Hamiltonian Zoo

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No Matter Where They're From

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I. MODEL HAMILTONIANS

1. Coulomb

$$H_{\text{Coulomb}} = \frac{q_1 q_2}{4\pi\epsilon_0 r} \tag{1}$$

A. Electronic Structure

- 1. Born-Oppenheimer Model
 - 2. Watson Model

B. Fine and Hyperfine Structure

- 1. Stark
- 2. Zeeman

$$H_{\text{Zeeman}} = -\frac{\mu_B \left(g_l \vec{L} + g_s \vec{S} \right)}{\hbar} \cdot \vec{B}$$
 (2)

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3. Complete magnetic dipole interaction

$$H_{\text{Magnetic Dipole}} = \frac{g_{I}\mu_{N}\mu_{B}\mu_{0}}{2\pi} \left(\frac{1}{L_{z}} \sum_{i} \frac{\hat{l}_{zi}}{r_{i}^{3}} \vec{I} \cdot \vec{L} + \frac{2g_{s}}{S_{z}} \sum_{i} \frac{\hat{s}_{zi}}{r_{i}^{3}} \left(3\left(\vec{I} \cdot \hat{r}\right)\left(\vec{S} \cdot \hat{r}\right) - \vec{I} \cdot \vec{S} \right) + \frac{g_{s}}{3S_{z}} \sum_{i} \hat{s}_{zi} \delta^{3}\left(\vec{r}_{i}\right) \vec{I} \cdot \vec{S} \right)$$
(3)

- 4. Frosch-Foley
 - $5. \quad Spin-orbit$
 - 6. Orbit-orbit
- 7. Spin-other-orbit (Gaunt)
 - 8. Fermi Contact

C. Special Relativity

- 1. Dirac
- 2. Klein-Gordon
- 3. Darwin (1e⁻)
- 4. Darwin $(2e^-)$
- 5. Mass Velocity
 - $6. \quad \textit{Breit-Pauli}$
- 7. Dirac-Coulomb-Breit

D. Quantum Electrodynamics

1. Aracki-Sucher

$$H_{\text{Aracki-Sucher}} = -\frac{7\alpha_{\text{FS}}^3}{6\pi} \sum_{i>j} \lim_{a\to 0} \left(\frac{\theta(r_{ij} - a)}{r_{ij}^3} + 4\pi(\gamma + \ln a)\delta(\vec{r}_{ij}) \right)$$
(4)

2. One-Loop

3. Schwinger-Dyson

E. Nuclear Motion

$$H_{\text{Nuc}} = -\frac{\hbar^2}{2} \sum_{i=1}^{N} \sum_{\alpha=1}^{3} \frac{1}{M_i} \frac{\partial^2}{\partial R_{i\alpha}^2} + V(\mathbf{R}_1, \dots, \mathbf{R}_N)$$
 (5)

- 1. Rigid Rotor Harmonic Oscillator
 - 2. Centrifugal distortion
 - 3. Centrifugal distortion

F. Long-range Interactions

- 1. van der Waals
- 2. Lenard-Jones
- 3. Casimir-Polder
- 4. Resonance dipole-dipole
 - 5. Meath
 - 6. Axilrod-Teller

G. Hubbard Models

- 1. Bose-Hubbard (Boson Hubbard)
- 2. Fermi-Hubbard (Fermi Hubbard)
 - 3. Jaynes-Cummings-Hubbard

H. Open Quantum Systems

- 1. Rabi model
- 2. Spin-Boson
- 3. Feynman-Vernon
- 4. Leggett-Caldeira
- 5. Ishizaki-Fleming

2. Richardson-Gaudin

3. Exactly solvable pairing Hamiltonians

SU(2), Rank 1 algebra

$$H_{SU(2)} = \sum_{i} \epsilon_i n_i - g \sum_{ij} P_i^+ P_j \tag{7}$$

SO(5), Rank 2 algebra

$$H_{SO(5)} = \sum_{i} \epsilon_i n_i - g \sum_{ijk} P_{ik}^+ P_{jk}$$
(8)

SO(8), Rank 4 algebra

$$H_{SO(8)} = \sum_{i} \epsilon_{i} n_{i} - g_{T} \sum_{ijk} P_{ik}^{+} P_{jk} - g_{S} \sum_{ijk} D_{ik}^{+} D_{jk}$$
(9)

4. t-J model

J. Models of Superfluidity

1. 2D p-wave Fermi superfluid

$$H_{2DFSF} = \sum_{i} \epsilon_{i} a_{i}^{\dagger} a_{i} + \frac{i\Delta_{i}}{2} \left(a_{i}^{\dagger} a_{-i}^{\dagger} + \text{H.c.} \right)$$
 (10)

Matrix Zoo

- K. Ising
- L. Heisenberg
- M. J1-J2 Model
- N. Majumdar-Ghosh
 - O. AKLT Model
 - P. Kitaev Models
 - 1. Toric Code
 - 2. Ocko-Yoshida
 - 3. Honeycomb Model

$$H_{\text{Honeycomb}} = -J_x \sum_{x-\text{links}} x_i x_j - J_y \sum_{y-\text{links}} y_i y_j - J_z \sum_{z-\text{links}} z_i z_j$$
 (11)

Q. 2-level systems (spin-1/2 particles)

$$z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$
 (12)

$$s_{+} \equiv \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \frac{x + iy}{2} \tag{13}$$

$$s_{-} \equiv \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = \frac{x - iy}{2} \tag{14}$$

$$s_{\alpha} \equiv \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \frac{1 + z}{2} \equiv b \tag{15}$$

$$s_{\beta} \equiv \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \frac{1 - z}{2} \tag{16}$$

R. 3-level systems (spin-1 particles)

$$z_{3} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, x = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, y = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

$$(17)$$

$$\lambda_{1} \equiv \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_{2} \equiv \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_{3} \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_{4} \equiv \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \tag{18}$$

$$\lambda_5 \equiv \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \lambda_6 \equiv \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \lambda_7 \equiv \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \lambda_8 \equiv \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$
(19)

$$s_{3+} \equiv \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} = \frac{x_3 + iy_3}{2} \tag{20}$$

$$s_{3-} \equiv \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} = \frac{x_3 - iy_3}{2} \tag{21}$$

$$s_{3,1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \frac{1 + z}{2} \equiv t \tag{22}$$

$$s_{3,2} \equiv \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{23}$$

$$s_{3,3} \equiv \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \frac{1 - z}{2} \equiv t \tag{24}$$

S. 4-level systems (spin-3/2 particles)

$$z_{4} = \begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -3 \end{pmatrix}, x_{4} = \begin{pmatrix} 0 & \sqrt{3} & 0 & 0 \\ \sqrt{3} & 0 & 2 & 0 \\ 0 & 2 & 0 & \sqrt{3} \\ 0 & 0 & \sqrt{3} & 0 \end{pmatrix}, y_{4} = \begin{pmatrix} 0 & -\sqrt{3}i & 0 & 0 \\ \sqrt{3}i & 0 & 2 & 0 \\ 0 & 2 & 0 & -\sqrt{3}i \\ 0 & 0 & \sqrt{3}i & 0 \end{pmatrix}$$

$$(25)$$

$$\gamma^{0} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \gamma^{1} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & -1 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}, \gamma^{2} = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}, \gamma^{3} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \gamma^{5} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$(26)$$

$$s_{4+} \equiv \begin{pmatrix} 0 & \sqrt{3} & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & \sqrt{3} \\ 0 & 0 & 0 & 0 \end{pmatrix} \equiv \frac{x_{3/2} + iy_{3/2}}{2} \equiv \frac{x_{4\times 4} + iy_{4\times 4}}{2} \tag{27}$$

$$s_{4-} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 \\ \sqrt{3} & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & \sqrt{3} & 0 \end{pmatrix} \equiv \frac{x_4 - iy_4}{2} \tag{28}$$

$$\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix} = \frac{1}{2} \left(\mathbb{1} - z_1 + z_1 z_2 + z_2 \right) \tag{33}$$

$$\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix} = \frac{1}{2} \left(\mathbf{1} - z_2 + z_1 z_2 + z_1 \right) \tag{34}$$

$$|10\rangle\langle01| = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \frac{1}{4} \left(x_1 x_2 - i y_1 x_2 + i x_1 y_2 + y_1 y_2 \right)$$
(35)

$$|10\rangle\langle01| = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \frac{1}{4} \left(x_1 x_2 - i x_1 y_2 + i y_1 x_2 + y_1 y_2 \right)$$
(36)

$$CNOT = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0
\end{pmatrix} = \frac{1}{2} (1 + z_1 + x_2 - z_1 x_2) \tag{37}$$

T. 5-level systems (spin-2 particles)

$$s_{5+} \equiv \begin{pmatrix} 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{6} & 0 & 0 \\ 0 & 0 & 0 & \sqrt{6} & 0 \\ 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \equiv \frac{x_{5\times5} + iy_{5\times5}}{2} \equiv \frac{x_{(5)} + iy_{(5)}}{2}$$

$$(39)$$

$$s_{5-} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 \\ 0 & \sqrt{6} & 0 & 0 & 0 \\ 0 & 0 & \sqrt{6} & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \end{pmatrix} \equiv \frac{x_{5\times5} + iy_{5\times5}}{2} \equiv \frac{x_{(5)} + iy_{(5)}}{2}$$

$$(40)$$

U. 6-level systems (spin-3/2 particles)

II. OTHER HAMILTONIANS

A. Feynman Hamiltonian

Wavefunction Zoo

B. Electronic Structure Ansatze

- 1. Hartree Product
- 2. Configuration Interaction
 - 3. Coupled Cluster

CC(n)

Bruckner-CC(n)

EOM-CC(n)

EOM-IP-CC(n)

FS-CC(n)

DLPNO-CC(n)

MR-EOM

4. Geminals

AP1roG/pCCD

APIG

APSetG

APG

Potential Zoo

	C.	Diatomics
Morse		
Lenard-Jones		
$Morse/Long ext{-}range$		
Tiemann		
Morse-Rosen		
	D.	Triatomics
Jensen		
Schwenke		
PolyMLR		

E. Torsion Potentials

Functional Zoo

Function Zoo

Master Equation Zoo

Particle Zoo

https://en.wikipedia.org/wiki/Particle_zoo

$Complexity\ Zoo\ \ ({\rm founded\ by\ Scott\ Aarsonson\ of\ UWaterloo})$

 $https://complexityzoo.uwaterloo.ca/Complexity_Zoo$

$Algorithms \ Zoo \ \ ({\rm founded} \ {\rm by} \ {\rm Stephen} \ {\rm Jordan} \ {\rm of} \ {\rm NIST})$

https://math.nist.gov/quantum/zoo/