HACS—A Formalism for Compiler Development

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DIKU “COPLAS”

Outline

Compilers and translators

- Programming Culture

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- Programming Culture bordering on Art.

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- We teach semantic techniques

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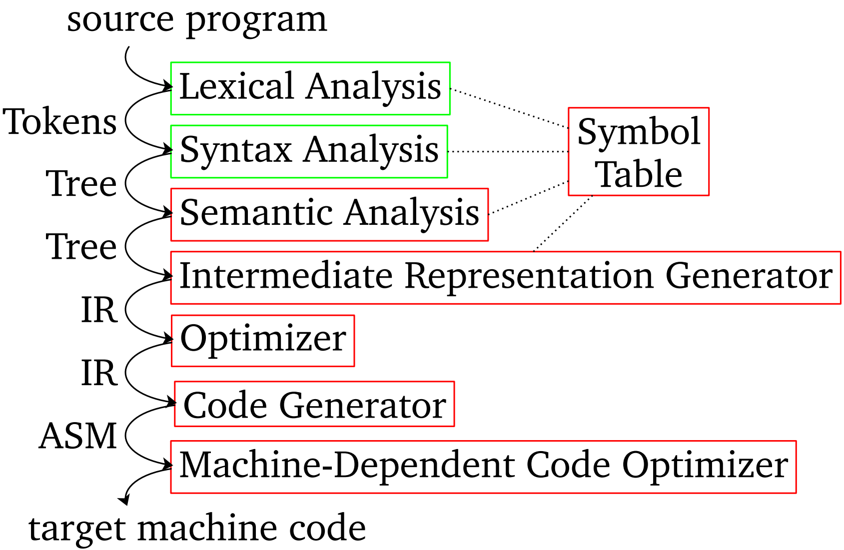
source program
Lexical Analysis
Tokens
Syntax Analysis
Tree
Semantic Analysis
Tree
Intermediate Representation Generator
IR
Optimizer
IR
Code Generator
ASM
Machine-Dependent Code Optimizer
target machine code
Symbol Table

What formalizations are we using?

Lexical Analysis. Regular Expressions.

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Symbol Table. Side effects and explicit scope structures.

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HACS—A New *Formally Founded* Compiler Generator

1. Designed to directly support existing formal notations.
2. Formally defined in terms of a “core” language, which is in fact CRSX.
3. Integrated with special purpose compiler support algorithms.

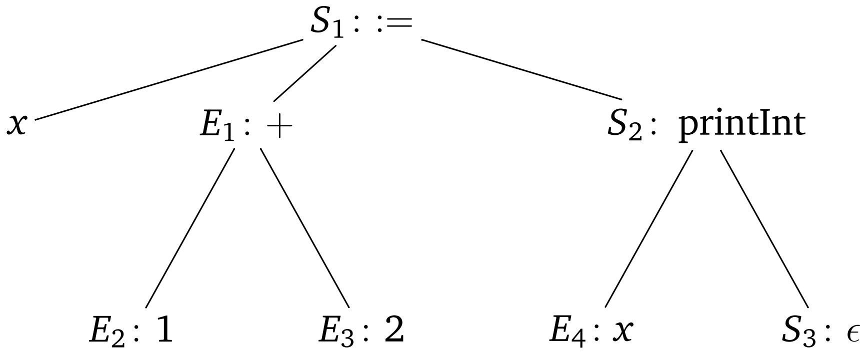
On Attribute Grammars

- Based on Knuth's seminal 1968 "Semantics of Context-Free Languages"
- Dominate teaching in compiler construction
- Can be adapted to imperative programming ($ in yacc)
- Compatible with rewriting, can be implemented with rewriting (ASF+SDF)

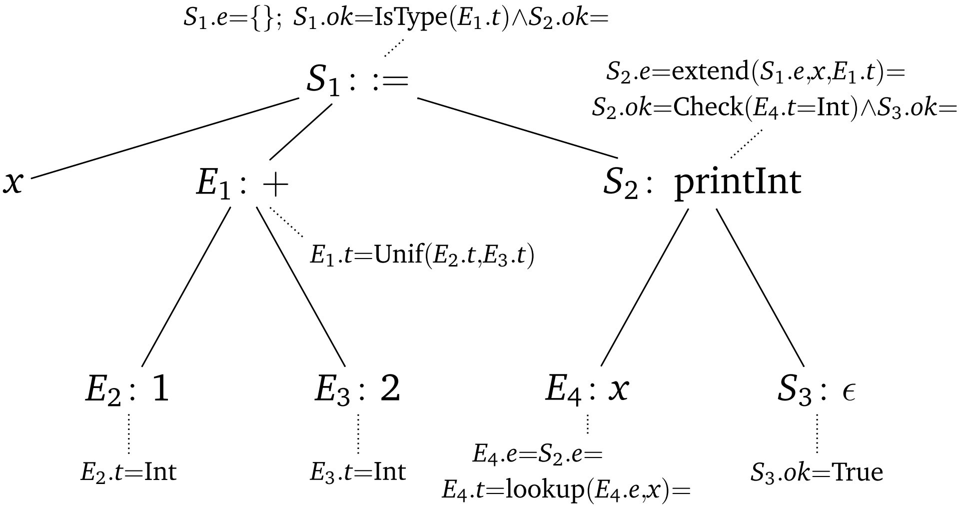
Attribute Grammar = Syntax-Directed Definition

PRODUCTION	SEMANTIC RULES	
$S \rightarrow \textbf{name} := E_1; S_2$	$E_1.e = S.e; S_2.e = \text{extend}(S.e, \textbf{name}.sym, E_1.t);$ $S.ok = \text{IsType}(E_1.t) \wedge S_2.ok$	$(S1)$
$\mid \{\ S_1\ \}\ S_2$	$S_1.e = S.e; S_2.e = S.e; S.ok = S_1.ok \wedge S_2.ok$	$(S2)$
$\mid \text{printInt}\ \ E_1; S_2$	$E_1.e = S.e; S.ok = \text{Check}(E_1.t = \text{Int}) \wedge S_2.ok$	$(S3)$
$\mid \text{printFloat}\ \ E_1; S_2$	$E_1.e = S.e; S.ok = \text{Check}(E_1.t = \text{Float}) \wedge S_2.ok$	$(S4)$
$\mid \epsilon$	$S.ok = \text{True}$	$(S5)$
$E \rightarrow E_1 + E_2$	$E_1.e = E.e; E_2.e = E.e; E.t = \text{Unif}(E_1.t, E_2.t)$	$(E1)$
$\mid E_1 * E_2$	$E_1.e = E.e; E_2.e = E.e; E.t = \text{Unif}(E_1.t, E_2.t)$	$(E2)$
$\mid \textbf{int}$	$E.t = \text{Int}$	$(E3)$
$\mid \textbf{float}$	$E.t = \text{Float}$	$(E4)$
$\mid \textbf{name}$	$E.t = \text{if defined}(E.e, \textbf{name}.sym)$ $\text{then lookup}(E.e, \textbf{name}.sym)$ else TypeErr	$(E5)$

Example: “`x:=1+2; printInt x`”



Example: "`x:=1+2; printInt x`" with Attributes



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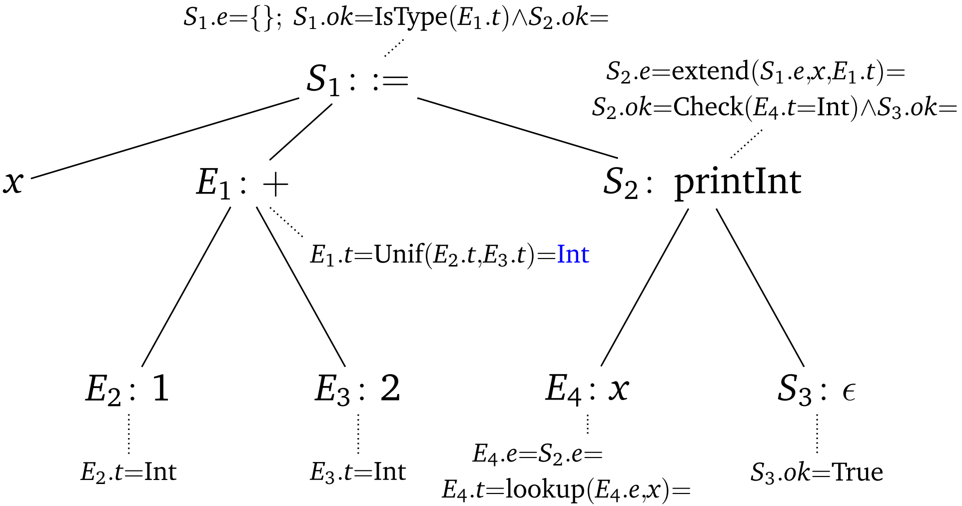

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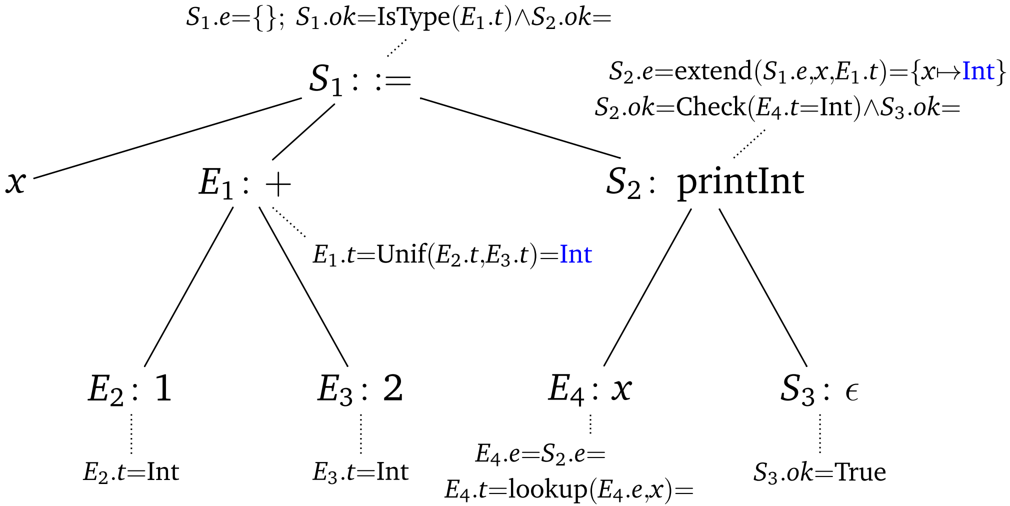

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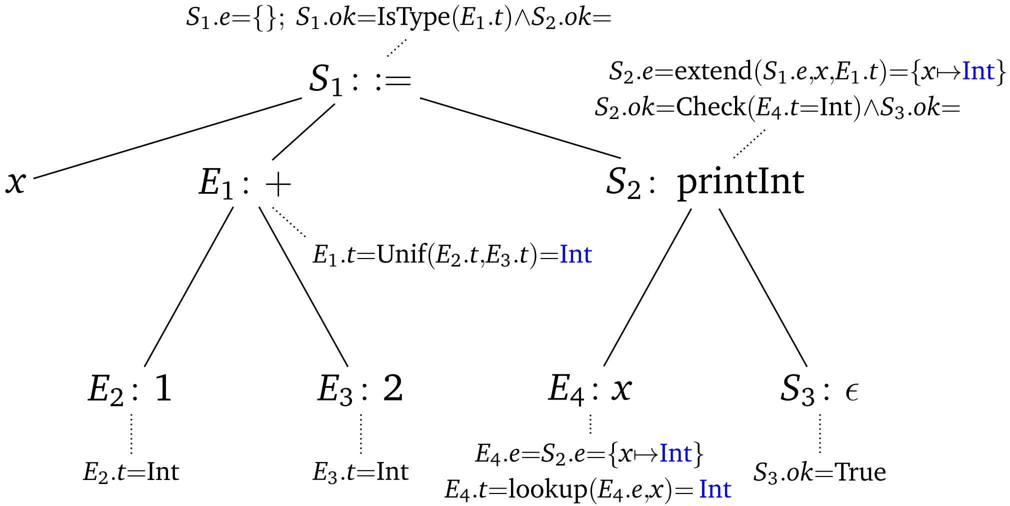

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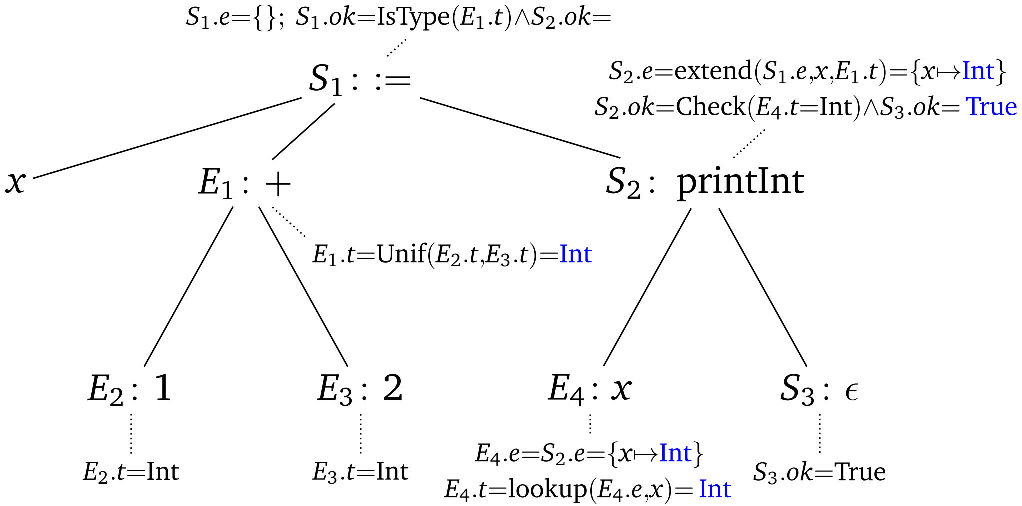

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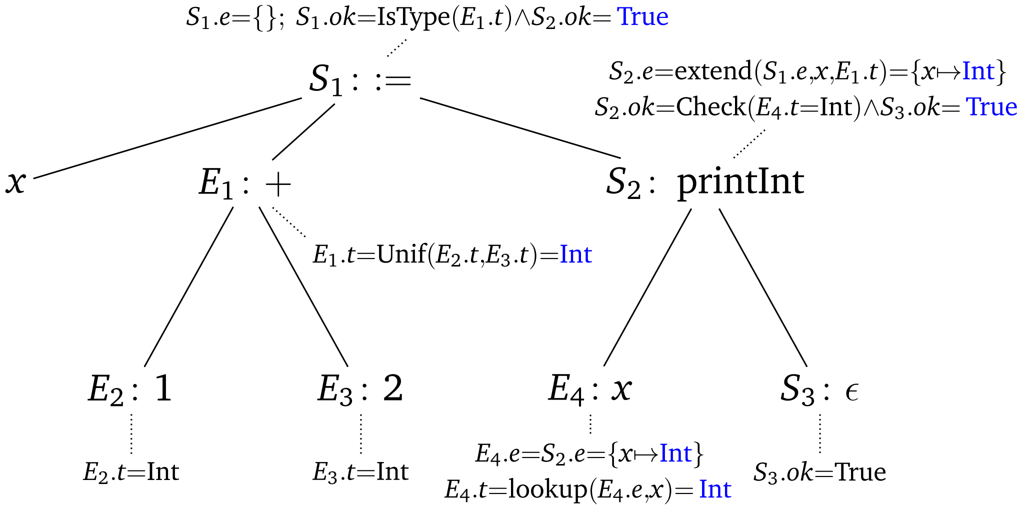

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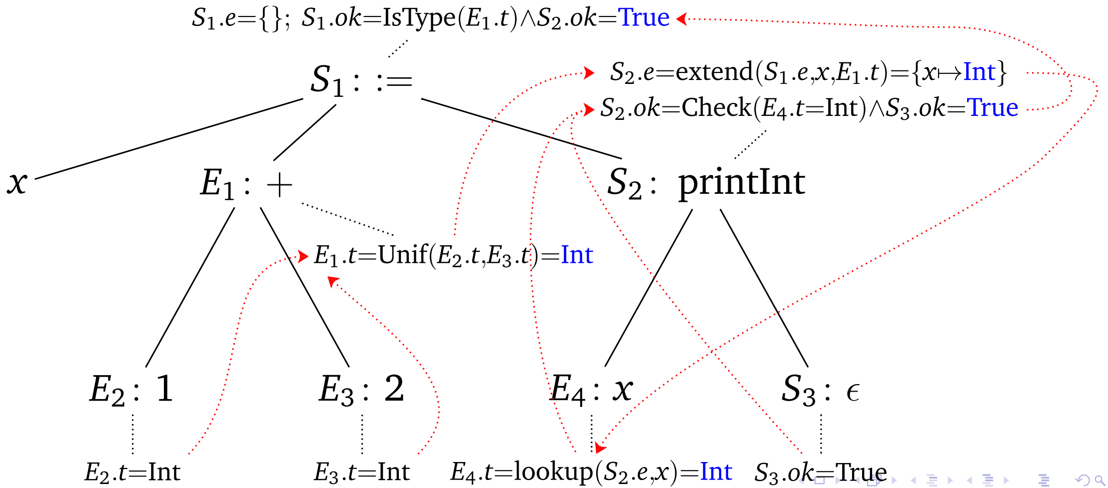

Attribute Propagation Patterns

- Synthesized attributes mostly depend only on synthesis. (An AG is "S-attributed" if this is exclusively the case.)
- Cross-synthesized-inherited attributes depend left to right. (It is "L-attributed," like inference systems.)

HACS Features

- Parser generation notation.
  Supports Higher-Order Abstract Syntax.
- Rules for synthesis.
- Rules for recursive compilation schemes.
- Polymorphic sort system.
- All rules and sorts use native user syntax.

Words, words, words. . .

Let’s code.

HACS Program (I)

```
module "edu.nyu.cims.cc.Check" {

// PARSER.

space [ \t\n] ;
token INT    | [0−9]+ ;
token FLOAT  | [0−9]+ [.] [0−9]∗ ([Ee] [−+]? [0−9]+)? ;
token ID     | [a−z] [a−zA−Z0−9_]∗ ;
sort Name | symbol ⟦⟨ID⟩⟧ ;

sort S  | ⟦ ⟨[x:Name]⟩ := ⟨E⟩; ⟨S[x:Name]⟩ ⟧ | ⟦ { ⟨S⟩ } ⟨S⟩ ⟧
        | ⟦ printInt ⟨E⟩; ⟨S⟩ ⟧ | ⟦ printFloat ⟨E⟩; ⟨S⟩ ⟧
        | ⟦⟧ ;

sort E  | ⟦ ⟨E⟩ + ⟨E@1⟩ ⟧| ⟦ ⟨E@1⟩ ∗ ⟨E@2⟩ ⟧@1
        | ⟦ ⟨INT⟩ ⟧@2 | ⟦ ⟨FLOAT⟩ ⟧@2 | ⟦⟨Name⟩ ⟧@2
        | sugar ⟦ ( ⟨E#1⟩ ) ⟧@2 →E#1 ;
```

HACS Program (II)

// SEMANTIC SORTS & HELPERS.

```
sort Bool | ⟦T⟧ | ⟦F⟧;
| scheme ⟦⟨Bool⟩∧⟨Bool⟩⟧;  ⟦T∧T⟧ → ⟦T⟧;  default ⟦⟨Bool#1⟩∧⟨Bool#2⟩⟧ →⟦F⟧;

sort Type | Int | Float | TypeErr;
| scheme Unif(Type,Type);
Unif(Int, Int) → Int;          Unif(Int, Float) → Float;
Unif(Float, Int) → Float;   Unif(Float, Float) → Float;
default Unif(#1, #2) →TypeErr ;

sort Bool;
| scheme IsType(Type); IsType(Int) →⟦T⟧;  IsType(Float) → ⟦T⟧;  IsType(TypeErr) →⟦F⟧;
| scheme CheckInt(Type); CheckInt(Int) →⟦T⟧;  default CheckInt(#) →⟦F⟧;
| scheme CheckFloat(Type); CheckFloat(Float) →⟦T⟧; default CheckFloat(#) →⟦F⟧;
```

HACS Program (III)

```
// TYPE CHECKING.

sort CheckResult | ⟦Yes⟧;
| scheme Check(S) ;
Check(#s↑ok(⟦T⟧)) →⟦Yes⟧ ;
Check(#s↑ok(⟦F⟧)) →error⟦Type Error.⟧ ;

// Attributes.
attribute ↑ok(Bool);
attribute ↑t(Type);
attribute ↓e{Name:Type};

// Rules for S sort:
// – Synthesizes attribute ok.
// – Inherited attribute S.e is distributed by Te scheme (and helper Te2).
sort S | ↑ok | scheme ⟦Te ⟨S⟩⟧ ↓e | scheme ⟦Te2 ⟨S⟩⟧ ↓e ;
```

HACS Program (IV)

// (S1) $S \to \textbf{name} := E_1; S_2$ has three stages:
// 1. $E_1.e = S.e$ and recurse over E_1 (to get automatic propagation).
$[\![$ Te x := $\langle$E#1$\rangle$; $\langle$S#2[x]$\rangle$ $]\!]$ $\to$ $[\![$ Te2 x := Te$\langle$E#1$\rangle$; $\langle$S#2[x]$\rangle$ $]\!]$;
// 2. When $E_1.t$ is available then $S_2.e = \text{extend}(S.e, \textbf{name}.sym, E_1.t)$ and recurse over S_2.
$[\![$ Te2 x := $\langle$E#1$\uparrow$t(#t1)$\rangle$; $\langle$S#2[x]$\rangle$ $]\!]$ $\to$ $[\![$ x := $\langle$E#1$\rangle$; Te$\langle$S#2[x]$\downarrow$e{x:#t1}$\rangle$ $]\!]$;
// 3. When $S_2.ok$ is (also) available then we can synthesize $S.ok = \text{IsType}(E_1.t) \wedge S_2.ok$.
$[\![$ x := $\langle$E#1$\uparrow$t(#t1)$\rangle$; $\langle$S#2[x]$\uparrow$ok(#ok2)$\rangle$ $]\!]$$\uparrow$ok($[\![$$\langle$Bool IsType(#t1)$\rangle$$\wedge$$\langle$Bool#ok2$\rangle$$]\!]$) ;

// (S2) $S \to \{S_1\}\, S_2$:
// 1. $S_1.e = S.e$ and $S_2.e = S.e$ are propagated and recursed over.
$[\![$ Te { $\langle$S#1$\rangle$ } $\langle$S#2$\rangle$ $]\!]$ $\to$ $[\![$ { Te$\langle$S#1$\rangle$ } Te$\langle$S#2$\rangle$ $]\!]$;
// 2. When $S_1.ok$ and $S_2.ok$ are available then synthesize $S.ok = S_1.ok \wedge S_2.ok$.
$[\![$ { $\langle$S#1$\uparrow$ok(#ok1)$\rangle$ } $\langle$S#2$\uparrow$ok(#ok2)$\rangle$ $]\!]$$\uparrow$ok($[\![$$\langle$Bool#ok1$\rangle$ $\wedge$$\langle$Bool#ok2$\rangle$$]\!]$) ;

HACS Program (V)

// $(S3) \quad S \to \text{printInt } E_1; S_2$:
// 1. $E_1.e = S.e$ and $S_2.e = S.e$ are propagated and recursed over.
$[\![$ Te printInt $\langle$E#1$\rangle$; $\langle$S#2$\rangle$ $]\!] \to$ $[\![$ printInt Te$\langle$E#1$\rangle$; Te$\langle$S#2$\rangle$ $]\!]$;
// 2. When $S_1.t$ and $S_2.ok$ are available then synthesize $S.ok = \text{Check}(\text{Int} = E_1.t) \wedge S_2.ok$.
$[\![$ printInt $\langle$E#1↑t(#t1)$\rangle$; $\langle$S#2↑ok(#ok2)$\rangle$ $]\!]$↑ok($[\![$ $\langle$Bool CheckInt(#t1)$\rangle$ $\wedge\langle$Bool#ok2$\rangle$ $]\!]$) ;

// $(S4) \quad S \to \text{printFloat } E_1; S_2$:
// 1. $E_1.e = S.e$ and $S_2.e = S.e$ are propagated and recursed over.
$[\![$ Te printFloat $\langle$E#1$\rangle$; $\langle$S#2$\rangle$ $]\!] \to$ $[\![$ printFloat Te$\langle$E#1$\rangle$; Te$\langle$S#2$\rangle$ $]\!]$;
// 2. When $S_1.t$ and $S_2.ok$ are available then synthesize $S.ok = \text{Check}(\text{Float} = E_1.t) \wedge S_2.ok$.
$[\![$ printFloat $\langle$E#1↑t(#t1)$\rangle$; $\langle$S#2↑ok(#ok2)$\rangle$ $]\!]$↑ok($[\![$ $\langle$Bool CheckFloat(#t1)$\rangle$ $\wedge\langle$Bool#ok2$\rangle$ $]\!]$) ;

// $(S5) \quad S \to \epsilon$: No distribution necessary; synthesize $S.ok = \text{True}$.
$[\![$ Te $]\!]$ $\to$ $[\![]\!]$;
$[\![$ $]\!]$ ↑ok($[\![$T$]\!]$) ;

HACS Program (VI)

// Rules for E sort:
// – Synthesizes attribute $E.t$.
// – Inherited attribute $E.e$ is distributed by Te scheme (with helper Te2).
sort $\mathtt{E} \mid \uparrow\mathtt{t} \mid$ **scheme** $[\![\mathtt{Te}\ \langle\mathtt{E}\rangle]\!]\ \downarrow\mathtt{e}$;

// $(E1)\quad E \to E_1 + E_2$:
// 1. $E_1.e = E.e$ and $E_2.e = E.e$ recursively propagated (note use of parenthesis sugar).
$[\![\ \mathtt{Te}\ (\ \langle\mathtt{E\#1}\rangle + \langle\mathtt{E\#2}\rangle\ )\ ]\!] \to\ [\![\ (\mathtt{Te}\langle\mathtt{E\#1}\rangle) + (\mathtt{Te}\langle\mathtt{E\#2}\rangle)\ ]\!]$;
// 2. When $E_1.t$ and $E_2.t$ available then synthesize $E.t = \mathrm{Unif}(E_1.t, E_2.t)$.
$[\![\ \langle\mathtt{E\#1}\uparrow\mathtt{t(\#t1)}\rangle + \langle\mathtt{E\#2}\uparrow\mathtt{t(\#t2)}\rangle\ ]\!]\uparrow\mathtt{t(Unif(\#t1,\#t2))}$;

// $(E2)\quad E \to E_1 * E_2$:
// 1. $E_1.e = E.e$ and $E_2.e = E.e$ recursively propagated (note use of parenthesis sugar).
$[\![\ \mathtt{Te}\ (\ \langle\mathtt{E\#1}\rangle * \langle\mathtt{E\#2}\rangle\ )\ ]\!] \to\ [\![\ (\mathtt{Te}\langle\mathtt{E\#1}\rangle) * (\mathtt{Te}\langle\mathtt{E\#2}\rangle)\ ]\!]$;
// 2. When $E_1.t$ and $E_2.t$ available then synthesize $E.t = \mathrm{Unif}(E_1.t, E_2.t)$.
$[\![\ \langle\mathtt{E\#1}\uparrow\mathtt{t(\#t1)}\rangle + \langle\mathtt{E\#2}\uparrow\mathtt{t(\#t2)}\rangle\ ]\!]\uparrow\mathtt{t(Unif(\#t1,\#t2))}$;

HACS Program (VII)

// (*E*3) $E \rightarrow$ **int**:
// 1. Propagation leaf: set $E.t$ = Int directly.
$[\![$ Te $\langle$INT#1$\rangle$ $]\!] \rightarrow$ $[\![$ $\langle$INT#1$\rangle$ $]\!]\uparrow$t(Int) ;
// 2. Synthesize $E.t$ = Int.
$[\![$ $\langle$INT#1$\rangle$ $]\!]\uparrow$t(Int) ;

// (*E*4) $E \rightarrow$ **float**:
// 1. Propagation leaf: set $E.t$ = Float directly.
$[\![$ Te $\langle$FLOAT#1$\rangle$ $]\!]\rightarrow$ $[\![$ $\langle$FLOAT#1$\rangle$ $]\!]\uparrow$t(Float) ;
// 2. Synthesize $E.t$ = Float.
$[\![$ $\langle$FLOAT#1$\rangle$ $]\!]\uparrow$t(Float) ;

//(*E*5) $S \rightarrow$ **name**: There are two disjoint propagation cases:
// a. defined($E.e$, **name**.sym) so $E.t$ = lookup($E.e$, **name**.sym).
$[\![$ Te x $]\!]$ $\downarrow$e{x : #t} $\rightarrow$ $[\![$ x $]\!]$ $\uparrow$t(#t) ;
// b. $\neg$defined($E.e$, **name**.sym) so $E.t$ = TypeErr.
$[\![$ Te x $]\!]$ $\downarrow$e{$\neg$x} $\rightarrow$ $[\![$ x $]\!]$ $\uparrow$t(TypeErr) ;
}

HACS, Revisited

- Lexical and parsing specification (standard).
- Rewrite rules.
- Synthesis rules.

HACS Rewrite Rules with Explicit Propagation

$$F\left(\cdots \begin{matrix} A(\cdot)\uparrow a(\cdot) \\ X[\cdot]\uparrow x(\cdot) \end{matrix} \cdots\right)\downarrow c(\cdot) \to \cdots \begin{matrix} B(\cdot)\uparrow b(\cdot) \\ Y[\cdot]\uparrow y(\cdot) \end{matrix} \cdots \begin{matrix} G(\cdot)\downarrow g(\cdot) \\ Z[\cdot]\downarrow z(\cdot) \end{matrix} \cdots$$

becomes

$$F\left(\cdots \begin{matrix} A(\cdot) \\ X[\cdot] \end{matrix} \cdots\right)\downarrow c(\cdot) \to F'\left(\cdots \begin{matrix} \mathrm{Needs}_a(A(\cdot))) \\ \mathrm{Needs}_x(X[\cdot]) \end{matrix} \cdots\right)\downarrow c(\cdot)$$

$$F'\left(\cdots \begin{matrix} A(\cdot)\uparrow a(\cdot) \\ X[\cdot]\uparrow x(\cdot) \end{matrix} \cdots\right)\downarrow c(\cdot) \to \cdots \begin{matrix} B(\cdot)\uparrow b(\cdot) \\ Y[\cdot]\uparrow y(\cdot) \end{matrix} \cdots \begin{matrix} G(\cdot)\downarrow g(\cdot) \\ Z[\cdot]\downarrow z(\cdot) \end{matrix} \cdots$$

HACS Synthesis Rules with Explicit Propagation

$$A\Big(\cdots \begin{matrix} B(\cdot)\uparrow b(\cdot) \\ X[\cdot]\uparrow x(\cdot) \end{matrix} \cdots\Big)\uparrow a(\cdot)$$

becomes

$$\mathrm{Needs}_a\Big(A\Big(\cdots \begin{matrix} B(\cdot) \\ X[\cdot] \end{matrix} \cdots\Big)\Big) \to \mathrm{Collect}_R\Big(A\Big(\cdots \begin{matrix} \mathrm{Needs}_b(B(\cdot)) \\ \mathrm{Needs}_x(X[\cdot]) \end{matrix} \cdots\Big)\Big)$$

$$\mathrm{Collect}_R\Big(A\Big(\cdots \begin{matrix} B(\cdot)\uparrow b(\cdot) \\ X[\cdot]\uparrow x(\cdot) \end{matrix} \cdots\Big)\Big) \to A\Big(\cdots \begin{matrix} B(\cdot)\uparrow b(\cdot) \\ X[\cdot]\uparrow x(\cdot) \end{matrix} \cdots\Big)\uparrow a(\cdot)$$

with one R per $A\big(\cdots {\scriptstyle\begin{matrix} B(\cdot) \\ X[\cdot] \end{matrix}} \cdots\big)$.

First- and second-order term rewriting

First-order:

$$\begin{array}{rcl} \texttt{append}(\texttt{nil}, l) & \rightarrow & l \\ \texttt{append}(\texttt{cons}(x, l_1), l_2) & \rightarrow & \texttt{cons}(x, \texttt{append}(l_1, l_2)) \end{array}$$

First- and second-order term rewriting

First-order:

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Second-order:

$$\begin{aligned}\mathtt{map}(x.F(x), \mathtt{nil}) &\rightarrow \mathtt{nil} \\ \mathtt{map}(x.F(x), \mathtt{cons}(y, l)) &\rightarrow \mathtt{cons}(F(y), \mathtt{map}(x.F(x), l))\end{aligned}$$

First- and second-order term rewriting

First-order:

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\begin{aligned}
\mathtt{append}(\mathtt{nil}, l) &\rightarrow l \\
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\end{aligned}
$$

Second-order:

$$
\begin{aligned}
\mathtt{map}(x.F(x), \mathtt{nil}) &\rightarrow \mathtt{nil} \\
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\end{aligned}
$$

Not permitted: x of function type!

Second-order Term Rewriting

$$
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\mathtt{map}(x.F(x), \mathtt{nil}) & \to & \mathtt{nil} \\
\mathtt{map}(x.F(x), \mathtt{cons}(y, l)) & \to & \mathtt{cons}(F(y), \mathtt{map}(x.F(x), l)) \\
\mathtt{plus}(\mathtt{0}, y) & \to & y \\
\mathtt{plus}(\mathtt{s}(x), y) & \to & \mathtt{s}(\mathtt{plus}(x, y))
\end{array}
$$

Second-order Term Rewriting

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\end{array}$$

Reduction:

$$\mathtt{map}(x.\mathtt{plus}(x, \mathtt{s}(x)), \mathtt{cons}(\mathtt{s}(z), \mathtt{cons}(\mathtt{0}, \mathtt{nil})))$$

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\end{array}$$

Reduction:

$$\mathtt{map}(x.\mathtt{plus}(x, \mathtt{s}(x)), \mathtt{cons}(\mathtt{s}(z), \mathtt{cons}(\mathtt{0}, \mathtt{nil})))$$

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Second-order Term Rewriting

$$\begin{array}{rcl} \mathtt{map}(x.F(x), \mathtt{nil}) & \to & \mathtt{nil} \\ \mathtt{map}(x.F(x), \mathtt{cons}(y, l)) & \to & \mathtt{cons}(F(y), \mathtt{map}(x.F(x), l)) \\ \mathtt{plus}(\mathtt{0}, y) & \to & y \\ \mathtt{plus}(\mathtt{s}(x), y) & \to & \mathtt{s}(\mathtt{plus}(x, y)) \end{array}$$

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Attributes and Sorts

Given: set of sorts

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  - example: $\mathrm{GE}(\mathrm{plus}(3 \uparrow \{\}, x) \uparrow \{\}) : \mathrm{E}^{\downarrow}$ but $\mathrm{GE}(\mathrm{plus}(3 \uparrow \{\}, x) \uparrow \{\}) \downarrow \{\mathtt{e} : \{x : \mathtt{Int}\}\} : \mathrm{E}$

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Overview

1. Compilers
2. Attribute Grammars
3. HACS by Example
4. Core HACS
5. Conclusions

Relevant questions

Relevant questions

- termination

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Questions?