
CxSOM

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1 Introduction

The CxSOM software suite enables to model consensus based multi-SOMs, explored in the BISCUIT team of the Loria lab.

The current documentation is at work, and mainly technical stuff are reported here. See the README at the package root for an introduction, as well as examples.

2 The model

The computation enabled by CxSOM consists of successively updating data, keeping trace of the data values in history files.

2.1 Data instances (DI)

The elementary piece of data handled in CxSOM is called a *data instance (DI)*. Each of the computed DIs ends as a record in a file, once it has been computed.

DIs are denoted by a triplet $[T, X, t]$, that reads as “the instance of variable X hosted by the timeline T at the time instant t of that specific timeline”. Only elementary $[T, X, t]$ DIs matter, the concepts of *variable*, *timeline* and *time instant* only serve for describing the computation.

Each DIs is a value that is stored in a file once definitively computed. A value has a type, that can be:

- **Scalar**: A floating point value, usually in $[0, 1]$.
- **Pos1D**: A value in $[0, 1]$ corresponding to a position in a 1D map.
- **Pos2D**: A value in $[0, 1]^2$ corresponding to a position in a 2D map.
- **Array= n** : A value in \mathbb{R}^n , usually $[0, 1]^n$, $n \in \mathbb{N}^*$.
- **Map1D< \mathcal{X} >= n** : A 1D map of type \mathcal{X} , i.e. a value in \mathcal{X}^n , $n \in \mathbb{N}^*$.
- **Map2D< \mathcal{X} >= n** : A 2D squared map of type \mathcal{X} , i.e. a value in $(\mathcal{X}^n)^n$, $n \in \mathbb{N}^*$.

where type $\mathcal{X} \in \{\text{Scalar}, \text{Pos1D}, \text{Pos2D}, \text{Array}=k\}$.

In CxSOM, the type is associated to a variable, i.e. all the $[T, X, \bullet]$ are DIs with the same type.

Every DI is also doted with a status variable $\langle [T, X, t] \rangle \in \{\text{busy}, \text{ready}\}$:

- **ready**: The computation of the value of the DI is definitively done. The DI value will not change anymore.
- **busy**: The definitive value of the DI is still to be determined.

More generally, in the following, $\langle x \rangle$ reads as “the status of x ”.

2.2 Updates

An *update* of some DI is the (re)computation by the CxSOM computer of the value of that DI. There are at most two updates for a DI, one for the first update (initialization) that is optional, and one, mandatory, for other updates.

2.2.1 An update is a function taking arguments and computing a result

An update u for a DI $[T, X, t]$ is made of:

- The description of some computation, i.e. the function called to realize an update.
- The result res_u , i.e. $\text{res}_u \stackrel{\text{def}}{=} [T, X, t]$ itself, that is computed by the function of the update.
- Other DIs, that serve as arguments to the function (their value is read when the function is called). Among arguments, a distinction is made between *in-arguments* and *out-arguments*:
 - The in-arguments in_u of the update u such as $\text{res}_u = [T, X, t]$ is the set of DIs used as arguments of u of the form $[T, \bullet, t]$. All the DIs handled in the simulation of the form $[T, \bullet, t]$ define a *time step* \mathcal{S}_T^t of the simulation, so the in-arguments are the DIs used for the computation of an update which belong to the same timestep as the result.
 - The out-arguments out_u of the update u are the other DIs used as arguments for the computation of the update.

2.2.2 Time steps

As just introduced in the previous section, let us call a *time step* all the DIs $[T, \bullet, t]$. So a time step is only parametrized by a timeline T and an instant t , thus denoted by \mathcal{S}_T^t .

As detailed in next paragraph, all the DIs in a timestep will be updated several times until they reach some kind of consensus on the final values of the DIs.

Let us denote all the updates defined for a timestep by $\mathcal{U}_T^t \stackrel{\text{def}}{=} \{u \mid \text{res}_u \in \mathcal{S}_T^t\}$. Let us denote by $\mathcal{U}_T^t \stackrel{\text{def}}{=} \{u \mid \text{res}_u \in \mathcal{S}_T^t\}$ the set of their out-arguments. One immediate condition for activating the computation related to some timestep \mathcal{S}_T^t is to have $\forall [T', X', t'] \in \mathcal{O}_T^t, \langle [T', X', t'] \rangle = \text{ready}$.

Another condition is less obvious. Indeed, there may be some in-arguments in a timestep for which no update are defined. Such arguments need to be set externally, they are typically inputs provided to our simulation. Let us call them *unbound* DIs of a timestep \mathcal{S}_T^t , and denote them by $\mathcal{F}_T^t \stackrel{\text{def}}{=} \mathcal{S}_T^t \setminus \{\text{res}_u \mid u \in \mathcal{U}_T^t\}$. The second condition for activating the computation related to some timestep \mathcal{S}_T^t is then to have $\forall [T, X', t] \in \mathcal{F}_T^t, \langle [T, X', t] \rangle = \text{ready}$.

So once we have

$$(\forall [T', X', t'] \in \mathcal{O}_T^t, \langle [T', X', t'] \rangle = \text{ready}) \wedge (\forall [T, X', t] \in \mathcal{F}_T^t, \langle [T, X', t] \rangle = \text{ready})$$

the computation of the DIs of a time step \mathcal{S}_T^t which are still **ready** can be processed. This is the relaxation of the timestep.

2.2.3 Datation and relaxation

Before it has a definitive value (i.e. before being in the **ready** status), the value of a DI may change several times. Indeed, all DIs in a timestep $\mathcal{S}_T^t = [T, \bullet, t]$ are updated until all of them get **ready**, and during that process, the values of the DIs may be recomputed several times. In order

to handle the dependencies of the DIs within a timestep, we need to timestamp the values of the DIs, in order to determine those of them that have to be updated. Timestamp play the role of file dates in makefile, when the date of a target is compared to the date of its dependencies. Here, in order to know if an update u needs to be performed, we have to consider two things:

- Do we have $\forall [T, X, t] \in \text{out}_u, \langle [T, X, t] \rangle = \text{ready}$? If not, the update cannot be done, it is considered as *impossible*.
- Have the in-arguments been updated since the last computation of the result? If the answer is yes, the result needs to be recomputed.

To compute the second condition, every DI comes with a *datation* denoted by $d_{[T, X, t]} \in \mathbb{N}$.

An update then stores, for each of its in-arguments, the datation it had at the last computation of the result. When the simulator considers an update, it compares the current datation of the in-arguments to the ones stored by the update. If some are newer (or if the result has never been computed so far) the result is recomputed, and

$$d_{\text{res}_u} = 1 + \max_{[T, X, t] \in \text{in}_u} d_{[T, X, t]} \quad (1)$$

Considering an update for computation can thus lead to the following status of the update u , denoted by $\langle u \rangle$:

- $\langle u \rangle = \text{impossible}$: $\langle \text{res}_u \rangle = \text{busy}$, computation is not feasible yet, since some out-arguments are *busy*.
- $\langle u \rangle = \text{uptodate}$: $\langle \text{res}_u \rangle = \text{busy}$, nothing changed in the in-arguments input dates from last update, or the new value was not a significant modification. The datation d_{res_u} has not been modified.
- $\langle u \rangle = \text{updated}$: $\langle \text{res}_u \rangle = \text{busy}$, the computation has modified the value of res_u significantly.
- $\langle u \rangle = \text{done}$: $\langle \text{res}_u \rangle = \text{ready}$, the computation has modified the value res_u definitively.
- $\langle u \rangle = \text{none}$: Update status is not determined yet (used for initialization only).

The datation mechanism enables to control the update of all the DIs in a given timestep, i.e. all the DIs like $[T, \bullet, t]$ for the time instant t of the timeline T . When no more significant writes of the results can be done, the whole timestep is stable. So the relevance of the datation mechanism is to enable a *relaxation* of the DIs inside a timestep until stabilization of all of them is reached.

2.2.4 Update computation cycle

When a specific update is defined in CxSOM (as average, learning, etc.), the update is inherited from a base `cxsom:update:Base` class. In the subclass, the specificity of the computation has to be implemented, while the base class handles the details of the full update cycle, as datation and status. Algorithm 1 shows the cycle, the `on_*` calls are the method that subclass need to override in order to implement a specific update computation.

2.3 Timesteps and tasks

2.3.1 A computational structure for timesteps

In the simulator, the computation of the updates are put in a pool task and computed in parallel. However, since relaxation is related to the computation of updates belonging to the same timestep,

Algorithm 1 One update cycle for update u

Require: a boolean attribute `out_ok`, telling if all out-args have been read successfully previously.

Require: a boolean attribute `is_init`, telling if the update is an initialization.

```
1: if  $\langle \text{res}_u \rangle = \text{ready}$  then
2:   return done
3: end if
4: on_computation_start()
5: if  $\neg \text{out\_ok}$  then
6:   // out-arguments need to be read.
7:   for all  $[T, X, t] \in \text{out}_u$  do
8:     if  $\langle [T, X, t] \rangle = \text{ready}$  then
9:       on_read_out_arg( $[T, X, t]$ )
10:    else
11:      out_ok  $\leftarrow$  false
12:      on_read_out_arg_aborted()
13:      return impossible
14:    end if
15:  end for
16:  out_ok  $\leftarrow$  true // All in-args have been successfully read.
17: end if
18: for all  $[T, X, t] \in \text{in}_u$  do
19:   // Datation issues are handled in this loop, but not detailed here.
20:   on_read_in_arg( $[T, X, t]$ )
21: end for
22: if none of the in-arguments have been updated since last cycle then
23:   if is_init then
24:     return updated // updates used as initialization are never considered as up-to-date.
25:   else
26:     return uptodate
27:   end if
28: end if
29: // From here, we know that the result has to be recomputed.
30:  $x \leftarrow \text{none}$  // This is the status we plan to return, it is set next.
31:  $\alpha \leftarrow \text{on\_write\_result}(\text{res}_u)$  //  $\alpha$  is a Boolean telling if the value change is significant.
32: if we have just written a new significant value in  $\text{res}_u$  then
33:    $x \leftarrow \text{updated}$ 
34: else
35:   if is_init then
36:     return  $x \leftarrow \text{updated}$  // Even not significant changes are considered as an actual update for
    initialization updates.
37:   else
38:     return  $x \leftarrow \text{uptodate}$ 
39:   end if
40: end if
41: // Before returning, we have to check if in-arg may change in the future
42: if all in-arguments are ready then
43:   if is_init then
44:      $x \leftarrow \text{updated}$  // As  $u$  an init, we return updated since further change can be consider by the
    usual update coming next for updating  $\text{res}_u$ .
45:   else
46:      $x \leftarrow \text{done}$  // We have definitively computed the result...
47:      $\langle \text{res}_u \rangle \leftarrow \text{ready}$  // The result DI value is locked.
48:   end if
49: end if
50: return  $x$ 
```

this task management needs an intermediate level, which is a computational internal structure representing the timesteps.

Let us recall the notation $\mathcal{S}_T^t \stackrel{\text{def}}{=} \{[T', X', t'] \in \text{simulation} \mid T' = T \wedge t' = t\}$ for the timestep at instant t in the timeline T . For each $[T, X, t] \in \mathcal{S}_T^t$, the internal structure used in the simulator gathers at least an *usual update*, and an optional *initialization update*. The initialization update, if present, is used for the first feasible setting of the value of $[T, X, t]$ instead of the usual update. Otherwise, the usual update is used throughout the successive setting of the value of $[T, X, t]$ during the relaxation.

In order to avoid further confusions, let us denote by \bar{u} an update handled by a time step \mathcal{S}_T^t : it can be either a single update u (so u is the usual update) or a pair (u, u') if an initialization update is defined (then u' is the initialization update). In the case for which $\bar{u} = (u, u')$ is a pair, $\text{res}_u = \text{res}_{u'}$ obviously stands. In the case of a pair as well, the simulator uses u' until it gets a first computation of $\text{res}_{\bar{u}}$, and then it will switch to the use of u for next computations of $\text{res}_{\bar{u}}$.

As at least a call to the usual update u is required to be done for \bar{u} , we have particularized the status returned by initialization updates in algorithm 1, so that the evaluation of an initialization update can never lead to the definitive computation of res_u during the relaxation of the timestep.

2.3.2 Timestep status

The timestep structures \mathcal{S}_T^t at the simulator level have a status

$$\langle \mathcal{S}_T^t \rangle \in \{\text{unbound}, \text{blocked}, \text{relaxing}, \text{checking}, \text{done}\}.$$

The status $\langle \mathcal{S}_T^t \rangle$ depends on the status of the updates of the timestep, observed when the updates are realized. The meaning of the $\langle \mathcal{S}_T^t \rangle$ is:

- $\langle \mathcal{S}_T^t \rangle = \text{unbound}$: The timestep as unbound DIs, i.e. $\mathcal{F}_T^t \neq \emptyset$.
- $\langle \mathcal{S}_T^t \rangle = \text{blocked}$: The timestep is blocked due to impossible updates.
- $\langle \mathcal{S}_T^t \rangle = \text{relaxing}$: The timestep is under unstable computation.
- $\langle \mathcal{S}_T^t \rangle = \text{checking}$: Every update seem stable, we are checking this.
- $\langle \mathcal{S}_T^t \rangle = \text{done}$: The timestep is done, all updates \bar{u} have lead to $\text{res}_{\bar{u}} = \text{ready}$ and have quit the simulator.

2.3.3 Timestep update queues

The timestep structures \mathcal{S}_T^t at the simulator level are doted each with 5 update queues, where the updates \bar{u} are stored. Each the $\bar{u} \in \mathcal{S}_T^t$ belongs to one of the 4 queues of \mathcal{S}_T^t .

When a timestep is asked by the simulator to provide computation, it extracts updates from some of the queues. When the update is performed by the simulator, it is given back to the timestep, with the return status resulting from algorithm 1. According to this status, the update is stored in the appropriate queue, and some supplementary transfers from one queue to another queue may happen. This will be detailed further, as the *timestep state machine*.

For now, let us present the 4 queues used by a timestep \mathcal{S}_T^t .

$$\begin{aligned} [\mathcal{S}_T^t]_{\text{new}} &: \{ \bar{u} \in \mathcal{S}_T^t \mid \langle \bar{u} \rangle \text{ Newly added to the queue.} \} \\ [\mathcal{S}_T^t]_{\text{unstable}} &: \{ \bar{u} \in \mathcal{S}_T^t \mid \langle \bar{u} \rangle \text{ needs to be known.} \} \\ [\mathcal{S}_T^t]_{\text{impossible}} &: \{ \bar{u} \in \mathcal{S}_T^t \mid \langle \bar{u} \rangle \text{ has been detected as impossible} \} \\ [\mathcal{S}_T^t]_{\text{stable}} &: \{ \bar{u} \in \mathcal{S}_T^t \mid \bar{u} \text{ have been seen stable for the first time are here.} \} \\ [\mathcal{S}_T^t]_{\text{confirmed}} &: \{ \bar{u} \in \mathcal{S}_T^t \mid \bar{u} \text{ for which stability is confirmed.} \} \end{aligned}$$

2.3.4 Providing jobs

A timestep is periodically asked by the simulator to provide tasks, i.e. to offer updates that will be evaluated by the simulator. These updates are extracted from the timestep and inserted in a pool task, waiting for execution. The timestep keeps trace of their existency while they are outside, waiting for being executed.

So when a timestep \mathcal{S}_T^t is asked for new updates to be done, it provides (and extracts) :

- all the $\bar{u} \in [\mathcal{S}_T^t]_{\text{unstable}} \cup [\mathcal{S}_T^t]_{\text{stable}}$ if $\langle \bar{u} \rangle \in \{\text{relaxing}, \text{checking}\}$
- nothing otherwise.

2.3.5 The timestep state machine

When an update is executed, it is given back to the timestep by the simulator, with a status report given by algorithm 1.

The reporting of an update makes the update to be inserted in a queue of the timesteps. This is done as detailed in algorithm 2.

Algorithm 2 Reporting update \bar{u} to the timestep \mathcal{S}_T^t

Require: $\langle \bar{u} \rangle$ is the status of the update execution.

Require: $\langle \mathcal{S}_T^t \rangle$ is the current status of the timestep.

```

1: if  $\langle \bar{u} \rangle = \text{impossible}$  then
2:    $[\mathcal{S}_T^t]_{\text{impossible}} \leftarrow [\mathcal{S}_T^t]_{\text{impossible}} \cup \{\bar{u}\}$ 
3: else if  $\langle \bar{u} \rangle = \text{updated}$  then
4:    $[\mathcal{S}_T^t]_{\text{unstable}} \leftarrow [\mathcal{S}_T^t]_{\text{unstable}} \cup \{\bar{u}\}$ 
5: else if  $\langle \bar{u} \rangle = \text{done}$  then
6:   // The update is not handled anymore, since  $\langle \text{res}_{\bar{u}} \rangle = \text{ready}$ .
7: else if  $\langle \bar{u} \rangle = \text{uptodate}$  then
8:   if  $\langle \mathcal{S}_T^t \rangle = \text{checking}$  and  $\bar{u}$  was in  $[\mathcal{S}_T^t]_{\text{stable}}$  then
9:      $[\mathcal{S}_T^t]_{\text{confirmed}} \leftarrow [\mathcal{S}_T^t]_{\text{confirmed}} \cup \{\bar{u}\}$ 
10:  else
11:     $[\mathcal{S}_T^t]_{\text{stable}} \leftarrow [\mathcal{S}_T^t]_{\text{stable}} \cup \{\bar{u}\}$ 
12:  end if
13: end if
14: update_status( $\mathcal{S}_T^t$ ) // See algorithm 3.
```

2.3.6 Update patterns

Defining an update for all DIs (i.e. at each time t) would be exhausting. It can be done for specific time instants (usually $t = 0$), but a generic update definition is required for the general cases.

This is what *update patterns* (UP) are introduced for. Let us denote a *relative data instance* (RDI) as $\{T, X, \tau\}$, where $\tau \in \mathbb{Z}$ is a relative time shift. A RDI can be *anchored* as a DI at time t . This consists in turning $\{T, X, \tau\}$ into $[T, X, t + \tau]$.

We also consider RDI with a relative time factor, rather than a time shift. No difference is made here, in terms of notations, between the two. Such a RDI is anchored as a DI at time t by turning $\{T, X, \tau\}$ into $[T, X, t \times \tau]$.

The definition is then similar to the one of an update, presented in section 2.2.1. An UP π is made of:

Algorithm 3 `update_status`(\mathcal{S}_T^t)

```
1: if has_unbound( $\mathcal{S}_T^t$ ) then
2:    $\langle \mathcal{S}_T^t \rangle \leftarrow$  unbound // See algorithm 4.
3: else if  $[\mathcal{S}_T^t]_{\text{impossible}} \neq \emptyset$  then
4:    $[\mathcal{S}_T^t]_{\text{stable}} \leftarrow [\mathcal{S}_T^t]_{\text{stable}} \cup [\mathcal{S}_T^t]_{\text{confirmed}}$ 
5:    $[\mathcal{S}_T^t]_{\text{confirmed}} \leftarrow \emptyset$  // Blocking out-arguments lead to reconsider the  $\mathcal{S}_T^t$  stability.
6:   if all the out-arguments of the  $\bar{u} \in \mathcal{S}_T^t$  are ready then
7:      $[\mathcal{S}_T^t]_{\text{unstable}} \leftarrow [\mathcal{S}_T^t]_{\text{unstable}} \cup [\mathcal{S}_T^t]_{\text{impossible}}$ 
8:      $[\mathcal{S}_T^t]_{\text{impossible}} \leftarrow \emptyset$  // Impossible updates become instable, i.e. evaluable.
9:      $\langle \mathcal{S}_T^t \rangle \leftarrow$  relaxing
10:  else
11:     $\langle \mathcal{S}_T^t \rangle \leftarrow$  blocked
12:    // At this level, the simulator consider the timesteps  $\mathcal{S}_T^{t'}$ , owning busy out-arguments as blockers of  $\mathcal{S}_T^t$ .
13:  end if
14: else if  $[\mathcal{S}_T^t]_{\text{unstable}} \neq \emptyset$  then
15:   //  $[\mathcal{S}_T^t]_{\text{impossible}} = \emptyset$ 
16:    $[\mathcal{S}_T^t]_{\text{stable}} \leftarrow [\mathcal{S}_T^t]_{\text{stable}} \cup [\mathcal{S}_T^t]_{\text{confirmed}}$ 
17:    $[\mathcal{S}_T^t]_{\text{confirmed}} \leftarrow \emptyset$  // A single unstable argument leads to reconsider the  $\mathcal{S}_T^t$  stability.
18:    $\langle \mathcal{S}_T^t \rangle \leftarrow$  relaxing
19: else if  $[\mathcal{S}_T^t]_{\text{stable}} \neq \emptyset$  then
20:   //  $[\mathcal{S}_T^t]_{\text{impossible}} = \emptyset$ ,  $[\mathcal{S}_T^t]_{\text{unstable}} = \emptyset$ 
21:    $\langle \mathcal{S}_T^t \rangle \leftarrow$  checking //  $\mathcal{S}_T^t$  updates are all stable, stability confirmation is in progress.
22: else
23:   //  $[\mathcal{S}_T^t]_{\text{impossible}} = \emptyset$ ,  $[\mathcal{S}_T^t]_{\text{unstable}} = \emptyset$ ,  $[\mathcal{S}_T^t]_{\text{stable}} = \emptyset$ 
24:   // Every update stability is confirmed. We can set all the results as ready
25:   for all  $\bar{u} \in [\mathcal{S}_T^t]_{\text{confirmed}}$  do
26:      $\langle \text{res}_u \rangle \leftarrow$  ready
27:   end for
28:    $\langle \mathcal{S}_T^t \rangle \leftarrow$  done
29:   for all  $\mathcal{S}_T^{t'}$ , for which  $\mathcal{S}_T^t$  is a blocker do
30:     update_status( $\mathcal{S}_T^{t'}$ )
31:   end for
32: end if
```

Algorithm 4 `has_unbound`(\mathcal{S}_T^t)

```
1: // The readiness of the DIs that are unbound in-arguments of some update in the  $[\mathcal{S}_T^t]_{\text{new}}$  queue are checked. If the update has no busy unbound in-arguments, it is moved into the  $[\mathcal{S}_T^t]_{\text{unstable}}$  queue.
2: return  $\mathcal{F}_T^t \neq \emptyset$ 
```

- The decription of some computation, i.e. the function called to realize an update.
- The result res_π , which is a zero-shift DRI $\{T, X, 0\}$.
- Other RDIs or DIs, thats serve as arguments to the function.

Anchoring an UP π such as $\text{res}_\pi = \{T, X, 0\}$ at time t consists in defining an update by anchoring the result and all the arguments which are RDIs, in order to get a regular update for $[T, X, t]$.

2.4 Simulation

The simulator compute DIs. It does it by asking tasks to its active timesteps, as explained in section 2.3.4. When it is out of work, it tries to generate more DIs to compute by anchoring available UPs, thus creating new active timesteps. This is described in algorithm 5.

Algorithm 5 `get_one_job()`

Require: A task queue (jobs) denoted by \mathcal{J} , used and updated by the algorithm.

```

1: if  $\mathcal{J} = \emptyset$  then
2:   // Let us try to fill the task queue  $\mathcal{J}$ .
3:   for all  $\mathcal{S}_T^t$  which are handled currently do
4:     // We check for eventual unbound DIs
5:     if has_unbound( $\mathcal{S}_T^t$ ) then
6:        $\langle \mathcal{S}_T^t \rangle \leftarrow \text{unbound}$  // See algorithm 4.
7:     end if
8:     if  $\langle \mathcal{S}_T^t \rangle \in \{\text{relaxing}, \text{checking}\}$  then
9:       push( $\mathcal{J}$ ,  $[\mathcal{S}_T^t]_{\text{unstable}} \cup [\mathcal{S}_T^t]_{\text{stable}}$ ) // see section 2.3.4
10:    end if
11:  end for
12:  if  $\mathcal{J} = \emptyset$  then
13:    // No active patterns are able to provide new jobs.
14:    // We create new timesteps by anchoring the UPs.
15:    anchor_UPs() // See algorithm 6.
16:    // We may have new  $\mathcal{S}_T^t$ s, let us retry to ask them jobs.
17:    for all  $\mathcal{S}_T^t$  which are handled currently do
18:      // We check for eventual unbound DIs
19:      if has_unbound( $\mathcal{S}_T^t$ ) then
20:         $\langle \mathcal{S}_T^t \rangle \leftarrow \text{unbound}$  // See algorithm 4.
21:      end if
22:      if  $\langle \mathcal{S}_T^t \rangle \in \{\text{relaxing}, \text{checking}\}$  then
23:        push( $\mathcal{J}$ ,  $[\mathcal{S}_T^t]_{\text{unstable}} \cup [\mathcal{S}_T^t]_{\text{stable}}$ ) // see section 2.3.4
24:      end if
25:    end for
26:  end if
27: end if
28: if  $\mathcal{J} = \emptyset$  then
29:   return // There is really no way to get a new job.
30: else
31:    $j \leftarrow \text{pop}(\mathcal{J})$ 
32:   return  $j$  // That job will be done.
33: end if

```

Algorithm 6 `anchor_UPs()`

Require: The set of defined UPs is referred to as \mathcal{P} .

- 1: Determine the used variables $\mathcal{V} \stackrel{\text{def}}{=} \{(T, X) \mid \pi \in \mathcal{P} \wedge \text{res}_\pi = \{T, X, \tau\}\}$.
 - 2: For each timeline T mentioned in \mathcal{V} , compute $\mathcal{O}T$ as the first time instant for which a variable $(T, X) \in \mathcal{V}$ is **busy** (i.e. not computed so far).
 - 3: **for all** $\pi \in \mathcal{P}$ **do**
 - 4: Consider $\{T, X, 0\} = \text{res}_\pi$
 - 5: **if** $\langle [T, X, \mathcal{O}T] \rangle = \text{busy}$ **then**
 - 6: Anchor π at time instant $\mathcal{O}T$, thus creating a new timestep in the simulator for the next instant to be computed in that timeline.
 - 7: **else**
 - 8: // The DI computed by π is already available at $\mathcal{O}T$, some other UPs are late, they will be instantiated first.
 - 9: **end if**
 - 10: **end for**
-

3 Available update functions

3.1 Initializations

This sets the variable X so that each of the float it contains is 3.14, Y so that each of the float it contains is a random value in $[0, 1]$, Z as a copy of X .

```
"X" << fx::clear() | kwd::use("value", 3.14);
"Y" << fx::random();
"Z" << fx::copy("X");
```

3.2 Matching a value against weights

In SOM-like computation, an input ξ is given and is matched against all the weights w_i in the map. The result is an activity vector, its elements a_i are in $[0, 1]$, corresponding to each weight, that is 1 if the weight and the input perfectly match.

The formula is, for a map of weights w_i and an input $a_i, \forall i, a_i = \mu(\xi, w_i)$. Two matching functions are defined, a triangle matching μ_Δ and a Gaussian matching μ_G .

$$\mu_G(\xi, w) \stackrel{\text{def}}{=} \exp\left(\frac{(\xi - w)^2}{2\sigma^2}\right)$$
$$\mu_\Delta(\xi, w) \stackrel{\text{def}}{=} \max\left(1 - \frac{|\xi - w|}{r}, 0\right)$$

```
kwd::type("Xi", "Pos2D");
kwd::type("Wgt", "Map1D<Pos2D>=100");
kwd::type("ActT", "Map1D<Scalar>=100");
kwd::type("ActG", "Map1D<Scalar>=100");

"ActT" << fx::match_triangle("Xi", "Wgt") | kwd::use("r", .3);
"ActG" << fx::match_gaussian("Xi", "Wgt") | kwd::use("sigma", .3);
```

3.3 Merging activities

The average is also the contextual merge. So we have two names for the same operation.

```

kwd::type("Ac1", "Map1D<Scalar>=100");
kwd::type("Ac2", "Map1D<Scalar>=100");
kwd::type("Ac3", "Map1D<Scalar>=100");
kwd::type("Ac", "Map1D<Scalar>=100");
"Ac" << fx::average({"Ac1", "Ac2", "Ac3"});
"Ac" << fx::context_merge({"Ac1", "Ac2", "Ac3"});

```

A specific operation is allowed for merging contextual and external activity.

```

kwd::type("Ae", "Map1D<Scalar>=100");
kwd::type("Ag", "Map1D<Scalar>=100");
"Ag" << fx::merge("Ae", "Ac"); | kwd::use("beta", .5),

```

The formula is

$$\forall i, a_i^g = \sqrt{a_i^e + (1 - \beta)a_i^c}$$

3.4 Learning

Learning concerns the weights w_i of a map, once a best matching unit (BMU) π is defined. Learning consists of moving the weights to the current input ξ , in the surrounding of the best matching unit. The neighboring function is $\mu(i, \pi)$, using Gaussian and triangular matchings μ_G or μ_Δ . The learning rule is:

$$\forall i, w_i^{t+1} = (1 - \alpha h)w_i^t + \alpha h \xi, \text{ with } h = \mu(i, \pi)$$

```

kwd::type("Xi", "Pos2D");
kwd::type("BMU", "Pos1D");
kwd::type("Wgt", "Map1D<Pos2D>=100");

"Wgt" << fx::learn_gaussian("Xi", kwd::prev("Wgt"),
                           "BMU") | kwd::use("alpha", .05), kwd::use("r", .3);
"Wgt" << fx::learn_triangle("Xi", kwd::prev("Wgt"),
                            "BMU") | kwd::use("alpha", .05), kwd::use("sigma", .3);

```

3.5 BMU computation

A scalar activity distribution over a map serves as a basis for computing the BMU. The basic operation is the argmax, i.e. the position on the map where the activity is maximal. In case of ex-aequos, a random choice can be performed (see the "random-bmu" parameter). The activity distribution can be convoluted by a gaussian kernel before the argmax computation. The variance σ of that kernel is expressed in units such as the map side as a size of 1, whatever the number of units actually in the map.

```

kwd::type("Act", "Map1D<Scalar>=100");
kwd::type("BMU_noconv", "Pos1D");
kwd::type("BMU_conv", "Pos1D");

"Act" << fx::random()
"BMU_noconv" << fx::argmax("Act") | kwd::use("random-bmu", 1.0);
"BMU_conv" << fx::conv_argmax("Act") | kwd::use("random-bmu", 0.0),
          kwd::use("sigma", .05);

```

The BMU can also be updated from a previous BMU position, by modifying it toward the argmax (after convolution) with a step δ . This is used for relaxation processes. The number of convergence steps can be measured thanks to the "converge" operator.

```

kwd::type("Act", "Map1D<Scalar>=100");
kwd::type("BMU", "Pos1D");
kwd::type("NbSteps", "Scalar");

"Act" << fx::random()

```

```

"BMU" << fx::toward_conv_argmax("Act",
                                kwd::prev("BMU")) | kwd::use("sigma", .05),
                                kwd::use("delta", .05);
"NBSteps" << fx::converge({kwd::data("BMU")}));

```

4 Implementation

4.1 Variables

Variables are stored in `.var` files. They are binary files. When unsigned integers are mentioned in the following, they are 8-byte unsigned values, starting from most significant bytes first (big endian). The file is organized as follows, in that order:

- 64 bytes: They contain an ascii version of the type, ended by `'\n'`, and complemented with 0 bytes (padding) until the whole description's length is exactly 64-bytes.
- 8 bytes: An unsigned integer representing the cache size. This is used by the simulator to determine the size of the cache to be associated to that variable during the simulation.
- 8 bytes: An unsigned integer representing the buffer size s . Indeed, the file stores an history of the variable, i.e. the values from $[T, X, 0]$ to $[T, X, t]$. Although this theoretically represents $t + 1$ values, all of them may not be present in the file. The file is rather a circular buffer, with a limited size s . So only values from $[T, X, t - s + 1]$ to $[T, X, t]$ are stored. Once determined in the file, the value of the buffer size cannot be changed.
- 8 bytes: The highest time in the file. This is the value t mentioned above. If the file is empty (i.e. even the first $[T, X, 0]$ is not stored yet), the 8 bytes are `0xFFFFFFFFFFFFFFF`.
- 8 bytes: The next free position in the file. The file contain a range of s values, but it is a circular buffer. This bytes tells which index (starting from zero) is the next free position (i.e. where the $[T, X, t + 1]$ has to be stored).
- from 0 to at most $s \times (d + 1)$ bytes: This is the data. Each datum is a $[T, X, t']$ value, *preceded* by a boolean byte. So if d is the number of bytes required for storing a datum, a slot requires $d + 1$ bytes. The file buffer contains then from 0 to at most s slots for storing values. If the boolean byte is 0 in a slot, the datum in that slot is considered as undetermined yet (i.e. `busy`). Otherwise, the d bytes following the boolean byte describe the datum value stored in this slot.