Hamiltonian Zoo

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and Whoever Else Contributes
No Matter Where They're From

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I. MODEL HAMILTONIANS

1. *Coulomb*

$$H_{\text{Coulomb}} = \frac{q_1 q_2}{4\pi\epsilon_0 r} \tag{1}$$

A. Electronic Structure

1. *Born-Oppenheimer Model*

2. *Watson Model*

B. Fine and Hyperfine Structure

1. *Stark*

2. *Zeeman*

$$H_{\text{Zeeman}} = -\frac{\mu_B \left(g_l \vec{L} + g_s \vec{S} \right)}{\hbar} \cdot \vec{B} \tag{2}$$

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3. Complete magnetic dipole interaction

$$H_{\text{Magnetic Dipole}} = \frac{g_I \mu_N \mu_B \mu_0}{2\pi} \left(\frac{1}{L_z} \sum_i \frac{\hat{l}_{zi}}{r_i^3} \vec{I} \cdot \vec{L} + \frac{2g_s}{S_z} \sum_i \frac{\hat{s}_{zi}}{r_i^3} \left(3 \left(\vec{I} \cdot \hat{r} \right) \left(\vec{S} \cdot \hat{r} \right) - \vec{I} \cdot \vec{S} \right) + \frac{g_s}{3S_z} \sum_i \hat{s}_{zi} \delta^3 \left(\vec{r}_i \right) \vec{I} \cdot \vec{S} \right) \tag{3}$$

4. Frosch-Foley

5. Spin-orbit

6. Orbit-orbit

7. Spin-other-orbit (Gaunt)

8. Fermi Contact

C. Special Relativity

1. Dirac

2. Klein-Gordon

3. Darwin ($1e^-$)

$$H_{\text{Darwin}-1} = \alpha_{\text{FS}}^2 \frac{\pi Z}{2} \sum_i \delta \left(\vec{r}_i \right) \tag{4}$$

4. Darwin ($2e^-$)

$$H_{\text{Darwin}-2} = \alpha_{\text{FS}}^2 \pi \sum_{i<j} \delta \left(\vec{r}_{ij} \right) \tag{5}$$

5. Mass Velocity

$$H_{\text{Mass–Velocity}} = -\alpha_{\text{FS}}^2 \frac{1}{8} \sum_i \nabla_i^4 . \tag{6}$$

6. Breit-Pauli

7. Dirac-Coulomb-Breit

D. Quantum Electrodynamics

1. Aracki-Sucher

$$H_{\mathrm{Aracki-Sucher}} = -\alpha_{\mathrm{FS}}^{3} \frac{7}{6\pi} \sum_{i>j} \lim_{a \to 0} \left(\frac{\theta\left(r_{ij} - a\right)}{r_{ij}^{3}} + 4\pi \left(\gamma + \ln a\right) \delta(\vec{r}_{ij}) \right) \tag{7}$$

2. One-Loop

$$H_{1-\mathrm{loop}} = \alpha_{\mathrm{FS}}^{4} \pi Z^{2} \left(\frac{427}{96} - \ln 2 \right) \sum_{i} \delta\left(\vec{r}_{i}\right) \tag{8}$$

3. Schwinger-Dyson

E. Nuclear Motion

$$H_{\mathrm{Nuc}} = -\frac{\hbar^{2}}{2} \sum_{i=1}^{N} \sum_{\alpha=1}^{3} \frac{1}{M_{i}} \frac{\partial^{2}}{\partial R_{i\alpha}^{2}} + V\left(\boldsymbol{R}_{1}, \ldots, \boldsymbol{R}_{N}\right) \tag{9}$$

1. Rigid Rotor Harmonic Oscillator

2. Centrifugal distortion

3. Centrifugal distortion

F. Long-range Interactions

1. van der Waals

2. Lenard-Jones

3. Casimir-Polder

4. Resonance dipole-dipole

5. Meath

6. Axilrod-Teller

G. Hubbard Models

1. Bose-Hubbard (Boson Hubbard)

2. Fermi-Hubbard (Fermi Hubbard)

3. Jaynes-Cummings-Hubbard

4. Tavis-Cummings-Hubbard

H. Open Quantum Systems

1. Rabi model

2. Spin-Boson

3. Feynman-Vernon

4. Leggett-Caldeira

2. Richardson-Gaudin

3. Exactly solvable pairing Hamiltonians

SU(2), Rank 1 algebra

$$H_{\mathrm{SU}(2)} = \sum_i \epsilon_i n_i - g \sum_{ij} P_i^+ P_j \tag{11}$$

SO(5), Rank 2 algebra

$$H_{\mathrm{SO}(5)} = \sum_i \epsilon_i n_i - g \sum_{ijk} P_{ik}^+ P_{jk} \tag{12}$$

SO(8), Rank 4 algebra

$$H_{\mathrm{SO}(8)} = \sum_i \epsilon_i n_i - g_T \sum_{ijk} P_{ik}^+ P_{jk} - g_S \sum_{ijk} D_{ik}^+ D_{jk} \tag{13}$$

4. t-J model

J. Models of Superfluidity

1. 2D p-wave Fermi superfluid

$$H_{2DFSF} = \sum_i \epsilon_i a_i^\dagger a_i + \frac{\mathrm{i}\Delta_i}{2} \left(a_i^\dagger a_{-i}^\dagger + \mathrm{H.c.} \right) \tag{14}$$

K. Spin models

1. Ising

2. Transverse Ising

3. XY

4. XYZ

L. Heisenberg

M. J1-J2 Model

N. Majumdar-Ghosh

O. AKLT Model

P. Kitaev Models

1. Toric Code

2. Ocko-Yoshida

3. Honeycomb Model

$$H_{\text{Honeycomb}} = -J_x \sum_{x-\text{links}} x_i x_j - J_y \sum_{y-\text{links}} y_i y_j - J_z \sum_{z-\text{links}} z_i z_j \tag{15}$$

Matrix Zoo

Q. 2-level systems (spin-½ particles)

$$z \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, x \equiv \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, y \equiv \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix} \tag{16}$$

$$s_+ \equiv \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \frac{x + \mathrm{i}y}{2} \tag{17}$$

$$s_- \equiv \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = \frac{x - \mathrm{i}y}{2} \tag{18}$$

$$s_\alpha \equiv \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \frac{\mathbb{1} + z}{2} \equiv b \tag{19}$$

$$s_\beta \equiv \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \frac{\mathbb{1} - z}{2} \tag{20}$$

R. 3-level systems (spin-1 particles)

$$z_3 \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, x \equiv \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, y \equiv \begin{pmatrix} 0 & -\mathrm{i} & 0 \\ \mathrm{i} & 0 & -\mathrm{i} \\ 0 & \mathrm{i} & 0 \end{pmatrix} \tag{21}$$

$$\lambda_1 \equiv \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_2 \equiv \begin{pmatrix} 0 & -\mathrm{i} & 0 \\ \mathrm{i} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_3 \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_4 \equiv \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \tag{22}$$

$$\lambda_5 \equiv \begin{pmatrix} 0 & 0 & -\mathrm{i} \\ 0 & 0 & 0 \\ \mathrm{i} & 0 & 0 \end{pmatrix}, \lambda_6 \equiv \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \lambda_7 \equiv \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\mathrm{i} \\ 0 & \mathrm{i} & 0 \end{pmatrix}, \lambda_8 \equiv \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \tag{23}$$

$$s_{3+} \equiv \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} = \frac{x_3 + \mathrm{i}y_3}{2} \tag{24}$$

$$s_{3-} \equiv \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} = \frac{x_3 - \mathrm{i}y_3}{2} \tag{25}$$

$$s_{3,1} \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \frac{\mathbb{1} + z}{2} \equiv t \tag{26}$$

$$s_{3,2} \equiv \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{27}$$

$$s_{3,3} \equiv \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \frac{\mathbb{1} - z}{2} \equiv t \tag{28}$$

$$|1\rangle\langle 2| \equiv \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} = \frac{1}{2}\left(\lambda_6 + \mathrm{i}\lambda_7\right) \tag{29}$$

$$\lambda_7\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{30}$$

$$\lambda_7\lambda_5 = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_3\lambda_7 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_7\lambda_3 + \lambda_3\lambda_7 = \lambda_1 \tag{31}$$

$$\lambda_7\lambda_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} = -\lambda_6\lambda_1, \lambda_2\lambda_7 = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = -\lambda_1\lambda_6, \lambda_7\lambda_2 + \lambda_2\lambda_7 = -\lambda_4, \lambda_6\lambda_1 + \lambda_1\lambda_6 = \lambda_4 \tag{32}$$

$$\lambda_5^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = |\lambda_1| \tag{33}$$

$$\lambda_5\lambda_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \lambda_2\lambda_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_3\lambda_2 + \lambda_2\lambda_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = \lambda_6 \tag{34}$$

$$\lambda_2^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = |\lambda_3| \tag{35}$$

S. 4-level systems (spin-3/2 particles)

$$z_4 \equiv \begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -3 \end{pmatrix}, x_4 \equiv \begin{pmatrix} 0 & \sqrt{3} & 0 & 0 \\ \sqrt{3} & 0 & 2 & 0 \\ 0 & 2 & 0 & \sqrt{3} \\ 0 & 0 & \sqrt{3} & 0 \end{pmatrix}, y_4 \equiv \begin{pmatrix} 0 & -\sqrt{3}\mathrm{i} & 0 & 0 \\ \sqrt{3}\mathrm{i} & 0 & 2 & 0 \\ 0 & 2 & 0 & -\sqrt{3}\mathrm{i} \\ 0 & 0 & \sqrt{3}\mathrm{i} & 0 \end{pmatrix} \tag{36}$$

$$\gamma^0 \equiv \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \gamma^{1\equiv} \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & -1 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}, \gamma^2 \equiv \begin{pmatrix} 0 & 0 & 0 & -\mathrm{i} \\ 0 & 0 & \mathrm{i} & 0 \\ 0 & \mathrm{i} & 0 & 0 \\ -\mathrm{i} & 0 & 0 & 0 \end{pmatrix}, \gamma^3 \equiv \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \gamma^5 \equiv \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \tag{37}$$

$$s_{4+} \equiv \begin{pmatrix} 0 & \sqrt{3} & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & \sqrt{3} \\ 0 & 0 & 0 & 0 \end{pmatrix} \equiv \frac{x_{3/2} + \mathrm{i}y_{3/2}}{2} \equiv \frac{x_{4\times4} + \mathrm{i}y_{4\times4}}{2} \tag{38}$$

$$s_{4-} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 \\ \sqrt{3} & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & \sqrt{3} & 0 \end{pmatrix} \equiv \frac{x_4 - \mathrm{i}y_4}{2} \tag{39}$$

$$s_{4,1} \equiv \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \frac{1}{2}\left(z_2 + z_1 z_2 + z_1 - \mathbb{1}\right) \tag{40}$$

$$s_{4,2} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \tag{41}$$

$$s_{4,3} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \tag{42}$$

$$s_{4,4} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \tag{43}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \frac{1}{2}\left(\mathbb{1} - z_1 + z_1 z_2 + z_2\right) \tag{44}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \frac{1}{2}\left(\mathbb{1} - z_2 + z_1 z_2 + z_1\right) \tag{45}$$

$$|10\rangle\langle 01| = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \frac{1}{4}\left(x_1x_2 - \mathrm{i}y_1x_2 + \mathrm{i}x_1y_2 + y_1y_2\right) \tag{46}$$

$$|10\rangle\langle 01| = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \frac{1}{4}\left(x_1x_2 - \mathrm{i}x_1y_2 + \mathrm{i}y_1x_2 + y_1y_2\right) \tag{47}$$

$$\mathrm{CNOT} \equiv \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} = \frac{1}{2}\left(\mathbb{1} + z_1 + x_2 - z_1x_2\right) \tag{48}$$

T. 5-level systems (spin-2 particles)

$$z_5 \equiv \begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -2 \end{pmatrix}, x_5 \equiv \begin{pmatrix} 0 & 2 & 0 & 0 & 0 \\ 2 & 0 & \sqrt{6} & 0 & 0 \\ 0 & \sqrt{6} & 0 & \sqrt{6} & 0 \\ 0 & 0 & \sqrt{6} & 0 & 2 \\ 0 & 0 & 0 & 2 & 0 \end{pmatrix}, y_5 \equiv \begin{pmatrix} 0 & -2\mathrm{i} & 0 & 0 & 0 \\ 2\mathrm{i} & 0 & -\sqrt{6}\mathrm{i} & 0 & 0 \\ 0 & \sqrt{6}\mathrm{i} & 0 & -\sqrt{6}\mathrm{i} & 0 \\ 0 & 0 & \sqrt{6}\mathrm{i} & 0 & -2\mathrm{i} \\ 0 & 0 & 0 & 2\mathrm{i} & 0 \end{pmatrix} \tag{49}$$

$$s_{5+} \equiv \begin{pmatrix} 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{6} & 0 & 0 \\ 0 & 0 & 0 & \sqrt{6} & 0 \\ 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \equiv \frac{x_{5\times5} + \mathrm{i}y_{5\times5}}{2} \equiv \frac{x_{(5)} + \mathrm{i}y_{(5)}}{2} \tag{50}$$

$$s_{5-} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 \\ 0 & \sqrt{6} & 0 & 0 & 0 \\ 0 & 0 & \sqrt{6} & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \end{pmatrix} \equiv \frac{x_{5\times5} + \mathrm{i}y_{5\times5}}{2} \equiv \frac{x_{(5)} + \mathrm{i}y_{(5)}}{2} \tag{51}$$

$$s_{5,1} \equiv \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} = \tag{52}$$

$$s_{5,2} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} = \tag{53}$$

$$s_{5,3} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} = \tag{54}$$

$$s_{5,4} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} = \tag{55}$$

$$s_{5,5} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} = \tag{56}$$

U. 6-level systems (spin-3/2 particles)

$$|0_b 1_t\rangle\langle 1_b 0_t| \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} = \frac{1}{4}\left(x\lambda_1 + \mathrm{i}y\lambda_1 - \mathrm{i}x\lambda_2 + y\lambda_2\right) \tag{57}$$

$$|1_b 0_t\rangle\langle 0_b 1_t| \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} = \frac{1}{4}\left(x\lambda_1 - \mathrm{i}y\lambda_1 + \mathrm{i}x\lambda_2 + y\lambda_2\right) \tag{58}$$

$$|0_b 1_t\rangle\langle 1_b 0_t| + |1_b 0_t\rangle\langle 0_b 1_t| \equiv \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} = \frac{1}{2}\left(x\lambda_1 + y\lambda_2\right) \tag{59}$$

$$|2_t 0_b\rangle\langle 0_t 1_b| \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} = \tag{60}$$

$$|2_t 0_b\rangle\langle 0_t 1_b| + |0_t 1_b\rangle\langle 2_t 0_b| \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} = \frac{1}{2}\left(\lambda_5 y + \lambda_4 x\right) \tag{61}$$

$$|1_t 1_b\rangle\langle 2_t 1_b| \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} = \frac{1}{4}\left(\lambda_6 - z\lambda_6 + \mathrm{i}\lambda_7 - \mathrm{i}z\lambda_7\right) = |1_b 1_t\rangle\langle 1_b 2_t| \tag{62}$$

$$|1_t 1_b\rangle\langle 2_t 1_b| + |2_t 1_b\rangle\langle 1_t 1_b| \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} = \frac{1}{2}\left(\lambda_6 - z\lambda_6\right) \tag{63}$$

$$\equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} = \frac{1}{4}\left(z\lambda_1 + \lambda_1 - \mathrm{i}z\lambda_2 - \mathrm{i}\lambda_2\right) \tag{64}$$

$$\equiv \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} = \frac{1}{4}\left(z\lambda_1 + \lambda_1 + \mathrm{i}z\lambda_2 + \mathrm{i}\lambda_2\right) \tag{65}$$

$$\equiv \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} = \frac{1}{2}\left(z\lambda_1 + \lambda_1\right) \tag{66}$$

$$|0_b 2_t\rangle\langle 1_b 0_t| \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} = \frac{1}{4}\left(x\lambda_4 + \mathrm{i}y\lambda_4 - \mathrm{i}x\lambda_5 + y\lambda_5\right) \tag{67}$$

$$|0_b 2_t\rangle\langle 1_b 0_t| + |1_b 0_t\rangle\langle 0_b 2_t| \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} = \frac{1}{2}\left(x\lambda_4 + y\lambda_5\right) \tag{68}$$

$$\equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} = \frac{1}{4}\left(z\lambda_4 + \lambda_4 - \mathrm{i}z\lambda_5 - \mathrm{i}\lambda_5\right) \tag{69}$$

$$\equiv \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} = \frac{1}{4}\left(z\lambda_4 + \lambda_4\right) \tag{70}$$

$$|1_b 1_t\rangle\langle 1_b 2_t| \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} = \tag{71}$$

$$|1_b 1_t\rangle\langle 1_b 2_t| + |1_b 2_t\rangle\langle 1_b 1_t| \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} = \tag{72}$$

$$|1_{t,1}\rangle\langle 2_{t,1}|x_2 + |2_{t,1}\rangle\langle 1_{t,1}|x_2 \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} = \lambda_6 x_2 = -\frac{1}{2}z\lambda_1 + \frac{1}{2}\lambda_1 - \frac{1}{\sqrt{3}}x\lambda_8 + \frac{1}{3}x \neq x_2\lambda_6 \tag{73}$$

$$x_1|1_{t,2}\rangle\langle 2_{t,2}| \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} = \frac{1}{2}x\left(\lambda_6 + \mathrm{i}\lambda_7\right) \tag{74}$$

$$x_1|1_{t,2}\rangle\langle 2_{t,2}| + x_1|2_{t,2}\rangle\langle 1_{t,2}| \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} = x\lambda_6 \tag{75}$$

$$|1_t 0_b\rangle\langle 2_t 0_b| \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} = \frac{1}{4}\left(x\lambda_6 + \mathrm{i}y\lambda_6 - \mathrm{i}x\lambda_7 + y\lambda_7\right) = \frac{1}{4}\left(\lambda_6 + \lambda_6 z + \mathrm{i}\lambda_7 + \mathrm{i}\lambda_7 z\right) \neq \frac{1}{4}\left(\lambda_6 + z\lambda_6 + \mathrm{i}\lambda_7 + \mathrm{i}z\lambda_7\right) \tag{76}$$

$$|0_b 1_t\rangle\langle 0_b 2_t| \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \neq \frac{1}{4}\left(\lambda_6 + z\lambda_6 + \mathrm{i}\lambda_7 + \mathrm{i}z\lambda_7\right) = \frac{1}{4}\left(\lambda_6 + \lambda_6 z + \mathrm{i}\lambda_7 + \mathrm{i}\lambda_7 z\right) \tag{77}$$

$$|0_b 1_t\rangle\langle 0_b 2_t| + |0_b 2_t\rangle\langle 0_b 1_t| \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} = \frac{1}{2}\left(\lambda_6 + \lambda_6 z\right) \tag{78}$$

II. OTHER HAMILTONIANS

A. Feynman Hamiltonian

Wavefunction Zoo

B. Electronic Structure Ansatze

1. Hartree Product

2. Configuration Interaction

3. Coupled Cluster

CC(n)

Bruckner-CC(n)

EOM-CC(n)

EOM-IP-CC(n)

FS-CC(n)

DLPNO-CC(n)

MR-EOM

4. Geminals

AP1roG/pCCD

APIG

APSetG

APG

Potential Zoo

C. Diatomics

Morse

Lenard-Jones

Morse/Long-range

Tiemann

Morse-Rosen

D. Triatomics

Jensen

Schwenke

PolyMLR

E. Torsion Potentials

Functional Zoo

Function Zoo

Master Equation Zoo

Particle Zoo

https://en.wikipedia.org/wiki/Particle_zoo

Complexity Zoo **(founded by Scott Aarsonson of UWaterloo)**

https://complexityzoo.uwaterloo.ca/Complexity_Zoo

Algorithms Zoo **(founded by Stephen Jordan of NIST)**

https://math.nist.gov/quantum/zoo/