# Assignment 3

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## & Theoratical exercises

### Exercise 3.1 (Section 8.4, E16)

Based on the setting of this problem, the two samples are two dependent samples (Matched Pairs). Therefore, their population deviations $\sigma_1$ and $\sigma_2$ are considered to be the same.

#### 1) The hypotheses in terms of the population parameter of interest:

- $H_0$: $\mu_d = 0$; (original claim)
- $H_a$: $\mu_d \neq 0$;

#### 2) The significance level:

- $\alpha = 5\%$

#### 3) The test statistic and its distribution under the null hypothesis:

According to the problem, the standard deviation of the population is unknown. Therefore, in this test, we replace it by the estimator $S_d$. The test statistics has t-distribution with (n-1 = 11) degrees of freedom.

#### 4) The observed value of the test statistic (the observed score);

The observed value of test statistics is:

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - 0}{S_d/\sqrt{n}} = -0.984$$

#### 5) The P-value or the critical values;

Since we assume technology is not available now, so we only calculated the critical values $-t_{11,0.025}$ = -2.201 and $t_{11,0.025}$ = 2.201

#### 6) Whether or not the null hypothesis is rejected and why.

Since $t > -t_{11,0.025}$, the test statistics, therefore, is not in the critical region.

Thus,
- we fail to reject the null hypothesis;
- there is not enough evidence warrant the rejection of the claim that males aged 12-16 correctly report their heights.

### Exercise 3.2 (Section 6.2, E32)

According to the problem, we know that - E = 0.03 - Confidence level = 99% - $\hat{p}$ = 15%

Therefore,

- $\alpha$ /2 = (1-99%) / 2 = 0.005
- $\hat{q}$ = 1 - 15% = 85%

and so

- $Z_\alpha/2 = 0.5199$

Thus, the sample size is

$$
\begin{aligned}
n &= \frac{\left[z_{\alpha/2}\right]^2 \hat{p}\hat{q}}{E^2} = \frac{[0.5199]^2(0.15)(0.85)}{0.03^2} \\
&= 38.291 = 39
\end{aligned}
$$

## & R-exercises

### Exercise 3.3

#### a)

```
t.test(Alice, Bob, mu = 0, alternative = "two.sided", paired=TRUE, conf.level = 0.95)
```

```
##
##  Paired t-test
##
## data:  Alice and Bob
## t = 3.4938, df = 49, p-value = 0.00102
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  0.1142398 0.4235934
## sample estimates:
## mean of the differences
##               0.2689166
```

According to the above results:

- An estimate for the difference of mean working time per evening of Alice and Bob: 0.2689
- A 95% confidence interval for the difference of mean working time per evening of Alice and Bob: [0.1142398, 0.4235934]

#### b)

```r
t.test(Alice, Bob, mu = 0, alternative = "two.sided", paired = TRUE, conf.level = 0.90)
```

```
##
##  Paired t-test
##
## data:  Alice and Bob
## t = 3.4938, df = 49, p-value = 0.00102
## alternative hypothesis: true difference in means is not equal to 0
## 90 percent confidence interval:
##  0.1398727 0.3979606
## sample estimates:
## mean of the differences
##               0.2689166
```

According to the results of above the t-test command, we know that the p-value is 0.001. Since the p-value=0.001 $<$ $\alpha$=0.01, our conclusions are as follows:

- reject the null-hypothesis;
- there is sufficient evidence to warrant rejection of the claim that on average Alice and Bob both work the same amount of time.

### c)

```r
t.test(Alice, Bob, mu = 0, alternative = "greater", paired = TRUE, conf.level = 0.90)
```

```
##
##  Paired t-test
##
## data:  Alice and Bob
## t = 3.4938, df = 49, p-value = 0.0005101
## alternative hypothesis: true difference in means is greater than 0
## 90 percent confidence interval:
##  0.1689274       Inf
## sample estimates:
## mean of the differences
##               0.2689166
```

According to the results of the above t-test command, we know that the p-value is 0.0005. Since the p-value=0.0005 $<$ $\alpha$=0.01, our conclusions are as follows:

- reject the null-hypothesis;
- there is sufficient evidence to support the claim that Alice works, on average, much more than Bob.

### d)

#### (1) Changes

Yes, because this time the data contained in *Assign3.RData* is no longer a dependent (matched pairs) sample in the sense that the data of Alice and Bob correspond to different workdays. Consequently, as mentioned in the problem, the $\sigma_1$ of sample vector `Alice` and the $\sigma_2$ of sample vector `Bob` is different.

**(2) Rre-test**

```
t.test(Alice, Bob, mu = 0, alternative = "two.sided", paired = FALSE, var.equal = FALSE, conf.level = 0
```

```
##
##  Welch Two Sample t-test
##
## data:  Alice and Bob
## t = 3.3647, df = 68.832, p-value = 0.001256
## alternative hypothesis: true difference in means is not equal to 0
## 90 percent confidence interval:
##  0.1356601 0.4021731
## sample estimates:
## mean of x mean of y
##  4.235244  3.966327
```

According to the results of the above t-test command with a new setting, we know that the p-value is 0.001. Since the p-value=0.001 $<$ $\alpha$=0.01, our conclusions are the same as a):

- reject the null-hypothesis;
- there is sufficient evidence to warrant rejection of the claim that on average Alice and Bob both work the same amount of time.

## Exercise 3.4

### a)

A **point estimate** that we obtained for the difference in the proportion of evenings that Alice and Bob have worked more than 4 hours is **0.36**.

### b)

```
prop.test(x = c(length(Alice[Alice>4]), length(Bob[Bob>4])),
          n = c(50, 50), alternative = "greater", conf.level = 0.95)
```

```
##
##  2-sample test for equality of proportions with continuity
##  correction
##
## data:  c(length(Alice[Alice > 4]), length(Bob[Bob > 4])) out of c(50, 50)
## X-squared = 12.267, df = 1, p-value = 0.0002306
## alternative hypothesis: greater
## 95 percent confidence interval:
##  0.1917075 1.0000000
## sample estimates:
## prop 1 prop 2
##   0.80   0.44
```

The results of the above two proportions test results in a p-value of 0.0002 which is smaller than the significance level $\alpha = 0.05$.

Therefore, our conclusions are as follows: - We reject the null hypothesis; - There is sufficient evidence to support the claim that the proportion of evenings on which she worked more than 4 hours is larger than the proportion of evenings on which Bob worked more than 4 hours.

## *Appendix*

```r
# E3.3
load("Assign3.RData")
```