— LAMBDA CALCULUS — REVIEW

1. Reducible expression (REDEX)

$(\lambda x. E)\, M$ is called a redex

$(\lambda x. E)\, M \to_\beta E_1$

$\lambda x. E$

- $(\lambda x. x)\, y \to_\beta y$
- $(\lambda x. x)\, (\lambda z. z) \to_\beta$
- $(\lambda \underset{x}{x}. \underset{E}{\lambda y. x})\, \underset{M}{z}\ w \to_\beta (\lambda y. z)\, w \to_\beta z$

2. Substitution

Each β-reduction triggers a substitution

$(\lambda x. E)\, M \to \underset{E_1}{E[M/x]}$

- Substitution may require renaming of formal parameters to avoid making free variables bound
- In our examples, we will be performing renaming only when necessary

$(\lambda x. \underline{\lambda y. x})\ z\ w \to_\beta \underline{(\lambda y. z)\ w} \to z$

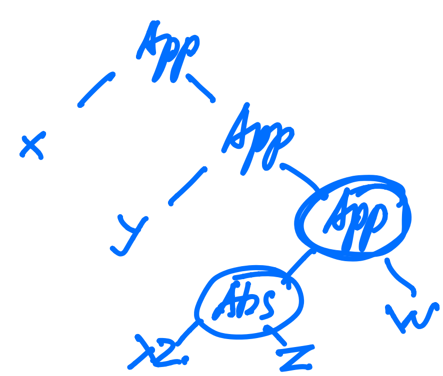
- Automated algorithm does aggressive renaming (Lec 15, slide 20)

  $\underline{(\lambda y. E)}[M/x]$

3. Normal Form

3.1 Canonical normal form $\underline{NF}$

- $\lambda z. z$ is in NF
- $z\ v\ w$ is in NF
- $z\ (v\ w)$ is in NF
- $\underline{(\lambda x. x)}\ \underline{z}$ is not in NF
- $\underbrace{x}_{E_1}\ \underbrace{(\underline{(\lambda x. x)\ (\lambda y. y)})}_{E_2}$ is not in NF
- $\underbrace{x}_{E_1}\ \underbrace{(\underbrace{y}_{E_3}\ \underbrace{(\underline{(\lambda z. z)\ w})}_{E_4})}_{E_2}$ is not in NF



3.2 Weak normal forms (lazy normal forms)

HNF:

- x is in HNF
- $\cancel{x}\ \underline{E_1}\ \underline{E_2}\ \underline{E_3} \dots \underline{E_n}$ is in HNF
- $\lambda x. E$ is in HNF if E is in HNF

- $\lambda z. z$ is in HNF
- $\underline{x}\ (\underline{(\lambda x. x)(\lambda y. y)})$ is in HNF

WHNF:

- x is in WHNF
- $\underline{x}\ E_1\ E_2 \dots E_n$ is in WHNF
- $\lambda x. \underline{E}$ is WHNF

- $\lambda x. \underline{(\lambda y. y)\ z}$ is in WHNF

4. Combinators are terms with no free variables

$$I = \lambda x.x$$

$$S = \lambda x.\lambda y.\lambda z.\, xz(yz)$$

$$K = \lambda x.\lambda y.x$$

$$\rightarrow SKSK = ((SK)S)K$$

$$(\lambda x.\lambda y.\lambda z.\, xz(yz))KSK \to_\beta$$

$$(\lambda y.\lambda z.\, Kz(yz))SK \to_\beta$$

$$(\lambda z.\, Kz(Sz))K \to_\beta$$

$$KK(SK) =$$

$$(\lambda x.\lambda y.x)K(SK) \to_\beta^{*} K$$