<3장 문제> 4 20 21 23

4. A particle starts at the origin of the real line and moves along the line in jumps of one unit. For each jump the probability is p that the particle will jump one unit to the left and the probability is $1-p$ that the particle will jump one unit to the right. Let X_n be the position of the particle after n units. Find $\mathbb{E}(X_n)$ and $\mathbb{V}(X_n)$. (This is known as a **random walk.**)

$$\begin{cases} p : -1 & x \text{번} \\ 1-p : +1 & n-x\text{번} \end{cases} \qquad -p+1-p = 1-2p$$

$$E[X_n] = E[\sum_{i=1}^{n} X_i] = \sum_{i=1}^{n} E[X_i] = \sum_{i=1}^{n} (1-2p) = n\times(1-2p)$$

$$V[X_n] = E[X_n^2] - E[X_n]^2 = \sum_{i=1}^{n} E[X_i^2] - n^2(1-4p+4p^2)$$
$$= n^2 - n^2 + 4pn^2 - 4p^2n^2$$
$$= 4pn^2 - 4p^2n^2$$

20. Prove Lemma 3.21.

3.21 Lemma. *If a is a vector and X is a random vector with mean μ and variance Σ, then $\mathbb{E}(a^TX) = a^T\mu$ and $\mathbb{V}(a^TX) = a^T\Sigma a$. / If A is a matrix then $\mathbb{E}(AX) = A\mu$ and $\mathbb{V}(AX) = A\Sigma A^T$.*

Vector 일때 와 Matrix일때는 어떻게 되는가!

i) $E[X] = \mu$, $V[X] = \Sigma$ 이고 a 가 Vector라면 $E[a^TX] = a^T\mu$, $V[a^TX] = a^T\Sigma a$ 인가?

$$X = [X_1 \ X_2 \ \cdots \ X_N] \qquad a = [a_1 \ a_2 \ \cdots \ a_N]$$

$$E[X] = E[X_1 X_2 \cdots X_N] = [E[X_1] \ E[X_2] \ \cdots \ E[X_N]]$$

$$E[a^TX] = E\left(\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{bmatrix} [X_1 \ X_2 \ \cdots \ X_N]\right)$$

$$V[X] = V[X_1 \ X_2 \ \cdots \ X_N] = [V[X_1] \ V[X_2] \ \cdots \ V[X_N]]$$

다음장에 계속..

$$E[a^TX] = E\left(\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{bmatrix} [X_1 \ X_2 \ \cdots \ X_N]\right) = E\begin{bmatrix} a_1X_1 & a_1X_2 & \cdots & a_1X_N \\ a_2X_1 & a_2X_2 & \cdots & a_2X_N \\ \vdots & \vdots & & \vdots \\ a_NX_1 & a_NX_2 & & a_NX_N \end{bmatrix}$$

$$= \begin{bmatrix} a_1E[X_1] & a_1E(X_2] & \cdots & a_1E[X_N] \\ a_2E[X_1] & a_2E(X_2] & \cdots & a_2E[X_N] \\ & & & \\ a_NE[X_1] & a_NE(X_2] & \cdots & a_NE[X_N] \end{bmatrix} = a^T\mu$$

ii) $V[a^TX] = E[(a^TX - a^T\mu)(a^TX - a^T\mu)^T]$

$= E[a^T(X-\mu)(X-\mu)^Ta]$

$= a^T E[(X-\mu)(X-\mu)^T] a$

$= a^T V[X] a$

$= a^T \Sigma a$

21 Let X and Y be random variables. Suppose that $\mathbb{E}(Y|X) = X$. Show that $\mathsf{Cov}(X,Y) = \mathbb{V}(X)$.

$\mathsf{Cov}(X,Y) = \mathbb{E}\Big((X-\mu_X)(Y-\mu_Y)\Big)$

$= E[XY] - E[X]E[Y]$

$V[X] = E[(X-\mu_x)^2]$

$= E[X^2] - E[X]^2$

원래 $E_X[E_Y[Y|X]] = E[Y]$인데 이 값이 $E[X]$ 니까 $E[X] = E[Y]$

$\Rightarrow Cov(X,Y) = E[X^2] - E[X]^2$

$\updownarrow$

$E[XY]$ 인가?

다시 해보기!

23. Find the moment generating function for the 1. Poisson, 2. Normal, and 3. Gamma distributions.

3.29 Definition. *The* **moment generating function** MGF, *or* **Laplace transform**, *of X is defined by*

프사이

$$\psi_X(t) = \mathbb{E}(e^{tX}) = \int e^{tx}\, dF(x)$$

오.. 신기

where t varies over the real numbers.

1) poisson

$X \sim \text{poisson}(\lambda)$

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

$$E[e^{tx}] = \sum_{t=0}^{\infty} e^{tx} \cdot \frac{\lambda^x e^{-\lambda}}{x!} = e^{-\lambda} \sum_{t=0}^{\infty} \frac{e^{tx}\lambda^x}{x!}$$

$$= e^{-\lambda} \sum_{t=0}^{\infty} \frac{(\lambda e^t)^x}{x!}$$

$$= e^{-\lambda} e^{\lambda e^t}$$

$$= e^{\lambda(e^t - 1)}$$

2) 나머지는 나중에..