Homework assignment 4 – Symbolic Systems I – UvA, June 2020

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Question 1

<u>TBox</u>
$Person$
$Country$
$City$
$Republic \sqsubseteq Country$
$ConstitutionalMonarchy \sqsubseteq Country$
$Republic \sqcap ConstitutionalMonarchy \sqsubseteq \bot$
$Person \sqcap Country \sqsubseteq \bot$
$City \sqcap Country \sqsubseteq \bot$
$Person \sqcap City \sqsubseteq \bot$
$Country \sqsubseteq \exists hasCapital.City$
$Country \sqsubseteq \forall hasCapital.City$
$Country \sqsubseteq \exists hasHead.Person$
$City \sqsubseteq \exists hasHead.Person$
$President \sqsubseteq Person$
$Monarch \sqsubseteq Person$
$Mayor \sqsubseteq Person$
$ConstitutionalMonarchy \sqsubseteq \forall hasHead.Monarch$
$City \sqsubseteq \forall hasHead.Mayor$

<u>ABox</u>
$NL : Country$
$NL : ConstitutionalMonarchy$
$(NL, WillemAlexander) : hasHead$
$(Amsterdam, Femke) : hasHead$

Continued:
GCI's:

1. $(Republic \sqcap Country) \sqsubseteq \forall hasHead.\neg Mayor$
   It may be noted that republic is already a subset of country. So the following is correct as well:
   $Republic \sqsubseteq \forall hasHead.\neg Mayor$
   No, this does not follow from the knowledge base.

   $Republic \sqsubseteq Country$
   $Country \sqsubseteq \forall hasHead.Person$
   $President \sqsubseteq Person$
   $Monarch \sqsubseteq Person$
   $Mayor \sqsubseteq Person$

   From this KB it follows that a republic country always has a head which is a person. This person could in this knowledge base be of the set President, Monarch and/or Mayor. To make this statement follow from the knowledge base the following statements could be added:
   $Republic \sqsubseteq \forall hasHead.President \sqcup Monarch$
   $President \sqcap Mayor \sqsubseteq \bot$

$Monarch \sqcap Mayor \sqsubseteq \perp$

2. $Femke : Mayor$
   Yes, this follows from the knowledge base.
   $(Amsterdam, Femke) : hasHead$
   $City \sqsubseteq \forall hasHead.Mayor$
   Therefore, Femke is in the set Mayor.
3. $NL : \neg Republic$
   Yes, this follows from the knowledge base.
   $NL : ConstitutionalMonarchy$
   $Republic \sqcap ConstitutionalMonarchy \sqsubseteq \perp$
   So, NL cannot be Republic.
4. $WillemAlexander : \neg Mayor$
   No, this does not follow from the knowledge base.
   $(NL, WillemAlexander) : hasHead$
   $NL : ConstitutionalMonarchy$
   $ConstitutionalMonarchy \sqsubseteq \forall hasHead.Monarch$
   But, there is never mentioned that a Monarch cannot also be a Mayor.
   Therefore, the following statement needs to be added:
   $Monarch \sqcap Mayor \sqsubseteq \perp$

   For consistency, the following statements could be added but this does not influence the statement $WillemAlexander : \neg Mayor$.
   $Monarch \sqcap President \sqsubseteq \perp$
   $Mayor \sqcap President \sqsubseteq \perp$

Question 2

Constructing a complex concept of a 3-CNF formula can be done by replacing every propositional variable $x_1, ..., x_n$ in ϕ to the concept names $A_1, ..., A_n$.
So for instance if ϕ is :
$\phi = (x_1 \vee x_2 \vee \neg x_3) \wedge (x_3 \vee \neg x_4 \vee x_5)$
Then constructing this into a complex concept C_ϕ:
$C_\phi \equiv (A_1 \sqcup A_2 \sqcup \neg A_3) \sqcap (A_3 \sqcup \neg A_4 \sqcup A_5)$
Furthermore, we could set the first clause concept $C_1 \equiv A_1 \sqcup A_2 \sqcup \neg A_3$.

A 3-CNF formula is satisfiable if every clause in the formula is true. A clause in a 3-CNF formula is true if at least one literal, being x_i or $\neg x_i$, in the clause is true.
If for an interpretation $I = (\Delta^I, \cdot^I)$ x_i is true, then $A_i^I \neq \emptyset$. Furthermore, if in a clause concept one concept is non empty, then we can conclude that the entire clause is non empty: $C_1 \equiv A_1 \sqcup A_2 \sqcup \neg A_3$ and $A_1^I \neq \emptyset$ leads to $C_1^I \neq \emptyset$.
$C_\phi \equiv A_1 \sqcup A_2 \sqcup \neg A_3) \sqcap (A_3 \sqcup \neg A_4 \sqcup A_5) \equiv C_1 \sqcap C_2$
For this example, $C_\phi \neq \emptyset$ iff both $C_1 \neq \emptyset$ and $C_2 \neq \emptyset$. $C_1 \neq \emptyset$ and $C_2 \neq \emptyset$ iff for both clauses one literal is true. So therefore, if the 3-CNF formula ϕ is satisfiable there exists an interpretation I for which $C_\phi^I \neq \emptyset$

Question 3

For all $n \in \{1, ..., m\}$

KB_n
<u>TBox</u>
Add the TBox of all the KB_i, with $0 < i < n$
$A_{n-1} \sqsubseteq \forall r.((B_{n,a} \sqcap A_n) \sqcup (B_{n,b} \sqcap A_n))$

$\exists r.(B_{n,a} \sqcap A_n)$
$\exists r.(B_{n,b} \sqcap A_n)$

$B_{n,a} \sqcap B_{n,b} \sqsubseteq \perp$

$\forall r.B_{n,a}$
$\forall r.B_{n,b}$

<u>ABox</u>
$x : A_0$

Question 4