form

$$P_i = \frac{1}{T}\int_0^T (I_{ba} + i_{pa})(E_{ba} + I_{ba}R_b + e_{pa} + e_{zb})\, dt \qquad \text{(A-4)}$$

$$= I_{ba}E_{ba} + I_{ba}^2 R_b + I_{ba}\frac{1}{T}\int_0^T e_{pa}\, dt + I_{ba}\frac{1}{T}\int_0^T e_{zb}\, dt +$$

$$E_{ba}\frac{1}{T}\int_0^T i_{pa}\, dt + I_{ba}R_b\frac{1}{T}\int_0^T i_{pa}\, dt + \frac{1}{T}\int_0^T i_{pa}e_{pa}\, dt + \frac{1}{T}\int_0^T i_{pa}e_{zb}\, dt \qquad \text{(A-5)}$$

Since i_{pa}, e_{pa}, and e_{zb} are measured with respect to average values, they do not contain steady components. Therefore, all but the last two integrals of Eq. (A-5) are zero. The power input is

$$P_i = I_{ba}E_{ba} + I_{ba}^2R_b + \frac{1}{T}\int_0^T i_{pa}e_{pa}\,dt + \frac{1}{T}\int_0^T i_{pa}e_{zb}\,dt \tag{A-6}$$

The first term of Eq. (A-6) represents the d-c power expended in the tube, or d-c plate dissipation. The second term represents the d-c power developed in the load. The third term represents the a-c power expended in the tube, or a-c plate dissipation. The last term represents the a-c power developed in the load.

The total plate dissipation is

$$P_p = I_{ba}E_{ba} + \frac{1}{T}\int_0^T i_{pa}e_{pa}\,dt \tag{A-7}$$

The a-c power output is

$$P_o = \frac{1}{T}\int_0^T i_{pa}e_{zb}\,dt \tag{A-8}$$

Since the plate supply voltage is constant, rise in voltage across the load is accompanied by an equal reduction in plate voltage, and so the alternating plate and load voltages are equal in magnitude, but opposite in phase.

$$e_{pa} = -e_{zb} \tag{A-9}$$

Therefore

$$\frac{1}{T}\int_0^T i_{pa}e_{pa}\,dt = -P_o \tag{A-10}$$

and

$$P_p = I_{ba}E_{ba} - P_o \tag{A-11}$$

Since the resistive component of load impedance causes the instantaneous plate voltage to fall with increase of plate current, e_{pa} has a component that is in phase opposition to i_{pa} and the second term of Eq. (A-7) is negative. Excitation therefore causes the plate dissipation to decrease. Equation (A-11) shows that the reduction of plate dissipation is equal to the power output. This fact and the negative value of the second term of Eq. (A-7) can be interpreted as indicating that the tube acts as a source of power delivered to the load or, more correctly, that the tube converts d-c power furnished by the plate supply into a-c power in the load.

A-3. Linear Modulation.—Application of a sinusoidal carrier excitation voltage to a circuit containing an element that conducts in only one direction results in the production of pulses of current. By Fourier analysis the current may be analyzed into a steady component and components having frequencies equal to the applied frequency and its harmonics. The addition of a steady biasing voltage in series with the carrier excitation voltage changes the portion of the cycle during which current flows. It thereby changes the amplitude of the current pulses and, therefore, of the fundamental component of current. Expressions for the fundamental component of current I_k and for the fundamental component of output voltage E_k across a resistance load may be derived as follows. Let the carrier excitation voltage be $E_2 \cos \omega_k t$, and let the biasing voltage be E_b. Current flows only when the total instantaneous applied voltage $E_2 \cos \omega_k t + E_b$, exceeds zero. If θ_0 is the value of $\omega_k t$ at which the current is cut off, then $E_2 \cos \theta_0 + E_b = 0$, or $\cos \theta_0 = -E_b/E_2$. If the circuit contains only resistance and the characteristic curve of the rectifier is linear, the instantaneous current is proportional to the instantaneous applied voltage when this voltage is positive, and is zero during the remainder of the cycle. Thus the current is

$$i = KE_2\left(\cos \omega_k t + \frac{E_b}{E_2}\right) = I_{\max}(\cos \omega_k t - \cos \theta_0), \qquad -\theta_0 < \omega_k t < +\theta_0 \tag{A-12}$$

in which K is a constant of proportionality, and $I_{\max} = KE_2 =$ the crest instantaneous current when $E_b = 0$. Expressed as a Fourier series the current is

$$i = A_0 + A_1 \sin \omega_k t + B_1 \cos \omega_k t + \cdots \tag{A-13}$$

$$A_1 = \frac{1}{\pi}\int_{-\pi}^{\pi} i \sin \omega_k t \, d(\omega_k t) = \frac{I_{\max}}{\pi}\int_{-\theta_0}^{+\theta_0} (\cos \omega_k t - \cos \theta_0) \sin \omega_k t \, d(\omega_k t) = 0 \tag{A-14}$$

$$B_1 = \frac{I_{\max}}{\pi}\int_{-\theta_0}^{+\theta_0} (\cos \omega_k t - \cos \theta_0) \cos \omega_k t \, d(\omega_k t) = \frac{I_{\max}}{\pi}(\theta_0 - \sin \theta_0 \cos \theta_0) \tag{A-15}$$

Therefore, the amplitude of the fundamental component of current is

$$I_k = \frac{I_{\max}}{\pi}(\theta_0 - \sin \theta_0 \cos \theta_0) \tag{A-16}$$

The amplitude of the fundamental component of output voltage across a resistance load is

$$E_k = \frac{I_{\max} R_e}{\pi}(\theta_0 - \sin \theta_0 \cos \theta_0) \tag{A-17}$$

in which R_e is the effective resistance of the load at carrier frequency. The solid curve of Fig. 9-10 was plotted by means of Eq. (A-17).

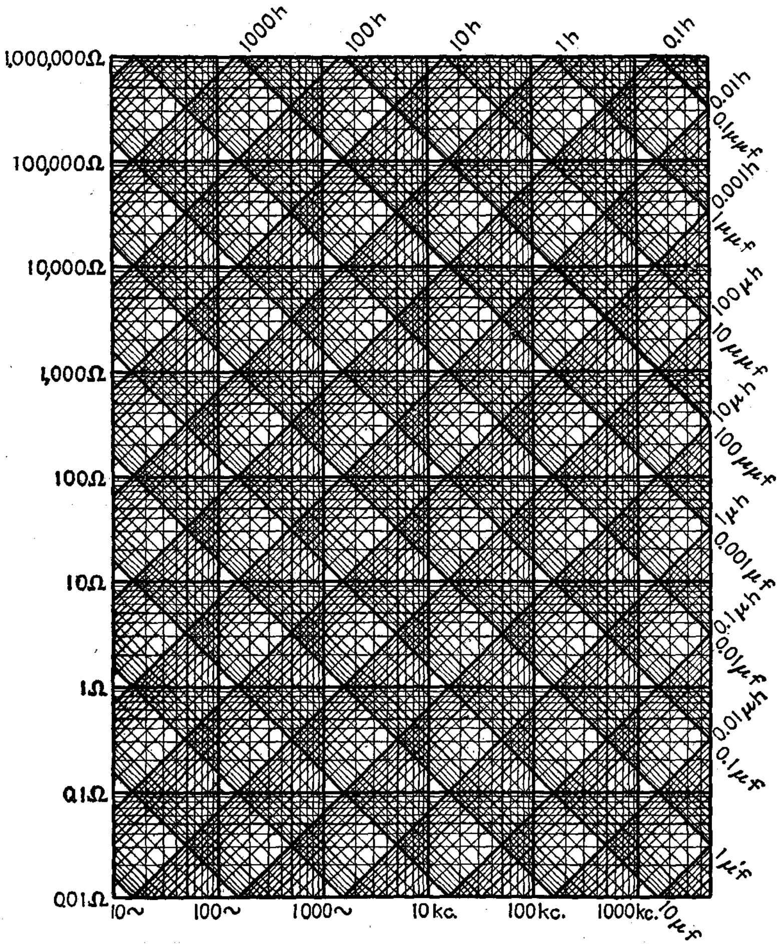


FIG. A-6.—Chart for the determination of inductive and capacitive reactance and of resonance frequency. The reactance at any frequency is determined by the intersection of the vertical line corresponding to the frequency with the diagonal line corresponding to the capacitance or inductance. The intersection of an inductance line with a capacitance line gives the frequency at which the inductance and capacitance are in resonance.

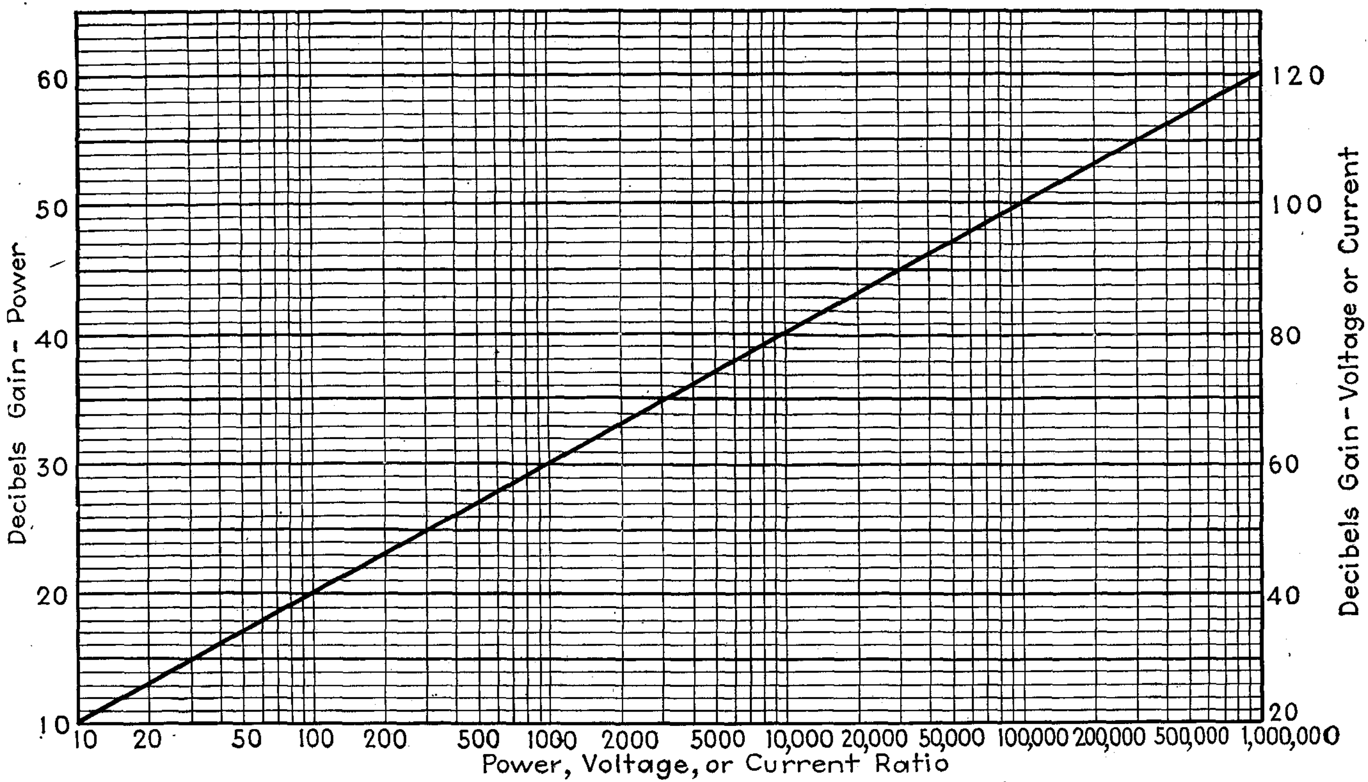


FIG. A-7.—Chart for the determination of decibel gain.

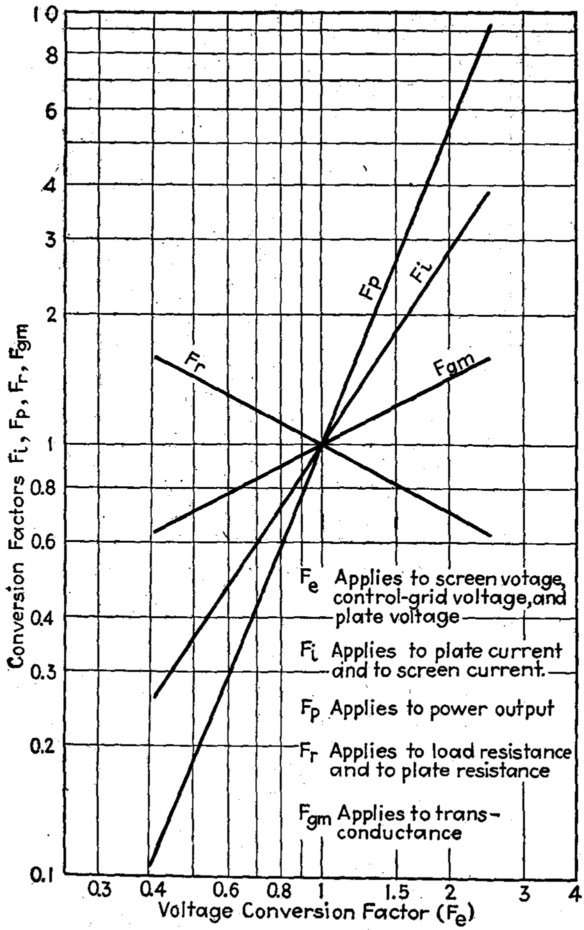


FIG. A-8.—Conversion factors for power amplifier triodes and pentodes.

In using these curves, the ratio of the new plate voltage to the published plate voltage nearest the desired new operating point is first determined. This ratio, the voltage conversion factor F_e, is then used to determine, from the curves, the factors F_i, F_p, F_r, and F_{gm}. F_e is also used to determine the new screen and control-grid voltages. (*Courtesy of RCA Manufacturing Company, Inc.*)

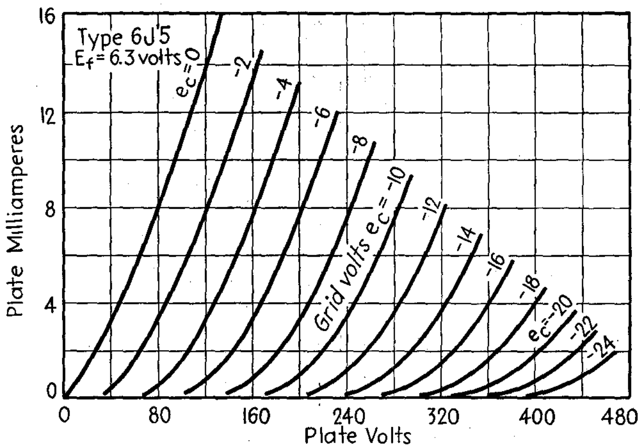


FIG. A-9.—Average plate characteristics for the type 6J5 triode.

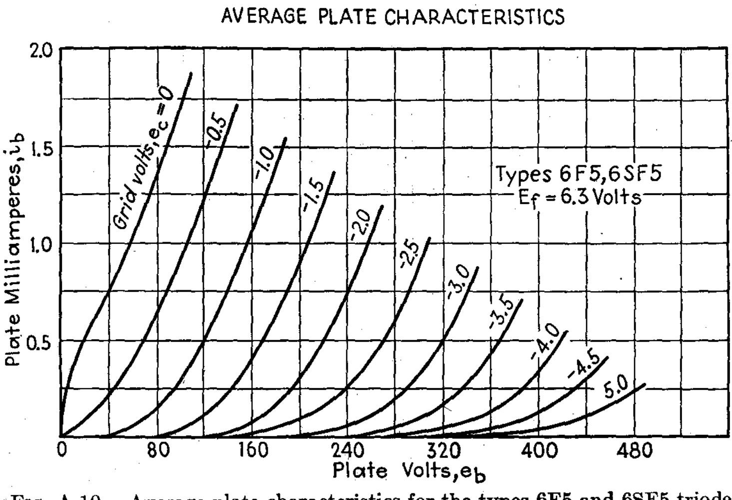


FIG. A-10.—Average plate characteristics for the types 6F5 and 6SF5 triodes.

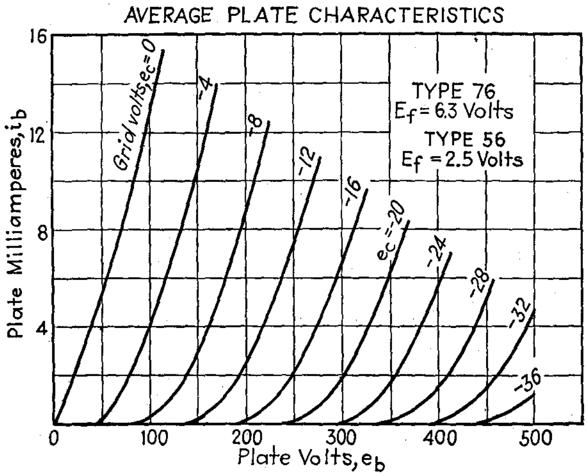


FIG. A-11.—Average plate characteristics for the types 76 and 56 triodes.

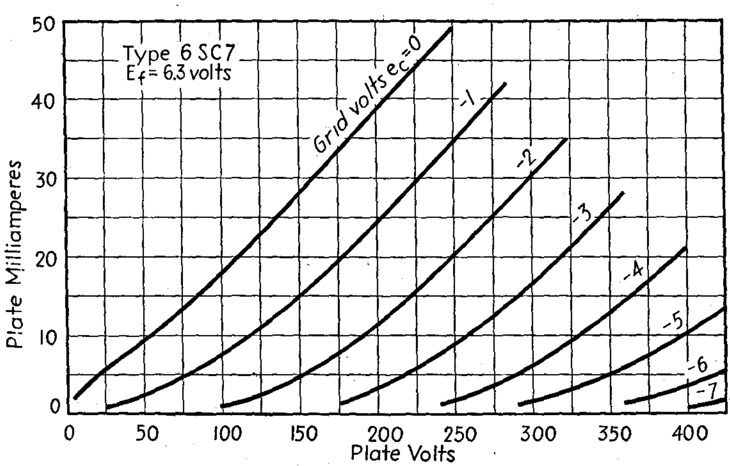


FIG. A-12.—Average plate characteristics for each unit of the type 6SC7 twin triode.

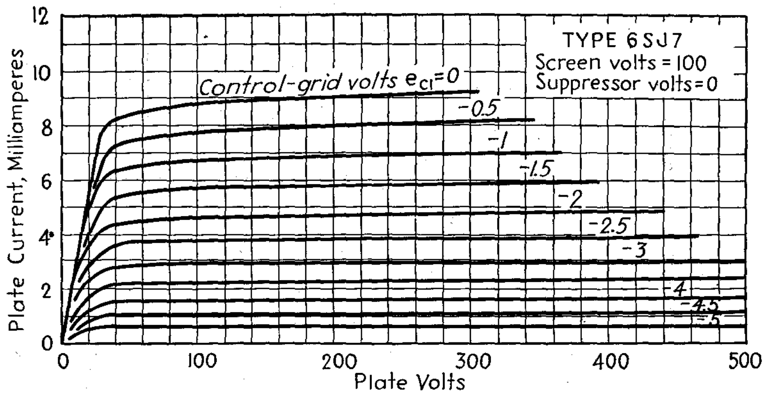


FIG. A-13.—Average plate characteristics for the type 6SJ7 pentode with 100-volt screen voltage.

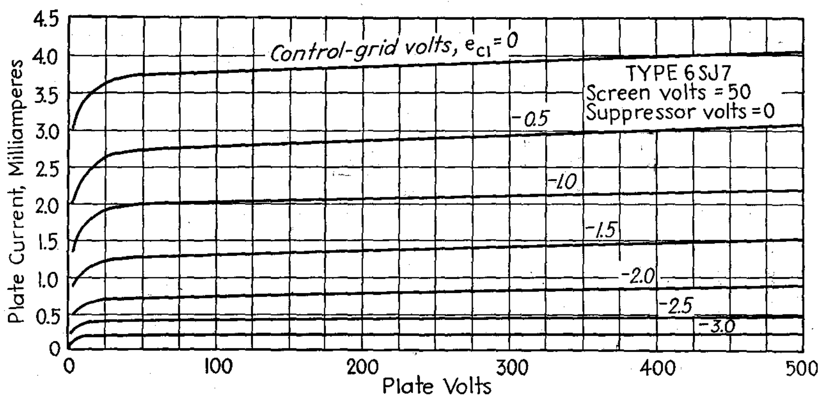


FIG. A-14.—Average plate characteristics for the type 6SJ7 pentode with 50-volt screen voltage.

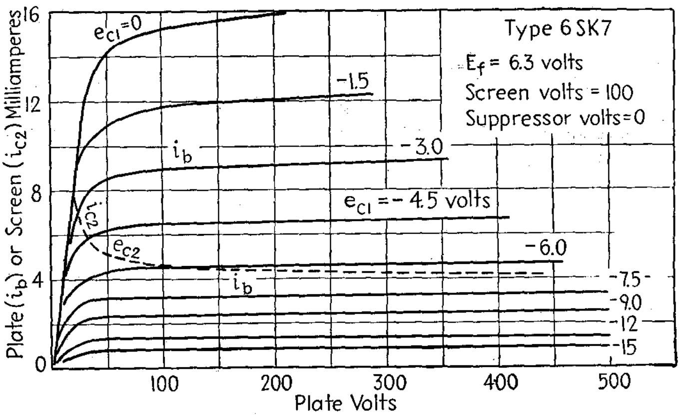


FIG. A-15.—Average plate characteristics for the type 6SK7 pentode.

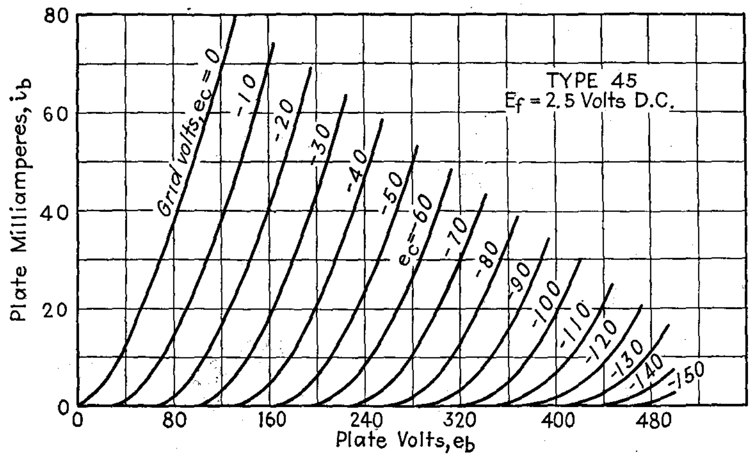


FIG. A-16.—Average plate characteristics for the type 45 triode.

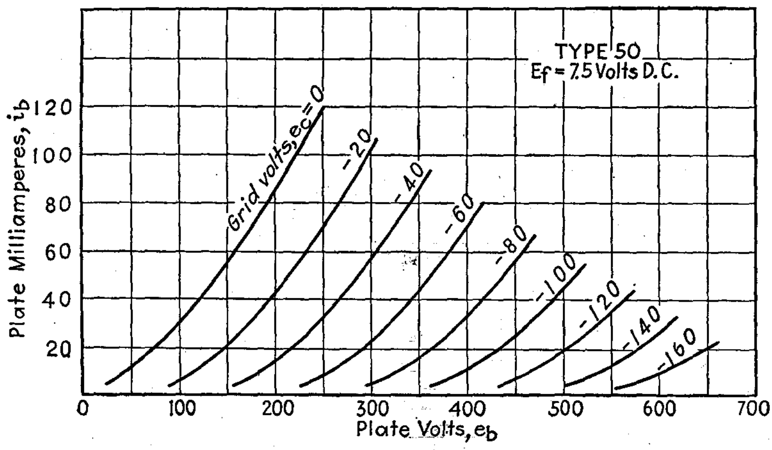


FIG. A-17.—Average plate characteristics for the type 50 triode.

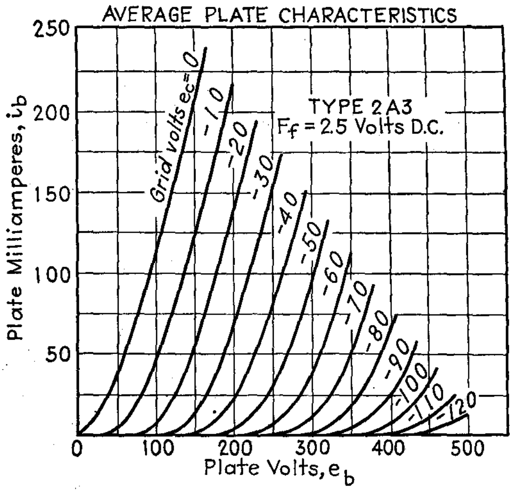


FIG. A-18.—Average plate characteristics for the type 2A3 triode.

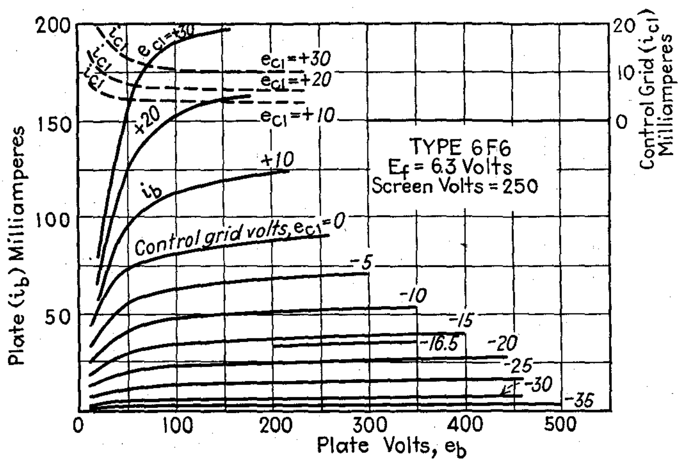


FIG. A-19.—Average plate characteristics for the type 6F6 triple-grid tube, connected as a pentode. See Fig. 3-4 for characteristics for triode connection.

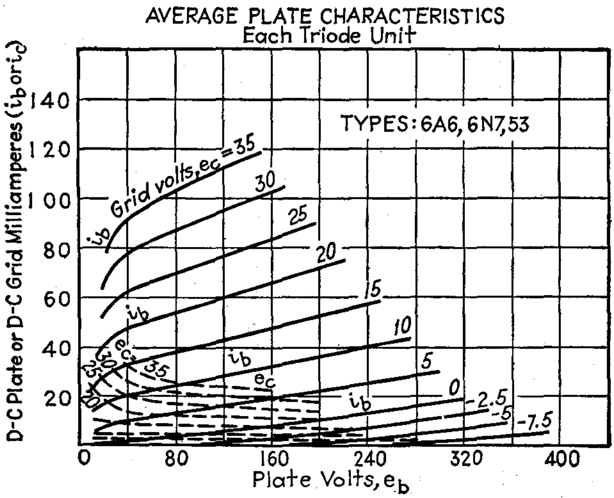


FIG. A-20.—Average plate characteristics for types 6A6, 6N7, and 53 twin triodes.

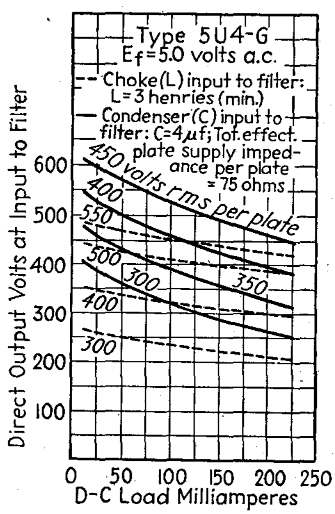


FIG. A-21.—Operation characteristics for the type 5U4-G rectifier.

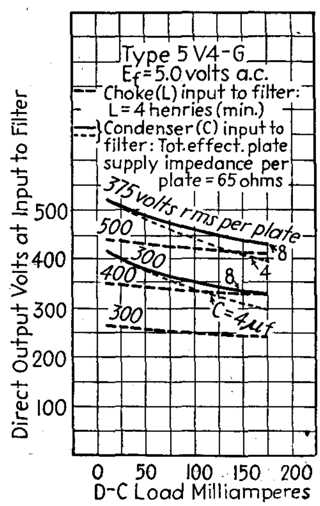


FIG. A-22.—Operation characteristics for the type 5V4-G rectifier.

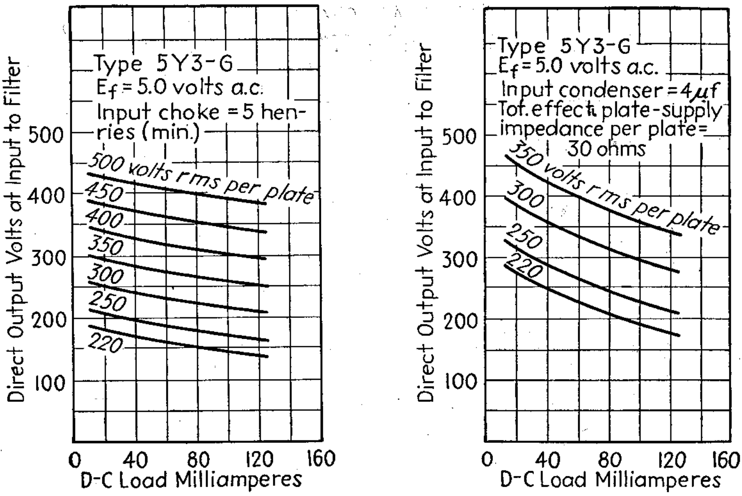


FIG. A-23.—Operation characteristics for the type 5Y3-G rectifier.

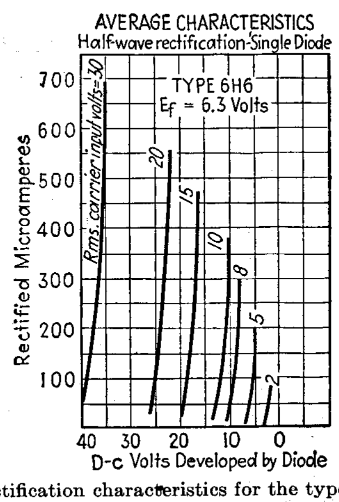


FIG. A-24.—Rectification characteristics for the type 6H6 twin diode.

Operating Data for Typical Amplifier Tubes (Receiving Type)

Type	Classification by construction	Type of cathode	Filament or heater voltage	Filament or heater current, amp	Maximum plate voltage	Maximum screen voltage	Allowable plate dissipation, watts	Allowable screen dissipation, watts	C_{gk},* $\mu\mu f$	C_{gp}, $\mu\mu f$	C_{pk},* $\mu\mu f$	Connection and class of operation	Static operating voltages and currents: Grid voltage	Screen voltage	Screen current, ma	Plate voltage	Plate current, ma per tube	μ	r_p, ohms	g_m, μmhos	Load resistance, ohms†	P_0 watts‡	Total harmonic content, per cent
2A3	Power triode	Filament	2.5	2.5	250	....	....	....	9	13	4	Class A1, single tube	−45	...	...	250	60	4.2	800	5,250	2,500	3.5	5.0
												Class AB1, two tubes	−62	...	...	300	40	....	....	.....	3,000	15.0	2.5
2A5	Triple-grid power amplifier	Heater	2.5	1.75	375	285	11	3.75	...	......	....	Single Class A1 pentode	−20	285	7.0	285	38	200	78,000	2,560	7,000	4.8	9.0
					350	....	10	....	...	......	....	Single Class A1 triode	−20	...	...	250	31	6.8	2,600	2,600	4,000	0.85	6.5
6F6		Heater	6.3	0.7	375	285	11	3.75	...	......	....	Class AB2 pentodes	−26	250	2.5	375	17	....	....	.....	10,000	18.5	3.5
					350	....	10	....	...	......	....	Class AB2 triodes	−38	...	...	350	24	....	....	.....	6,000	13.0	2.0
6J5	Detector, voltage amplifier triode	Heater	6.3	0.3	300	....	2.5	....	3.4	3.4	3.6	Voltage amplifier	− 8	...	...	250	9	20	7,700	2,600			
6L6	Beam power pentode	Heater	6.3	0.9	360	270	19	2.5	...	......	....	Single, Class A1	−14	250	5.0	250	72	135	22,500	6,000	2,500	6.5	10.0
					360	270	19	2.5	...	......	....	Push-pull, Class A1	−16	250	5.0	250	60	....	....	.....	5,000	14.5	2.0
					360	270	19	2.5	...	......	....	Push-pull, Class AB1	−22.5	270	2.5	360	44	....	....	.....	6,600	26.5	2.0
6SC7	Twin high-mu triode	Heater	6.3	0.3	250	....	....	....	3	2.4	4	Voltage Amplifier	− 2	...	...	250	2	70	53,000	1,325			
6SF5	High-mu triode	Heater	6.3	0.3	250	....	....	....	4	2.4	3.6	Voltage Amplifier	− 2	...	...	250	0.9	100	66,000	1,500			
6SJ7	Triple-grid detector amplifier	Heater	6.3	0.3	300	125	2.5	0.3	6	0.005	7	Pentode voltage amplifier	− 3	100	0.8	250	3	Greater than 1,650	Greater than 1,000,000	1,650			
					250	....	2.5	....	3.4	2.8	11	Triode voltage amplifier	− 8.5	...	...	250	9.2	19	7,600	2,500			
6SK7	Triple-grid supercontrol (variable-mu) amplifier	Heater	6.3	0.3	300	125	4.0	0.4	6	0.005	7	Pentode voltage amplifier	− 3	100	2.4	250	9.2	Approximately 1,600	Approximately 800,000	2,000			
45	Power triode	Filament	2.5	1.5	275	....	....	....	4	7	3	Single Class A1	−50	...	...	250	34	3.5	1,610	2,175	3,900	1.6	5.0
												Push-pull Class AB2	−68	...	...	275	14	....	....	.....	3,200	18.0	5.0
50	Power triode	Filament	7.5	1.25	450	....	....	....	4.2	7.1	3.4	Single Class A1	−84	...	...	450	55	3.8	1,800	2,100	4,350	4.6	5.0
													−54	...	...	300	35	3.8	2,000	1,900	4,600	1.6	5.0

* C_{gk} and C_{pk} are total input and output capacitances (including capacitance to suppressor and screen) in the case of pentodes.
† Plate-to-plate load resistance in push-pull operation. ‡ Power output for two tubes in push-pull operation.

Operating Data for Typical Power Thyratrons

Type	No. of electrodes	Cathode: Type	Cathode: Voltage	Cathode: Current, amps	Cathode: Heating time	Tube drop, volts: Max.	Tube drop, volts: Min.	Approx. starting characteristics: Anode voltage	Approx. starting characteristics: Shield-grid voltage	Approx. starting characteristics: Control-grid voltage	Deionization time, μsec	Maximum ratings: Max. peak inverse voltage	Maximum ratings: Max. peak forward voltage	Maximum ratings: Amps max. inst. anode current at 25 cycles and above	Maximum ratings: Amps max. inst. anode current below 25 cycles	Maximum ratings: Average current, amps
FG-17	3	Fil.	2.5	5.0	5 sec	16		50	....	−0	1,000	5,000	2,500	2.0	1.0	0.5
								100	....	−2.25						
								1,000	....	−6.5						
FG-27-A	3	Fil.	5.0	4.5	1 min	16		70	....	0	1,000	1,000	1,000	10	5.0	2.5
								100	....	−2.25						
								1,000	....	−8.0						
FG-33	3	Heater	5.0	4.5	5 min	24	10	100	....	+10	1,000	1,000	1,000	15	5.0	2.5
								1,000	....	+10						
FG-41	3	Heater	5.0	20	5 min	24	10	250	....	0	100	10,000	10,000	75	25	12.5
								1,000	....	−1.5						
								10,000	....	−4.5						
FG-57	3	Heater	5.0	4.5	5 min	16		60	....	0	1,000	1,000	1,000	15	5.0	2.5
								100	....	−1.75						
								1,000	....	−6.5						
FG-67	3	Heater	5.0	4.5	5 min	24	10	100	....	+3.0	100	1,000	1,000	15	5.0	2.5
								1,000	....	−0.5						
FG-81-A	3	Fil.	2.5	5.0	5 sec	24	8	50	....	0	1,000	500	500	2.0	1.0	0.5
								100	....	−3.75						
								500	....	−5.25						
FG-95	4	Heater	5.0	4.5	5 min	26	10	100	0	+3.5	1,000	1,000	1,000	15	5.0	2.5
								1,000	0	−9.0						
FG-97	4	Fil.	2.5	5.0	5 sec	26	10	100	0	+1.0	1,000	1,000	1,000	2.0	1.0	0.5
								1,000	0	−10.0						
FG-98-A	4	Fil.	2.5	5.0	5 sec	26	8	60	0	0	1,000	500	500	2.0	1.0	0.5
								500	0	−10.0						

Operating Data for Typical Power Thyratrons.—(*Continued*)

Type	No. of electrodes	Cathode				Tube drop, volts		Approx. starting characteristic			Deionization time, μsec	Maximum ratings				
		Type	Voltage	Current, amps	Heating time	Max.	Min.	Anode voltage	Shield-grid voltage	Control-grid voltage		Max. peak inverse voltage	Max. peak forward voltage	Amps max. inst. anode current at 25 cycles and above	Amps max. inst. anode current below 25 cycles	Average current, amps
FG-105	4	Heater	5.0	10.0	5 min	20	10	100	0	+1.0	1,000	2,500	2,500	40	12.8	6.4
FG-154	4	Fil.	5.0	7.0	10 sec	30	10	60	0	0	1,000	500	500	10.0	5.0	2.5
								100	0	−3.5						
FG-172	4	Heater	5.0	10.0	5 min	24	6	100	0	+1.0	1,000	1,000	1,000	40	13.0	6.4
								1,000	0	−9.0						
FG-178-A	3	Fil.	2.5	2.25	6 sec	24	8	25	....	0	1,000	500	500	500	250	125
								100	....	−3.0						
								500	....	−7.0						
GL-414	4	Heater	5.0	20.0	10 min	24	10	100	0	−1.0	1,000	2,000	,2000	100	25	12.5
								2,000	0	−14.0						
KU-627	3	Fil.	2.5	6	10 sec	12		100	....	−2	1,000	2,500	1,250	2.5		0.64
								1,000	....	−8						
KU-628	3	Fil.	5	11.5	40 sec	12		100	....	−2	1,000	2,500	1,250	8		2
								1,000	....	−8						
WL-672	4	Heater	5.0	6.0	5 min	12		1,000	+10	−18	1,000	1,500	1,500	30		2.5
								100	+10	−3						
								1,000	0	−10						
								100	0	+3						
								1,000	−5	+12						
								100	−5	+21						
								1,000	−30	+170						
								100	−30	+175						
WL-677	3	Heater	5	9.5	5 min	12		1,000	....	−6	1,000	10,000	10,000	15		4
								500	....	−30						

Operating Data for Typical Ignitrons

	FG-235-A	FG-238-B	FG-258-A	FG-259-B	FG-271	GL-415	GL-427	WL-651/656	WL-652/657	WL-655/658	WL-681/686
Tube drop	8–20	12½–19	8–20	12½–20	8–20	8–20	10–15	8–20	8–20	8–20	8–20
Maximum demand, kva	1,200	2,400	2,400	1,200	600	105	.....	1,200 400	600 200	2,400 800	300 100
Corresponding average anode current, amps	75.6	135	192	75	30.2	3	5	75.6 140	30.2 56	192 355	12.1 22.4
Max. surge peak anode current at 250 v.	......	......	......	......	.....	.....	.....	13,500	6,750	27,000	3,360
Max. surge peak anode current at 500 v.	......	......	......	......	.....	.....	.....	6,750	3,375	13,500	1,680
Ignitor voltage, max. inst. pos. allowed	900	anode volt.	900	anode volt.	900	900	350	900	900	900	900
Ignitor voltage, max. inst. pos. required	200	150	200	150	200	200	150	200	200	200	200
Ignitor voltage, max. inst. neg. allowed	5	5	5	5	5	5	5	5	5	5	5
Ignitor current, max. inst. allowed, amps	100	100	100	100	100	100	100	100	100	100	100
Ignitor current, max. inst. required, amps	40	40	40	40	40	40	30	40	30	40	40
Ignitor current, average allowed	1	2.0	1	2.0	1	1	2	1	1	1	1
Ignition time, μsec	100	100	100	100	100	100	100	100	100	100	100

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G

SYMBOL INDEX

The following index lists the pages on which each symbol, used with a particular connotation, is defined. It does not include most of the special symbols for electrode voltages and currents listed on pages 70 and 71, some symbols used only in the course of derivations or in a single figure, or well-known standard symbols such as π or the vector operator j.

ANSWERS TO PROBLEMS

1-1. (*a*) 4.24×10^{-10} sec. (*b*) 4×10^{-10} erg.

1-2. (*a*) Electron returns to first electrode. It moves to a maximum distance of 0.237 cm from the first electrode. At the instant of field reversal it is 0.118 cm from the first electrode. (*b*) The energy acquired before field reversal is returned to the source of applied potential. That acquired during the return to the first electrode is delivered to the cathode and is converted principally into heat.

1-3. (*a*) The energy lost by the electron in moving to the second electrode against the field is 6.4×10^{-10} erg. As this is less than the initial energy, the electron will reach the second electrode. (*b*) 6.4×10^{-10} erg is delivered to the source of applied potential. The remainder is delivered to the second electrode and is converted mainly into heat.

1-4. 192.7 volts.

1-5. 11.38 gausses.

3-1.[1] $r_p = 11{,}600\ \Omega$; $\mu = 19.5$; $g_m = 1680\ \mu$mhos.

3-2.[1] $g_m = 1950\ \mu$mhos ($2000\ \mu$mhos from slope of transfer characteristic); $r_p = 350{,}000\ \Omega$; $\mu = 680$.

3-3.[1] (*a*) $\mu \cong 8$; $g_m \cong 1700\ \mu$mhos; $r_p = \mu/g_m \cong 4700\ \Omega$.

(*b*) $\mu \cong 15$; $r_p \cong 10{,}000\ \Omega$; $g_m = \mu/r_p \cong 1500\ \mu$mhos.

3-4. 60, 100, 900; 120, 200, 1800; 180, 300, 2700; 160, 960, 1000; 40, 800, 840; 740, 860, 940, 1060; 220, 260, 1020, 1100, 1860, 1900; 20, 140, 700, 780, 1700, 1740.

3-8.[1] (*a*) At $e_b = 250$ *v*. and $E_c = -8v.$, $\mu = 3.5$; $r_p = 1550\ \Omega$; $\partial r_p/\partial e_b = -16$ Ω/volt; $a_1 = 2.5$ ma/volt; $a_2 = 4.07 \times 10^{-2}$ ma/(volt)^2.

(*b*) $a_1 = 1.25$ ma/volt; $a_2 = 5.1 \times 10^{-3}$ ma/(volt)^2.

(*c*) $a_1 = 0.83$ ma/volt; $a_2 = 1.5 \times 10^{-3}$ ma/(volt)^2.

4-7.[1] (*a*) $\mu = 3.5$; $r_p = 1785\ \Omega$; $g_m = 1960\ \mu$mhos.

(*b*) $R_b = 425\ \Omega$.

(*d*) For a 50-volt grid swing, $H_1 = 24.8$ ma and $H_2 = 1.88$ ma. For a 40-volt grid swing, $H_1 = 20$ ma and $H_2 = 1.25$ ma.

(*e*) $P_o = 1.54$ watts and 1.0 watt.

5-2. Voltage gain = 40 db; current gain = 20 db; power gain = 30 db.

6-2. (*c*) $\mu = 19$ and $r_p = 12{,}800\ \Omega$ (determined from the plate diagram[1]); $A_1 =$ 6.42 at 60 cps, 9.15 at 100 cps, and 14.52 at 1000 cps; $A_2 = 15.1$; over-all gain = 39.7 db at 60 cps, 42.8 db at 100 cps, and 46.8 db at 1000 cps.

(*d*) $A_1 = 19.85$ at 60 cps, 27.7 at 100 cps, and 40.3 at 1000 cps; $A_2 = 48.4$; over-all gain = 59.6 db at 60 cps, 62.6 db at 100 cps, and 65.8 db at 1000 cps.

(*f*) $I_{bo} = 2$ ma, $R_{cc} = 3000\ \Omega$.

6-3. Assume that the operating plate voltage is 250 volts and the bias -2 volts (in practice the plate *supply* voltage would probably be 250 volts and the bias about -1.5 volts). Then $\mu = 100$ and $r_p = 66{,}000\ \Omega$.

Second stage: Use $r_o = 500{,}000\ \Omega$. Then $r_h = 58{,}300\ \Omega$. At 10,000 cps, $\omega r_h C_2 =$ 0.0183 and $A = A_m = 87.4$.

[1] For accurate results it is recommended that the student use characteristic curves contained in the RCA Receiving Tube Handbook, or a similar handbook in which the curves are plotted on closely ruled graph paper.

First stage: $C_2 = 222\ \mu\mu f$. If $r_1 = 50{,}000\ \Omega$ and $r_2 = 250{,}000\ \Omega$, $r_h = 25{,}600\ \Omega$; then $A_m = 38.4$, $\omega r_h C_2 = 0.356$ at 10,000 cps; and $A = 0.93A_m = 35.7$. $r_l = 278{,}000\ \Omega$. Make $\omega r_l C_c = 10$ at 100 cps to make $A = A_m$ at this frequency. Then $C_c = 0.057\ \mu f$. Use $C_c = 0.06 \mu f$. If r_1 is decreased to 25,000 Ω and $r_2 = 250{,}000\ \Omega$, $r_h = 16{,}900\ \Omega$; then $A_m = 25.4$, $\omega r_h C_2 = 0.236$ at 10,000 cps, and $A = 0.97A_m = 24.6$. $r_l = 268{,}000\ \Omega$. If $C_c = 0.06\ \mu f$, $\omega r_l C_c = 10$ at 100 cps, and $A = A_m = 25.4$ at this frequency.

6-4. A of second stage is constant at 15.1 below 100,000 cps; A of first stage is 14.23 at 50,000 cps and 13.42 at 100,000 cps.

6-5.[1] Crest value of fundamental component of output voltage $= (210 - 40)/2 = 85$ volts; $A = 85/5.5 = 15.47$; per cent $H_2 = (250 - 264)/(2 \times 170) = 4.12$.

6-6.[1] (*a*) Crest value of fundamental component of output voltage $= (208 - 90)/2 = 59$ volts; $A = 59/0.75 = 78.6$; per cent $H_2 = 1.0$. (*b*) $r_b = r_1r_2/(r_1 + r_2) = 333{,}333\ \Omega$; $R_b = r_1 = 500{,}000\ \Omega$; crest fundamental output voltage $= 53$ volts; $A = 70.6$; per cent $H_2 = 1.7$.

6-7.[1] (*a*) Crest value of fundamental output voltage $= (188 - 38)/2 = 75$ volts; $A = 75$. (*b*) $r_p = 208{,}000\ \Omega$; $\mu = 96$; $A = 79.5$. (*c*) $I_{bc} = 0.125$ ma; $R_{cc} = 12{,}000\ \Omega$. (*d*) $A = 40.5$.

6-9.

Frequency	A	Frequency	A	Frequency	A
20	21	550	60	10,000	102
50	40	1000	60.1	12,600	109
100	52	2000	62.5	16,500	60
200	57.5	5000	65	20,000	36
400	59.7	7000	78		

6-10. $k = 0.034$.

7-3.[1] (*a*) Operating point is determined by allowable plate dissipation. $E_c = -63$ volts. (*b*) Load line intersects the voltage axis at 490 volts. (*c*) Opt. $P_o = 2.48$ watts. (*d*) $r_b = 5780\ \Omega$. (*e*) $r_p = 1500\ \Omega$. (*f*) $I_p = 20.8$ ma; $P_o = 2.5$ watts. (*g*) $P_o = 2.47$ watts. (*h*) $P_i = 9.99$ watts; $\eta_p = 24.8$ per cent. (*i*) Power sensitivity $= 3.13 \times 10^{-4}$ mho. (*j*) $n = 12{:}1$.

7-4.[1] The following values are found by the use of Eq. (7-46): $r_p = 2000\ \Omega$; $I_{bo} = 18$ ma; $E_{bo} = 278$ volts; $E_c \cong -65$ volts; Opt. $P_o = 1.34$ watts; per cent $H_2 = 4.5$; r_b (determined from the dynamic load line) $= 13{,}300\ \Omega$. (Note that the allowable plate dissipation was made low in order that the problem could be solved by the use of plate characteristics normally available.)

7-6.[1] $r_{bb} = 2360\ \Omega$; per cent $H_3 = 2.75$; per cent $H_5 = 0.4$; $P_o = 2.24$ watts; $\eta_p = 44.5$ per cent; $n = 6.89{:}1$.

9-8.[1] (*a*) Fundamental modulation-frequency output is 15.0 volts; second-harmonic output is negligible. (*b*) Fundamental modulation-frequency output is 19 volts; second-harmonic output is negligible.

9-9.[1] (*a*) Fundamental modulation-frequency output is 14.0 volts; second-harmonic output is 0.75 volt (5.35 per cent). (*b*) Fundamental modulation-frequency output is 16.4 volts; second-harmonic output is 1.6 volts (9.75 per cent).

10-3. (*a*) $e_{c3} = -10$ volts. (*b*) $\Delta e_{c2}/\Delta e_{c2} = \frac{1}{2}$. (*c*) $\rho_o = -6900\ \Omega$. (*d*) $C = 1.45\ \mu f$; $L = 1.75h$.

[1] See footnote, p. 713.

6-11.[1] A type 2A3 triode operated at $E_{bo} = 250$ volts and $E_c = -45$ volts will deliver the 50 ma crest current. Crest fundamental alternating current of $(120 - 15)/2 = 52.5$ ma with 5 per cent second harmonic is obtained with a load resistance of 2500 Ω and a grid bias of -43.5 volts (-45 volts relative to mid-point of filament). $E_{gm2} = 43$ volts for 50 ma crest current. Required voltage amplification of first stage $= \frac{43}{1} = 43$. For the first stage use 6SF6 tube with 500,000-Ω plate-coupling resistor. For $E_{bb} = 250$ volts and $E_c = -1.5$ volts, graphically determined $A_1 = 71$. Input voltage to amplifier for full deflection = 0.605 volt. Use direct coupling in first stage and 2500-Ω resistor in series with oscillograph element. The circuit is shown in Fig. 6-55. Circuit of the form of Fig. 5-5 could also be used, but degeneration caused by second-stage plate current in the voltage divider or cathode resistor would reduce the sensitivity.

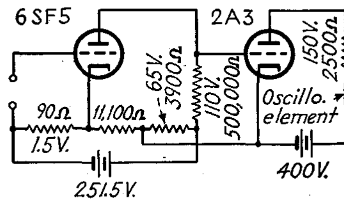

FIG. 6-55.—Solution for Prob. 6-11.

6-12.[1] (*a*) 2A3 triode or 6L6 pentode. (*b*) For 6L6 pentode operated on 200-volt supply with 2000-Ω load (field), plate current is 50 ma at $e_c = -16$ volts and 60 ma at -13.5 volts (determined from dynamic transfer characteristic). (*c* and *d*) Required voltage amplification for first stage = 2.5/0.05 = 50. Use the circuit of Fig. 6-37 without the exciter. Use a 6SF5 tube in the first stage; $E_{ob} = 100$ volts, $E_c = -1.0$ volt, and 200,000-Ω coupling resistor. A of first stage (determined graphically) is 50. With 500,000-Ω coupling resistor, A of first stage is 62.

7-2.[1] (*a*) $\mu = 3.6$ by graphical determination. Opt. $E_c = -0.7 \times 200/3.6 = -39$ volts. Use $E_c = -40$ volts. Zero-signal plate dissipation $= 200 \times 21 \times 10^{-3} = 4.2$ watts, which is within the allowable value. (*b*) Load line for 5 per cent second harmonic intersects the voltage axis at 328 volts, and the current axis at 54 ma. (*c*) $P_o = 0.94$ watt. (*d*) Opt. $r_b = 6100$ Ω. (*e*) $r_p = 1780$ Ω.

From the following graphically determined values it can be seen that some increase in power output can be obtained, at the expense of plate circuit efficiency, by changing the grid bias from -40 volts to -35 volts:

E_c volts	Zero-signal P_p, watts	P_o at 5 per cent H_2, watts	r_b ohms	r_p ohms	r_b/r_p	η_p per cent
−40	4.2	0.93	6100	1790	3.4	22.1
−37.5	5.2	0.97	4850	1600	3.03	18.6
−35	6.3	1.05	3540	1500	2.36	16.7
−32.5	7.5	1.04	2640	1400	1.89	13.9
−30	8.8	1.03	1710	1300	1.32	11.7

(*f*) $I_p = 0.707\mu E_c/(r_p + r_b) = 17.7$ ma for $E_c = -35$ volts; $P_o = I_p^2 r_b = 1.11$ watts. (*g*) $P_o = 1.17$ watts; (*h*) $P_i = 6.3$ watts; plate-circuit efficiency is listed in the foregoing table. (*i*) P. S. $= 4.3 \times 10^{-4}$ mho. (*j*) $n = 9.4$ for $r_b = 3540$ Ω.

13-1. Wrong type of amplifier tube; crest phototube voltage too high for gas phototube; coupling resistance too low; relay should be shunted by a condenser; circuit should include bias adjustment; polarity of phototube is incorrect (coupling resistor should be shunted by a condenser for best results)

[1] See footnote, p. 713.

13-2. (*a*) See Fig. 13-83. (*b*) See Fig. 13-83. (*c*) For 72-ft-candle illumination, phototube flux = 0.5 lumen. Phototube current = 3.5 μa. Voltage drop across coupling resistance = 28 volts. Bias caused by coupling-resistance drop = -28 volts. For E_{bb} = 100 volts and i_b = 7 ma, e_c = -1 volt (determined from transfer characteristic for 1000-Ω load). Required applied biasing voltage = $+27$ volts. (*d*) For E_{bb} = 100 volts and i_b = 8 ma, e_c = -0.5 volt. Δe_c = 0.5 volt. Δi = 0.0625 μa. ΔL = 0.00893 lumen. Change in illumination = 1.285 ft-candles.

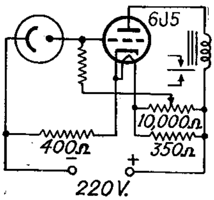

FIG. 13-83.—Solution for Prob. 13-2.

13-3. (*a*) Use the circuit of Fig. 13-11. (*b*) Relay closes at e_c = -1 volt; opens at e_c = -2 volts. (*d*) i = 9.4 μa when L = 0.2 lumen. E_r = $+9.4$ volts. E_{cc} = -11.4 volts. (*e*) To close relay, E_r = 10.4 volts, i = 10.4 μa, L = 0.22 lumen, ΔL = 0.02 lumen. Change in illumination required to close relay = 3.2 ft-candles. A common voltage supply may be used for anode and plate. C supply voltage causes the anode voltage to be one or two volts greater than 90 volts, but this will have negligible effect upon the computations.

10-5. 0.83 by planimeter; 0.85 by selected ordinates.

14-1. A 25Z6 tube may be used. $\hat{r}_p$ = 138 Ω; Assume R_s = 5 Ω; R = 14,000 Ω; $\hat{R}_s/R$ = 0.0102; ωRC = 105; $C \cong 2$ μf; $\bar{R}_s/R$ = 0.0116; E_{dc}/E_m = 1.7; E_{dc} = 288 volts; $\hat{I}_p$ = 160 ma; peak inverse voltage at no load = 338 volts.

14-2. Use full-wave, single-phase rectifier with 5Y3G tube. For two-stage L-section filter, required LC = 64.3. For 20,000-Ω bleeder, $R_{min.}$ = 2600 Ω, and L_1 should exceed 5.2 henrys at full load. L_1 should exceed 20 henrys at minimum load. Use 25-henry choke. If L_1 drops to 12.5 henrys at full load, required condenser capacitance per stage is 5.15 μf. Use 6 μf. Resonant frequency is 18.4 cps at full load. Voltage drop in chokes at full load is approximately 17 volts. Required secondary voltage is 425 volts each side of center. Effective output impedance of the filter at 50 cps is 560 Ω. Terminal voltage rises to about 365 volts at minimum load.

14-3. A 5Y3-GT tube may be used. r_p = 344 Ω; assume R_s = 125 Ω; Assume two chokes will be used having a resistance of 75 Ω each; R = 4150 Ω; Use C_1 = 4 μf; ωRC = 6.25; $\hat{r}_p$ = 320 Ω; $\hat{R}_s/R$ = 0.113; ρ_1 = 0.09; $\bar{R}_s/R$ = 0.124; E_{dc}/E_m = 0.7; E_{rms} = .707(400 + 15)/.7 = 420 volts each side of center. Remainder of solution is similar to that of Prob. 14-2.

14-4. The peak inverse anode voltage will be somewhat greater than 4700 volts. A type 866A/866 mercury-vapor rectifier tube may be used (see transmitting tube manual for tube data). Use a 50,000-Ω bleeder and two chokes, having an estimated resistance of 75 Ω each. α_1 = 472; LC = 40 $\times$ 10^{-6} henrys $\times$ farads; R_t = 5520 Ω at full load; L_1 should exceed 11 henrys at full load; R_t = 18,950 Ω at minimum load; L_1 should exceed 18.9 henrys at minimum load. Use chokes having an inductance of 25 henrys at no load. If the inductance falls to half this value at full load, the required value of C is 3.2 μf. Use 4-μf condensers. Resonance frequency = 22.5 cps at full load. For a tube drop of 15 volts, the required full-load secondary voltage per anode is 1730 volts.

[1] See footnote, p. 713.