

Hamiltonian Zoo

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No Matter Where They're From

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I. MODEL HAMILTONIANS

1. *Coulomb*

$$H_{\text{Coulomb}} = \frac{q_1 q_2}{4\pi\epsilon_0 r} \quad (1)$$

A. Electronic Structure

1. *Born-Oppenheimer Model*

2. *Watson Model*

B. Fine and Hyperfine Structure

1. *Stark*

2. *Zeeman*

$$H_{\text{Zeeman}} = -\frac{\mu_B (g_l \vec{L} + g_s \vec{S})}{\hbar} \cdot \vec{B} \quad (2)$$

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3. Complete magnetic dipole interaction

$$H_{\text{Magnetic Dipole}} = \frac{gI\mu_N\mu_B\mu_0}{2\pi} \left(\frac{1}{L_z} \sum_i \frac{\hat{l}_{zi}}{r_i^3} \vec{I} \cdot \vec{L} + \frac{2g_s}{S_z} \sum_i \frac{\hat{s}_{zi}}{r_i^3} (3(\vec{I} \cdot \hat{r})(\vec{S} \cdot \hat{r}) - \vec{I} \cdot \vec{S}) + \frac{g_s}{3S_z} \sum_i \hat{s}_{zi} \delta^3(\vec{r}_i) \vec{I} \cdot \vec{S} \right) \quad (3)$$

4. *Frosch-Foley*

5. *Spin-orbit*

6. *Orbit-orbit*

7. *Spin-other-orbit (Gaunt)*

8. *Fermi Contact*

C. Special Relativity

1. *Dirac*

2. *Klein-Gordon*

3. *Darwin ($1e^-$)*

4. *Darwin ($2e^-$)*

5. *Mass Velocity*

6. *Breit-Pauli*

7. *Dirac-Coulomb-Breit*

D. Quantum Electrodynamics

1. *Aracki-Sucher*

$$H_{\text{Aracki-Sucher}} = -\frac{7\alpha_{\text{FS}}^3}{6\pi} \sum_{i>j} \lim_{a \rightarrow 0} \left(\frac{\theta(r_{ij} - a)}{r_{ij}^3} + 4\pi (\gamma + \ln a) \delta(\vec{r}_{ij}) \right) \quad (4)$$

2. *One-Loop*

3. *Schwinger-Dyson*

E. Nuclear Motion

$$H_{\text{Nuc}} = -\frac{\hbar^2}{2} \sum_{i=1}^N \sum_{\alpha=1}^3 \frac{1}{M_i} \frac{\partial^2}{\partial R_{i\alpha}^2} + V(\mathbf{R}_1, \dots, \mathbf{R}_N) \quad (5)$$

1. *Rigid Rotor Harmonic Oscillator*

2. *Centrifugal distortion*

3. *Centrifugal distortion*

F. Long-range Interactions

1. *van der Waals*

2. *Lenard-Jones*

3. *Casimir-Polder*

4. *Resonance dipole-dipole*

5. *Meath*

6. *Axilrod-Teller*

G. Hubbard Models

1. *Bose-Hubbard (Boson Hubbard)*

2. *Fermi-Hubbard (Fermi Hubbard)*

3. *Jaynes-Cummings-Hubbard*

H. Open Quantum Systems

1. *Rabi model*

2. *Spin-Boson*

3. *Feynman-Vernon*

4. *Leggett-Caldeira*

5. *Ishizaki-Fleming*

2. Richardson-Gaudin

3. Exactly solvable pairing Hamiltonians

SU(2), Rank 1 algebra

$$H_{\text{SU}(2)} = \sum_i \epsilon_i n_i - g \sum_{ij} P_i^+ P_j \quad (7)$$

SO(5), Rank 2 algebra

$$H_{\text{SO}(5)} = \sum_i \epsilon_i n_i - g \sum_{ijk} P_{ik}^+ P_{jk} \quad (8)$$

SO(8), Rank 4 algebra

$$H_{\text{SO}(8)} = \sum_i \epsilon_i n_i - g_T \sum_{ijk} P_{ik}^+ P_{jk} - g_S \sum_{ijk} D_{ik}^+ D_{jk} \quad (9)$$

4. *t*-J model

J. Models of Superfluidity

1. 2D *p*-wave Fermi superfluid

$$H_{2DFS} = \sum_i \epsilon_i a_i^\dagger a_i + \frac{i\Delta_i}{2} (a_i^\dagger a_{-i}^\dagger + \text{H.c.}) \quad (10)$$

II. SPIN HAMILTONIANS

A. Ising

B. Heisenberg

C. J1-J2 Model

D. Majumdar-Ghosh

E. AKLT Model

F. Kitaev Models

1. *Toric Code*

2. *Ocko-Yoshida*

3. *Honeycomb Model*

$$H_{\text{Honeycomb}} = -J_x \sum_{x\text{-links}} x_i x_j - J_y \sum_{y\text{-links}} y_i y_j - J_z \sum_{z\text{-links}} z_i z_j \quad (11)$$

G. 2-level systems (spin-1/2 particles)

$$z \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, x \equiv \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, y \equiv \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad (12)$$

$$s_+ \equiv \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \frac{x + iy}{2} \quad (13)$$

$$s_- \equiv \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = \frac{x - iy}{2} \quad (14)$$

$$s_\alpha \equiv \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \frac{\mathbb{1} + z}{2} \equiv b \quad (15)$$

$$s_\beta \equiv \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \frac{\mathbb{1} - z}{2} \quad (16)$$

H. 3-level systems (spin-1 particles)

$$z_3 \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, x \equiv \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, y \equiv \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad (17)$$

$$s_{3+} \equiv \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} = \frac{x_3 + iy_3}{2} \quad (18)$$

$$s_{3-} \equiv \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} = \frac{x_3 - iy_3}{2} \quad (19)$$

$$s_{3,1} \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \frac{\mathbb{1} + z}{2} \equiv t \quad (20)$$

$$s_{3,2} \equiv \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (21)$$

$$s_{3,3} \equiv \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \frac{\mathbb{1} - z}{2} \equiv t \quad (22)$$

I. 4-level systems (spin-3/2 particles)

$$z_4 \equiv \begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -3 \end{pmatrix}, x_4 \equiv \begin{pmatrix} 0 & \sqrt{3} & 0 & 0 \\ \sqrt{3} & 0 & 2 & 0 \\ 0 & 2 & 0 & \sqrt{3} \\ 0 & 0 & \sqrt{3} & 0 \end{pmatrix}, y_4 \equiv \begin{pmatrix} 0 & -\sqrt{3}i & 0 & 0 \\ \sqrt{3}i & 0 & 2 & 0 \\ 0 & 2 & 0 & -\sqrt{3}i \\ 0 & 0 & \sqrt{3}i & 0 \end{pmatrix} \quad (23)$$

$$s_{4+} \equiv \begin{pmatrix} 0 & \sqrt{3} & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & \sqrt{3} \\ 0 & 0 & 0 & 0 \end{pmatrix} \equiv \frac{x_{3/2} + iy_{3/2}}{2} \equiv \frac{x_{4 \times 4} + iy_{4 \times 4}}{2} \quad (24)$$

$$s_{4-} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 \\ \sqrt{3} & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & \sqrt{3} & 0 \end{pmatrix} \equiv \frac{x_4 - iy_4}{2} \quad (25)$$

$$s_{4,1} \equiv \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \frac{1}{2} (z_2 + z_1 z_2 + z_1 - \mathbb{1}) \quad (26)$$

$$s_{4,2} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \quad (27)$$

$$s_{4,3} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \quad (28)$$

$$s_{4,4} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \quad (29)$$

$$\equiv \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \frac{1}{2} (\mathbb{1} - z_1 + z_1 z_2 + z_2) \quad (30)$$

$$\equiv \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \frac{1}{2} (\mathbb{1} - z_2 + z_1 z_2 + z_1) \quad (31)$$

J. 5-level systems (spin-2 particles)

$$z_5 \equiv \begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -2 \end{pmatrix}, x_5 \equiv \begin{pmatrix} 0 & 2 & 0 & 0 & 0 \\ 2 & 0 & \sqrt{6} & 0 & 0 \\ 0 & \sqrt{6} & 0 & \sqrt{6} & 0 \\ 0 & 0 & \sqrt{6} & 0 & 2 \\ 0 & 0 & 0 & 2 & 0 \end{pmatrix}, y_5 \equiv \begin{pmatrix} 0 & -2i & 0 & 0 & 0 \\ 2i & 0 & -\sqrt{6}i & 0 & 0 \\ 0 & \sqrt{6}i & 0 & -\sqrt{6}i & 0 \\ 0 & 0 & \sqrt{6}i & 0 & -2i \\ 0 & 0 & 0 & 2i & 0 \end{pmatrix} \quad (32)$$

$$s_{5+} \equiv \begin{pmatrix} 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{6} & 0 & 0 \\ 0 & 0 & 0 & \sqrt{6} & 0 \\ 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \equiv \frac{x_{5 \times 5} + iy_{5 \times 5}}{2} \equiv \frac{x_{(5)} + iy_{(5)}}{2} \quad (33)$$

$$s_{5-} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 \\ 0 & \sqrt{6} & 0 & 0 & 0 \\ 0 & 0 & \sqrt{6} & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \end{pmatrix} \equiv \frac{x_{5 \times 5} + iy_{5 \times 5}}{2} \equiv \frac{x_{(5)} + iy_{(5)}}{2} \quad (34)$$

$$s_{5,1} \equiv \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} = \quad (35)$$

$$s_{5,2} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} = \quad (36)$$

$$s_{5,3} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} = \quad (37)$$

$$s_{5,4} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} = \quad (38)$$

$$s_{5,5} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} = \quad (39)$$

Wavefunction Zoo

K. Electronic Structure Ansätze

1. Hartree Product

2. Configuration Interaction

3. Coupled Cluster

$CC(n)$

$Bruckner-CC(n)$

$EOM-CC(n)$

$EOM-IP-CC(n)$

$FS-CC(n)$

$DLPNO-CC(n)$

$MR-EOM$

4. Geminals

$AP1roG/pCCD$

$APIG$

$APSetG$

APG

Potential Zoo

L. Diatomics

Morse

Lenard-Jones

Morse/Long-range

Tiemann

Morse-Rosen

M. Triatomics

Jensen

Schwenke

PolyMLR

N. Torsion Potentials

Functional Zoo

Function Zoo

Master Equation Zoo

Particle Zoo

https://en.wikipedia.org/wiki/Particle_zoo

Complexity Zoo (founded by Scott Aaronson of UWaterloo)

https://complexityzoo.uwaterloo.ca/Complexity_Zoo

Algorithms Zoo (founded by Stephen Jordan of NIST)

<https://math.nist.gov/quantum/zoo/>