FONDAMENTI DELL'INFORMATICA - ESERCITAZIONE

30.09.2021

$A = \{a, b\}$ $\quad B = \{1, 3, 5, 7, 9\}$

$C = \{\emptyset\}$ $\quad D = \{1, \{b\}, \{1, \emptyset\}\}$

1)

Est $(P(D))$

$\{\}$

$P(D) = \{\emptyset, \{1\}, \underbrace{\{\{b\}\}}_{\|1\|}, \{\{1, \emptyset\}\}, \underbrace{\{1, \{b\}\}, \{1, \{1, \emptyset\}\}}_{\|2\|}$

$\{\{b\}, \{1, \emptyset\}\}, \underbrace{\{1, \{b\}, \{1, \emptyset\}\}}_{\|3\|} \}$ → D

2)

INT (B)

$B = \{\forall x \in \mathbb{N}, xy.z \neq 0, x \leq 9\}$

$X \mid X \in \mathbb{N}, x.y.z \neq 0, X \leq 9$

3)

Est $(A \cup D)$

$A \cup D = \{a, b, 1, \{b\}, \{1, \emptyset\}\}$

4) Est $(P(B) \cap P(D))$

$P(D) \cap P(B) = \{\emptyset, \{1\}\}$

Vero o Falso

- $a \in A$ ✓
- $3 \subseteq B$ ✗

- $\{3\} \subseteq B$ ✓
- $\{b\} \in D$ ✓
- $\{b\} \subseteq D$ ✗
- $\{b\} \in P(A) \setminus D$ ✗
- $\emptyset \subseteq A \cap D$ ✓
- $\emptyset \in C$ ✓

Prodotto Cartesiano

$$A \times C = \{\langle a, \emptyset \rangle, \langle b, \emptyset \rangle\}$$

$$A \times P(C) = \{\langle a, \emptyset \rangle, \langle a, \{\emptyset\} \rangle, \langle b, \emptyset \rangle, \langle b, \{\emptyset\} \rangle\}$$

$$|D \times B| = |D| \times |B| = 15$$

Partizioni

$B' = \{\{1\}, \{3\}, \{5\}, \{7, 9\}\}$

07.10.2021

Relazioni e Funzioni

1)

$A = \{1, 2, 3, 4, 5, 6\}$ $R_1 \subseteq A \times A$ $R_2 \subseteq A \times A$

$R_1 = \{\langle x, y\rangle \mid y = x + 2\}$ $R_2 = \{\langle x, y\rangle \mid x + y > 6\}$

$R_1 = \{\langle 1,3\rangle, \langle 2,4\rangle, \langle 3,5\rangle, \langle 4,6\rangle\}$

$R_2 = \{\langle 1,6\rangle, \langle 2,5\rangle, \langle 2,6\rangle, \langle 3,4\rangle, \langle 3,5\rangle$
$\langle 3,6\rangle, \langle 4,3\rangle, \langle 4,4\rangle, \langle 4,5\rangle, \langle 4,6\rangle$
$\langle 5,3\rangle, \langle 5,4\rangle, \langle 5,5\rangle, \langle 5,6\rangle, \langle 6,1\rangle$
$\langle 6,2\rangle, \langle 6,3\rangle, \langle 6,4\rangle, \langle 6,5\rangle, \langle 6,6\rangle\}$

2)

$g_{\mathbb{N}} : \mathbb{N} \to \mathbb{N}$, $g_{\mathbb{Z}} : \mathbb{Z} \to \mathbb{Z}$, $g_{misto} : \mathbb{Z} \to \mathbb{N}$, $f : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$

$g : \mathbb{Z} \to \mathbb{R}$, $h : \mathbb{N} \to \mathbb{N}$

	TOT	INJ	SUR	INV	BIJ	BIJ
$g_{\mathbb{N}}(x) = x^2$	✓	✓	✗	✓	✗	✗
$g_{\mathbb{Z}}(x) = x^2$	✓	✗	✗	✗	✗	✗
$g_{misto}(x) = x^2$	✓	✗	✗	✗	✗	✗
$f(x,y) : TR\left(\frac{x}{y}\right)$ ↓ SENZA DIVISIONI	✗	✗	✓	✗	✗	✗
$g(x) = 2^x$	✓	✓	✗	✓	✗	✗
$h(x) = x + 5$	✓	✓	✗	✓	✗	✗

3) funzioni composte

$f \circ h = /$ $\quad h \circ f = [TR(\frac{x}{y})] + 5$

$g \circ h = 2^{x+5}$ $\quad h \circ g = /$

$g \circ f = 2^{\frac{x}{y}}$ $\quad f \circ g = /$

14.10.2021

1)

$U = \{$ marco, giulio, sara, luca, daniela, carlo $\}$

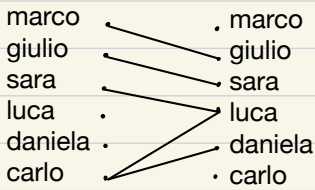
$R_1 \subseteq U \times U$ $\quad R_2 \subseteq U \times U$

$R_1 = \{$ <marco, giulio>, <giulio, sara>, <sara, luca>, <carlo, daniela>, <carlo, luca> $\}$

$R_2 = \{$ <giulio, sara>, <sara, luca>, <carlo, daniela>, <sara, giulio>, <luca, sara>, <daniela, carlo> $\}$

grafo bipartito | matrice booleana | grafo orientato

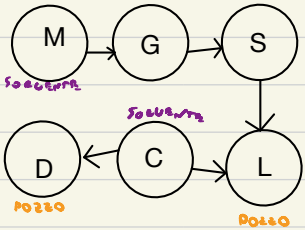
R_1



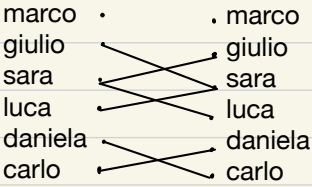
R_1

	M	G	S	L	D	C
M	0	1	0	0	0	0
G	0	0	1	0	0	0
S	0	0	0	1	0	0
L	0	0	0	0	0	0
D	0	0	0	0	0	0
C	0	0	0	1	1	0

R_1



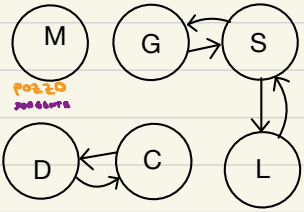
R_2



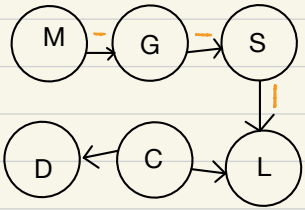
R_2

	M	G	S	L	D	C
M	0	0	0	0	0	0
G	0	0	1	0	0	0
S	0	1	0	1	0	0
L	0	0	1	0	0	0
D	0	0	0	0	0	1
C	0	0	0	0	1	0

R_2



R_1



$c\,l_4 = \emptyset$

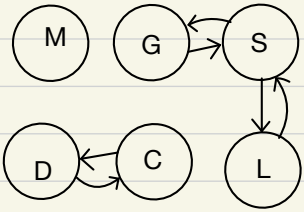
$c\,l_3 = \langle M,G,S,L \rangle$

$sc\,l_5 = \langle M,G,S,L,C,D \rangle$
$\langle M,G,M,G,S,L \rangle$
$\langle M,G,M,G,M,G \rangle$

R_1
connesso
aciclico

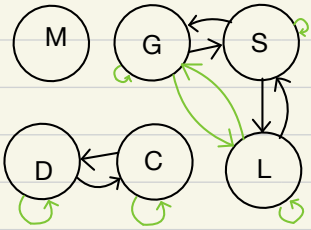
R_2
non connesso
ciclico

R_2

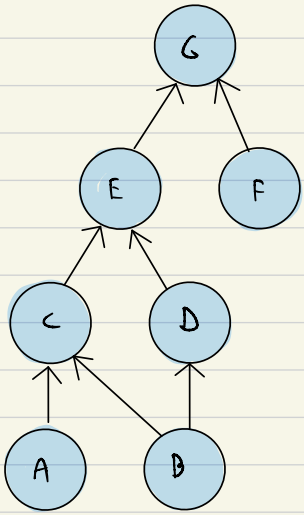


Proprietà:
simmetria
irriflessiva

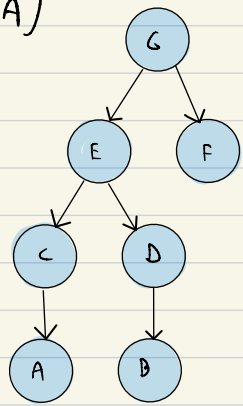
R_2 TRANSITIVA



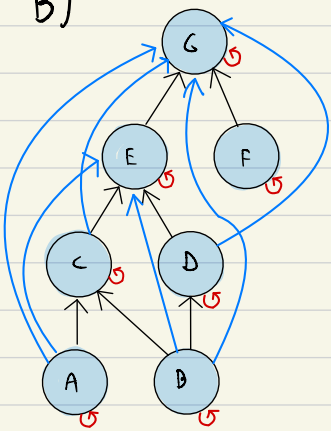
21.10.2021



A)



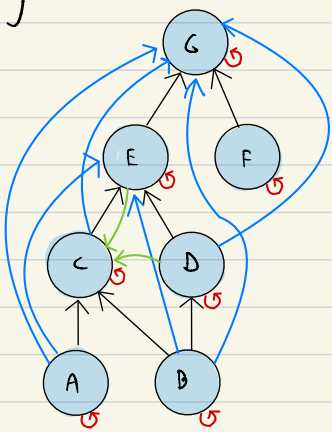
B)



TRANS.

RIPL.

E)

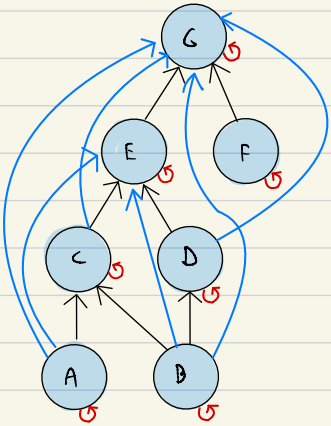


TRANS.

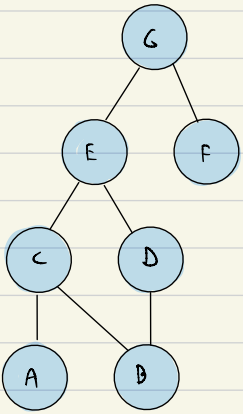
RIPL.

PRE ORDER

28.10.2021



G1
POSET



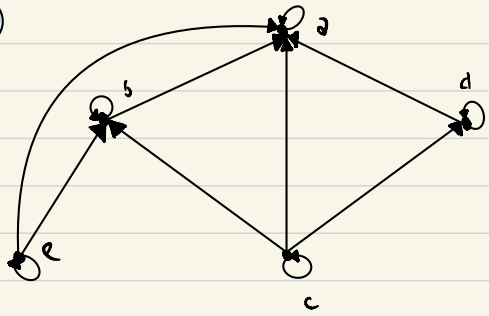
G 1
HASSE

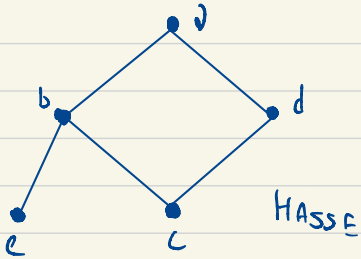
se tra, preso un cammino tra due nodi ne esiste un altro di lunghezza maggiore che li collega, il primo va eliminato

minimali	massimali	massimi	minimi
{A, D, F}	{G}	{G}	{ }

28.10.2021

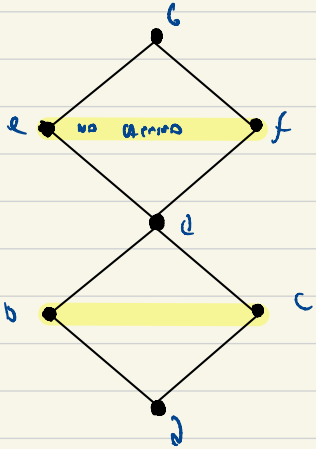
1

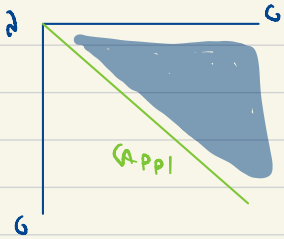




HASSE

2





$e \cup c = e$

$e \cap c = c$

$e \cup f = G$

$e \cap f = d$

$G \cap e = e \cap G$

CONTROLLO SOLO

METÀ DIAGONALE

3

	RETICOLO	COMPL.	DISTRIB.	LIMITATO	A. BOOLE
1	SÍ	SÍ a' = d, b' = c d' = a, c' = b	SÍ	SÍ M = a m = d	SÍ
2	SÍ	NO e non ha e'		SÍ	NO
3	NO				
4	SÍ	SÍ a' = e, e' = a b' = c, d' = c c' = b, d	NO	SÍ	NO
5	SÍ				
6	NO <b, c> non ha max e min				

02.12.2021

$A \to B \equiv \neg A \vee B \Rightarrow (A \to B) \to (\neg A \vee B)$

$(\neg A \vee B) \to (A \to B)$

$F\ (A \to B) \to (\neg A \vee B)$

$F \to$

$T\ (A \to B),\ F\ (\neg A \vee B)$

$T \to$

$FA, F\neg A \vee B \mid TB, F\neg A \vee B$

$F \vee$

$FA, F\neg A, FB \mid TB, F\neg A, FB$

$F \neg$

$FA, TA, FB \mid$

CHIUSO TAUTOLOGIA

$F\ (\neg A \vee B) \to (A \to B)$

$F \to$

$T \neg A \vee B,\ F\ A \to B$

$F \to$

$T \neg A \vee B,\ TA,\ FB$

$T \vee$

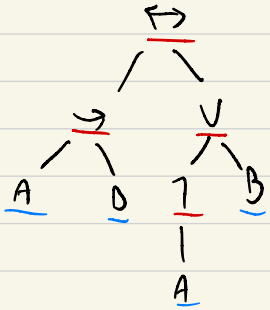
$T \neg A, TA, FB \mid TB, FB, TA$

$T \neg$

$FA, TA, FB \mid$

CHIUSO TAUTOLOGIA

$(A \to B) \leftrightarrow (\neg A \vee B)$



P	Q	$P \leftrightarrow Q$
0	0	1
0	1	0
1	0	0
1	1	1

A	B	¬A	¬A∨B	A→B	FORMULA Ø
0	0	1	1 1 0	0 1 0	1
0	1	1	1 1 1	0 1 1	1
1	0	0	0 0 0	1 0 0	1
1	1	0	0 1 1	1 1 1	1

TAVOLA DA

13.12.2021

Traduzione Frasi in Linguaggio di Primo Ordine

1) Ogni Pokemon ha un potere
2) Ash non ha catturato tutti i pokemon
3) se due pokemon hanno lo stesso allenatore allora sono amici

Segnatura

costanti : *Ash*,

funzioni :

predicati : *HaPotere*^{2}, Pokemon^{1}, *Potere*^{1}, HaCatturato^{2}, ÈAllenato^{2}, Allenatore,^{1} SonoAmici^{2}

Traduzione

1) $\forall x\,(\text{POKEMON}(x) \rightarrow (\exists y\,\text{HA POTERE}(x,y) \wedge \text{POTERE}(y)))$
2) $\exists x\,(\text{POKEMON}(x) \rightarrow \neg(\text{HA CATTURATO}(\text{ASH},x))$
3) $\forall x,y\,\exists z\,(\text{POKEMON}(x) \wedge \text{POKEMON}(y) \wedge \text{ALLENATORE}(z) \wedge \text{È ALLENATO}(x,z) \wedge \text{È ALLENATO}(y,z) \rightarrow \text{SONO AMICI}(x,y))$

RDANE GERMAIO

R $\langle x, y \rangle$ $x = x^2$ $y = x^2$ $y = x$

$\{0, 1, 2, 3, 4, 9\}$

$\langle 0,0 \rangle$ $\langle 1,1 \rangle$ $\langle 2,2 \rangle$ $\langle 3,3 \rangle$ $\langle 4,4 \rangle$ $\langle 9,9 \rangle$

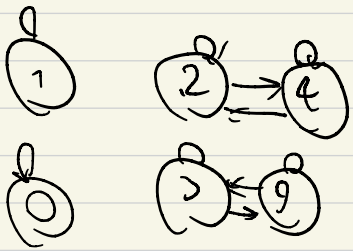
$\langle 2,4 \rangle$ $\langle 3,9 \rangle$ $\langle 9,3 \rangle$ $\langle 4,2 \rangle$

	0	1	2	3	4	9
0	1	0	0	0	0	0
1	0	1	0	0	0	0
2	0	0	1	0	1	0
3	0	0	0	1	0	1
4	0	0	1	0	1	0
9	0	0	0	1	0	1

• RIF • SIM • TRANS

RELAZIONE DI EQUIVALENZA

CLASSI DI EQ.:



$\neg((r \vee \neg r) \rightarrow \neg(p \vee q))$

$\vdash ((r \vee \neg r), \vdash (p \vee q)$

$$\frac{F\ ((A \rightarrow B) \wedge (A \rightarrow \neg B)) \rightarrow B}{F\rightarrow}$$

$$\frac{\vdash ((A \rightarrow B) \wedge (A \rightarrow \neg B)),\ F\ B}{\vdash\wedge}$$

$$\frac{\vdash (A \rightarrow B),\ \vdash (A \rightarrow \neg B),\ F\ B}{\vdash\rightarrow}$$

$$\frac{F B,\ \vdash (A \rightarrow \neg B), F A \mid \vdash (A \rightarrow \neg B), F B, \vdash B}{\vdash\rightarrow}$$

$$\frac{F B,\ F A,\ F A \mid F B,\ F A,\ \vdash \neg B}{\vdash\neg}$$

$$\{\emptyset\} \qquad F B,\ F A,\ F B$$

$\neg\ ((A \to B) \wedge (A \to \neg B)) \to B$ $\quad \neg \to$

$F[(A \to B) \wedge (A \to \neg B)]$ | $T\ B$

$F \wedge$

$F(A \to B)$ | $F(A \to \neg B)$