Formulaire de physique 1 FSA11

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Part I

Formules

1 Components of vectors

1.1 Vector decomposition

$$A = \sqrt{{A_x}^2 + {A_y}^2}$$

$$\theta = \arctan \frac{A_y}{A_x}$$

1.2 Scalar product

$$\vec{A}.\vec{B} = AB\cos\theta = |A||B|\cos\theta$$

$$\vec{A}.\vec{B} = A_x B_x + A_y B_y + A_z B_z$$

1.3 Vector product

$$C = AB\sin\theta$$

$$\vec{C} = \left(\begin{pmatrix} A_y & A_z \\ B_y & B_z \end{pmatrix}, - \begin{pmatrix} A_b & A_z \\ B_x & B_z \end{pmatrix}, \begin{pmatrix} A_x & A_y \\ B_x & B_y \end{pmatrix}, \right)$$

2 Specifics movements

2.1 Movement along a strait line MRU/MRUA

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

$$a_x = \lim_{\Delta t \to 0} \frac{v_x}{\Delta t} = \frac{dv_x}{dt}$$

$$v_x = v_{0x} + a_x t$$

$$x = x_0 + v_{0x} t + \frac{at^2}{2}$$

2.2 Projectile motion

$$v_x = v_0 \cos\alpha_0 \quad (1)$$

$$v_y = v_0 \sin\alpha_0 - gt \quad (2)$$

$$x = \int v_x dt \overset{(1)}{=} (v_0 \cos\alpha_0) t$$

$$y = \int v_y dt \overset{(2)}{=} (v_0 \sin\alpha_0) t - \frac{1}{2} g t^2$$

2.3 Uniform circular motion (MCU)

$$a_{rad} = \frac{v^2}{R}$$

$$T = \frac{2\pi R}{v}$$

$$a_{rad} = \frac{4\pi^2 R}{T^2}$$

$$a_{tan} = \frac{d|\vec{v}|}{dt}$$

$$F_{net} = ma_{rad} = m\frac{v^2}{R}$$

2.4 Rounding curve

If the curve is flat

$$m\frac{v^2}{R} \leq \mu_s mg$$

So

$$v_{max} = \sqrt{\mu_s g R}$$

If the curve is banked (β is the angle) We use here $n\cos\beta = mg$ instead of a more logical $n = mg\cos\beta$ because that's [1, p. 157] does to simplify the problem. So there is also $n\sin\beta = ma_{rad}$.

$$\begin{aligned} \tan\beta &= \frac{a_{rad}}{g} \\ \tan\beta &= \frac{v^2}{gR} \end{aligned}$$

2.5 Relative velocity

In one dimension:

$$v_{P/A-x} = v_{P/B-x} + v_{B/A-x}$$

In two dimensions:

$$\vec{v}_{P/A} = \vec{v}_{P/B} + \vec{v}_{B/A}$$

2.6 Newtons laws of motion

A body acted on by no net force moves with a constant velocity (may be zero) and a zero acceleration.

$$\sum \vec{F} = 0$$

If a net external force acts on a body, the body accelerates. The direction of acceleration is the same as the direction of the net force. The mass of the body times the acceleration of the body equals the net force vector.

$$\sum \vec{F} = m\vec{a}$$

If body A exerts a force on body B (an "action"), the body B exerts a force on body A (a "reaction"). These two forces have the same magnitude but are opposite in direction. These two forces act on different bodies.

$$\vec{F}_{A \text{ on } B} = -\vec{F}_{B \text{ on } A}$$

3 Fluid resistance and Terminal Speed

Fluid resistance at low speed:

$$f = kv$$

$$v_t = \frac{mg}{k}$$

Fluid resistance at high speed:

$$f = Dv^2$$

$$v_t = \sqrt{\frac{mg}{D}}$$

4 Kinetic and potential energy

$$K = \frac{mv^2}{2}$$

$$U = mgy$$

K and U are defined with a possible constant (like the derivatives)

4.1 Conservative work

When no extern force interact with the body, the work is conservative (there are no energy-loss).

$$E = K_1 + U_1 = K_2 + U_2 = cste$$

4.2 Non-conservative work

If an external force acts on the body (i.e. air drag), the work is non conservative and the previous equation becomes:

$$K_1 + U_1 + W_{ext} = K_2 + U_2$$

And W_{ext} is :

$$W_{ext} = (K_1 - K_2) + (U_1 - U_2) = F(y_1 - y_2)$$

4.3 Elastic potential energy

$$U_{el} = \frac{kx^2}{2}$$

k depends on the spring

5 Momentum, impulse & collisions

$$\vec{p} = m\vec{v}$$

$$\vec{J} = \sum \vec{F}\Delta t$$

The change in momentum of a particule during a time interval equals the impulseof the net force that acts on the particlue during that interval.

$$\vec{J} = \vec{p_2} - \vec{p_1}$$

5.1 Conservation of Momentum

If the vector sum of the external forces on a system is zero, the total momentul of the system is constant.

$$\vec{P} = \vec{p_1} + \vec{p_2} + ...$$

Caution ! With two momentums of different direction, whe must use the vector addition !

6 Gravitation

$$F_g = \frac{Gm_1m_2}{r^2}$$

$$G = 6.67x10^{-11}N.m^2/kg^2$$

The Weight of a body is the total gravitationnal force exerted on this body by all other bodies of the universe.

$$U = -\frac{Gm_Em}{r_e}$$

6.1 Satellite in circular orbit

$$\frac{Gm_Em}{r_E} = \frac{mv^2}{2} \Leftrightarrow v = \sqrt{\frac{Gm_E}{r}}$$

$$T = \frac{2\pi r}{v} = 2\pi r\sqrt{\frac{r}{Gm_E}} = \frac{2\pi r^{3/2}}{\sqrt{Gm_E}}$$

7 Equilibrium

2 conditions have to be respected to have equilibrium. First

$$\sum \vec{F} = \vec{0} \tag{3}$$

For the second condition, choose a arbitrary point O in space (it doesn't depends of your choice if (3) is verified)

$$\sum \vec{OP} \times \vec{F} = \vec{0}$$

where P is the position where the force is applied

References

[1] Young and Freedman, *University Physics*, 13th Edition