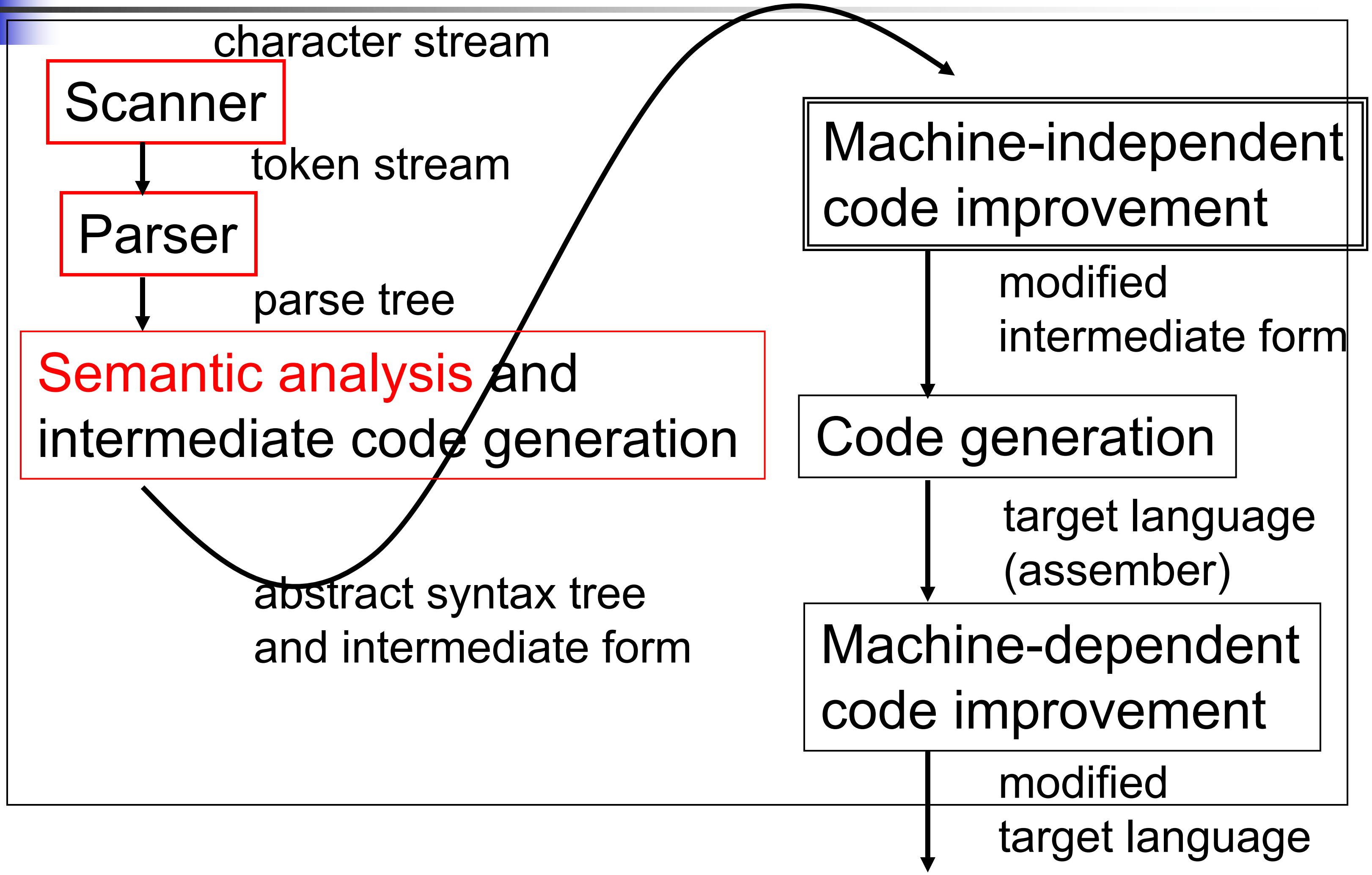
# Programming Language Syntax

Read: Scott, Chapter 2.1

# Lecture Outline

- Formal languages
- Regular expressions
- Context-free grammars
  - Derivation
  - Parse
  - Parse trees
  - Ambiguity
- Expression Grammars

# Last Class: Compiler



# Syntax and Semantics

- Syntax is the form or structure of expressions, statements, and program units of a given language
  - Syntax of a Java **while** statement:
    - `while (` *boolean_expr* `)` *statement*

- Semantics is the meaning of expressions, statements and program units of a given language
  - Semantics of `while (` *boolean_expr* `)` *statement*
    - Execute *statement* repeatedly (0 or more times) as long as *boolean_expr* evaluates to `true`

# Formal Languages

- Theoretical foundations – Automata theory
- A language is a set of strings (also called sentences) over a finite alphabet
- A generator is a set of rules that generate the strings in the language
- A recognizer reads input strings and determines whether they belong to the language
- Languages are characterized by the complexity of generation/recognition rules
  - E.g., regular languages
  - E.g., context-free languages

# Question

- What are the classes of formal languages?

- The Chomsky hierarchy:
  - Regular languages
  - Context-free languages
  - Context-sensitive languages
  - Recursively enumerable languages

# Formal Languages

- Generators and recognizers become more complex as languages become more complex
  - Regular languages
    - Describe PL tokens (e.g., keywords, identifiers, numeric literals)
    - Generated by Regular Expressions
    - Recognized by a Finite Automaton (scanner)
  - Context-free languages
    - Describe more complex PL constructs (e.g., expressions and statements)
    - Generated by a Context-free Grammar
    - Recognized by a Push-down Automaton (parser)
  - Even more complex constructs

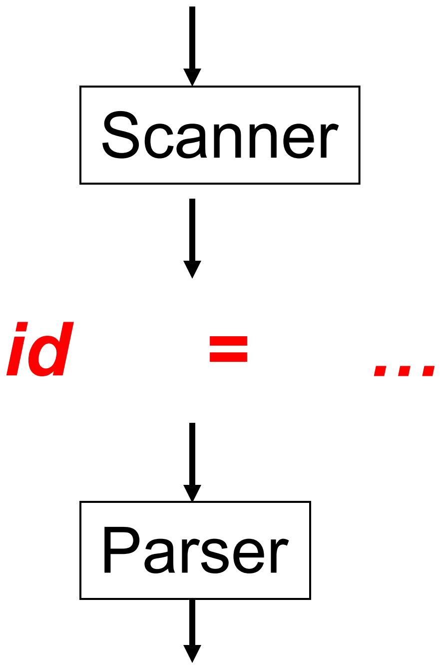
# Formal Languages

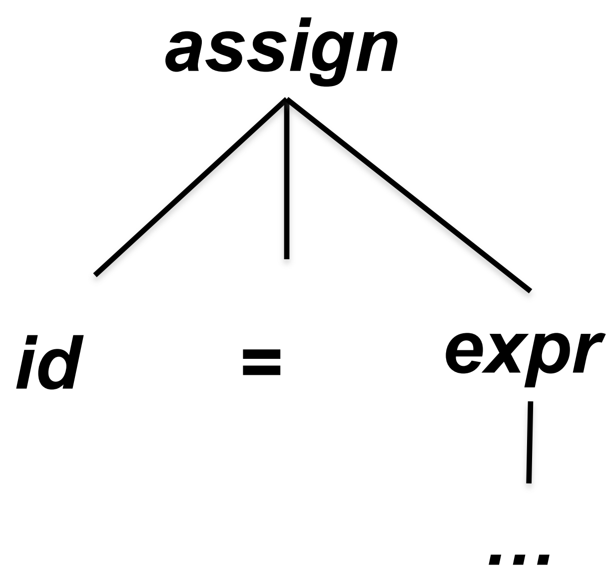
- Main application of formal languages: enable proof of relative difficulty of computational problems
- Our focus: formal languages provide the formalism for describing PL constructs
  - A compelling application of formal languages!
  - Building a scanner
  - Building a parser
  - Central issue: build efficient, linear-time parsers

# A Single Pass

***position = initial + rate * 60;***

- Scanner emits next token
- Parser consumes the token and continues building the parse tree (typically bottom up)



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# Regular Expressions

- Simplest structure
- Formalism to describe the simplest programming language constructs, the tokens
  - each symbols (e.g., “+”, “-”) is a token
  - an identifier (e.g., position, rate, initial) is a token
  - a numeric constant (e.g., 59) is a token
  - etc.
- Recognized by a finite automaton

# Regular Expressions

- A Regular Expression is one of the following:
  - A character, e.g., **`a`**
  - The empty string, denoted by **ε**
  - Two regular expressions next to each other, $R_1\ R_2$
    - Meaning: $R_1\ R_2$ generates the language of strings that are made up of any string generated by $R_1$, followed by any string generated by $R_2$
  - Two regular expressions separated by |, $R_1 \mid R_2$
    - Meaning: $R_1 \mid R_2$ generates the language that is the union of the strings generated by $R_1$ with the strings generated by $R_2$

# Question

- What is the language defined by reg. exp. `(a | b) (a a | b b)` ?

- We saw concatenation and alternation. What operation is still missing?

# Regular Expressions

- A Regular Expression is one of the following:
  - A character, e.g., **a**
  - The empty string, denoted by **ε**
  - $R_1 R_2$
  - $R_1 \mid R_2$
  - Regular expression followed by a Kleene star, R*
    - Meaning: the concatenation of zero or more strings generated by R
    - E.g., **a*** generates {**ε, a, aa, aaa,** … }
    - E.g., **(a|b)*** generates all strings of **a**’s and **b**’s

# Regular Expressions

- Precedence
  - Kleene * has highest precedence
  - Followed by concatenation
  - Followed by alternation |

  - E.g., `a b | c` is `(a b) | c` not `a (b | c)`
    - Generates `{ab,c}` not `{ab,ac}`
  - E.g., `a b*` generates `{a,ab,abb,…}` not `{ε, ab, abab, ababab,…}`

# Question

- What is the language defined by regular expression **`(0 | 1)* 1`** ?

- What about **`0* (1 0* 1 0*)*`** ?

# Regular Expressions in Programming Languages

- Describe tokens
- Let

  *letter* → **`a|b|c| … |z`**

  *digit* → **`1|2|3|4|5|6|7|8|9|0`**

- Which token is this?

  *1. letter ( letter | digit )**      ?

  *2. digit digit **      ?

  *3. digit *. digit digit **      ?

# Regular Expressions in Programming Languages

- Which token is this:

*number* $\rightarrow$ *integer* | *real*

*real* $\rightarrow$ *integer exponent* | *decimal* ( *exponent* | $\varepsilon$ )

*decimal* $\rightarrow$ *digit** ( `.` *digit* | *digit* `.` ) *digit**

*exponent* $\rightarrow$ ( `e` | `E` ) ( `+` | `-` | $\varepsilon$ ) *integer*

*integer* $\rightarrow$ *digit digit**

*digit* $\rightarrow$ `1` | `2` | `3` | `4` | `5` | `6` | `7` | `8` | `9` | `0`

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# Context-Free Grammars

- Unfortunately, regular languages cannot specify all constructs in programming
- E.g., can we write a regular expression that specifies valid arithmetic expressions?
  - `id * ( id + id * ( number - id ) )`
  - Among other things, we need to ensure that parentheses are matched!
  - Answer is no. We need context-free languages and context-free grammars!

# Grammar

- A grammar is a formalism to describe the strings of a (formal) language
- A grammar consists of a set of terminals, set of nonterminals, a set of productions, and a start symbol
  - Terminals are the characters in the alphabet
  - Nonterminals represent language constructs
  - Productions are rules for forming syntactically correct constructs
  - Start symbol tells where to start applying the rules

# Notation

Specification of identifier:

Regular expression: *letter* ( *letter* | *digit* )*

BNF: *<digit> ::= 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 0*

*<letter> ::= a | b | c | … | x | y | z*

*<id> ::= <letter> | <id> <letter> | <id> <digit>*

Textbook and slides: (also BNF)

*digit* → `1` | `2` | `3` | `4` | `5` | `6` | `7` | `8` | `9` | `0`

*letter* → `a` | `b` | `c` | `d` | … | `z`

*id* → *letter* | *id letter* | *id digit*

Nonterminals shown in *italic*

Terminals shown in **typewriter**

# Regular Grammars

- Regular grammars generate regular languages
- The rules in regular grammars are of the form:
  - Each left-hand-side (lhs) has exactly one nonterminal
  - Each right-hand-side (rhs) is one of the following
    - A single terminal symbol or
    - A single nonterminal symbol or
    - A <u>nonterminal followed by a terminal</u>

e.g., $\mathtt{1}\ \mathtt{2}^*\ |\ \mathtt{0}^+$

$$S \to A \mid B$$
$$A \to \mathtt{1} \mid A\ \mathtt{2}$$
$$B \to \mathtt{0} \mid B\ \mathtt{0}$$

# Question

- Is this a regular grammar:

$$S \rightarrow \mathtt{0}\ A$$
$$A \rightarrow S\ \mathtt{1}$$
$$S \rightarrow \varepsilon$$

  - No, this is a context-free grammar
    - It generates $\mathtt{0}^n\mathtt{1}^n$, the canonical example of a context-free language
    - rhs should be <u>nonterminal followed by a terminal</u>, thus, $S \rightarrow \mathtt{0}\ A$ is not a valid production

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# Context-free Grammars (CFGs)

- Context-free grammars generate context-free languages
  - Most of what we need in programming languages can be specified with CFGs
- Context-free grammars have rules of the form:
  - Each left-hand-side has exactly one nonterminal
  - Each right-hand-side contains an arbitrary sequence of terminals and nonterminals
- A context-free grammar

  e.g. $\mathbf{0}^n\mathbf{1}^n, n\geq 1$ $\quad S \rightarrow \mathbf{0}\ S\ \mathbf{1}$

  $S \rightarrow \mathbf{0}\ \ \mathbf{1}$

# Question

- Examples of a non-context-free languages?
  - E.g., $\mathbf{a}^n\mathbf{b}^m\mathbf{c}^n\mathbf{d}^m$ $n \geq 1, m \geq 1$
  - E.g., $w\mathbf{c}w$ where $w$ is in `(0|1)*`
  - E.g., $\mathbf{a}^n\mathbf{b}^n\mathbf{c}^n$ $n \geq 1$ (canonical example)

# Context-free Grammars

- Can be used to <u>generate</u> strings in the context-free language (derivation)
- Can be used to <u>recognize</u> well-formed strings in the context-free language (parse)

- In Programming Languages and compilers, we are concerned with two special CFGs, called LL and LR grammars

# Derivation

Simple context-free grammar for expressions:

$expr \rightarrow \texttt{id} \mid \texttt{(}\ expr\ \texttt{)} \mid expr\ op\ expr$

$op \rightarrow \texttt{+} \mid \texttt{*}$

We can generate (derive) expressions:

$$
\begin{aligned}
expr &\Rightarrow expr\ op\ \underline{expr} \\
&\Rightarrow expr\ \underline{op}\ \texttt{id} \\
&\Rightarrow \underline{expr}\ \texttt{+}\ \texttt{id} \\
&\Rightarrow expr\ op\ \underline{expr}\ \texttt{+}\ \texttt{id} \quad \longleftarrow \quad \text{sentential form} \\
&\Rightarrow expr\ \underline{op}\ \texttt{id}\ \texttt{+}\ \texttt{id} \\
&\Rightarrow \underline{expr}\ \texttt{*}\ \texttt{id}\ \texttt{+}\ \texttt{id} \\
&\Rightarrow \texttt{id}\ \texttt{*}\ \texttt{id}\ \texttt{+}\ \texttt{id} \quad \longleftarrow \quad \text{sentence, string or yield}
\end{aligned}
$$

# Derivation

- A derivation is the process that starts from the start symbol, and at each step, replaces a nonterminal with the right-hand-side of a production
  - E.g., *expr op <u>expr</u>* derives *expr op* **`id`**
    We replaced the right (underlined) *expr* with **`id`**
    due to production *expr* $\rightarrow$ **`id`**
- An intermediate sentence is called a sentential form
  - E.g., *expr op* **`id`** is a sentential form

# Derivation

- The resulting sentence is called yield
  - E.g., `id*id+id` is the yield of our derivation
- What is a left-most derivation?
  - Replaces the left-most nonterminal in the sentential form at each step
- What is a right-most derivation?
  - Replaces the right-most nonterminal in the sentential form at each step
- There are derivations that are neither left- nor right-most

# Question

- What kind of derivation is this:

$$
\begin{aligned}
expr &\Rightarrow expr\ op\ \underline{expr} \\
&\Rightarrow expr\ \underline{op}\ \mathtt{id} \\
&\Rightarrow \underline{expr}\ \mathtt{+}\ \mathtt{id} \\
&\Rightarrow expr\ op\ \underline{expr}\ \mathtt{+}\ \mathtt{id} \\
&\Rightarrow expr\ \underline{op}\ \mathtt{id}\ \mathtt{+}\ \mathtt{id} \\
&\Rightarrow \underline{expr}\ \mathtt{*}\ \mathtt{id}\ \mathtt{+}\ \mathtt{id} \\
&\Rightarrow \mathtt{id}\ \mathtt{*}\ \mathtt{id}\ \mathtt{+}\ \mathtt{id}
\end{aligned}
$$

  - A right-most derivation. At each step we replace the right-most nonterminal

# Question

- What kind of derivation is this:

$$
\begin{aligned}
expr &\Rightarrow expr\ op\ \underline{expr} \\
&\Rightarrow expr\ \underline{op}\ \mathtt{id} \\
&\Rightarrow \underline{expr}\ \mathtt{+}\ \mathtt{id} \\
&\Rightarrow \underline{expr}\ op\ expr\ \mathtt{+}\ \mathtt{id} \\
&\Rightarrow \mathtt{id}\ op\ \underline{expr}\ \mathtt{+}\ \mathtt{id} \\
&\Rightarrow \mathtt{id}\ \underline{op}\ \mathtt{id}\ \mathtt{+}\ \mathtt{id} \\
&\Rightarrow \mathtt{id}\ \mathtt{*}\ \mathtt{id}\ \mathtt{+}\ \mathtt{id}
\end{aligned}
$$

  - Neither left-most nor right-most

# Parse

Recall our context-free grammar for expressions:

$expr \rightarrow$ **id** | **(** $expr$ **)** | $expr\ op\ expr$

$op \rightarrow$ **+** | *****

- A parse is the reverse of a derivation

$$
\begin{aligned}
\texttt{id * id + id} &\Rightarrow expr\ \underline{\texttt{*}}\ \texttt{id + id} \\
&\Rightarrow expr\ op\ \underline{\texttt{id}}\ \texttt{+ id} \\
&\Rightarrow \underline{expr\ op\ expr}\ \texttt{+ id} \\
&\Rightarrow expr\ \underline{\texttt{+}}\ \texttt{id} \\
&\Rightarrow expr\ op\ \underline{\texttt{id}} \\
&\Rightarrow \underline{expr\ op\ expr} \\
&\Rightarrow expr
\end{aligned}
$$

# Parse

- A parse starts with the string of terminals, and at each step, replaces the right-hand-side (rhs) of a production with the left-hand-side (lhs) of that production. E.g.,

  … $\Rightarrow$ *expr op expr* **`+ id`**

  $\Rightarrow$ *expr* **`+ id`**

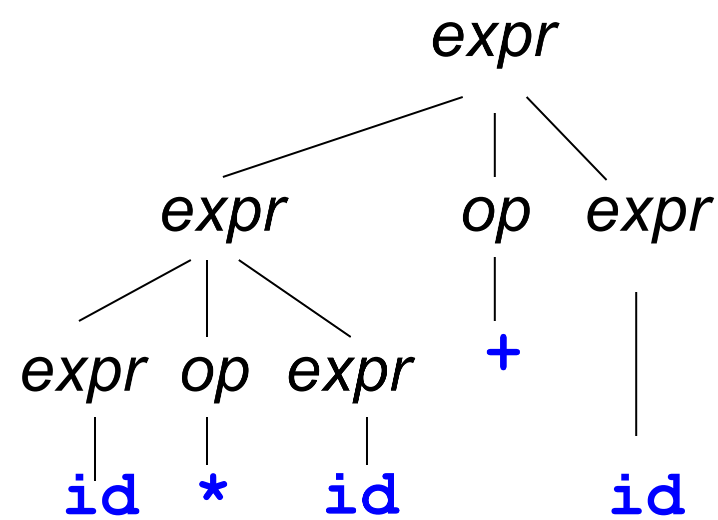
  Here we replaced *expr op expr* (the rhs of production *expr* $\rightarrow$ *expr op expr*) with *expr* (the lhs of the production)

# Parse Tree

$expr \rightarrow$ **id** | **(** $expr$ **)** | $expr\ op\ expr$

$op \rightarrow$ **+** | *****

$$
\begin{aligned}
expr &\Rightarrow expr\ op\ \underline{expr} \\
&\Rightarrow expr\ \underline{op}\ \mathbf{id} \\
&\Rightarrow \underline{expr}\ \mathbf{+}\ \mathbf{id} \\
&\Rightarrow expr\ op\ \underline{expr}\ \mathbf{+}\ \mathbf{id} \\
&\Rightarrow expr\ \underline{op}\ \mathbf{id}\ \mathbf{+}\ \mathbf{id} \\
&\Rightarrow \underline{expr}\ \mathbf{*}\ \mathbf{id}\ \mathbf{+}\ \mathbf{id} \\
&\Rightarrow \mathbf{id}\ \mathbf{*}\ \mathbf{id}\ \mathbf{+}\ \mathbf{id}
\end{aligned}
$$




Internal nodes are nonterminals. Children are the rhs of a rule for that nonterminal.
Leaf nodes are terminals.

# Ambiguity

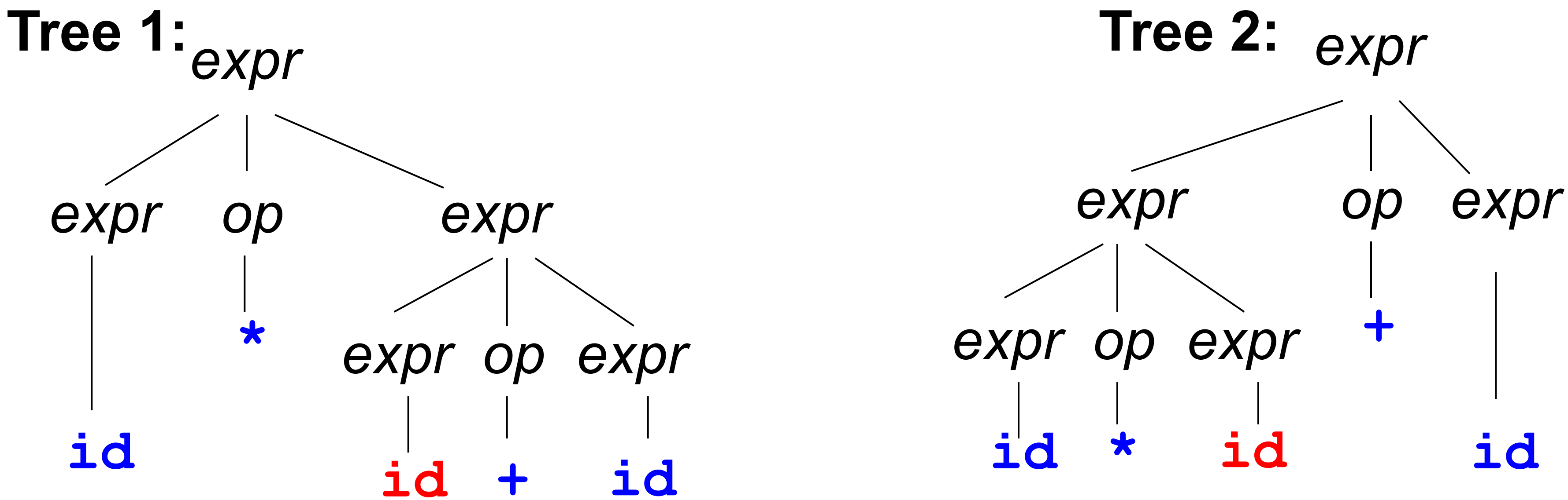
- Ambiguity
  - A grammar is ambiguous if some string can be generated by two or more distinct parse trees
  - There is no algorithm that can tell if an arbitrary context-free grammar is ambiguous
- Ambiguity arises in programming language grammars
  - Arithmetic expressions
  - If-then-else: the dangling else problem
- Ambiguity is bad

# Ambiguity

*expr* → **id** | **(** *expr* **)** | *expr op expr*

*op* → **+** | *****

- How many parse trees for **id * id + id** ?




- Which one is “correct”?

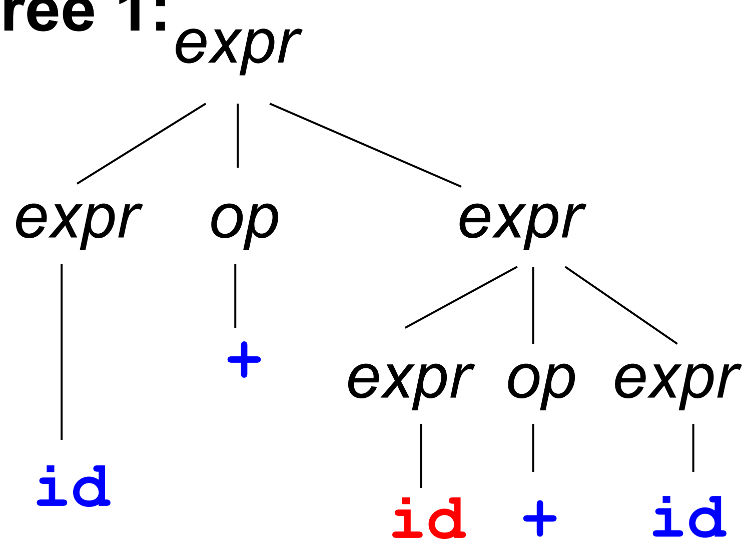
# Ambiguity

*expr* → **id** | **(** *expr* **)** | *expr op expr*

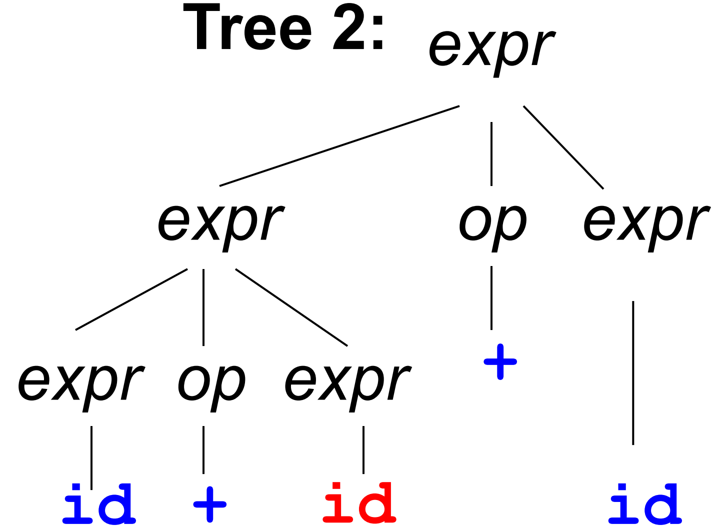
*op* → **+** | *****

- How many parse trees for **id + id + id** ?

**Tree 1:**




**Tree 2:**




- Which one is “correct”?

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# Expression Grammars

- Generate expressions
  - Arithmetic expressions
  - Regular expressions
  - Other
- Terminals: operands, operators, and parentheses

$expr \rightarrow$ **`id`** | **`(`** $expr$ **`)`** | $expr\ op\ expr$

$op \rightarrow$ **`+`** | **`*`**

# Handling Ambiguity

Our ambiguous grammar, slightly simplified:

$expr \rightarrow \mathtt{id} \mid \mathtt{(}\ expr\ \mathtt{)} \mid expr\ \mathtt{+}\ expr \mid expr\ \mathtt{*}\ expr$

- Rewrite the grammar into unambiguous one:

  $expr \rightarrow expr\ \mathtt{+}\ term \mid term$

  $term \rightarrow term\ \mathtt{*}\ factor \mid factor$

  $factor \rightarrow \mathtt{id} \mid \mathtt{(}\ expr\ \mathtt{)}$

- Forces left associativity of + and *
- Forces higher precedence of * over +

# Rewriting Expression Grammars: Intuition

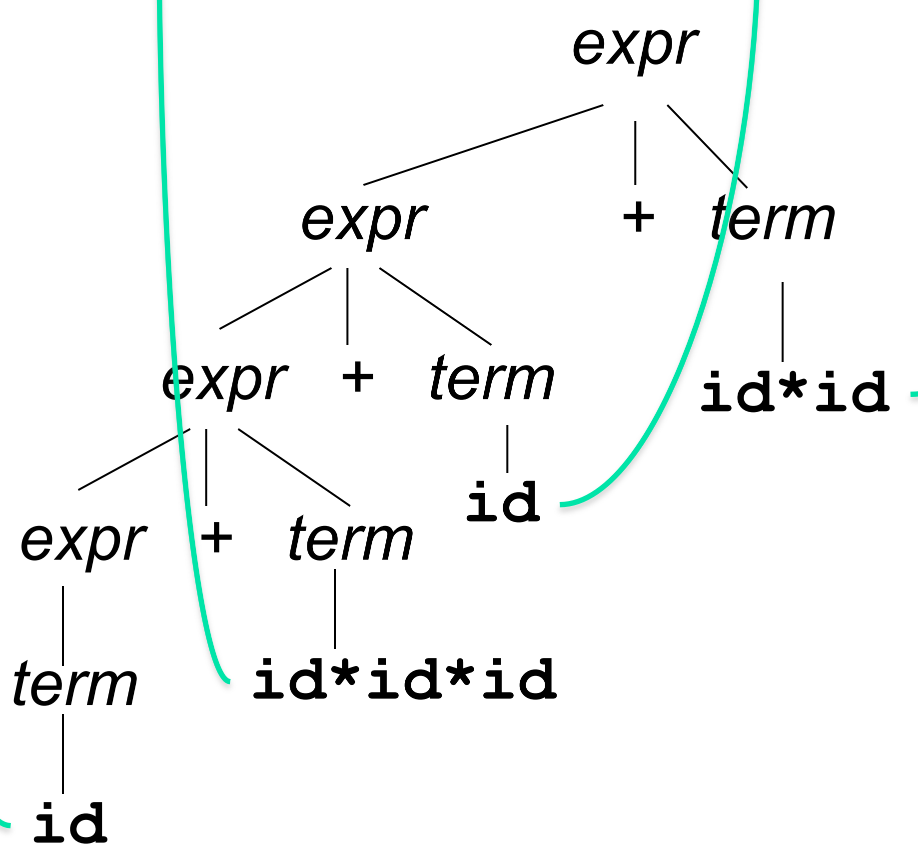
*expr* → **id** | **(** *expr* **)** | *expr* **+** *expr* | *expr* ***** *expr*

- A new nonterminal, *term*
- *expr* ***** *expr* becomes *term*. Thus, ***** gets pushed down the tree, forcing higher precedence of *****
- *expr* **+** *expr* becomes *expr* **+** *term*. Pushes leftmost **+** down the tree, forcing operand to associate with **+** on its left
  - *expr* → *expr* **+** *expr* becomes *expr* → *expr* **+** *term* | *term*

# Rewriting Expression Grammars: Intuition

*terms* in the sum

E.g., look at `id` + `id*id*id` + `id` + `id*id`



# Rewriting Expression Grammars: Intuition

- Another new nonterminal, *factor* and productions:
  - $term \rightarrow term$ `*` $factor \mid factor$
  - $factor \rightarrow$ `id` $\mid$ `(` $expr$ `)`

# Exercise

$$expr \rightarrow expr \times expr \mid expr \;\text{^}\; expr \mid \mathbf{id}$$

- How many parse trees for `id×id^id×id`?
  - No need to draw them all
- Rewrite this grammar into an equivalent unambiguous grammar where
  - ^ has higher precedence than ×
  - ^ is right-associative
  - × is left-associative

# The End