< 통계 1장 문제 > 11, 16, 17, 19

11. Suppose that A and B are independent events. Show that A^c and B^c are independent events.

$P(A,B) = P(A)P(B)$ 일때

$$A^c = \{\omega \in \Omega : \ \omega \notin A\}$$

$P(A^c, B^c) = P(A^c)P(B^c)$ 인가?

좌변

$P(A^c \cap B^c)$ 이렇게하면되나?

$= 1 - P(A \cup B)$

$= 1 - (P(A) + P(B) - P(A \cap B)$

우변

$P(A^c)P(B^c)$

$= (1 - P(A))(1 - P(B))$

$= 1 - P(A) - P(B) + P(A)P(B)$

$= 1 - P(A) - P(B) + P(A,B)$

16. Prove Lemma 1.14.

$$\mathbb{P}(A|B) = \frac{\mathbb{P}(AB)}{\mathbb{P}(B)}.$$

1.14 Lemma. *If A and B are independent events then $\mathbb{P}(A|B) = \mathbb{P}(A)$. Also, for any pair of events A and B,*

$$\mathbb{P}(AB) = \mathbb{P}(A|B)\mathbb{P}(B) = \mathbb{P}(B|A)\mathbb{P}(A).$$

$P(A|B) = P(AB)/P(B) = P(A)P(\cancel{B})/P(\cancel{B}) = P(A)$

$P(B|A) = P(B)$

$P(AB) = P(A)P(B) = P(A|B)P(B)$

$= P(B|A)P(A)$

17. $\mathbb{P}(ABC) = \mathbb{P}(A|BC)\mathbb{P}(B|C)\mathbb{P}(C).$

$$= \frac{P(A,B,C)}{P(\cancel{B,C})} \times \frac{P(\cancel{B,C})}{P(\cancel{C})} \times P(\cancel{C})$$

$$= P(A,B,C)$$

19. Suppose that 30 percent of computer owners use a Macintosh, 50 percent use Windows, and 20 percent use Linux. Suppose that 65 percent of the Mac users have succumbed to a computer virus, 82 percent of the Windows users get the virus, and 50 percent of the Linux users get the virus. We select a person at random and learn that her system was infected with the virus. What is the probability that she is a Windows user?

$P(X = Mac) = 0.3$

$P(X = Win) = 0.5$

$P(X = Lin) = 0.2$

$P(Virus \mid Mac) = 0.65$

$P(virus \mid Win) = 0.82 = \frac{P(virus, win)}{P(win)}$

$P(virus \mid Lin) = 0.5$

$$P(win \mid virus) = \frac{P(win, virus)}{P(virus)} = \frac{0.41}{0.3 \times 0.65 + 0.5 \times 0.82 + 0.2 \times 0.5}$$

$$= \frac{0.41}{0.195 + 0.41 + 0.1}$$

$$= \frac{0.410}{0.705} = 0.58156028$$

$$\doteq 0.58$$

$P(virus, win)$

$= 0.82 \times 0.5 = 0.41$