# <5장 문제> 3, 7, 9, 14

3. Let $X_1, \ldots, X_n$ be IID and let $\mu = \mathbb{E}(X_1)$. Suppose that the variance is finite. Show that $\overline{X}_n \xrightarrow{\text{qm}} \mu$.

$E[(\overline{X}_n - \mu)^2] = 0$ 인가? $\quad V(\overline{X}_n] = \frac{\sigma^2}{n} = E[\overline{X}_n^2] - E[\overline{X}_n]^2$

$$E[\overline{X}_n^2 - 2\mu \overline{X}_n + \mu^2] = E[\overline{X}_n^2] - 2\mu E[\overline{X}_n] + \mu^2$$
$$= E[\overline{X}_n^2] - \mu^2$$
$$= V[\overline{X}_n]$$
$$= \frac{\sigma^2}{n}$$

$n \to \infty$ 면 0으로 수렴

7. Let $\lambda_n = 1/n$ for $n = 1, 2, \ldots$. Let $X_n \sim \text{Poisson}(\lambda_n)$.

(a) Show that $X_n \xrightarrow{\text{P}} 0$.

(b) Let $Y_n = nX_n$. Show that $Y_n \xrightarrow{\text{P}} 0$.

$X_n \sim \text{poisson}(\lambda_n) \Rightarrow p(X_n) = \dfrac{\lambda_n^x e^{-\lambda_n}}{x!}$

(a) $P(|X_n| > \epsilon) \to 0$ 인가?

$$P(|X_n| > \epsilon) \le \frac{E(|X_n|)}{\epsilon} \times 2 = \frac{2\lambda}{\epsilon} \to 0$$

$\Rightarrow$ 0으로 수렴.

(b) $P(|n \cdot X_n| > \epsilon) \to 0$ 인가?

Markov $\quad 2 \times \dfrac{E(n \cdot |X_n|)}{\epsilon} = \dfrac{2n \times E(|X_n|)}{\epsilon} = \dfrac{2n \times \frac{1}{n}}{\epsilon} = \dfrac{2}{\epsilon}$ 음.. 안되네

Chebyshev

$$P\left(|X_n| > \frac{\epsilon}{n}\right) = P\left(|X_n - \mu| > \frac{\epsilon}{n} - \mu\right) \le \frac{\lambda^2}{\frac{\epsilon - \lambda n}{n}} = \frac{\frac{1}{n^2}}{\frac{\epsilon - \lambda n}{1}} = \frac{1}{n^2(\epsilon - 1)} \to 0$$

9 Suppose that $\mathbb{P}(X=1)=\mathbb{P}(X=-1)=1/2$. Define

$$X_n = \begin{cases} X & \text{with probability } 1-\frac{1}{n} \\ e^n & \text{with probability } \frac{1}{n}. \end{cases}$$

Does $X_n$ converge to $X$ in probability? Does $X_n$ converge to $X$ in distribution? Does $\mathbb{E}(X-X_n)^2$ converge to 0?

$P(X_n = X) = 1-\frac{1}{n} = 1-n^{-1}$

$P(X_n = e^n) = \frac{1}{n} = n^{-1}$

① $X_n$ Converge to $X$ in probability? $\mathbb{P}(|X_n - X| > \epsilon) \to 0$ as $n \to \infty$

$$|X_n - X| \begin{cases} 0 & \text{with probability } 1-\frac{1}{n} \\ e^n - X & \text{with probability } \frac{1}{n} \end{cases} \Rightarrow \underset{n\to\infty}{\begin{matrix} 1 \\ 0 \end{matrix}}$$

$P((|X_n - X|) > \epsilon) \Rightarrow P(0 > \epsilon) = 0$

② $X_n$ Converge to $X$ in distribution? $\lim_{n\to\infty} F_n(t) = F(t)$

# Delta Method

14. Let $X_1, \ldots, X_n \sim \text{Uniform}(0,1)$. Let $Y_n = \overline{X}_n^2$. Find the limiting distribution of $Y_n$.