

Assignment 3

Hongyu He (2632195) & Bruno Hoevelaken (2645065)

Group CS 6

& Theoretical exercises

Exercise 3.1 (Section 8.4, E16)

Based on the setting of this problem, the two samples are **two dependent samples** (Matched Pairs). Therefore, their **population deviations** σ_1 and σ_2 are considered to be the same.

1) The hypotheses in terms of the population parameter of interest:

- $H_0: \mu_d = 0$; (original claim)
- $H_a: \mu_d \neq 0$;

2) The significance level:

- $\alpha = 5\%$ (two-sided)

3) The test statistic and its distribution under the null hypothesis:

According to the problem, the standard deviation of the population is **unknown**. Therefore, in this test, we replace it by the estimator S_d . The test statistic (two-tailed) has t-distribution with ($n-1 = 11$) degrees of freedom (**df**).

4) The observed value of the test statistic (the observed score);

Under the assumption that the null hypothesis is true:

$$t = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}} = \frac{(\bar{X}_1 - \bar{X}_2) - 0}{S_d/\sqrt{n}} = -0.984$$

5) The P-value or the critical values

Since we assume technology is not available now, so we hereby only calculated the **critical values**:

- **df** = $n - 1 = 11$
- $-2.718 < -t_{11,0.025} < -2.201$ (Tech: $-t_{11,0.025} = -2.200985$)
- $+2.201 < +t_{11,0.025} < +2.718$ (Tech: $+t_{11,0.025} = +2.200985$)

6) Whether or not the null hypothesis is rejected and why.

Since $t > -t_{11,0.025}$ (Tech: > -2.200985), the test statistics, therefore, is not in the critical region.

Thus,

- we fail to reject the null hypothesis;
- there is not enough evidence warrant the rejection of the claim that males aged 12-16 correctly report their heights.

Exercise 3.2 (Section 6.2, E32)

According to the problem, we know that

- $E = 0.03$
- Confidence level = 99%
- $\hat{p} = 15\%$

Therefore,

- $\alpha / 2 = (1 - 99\%) / 2 = 0.005$
- $\hat{q} = 1 - 15\% = 85\%$

and so

- $Z_{\alpha/2} = 2.575$

Thus, the sample size is

$$\begin{aligned} n &= \frac{[z_{\alpha/2}]^2 \hat{p}\hat{q}}{E^2} = \frac{[2.575]^2 (0.15)(0.85)}{0.03^2} \\ &= 939.34 \geq 940 \end{aligned}$$

Therefore, the **sample size** n should be greater or equal to **940**.

& R-exercises

Exercise 3.3

a)

```
t.test(Alice, Bob, mu = 0, alternative = "two.sided", paired=TRUE, conf.level = 0.95)
```

```
##
## Paired t-test
##
## data: Alice and Bob
## t = 3.4938, df = 49, p-value = 0.00102
```

```
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  0.1142398 0.4235934
## sample estimates:
## mean of the differences
##                0.2689166
```

According to the above results:

- An **estimate** for the difference of mean working time per evening of Alice and Bob: **0.2689**
- A 95% confidence interval for the difference of mean working time per evening of Alice and Bob: **[0.1142398, 0.4235934]**

b)

b1) R command and results:

```
t.test(Alice, Bob, mu = 0, alternative = "two.sided", paired = TRUE, conf.level = 0.90)
```

```
##
## Paired t-test
##
## data: Alice and Bob
## t = 3.4938, df = 49, p-value = 0.00102
## alternative hypothesis: true difference in means is not equal to 0
## 90 percent confidence interval:
##  0.1398727 0.3979606
## sample estimates:
## mean of the differences
##                0.2689166
```

b2) The hypotheses in terms of the population parameter of interest:

- $H_0: \mu_d = 0$; (original claim)
- $H_a: \mu_d \neq 0$;

b3) The significance level:

- $\alpha = 10\%$

b4) The test statistic and its distribution under the null hypothesis:

- two-tailed test;
- two dependent samples (Matched Pairs);
- the test statistic is under a t-distribution.

b5) The observed value of the test statistic (the observed score);

Under the assumption that the null hypothesis is true, the test statistic is 3.4938.

b6) The P-value

- P-value = 0.00102

b7) Conclusions

According to the results of above the t-test command, we know that the **p-value** is **0.001**.

Since the **p-value=0.001** < $\alpha=0.01$, our conclusions are as follows:

- reject the null-hypothesis;
- there is sufficient evidence to warrant rejection of the claim that on average Alice and Bob both work the same amount of time.

c)

c1) R command and results:

```
t.test(Alice, Bob, mu = 0, alternative = "greater", paired = TRUE, conf.level = 0.90)
```

```
##
## Paired t-test
##
## data: Alice and Bob
## t = 3.4938, df = 49, p-value = 0.0005101
## alternative hypothesis: true difference in means is greater than 0
## 90 percent confidence interval:
##  0.1689274      Inf
## sample estimates:
## mean of the differences
##                0.2689166
```

c2) The hypotheses in terms of the population parameter of interest:

- $H_0: \mu_d = 0$;
- $H_a: \mu_d > 0$; (original claim)

c3) The significance level:

- $\alpha = 10\%$

c4) The test statistic and its distribution under the null hypothesis:

- right-tailed test;
- two dependent samples (Matched Pairs);
- the test statistic is under a t-distribution.

c5) The observed value of the test statistic (the observed score);

Under the assumption that the null hypothesis is true, the test statistic is 3.4938.

c6) The P-value

- P-value = 0.0005101

c7) Conclusions

According to the results of the above t-test command, we know that the **p-value** is **0.0005**.

Since the **p-value=0.0005** < $\alpha=0.01$, our conclusions are as follows:

- reject the null-hypothesis;
- there is sufficient evidence to support the claim that Alice works, on average, much more than Bob.

d)

(1) Changes

Yes, because this time the data contained in *Assign3.RData* is **no longer a dependent (matched pairs) sample** in the sense that the data of Alice and Bob correspond to different workdays.

Consequently, as mentioned in the problem, the σ_1 of sample vector Alice and the σ_2 of sample vector Bob is different.

(2) Rre-test

d1) R command and results:

```
t.test(Alice, Bob, mu = 0, alternative = "two.sided",
       paired = FALSE, var.equal = FALSE, conf.level = 0.90)

##
## Welch Two Sample t-test
##
## data: Alice and Bob
## t = 3.3647, df = 68.832, p-value = 0.001256
## alternative hypothesis: true difference in means is not equal to 0
## 90 percent confidence interval:
##  0.1356601 0.4021731
## sample estimates:
## mean of x mean of y
##  4.235244  3.966327
```

d2) The hypotheses in terms of the population parameter of interest:

- $H_0: \mu_d = 0$; (original claim)
- $H_a: \mu_d \neq 0$;

d3) The significance level:

- $\alpha = 10\%$

d4) The test statistic and its distribution under the null hypothesis:

- two-tailed test;
- two independent samples;
- σ_1 and σ_2 unknown and we assume that $\sigma_1 \neq \sigma_2$;
- the test statistic is under a t-distribution.

d5) The observed value of the test statistic (the observed score);

Under the assumption that the null hypothesis is true, the test statistic is 3.3647.

d6) The P-value

- P-value = 0.001256;

d7) Conclusions

According to the results of the above t-test command with a new setting, we know that the p-value is 0.001.

Since the **p-value=0.001** < $\alpha=0.01$, our conclusions are the same as a):

- reject the null-hypothesis;
- there is sufficient evidence to warrant rejection of the claim that on average Alice and Bob both work the same amount of time.

Exercise 3.4

a)

A **point estimate** that we obtained for the difference in the proportion of evenings that Alice and Bob have worked more than 4 hours is **0.36**.

b)

b1) R command and results:

```
prop.test(x = c(length(Alice[Alice>4]), length(Bob[Bob>4])),
          n = c(50, 50), alternative = "greater", conf.level = 0.95)

##
## 2-sample test for equality of proportions with continuity
## correction
##
## data:  c(length(Alice[Alice > 4]), length(Bob[Bob > 4])) out of c(50, 50)
## X-squared = 12.267, df = 1, p-value = 0.0002306
## alternative hypothesis: greater
## 95 percent confidence interval:
##  0.1917075 1.0000000
## sample estimates:
## prop 1 prop 2
##  0.80  0.44
```

b2) The hypotheses in terms of the population parameter of interest:

- $H_0: \mu_d = 0$;
- $H_a: \mu_d > 0$; (original claim)

b3) The significance level:

- $\alpha = 5\%$

b4) The test statistic and its distribution under the null hypothesis:

- right-tailed;
- two independent samples;
- the test statistic is under a t-distribution.

b5) The observed value of the test statistic (the observed score);

Under the assumption that the null hypothesis is true:

- the pooled sample proportion: $\bar{p} = \frac{x_1+x_2}{n_1+n_2} = 0.62$
- the test statistic: $z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{\bar{p}\bar{q}}{n_1} + \frac{\bar{p}\bar{q}}{n_2}}} = 3.708$.

b6) The P-value

- P-value = 0.0002306

b7) Conclusions

The results of the above two proportions test results in a **p-value** of **0.0002** which is **smaller than** the significance level $\alpha = \mathbf{0.05}$.

Therefore, our conclusions are as follows:

- We reject the null hypothesis;
- There is sufficient evidence to support the claim that the proportion of evenings on which she worked more than 4 hours is larger than the proportion of evenings on which Bob worked more than 4 hours.

Appendix

```
#E3.1
qt(0.05/2, 11)
abs(qt(0.05/2, 11))

# E3.4
load("Assign3.RData")
```

```

# a)
alice_propotion = mean(Alice > 4)
bob_propotion = mean(Bob > 4)

point_estimate = alice_propotion - bob_propotion

# b)
pooled_p = (length(Alice[Alice>4]) + length(Bob[Bob>4])) / (50*2)

test_statistic = (alice_propotion - bob_propotion) / sqrt(2*pooled_p*(1-pooled_p) / 50)

```