# ANALISI - ESERCITAZIONE

08.10.21

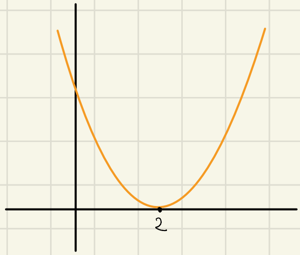
FUNZIONI

$f(x) = (x-2)^2$

$D: \mathbb{R}$

$Im: [0;+\infty)$

SURIETTIVA



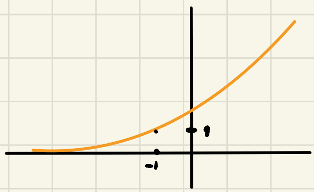

$g(x) = 2^{x+1}$

$D: \mathbb{R}$

$Im: (0;+\infty)$

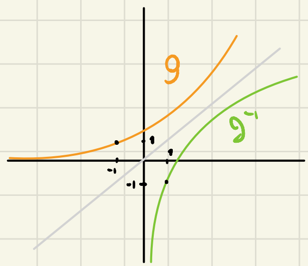
INIETTIVA
SURIETTIVA

BIUNIVOCA




$y = 2^{x+1}$ ; $x+1 = \log_2 y$ ; $x = \log_2 y - 1$

$g^{-1}(y) = \log_2 y - 1$




$h(x) = x^3 + 1$

$D: \mathbb{R}$

$Im: \mathbb{R}$

INIETTIVA
SURIETTIVA

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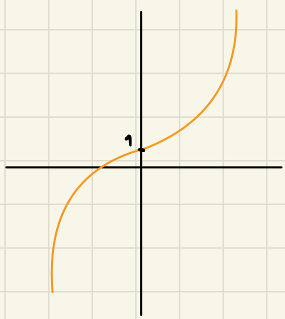
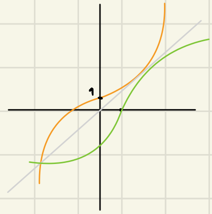


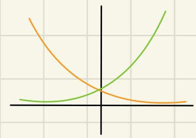
$y = x^3+1;\ x^3 = y-1;\ x = \sqrt[3]{y-1}$

$h^{-1}(y) = \sqrt[3]{y-1}$




$f(x) = e^{-x}$

$f^{-1}(y) = e^{x}$



$2^x \ln x > 1$

$$\frac{\cancel{2^x}\ln x}{\cancel{2^x}} > \frac{1}{2^x}$$

$\ln x > 2^{-x}$

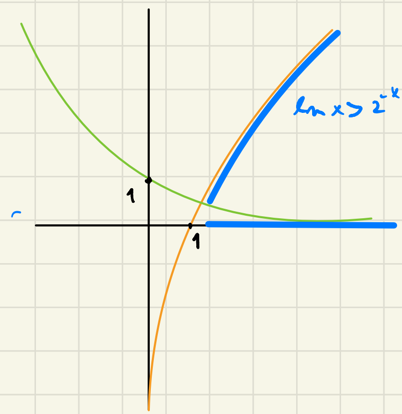
A) $(\alpha;+\infty)$ $\quad \alpha > 1$

B) $(\alpha;+\infty)$ $\quad \alpha \in (0;1)$

C) $(0;\alpha)$ $\quad \alpha > 1$

D) $(0;\alpha)$ $\quad \alpha \in (0;1)$

A




$e^{x}\sqrt[3]{x-1} > 1$

$\sqrt[3]{x-1} > e^{x}$

$(\alpha;+\infty)$ $\quad \alpha > 1$

$(f \circ g)(x) = f(g(x))$

$f(x) = e^{x}$ $\qquad f \circ g \Rightarrow f(1-x^2) = e^{1-x^2}$

$g(x) = 1 - x^2$ $\qquad g \circ f \Rightarrow g(e^x) = 1 - e^{2x}$

SUCCESSIONI

$$\lim_{n \to \infty} a^n = +\infty \qquad a > 1$$

$$\forall K > 0 \ \exists n_0 \in \mathbb{N} : \forall n \geq n_0 \ a^n > K$$

$K > 0$ FISSATO

$$a^n > K \iff n > \log_a K$$

$$n_0 = [\log_a K] + 1$$

PARTE INTERA

$$\lim_{n \to +\infty} a^n = 0 \qquad 0 < a < 1$$

$$\forall \varepsilon > 0 \ \exists n_0 \in \mathbb{N} : \forall n \geq n_0 \ |a^n| < \varepsilon$$

$\varepsilon$ FISSATO

$$|a^n| < \varepsilon \iff a^n < \varepsilon \iff n > \log_a \varepsilon$$

$$n_0 = [\log_a \varepsilon] + 1$$

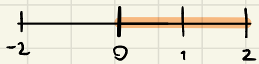
18.10.2021

1) $A = \{ |x| : x^2 + x < 2, \quad x \in \mathbb{R} \}$

$$x^2 - x - 2 < 0$$

$x_1 = -2$

$x_2 = 1$




INF = MIN = 0

SUP = +2

19.10.2021

studiare il carattere della seguente successione

$$\begin{cases} a_0 = 1 \\ a_{n+1} = 1 + \dfrac{a_n^2}{4} \end{cases}$$

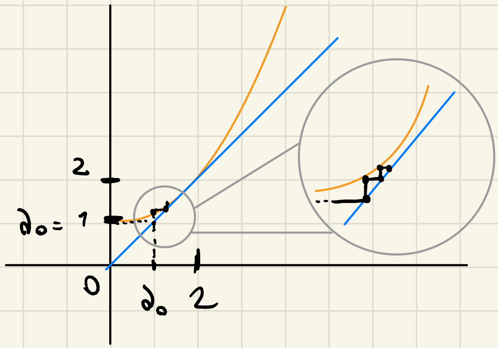
$$a_0 = 1 ; \quad a_1 = \frac{5}{4} ; \quad a_2 = 1 + \frac{25}{64}$$

è monotona crescente? $\Rightarrow$ $a_n \leq a_{n+1} \quad \forall n \in \mathbb{N}$ ?

$$a_n \leq 1 + \frac{a_n^2}{4} ; \quad a_n^2 - 4a_n + 4 \geq 0 ;$$

$$(a_n - 2)^2 \geq 0 \Rightarrow \forall n \in \mathbb{N}$$

$$a_{n+1} = f(a_n) \quad \text{dove} \quad f(x) = 1 + \frac{x^2}{4}$$




ricerco i punti fissi

$$1 + \frac{x^2}{4} = x \; ; \; (x-2)^2 = 0$$

$$x = 2$$

proviamo che $a_n \leq 2 \quad \forall n$

- $a_0 \leq 2$
- - $a_n \leq 2$ provo $a_{n+1} \leq 2$

$$a_{n+1} \Rightarrow 1 + \frac{a_n^2}{4} \leq 1 + \frac{4}{4} \leq 2$$

$$a_n \longrightarrow \ell \in \mathbb{R}$$

$$\begin{cases} a_0 = 1 \\ a_{n+1} = \sqrt{a_n} + 2 \end{cases} \Rightarrow f(x) = \sqrt{x} + 1$$

è ben definita? $a_n \geq 0 \quad \forall n \in \mathbb{N}$

- $a_0 \geq 0$
- SIA $a_n \geq 0$

ALLORA $a_{n+1} = \sqrt{a_n} + 2 \geq 0$

è monotona?

| | | |
|---|---|---|
| $1$ | $3$ | $\sqrt{3} + 2$ |
| $a_0$ | $a_1$ | $a_2$ |

è vero che $a_{n+1} \geq a_n \quad \forall n \in \mathbb{N}$?

$a_n \leq \sqrt{a_n} + 2 \; ; \; a_n - \sqrt{a_n} - 2 \leq 0$

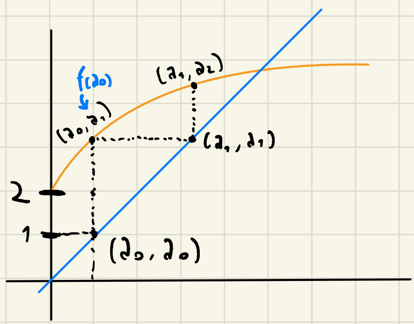
$$\sqrt{a_n} = \frac{1 \pm \sqrt{8+1}}{2} \left\langle \begin{array}{l} -1 \\ 2 \end{array} \right. \Rightarrow \sqrt{a_n} \leq 2$$

la successione è monotona crescente se e solo se $\sqrt{a_n} \leq 2$

proviamo che $\sqrt{a_n} \leq 2 \quad \forall n \in \mathbb{N}$

- $\sqrt{a_0} \leq 2$ ; $a_0 \leq 4$
- $a_{n+1} \leq 4$

$$a_{n+1} = \underbrace{\sqrt{a_n}}_{\leq 2\ \forall} + 2 \leq 4 \implies$$ è monotona crescente




punti fissi

$$\sqrt{x} + 2 = x \ ;$$

$$x = 4$$

$$a_n \longrightarrow \ell \in \mathbb{R}$$

22.10.2021

## SERIE GEOMETRICHE

$$\sum_{n=0}^{+\infty} 2\cdot 4^{-n} = 2\sum_{n=0}^{+\infty}\left(\frac{1}{4}\right)^n = 2\,\frac{1}{1-\frac{1}{4}} = 2\,\frac{4}{3} = \frac{8}{3}$$

$$\sum_{n=1}^{+\infty} 3^{1-n} = \sum_{n=1}^{+\infty} 3\cdot 3^{-n} = 3\sum_{n=1}^{+\infty}\left(\frac{1}{3}\right)^n = 3\left(\sum_{n=0}^{+\infty}\left(\frac{1}{3}\right)^n - \left(\frac{1}{3}\right)^0\right) =$$

$$3\cdot\left(\frac{1}{1-\frac{1}{3}} - 1\right) = 3\left(\frac{3}{2} - 1\right) = \frac{3}{2}$$

$$\sum_{n=2}^{+\infty} e^{1-2n} = e\sum_{n=2}^{+\infty} e^{-2n} = e\sum_{n=2}\left(\frac{1}{e^2}\right)^n =$$

$$e\left(\sum_{n=0}^{+\infty}\left(\frac{1}{e^2}\right)^n - \left(\frac{1}{e^2}\right)^0 - \left(\frac{1}{e^2}\right)^1\right) = e\left(\frac{1}{1-\frac{1}{e^2}} - 1 - \frac{1}{e^2}\right) =$$

$$= e\left(\frac{e^2}{e^2-1} - 1 - \frac{1}{e^2}\right) = \frac{e^2}{e^2-1} - e - \frac{1}{e}$$

$$\sum_{n=1}^{+\infty}\frac{2^{n+1}}{3^n} = \sum_{n=1}^{+\infty}\frac{2^n\cdot 2}{3^n} = 2\sum_{n=1}^{+\infty}\left(\frac{2}{3}\right)^n = 2\left(\sum_{n=0}^{+\infty}\left(\frac{2}{3}\right)^n - 1\right) = 4$$

$$\sum_{n=0}^{+\infty} (-1)^n \frac{3^n}{2^{2n-1}} = \sum_{n=0}^{+\infty} (-1)^n \frac{3^n}{2^{2n} \cdot 2^{-1}} = 2 \sum_{n=0}^{+\infty} (-1)^n \cdot \frac{3^n}{4^n} = 2 \sum_{n=0}^{+\infty} \left(-\frac{3}{4}\right)^n = 2 \cdot \frac{1}{1+\frac{3}{4}} =$$

$$= 2 \cdot \frac{4}{7} = \frac{8}{7}$$

SERIE TELESCOPICHE

$$\sum_{n=1}^{+\infty} \frac{1}{n^2+2n} = \sum_{n=1}^{+\infty} \frac{1/2}{n} + \frac{-1/2}{n+2} = \sum_{n=1}^{+\infty} \left(1/2\left(\frac{1}{n} - \frac{1}{n+2}\right)\right) = \frac{1}{2} \sum_{n=1}^{+\infty} \frac{1}{n} - \frac{1}{n+2}$$

$$\frac{1}{n(n+2)} = \frac{A}{n} + \frac{B}{n+2} = \frac{\overset{An+2A}{A(n+2)+B(n)}}{n(n+2)} = \frac{(A+B)n+2A}{n(n+2)} \quad \begin{cases} A+B=0 \;:\; B=-\frac{1}{2} \\ 2A=1 \;:\; A=\frac{1}{2} \end{cases}$$

$$S_1 = \frac{1}{2}\left(\frac{1}{1} - \frac{1}{3}\right)$$

$$S_2 = \frac{1}{2}\left(1 - \frac{1}{3} + \frac{1}{2} - \frac{1}{4}\right)$$

$$S_3 = \frac{1}{2}\left(1 - \cancel{\frac{1}{3}} + \frac{1}{2} - \frac{1}{4} + \cancel{\frac{1}{3}} - \frac{1}{5}\right)$$

$$S_4 = \frac{1}{2}\left(1 - \cancel{\frac{1}{3}} + \frac{1}{2} - \cancel{\frac{1}{4}} + \cancel{\frac{1}{3}} - \frac{1}{5} + \cancel{\frac{1}{4}} - \frac{1}{6}\right)$$

RIMANGONO

$$S_k = \frac{1}{2}\left(1 + \frac{1}{2} + \cdots + \cancel{\frac{1}{k-1}} - \frac{1}{k+1} + \cancel{\frac{1}{k}} - \frac{1}{k+2}\right)$$

$$S_k = \frac{1}{2}\left(1 + \frac{1}{2} + \frac{1}{k+1} + \frac{1}{k+2}\right)$$

$$\sum_{n=1}^{+\infty} \frac{1}{n^2+2n} = \lim_{k \to \infty} \frac{1}{2}\left(1 + \frac{1}{2} + \frac{1}{k+1} + \frac{1}{k+2}\right) =$$

# SERIE ARMONICHE

$$\sum_{n=2}^{+\infty} \frac{1}{n^{\alpha} \cdot \ln^{\beta} n}$$

convergente $\nearrow$ $\alpha > 1 \vee (\alpha = 1 \wedge \beta > 1)$

divergente $\searrow$ $\alpha < 1 \vee (\alpha = 1 \wedge \beta < 1$

$$\sum_{n=2}^{+\infty} \frac{n^{2\alpha}}{\ln n} = \sum_{n=2}^{+\infty} \frac{1}{n^{-2\alpha} \ln n}$$

CONVERGE $\iff -2\alpha > 1 \iff \alpha < -\frac{1}{2}$

29.10.2021

1) $\displaystyle\sum_{n=1}^{+\infty} \ln\left(\frac{n^4+1}{n^4}\right)$ $\qquad$ $\displaystyle\ln\left(\frac{n^4+1}{n^4}\right) = a_n$

$$= \ln\left(1+\frac{1}{n^4}\right) \sim \frac{1}{n^4} \qquad \sum_{n=1}^{+\infty} \frac{1}{n^4} \text{ converge}$$

2) $\displaystyle\sum_{n=1}^{+\infty} \frac{1}{\sqrt{n}} \arctan\left(\frac{1}{\sqrt{n}}\right)$ $\qquad$ $\displaystyle\frac{1}{\sqrt{n}} \arctan\left(\frac{1}{\sqrt{n}}\right) \sim \frac{1}{\sqrt{n}} \cdot \frac{1}{\sqrt{n}} = \frac{1}{n}$

$$\sum_{n=1}^{+\infty} \frac{1}{n} \quad \text{diverge}$$

3) $\displaystyle\sum_{n=4}^{+\infty} \frac{4+\sin n}{\sqrt{n^7-n^4}}$ $\qquad$ $-1 \le \sin n \le 1$

$$3 \le 4+\sin n \le 5$$

$$\frac{3}{\sqrt{n^7-n^4}} \le \frac{4+\sin n}{\sqrt{n^7-n^4}} \le \frac{5}{\sqrt{n^7-n^4}}$$

$$\frac{3}{\sqrt{n^7-n^4}} \sim \frac{3}{\sqrt{n^7}} = \frac{3}{\sqrt{n^{7/2}}}$$

$$\frac{5}{\sqrt{n^7 - n^3}} \sim \frac{5}{n^{7/2}} \qquad \sum \frac{1}{n^{7/2}} \text{ CONVERGE}$$

4) $$\sum_{n=2}^{+\infty} \frac{n^2}{2^n} \qquad a_n = \frac{n^2}{2^n} \qquad a_{n+1} = \frac{(n+1)^2}{2^{n+1}}$$

$$\lim_{n \to +\infty} \frac{a_{n+1}}{a_n} = \frac{(n+1)^2}{2^{n+1}} \cdot \frac{2^n}{n^2} = \frac{(n+1)^2}{\cancel{2^n} \cdot 2} \cdot \frac{\cancel{2^n}}{n^2} =$$

$$= \frac{(n+1)^2}{2n^2} \sim \frac{\cancel{n^2}}{2\cancel{n^2}} = \frac{1}{2} \qquad \text{CONVERGE}$$

LIMITI DI FUNZIONI

5) $$\lim_{x \to -\infty} \frac{x^3 - 4x}{5 - x^5} = \lim_{x \to -\infty} \frac{\cancel{x^3}}{-\cancel{x^5}} = \lim_{x \to -\infty} \left(-\frac{1}{x^2}\right) = 0$$

6) $$\lim_{x \to -\infty} \frac{\overset{0}{e^x} + \overset{-\infty}{x^3}}{\underbrace{x^2 + \ln(-x)}_{+\infty}} = \lim_{x \to -\infty} x = -\infty$$

7) $$\lim_{x \to 2} \frac{x^3 - x^2 + 2x - 8}{x^2 - 5x + 6} = \frac{0}{0}$$

TEOREMA ANNULATE RUFFINI

$(x-2)(x-3)$

$$\begin{array}{c|ccc|c} & 1 & -1 & 2 & -8 \\ 2 & & 2 & 2 & 8 \\ \hline & 1 & 1 & 4 & 0 \end{array}$$

$(x-2)(x^3 + x^2 + 4)$

$$\lim_{x \to 2} \frac{\cancel{(x-2)}(x^3 + x^2 + 4)}{\cancel{(x-2)}(x-3)} = \frac{10}{-1} = -10$$

8) $$\lim_{x \to 1^-} \frac{x^2 - 1}{x^3 - 3x + 2} = \frac{0}{0}$$

$$\begin{array}{c|ccc|c} & 1 & 0 & -3 & 2 \\ 1 & & 1 & 1 & -2 \\ \hline & 1 & 1 & -2 & 0 \end{array}$$

$$\lim_{x \to 1^-} \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}(\underbrace{x^2 + x - 2}_{<2})} = \frac{2^+}{0^-} = -\infty$$

9) $$\lim_{x \to 1} \frac{\sqrt{x+3} - 2}{x^2 - 1} = \frac{0}{0}$$

$$\frac{\sqrt{x+3} - 2}{x^2 - 1} \cdot \frac{\sqrt{x+3} + 2}{\sqrt{x+3} + 2} =$$

$$= \frac{x + 3 - 4}{(x^2 - 1)(\sqrt{x+3} + 2)} = \frac{\cancel{x - 1}}{(x^2 - 1)(\sqrt{x+3} + 2)} = \frac{1}{8}$$

$(x-1)(x+1)$

19.11.2021

## ASINTOTI

$y = L$ è asintoto orizzontale se $\lim_{x \to +\infty} f(x) = L$

$y = x_0$ è asintoto verticale se $\lim_{x \to x_0} f(x) = \pm\infty$

$y = mx + q$ è asintoto obliquo se $\lim_{x \to x_0} [f(x) - (mx+q)] = 0$

1) $\lim_{x \to \pm\infty} f(x) = \pm\infty$

2) $\exists \lim_{x \to \pm\infty} \frac{f(x)}{x} = m \in \mathbb{R} \setminus \{0\}$

3) $\exists f(x) - mx = q \in \mathbb{R}$

26.11.2021

$$f(x) = \begin{cases} a x + x^2 & x \leq 0 \\ b e^x + \sin x - 1 & x > 0 \end{cases}$$

$$f(0) = \lim_{x \to 0^-} f(x) = 0 \qquad \lim_{x \to 0^+} f(x) = b - 1$$

$$b - 1 = 0 \; ; \quad b = 1$$

$$f'(x) = \begin{cases} a + 2x & x < 0 \\ b e^x + \cos x & x > 0 \end{cases}$$

$$\lim_{x \to 0} f'(x) = a \qquad \lim_{x \to 0^+} f(x) = b + 1$$

$$a = b + 1 = 2$$

$$b = 1$$

$$f(x) = \begin{cases} e^{ax+b} & x \le 0 \\ 2\arctan x + a & x > 0 \end{cases}$$

$$f(0) = \lim_{x \to 0^-} f(x) = e^b \qquad \lim_{x \to 0^+} f(x) = a$$

$$e^b = a$$

$$f'(x) = \begin{cases} a e^{ax+b} & x < 0 \\ 2 \cdot \dfrac{1}{1+x^2} & x > 0 \end{cases}$$

$$\lim_{x \to 0^-} f'(x) = a e^b \qquad \lim_{x \to 0^+} f'(x) = 2$$

$$\begin{cases} a e^b = 2 \Rightarrow a^2 = 2 = \pm\sqrt{2} \\ b = e^a \end{cases} \qquad \begin{cases} a = \sqrt{2} \\ e^b = \sqrt{2} \end{cases} \qquad \begin{cases} a = -\sqrt{2} \\ e^b = -\sqrt{2} \end{cases}$$

IMP

$$\begin{cases} a = \sqrt{2} \\ b = \ln\sqrt{2} \end{cases}$$

$$f(x) = \begin{cases} \ln(1+x) & -1 < x < 1 \\ 1 + ax + bx^2 & x \geq 1 \end{cases}$$

DERIVABILE PER $x > -1$

$$f(x) = x e^{|x+2|} \qquad D: \mathbb{R}$$

$f$ È CONTINUA SU $\mathbb{R}$

$$f(x) = \begin{cases} x e^{x+2} & x+2 \geq 0 \; ; \; x \geq -2 \\ x e^{-x-2} & x+2 < 0 \; ; \; x < -2 \end{cases}$$

$$f'(x) = e^{x+2} + x e^{x+2} \qquad x > -2$$

$$f'(x) = e^{-2-x} - x e^{-x-2} \qquad x < -2$$

$$f'(x) = \begin{cases} e^{x+2}(x+1) & x > -2 \\ e^{-x-2}(1-x) & x < -2 \end{cases}$$

$$\lim_{x \to -2^-} f'(x) = 3 \qquad \lim_{x \to -2^+} f'(x) = -1$$

$\Rightarrow x = -2$ PUNTO ANGOLOSO

$f(x) = \sqrt[3]{x^2(x-1)}$ $\qquad D = \mathbb{R}$

$$f'(x) = \frac{1}{3}\left[x^2(x-1)\right]^{-\frac{2}{3}}\left[2x(x-1) + x^2\right] =$$

$$= \frac{3x^2 - 2x}{3\sqrt[3]{x^4(x-1)^2}} \Rightarrow \begin{array}{l} x \neq 0 \\ x \neq 1 \end{array}$$

$$\lim_{x \to 0} \frac{3x^2-2x}{3\sqrt[3]{x^4(x-1)^2}} = \frac{0}{0} = \lim_{x \to 0^{\pm}} \frac{\cancel{x}(3x-2)}{3\cancel{x}\sqrt[3]{x(x-1)^2}} = \frac{-2}{0^{\pm}} = \mp\infty$$

$x = 0$ CUSPIDE

$$\lim_{x \to 1^{\pm}} \frac{3x^2-2x}{3\sqrt[3]{x^4(x-1)^2}} = \frac{1}{0^+} = +\infty$$

$x = 1$ FLESSO TAN. VERTICALE

Rolle e Lagrange

$$f(x) = \begin{cases} x-1 & 0 \leq x \leq 1 \\ \ln x & 1 < x \leq 2 \end{cases} \qquad [0; 2]$$

$$f(1) = \lim_{x \to 1^-} f(x) = 0 \qquad \lim_{x \to 1^+} f(x) = 0$$

$f$ continua in $[0; 2]$

$$f'(x) = \begin{cases} 1 & 0 < x < 1 \\ \frac{1}{x} & 1 < x < 2 \end{cases} \qquad (0; 2)$$

$$f'(1) = \lim_{x \to 1^-} f'(x) = 1 \qquad \lim_{x \to 1^+} f'(x) = 1$$

$f$ derivabile in $(0; 2)$

LAGRANGE $\qquad \exists\, c \in (0; 2) \;/\; f'(c) = \dfrac{f(2) - f(0)}{2-0}$

$$f'(c) = \frac{\ln 2 + 1}{2}$$

$$f'(x) = \begin{cases} 1 & ✗ \\ \frac{1}{x} & ✓ \end{cases} \qquad 1 < c < 2 \implies \frac{1}{c} = \frac{\ln 2 + 1}{2};$$

$$c = \frac{2}{\ln 2 + 1}$$

Taylor

$$f(x) = \frac{x-3}{x^2+1} \qquad x_0 = 1 \qquad \text{OR} = 2$$

$$f(1) = \frac{1-3}{1+1} = -1 \qquad f'(x) = \frac{-x^2+6x+1}{(x^2+1)^2}$$

$$f'(1) = \frac{3}{2} \qquad f'' = \frac{(-2x+6)(x^2+1)^2 - 2(x^2+1)\cdot 2x \cdot (-x^2+6x+1)}{(x^2+1)^4}$$

$$f''(1) = \frac{4\cdot 4 - 2\cdot 2\cdot 6}{16} = \frac{16-48}{16} = \frac{-32}{16} = -2$$

$$P(x) = -1 + \frac{3}{2}(x-1) \; \frac{-2}{2!} \; (x-1)$$

**29.12.2021**

Esericizi per esame

Gerarchie delle successioni

Infiniti

$\ln n \quad n^b \quad a^n \quad n! \quad n^n$

Infinitesimi

$\frac{1}{n^n} \quad \frac{1}{n!} \quad \frac{1}{a^n} \quad \frac{1}{n^b} \quad \frac{1}{\ln n}$

$$\lim_{n \to \infty} \left(\frac{n+2}{n+1}\right)^{n^2} = \frac{\cancel{n}\left(1+\frac{2}{n}\right)}{\cancel{n}\left(1+\frac{1}{n}\right)} = \frac{\left(1+\frac{2}{n}\right)^{n^2}}{e^n} = \frac{e^{2n}}{e^n}$$

$$\frac{e^{2n}}{e^n} = e^{2n-n} = e^n \to +\infty$$

# 07.01.2022

$$\sum_{n=1}^{\infty} \frac{n+3}{2n^3+2n+2}$$

TEO. RAPP.

$$\lim_{n\to+\infty} \frac{n+1+3}{\cancel{2}(n+1)^3+\cancel{2}(n+1)+\cancel{2}} \cdot \frac{\cancel{2}n^3+\cancel{2}n+\cancel{2}}{n+3} =$$

$$\frac{n+4}{(n^3+1+3n^2+3n)+(n+1)+1} \cdot \frac{n^3+n+1}{n+3} = \frac{\cancel{n^4}}{\cancel{n^4}} = 1$$

TEO. RADICE.

$$\lim_{n\to+\infty} \left(\frac{n+3}{2n^3+2n+2}\right)^{\frac{1}{n}} = 1$$

S.2

$$\sum \frac{\sqrt[3]{n}}{\sqrt{n^2+n+1}} \qquad \frac{\sqrt[3]{n+1}}{\sqrt{(n+1)^2+n+1+1}} \cdot \frac{\sqrt{n^2+n+1}}{\sqrt[3]{n}}$$

TEO. RAPP.

LIMITE SUCC.

$$\lim_{n\to+\infty} \frac{\sqrt[3]{n}}{\sqrt{n^2+n+1}} \sim \frac{n^{\frac{1}{3}}}{n^2} = 0$$

TEO. RAD. $$\sqrt[n]{\frac{\sqrt[3]{n}}{\sqrt{n^2+n+1}}} = \frac{\sqrt[3n]{n}}{\sqrt[2n]{n^2+n+1}} \sim \frac{n^{\frac{1}{3n}}}{n^{\frac{1}{n}}} = \frac{n^0}{n^0} = 1$$

**11.01.2022**

1) $f(x) = (x^2 - x)e^{-x}$

$$f'(x) = \frac{(x^2 - x)}{e^x} = \frac{(2x-1)e^x - e^x(x^2-x)}{e^{x2}} = \frac{e^x(2x-1-x^2+x)}{e^{x2}} =$$

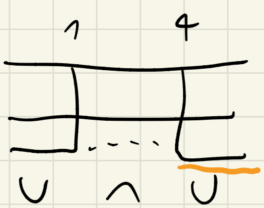
$$= \frac{\cancel{e^x}(3x - x^2 - 1)}{e^{x\cancel{x}}}$$

$$f''(x) = \frac{(3-2x)e^x - e^x(3x - x^2 - 1)}{e^{2x}} = \frac{\cancel{e^x}(3-2x-3x+x^2+1)}{e^{\cancel{2}x}} =$$

$$\frac{(4 - 5x + x^2)}{e^x} \geq 0 \qquad x^2 - 5x + 4 \geq 0$$

$\Delta = 25 - 16 = 9$

$x_{12} = \frac{5 \pm 3}{2} \left< \begin{matrix} 4 \\ 1 \end{matrix} \right.$




$e^x > 0 \quad \forall x \in \mathbb{R}$

$(k, +\infty)$

$k = 4$

2) $$\int_0^1 \frac{(x^2-x)e^{-x}}{x} = \int_0^1 \frac{x^2-x}{e^x x} = \frac{\cancel{x}(x-1)}{\cancel{x}e^x} = \frac{x-1}{e^x} =$$

$$\int_0^1 \frac{x}{e^x} - \int \frac{1}{e^x} =$$

$f(x) = x \quad , \quad f' = 1$

$\frac{1}{e^x} = g'(x) \; , \quad g(x) = -\frac{1}{e^x}$

$$-\frac{x}{e^x} + \int \frac{1}{e^x}$$

$$-\frac{x}{e^x} \cancel{-\frac{1}{e^x}} \cancel{+\frac{1}{e^x}}$$

$$\left[-\frac{1}{e^x}\right]_1^0 = -\frac{1}{e} - -\frac{0}{1} = -\frac{1}{e}$$

3) $$\sum_{n=1}^{+\infty} q^n$$

CONVERGE

$|q| < 1 \quad -1 \;\; 0 \;\; +1$

$(-1 ; +1)$

DIVERGE A $+\infty$

$q \geq 1 \; ; \quad [1, +\infty)$

$$\sum_{n=1}^{+\infty} (2e^x - 3)^n \qquad x \in \mathbb{R}$$

1. CONVERGE

$2e^x - 3 > -1$ ; $2e^x > +2$ ; $e^x > 1$ ; $x > \ln(1)$ ; $x > 0$

$2e^x - 3 < 1$ ; $2e^x < 4$ ; $e^x < 2$ ; $x < \ln(2)$

$(0; \ln(2))$

2. DIVERGE

$2e^x - 3 \geq 1 = e^x \geq \ln(2) \qquad [\ln(2); +\infty)$

3)

$$a_n = \frac{n^2 + \ln(n)}{n\sqrt{n} + e^{-n}} \sim \frac{n^2}{n\sqrt{n}} \qquad b_n = \frac{n\sqrt{n} - \ln n}{n + e^n} \sim \frac{n\sqrt{n}}{e^n}$$

$$\frac{a_n}{b_n} = \frac{n^2}{n\sqrt{n}} \cdot \frac{e^n}{n\sqrt{n}} = \lim_{x \to \infty} \frac{+e^n}{n^k} \to +\infty$$

$$\frac{b_n}{s_n} = \frac{n\sqrt{n}}{e^n} \cdot \frac{n\sqrt{n}}{n^4} = \frac{n^{\cancel{3}}}{\cancel{n}\, e^n} \to 0$$

$$b_n = o(s_n)$$

$$\begin{cases} a_1 = 1 \\ a_{n+1} = 1 + a_n^2/4 \end{cases}$$

① monotona crescente

$$\forall n \quad a_{n+1} \geq a_n$$

$$1 + \frac{a_n^2}{4} \geq a_n \, ; \quad 1 + \frac{a_n^2}{4} - a_n \geq 0$$

$$a_n^2 - 4a_n + 1 \geq 0 \, ; \quad (a_n - 2)^2 \geq 0 \quad \checkmark$$

② limitata $\quad \forall n \in \mathbb{N} \quad 1 \leq a_n \leq 2$

$$a_n \geq 1$$

① $a_1 = 1 \geq 1$

② $a_n \ge 1$

$\Rightarrow a_{n+1} \ge a_n \ge 1$

$a_n \le 2$

① $a_1 = 1 \le 2$

② $a_n \le 2$

$\Rightarrow a_{n+1} = 1 + \frac{a_n^2}{4} \le 1 + \frac{2^2}{4} = 1 + 1 = 2$

$\lim_{n \to +\infty} a_n = L \qquad \lim_{x \to +\infty} a_{n+1} = L$

$\lim_{n \to +\infty} 1 - \frac{a_n^2}{4} = 1 + \frac{2^2}{4} = L; \qquad L = 2$

$$\sum_{n=2}^{+\infty} \frac{(1-x)^{2n}}{n \ln^2 n}$$

modulo, termini positivi

$$\frac{|1+x|^{2n+2}}{(n+1)\ln^2(n+1)} \cdot \frac{n \ln^2 n}{|1+x|^{2n}} \to 1 \quad ; \quad |1+x|<1, \quad 0 \le x \le 2$$

SNUCR GS

## 14.01.2022

$$f(x) = \sqrt{x-4} - \frac{x}{2} \qquad D: x-4 \geq 0, \quad x \geq 4 \quad [4; +\infty)$$

$$\sqrt{x-4} - \frac{x}{2} = 0; \qquad \sqrt{x-4} = \frac{x}{2}; \qquad x-4 = \frac{x^2}{4};$$

$$4x - 16 = x^2; \quad -x^2 + 4x - 16 = 0; \quad x^2 - 4x + 16 = 0$$

$$\Delta < 0 \quad \nexists x \in \mathbb{R}$$

$$\sqrt{x-4} > \frac{x}{2}$$

$$\begin{cases} x-4 \geq 0 \\ \frac{x}{2} \geq 0 \\ x-4 > \frac{x^2}{4} \end{cases} \cup \begin{cases} x-4 \geq 0 \\ \frac{x}{2} < 0 \end{cases}$$

$$\nexists \in \mathbb{R} \qquad\qquad \begin{matrix} x \geq 4 \\ x < 0 \end{matrix} \quad \nexists x \in \mathbb{R}$$

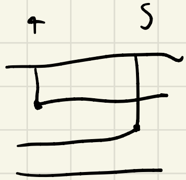
$$\lim_{x\to 4^+}\left(\sqrt{x-4}-\frac{x}{2}\right) = -2$$

$$\lim_{x\to +\infty}\left(\sqrt{x-4}-\frac{x}{2}\right) = +\infty-\infty \quad \text{F.I.}$$

$$f'(x) = \frac{1}{2\sqrt{x-4}} - \frac{1}{2} \geq 0$$

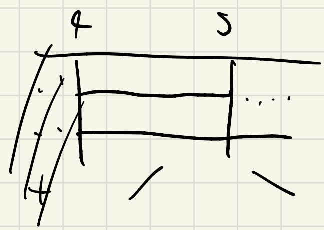
$$\frac{1-\sqrt{x-4}}{2\sqrt{x-4}} \geq 0 \qquad 1 \geq \sqrt{x-4}:$$

$$\begin{cases} x-4 \geq 0 & x \geq 4 \\ 1 \geq 0 & \forall x \in \mathbb{R} \\ x-4 \leq 1 & x \leq 5 \end{cases}$$



$4 \leq x \leq 5$

$D > 0 \qquad 2\sqrt{x-4} \geq 0; \quad x \geq 4$



$$f''(x) = \frac{-\frac{\cancel{2}}{\cancel{2}\sqrt{x-4}}}{(2\sqrt{x-4})^2} = -\frac{1}{(2\sqrt{x-4})^2\sqrt{x-4}} \geq 0$$

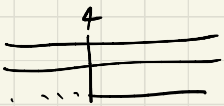
$-1 \geq 0 \quad \nexists x \in \mathbb{R}$

$(2\sqrt{x-4})^2\sqrt{x-4} > 0 \; ; \quad 4\sqrt{x-4}\,(x-4)$

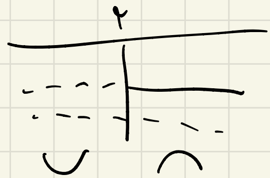
$4 > 0 \quad \forall x \in \mathbb{R}$

$\sqrt{x-4} > 0 \quad x \geq 4$

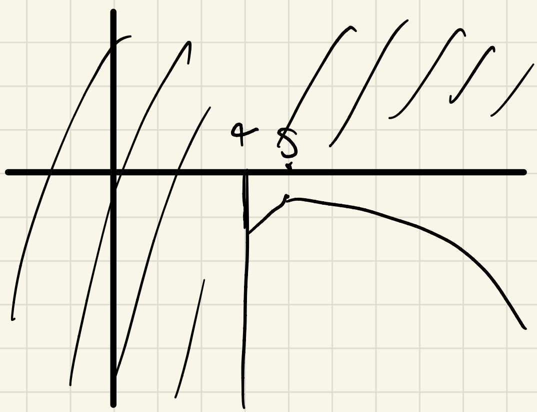
$x-4 > 0 \quad x \geq 4$




$x > 4$




$M(5; -\frac{1}{2})$




TAYLOR $\quad x_0 = 8$

$$f(x_0) + 1\left(f'(x_0)(x-x_0) + \frac{1}{2!} f''(x_0)(x-x_0)^2\right) =$$

$$-2 + 1\left(-\frac{1}{4}\right)(x-8) + \frac{1}{2}\left(-\frac{1}{32}\right)(x-8)^2) =$$

$$\cancel{-2} - \frac{x}{4} \cancel{+2} - \frac{1}{64}\left(x^2 - 16x + 64\right) =$$

$$-\frac{x}{4} - \frac{x^2}{64} + \frac{16x}{64} - 1 = \frac{\cancel{-16x}\ \cancel{+16x} - x^2 - 64}{64} = \frac{-x^2}{64} - 1$$

$$\lim_{n\to\infty} \frac{n^2 - n^2\sqrt{n} - 1}{n + i^n + \ln^2 n} = \lim_{n\to+\infty} \frac{-n^{\cancel{2}}\sqrt{n}}{\cancel{n}} = -n\sqrt{n} = -\infty$$

$a_1 = 2$

$a_{n+1} = \sqrt{a_n}$

① MONOTONA DECRESCENTE

$a_n \geq a_{n+1} \quad \forall n \in \mathbb{N}$

$a_n - \sqrt{a_n} \geq 0$

I) $a_1 - \sqrt{a_1} = 2 - \sqrt{2} \geq 0$

II) $a_n - \sqrt{a_n} > 0$

$\Rightarrow a_{n+1} - \sqrt{a_{n+1}} = \sqrt{a_n} - \sqrt[4]{a_n}$

$a_n > \sqrt{a_n} \Rightarrow \sqrt{\sqrt{a_2}} - \sqrt[4]{a_2} = 0$

$1 < a_n \leq 2 \quad \forall n \in \mathbb{N}$

- $a_n \leq 2$ $\quad a_1 = 2$ MONOTONA DECRESCENTE

$\hookrightarrow$ I. $a_1 = 2 \leq 2$

II. $a_n \leq 2$

$\Rightarrow \quad a_{n+1} \leq a_n \leq 2$

- $a_n > 1$

$\hookrightarrow$ I $a_1 = 2 > 1$

II. $a_n > 1$

$\Rightarrow \quad a_{n+1} = \sqrt{a_n} > \sqrt{1} = 1$

$$\lim_{n\to\infty} a_n = L \in \mathbb{R} \Rightarrow \lim_{n\to\infty} a_{n+1} = L$$

$L = \sqrt{L}$

$$\lim a_{n+1} = \lim \sqrt{a_n} = \sqrt{L}$$

$L = 0$ X

$L = 1$ ✓

$$\sum_{n\geq 1}^{+\infty} 5 \cdot 3^{1-n} = \sum_{n\geq 1}^{+\infty} 5 \cdot 3 \cdot 3^{-n} = 15 \sum_{n\geq 1} \left(\frac{1}{3}\right)^n \quad -1 < \frac{1}{3} < 1$$

$$= 15 \left[ \sum_{n\geq 0}^{+\infty} \left(\frac{1}{3}\right)^n - \left(\frac{1}{3}\right)^0 \right] = 15 \left[ \frac{1}{1-\frac{1}{3}} - 1 \right]$$

$$\sum_{n\geq 4}^{+\infty} \underbrace{\left(e^{\frac{1}{n}} - 1\right)} \cdot \frac{\ln\left(1 + \frac{1}{n^2}\right)}{n^{-4}}$$

$$\frac{e^x - 1}{x} \to 1 \qquad \frac{\ln(1+x)}{x} \to 1$$

$$e^x - 1 \sim x \qquad \ln(1+x) \sim x$$

$$\Downarrow \qquad \Downarrow$$

$$\frac{1}{n} \qquad \sim \frac{1}{n^2}$$

$$\frac{1}{n} \cdot \frac{1/n^2}{n^4} =$$

$f(x) = e^{-x}(x^2-1)$

D: $\mathbb{R}$ $\qquad \lim_{x\to+\infty} e^{-x}(x^2-1) = +\infty$

$$\lim_{x\to+\infty} \frac{(x^2-1)}{\underbrace{e^x}_{=+\infty}} = 0$$

vBus

A.V. no $\qquad$ A.O. $\quad y=0$ (dx)

A. Ob.

$$m = \lim_{x\to-\infty} \frac{e^x \cdot (x^{\cancel{2}})}{\cancel{x}} = e^x \cdot x = -\infty \quad \text{no, Ob}$$

$\int x e^x \, dx \qquad\qquad \int x e^{x^2} \, dx$

**17.01.2022**

$x \ln(x+1) - x^2$ $\qquad$ $[0, e-1]$

$$\frac{f(e-1) - f(0)}{e-1} =$$

$e-1 - (e^2+1-2e)$ $\qquad$ $0$

$$\frac{e-1-e^2-1+2e}{e-1} = \frac{-e^2+3e-2}{e-1}$$

$$-\frac{e^2-3e+2}{e-1} = -\frac{(e-2)\cancel{(e-1)}}{\cancel{e-1}} = -e+2$$

---

$$\sum_{n=1}^{+\infty} \frac{n^2}{n\ln n + 2n^{\alpha+1}} \sim \frac{\cancel{n^2}}{2\cancel{n}^{\alpha+1}} = \frac{1}{2n^{\alpha-1}} \sim \frac{1}{n^{\alpha-1}}$$

$\alpha+1-2 = \alpha-1$

SERIE ARMONICA

CONVERGE PER $\alpha > 1$; $\qquad \alpha-1>1$; $\alpha > 2$

$f(x) = \ln x - \ln^2 x$

$D: x > 0 \quad : (0; +\infty)$

$$\lim_{x \to 0} \left(\ln(x) - \ln^2(x)\right) = \left(\ln(x)\left(1 - \ln(x)\right)\right) = -\infty$$

$x = 0$ ASINTOTO

$$\lim_{x \to +\infty} \left(\ln(x) - \ln^2(x)\right) = \left(\overset{+\infty}{\ln(x)}\left(\overset{1-+\infty}{1 - \ln(x)}\right)\right) = -\infty$$

$$f'(x) = \frac{1}{x} - \frac{2\ln(x)}{x} = \frac{1}{x}\left(1 - 2\ln(x)\right) = \frac{1 - 2\ln x}{x} \geq 0$$

$$\frac{1 - 2\ln(x)}{x}$$

$N \geq 0 = 1 - 2\ln(x) \geq 0;$

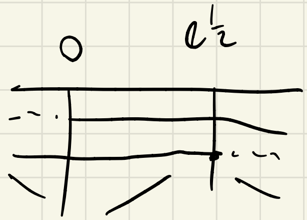
$2\ln(x) \leq 1$

$\ln(x) \leq \frac{1}{2}$

$\ln(x) \leq \ln e^{\frac{1}{2}}$

$x \leq e^{\frac{1}{2}}$

$D > 0 ; x > 0$

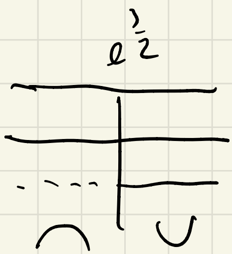


$$f''(x) = \frac{\frac{-2x}{x} - (1-2\ln x)}{x^2} = \frac{-2 - 1 + 2\ln x}{x^2} =$$

$$= \frac{2\ln x - 3}{x^2} \geq 0$$

$2\ln x - 3 \geq 0$; $\ln x \geq \frac{3}{2}$

$x \geq e^{\frac{3}{2}}$




$x^2 > 0 ; \forall x \in \mathbb{R} - \{0\}$

$$f(e^{\frac{3}{2}}) = \ln e^{\frac{3}{2}} - \ln^2(e^{\frac{3}{2}}) = \frac{3}{2} - \frac{3}{2}^2 = \frac{3}{4} - \frac{9}{4} = -\frac{3}{4}$$

$T(e^{\frac{3}{2}}; -\frac{3}{4})$

$$f'(e^{\frac{3}{2}}) = \frac{1 - 2\ln(x)}{x} = \frac{1 - \cancel{2}\cdot\frac{3}{\cancel{2}}}{e^{\frac{3}{2}}} = \frac{-2}{e^{\frac{3}{2}}}$$

$+\frac{3}{4} = -\frac{2}{e^{\frac{3}{2}}}(x - e^{\frac{3}{2}})$; $y = -\frac{2x}{e^{\frac{3}{2}}} + 2 - \frac{3}{4}$; $-\frac{2x}{e^{\frac{3}{2}}} + 5/4$

$f(x) = x \sin x$

$\int x \cos x$ $\quad f(x) = x \quad f'(x) = 1$

$g'(x) = \cos x \quad g(x) = -\cos x$

$-x\cos x - \int -\cos x \cdot 1 \; ; \; -x\cos x + \sin x + C$

$-\pi \cos \pi + \sin \pi + C = 2(-0\cos 0 + \sin 0 + C)$

$\pi + C = 2C \; ; \; \pi = C$

$-x\cos x + \sin x + \pi$

$\int_0^{\pi} -x\cos x + \sin x = \left[-x\cos x + \sin x\right]_0^{\pi} = -0 + 0 - \pi \cdot -1 + 0 =$

$+\pi$

$$\sum_{n=1}^{+\infty} \cos(\pi n)\,\sin\left(\frac{1}{n}\right)$$

CRITERIO DI LEIBNITZ

**19.01.2022**

TAYLOR

$$f(x) = \frac{1}{1+x^2} \qquad x_0 = 1$$

$$f'(x) = (1+x^2)^{-1} \; ; \; -(1+x^2)^{-2}\cdot 2x \; ; \; \frac{-2x}{(1+x^2)^2}$$

$$f''(x) = \frac{-2(1+x^2)^2 - (2(1+x^2)\cdot 2x \cdot -2x)}{(1+x^2)^4} = \frac{\cancel{(1+x^2)}(-2(1+x^2) - (2\cdot 2x\cdot -2x))}{(1+x^2)^{\cancel{4}}} =$$

$$= \frac{-2 - 2x^2 + 8x^2}{(1+x^2)^3} = \frac{6x^2-2}{(1+x^2)^3}$$

$$f(x_0) + f'(x_0) + f''(x_0)\frac{(x-x_0)^2}{2!} = \frac{1}{2} - \frac{1}{2}(x-1) + \frac{1}{4}(x-1)^2 + o(x-1)^2$$

$e^{x^3+x}$ $\qquad x_0 = 1$

$f' = e^{x^3+x} \cdot (3x^2+1)$

$f'' = e^{x^3+x} \cdot (3x^2+1)^2 + (6x)\, e^{x^3+x} =$

$e^{x^3+x}\left((3x^2+1)^2 + 6x\right)$

$f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2} f''(x_0)(x - x_0)^2$

$e^2 + e^2 \cdot 4\,(x-1) + 11\,e^2\,(x - x_0)^2 + 6\,(e^{x^3+x})$

$\to 0$

$$\frac{\sin x}{x} = 1 \qquad \frac{\tan x}{x} = 1 \qquad \frac{\arcsin x}{x} = 1$$

$$\frac{\arctan x}{x} = 1 \qquad \frac{1-\cos x}{x^2} = \frac{1}{2}$$

$$\frac{\ln(1+x)}{x} = 1 \qquad (1+x)^{\frac{1}{x}} = e \qquad \frac{e^x - 1}{x} = 1$$

$\to \infty$

$$\left(1+\frac{k}{x}\right)^x = e^k \qquad \frac{\ln x}{x} = 0$$

$\sum q^n$ $\qquad |q| < 1 \qquad \frac{1}{1-q}$

$q \geq 1 \qquad +\infty$

$q \leq -1 \qquad$ IND.

UNA SERIE A TERMINI DEFINITIVAMENTE POSITIVI

$a_n \geq 0$ NON PUÒ ESSERE INDETERMINATA.
definitivamente

$$\lim_{n \to +\infty} \frac{a_{n+1}}{a_n} = l$$

$l < 1$ CONVERGE

$l > 1$ DIVERGE

$l = 1$ IND.

SE $a_n \geq 0$ DEF. $\qquad \lim_{n \to +\infty} \sqrt[n]{a_n} = l$

$l <$ CONVERGE

$l >$ DIVERGE

$l = 1$ IND..

$\frac{1}{n^{\alpha}}$ $\alpha >$ CONVERGE

$\alpha \leq 1$ DIVERGE

$$\sum_{n=0}^{+\infty} \frac{n^3+\cos n+2n}{6n^2+1+4^n} \sim \frac{n^3}{4^n}$$

$$\frac{(n+1)^3}{4^{n+1}} \cdot \frac{4^n}{n^3} = \frac{(n+1)^3}{4\,\cancel{4^n}} \cdot \frac{\cancel{4^n}}{n^3} = \frac{1}{4}\left(\frac{n+1}{n}\right)^3 =$$

$$= \frac{1}{4}(1) = \frac{1}{4} < 1$$ CONVERGE

$$\sum_{n=1}^{\infty} \frac{\sin(n!)}{n^4} \qquad 0 \leq \frac{|\sin(n!)|}{n^4} \leq \frac{1}{n^4}$$ $\alpha > 1$ CONVERGE

$$\sum_{1}^{\infty} (-1)^n \frac{n^3+3}{n^4+2n} \qquad \frac{n^2+3}{n^4+2n} \sim \frac{\cancel{n^2}}{n^4} = \frac{1}{n^2}$$ $\alpha > 1$ CONVERGE

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} = (-1)^n \cdot a_n$$

$a_n \to 0$

$a_n > 0$

$a_n \leq a_{n+1}$

$\Downarrow$

CONVERGE

$$\sum_{n=2}^{+\infty} (-1)^n \frac{\ln(n)}{n}$$

$a_n > 0$

$a_n \to 0$

A. C.

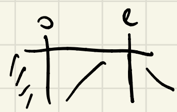
$$\frac{\ln(n)}{n} \geq \frac{1}{n} \Rightarrow \text{DIVERGE}$$

LEIBNITZ

$$a_{n+1} \leq a_n \to \frac{\ln(n+1)}{(n+1)} \leq \frac{\ln n}{n}$$

$f'(x) = \frac{1-\ln x}{x^2}$ $\quad 1 - \ln n \geq 0;\ \ln x \leq 1;\ x \leq e$

DEFINITIVAMENTE DECRESCENTE $\Leftarrow$




$\Downarrow$

$a_n$ DEF. PER $n \geq 3$

CONVERGE

$$2 > 2$$

$$n e^{1 - \frac{1}{n}} \cdot \frac{1}{n^{2}} \qquad \frac{n}{n^{2}} \to + \infty = 0$$

$$\frac{n}{n^{2}} = n^{1-a} = n^{-2} \qquad \frac{1}{n^{2}}$$ $a > 1$ converge

$$n^{1-2} \qquad \frac{1}{n^{2-1}}$$ $a \leq 1$ diverge

$$f x_0 ' \quad f'(x_0) \cdot (x - x_0)' \cdot f''(x_0) \cdot (x-x_0)^2 + o'(x-x_0)''$$

Mc laurin $\to x_0 = 0$

$$\int k = kx + c \qquad \int \sin x = -\cos x$$

$$\int x^a = \frac{x^{a+1}}{a+1} \qquad \int \cos x = \sin x$$

$$\int \frac{1}{x} = \ln |x| + c$$

$$F x \cdot G x \cdot - \int f' x \cdot g x$$

$(2x-3)$

$\int (2x-3)^2 \quad dx$

$\int \frac{\ln^2(2x-3)}{(2x-3)}$ $\quad \ln^2(4x-3) \cdot \frac{1}{(2x-3)}$

$\int (2x-3)^2$ $\quad \frac{1}{2} \cdot \int 2\,(2x-3)^2$

$\frac{1}{2} \cdot \frac{(2x-3)^3}{3}$ $\quad \frac{(2x-3)^3}{6} =$

$\frac{1}{2}\int e^{2x-3} \cdot 2$