Assignment 3

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All values are properly rounded to at most 2-3 digits after the decimal point, as requested in the example assignment.

& Theoratical exercises

Exercise 3.1 (Section 8.4, E16)

Based on the setting of this problem, the two samples are **two dependent samples** (Matched Pairs). Therefore, their **population deviations** σ_1 and σ_2 are considered to be the same.

1) The hypotheses in terms of the population parameter of interest:

- $\mathrm{H_0}$: $\mu_\mathrm{d} = 0$; (original claim)
- $\mathrm{H_a}$: $\mu_\mathrm{d} \neq 0$;

2) The significance level:

- $\alpha = 5\%$ (two-sided)

3) The test statistic and its distribution under the null hypothesis:

According to the problem, the standard deviation of the population is **unknown**. Therefore, in this test, we replace it by the estimator $\mathrm{S_d}$. The test statistic (two-tailed) has t-distribution with (n-1 = 11) degrees of freedom (`fd`).

4) The observed value of the test statistic (the observed score);

Under the assumption that the null hypothesis is true:

$$t = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}} = \frac{(\bar{X}_1 - \bar{X}_2) - 0}{S_d/\sqrt{n}} = -0.984$$

5) The P-value or the critical values

Since we assume technology is not available now, so we hereby only calculated the **critical values**:

- `df` = n - 1 = 11
- $-\ 2.718 < -\ \mathrm{t}_{11,0.025} < -\ 2.201$ (Tech: $-\ \mathrm{t}_{11,0.025} = -\ 2.201$)
- $+\ 2.201 < +\ \mathrm{t}_{11,0.025} < +\ 2.718$ (Tech: $+\ \mathrm{t}_{11,0.025} = +\ 2.201$)

6) Whether or not the null hypothesis is rejected and why.

Since t $> -$ $t_{11,0.025}$ (Tech: $> -$ 2.201), the test statistics, therefore, is not in the critical region.

Thus,

- we fail to reject the null hypothesis;
- there is not enough evidence to warrant the rejection of the claim that males aged 12-16 correctly report their heights.

Exercise 3.2 (Section 6.2, E32)

According to the problem, we know that

- E = 0.03
- Confidence level = 99%
- $\hat{p}$ = 15%

Therefore,

- α /2 = (1-99%) / 2 = 0.005
- $\hat{q}$ = 1 - 15% = 85%

and so

- $Z_\alpha/2$ = 2.575

Thus, the sample size is

$$n = \frac{\left[z_{\alpha/2}\right]^2 \hat{p}\hat{q}}{E^2} = \frac{[2.575]^2(0.15)(0.85)}{0.03^2}$$
$$= 939.34 \geq 940$$

Therefore, the **sample size** n should be greater or equal to **940**.

& R-exercises

Exercise 3.3

a)

```r
t.test(Alice, Bob, mu = 0, alternative = "two.sided", paired=TRUE, conf.level = 0.95)
```

```
##
##  Paired t-test
##
## data:  Alice and Bob
## t = 3.4938, df = 49, p-value = 0.00102
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  0.1142398 0.4235934
## sample estimates:
## mean of the differences
##               0.2689166
```

According to the results above:

- An **estimate** for the difference of mean working time per evening of Alice and Bob: **0.269**
- A 95% confidence interval for the difference of mean working time per evening of Alice and Bob: [**0.114, 0.424**]

b)

b1) R command and results:

```
t.test(Alice, Bob, mu = 0, alternative = "two.sided", paired = TRUE, conf.level = 0.90)
```

```
##
##  Paired t-test
##
## data:  Alice and Bob
## t = 3.4938, df = 49, p-value = 0.00102
## alternative hypothesis: true difference in means is not equal to 0
## 90 percent confidence interval:
##  0.1398727 0.3979606
## sample estimates:
## mean of the differences
##               0.2689166
```

b2) The hypotheses in terms of the population parameter of interest:

- $\mathrm{H_0}$: $\mu_\mathrm{d} = 0$; (original claim)
- $\mathrm{H_a}$: $\mu_\mathrm{d} \neq 0$;

b3) The significance level:

- $\alpha = 10\%$

b4) The test statistic and its distribution under the null hypothesis:

- two-tailed test;
- two dependent samples (Matched Pairs);
- the test statistic is under a t-distribution.

b5) The observed value of the test statistic (the observed score);

Under the assumption that the null hypothesis is true, the test statistic is 3.494.

b6) The P-value

- P-value = 0.001

b7) Conclusions

According to the results of above the t-test command, we know that the **p-value** is **0.001**.

Since the **p-value=0.001** $<$ α**=0.01**, our conclusions are as follows:

- reject the null-hypothesis;

- there is sufficient evidence to warrant rejection of the claim that on average Alice and Bob both work the same amount of time.

c)

c1) R command and results:

```
t.test(Alice, Bob, mu = 0, alternative = "greater", paired = TRUE, conf.level = 0.90)
```

```
##
##  Paired t-test
##
## data:  Alice and Bob
## t = 3.4938, df = 49, p-value = 0.0005101
## alternative hypothesis: true difference in means is greater than 0
## 90 percent confidence interval:
##  0.1689274       Inf
## sample estimates:
## mean of the differences
##               0.2689166
```

c2) The hypotheses in terms of the population parameter of interest:

- $\mathrm{H_0}$: $\mu_\mathrm{d} = 0$;
- $\mathrm{H_a}$: $\mu_\mathrm{d} > 0$; (original claim)

c3) The significance level:

- $\alpha = 10\%$

c4) The test statistic and its distribution under the null hypothesis:

- right-tailed test;
- two dependent samples (Matched Pairs);
- the test statistic is under a t-distribution.

c5) The observed value of the test statistic (the observed score);

Under the assumption that the null hypothesis is true, the test statistic is 3.494.

c6) The P-value

- P-value = 0.001

c7) Conclusions

According to the results of the above t-test command, we know that the **p-value** is **0.001**.

Since the **p-value=0.001** $<$ α**=0.01**, our conclusions are as follows:

- reject the null-hypothesis;
- there is sufficient evidence to support the claim that Alice works, on average, much more than Bob.

d)

(1) Changes

Yes, because this time the data contained in *Assign3.RData* **is no longer a dependent (matched pairs) sample** in the sense that the data of `Alice` and `Bob` correspond to different workdays.

Consequently, as mentioned in the problem, the σ_1 of sample vector `Alice` and the σ_2 of sample vector `Bob` is different.

(2) Rre-test

d1) R command and results:

```
t.test(Alice, Bob, mu = 0, alternative = "two.sided",
       paired = FALSE, var.equal = FALSE, conf.level = 0.90)
```

```
##
##  Welch Two Sample t-test
##
## data:  Alice and Bob
## t = 3.3647, df = 68.832, p-value = 0.001256
## alternative hypothesis: true difference in means is not equal to 0
## 90 percent confidence interval:
##  0.1356601 0.4021731
## sample estimates:
## mean of x mean of y
##  4.235244  3.966327
```

d2) The hypotheses in terms of the population parameter of interest:

- $\mathrm{H_0}$: $\mu_\mathrm{d} = 0$; (original claim)
- $\mathrm{H_a}$: $\mu_\mathrm{d} \neq 0$;

d3) The significance level:

- $\alpha = 10\%$

d4) The test statistic and its distribution under the null hypothesis:

- two-tailed test;
- two independent samples;
- $\sigma_1 \ and\ \sigma_2$ unknown and we assume that $\sigma_1 \neq \sigma_2$;
- the test statistic is under a t-distribution.

d5) The observed value of the test statistic (the observed score);

Under the assumption that the null hypothesis is true, the test statistic is 3.365.

d6) The P-value

- P-value $= 0.001$;

d7) Conclusions

According to the results of the above t-test command with a new setting, we know that the p-value is 0.001.

Since the **p-value=0.001** $<$ α**=0.01**, our conclusions are the same as a):

- reject the null-hypothesis;
- there is sufficient evidence to warrant rejection of the claim that on average Alice and Bob both work the same amount of time.

Exercise 3.4

a)

A **point estimate** that we obtained for the difference in the proportion of evenings that Alice and Bob have worked more than 4 hours is **0.36**. For further elaboration on how we obtained this, please look at the appendix.

b)

b1) R command and results:

```
prop.test(x = c(length(Alice[Alice>4]), length(Bob[Bob>4])),
          n = c(50, 50), alternative = "greater", conf.level = 0.95)
```

```
##
##  2-sample test for equality of proportions with continuity
##  correction
##
## data:  c(length(Alice[Alice > 4]), length(Bob[Bob > 4])) out of c(50, 50)
## X-squared = 12.267, df = 1, p-value = 0.0002306
## alternative hypothesis: greater
## 95 percent confidence interval:
##  0.1917075 1.0000000
## sample estimates:
## prop 1 prop 2
##   0.80   0.44
```

b2) The hypotheses in terms of the population parameter of interest:

- $\mathrm{H_0}$: $\mu_\mathrm{d} = 0$;
- $\mathrm{H_a}$: $\mu_\mathrm{d} > 0$; (original claim)

b3) The significance level:

- $\alpha = 5\%$

b4) The test statistic and its distribution under the null hypothesis:

- right-tailed;
- two independent samples;
- the test statistic is under a t-distribution.

b5) The observed value of the test statistic (the observed score);

Under the assumption that the null hypothesis is true:

- the pooled sample proportion: $\bar{p} = \frac{x_1+x_2}{n_1+n_2} = 0.62$
- the test statistic: $z = \frac{(\hat{p}_1-\hat{p}_2)-(p_1-p_2)}{\sqrt{\frac{\bar{p}\bar{q}}{n_1}+\frac{\bar{p}\bar{q}}{n_2}}} = 3.708.$

b6) The P-value

- P-value $= 0.001$

b7) Conclusions

The results of the above two proportions test results in a **p-value** of **0.001** which is **smaller than** the significance level $\alpha = \mathbf{0.05}$.

Therefore, our conclusions are as follows:

- We reject the null hypothesis;
- There is sufficient evidence to support the claim that the proportion of evenings on which she worked more than 4 hours is larger than the proportion of evenings on which Bob worked more than 4 hours.

Appendix

```r
#E3.1
qt(0.05/2, 11)
abs(qt(0.05/2, 11))

# E3.4
load("Assign3.RData")

# a)
alice_propotion = mean(Alice > 4)
bob_propotion = mean(Bob > 4)

point_estimate = alice_propotion - bob_propotion

# b)
pooled_p = (length(Alice[Alice>4]) + length(Bob[Bob>4])) / (50*2)

test_statistic = (alice_propotion - bob_propotion) / sqrt(2*pooled_p*(1-pooled_p) / 50)
```