Assignment 3

Hongyu He (2632195) & Bruno Hoevelaken (2645065)

Group CS 6

& Theoratical exercises

Exercise 3.1 (Section 8.4, E16)

Based on the setting of this problem, the two samples are two dependent samples (Matched Pairs). Therefore, their population deviations σ_1 and σ_2 are considered to be the same.

1) The hypotheses in terms of the population parameter of interest:

- H_0: $\mu_d = 0$; (original claim)
- H_a: $\mu_d \neq 0$;

2) The significance level:

- $\alpha = 5\%$

3) The test statistic and its distribution under the null hypothesis:

According to the problem, the standard deviation of the population is unknown. Therefore, in this test, we replace it by the estimator S_d. The test statistics has t-distribution with (n-1 = 11) degrees of freedom.

4) The observed value of the test statistic (the observed score);

Under the assumption that the null hypothesis is true:

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - 0}{S_d/\sqrt{n}} = -0.984$$

5) The P-value or the critical values;

Since we assume technology is not available now, so we only calculated the critical values:

- `df` = n - 1 = 11
- $-2.718 < -t_{11,0.025} < -2.201$
- $+2.201 < +t_{11,0.025} < +2.718$

6) Whether or not the null hypothesis is rejected and why.

Since t > -$t_{11,0.025}$, the test statistics, therefore, is not in the critical region.

Thus,

- we fail to reject the null hypothesis;
- there is not enough evidence warrant the rejection of the claim that males aged 12-16 correctly report their heights.

Exercise 3.2 (Section 6.2, E32)

According to the problem, we know that

- E = 0.03
- Confidence level = 99%
- $\hat{p}$ = 15%

Therefore,

- α /2 = (1-99%) / 2 = 0.005
- $\hat{q}$ = 1 - 15% = 85%

and so

- $Z_\alpha/2$ = 2.575

Thus, the sample size is

$$\begin{aligned} n &= \frac{\left[z_{\alpha/2}\right]^2 \hat{p}\hat{q}}{E^2} = \frac{[2.575]^2(0.15)(0.85)}{0.03^2} \\ &= 939.34 \geq 940 \end{aligned}$$

Therefore, the sample size n should be greater or equal to 940.

& R-exercises

Exercise 3.3

a)

```
t.test(Alice, Bob, mu = 0, alternative = "two.sided", paired=TRUE, conf.level = 0.95)
```

```
##
##  Paired t-test
##
## data:  Alice and Bob
## t = 3.4938, df = 49, p-value = 0.00102
```

```
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  0.1142398 0.4235934
## sample estimates:
## mean of the differences
##               0.2689166
```

According to the above results:

- An estimate for the difference of mean working time per evening of Alice and Bob: 0.2689
- A 95% confidence interval for the difference of mean working time per evening of Alice and Bob: [0.1142398, 0.4235934]

b)

```r
t.test(Alice, Bob, mu = 0, alternative = "two.sided", paired = TRUE, conf.level = 0.90)
```

```
##
##  Paired t-test
##
## data:  Alice and Bob
## t = 3.4938, df = 49, p-value = 0.00102
## alternative hypothesis: true difference in means is not equal to 0
## 90 percent confidence interval:
##  0.1398727 0.3979606
## sample estimates:
## mean of the differences
##               0.2689166
```

According to the results of above the t-test command, we know that the p-value is 0.001. Since the p-value=0.001 $<$ α=0.01, our conclusions are as follows:

- reject the null-hypothesis;
- there is sufficient evidence to warrant rejection of the claim that on average Alice and Bob both work the same amount of time.

c)

```r
t.test(Alice, Bob, mu = 0, alternative = "greater", paired = TRUE, conf.level = 0.90)
```

```
##
##  Paired t-test
##
## data:  Alice and Bob
## t = 3.4938, df = 49, p-value = 0.0005101
## alternative hypothesis: true difference in means is greater than 0
## 90 percent confidence interval:
##  0.1689274       Inf
## sample estimates:
## mean of the differences
##               0.2689166
```

According to the results of the above t-test command, we know that the p-value is 0.0005. Since the p-value=0.0005 < α=0.01, our conclusions are as follows:

- reject the null-hypothesis;
- there is sufficient evidence to support the claim that Alice works, on average, much more than Bob.

d)

(1) Changes

Yes, because this time the data contained in *Assign3.RData* is no longer a dependent (matched pairs) sample in the sense that the data of Alice and Bob correspond to different workdays. Consequently, as mentioned in the problem, the σ_1 of sample vector `Alice` and the σ_2 of sample vector `Bob` is different.

(2) Rre-test

```
t.test(Alice, Bob, mu = 0, alternative = "two.sided",
       paired = FALSE, var.equal = FALSE, conf.level = 0.90)
```

```
##
##  Welch Two Sample t-test
##
## data:  Alice and Bob
## t = 3.3647, df = 68.832, p-value = 0.001256
## alternative hypothesis: true difference in means is not equal to 0
## 90 percent confidence interval:
##  0.1356601 0.4021731
## sample estimates:
## mean of x mean of y
##  4.235244  3.966327
```

According to the results of the above t-test command with a new setting, we know that the p-value is 0.001. Since the p-value=0.001 < α=0.01, our conclusions are the same as a):

- reject the null-hypothesis;
- there is sufficient evidence to warrant rejection of the claim that on average Alice and Bob both work the same amount of time.

Exercise 3.4

a)

A **point estimate** that we obtained for the difference in the proportion of evenings that Alice and Bob have worked more than 4 hours is **0.36**.

b)

```
prop.test(x = c(length(Alice[Alice>4]), length(Bob[Bob>4])),
          n = c(50, 50), alternative = "greater", conf.level = 0.95)
```

```
##
##  2-sample test for equality of proportions with continuity
##  correction
##
## data:  c(length(Alice[Alice > 4]), length(Bob[Bob > 4])) out of c(50, 50)
## X-squared = 12.267, df = 1, p-value = 0.0002306
## alternative hypothesis: greater
## 95 percent confidence interval:
##  0.1917075 1.0000000
## sample estimates:
## prop 1 prop 2
##   0.80   0.44
```

The results of the above two proportions test results in a p-value of 0.0002 which is smaller than the significance level $\alpha = 0.05$.

Therefore, our conclusions are as follows:

- We reject the null hypothesis;
- There is sufficient evidence to support the claim that the proportion of evenings on which she worked more than 4 hours is larger than the proportion of evenings on which Bob worked more than 4 hours.

Appendix

```
# E3.3
load("Assign3.RData")
```