Hamiltonian Zoo

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No Matter Where They're From

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I. MODEL HAMILTONIANS

1. *Coulomb*

$$H_{\text{Coulomb}} = \frac{q_1 q_2}{4\pi\epsilon_0 r} \tag{1}$$

A. Electronic Structure

1. *Born-Oppenheimer Model*

2. *Watson Model*

B. Fine and Hyperfine Structure

1. *Stark*

2. *Zeeman*

$$H_{\text{Zeeman}} = -\frac{\mu_B \left(g_l \vec{L} + g_s \vec{S}\right)}{\hbar} \cdot \vec{B} \tag{2}$$

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3. Complete magnetic dipole interaction

$$H_{\mathrm{Magnetic\,Dipole}} = \frac{g_I \mu_N \mu_B \mu_0}{2\pi} \left(\frac{1}{L_z} \sum_i \frac{\hat{l}_{zi}}{r_i^3} \vec{I} \cdot \vec{L} + \frac{2g_s}{S_z} \sum_i \frac{\hat{s}_{zi}}{r_i^3} \left(3 \left(\vec{I} \cdot \hat{r} \right) \left(\vec{S} \cdot \hat{r} \right) - \vec{I} \cdot \vec{S} \right) + \frac{g_s}{3S_z} \sum_i \hat{s}_{zi} \delta^3 \left(\vec{r}_i \right) \vec{I} \cdot \vec{S} \right) \tag{3}$$

4. Frosch-Foley

5. Spin-orbit

6. Orbit-orbit

7. Spin-other-orbit (Gaunt)

8. Fermi Contact

C. Special Relativity

1. Dirac

2. Klein-Gordon

3. Darwin ($1e^-$)

4. Darwin ($2e^-$)

5. Mass Velocity

6. Breit-Pauli

7. Dirac-Coulomb-Breit

D. Quantum Electrodynamics

1. Aracki-Sucher

$$H_{\text{Aracki–Sucher}} = -\frac{7\alpha_{\text{FS}}^3}{6\pi}\sum_{i>j}\lim_{a\to 0}\left(\frac{\theta\left(r_{ij}-a\right)}{r_{ij}^3} + 4\pi\left(\gamma+\ln a\right)\delta(\vec{r}_{ij})\right) \tag{4}$$

2. One-Loop

3. Schwinger-Dyson

E. Nuclear Motion

$$H_{\mathrm{Nuc}} = -\frac{\hbar^2}{2}\sum_{i=1}^{N}\sum_{\alpha=1}^{3}\frac{1}{M_i}\frac{\partial^2}{\partial R_{i\alpha}^2} + V(\boldsymbol{R}_1,\ldots,\boldsymbol{R}_N) \tag{5}$$

1. Rigid Rotor Harmonic Oscillator

2. Centrifugal distortion

3. Centrifugal distortion

F. Long-range Interactions

1. van der Waals

2. Lenard-Jones

3. Casimir-Polder

4. Resonance dipole-dipole

5. Meath

6. Axilrod-Teller

G. Hubbard Models

1. Bose-Hubbard (Boson Hubbard)

2. Fermi-Hubbard (Fermi Hubbard)

3. Jaynes-Cummings-Hubbard

H. Open Quantum Systems

1. Rabi model

2. Spin-Boson

3. Feynman-Vernon

4. Leggett-Caldeira

5. Ishizaki-Fleming

2. Richardson-Gaudin

3. Exactly solvable pairing Hamiltonians

SU(2), Rank 1 algebra

$$H_{\mathrm{SU}(2)} = \sum_i \epsilon_i n_i - g \sum_{ij} P_i^{+} P_j \tag{7}$$

SO(5), Rank 2 algebra

$$H_{\mathrm{SO}(5)} = \sum_i \epsilon_i n_i - g \sum_{ijk} P_{ik}^{+} P_{jk} \tag{8}$$

SO(8), Rank 4 algebra

$$H_{\mathrm{SO}(8)} = \sum_i \epsilon_i n_i - g_T \sum_{ijk} P_{ik}^{+} P_{jk} - g_S \sum_{ijk} D_{ik}^{+} D_{jk} \tag{9}$$

4. t-J model

J. Models of Superfluidity

1. 2D p-wave Fermi superfluid

$$H_{2DFSF} = \sum_i \epsilon_i a_i^{\dagger} a_i + \frac{\mathrm{i}\Delta_i}{2} \left(a_i^{\dagger} a_{-i}^{\dagger} + \mathrm{H.c.} \right) \tag{10}$$

II. SPIN HAMILTONIANS

A. Ising

B. Heisenberg

C. J1-J2 Model

D. Majumdar-Ghosh

E. AKLT Model

F. Kitaev Models

1. Toric Code

2. Ocko-Yoshida

3. Honeycomb Model

$$H_{\text{Honeycomb}} = -J_x \sum_{x-\text{links}} x_i x_j - J_y \sum_{y-\text{links}} y_i y_j - J_z \sum_{z-\text{links}} z_i z_j \tag{11}$$

G. 2-level systems (spin-1/2 particles)

$$z \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, x \equiv \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, y \equiv \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix} \tag{12}$$

$$s_+ \equiv \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \frac{x + \mathrm{i}y}{2} \tag{13}$$

$$s_- \equiv \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = \frac{x - \mathrm{i}y}{2} \tag{14}$$

$$s_\alpha \equiv \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \frac{\mathbb{1} + z}{2} \equiv b \tag{15}$$

$$s_\beta \equiv \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \frac{\mathbb{1} - z}{2} \tag{16}$$

H. 3-level systems (spin-1 particles)

$$z_3 \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, x \equiv \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, y \equiv \begin{pmatrix} 0 & -\mathrm{i} & 0 \\ \mathrm{i} & 0 & -\mathrm{i} \\ 0 & \mathrm{i} & 0 \end{pmatrix} \tag{17}$$

$$s_{3+} \equiv \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} = \frac{x_3 + \mathrm{i}y_3}{2} \tag{18}$$

$$s_{3-} \equiv \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} = \frac{x_3 - \mathrm{i}y_3}{2} \tag{19}$$

$$s_{3,1} \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \frac{\mathbb{1} + z}{2} \equiv t \tag{20}$$

$$s_{3,2} \equiv \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{21}$$

$$s_{3,3} \equiv \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \frac{\mathbb{1} - z}{2} \equiv t \tag{22}$$

I. 4-level systems (spin-3/2 particles)

$$z_4 \equiv \begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -3 \end{pmatrix}, x_4 \equiv \begin{pmatrix} 0 & \sqrt{3} & 0 & 0 \\ \sqrt{3} & 0 & 2 & 0 \\ 0 & 2 & 0 & \sqrt{3} \\ 0 & 0 & \sqrt{3} & 0 \end{pmatrix}, y_4 \equiv \begin{pmatrix} 0 & -\sqrt{3}\mathrm{i} & 0 & 0 \\ \sqrt{3}\mathrm{i} & 0 & 2 & 0 \\ 0 & 2 & 0 & -\sqrt{3}\mathrm{i} \\ 0 & 0 & \sqrt{3}\mathrm{i} & 0 \end{pmatrix} \tag{23}$$

$$s_{4+} \equiv \begin{pmatrix} 0 & \sqrt{3} & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & \sqrt{3} \\ 0 & 0 & 0 & 0 \end{pmatrix} \equiv \frac{x_{3/2} + \mathrm{i}y_{3/2}}{2} \equiv \frac{x_{4\times 4} + \mathrm{i}y_{4\times 4}}{2} \tag{24}$$

$$s_{4-} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 \\ \sqrt{3} & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & \sqrt{3} & 0 \end{pmatrix} \equiv \frac{x_4 - \mathrm{i}y_4}{2} \tag{25}$$

$$s_{4,1} \equiv \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \frac{1}{2}\left(z_2 + z_1 z_2 + z_1 - \mathbb{1}\right) \tag{26}$$

$$s_{4,2} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \tag{27}$$

$$s_{4,3} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \tag{28}$$

$$s_{4,4} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \tag{29}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \frac{1}{2}\left(\mathbb{1} - z_1 + z_1 z_2 + z_2\right) \tag{30}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \frac{1}{2}\left(\mathbb{1} - z_2 + z_1 z_2 + z_1\right) \tag{31}$$

$$|10\rangle\langle 01| = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \frac{1}{4}\left(x_1 x_2 - \mathrm{i}y_1 x_2 + \mathrm{i}x_1 y_2 + y_1 y_2\right) \tag{32}$$

$$|10\rangle\langle 01| = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \frac{1}{4}\left(x_1 x_2 - \mathrm{i}x_1 y_2 + \mathrm{i}y_1 x_2 + y_1 y_2\right) \tag{33}$$

J. 5-level systems (spin-2 particles)

$$z_5 \equiv \begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -2 \end{pmatrix}, x_5 \equiv \begin{pmatrix} 0 & 2 & 0 & 0 & 0 \\ 2 & 0 & \sqrt{6} & 0 & 0 \\ 0 & \sqrt{6} & 0 & \sqrt{6} & 0 \\ 0 & 0 & \sqrt{6} & 0 & 2 \\ 0 & 0 & 0 & 2 & 0 \end{pmatrix}, y_5 \equiv \begin{pmatrix} 0 & -2\mathrm{i} & 0 & 0 & 0 \\ 2\mathrm{i} & 0 & -\sqrt{6}\mathrm{i} & 0 & 0 \\ 0 & \sqrt{6}\mathrm{i} & 0 & -\sqrt{6}\mathrm{i} & 0 \\ 0 & 0 & \sqrt{6}\mathrm{i} & 0 & -2\mathrm{i} \\ 0 & 0 & 0 & 2\mathrm{i} & 0 \end{pmatrix} \tag{34}$$

$$s_{5+} \equiv \begin{pmatrix} 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{6} & 0 & 0 \\ 0 & 0 & 0 & \sqrt{6} & 0 \\ 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \equiv \frac{x_{5\times 5} + \mathrm{i}y_{5\times 5}}{2} \equiv \frac{x_{(5)} + \mathrm{i}y_{(5)}}{2} \tag{35}$$

$$s_{5-} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 \\ 0 & \sqrt{6} & 0 & 0 & 0 \\ 0 & 0 & \sqrt{6} & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \end{pmatrix} \equiv \frac{x_{5\times 5} + \mathrm{i}y_{5\times 5}}{2} \equiv \frac{x_{(5)} + \mathrm{i}y_{(5)}}{2} \tag{36}$$

$$s_{5,1} \equiv \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} = \tag{37}$$

$$s_{5,2} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} = \tag{38}$$

$$s_{5,3} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} = \tag{39}$$

$$s_{5,4} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} = \tag{40}$$

$$s_{5,5} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} = \tag{41}$$

III. OTHER HAMILTONIANS

A. Feynman Hamiltonian

Wavefunction Zoo

B. Electronic Structure Ansatze

1. Hartree Product

2. Configuration Interaction

3. Coupled Cluster

CC(n)

Bruckner-CC(n)

EOM-CC(n)

EOM-IP-CC(n)

FS-CC(n)

DLPNO-CC(n)

MR-EOM

4. Geminals

AP1roG/pCCD

APIG

APSetG

APG

Potential Zoo

C. Diatomics

Morse

Lenard-Jones

Morse/Long-range

Tiemann

Morse-Rosen

D. Triatomics

Jensen

Schwenke

PolyMLR

E. Torsion Potentials

Functional Zoo

Function Zoo

Master Equation Zoo

Particle Zoo

https://en.wikipedia.org/wiki/Particle_zoo

Complexity Zoo **(founded by Scott Aarsonson of UWaterloo)**

https://complexityzoo.uwaterloo.ca/Complexity_Zoo

Algorithms Zoo **(founded by Stephen Jordan of NIST)**

https://math.nist.gov/quantum/zoo/