Declarative Array Programming with Single Assignment C (SAC)

Language Design and Compiler Technology

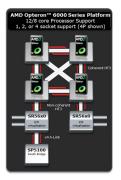
Clemens Grelck



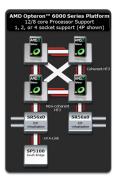
2nd HIPERFIT Workshop Copenhagen, Denmark Dec 1/2, 2011





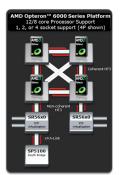


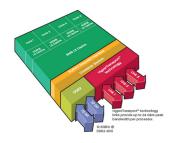






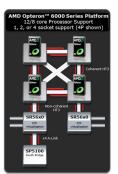




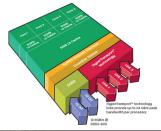
















Hardware in the many-core era is a zoo:

- Vastly different numbers of cores
- Vastly different core architectures: power, genericity
- Vastly different memory architectures

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Programming diverse hardware is uneconomic:

- Diverse low-level programming models
- Each requires expert knowledge
- ▶ Heterogeneous combinations of the above ?



Genericity through abstraction:

- Program what to compute, not exactly how
- Leave execution organisation to compiler and runtime system
- ▶ Put expert knowledge into compiler, not into applications



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Genericity through abstraction:

- Program what to compute, not exactly how
- Leave execution organisation to compiler and runtime system
- ▶ Put expert knowledge into compiler, not into applications
- Let programs remain architecture-agnostic
- Compile one source to diverse target hardware
- Promote multidimensional arrays as main data structure
- Pursue data-parallel approach to automatically exploit concurrency



Factorial imperative:

```
int fac( int n)
{
   int f = 1;
   while (n > 1) {
     f = f * n;
     n = n - 1;
   }
   return f;
}
```

Factorial functional:

Factorial imperative:

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Factorial functional:

```
fac n = prod( 1 + iota( n));
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Factorial imperative:

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Factorial functional:

Factorial data parallel:

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fac n = prod( 1 + iota( n));
```

10



Factorial imperative:

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Factorial functional:

```
fac n = if n \le 1
        then 1
        else n * fac (n - 1)
```

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10

0 1 2 3 4 5 6 7 8 9

1 2 3 4 5 6 7 8 9 10
```

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Factorial functional:

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fac n = if n <= 1
            then 1
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```

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fac n = prod(1 + iota(n));

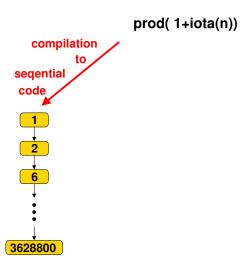
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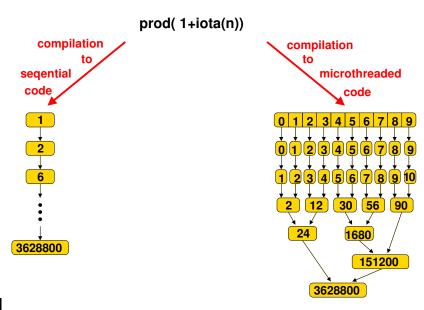
0 1 2 3 4 5 6 7 8 9 10

3628800
```

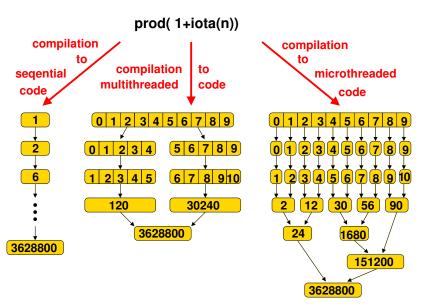
The Essence of (Data Parallel) Array Programming prod(1+iota(n))



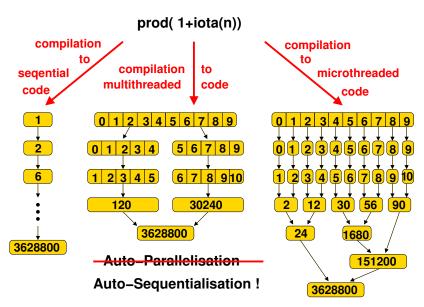














SAC



High-level functional, data-parallel programming with vectors, matrices, arrays



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Suitability to achieve high performance in sequential and parallel execution

High-level functional, data-parallel programming with vectors, matrices, arrays Easy to adopt for programmers SAC with an imperative background Suitability to achieve high performance in sequential and parallel execution



Introductory Example: gcd in SAC

Euclid's algorithm:

```
int gcd( int high, int low)
  if (high < low) {
         = low;
   mem
   low = high;
   high = mem;
  while (low != 0) {
   remain = high % low;
   high = low;
    low = remain;
  return high;
```

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 - Branches are syntactic sugar for conditional expressions
 - Loops are syntactic sugar for tail-end recursive functions
 - Data flow determines execution order



Execution Model

Contextfree substitution of expressions

Role of Functions

- Map argument values to result values
- ▶ No side effects
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- Variables are placeholders for values
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Control flow constructs

- ▶ Branches are syntactic sugar for conditional expressions
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Nature of Arrays

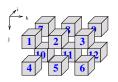
- Pure values, mapping indices to (other) values
- ▶ No state, no fixed memory representation



 $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \qquad \begin{array}{ccc} \text{dim:} & 2 \\ \text{shape:} & [3,3] \\ \text{data:} & [1,2,3,4,5,6,7,8,9] \end{array}$



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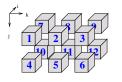


dim: 3

shape: [2,2,3]

data: [1,2,3,4,5,6,7,8,9,10,11,12]

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[1, 2, 3, 4, 5, 6]

dim: 3

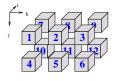
shape: [2,2,3]

data: [1,2,3,4,5,6,7,8,9,10,11,12]

dim: 1

shape: [6]

data: [1,2,3,4,5,6]



dim: 3

shape: [2,2,3]

data: [1,2,3,4,5,6,7,8,9,10,11,12]

[1, 2, 3, 4, 5, 6]

42

dim: 1

shape: [6]

data: [1,2,3,4,5,6]

dim: 0

shape: [] data: [42]

► Defining a vector:

```
vec = [1,2,3,4,5,6];
```

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▶ Defining a higher-dimensional array:

```
mat = [vec,vec];
mat = reshape([3,2], vec);
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Querying for the shape of an array:

```
shp = shape(mat); \rightarrow [3,2]
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- Querying for the shape of an array:
 - $shp = shape(mat); \rightarrow [3,2]$
- Querying for the rank of an array:

```
rank = dim(mat); \rightarrow 2
```

▶ Defining a vector: vec = [1.2.3.4.5.6];

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Querying for the shape of an array:

```
shp = shape(mat); \rightarrow [3,2]
```

► Querying for the rank of an array: rank = dim(mat); → 2

► Selecting elements:

```
x = sel([4], vec); \rightarrow 5

y = sel([2,1], mat); \rightarrow 6

x = vec[[4]]; \rightarrow 5

y = mat[[2,1]]; \rightarrow 6
```

With-Loops: Versatile Array Comprehensions

```
A = with {
    ([1,1] <= iv < [4,4]) : e(iv);
}: genarray( [5,4], def );</pre>
```

- Multidimensional array comprehensions
- Mapping from index domain into value domain

[0,0]	[0,1]	[0,2]	[0,3]
[1,0]	[1,1]	[1,2]	[1,3]
[2,0]	[2,1]	[2,2]	[2,3]
[3,0]	[3,1]	[3,2]	[3,3]
[4,0]	[4,1]	[4,2]	[4,3]



index domain

value domain



With-Loops: Versatile Array Comprehensions

```
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```

Variations:

- Multiple generators
- Strided generators
- Multiple operators
- Other defaults
- Reductions
- etc



Principle of Abstraction

Characteristics:

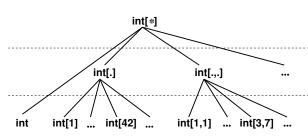
- Use with-loops to define elementary array operations
- Array versions of scalar built-in functions and operators
- Structural operations like rotation and shifting
- Standard reductions
- and much more

Principle of Abstraction

Characteristics:

- Use with-loops to define elementary array operations
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- Structural operations like rotation and shifting
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- and much more

Hierarchy of array types with subtyping and overloading:



AUD Class: rank: dynamic shape: dynamic

shape: dynamic

rank: static shape: dynamic

AKS Class: rank: static shape: static

Principle of Composition

Characteristics:

- Step-wise composition of functions
- from previously defined functions
- or basic building blocks (with-loop defined)

Example: convergence test

```
bool
is_convergent (double[*] new, double[*] old, double eps)
{
  return( all( abs( new - old) < eps));
}</pre>
```



```
is_convergent([1,2,3,8],[3,2,1,4], 3)
```



```
is_convergent([1,2,3,8], [3,2,1,4], 3)

all(abs([1,2,3,8] - [3,2,1,4]) < 3)
```

```
is_convergent( [1,2,3,8], [3,2,1,4], 3 )
all( abs( [1,2,3,8] - [3,2,1,4]) < 3 )
all( abs( [-2,0,2,4]) < 3 )</pre>
```

Shape- and Rank-Generic Programming

2-dimensional convergence test:

is_convergent(
$$\begin{pmatrix} 1 & 2 \\ 3 & 8 \end{pmatrix}$$
, $\begin{pmatrix} 3 & 2 \\ 1 & 7 \end{pmatrix}$, 3)

Shape- and Rank-Generic Programming

2-dimensional convergence test:

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3-dimensional convergence test:

is_convergent(
$$\begin{pmatrix} \begin{pmatrix} 1 & 2 \\ 3 & 8 \end{pmatrix} \\ \begin{pmatrix} 6 & 7 \\ 2 & 8 \end{pmatrix} \end{pmatrix}$$
, $\begin{pmatrix} \begin{pmatrix} 2 & 1 \\ 0 & 8 \end{pmatrix} \\ \begin{pmatrix} 1 & 1 \\ 3 & 7 \end{pmatrix} \end{pmatrix}$, 3)

- NO large collection of built-in operations
 - ► Simplified compiler design



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 - Simplified compiler design
- ► INSTEAD: library of array operations
 - Improved maintainability
 - Improved extensibility



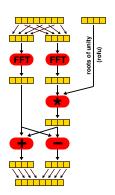
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- ► INSTEAD: library of array operations
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 - High confidence in correctness
 - Good readability of code
- General intermediate representation for array operations
 - ► Basis for code optimization
 - Basis for implicit parallelization



Case Study: 1-Dimensional Complex FFT (NAS-FT)



```
complex[.] FFT(complex[.] v, complex[.] rofu)
  even = condense(2, v):
  odd = condense(2, drop([1], v));
 even = FFT( even. rofu):
  odd = FFT( odd, rofu);
 rofu = condense( len(rofu) / len(odd), rofu)
 left = even + odd * rofu:
 right = even - odd * rofu;
 return left ++ right;
}
```

Case Study: 3-Dimensional Complex FFT (NAS-FT)

Algorithmic idea:



Implementation:

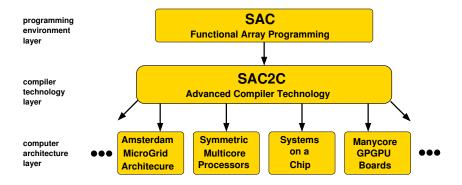
```
complex[.,.,.] FFT( complex[.,.,.] a, complex[.] rofu)
{
   b = { [.,y,z] -> FFT( a[.,y,z], rofu) };
   c = { [x,.,z] -> FFT( b[x,.,z], rofu) };
   d = { [x,y,.] -> FFT( c[x,y,.], rofu) };
   return d;
}
typedef double[2] complex;
```



The Same in Fortran

```
if (1 .eq. m) goto 160
                                    call cfftz (is, logd(1) subroutine cffts3(is, d, x, roue_1) fftz2 (is, 1 + 1, m, n, ff
                                                                                                          fftblockpad, u, y,
                                                 d(1), y, y(implifit none
subroutine fft(dir, x1, x2) >
                                                                                           enddo
                                                             include 'global.h'
                                     do j = 1, fftblock
implicit none
                                                                                           goto 180
                                                             integer is, d(3), logd(3)60
include 'global.h'
                                        do i = 1, d(1)
                                                                                           do i = 1, n
                                           xout(i, j+jj,k) = double1 complex x(d(1),d(2),d(3)) do i = 1. fftblock
integer dir
                                                             double complex xout(d(1),d(2),d(3)(i,j) = y(i,j)
double complex x1(ntotal), x2(ntotal) enddo
                                                             double complex y(fftblockpad, deado2)
double complex scratch(fftblockpadadefault
                                                             integer i, j, k, ii
                         *maxdenddo
                                                                                           continue
if (dir .eq. 1) then
                              return
                                                             do i = 1, 3
                                                                                           return
                                                                logd(i) = ilog2(d(i))
   call cffts1(1, dims(1,1),
   call cffts2(1, dims(1,2),
                                                                                           subroutine fftz2 (is, 1, m, n, ny
   call cffts3(1, dims(1,3), XTP
                                                             do i = 1, d(2)
                                                               do ii = 0, d(1) - fftblock.
                              include 'global.h'
                                                                                           integer is,k,l,m,n,ny,ny1,n1,li,l
   call cffts3(-1, dims(1,3),integet,iscratch), logd(3)
                                                                  do k = 1, d(3)
                                                                                           double complex u,x,y,u1,x11,x21
   call cffts2(-1, dims(1,2),d%hbl%1compresch(d(1),d(2),d(3))
                                                                     do i = 1, fftblock
                                                                        y(i,k,1) = x(i+ii,dimension u(n), x(ny1,n), y(ny1,n)
   call cffts1(-1, dims(1,1), dblb1&2complex bout(d(1),d(2),d(3))
                                                                                           n1 = n / 2
endif
                              double complex y(fftblockpad, d(2)endo
                                                                                           1k = 2 ** (1 - 1)
                              integer i, j, k, ii
                                                                  call cfftz (is, logd(3),1i = 2 ** (m - 1)
subroutine cffts1(is, d, x, xqut; x)1. 3
                                                                       d(3), y, y(1, 1, 2)1 = 2 * 1k
implicit none
                                 logd(i) = ilog2(d(i))
                                                                  do k = 1, d(3)
include 'global.h'
                              end do
                                                                     do i = 1, fftblock
                              do k = 1, d(3)
                                                                        xout(i+ii,j,k) = y(i
integer is, d(3), logd(3)
double complex x(d(1),d(2),d(3)) do ii = 0, d(1) - fftblock, fftblock enddo
                                                                                              i21 = i * 1i + 1
double complex x(d(1),d(2),d(3)) do j = 1, d(2)
double complex xout(d(1),d(2),d(3)) do i = 1, fftblock
                                                                                              i22 = i21 + 1k
                                                               enddo
double complex y(fftblockpad, d(1), 2) y(i,j,1) = x(i+ii
                                                                                              if (is .ge. 1) then
                                                                                                n1 = n(kn+i)
integer i, j, k, jj
                                                             end
                                                                                              else
do i = 1, 3
                                                             subroutine cfftz (is, m, n, x, y)u1 = dconjg (u(ku+i))
                                    call cfftz (is, logd(2),
   logd(i) = ilog2(d(i))
                                         d(2), y, y(1, 1, 2) implicit none
end do
                                                             include 'global.h'
                                                                                             do k = 0, lk - 1
do k = 1, d(3)
                                    do j = 1, d(2)
                                                                                                do i = 1, nv
                                                             integer is,m,n,i,j,l,mx
   do jj = 0, d(2) - fftblock, fftblock i = 1, fftblock
                                                                                                  x11 = x(j,i11+k)
                                                            vdouble) complex x, y
      do | = 1, fftblock
                                          xout(i+ii, j,k) =
                                                                                                  x21 = x(j, i12+k)
                                                             dimension x(fftblockpad,n), y(fftblockpad,n) = x11 + x21
         do i = 1, d(1)
            y(j,i,1) = x(i,j+jj,k) enddo
                                                             mx = u(1)
                                                                                                  y(j,i22+k) = u1 * (x11 - x2)
                                 ánádo
         enddo
                                                             do 1 = 1, m, 2
                              anddo
      enddo
                                                               call fftz2 (is, l. m. n. fftbbodho
                              return
                              and
                                                                            fftblockpad, u,emddo)
```

end





- ► Challenge 1: Stateless Arrays
 - ▶ How to avoid copying?
 - ▶ How to avoid boxing small arrays?
 - ▶ How to do memory management efficiently?



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 - ► How to do memory management efficiently?
- Challenge 2: Compositional Specifications
 - ► How to avoid temporary arrays?
 - ► How to avoid multiple array traversals?



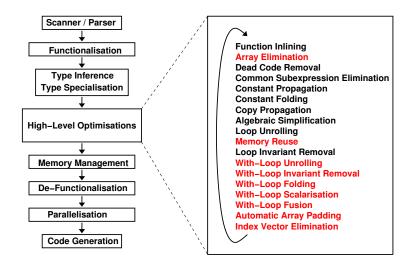
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 - How to represent arrays with different static knowledge?



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- ► Challenge 4: Organisation of Concurrent Execution
 - ▶ How to schedule index spaces to threads ?
 - ▶ When to synchronise (and when not) ?
 - Where does parallel execution pay off?
 - Granularity control ?



Challenge 5: Implementing a Fully-Fledged Compiler





SAC as a Compiler Technology Project

Large-scale (academic) project:

- ▶ **SAC** compiler + runtime library:
 - ► 300,000 lines of code
 - ▶ about 1000 files
 - ▶ about 250 compiler passes
 - + standard prelude
 - + standard library
- More than 15 years of research and development
- More than 30 people involved over the years
- Mostly BSc/MSc students, 5 PhDs



The SAC Project: Credits

Involved Universities:

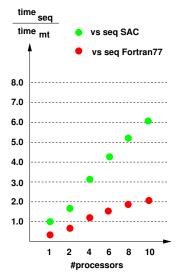
- ▶ University of Kiel, Germany (1994–2005)
- University of Toronto, Canada (since 2000)
- University of Lübeck, Germany (2001–2008)
- University of Hertfordshire, England (2004–2012)
- University of Amsterdam, Netherlands (since 2008)
- Heriot-Watt University, Scotland (since 2011)

Main Contributors:

- Sven-Bodo Scholz (Kiel, Herts, Heriot-Watt)
- Clemens Grelck (Kiel, Lübeck, Herts, Amsterdam)
- Stephan Herhut (Kiel, Herts, now at Intel, Santa Clara)
- ► Kai Trojahner (Lübeck, now at RTT AG, München)
- Dietmar Kreye (Kiel, now at sd&m AG, Hamburg)
- ► Robert Bernecky (Toronto)
- Jing Guo (Herts)



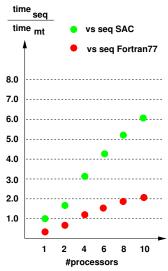
Runtime Performance: Standard Multiprocessor



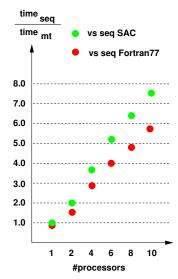
NAS benchmark FT



Runtime Performance: Standard Multiprocessor



NAS benchmark FT

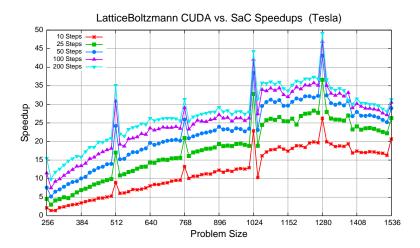


NAS benchmark MG



Runtime Performance: NVidia Tesla

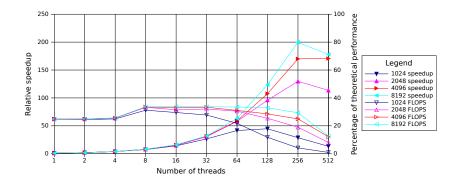
Lattice-Boltzmann:





Runtime Performance: Ultra Sparc T3-4 Server

Matrix Multiplication:





Summary

Language design:

- ► Functional state-less semantics but C-like syntax
- Architecture-agnostic high-level parallel programming
- Shape- and rank-generic array programming
- Index-free (index-less) array programming

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- ► Functional state-less semantics but C-like syntax
- Architecture-agnostic high-level parallel programming
- Shape- and rank-generic array programming
- Index-free (index-less) array programming

Language implementation:

- Fully-fledged compiler, not an embedded DSL
- Large-scale machine-independent optimisation
- Automatic parallelisation for various architectures
- Automatic granularity adaptation and control
- ► Automatic memory management



The End

Questions?

Check out www.sac-home.org !!

