# CxSOM

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## Contents

1 Introduction		1		
2	The model			1
	2.1	Data i	instances (DI)	1
2.2 Updates		Updat	es	2
		2.2.1	An update is a function taking arguments and computing a result	2
		2.2.2	Datation and relaxation	2
		2.2.3	Update computation cycle	3
	2.3	Times	teps and tasks	3
		2.3.1	A computational structure for timesteps	3

# 1 Introduction

The CxSOM software suite enables to model consensus based multi-SOMs, explored in the BISCUIT team of teh Loria lab.

The current documentation is at work, and mainly technical stuff are reported here. See the README at the package root for an introduction, as well as examples.

# 2 The model

The computation enabled by CxSOM consists of successively updating data, keeping trace of the data values in history files.

## 2.1 Data instances (DI)

The elementary piece of data handled in CxSOM is called a *data instance* (DI). Each of the computed DIs ends as a record in a file, once it has been computed.

DIs are denoted by a triplet [T, X, t], that reads as "the instance of variable X hosted by the timeline T at the time instant t of that specific timeline". Only elementary [T, X, t] DIs matter,

the concepts of variable, timeline and time instant only serve for describing the computation.

Each DIs is a value that is stored in a file once definitively computed. A value has a type, that can be:

- Scalar: A floating point value, usually in [0, 1].
- Pos1D: A value in [0,1] corresponding to a position in a 1D map.
- Pos2D: A value in  $[0,1]^2$  corresponding to a position in a 2D map.
- Array=n: A value in  $\mathbb{R}^n$ , usually  $[0, 1]^n$ ,  $n \in \mathbb{N}^*$ .
- Map1D< $\mathcal{X}$ >=n: A 1D map of type  $\mathcal{X}$ , i.e. a value in  $\mathcal{X}^n$ ,  $n \in \mathbb{N}^*$ .
- Map2D< $\mathcal{X}$ >=n: A 2D squared map of type  $\mathcal{X}$ , i.e. a value in  $(\mathcal{X}^n)^n$ ,  $n \in \mathbb{N}^*$ .

where type  $\mathcal{X} \in \{\text{Scalar}, \text{Pos1D}, \text{Pos2D}, \text{Array}=k\}.$ 

In CxSOM, the type is associated to a variable, i.e. all the  $[T, X, \bullet]$  are DIs with the same type.

Every DI is also doted with a status variable  $\langle [T, X, t] \rangle \in \{\text{busy}, \text{ready}\}:$ 

- ready: The computation of the value of the DI is definitively done. The DI value will not change anymore.
- busy: The definitive value of the DI is still to be determined.

More generally, in the following,  $\langle x \rangle$  reads as "the status of x".

## 2.2 Updates

An *update* of some DI is the (re)computation by the CxSOM computer of the value of that DI. There are at most two updates for a DI, one for the first update (initialization) that is optional, and one, mandatory, for other updates.

#### 2.2.1 An update is a function taking arguments and computing a result

An update u for a DI [T, X, t] is made of:

- The decription of some computation, i.e. the function called to realize an update.
- The result  $\operatorname{res}_u$ , i.e.  $\operatorname{res}_u \stackrel{\text{def}}{=} [T, X, t]$  itself, that is computed by the function of the update.
- Other DIs, thats serve as arguments to the function (their value is read when the function is called). Among arguments, a distinction is made between *in-arguments* and *out-arguments*:
  - The in-arguments in<sub>u</sub> of the update u such as  $\operatorname{res}_u = [T, X, t]$  is the set of DIs used as arguments of u of the form  $[T, \bullet, t]$ . All the DIs handled in the simulation of the form  $[T, \bullet, t]$  define a time step  $\mathcal{S}_T^t$  of the simulation, so the in-arguments are the DIs used for the computation of an update which belong to the same timestep as the result.
  - The out-arguments  $\operatorname{out}_u$  of the update u are the other DIs used as arguments for the computation of the update.

#### 2.2.2 Datation and relaxation

Before it has a definitive value (i.e. before being in the ready status), the value of a DI may change several times. Indeed, all DIs in a timestep  $\mathcal{S}_T^t = [T, \bullet, t]$  are updated until all of them get ready, and during that process, the values of the DIs may be recomputed several times. In order to handle the dependancies of the DIs within a timestep, we need to timestamp the values of the DIs, in order to determine those of them that have to be updated. Timestamp play the role of file dates in makefile, when the date of a target is compared to the date of its dependancies. Here, in order to know if an update u needs to be performed, we have to consider two things:

- Do we have  $\forall [T, X, t] \in \text{out}_u$ ,  $\langle [T, X, t] \rangle = \text{ready}$ ? If not, the update cannot be done, it is considered as *impossible*.
- Have the in-arguments been updated since the last computation of the result? If the answer is yes, the result needs to be recomputed.

To compute the second condition, every DI comes with a datation denoted by  $d_{[T, X, t]} \in \mathbb{N}$ .

An update then stores, for each of its in-arguments, the datation it had at the last computation of the result. When the simuator considers an update, it compares the current datation of the in-arguments to the ones stored by the update. If some are newer (or if the result has never been computed so far) the result is recomputed, and

$$d_{res_u} = 1 + \max_{[T, X, t] \in in_u} d_{[T, X, t]}$$
(1)

Considering an update for computation can thus lead to the following status of the update u, denoted by  $\langle u \rangle$ :

- $\langle u \rangle$  = impossible:  $\langle res_u \rangle$  = busy, computation is not feasible yet, since some out-arguments are busy.
- $\langle u \rangle$  = uptodate:  $\langle \text{res}_u \rangle$  = busy, nothing changed in the in-arguments input dates from last update, or the new value was not a significant modification. The datation  $d_{\text{res}_u}$  has not been modified.
- $\langle u \rangle = \text{updated}$ :  $\langle \text{res}_u \rangle = \text{busy}$ , the computation has modified the value of  $\text{res}_u$  significantly.
- $\langle u \rangle = \text{done: } \langle \text{res}_u \rangle = \text{ready}$ , the computation has modified the value  $\text{res}_u$  definitively.
- $\langle u \rangle$  = none: Update status is not determined yet (used for initialization only).

The datation mechanism enables to control the update of all the DIs in a given timestep, i.e. all the DIs like  $[T, \bullet, t]$  for the time instant t of the timeline T. When no more significant writes of the results can be done, the whole timestep is stable. So the relevance of the datation mechanism is to enable a *relaxation* of the DIs inside a timestep until stabilization of all of them is reached.

#### 2.2.3 Update computation cycle

When a specific update is defined in CxSOM (as average, learning, etc.), the update is inherited from a base cxsom::update::Base class. In the subclass, the specificity of the computation has to be implemented, while the base class handles the details of the full update cycle, as datation and status. Algorithm 1 shows the cycle, the on\_\* calls are the method that subclass need to override in order to implement a specific update computation.

### ${f Algorithm~1}$ One update cycle for update u

Require: a boolean attribute out\_ok, telling if all out-args have been read successfully previously. Require: a boolean attribute is\_init, telling if the update is an initialization.

```
1: if \langle res_u \rangle = ready then
       return done
 3: end if
 4: on_computation_start()
 5: if ¬out₋ok then
       // out-arguments need to be read.
       for all [T, X, t] \in \text{out}_u do
 7:
         if \langle [T, X, t] \rangle = \text{ready then}
 8:
            on_read_out_arg ([T, X, t])
 9:
         else
10:
            out\_ok \leftarrow false
11:
12:
            on_read_out_arg_aborted()
            return impossible
13:
         end if
14:
       end for
15:
16:
       out_ok ← true // All in-args have been successfully read.
17: end if
    for all [T, X, t] \in \text{in}_u do
18:
       // Datation issues are handled in this loop, but not detailed here.
19:
       on_read_in_arg ([T, X, t])
21: end for
22: if none of the in-arguments have been updated since last cycle then
23:
       if is_init then
24:
         return updated // updates used as initialization are never considered as up-to-date.
25:
       else
         return uptodate
26:
27:
       end if
28: end if
29: // From here, we know that the result has to be recomputed.
30: x \leftarrow \text{none} // This is the status we plan to return, it is set next.
31: \alpha \leftarrow \text{on\_write\_result}(\text{res}_u) // \alpha is a Boolean telling if the value change is significant.
    if we have just written a new significant value in res_u then
       x \leftarrow \mathtt{updated}
33:
34: else
       if is_init then
35:
         return x \leftarrow updated // Even not significant changes are considered as an actual update for
36:
         initialization updates.
37:
       else
         return x \leftarrow \texttt{uptodate}
38:
       end if
39:
40: end if
41: // Before returning, we have to check if in-arg may change in the future
42: if all in-arguments are ready then
43:
       if is_init then
         x \leftarrow \text{updated} // As u an init, we return updated since further change can be consider by the
44:
         usual update comming next for updating res_u.
45:
46:
         x \leftarrow \text{done} // We have definitively computed the result...
          \langle \operatorname{res}_u \rangle \leftarrow \operatorname{ready} // \operatorname{The result DI value is locked}.
47:
       end if
48:
49: end if
50: \mathbf{return} \ x
```

# 2.3 Timesteps and tasks

## 2.3.1 A computational structure for timesteps

In the simulator, the computation of the updates are put in a pool task and computed in parallel. However, since relaxation is related to the computation of updates belonging to the same timestep, this task management needs an intermediate level, which is a computational internal structure representing the timesteps.

Let us recall the notation  $\mathcal{S}_T^t \stackrel{\text{def}}{=} \{[T',\,X',\,t'] \in \text{simulation} \mid T' = T \wedge t' = t\}$  for the timestep at instant t in the timeline T. For each  $[T,\,X,\,t] \in \mathcal{S}_T^t$ , the internal structure used in the simulator gathers at least an usual update, and an optional initialization update. The initialization update, if present, is used for the first feasible setting of the value of  $[T,\,X,\,t]$  instead of the usual update. Otherwise, the usual update is used throughout the successive setting of the value of  $[T,\,X,\,t]$  during the relaxation.