1. Substitution

$$(\lambda x. E) \, M \Rightarrow E[M/x]$$

$$((\text{Lambda } (x) \, \underbrace{(+ 1 \, x)}_{E}) \, \underbrace{5}_{M}) \Rightarrow$$

$$(+ 1 \, x)[5/x] \Rightarrow (+ 1 \, 5)$$

2. Free and Bound Variables

$$(\lambda x. x \, y)$$

$$(\lambda x. \lambda y. x \, y) \, (y \, w) \Rightarrow$$

$$\lambda y. (y \, w) \, y$$

$$(\lambda y. x \, y) \, [(y \, w)/x]$$

$$\lambda 1. \underbrace{((x \, y)[1/y])}_{recurse} [(y \, w)/x] \Rightarrow$$

$$\lambda 1. (x \, 1)[(y \, w)/x] \Rightarrow$$

$$\lambda 1. (y \, w) \, 1$$

3. Substitution Algorithm (Slide 20)

$$(\lambda x. E)\ M \Rightarrow E[M/x]$$

E can be variable or Abs, or App
recursively defined

Algorithm aggressively renames parameters.

$$(\lambda y. \lambda z.\ x\ y\ z)\ [v/x] \Rightarrow$$

$$(\lambda y. \lambda z.\ v\ y\ z)$$

4. $(\lambda x. \lambda y. \lambda z.\ x\ y\ z)\ v$

$$\Rightarrow \lambda y. \lambda z.\ v\ y\ z$$